Non-Gaussian dynamic Bayesian modelling for panel data

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Abstract

A first order autoregressive non-Gaussian model for analysing panel data is proposed. The main feature is that the model is able to accommodate fat tails and also skewness, thus allowing for outliers and asymmetries. The modelling approach is to gain sufficient flexibility, without sacrificing interpretability and computational ease. The model incorporates individual effects and we pay specific attention to the elicitation of the prior. As the prior structure chosen is not proper, we derive conditions for the existence of the posterior. By considering a model with individual dynamic parameters we are also able to formally test whether the dynamic behaviour is common to all units in the panel. The methodology is illustrated with two applications involving earnings data and one on growth of countries.

JEL CLASSIFICATION: C11; C23.

KEYWORDS: autoregressive modelling; growth convergence; individual effects; labour earnings; prior elicitation; posterior existence; skewed distributions.

1 Introduction

Autoregressive models are used extensively in the analysis of panel or longitudinal data (Arellano, 2003; Baltagi, 2001; Hsiao and Pesaran, 2004; Liu and Tiao, 1980; Mátyás and Sevestre, 1995; Sáfadi and Morettin, 2003). In this paper we will assume that the data available $\boldsymbol{y} = \{y_{it}\}$ form a (possibly unbalanced) panel of $i = 1, \ldots, m$ individuals for each of which we have T_i consecutive observations, and focus on the first order autoregressive model:

$$y_{it} = \beta_i (1-\alpha) + \alpha\, y_{i\,t-1} + \lambda^{-\frac{1}{2}} \varepsilon_{it}, \tag{1}$$

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where the errors $\{\varepsilon_{it}\}$ are independent and identically distributed (iid) random quantities centred at zero with unit precision, and α is the parameter governing the dynamic behaviour of the panel. The intercepts $\boldsymbol{\beta} = \{\beta_1, \ldots, \beta_m\}$ are often called individual effects. It is also assumed that the process is stationary, *i.e.* $|\alpha| < 1$.

The common approach is to assume that the error term in (1) is standard Gaussian, $\varepsilon_{it} \sim \mathrm{N}(\varepsilon_{it} \mid 0, 1)$. From a Bayesian perspective, which will be pursued in this paper, prior distributions must be specified for the model parameters, $\{\alpha, \boldsymbol{\beta}, \lambda\}$, and any parameters indexing the error distribution. Then, inference is carried out using numerical techniques, usually Markov chain Monte Carlo (MCMC). See e.g. Chib and Greenberg (1994); Nandram and Petruccelli (1997); Wang and Ghosh (2002). To account for tail behaviour heavier than Normal, a useful approach is to specify a Student-t_ν distribution for the errors, with ν degrees of freedom (often fixed at a small value) and to incorporate this into the sampler through a scale mixture of Normals representation of the Student (augmenting with the mixing variables).

In order to achieve more flexibility, Hirano (2002) proposes a semiparametric approach, with a nonparametric distribution on ε_{it} using a Dirichlet process prior. However, such a fully nonparametric approach sacrifices interpretability and computational ease for the sake of flexibility. Here we propose a flexible, yet fully parametric, framework, which allows us to fully maintain control of the properties of the error distribution and facilitates computation, interpretation, communication and testing.

Section 2 describes the models and the prior elicitation we propose, and provides conditions for posterior propriety. These models are used in Section 3 in the context of three applications: two involving earnings data (regional averages and individual earnings) and one on GDP growth of countries. A final section concludes.

2 The models

The Student-t_ν model described briefly above has been used successfully in a number of applications. However, asymmetries are often found in real data which this formulation cannot account for. Also, it seems sensible to let the data determine the tail behaviour, *i.e.* not to fix ν in advance but estimate it along with the rest of the parameters.

There are a number of different classes of distributions for modelling unimodal skewed data, and the most common approach is to modify an originally symmetric distribution. The most pervasive mechanism is the hidden truncation idea (Arnold and Beaver, 2002), which underlies the skew-Normal distribution introduced in Azzalini (1985) and is applied to (multivariate) Student-t distributions in Azzalini and Capitanio (2003). An alternative class of skewed t distributions was proposed in Jones and Faddy (2003). Arnold and Beaver (2002) and Genton (2004) present an overview of these and other approaches.

Fernández and Steel (1998) introduced an alternative way of skewing univariate distributions, namely by scaling a symmetric distribution (around the origin) by reciprocal weights. We will use these families of distributions in the sequel for a number of reasons: their parameters separately control for location, scale, skewness and tail behaviour, with the skewness parameter being a simple transformation of an interpretable skewness measure, they are easy to use, and they allow for any amount of skewness at either side of the mode.

Formally, Fernández and Steel (1998) consider a unimodal probability density function f, symmetric around zero, such that $f(s) = f(|s|)$ and define the skew version of f, indexed by $\gamma \in \mathbb{R}_+$, as

$$Sf(s \mid \gamma) = \frac{2}{\gamma + \gamma^{-1}}\Big[f(s\gamma)\, I_{(-\infty,0]}(s) + f(s\gamma^{-1})\, I_{(0,\infty)}(s)\Big]. \tag{2}$$

This skew version obviously does not have zero mean (for $\gamma \neq 1$), but still has a unique mode at zero. Further, γ controls the amount of skewness (in particular, the relative amount of probability mass to the right of the mode) given that

$$\varphi = \gamma^2 = \frac{\mathrm{P}[s > 0 \mid \gamma]}{\mathrm{P}[s < 0 \mid \gamma]}\,.$$

Clearly, $Sf(s \mid \gamma) = Sf(-s \mid 1/\gamma)$, so that $\gamma > 1$ and $1/\gamma$ introduce the same amount of right and left skewness, respectively. By contrasting (2) with the original symmetric distribution (the special case when $\gamma = 1$), we can test for symmetry.

In particular, we will focus on the skew versions of the Normal and Student-t_ν (or t) distributions, leading to

$$\mathrm{SN}(\varepsilon \mid \gamma) = \frac{2}{\gamma + \gamma^{-1}}\sqrt{\frac{1}{2\pi}}\,\exp\Big[-\frac{1}{2}\varepsilon^2\Big(\gamma^2 I_{(-\infty,0]}(\varepsilon) + \gamma^{-2}\, 1_{(0,\infty)}(\varepsilon)\Big)\Big], \tag{3}$$

and

$$\mathrm{Skt}(\varepsilon \mid \gamma, \nu) = \frac{2}{\gamma + \gamma^{-1}}\,\frac{\Gamma[(\nu+1)/2]}{\Gamma[\nu/2]}\sqrt{\frac{1}{\nu\pi}}\Big[1 + \frac{1}{\nu}\varepsilon^2\Big(\gamma^2 I_{(-\infty,0]}(\varepsilon) + \gamma^{-2}\, 1_{(0,\infty)}(\varepsilon)\Big)\Big]^{-\frac{\nu+1}{2}}. \tag{4}$$

Thus, we will use (1) with ε_{it} distributed according to (3) or (4), for unit $i = 1, \ldots, m$, and with $T_1, \ldots, T_m$ consecutive measurements in time. This parameterisation allows for a clear interpretation of α as the parameter governing the dynamics of the panel, λ as driving the precision in the measurements and β_i as an individual location (level) or "individual effect". In addition, γ will control the skewness and, in the case of (4), ν determines the tail behaviour. Since the error distributions have a mode at zero, individual effects are interpreted as the corresponding modes. Further, these are assumed to be related according to

$$\beta_i \sim \mathrm{N}(\beta_i \mid \beta, \tau)\,, \tag{5}$$

which is a commonly used random effects specification, found e.g. in Liu and Tiao (1980), Nandram and Petruccelli (1997) and Gelman (2006), where β is a common mean and τ the precision. Within a Bayesian framework, (5) is merely a hierarchical specification of the prior on the β_i's. Also, we assume that the initial observed value for individual i is y_{i0}, on which we condition throughout, and that the process started a long time ago.

2.1 The prior

For the full skew-t model we will specify a prior of the product form

$$\pi(\alpha, \beta, \tau, \lambda, \gamma, \nu) = \pi(\alpha)\,\pi(\beta)\,\pi(\tau)\,\pi(\lambda)\,\pi(\gamma)\,\pi(\nu)\ . \tag{6}$$

Prior for (β, λ):

As we do not want to include strong prior beliefs, we will use the standard “diffuse” prior for the individual effects' mean β and for the observational precision, λ

$$\pi(\beta)\,\pi(\lambda) \propto \lambda^{-1}\ . \tag{7}$$

Prior for τ :

It is well known that a flat prior on the log of the precision in (5), *i.e.* $\pi(\tau) \propto \tau^{-1}$, will yield an improper posterior (Hill, 1965; Sun *et al.*, 2001). Gelman (2006) analyses this problem in the context of hierarchical Gaussian models and proposes alternative distributions. Fernández *et al.* (1997) give conditions under which the posterior is proper in the context of panel data with unobserved heterogeneity in the location.

Here, we propose to elicit a proper prior, $\pi(\tau)$, centred at a value τ_0. Specifically, we will use a Gamma distribution with mode at τ_0 (for $a_\tau > 1$) and use the shape parameter a_τ to control its spread, *i.e.* $\tau \sim$ Ga$\,(a_\tau, (a_\tau - 1)/\tau_0)$ or

$$\pi\,(\tau \mid a_\tau, \tau_0) = \frac{\left[(a_\tau - 1)/\tau_0\right]^{a_\tau}}{\Gamma[a_\tau]}\,\tau^{a_\tau - 1}\,\exp\!\left[-\frac{(a_\tau - 1)}{\tau_0}\,\tau\right]. \tag{8}$$

Alternative forms for this prior are easily accommodated within the sampler, given that this will only affect the conditional distribution on $\{\beta, \tau\}$. As stated above, using the “noninformative” prior on τ with density proportional to τ^{-1} would lead to an improper posterior. One common choice is to use the conditionally-conjugate prior in (8), set its mean at one (*i.e.* make $\tau_0 = (a_\tau - 1)/a_\tau$) and then fix a_τ at a value close to zero, in order to let its dispersion grow

(Spiegelhalter *et al.*, 2004) and to approximate the noninformative prior mentioned above. This specification allocates a lot of prior mass to low values of the precision. Gelman (2006) argues that the resulting posterior can be quite sensitive to the particular choice of (small) a_τ if the data allow mass for τ close to zero.

Therefore, we take $a_\tau = 2$ which avoids a lot of prior mass close to zero and we calibrate the prior to the scaling of the data by choosing the mode τ_0 equal to c/s_β^2, where s_β^2 is the (between-group) sample variance of the group means and $c > 1$ to account for the influence of the within-group variation. We take $c = 2$ in our examples, which works well. This dependence of the prior on a (fairly minor) aspect of the data is inevitable, if we wish to avoid inadvertently informative priors and make sure that the prior information is never greatly at odds with the data. However, the fairly small value of a_τ (which is combined with $m/2$ in the conditional posterior) means that the prior is still relatively vague and prior mass is spread over a wide range of (reasonable) values.

Prior for γ:

To specify a prior for γ we focus on a more readily interpretable quantity, namely the (scale-free) amount of skewness, AG, measured as in Arnold and Groeneveld (1995) by one minus twice the probability mass left of the mode. In the case of the skew family defined by (2) this is simply a one-to-one function of the relative amount of mass to the right of the mode, $\varphi = \gamma^2$

$$AG = \frac{\varphi - 1}{\varphi + 1}\ .$$

This means that we are able to derive equivalent priors, starting from either parameterisation of the skewness parameter. Clearly, $-1 < AG < 1$, with negative (positive) values corresponding to left (right) skewness and $AG = 0$ for the original, symmetric model.

We propose a Beta prior distribution on AG, rescaled to (-1,1),

$$\pi\,(AG \mid a, b) = \frac{2^{1-a-b}}{B(a, b)}\,(1 + AG)^{a-1}(1 - AG)^{b-1} \quad a, b > 0\ ,$$

thus implying

$$\pi\,(\varphi \mid a, b) = \frac{1}{B(a, b)}\ \varphi^{a-1}\,(1 + \varphi)^{-(a+b)}\ ,$$

an inverted Beta distribution with parameters $(a, b, 1)$ (Zellner, 1971, p. 376), which, in turn, implies that $\frac{a}{b}\varphi$ follows a Snedecor $F_{(2a,2b)}$ distribution with $(2a, 2b)$ degrees of freedom. In particular, setting $a = b$ implies symmetry in the distribution for AG, which we think is sensible in the absence of strong prior beliefs. Thus, $\varphi \sim F_{(2a,2a)}$ and, therefore, $\mathrm{P}[\varphi < x] = \mathrm{P}[\varphi > 1/x]$, which extends directly to the parameterisation on γ, addressing in a natural way the symmetry between γ and γ^{-1} described in the discussion following (2). Figure 1 depicts the implied prior on γ for different values of the hyperparameter a.

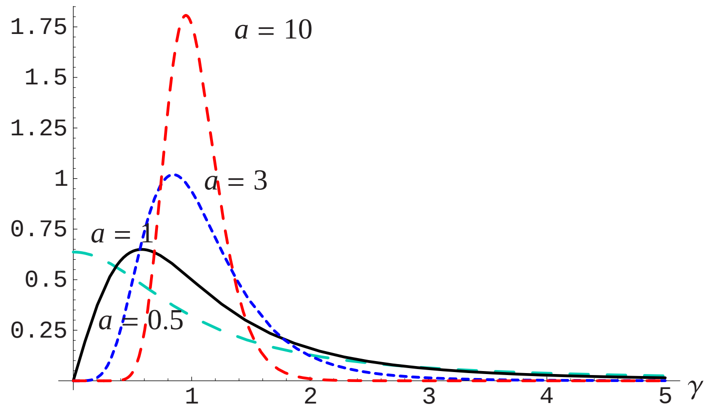


Figure 1. *The implied prior for the skewness parameter, γ, derived from a re-scaled symmetric beta distribution on the amount of skewness AG.*

If we further let $a = 1$ (*i.e.* a flat prior on AG), the implied distribution in terms of γ, is

$$\pi(\gamma) = 2\gamma \left(1+\gamma^2\right)^{-2} . \tag{9}$$

Note that even though the implied prior for φ with this specification is proper (indeed, an $F_{(2,2)}$), it does not have a mean but still retains its median at one.

Prior for α:

In order to restrict the values of the dynamics parameter to the stationarity region, the prior on α is taken as

$$\pi\left(\alpha \mid a_\alpha, b_\alpha\right) = \frac{2^{1-a_\alpha-b_\alpha}}{B(a_\alpha, b_\alpha)} (1+\alpha)^{a_\alpha-1}(1-\alpha)^{b_\alpha-1} \qquad |\alpha| < 1 , \quad a_\alpha, b_\alpha > 1 , \tag{10}$$

i.e. a Beta distribution re-scaled to (-1,1). This specification is the same as in Liu and Tiao (1980), but we now make the hyperparameters $\{a_\alpha, b_\alpha\}$ stochastic (*i.e.* we use a hierarchical prior on α) to permit greater flexibility. As we wish to remain relatively non-informative on the dynamic behaviour of the panel, we propose identical Gamma priors, Ga$(\cdot \mid r, q)$, for the hyperparameters $\{a_\alpha, b_\alpha\}$, thus centring the induced marginal prior for α at zero and making it symmetric.

For technical reasons described in Section 2.2, and to keep $\pi(\alpha)$ symmetric, we will impose the restriction $a_\alpha, b_\alpha > 1$. To set specific values for $\{r, q\}$ we balance the marginal odds $\mathrm{P}[|\alpha| \le 0.5]/\,\mathrm{P}[|\alpha| \ge 0.5]$ and control for $\mathrm{P}[|\alpha| > 0.9]$. These region probabilities are shown in Table 1 for different combinations of the parameters. Figure 2 shows the induced marginal prior densities on α for these choices. Overall, we think it is sensible to set $\{r, q\} = \{2, 0.1\}$, allocating almost 2/3 of the mass within the region defined by $|\alpha| \le 0.5$ and around

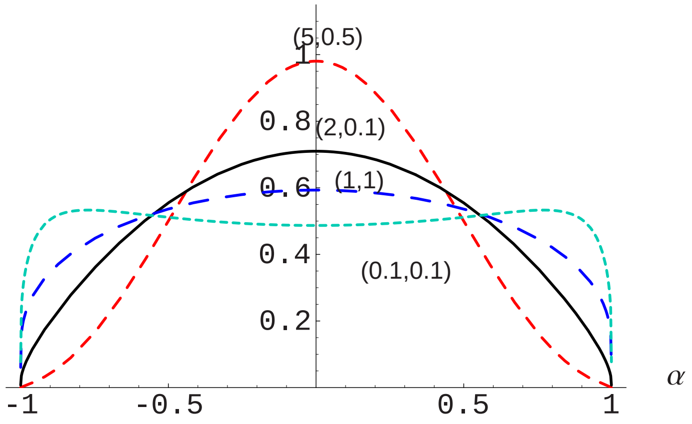


Figure 2. *Induced marginal priors for α, with Gamma hyperpriors Ga$(\cdot \mid r, q)$, and different combinations of $\{r, q\}$.*

1/40 to the region where $|\alpha| \geq 0.9$. Therefore, we will use

$$\pi(a_\alpha) \propto a_\alpha\, e^{-0.1\, a_\alpha}\ ,\ a_\alpha > 1 \qquad \text{and} \qquad \pi(b_\alpha) \propto b_\alpha\, e^{-0.1\, b_\alpha}\ ,\ b_\alpha > 1\ . \tag{11}$$

Table 1. *Induced marginal prior probabilities for α with Gamma,Ga$(\cdot \mid r, q)$, hyperpriors.*

$\{r, q\}$	(5,0.5)	(2,0.1)	(1,1)	(0.1,0.1)
$\mathrm{P}[|\alpha| \leq 0.5]$	0.805	0.658	0.575	0.495
$\mathrm{P}[|\alpha| \geq 0.9]$	0.004	0.026	0.056	0.085

Prior for ν:

Finally, for the (skew)-t model, we specify a Gamma$(2, 0.1)$ prior for the degrees of freedom, given by

$$\pi(\nu) = \frac{\nu}{100}\, e^{-\nu/10}\ . \tag{12}$$

This distribution assigns some mass to large values of ν (virtually implying Normality) as well as to small values of the degrees of freedom, thus also allowing for thicker tails.

Priors for restrictions of the skew-t model (*i.e.* Normal, skew-Normal or Student-t_ν models) are obtained by deleting the corresponding irrelevant factors in (6).

Once data become available, (1) defines the likelihood which, using the scale mixture of normals representation for the skew-t model, becomes

$$l(\alpha, \boldsymbol{\beta}, \gamma, \lambda, \boldsymbol{\omega}, \nu, a_\alpha, b_\alpha) \propto \lambda^{\frac{\mathcal{T}}{2}} \left[\frac{(\nu/2)^{\nu/2}}{\Gamma[\nu/2]}\right]^{\mathcal{T}} \prod_{i=1}^{m} \prod_{t=1}^{T_i} \omega_{it}^{\frac{\nu+1}{2}-1} \left[\frac{2^{1-a_\alpha-b_\alpha}}{B(a_\alpha, b_\alpha)}(1+\alpha)^{a_\alpha-1}(1-\alpha)^{b_\alpha-1}\right] \times$$
$$\left[\frac{2}{\gamma+\gamma^{-1}}\right]^{\mathcal{T}} \exp\left[-\frac{1}{2}\sum_{i=1}^{m}\sum_{t=1}^{T_i} \omega_{it}\left(\nu + \varepsilon_{it}^2\left(\gamma^2\, I_{(-\infty,0]}(\varepsilon_{it}) + \gamma^{-2}\, I_{(0,\infty)}(\varepsilon_{it})\right)\right)\right],$$

where $\omega = \{\omega_{it}, i = 1, \ldots, m, t = 1, \ldots, T_i\}$ and the ω_{it} are iid, Gamma$(\nu/2, \nu/2)$ distributed mixing parameters, $\varepsilon_{it} = \lambda^{1/2}\left(y_{it} - \beta_i(1-\alpha) - \alpha y_{it-1}\right)$, and $\mathcal{T} = \sum_{i=1}^{m} T_i$, the total available observations. This will be combined with the prior structure defined by (6) through (12).

2.2 Propriety of the posterior

Note that (7) makes the joint prior improper. The following theorem states a sufficient condition under which the resulting posterior is proper (*i.e.* well-defined as described in Fernández *et al.*, 1997). The proof is deferred to Appendix A.

Theorem 1.
Consider the Bayesian model defined by (1) *(with either* (3) *or* (4)*),* (5) *and* (6) *through* (12)*. If $\mathcal{T} > m + 1$, then the joint posterior is proper .*

As will be clear from the proof in the Appendix, the condition $b_\alpha > 1$ in (11) is important in avoiding non-integrability due to the implied prior on $\beta_i(1-\alpha)$ exploding as $\alpha \to 1$. We also restrict $a_\alpha > 1$, in order to maintain symmetry on the induced prior for α.

The condition in Theorem 1 is very mild and trivial to check. Indeed, for example, as long as $T_i \geq 1$ with strict inequality for at least two units, the posterior will be proper.

2.3 A model with individual dynamics

In the next section we will analyse three data sets: one comprising average earnings in 14 metropolitan areas (Liu and Tiao, 1980), another with annual labour earnings of young male household heads (Hirano, 2002) and the third one on GDP of 25 OECD countries. In these studies, panels were formed not necessarily according to statistical criteria, but rather geographical, demographical or political affinity, thus we think is sensible to verify whether the dynamic properties should indeed be pooled. We feel this situation is likely to occur in practice, so we entertain the “non-pooled” version of (1),

$$y_{it} = \beta_i(1-\alpha_i) + \alpha_i y_{it-1} + \lambda^{-\frac{1}{2}}\varepsilon_{it}\,, \tag{13}$$

with individual dynamics parameters α_i, $i = 1, \ldots, m$ arising independently from the same distribution (10). In order to make the comparison fair, the same prior structure and specifications for the error term ε_{it} will be maintained.

We have a similar result for posterior existence, with the proof sketched in Appendix A.

Theorem 2.
Consider the Bayesian model defined by (13) *(with either* (3) *or* (4)*),* (5) *and* (6) *through* (12)*. If* $\mathcal{T} > 2\,m$*, then the joint posterior is proper .*

This sufficient condition is somewhat more demanding than the one in Theorem 1, but is still straightforward to check and quite easily satisfied in practice. If we have more than two observations per individual on average, we are sure that the posterior exists.

3 Applications

Throughout, we will use the pooled model in (1) and the non-pooled model in (13), each with the Normal (N), Student-t_ν (t), skew-Normal (SN; see (3)) and Skew-t (Skt; see (4)) error distributions. As the posteriors are not of a known form, we implement (Metropolis-Hastings within Gibbs) MCMC samplers to estimate the parameters. Details of the samplers used as well as the Matlab code for implementing them are available upon request.

The MCMC samplers were ran for 1.7×10^5 repetitions, dropping the first 2×10^4 (the burn-in) and then recording every tenth draw, thus ending up with chains of length 15,000. The parameters of the proposal Metropolis-Hastings distributions were tuned as to achieve acceptance rates of around 1/3. We will use the formal tool of Bayes factor (BF) to compare alternative models, calculating the required marginal likelihoods with the method of Newton and Raftery (1994), with their $\delta = 0.1$. As a check on the numerical accuracy of the latter method, we also compute Bayes factors through the Savage-Dickey density ratios (Verdinelli and Wasserman, 1995), where this is feasible. We have successfully used the Bayesian models and algorithms on two simulated panels (not reported): one set of pooled data and the other non-pooled.

3.1 Regional average earnings

The data, described in Liu and Tiao (1980), consist of yearly averages of the hourly earnings of production workers within each of 14 metropolitan areas in California (transformed to growth rates). Each of the series ends in 1977 but have different beginnings, the longest beginning in 1945 and the shortest in 1963. This data set has also been analysed by Nandram and Petruccelli (1997).

Table 2 shows the log BF in favour of the Normal model with common dynamics. Thus, negative numbers indicate support for the alternative model in question.[1] Our model com-

[1]For example, the BF for the Normal model versus the skew-Normal, both with common dynamics, is $\exp(-8.04) = 0.0003$. This means that, with unitary prior odds, the posterior odds are more than 3000 to one in favour of the skew-Normal model.

Table 2. *Regional average earnings. Estimated log Bayes factors in favour of the Normal model with common dynamics.*

	Common α			Different α_i			
	SN	t	Skt	N	SN	t	Skt
log BF	-8.04	-8.16	-11.54	0.68	-10.17	-8.42	-13.46

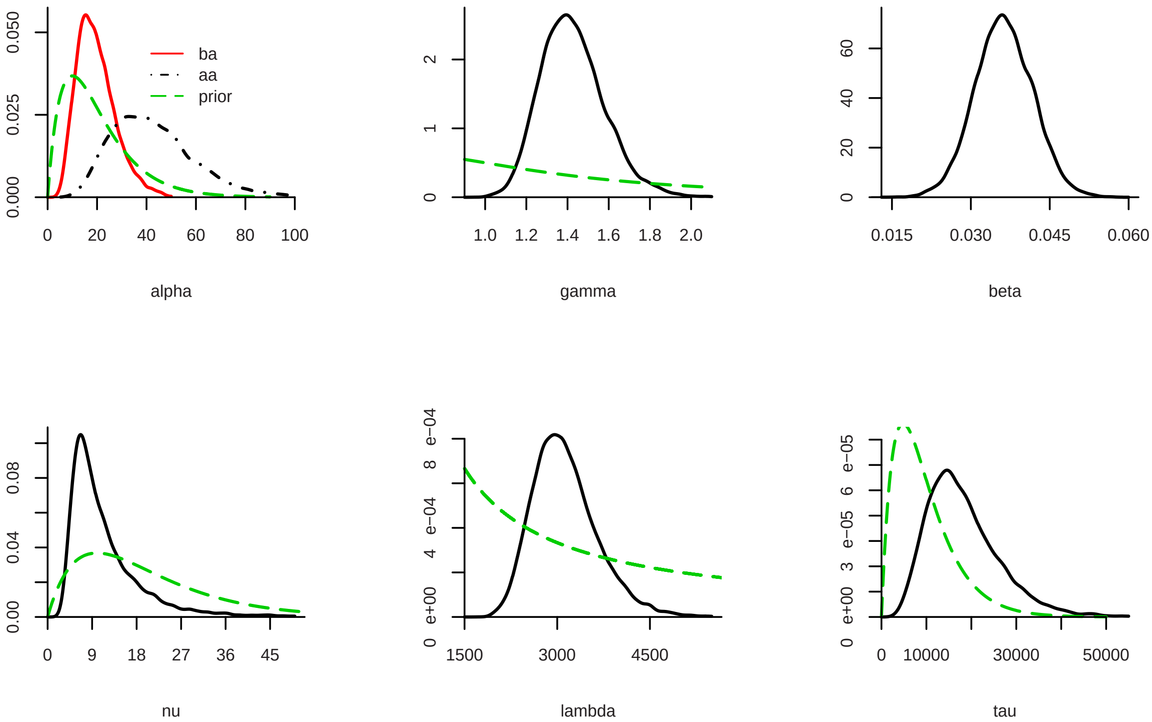


Figure 3. *Regional average earnings data set with the skew t model with individual dynamics. Priors (dashed) and estimated marginal posterior distributions (solid).*

parison results do not reject pooling for any symmetric model, but give mild evidence against it for any skew distribution. This is consistent with the findings of Nandram and Petruccelli (1997), who assume Normality in the error term[2]; in this case, our BF's in favour of pooling are 2.0 for the Normal and 0.12 for the SN model. Nandram and Petruccelli (1997) find that pooled and non-pooled estimates of the dynamics parameters are similar for all but four of the series; in our Normal case the estimated posterior mean of the common α is 0.42, and when we allow for a different value for each region, the intervals of length two posterior standard deviations around the posterior mean and the posterior credible intervals[3] of size 0.95

[2] Nandram and Petruccelli (1997) do not compute Bayes factors, but consider one-step-ahead prediction performance as a measure of model adequacy.

[3] All posterior credible intervals of size p reported hereinafter are constructed taking the corresponding $(1 -$

for each region contain 0.42.

In the modelling scenario with common dynamics, the BF does not distinguish between the SN and the t models. This could indicate that the model uses heavy tails to account for asymmetry when the latter feature is not introduced in the model and vice versa. This interpretation is corroborated by the fact that if the model only includes one of those features, the latter is invariably exaggerated with respect to the results for the model with both features. When both skewness and thick tails are included, BF's indicate evidence in favour of the Skt models versus the symmetric Student-t_ν models. The difference in log BF's is 3.4 and 5.0 for the models with common and individual dynamics. In this case, we can compute the log Savage-Dickey density ratios, which are 2.4 and 2.9 for the pooled and non-pooled models, respectively. Overall, the Skt model with different dynamics is preferred (BF=7 versus the Skt with common α), with estimated posterior medians of $\mathrm{Med}[\gamma \mid \boldsymbol{y}] = 1.38$ and $\mathrm{Med}[\nu \mid \boldsymbol{y}] = 10.1$. The estimated marginal posterior density functions are depicted in Figure 3. A more interpretable measure of skewness is AG, the posterior distribution of which is plotted in Figure 4 (a), clearly indicating the overwhelming evidence in favour of right skewness. The usual assumption of Normality (as used by Liu and Tiao, 1980 and Nandram and Petruccelli, 1997) is decisively rejected by the data (with BF=1.0×10^5 for the pooled case and BF=1.4×10^6 for the unpooled case).

Of special interest is the behaviour of predictive distributions. Figure 4 (b) depicts the posterior predictive distributions of the error term for the Normal and Skt models. The unequal distribution of the mass at either side of the mode is apparent for the Skt model; also, the Normal model must adopt a much larger dispersion than the Skt in order to accommodate the observations in the tails.

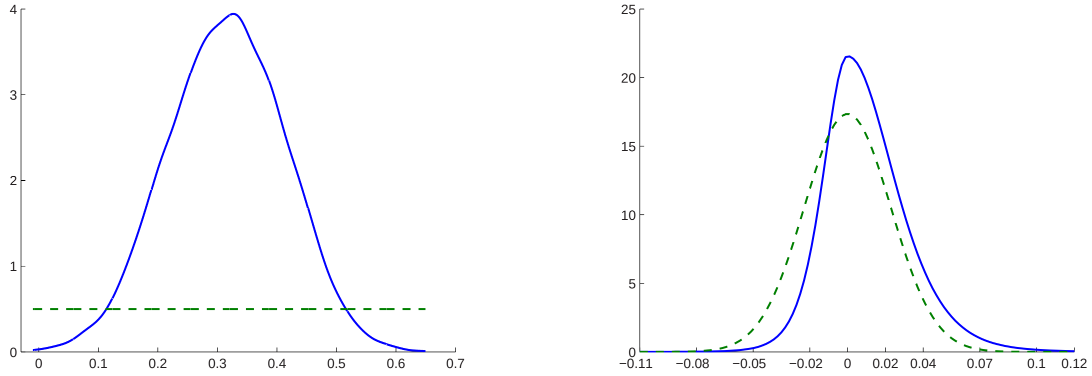
3.2 Individual labour earnings data

The data is drawn from the Panel Study of Income Dynamics (PSID) and records annual labour earnings for males who were heads of households between the ages of 24 and 33, during the period from 1967 to 1991. The data is filtered as described in Hirano (2002)[4], who constructs three subsets from the resulting 10 years of observations for 516 individuals, according to their education: high school dropouts (HSD, $m = 37$), high school graduates (HSG, $m = 100$) and college graduates (CG, $m = 122$). The full sample with $m = 516$ will be denoted by TOT.

As illustrated in Table 3 the data overwhelmingly favour the heavy-tailed models in both scenarios. In contrast with the previous example, the feature that provides the strongest

$p)/2$ and $(1 + p)/2$ percentiles.

[4]The log of the earnings is regressed on a constant, an indicator for race, indicators for different education levels and calendar year. The residuals of the least-squares regression are then kept as the "raw data".



(a) *Amount of skewness* **(b)** *Predictive densities*

Figure 4. *Regional average earnings. Panel (a): posterior density of the skewness measure AG under the preferred Skt model with individual dynamics (solid) and prior (dashed). Panel (b): posterior predictive densities of the error term for the Normal (dashed) and Skt (solid) models with common dynamics.*

evidence against Normality is tail behaviour. Also, a common dynamics behaviour is not supported by the data regardless of the model selected.

Table 3. *Individual labour earnings. Estimated log Bayes factors in favour of the Normal model with common dynamics, for all subsets and the whole sample.*

	Common α			Different α_i			
Sample	SN	*t*	Skt	N	SN	*t*	Skt
HSD	-31.15	-93.00	-96.02	-3.45	-33.01	-104.58	-105.41
HSG	-74.10	-230.96	-237.06	-14.01	-95.56	-276.98	-279.22
CG	-31.17	-229.10	-229.05	-59.95	-84.28	-316.67	-313.45
TOT	-248.47	-1157.26	-1177.43	-137.34	-404.97	-1510.07	-1518.25

These results are in line with those in Hirano (2002), who proposes nonparametric modelling of the error term which allows for more flexibility in the distribution. He also finds evidence of heavy tails and some left skewness, but does not provide a formal way to test or measure these characteristics.

Using Hirano's parameterisation, namely $\theta_i = \beta_i(1-\alpha)$ with $\theta_i \sim \mathrm{N}\left(\cdot \mid \psi y_{i1}, \Omega^{-1}\right)$, the estimated posterior medians of $\{\alpha, \lambda, \psi, \Omega\}$ for the Normal model with common dynamics and the non-pooled Skt model (including the *t* for HSD, where it is close, and for CG, where it is preferred) are shown in Table 4. For α we also present the mean (which exists since α has bounded support) and the mode. For the Skt and *t* models with individual dynamics the estimated overall mean $\bar{\alpha}$ (mode $\tilde{\alpha}$) are calculated (using the Rao-Blackwell idea) through

$$\bar{\alpha} = \frac{a_\alpha - b_\alpha}{a_\alpha + b_\alpha} \quad \text{and} \quad \tilde{\alpha} = \frac{a_\alpha - b_\alpha}{a_\alpha + b_\alpha - 2}\,.$$

When comparing his semiparametric model with the usual Normal model, Hirano (2002) finds higher values of the dynamics parameter for all but the HSD sample, and less variability in the individual effects distribution for CG and TOT. Our estimates of the dynamics parameter are also higher in the preferred models than in the Normal case[5]; however, Hirano's are larger than ours, mainly due to differences in the prior. Overall, our results are consistent with those in Hirano (2002).

Table 4. *Individual labour earnings. Posterior medians of the parameterisation in Hirano (2002). For α the mean ($\overline{\alpha}$) and mode ($\tilde{\alpha}$) are also presented*

Subset	Model	$\overline{\alpha}$	$\tilde{\alpha}$	α	$\lambda^{-\frac{1}{2}}$	ψ	$\Omega^{\frac{1}{2}}$
HSD	N	0.31	0.33	0.32	0.32	0.31	0.19
	t	0.39	0.41	0.39	0.14	0.29	0.20
	Skt	0.36	0.38	0.37	0.14	0.39	0.20
HSG	N	0.35	0.37	0.35	0.29	0.32	0.21
	Skt	0.40	0.44	0.41	0.13	0.37	0.19
CG	N	0.39	0.41	0.40	0.25	0.41	0.21
	t	0.48	0.52	0.49	0.11	0.45	0.17
	Skt	0.48	0.53	0.50	0.11	0.46	0.16
TOT	N	0.38	0.39	0.38	0.27	0.34	0.20
	Skt	0.44	0.54	0.47	0.14	0.29	0.20

From Table 3, the fat-tailed models with individual-specific dynamics are the most favoured alternatives for all data sets. Figure 5 displays the posterior density functions for the parameters of the Skt model using the HSD data. The log BF's in favour of the symmetric *t* model against the Skt are -0.82, -2.24, 3.22 and -8.18, while the log Savage-Dickey density ratios are 1.3, -2.22, 2.50 and -10.53, for the HSD, HS, CG and the whole sample, respectively. Thus, we cannot clearly distinguish between the Skt and the *t* model for the HSD subset. This feature is illustrated in the top centre panel of Figure 5, where the estimated marginal posterior for the skewness parameter is seen to have considerable mass in the neighborhood of one, without being squarely centred over one (it indicates mild negative skewness, as found in Hirano, 2002).

Estimated 95% credible intervals for this case are shown in Table 5. This also highlights the main feature of the data sets: extremely heavy tails. Indeed, estimated posterior intervals for all sets (not shown) are roughly within (1,3).

Figure 6 depicts the estimated posterior predictive densities of the error term for the Normal and Skt models with individual dynamics for the HSD subset, both in regular and log scales. It is interesting to note how in the fitting of the Normal model, the precision

[5]In contrast, the preferred non-pooled Skt model in the previous application leads to smaller values for α than the Normal model. In particular, we find that $\mathrm{E}(\overline{\alpha}|\boldsymbol{y}) = 0.37$ versus $\mathrm{E}(\alpha|\boldsymbol{y}) = 0.42$ under Normality.

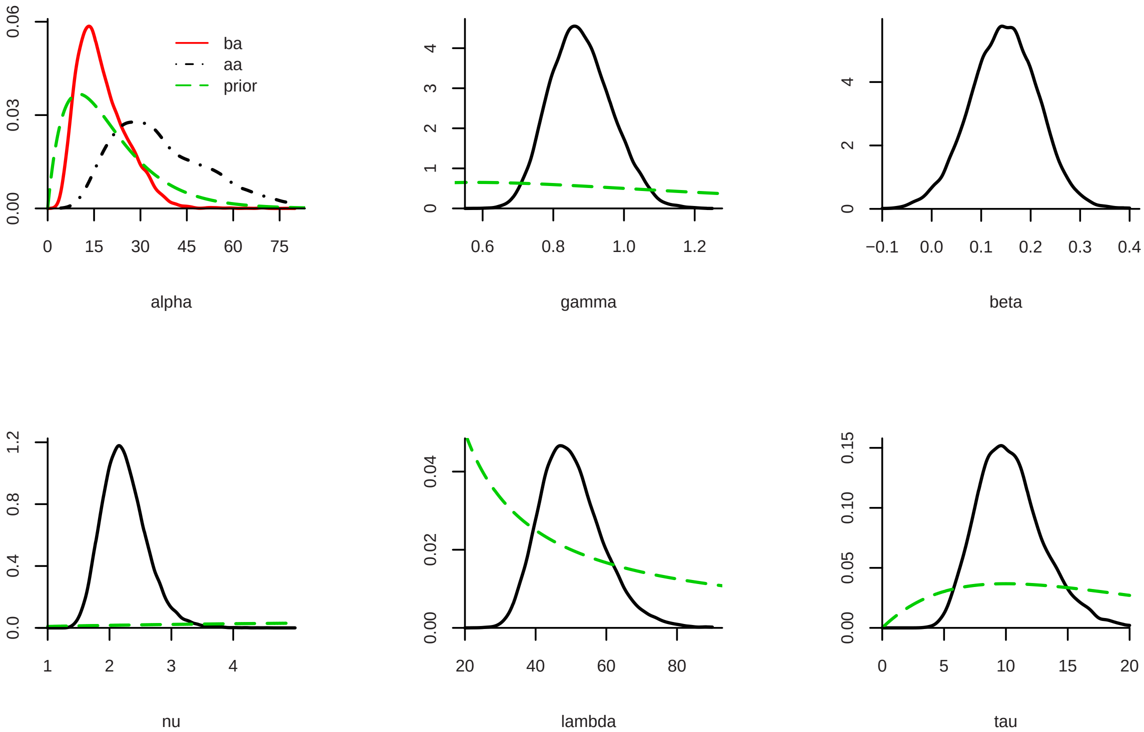


Figure 5. *Individual labour earnings. HSD subset with the skew-t model with individual dynamics. Priors (dashed) and estimated marginal posterior distributions (solid).*

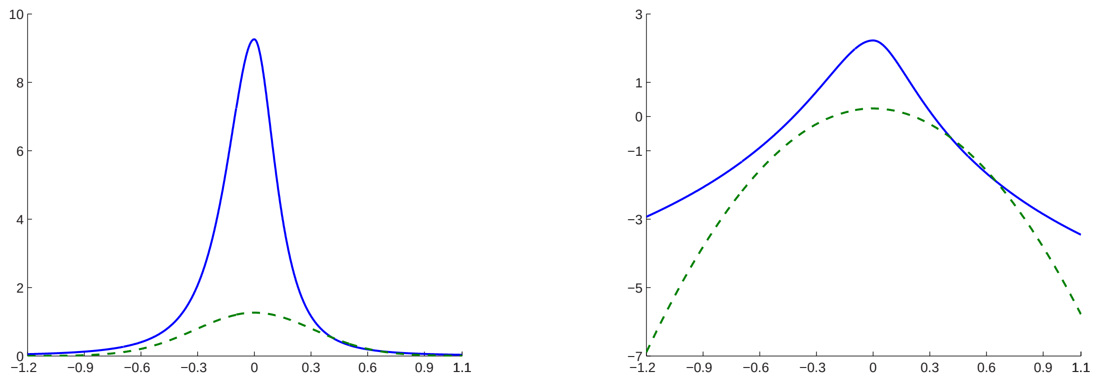
Table 5. *Individual labour earnings. Estimated posterior 95% credible intervals for the HSD subset using the Skt and t models with individual dynamics.*

	γ	λ	ν	β	τ
Skt	(0.73,1.06)	(35,71)	(1.6,3.1)	(0.01,0.28)	(6,16)
t	–	(35,71)	(1.6,2.9)	(-0.02,0.20)	(6,16)

parameter is much smaller, as it tries to account for the heavier tail behaviour. In contrast, the predictive distribution for the Skt model is more concentrated around the mode, also allowing for different tail behaviour to either side of the mode. It is apparent from the logarithmic density plots in Figure 6 (b) that the tails of the Skt model are much heavier.

3.3 GDP growth

There is a vast literature on the use of panel data to analyse economic growth and convergence, see e.g. Barro and Sala-i-Martin (2004); Durlauf and Quah (1999); Evans (1998); Evans and Karras (1996); Gaulier *et al.* (1999); Islam (1995); Lee *et al.* (1998); Temple (1999) and references therein. Here we analyse annual growth of real GDP per capita data of 25 OECD countries taken from the Penn World Tables (Heston *et al.*, 2002) for 1950-2000



(a) *Actual scale.* **(b)** *Log scale.*

Figure 6. *Individual labour earnings, HSD subset. Estimated posterior predictive densities of the error term for the Normal (dashed) and Skt (solid) models with individual dynamics.*

(see Appendix B). The panel is unbalanced as, for instance, there is no available data for Germany previous to 1970.

Using our model (1) on growth rates of GDP gives us the model examined in *e.g.* Ho (2006), where we can interpret $1/\alpha$ as a measure of the speed of GDP convergence[6] and β_i as the steady state GDP growth level for country i.

As shown in Table 6, there is no evidence in favour of skewness, but overwhelming support for heavy tails. The most favoured models are the ones with Student-t tails and individual dynamics. Evidence against pooling is definitive, with a log BF of around 14, irrespective of the error distribution.

Table 6. *GPD growth of OECD countries. Estimated log Bayes factors in favour of the Normal model with common dynamics.*

	Common α			Different α_i			
	SN	t	Skt	N	SN	t	Skt
log BF	-0.59	-85.55	-85.75	-15.24	-15.40	-99.74	-99.95

For the non-pooled Skt model, most of the estimated means of country-specific dynamics coefficients are within (0.35,0.45), with one group of *rapid convergence* ($\alpha_i \leq 0.35$) constituted by Denmark, Spain, Luxembourg, Mexico, New Zealand, Portugal, Turkey and USA (ranging from values for α centred around 0.1 for Turkey to values around 0.33 for Portugal) and a group of *slow convergence* ($\alpha_i \geq 0.45$) formed by France (median of α is 0.45), Germany (0.46), Italy (0.45) and Japan (0.53). All of the estimated median growth steady states are within (0.055, 0.061), except for Spain (0.063), Ireland (0.062), Japan (0.065), Luxembourg (0.0614) and Portugal (0.062). Although there is no strong evidence in favour of $\gamma \neq 1$,

[6] This is so-called beta convergence, or convergence within an economy. For an analysis of output convergence across economies, see *e.g.* Bernard and Durlauf (1996) and Pesaran (2006).

around 89% of the posterior mass of γ (AG) is to the right of 1 (0), which could be caused by the model accounting for these few countries with growth rates above the average. Fat tails account for the greatest evidence against Normality, with (3.0, 4.7) a 95% posterior credible interval for ν.

Based on the results above, we decided to break the data up into four disjoint subsets, as listed in Appendix B: core-EU (CEU), with 13 countries; rest-EU (REU), with $m = 6$; North America (NA), $m = 3$; and East Asia (EA), $m = 3$, and to estimate a separate model for each group. The resulting BF's support pooling within all subsets, and only suggest weak evidence against pooling for the REU and EA subsets when using either t models (log BF = -1.7 and -1.4, respectively). As shown in Table 7, there is, again, no real evidence of skewness in any subset, the salient feature against Normality being fat tails ($\mathrm{Med}[\nu_{CEU} \mid \boldsymbol{y}] = 3.8$, $\mathrm{Med}[\nu_{REU} \mid \boldsymbol{y}] = 4.4$, $\mathrm{Med}[\nu_{EA} \mid \boldsymbol{y}] = 13.2$). This is true for all but the NA subset, for which the pooled Normal model is not definitively rejected ($\mathrm{Med}[\nu_{NA} \mid \boldsymbol{y}] = 20.2$), with a log BF in favour of the Normal pooled model against the Skt of around -1.1. The posterior credible intervals for α of the two European subsets do not overlap, creating two well separated clubs of slow and rapid growth dynamics, which ties in the concept of convergence clubs in Quah (1997) and the evidence in favour of different regimes in Durlauf and Johnson (1995) and Canova (2004). This is not the case for the other two subsets, which might be merged in a subsequent analysis. An alternative to this grouping into subsets is to use a double-threshold model as in Hansen (1999) and Ho (2006), which allows for different speeds of convergence depending on the income levels.

Table 7. *GPD growth of OECD countries. Estimated 95% credible intervals for the parameters using the pooled Normal and Skt models for the four subsets and the whole sample (figures for λ are in thousands and in tens of thousands for τ).*

		α	γ	ν	β	τ $(\times 10^4)$	λ $(\times 10^3)$
CEU	*N*	(0.26, 0.40)	–	–	(0.059, 0.067)	(1.09, 6.12)	(0.90, 1.12)
	Skt	(0.36, 0.50)	(0.97, 1.19)	(3 , 5)	(0.050, 0.064)	(1.14, 6.23)	(1.65, 2.49)
REU	*N*	(-0.07,0.14)	–	–	(0.064, 0.076)	(1.12, 11.3)	(0.36, 0.50)
	Skt	(0.03, 0.24)	(0.88, 1.19)	(3, 8)	(0.058, 0.080)	(4.11, 61.4)	(0.61, 1.08)
NA	*N*	(0.20, 0.48)	–	–	(0.049, 0.067)	(28, 621)	(0.56, 0.88)
	Skt	(0.20, 0.48)	(0.74, 1.10)	(6, 60)	(0.050, 0.084)	(85, 1926)	(0.63, 1.14)
EA	*N*	(0.21, 0.50)	–	–	(0.039, 0.085)	(0.09,1.32)	(0.64, 1.01)
	Skt	(0.27, 0.57)	(0.85, 1.34)	(5, 44)	(0.030, 0.080)	(0.02, 3.97)	(0.75, 1.41)
OECD	*N*	(0.19, 0.29)	–	–	(0.059, 0.069)	(0.49, 1.63)	(0.64, 0.75)
	Skt	(0.29, 0.40)	(0.97,1.13)	(3, 5)	(0.054, 0.068)	(0.05, 1.69)	(1.20, 1.60)

In the case of the European countries, estimates for the dynamics parameter are clearly lower under Normality than when allowing for fat tails and skewness, thus erroneously suggesting a more rapid convergence in the former case. It is also apparent how the estimated observation precision, λ, must be much lower in the Normal case in order for the model to accommodate the observations in the tails. Remarkably, the estimates of the hyperparameter β are fairly similar, indicating that at least the mean of the distribution of the individual effects is not much affected by the choice of error distribution. In the case of CEU the same holds for the entire individual effects distribution.

Figure 7 plots the posterior distributions for the measure of speed of convergence, $1/\alpha$, estimated from the pooled Skt model, for each subset; different speeds of convergence are apparent for the two European subsets, while the difference is not so clear for the American and Asian groups. Clearly, forcing the dynamic behaviour of the REU cluster to be the same as that for the other groups (resulting in the graph labelled OECD) flies in the face of the data evidence. The faster convergence rate of the REU group is in line with the relatively recent industrialisation of most of these countries and the results in Canova (2004).

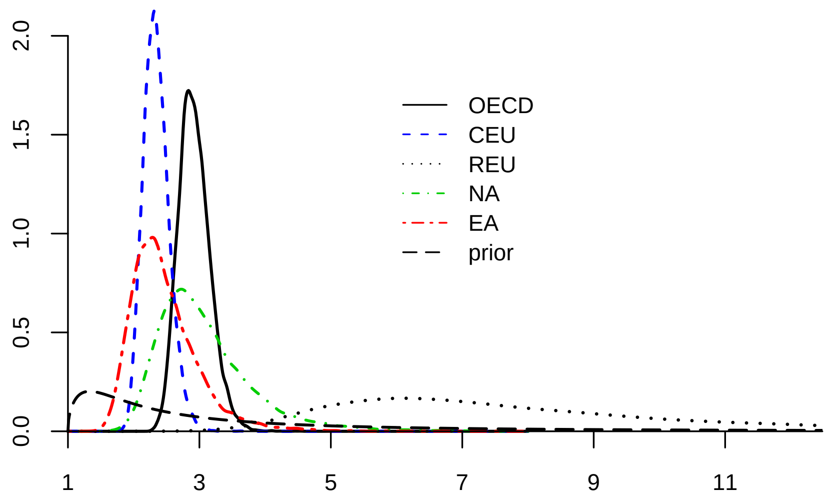


Figure 7. *GDP growth of OECD countries. Posterior distributions and prior for $1/\alpha$, the measure of speed of convergence, from the pooled SKt model and the four subsets.*

4 Conclusions

We propose a first-order autoregressive model with random effects which is capable of accommodating skewness and fat tails in a parametric fashion. Inference is carried out from a Bayesian perspective and a sensible prior structure is proposed that does not incorporate strong prior beliefs and which allows for comparisons across models. Further, an alternative model, which allows the dynamics to change over individuals, is considered. Estimation is done numerically through MCMC techniques.

The methodology is illustrated with three real data sets. The application to regional average earnings data strongly favours skewed models with moderately fat tails, whereas the evidence against Normality for the other data sets mostly centers around fat tails. However, in some cases fat tails may partially account for skewness and vice versa, and therefore it is important to treat both quantities as unknown and estimate them simultaneously. The applications to individual earnings data and to the growth of countries provide conclusive evidence against the assumption of common dynamics. The latter data, however, allow for pooling once the individual countries are clustered into four, more homogeneous, groups. Interestingly, for those cases where fat tails are strongly supported, the usual Normal model leads to substantially smaller values for the dynamics parameter. This is of particular interest for the application to economic growth, where this parameter is directly linked to the speed of GDP convergence.

Several extensions to the model are possible. In some cases, the interest of the researcher lies primarily in the effect of certain covariates. For instance, it may be of special interest to investigate the degree of influence of race, schooling, etc. on the income dynamics of individuals; or the effect of human capital, investment, etc. on economic growth. In that case, it is natural to include a set of covariates grouped in a $t_i \times p$ matrix X^i, such that

$$y_{it} = \beta_i (1-\alpha) + \alpha y_{it-1} + \mathbf{x}_t^i \boldsymbol{\delta} + \lambda^{-\frac{1}{2}} \varepsilon_{it} \,,$$

where $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_p)'$ are the additional parameters, with prior $\pi(\boldsymbol{\delta})$, and $\mathbf{x}_t^i$ is the t-th row of X^i. This can be accommodated straightforwardly within the procedure described in the paper, just by adding an extra step in the sampler for the new parameters. Such an extension would be natural for all the applications treated in the paper. Recall that Hirano's data (also used in Subsection 3.2) are in fact residuals from a least-squares regression on a number of covariates, and including these covariates in the model would be preferable, both from a methodological perspective and an empirical one. Inference on $\boldsymbol{\delta}$ might well depend quite crucially on the distributional assumptions on the error term.

Another extension that is fairly easy to implement is to include in the model a distribution for y_{i0}, the initial value, rather than conditioning on a fixed value. This could be particularly important in cases where the time dimension of the panels are relatively short (such as the Hirano data).

We feel that testing for heterogeneity from a Bayesian perspective offers a mayor advantage. Bayes factors are well-defined for any number of individuals m and repeated measures through time T_i, as long as the posteriors and priors on model-specific parameters are proper. In contrast, most of the alternative frequentist tests are based on semi-asymptotic results (see e.g. Pesaran *et al.*, 1995, and references therein) whose conditions are often not met in practice or are difficult to check.

With our methodology we can easily test whether to pool the dynamics across the whole panel. Of course, trying to find clusters of units exhibiting similar dynamic behaviour can be of paramount interest in some applications, such as growth economics (Arbia and Piras, 2005; Canova, 2004; Quah, 1997). In this paper, we have used the model with individual dynamics to suggest a useful clustering of the individuals into relatively homogeneous groups. Whereas that often works reasonably well in practice, this approach can be deemed somewhat ad-hoc.[7] Thus, we also intend to make the clustering mechanism part of the model and allow for a free selection of the number and composition of clusters within the model, following previous approaches in Canova (2004) and Frühwirth-Schnatter and Kaufmann (2004). Using a hierarchical framework, this would be a natural extension of the model and algorithm and is the topic of ongoing research.

Acknowledgements

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Appendix A Proofs

A.1 Proof of Theorem 1

The proof is done in two steps. First, we will prove that the posterior is proper for $\gamma > 0$ if and only if (iff) it is proper for $\gamma = 1$. Then, we will prove propriety of the posterior for the simpler cases of the Normal and Student distributions.

For step one, we use Theorem 1 of Fernández and Steel (1998) which states that the posterior of the skew model is proper iff it is proper when $\gamma = 1$. So we write

$$\mathcal{I} = \int \prod_{i=1}^{m} \prod_{t=1}^{T_i} f\left(y_{it} \mid \alpha, \beta, \gamma, \tau, \lambda, \nu\right) \, \mathrm{d}P_\alpha \, \mathrm{d}P_\beta \, \mathrm{d}P_\gamma \, \mathrm{d}P_\tau \, \mathrm{d}P_\lambda \, \mathrm{d}P_\nu \, .$$

[7] Although it perhaps has the advantage that it is not totally mechanical and considerations beyond statistical fit can be taken into account, as was done in our application in Subsection 3.3.

Clearly $f(s) = f(|s|)$ is decreasing in $|s|$, so the same upper and lower bounds for the sampling density as in Fernández and Steel (1998) hold, and using $g_i(\boldsymbol{\theta}) = \beta_i(1-\alpha) + \alpha\, y_{i\,t-1}$ these bounds are given by

$$\frac{2\,\lambda^{\frac{1}{2}}}{\gamma + \gamma^{-1}}\; f\left(\frac{\lambda^{\frac{1}{2}}\; |y_{i\,t} - g_i(\boldsymbol{\theta})|}{h(\gamma)}\right),$$

with

$$h(\gamma) = \begin{cases} \max\left\{\gamma, \gamma^{-1}\right\} & \text{for the upper bound} \\ \min\left\{\gamma, \gamma^{-1}\right\} & \text{for the lower bound.} \end{cases}$$

Therefore, $\mathcal{I} < \infty$ iff $\mathcal{I} < \infty$ under $\gamma = 1$.

In step two we write the model as $\boldsymbol{y} = X\,\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, with

$$\boldsymbol{y} = \begin{pmatrix} y_{1\,1} \\ \vdots \\ y_{1\,T_1} \\ y_{2\,1} \\ \vdots \\ y_{m\,T_m} \end{pmatrix}, \qquad X = \begin{pmatrix} \mathbf{1}_{T_1} & 0_{T_1,m-1} & & \boldsymbol{y}_1 \\ 0_{T_2,1} & \mathbf{1}_{T_2} & 0_{T_2,m-2} & \boldsymbol{y}_2 \\ \vdots & \ddots & \dots & \vdots \\ 0_{T_m,m-1} & & \mathbf{1}_{T_m} & \boldsymbol{y}_m \end{pmatrix} \quad \text{and} \quad \boldsymbol{\theta} = \begin{pmatrix} \beta_1(1-\alpha) \\ \beta_2(1-\alpha) \\ \vdots \\ \beta_m(1-\alpha) \\ \alpha \end{pmatrix}$$

where $\boldsymbol{y}_i = \left\{y_{i0}, \dots, y_{iT_i-1}\right\}$, $\mathbf{1}_k$ is a k-dimensional vector of ones and $0_{A,B}$ is an $A \times B$ matrix of zeros. So, $\boldsymbol{y} \in \mathbb{R}^{\mathcal{T}}$, X is a full column rank matrix of size $\mathcal{T} \times (m+1)$ and $\boldsymbol{\theta} \in \mathbb{R}^{m+1}$, with $\mathcal{T} = \sum_{i=1}^{m} T_i$ the total number of observations.

We now use Theorem 3 of Fernández *et al.* (1997), which states that the posterior distribution exists if $\pi(\theta_1, \dots, \theta_m)$ is bounded, $\pi(\alpha)$ is proper and $\mathcal{T}$ is strictly greater than the rank of X.

Normal case After integrating out λ we have that the posterior is proper iff the integral

$$\mathcal{I} = \int \left[\left(\boldsymbol{y} - X\,\boldsymbol{\theta}\right)' \left(\boldsymbol{y} - X\,\boldsymbol{\theta}\right)\right]^{-\mathcal{T}/2} \pi(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta}$$

is finite.

From the prior specification, we have for $i = 1, \dots, m$

$$\theta_i \mid \alpha, \beta, \tau \sim \mathrm{N}\left(\theta_i \mid \beta(1-\alpha), \tau/(1-\alpha)^2\right)$$

and so

$$\pi(\theta_i) = \int \mathrm{N}\left(\theta_i \mid \beta(1-\alpha), \tau/(1-\alpha)^2\right) \pi(\beta,\tau)\,\pi(\alpha)\, \mathrm{d}\beta\, \mathrm{d}\tau\, \mathrm{d}\alpha$$

with $\pi(\beta, \tau) = c\, \tau^{a_\tau - 1} \exp[-b_\tau \tau]$. Given that $\mathrm{N}(\cdot \mid \mu, \varphi) < \varphi^{1/2}$, we have that

$$\pi(\theta_i) \leq c \int \frac{\tau^{\frac{1}{2}}}{(1-\alpha)}\, \tau^{a_\tau - 1} \exp[-b_\tau \tau]\, \pi(\alpha)\, \mathrm{d}\tau\, \mathrm{d}\alpha$$
$$\leq c' \int (1+\alpha)^{a_\alpha - 1} (1-\alpha)^{b_\alpha - 2}\, \mathrm{d}\alpha,$$

which is bounded if $b_\alpha > 1$, as imposed in (11).

Therefore, given that the rank of X is $m+1$, $\mathcal{I} < \infty$ if $\mathcal{T} > m+1$.

Student case Let us consider the t distribution as a scale mixture of Normals as in Fernández and Steel (2000). So, after integrating out λ, the posterior is proper if the integral

$$\mathcal{I} = \int \left[\left(\boldsymbol{y} - X\boldsymbol{\theta} \right)' \Omega \left(\boldsymbol{y} - X\boldsymbol{\theta} \right) \right]^{-\mathcal{T}/2} \pi(\boldsymbol{\theta})\, \pi(\Omega \mid \nu)\, \pi(\nu)\, \mathrm{d}\boldsymbol{\theta}\, \mathrm{d}\Omega\, \mathrm{d}\nu$$

is finite, where $\Omega = \mathrm{diag}\,\{\omega_{it}\}$, $t = 1 \ldots, T_i$, $i = 1, \ldots, m$ is a diagonal matrix with the $\mathcal{T}$ (augmented) mixing parameters and $\pi(\omega_{it} \mid \nu) = \mathrm{Ga}\,(\omega_{it} \mid \nu/2, \nu/2)$, i.i.d.

Now we use the same argument as in the Normal case to prove that $\pi(\theta_1, \ldots, \theta_m)$ is bounded, provided that $b_\alpha > 1$. Further, Theorem 4(*ii*) of Fernández and Steel (2000) provides an upper bound for the integral above if $\mathcal{T} > m+1$.

A.2 Proof of Theorem 2

This proof is almost identical to that of Theorem 1, except that the parameterisation in step two now becomes

$$X = \begin{pmatrix} \mathbf{1}_{T_1} & 0_{T_1,m-1} & & & \boldsymbol{y}_1 & 0_{T_1,m-1} & \\ 0_{T_2,1} & \mathbf{1}_{T_2} & 0_{T_2,m-1} & & & \boldsymbol{y}_2 & 0_{T_2,m-2} \\ \vdots & \ddots & \ldots & \ldots & \ddots & & \vdots \\ 0_{T_m,m-1} & & \mathbf{1}_{T_m} & & 0_{T_m,m-1} & & \boldsymbol{y}_m \end{pmatrix} \quad \text{and} \quad \boldsymbol{\theta} = \begin{pmatrix} \beta_1 (1-\alpha_1) \\ \beta_2 (1-\alpha_2) \\ \vdots \\ \beta_m (1-\alpha_m) \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{pmatrix}$$

so that X is now a full column rank matrix of size $\mathcal{T} \times 2m$ and $\boldsymbol{\theta} \in \mathbb{R}^{2m}$. Following the same reasoning as in Theorem 1, a sufficient condition for posterior existence is $\mathcal{T} > 2m$.

Appendix B List of OECD countries

Core-EU		Rest-EU	North America	East Asia
Austria	Great Britain	Spain	Canada	Australia
Belgium	Germany	Greece	Mexico	Japan
Switzerland	Iceland	Ireland	USA	New Zealand
Denmark	Italy	Luxembourg		
Finland	Netherlands	Portugal		
France	Norway	Turkey		
	Sweden			

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