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14 March 2013

Online at https://mpra.ub.uni-muenchen.de/45036/
MPRA Paper No. 45036, posted 02 Apr 2013 04:12 UTC
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Intergenerational Transmission of Preferences

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ABSTRACT:
This paper examines the effects of compensatory-discrimination policies in a caste-based segregated economy where some high-paid positions in a certain industry are reserved for low-caste insiders as a consequence of the implementation of the policies. Cultural attitudes towards preferences for work-loving and leisure-loving traits evolve endogenously. The economy will converge to the efficient (inefficient) equilibrium with larger (smaller) fractions of work lovers among the insider and outsider populations if the profits in the industry with the purview of the reservation policy are sufficiently low (high) or the profits in the industry without the purview of the reservation policy are sufficiently high (low). Changes in the degree of compensatory-discrimination policies will affect the dynamics for insiders and outsiders differently.

Keywords: Caste system, cultural transmission, job reservations

JEL classification : A12, C62, J71

Acknowledgement: We are highly indebted to Arye L. Hillman and Hillel Rapoport for their comments and suggestions. We also thank David de la Croix, Shu-Hua Chen, Julio Davilla, Fabio Mariani and participants at the 2013 CEANA annual meeting and the 2012 Annual Conference of the Association of Public Economic Theory for helpful comments and suggestions. The first author would like to thank the financial support provided by the Program for Globalization Studies at the Institute for Advanced Studies in Humanities at the National Taiwan University. The usual disclaimer applies.
1. Introduction

With the intent of compensating the present generations of backward groups for past discrimination and injustice affirmative action or compensatory-discrimination policies have been implemented in some countries like the United States, South Africa, Malaysia, Brazil, Nigeria, India, and Sri Lanka. A large number of studies investigate this policy in various contexts.¹ Most of the theoretical studies explore the impact of affirmative action in a framework of job discrimination or wage discrimination relying on the theories of taste-based discrimination or statistical discrimination.²,³ There is however no prior study, to the best of our knowledge, which examines its role in shaping preferences for working attitude and its effect on the structural change in industry. This paper is therefore an attempt to assess the impact of the compensatory-discrimination policy - the caste-based reservations in India – on economic growth through the channels of intergenerational transmission of working attitude and the structural change in industry. The Indian case is although the institutional background of our study, the conclusions apply in principle more generally.

The Indian Hindu caste system consists of four distinct social classes (called Varna) which are arranged in a hierarchical order according to prestige, economic dominance and educational privileges. In the pre-compensatory discrimination regime, the low-caste group, placed at the bottom of the caste hierarchy, had denied access to education and good jobs.⁴ The caste system is however legislated out of existence in 1951, shortly after independence, but the introduction of caste-based "reservations" (the quotas imposed in the legislature, government-sponsored educational institutions and public sector jobs) by the government of India, which aimed at combatting caste-based inequalities, has further reinforced caste identities in social and political life (Mendelsohn and Vicziany, 1998).⁵ This paper focuses on the compensatory-discrimination job reservation policy.

¹ Tummala (1999) explains the rationale behind this policy and evaluates its consequences in India, the U.S. and South Africa. Also see for example, Sowell (2004) and Weisskopf (2004), for a worldwide comparison of affirmative action policy. For an overview of studies on affirmative action in the U.S. context, see Holzer and Neumark (2000) and on Indian caste-based reservations, see Haq and Ojha (2010).
² Taste-based discrimination theory is pioneered by Becker (1957) and statistical discrimination theory is initially studied by Phelps (1972) and Arrow (1973).
³ For the theoretical models on affirmative action in this vein, see for example, Welch (1976), Milgrom and Oster (1987), Kahn (1991), Lundberg (1991), Coate and Loury (1993).
⁴ The low-caste group consists of Scheduled Castes and Scheduled Tribes (defined under article 366 of India’s Constitution).
⁵ Since 1989 the list of beneficiaries of reservation policy has been expanded including the "Other Backward Classes" (OBCs) belonging to different castes and communities whose position was marginally better than that of the lower-caste group but worse than that of the higher-caste group.
Critics say that compensatory-discrimination policies do not have desired effect even after six decades of its implementation. The beneficiaries of this policy are the politically powerful groups (Pande, 2003) or they cluster at the upper income ends of the low-caste group (Aikara, 1996). Prevalent caste-gap in earning has been attributed to discrepancies in human and physical capital possessions (Deshpande, 2001; Borooah, 2005) and to differences in income generation structures faced by different caste groups (Kijima, 2006), the latter insinuating caste-based discrimination in the labour market.

Studies on caste-based job discrimination have drawn an increasing attention in the pervasive literature on labour market discrimination across gender and races all over the world. Major focus of research has been on explaining adverse discrimination towards the lower castes. Owing to a lack of caste network externality in the informal sector higher-paying jobs (Munshi and Rosenzweig, 2006) and low expectations of accessing to good jobs (Ito, 2009), the low-caste members usually end up with lower-paying jobs.

The current reservation policy has benefited the low-caste agents in the sense of affording them a share of regular salaried and wage employment in the public sector. The purview of reservation policy has been excluded in the private sector in India since the 1950s. Recently, there has been a lot of debate on the need of extending job reservations for the disadvantaged groups to the private sector. However, this proposal has not been upheld by corporate industry and some academics and has caused a political confrontation between a section of the political class and the industry leaders (Bhambhri, 2005; Thorat, 2005). Private industry contends that forcing them to hire incompetent workers will result in an inefficient allocation of labor and this, in turn, will reduce productivities and competitiveness of firms and thereby hamper economic growth.

Following this policy trend we analyze the issue by studying how the compensatory-discrimination policy impacts on economic growth if the requirement of job reservation has been applied to the private sector. Reservation policy is modelled as a constraint on employer requiring them to reserve some places in the high-paid jobs with a relaxation in the requirement of work effort for the low-caste. With job reservations, workers from different caste groups have an access to equal wages but with different work efforts. There is therefore a perception of an unequal distribution of job opportunities, skewed to the

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7 Although our model is based on a special feature of Indian caste-based reservation policy, conclusions have broader applicability in the sense that affirmative action in the US context portrays similar situation when the employers lower the required standard for the minorities in order to fulfill the affirmative action constraint on them of assigning the workers from different groups to highly rewarded jobs at the same rate.
advantage of lower castes. The same wage rate which fails to account for different levels of effort on work is perceived as unfair. This unfairness may generate disutility among the high-caste workers when they compare themselves to the low-caste workers exerting lower efforts, but have the same earnings. The unfairness caused by reservation policy may also distort the high-caste parents' decision regarding the socialization effort of transmitting working attitude to their children.\(^8\) Our attempt to evaluate the impact of the job reservation policy on preference of working attitude across castes can therefore make an important contribution to understanding how the working attitude preference dynamics can move the economy towards an efficient equilibrium.

We adopt the framework of cultural transmission of preferences developed by Bisin and Verdier (1998, 2001), which itself builds on population dynamics models of cultural transmission developed in Evolutionary Anthropology and Socio-biology (Cavalli-Sforza-Feldman, 1981; Boyd-Richerson, 1985) and on the work on socialization by Coleman (1994).\(^9\) Individuals acquire different preferences through socialization, and the internalization of different social norms leads to different equilibrium outcomes in the long run. Following the argument of Bisin and Verdier, the cultural transmission of 'working attitude' preferences within a caste group will be defined as the deliberate inculcation by rational parents who use their own preferences in evaluating ex-ante well-being of their children.

In this paper, we develop an overlapping generations model (OLG) with a firm-agent relation, rational expectations and cultural transmission of working attitude preferences. At each period, firms, when matched with the populations of insiders (a low-caste group) who are entitled to a reserved quota and outsiders (a high-caste group) who do not have an access to such quotas, has to assign an employment strategy to the agents of each group. Agents of insider and outsider groups can be of two types of preferences: work-loving and leisure-loving and respectively make high and low efforts on work. Firms can allocate agents to the industries with and without the purview of reservation policy. Reserved jobs cause disutility

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\(^8\) Experiment conducted by Hoff and Pandey (2006) suggests that the expectations of unfairness on the basis of caste identity undermine the low-caste individuals' motivation to perform well. With compensatory-discrimination policies, fairness concern is however raised by the higher castes, as claimed by this paper.

\(^9\) As shown in the cultural transmission literature, preferences and norms of behaviour, which are often formed by altruism (Rapoport and Vidal, 2007; Chen, 2009, 2010), cooperation (Bisin et al., 2004) and envy (Teraji, 2007), are transmitted through interactions across and within generations. For an overview of literature on cultural transmission, see Bisin and Verdier (2008). Explaining the emergence and long run survival of some norms even when the execution of those apparently run counter to individual self-interest has become one of the important issues addressed by cultural transmission models (Bisin et al., 2004).
for the high-effort making outsiders towards the comparatively low-effort making insiders having the same earnings.

Cultural values on working attitude are transmitted via parental effort. The incentives of insider and outsider parents to shape the preferences of working attitudes of their children depend on economic factors and, hence, directly on the expected payoffs under different strategies of firms. The parents' socialization decision is however influenced by perceived utility filtered through their own eyes. Because insider agents with leisure-loving trait have a chance of being assigned to high-paid positions under the reservation policy and this hurts outsider agents with work-loving trait, the perceived utility varies not only across the agents with different work efforts, but also across different groups. Cultural preferences of working attitude are therefore different among insider and outsider parents; and among parents with different traits.

The parents, being rational, choose their socialization efforts optimally in response to changes in the distribution of preferences in the population. In particular, the parents in majority cultural group (i.e., parents' trait is common to majority of the population) have, ceteris paribus, lower incentives to exert effort to socialize their children to their own preferences as compared to parents in minority cultural group, since 'oblique transmission' (socialization by the society) is a substitute for 'vertical transmission' (socialization inside the family). In our model, the distribution of preferences towards working attitudes evolve endogenously and lead to a stable interior steady state, specifying a heterogeneous distribution of preferences for working attitude in the population, thereby providing a prediction of the long-run economic performance. Our model setting allows us to trace the dynamics of insiders and outsiders under varying conditions.

The results show that there exist two stable equilibria: an 'efficient equilibrium' with large fractions of work-loving agents among the insider and outsider populations and an 'inefficient equilibrium' with large fractions of leisure-loving insider and outsider agents. We find that profits in the industries with and without job reservations are the most important determinants of the long-run economic performance. This is because these profits will affect firms’ decisions on labor allocation between industries, which will in turn affect parents’ socialization efforts on shaping their children’s working attitudes. The high profits for firms in industry with job reservation (or the low profits for firms in industry without job reservation) will induce firms to allocate workers in industry with job reservation. Therefore, both leisure-lover insider and outsider parents have strong incentives to transmit their traits to their children, causing the economy to converge to the inefficient equilibrium. If the profits
for firms in industry with job reservation are low (or the profits for firms in industry without job reservation are high), the situation will be reversed and the economy will converge to the efficient equilibrium.

Changes in the degree of the reservation policy affect the dynamics for insiders and outsiders differently. A decrease in the degree of the reservation policy reduces the outsiders’ disutility from working with insiders in industry with job reservation, thereby increasing the incentives for work-loving outsider parents to transmit their trait to their children, which can lead the dynamics for outsiders converge to the efficient equilibrium. On the other hand, an increase in the level of reserved jobs may prevent the dynamics for insiders from being trapped in the inefficient equilibrium. This finding seems unreasonable at a first glance because the higher level of reserved quota will cause a larger distortion in labor allocation. The high level of reserved jobs that raises the chances of offering high-paid positions to insiders making low efforts tends to induce the leisure-loving insider parents in transmitting their own trait to their children and undermines the motivation of work-loving outsider parents to exert efforts on their children socialization to the same trait. Both effects will push the economy to converge to the inefficient equilibrium. However, the driving force behind the convergence to the efficient equilibrium when the level of reserved jobs is high is firms’ strategy. An increase in the level of reserved jobs reduces firms’ profits obtained in industry with job reservation and induces firms to allocate agents to industry without job reservation. This structural change in industry will push the economy to converge to the efficient equilibrium. Therefore, this paper also provides an alternative explanation why compensatory-discrimination policies may not generate desired effects and different degrees of the reservation policy may cause very different long-run economic performances for insiders and outsiders.

The rest of the paper is organized as follows. The next section describes the model setting. Section 3 analyses the insider and outsider agents' optimal socialization effort choice. In section 4, we analyze the firm's optimal employment strategy. Section 5 presents the preference dynamics for the insiders and outsiders under different policy expectations. In section 6, convergence to the efficient and inefficient equilibria is described and the stability conditions are examined under three possible situations. The final section concludes.
2. The model

Consider an overlapping generations model with two dynasties in the economy – low-caste dynasty and high-caste dynasty, each of which extends over infinite generations \([t = ..., -2, -1, 0, 1, 2, ...]\) discrete time. Each of the agents belonging to either dynasty has a two-period life span. In the first period (childhood) he or she is educated in certain type of preferences of working attitude, while in the latter period (adulthood) he or she actively participates in the labour market as well as makes an effort attempting to transmit certain type of preferences to his or her only child. A compensatory-discrimination policy has been implemented at \(t = 0\) and our analysis in this paper is confined to post-compensatory discrimination regime.

2.1. Firms

There are two types of industry: a high-productive industry \((P_1)\) without the purview of reservation policy and a low-productive industry \((P_2)\) with the purview of reservation policy. Firms decide which industry to allocate agents and there is no unemployment. The low castes who are entitled to employment quota are referred as insiders \((I)\) and the high castes without entitlement to quota are referred as outsiders \((O)\). Populations of either group remain stationary with \(n_{it} + n_{ot} = 1\), where \(n_{it}\) and \(n_{ot}\) are respectively the proportions of low-caste and high-caste agents in the population at period \(t\).

In industry \(P_1\), wage rates are set equal to the effort each agent makes - \(\bar{w}_1\) for the high effort \((\bar{e})\) and \(w_i\) for the low effort \((e)\) with \(\bar{w}_1 > w_i\). Firms make high profits, \(\pi_i^h\) : if the agent exerts a high effort and low profits, \(\pi_i^l\) : if the agent exerts a low effort.

In industry \(P_2\), some places in the high-paid positions, say \(Q\), are reserved with a relaxation in the efforts required for the insiders. The wage rates \(\bar{w}_2\) and \(w_2\), such that \(\bar{w}_1 > \bar{w}_2 > w_i > w_2 > 0\), are offered to the agents who make high and low efforts, respectively. However, with the purview of reservation policy, an insider who makes a low effort now has a probability, \(\alpha \in (0,1)\), of being assigned to a high-paid position. Therefore, the expected wage for an insider agent who makes a low effort is \(\alpha \bar{w}_2 + (1-\alpha)w_2\). We assume that the chance of having a high-paid position for an insider who makes a low effort is increasing with the proportion of reserved places; that is, \(\alpha = \alpha(Q)\) with \(\alpha'(Q) > 0\). A firm
will obtain the profits $\pi^h_2$ if the agent makes a high effort. If the agent makes a low effort, the profits received by a firm will be $\pi^l_2$ when the firm pays the low wage rate and 0 when the firm pays the high wage rate. Therefore, if the agents makes a low effort, the expected profit for the firm will be $(1 - \alpha(Q))\pi^l_2$. The profits for firms are summarized in Table 1. The order of profits is assumed to be $\pi^h_1 > \pi^h_2 \geq (1 - \alpha(Q))\pi^l_2 > \pi^l_1 > 0$.

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>outsiders</th>
<th>$P_2$</th>
<th>insiders</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}$</td>
<td>$\pi^h_1$</td>
<td>$\bar{e}$</td>
<td>$\pi^h_2$</td>
<td>$\pi^l_2$</td>
</tr>
<tr>
<td>$\underline{e}$</td>
<td>$\pi^l_1$</td>
<td>$\underline{e}$</td>
<td>$\pi^l_2$</td>
<td>$(1 - \alpha(Q))\pi^l_2$</td>
</tr>
</tbody>
</table>

2.2. Agents’ preferences

There are two types of preferences among agents: work-loving (W) and leisure-loving (L). Agents decide to make low or high efforts on work. We assume that there is an additional cost ($\mu$) for a leisure lover to exert a high effort on work. Table 2 gives the payoffs to an insider agent:

<table>
<thead>
<tr>
<th></th>
<th>Work lover</th>
<th>Leisure lover</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_1$</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td>$\bar{w}_1$</td>
<td>$\bar{e}$</td>
</tr>
<tr>
<td>$\underline{e}$</td>
<td>$\underline{w}_1$</td>
<td>$\underline{e}$</td>
</tr>
<tr>
<td>$\alpha(Q)\bar{w}_2 + (1 - \alpha(Q))\underline{w}_2$</td>
<td>$\bar{w}_2 - \mu$</td>
<td>$\alpha(Q)\bar{w}_2 + (1 - \alpha(Q))\underline{w}_2$</td>
</tr>
</tbody>
</table>

Assume that outsiders making high efforts on work suffer from disutility ($\gamma$) from working with insider making low efforts in industry $P_2$ since they need to take extra work.\footnote{Types of preferences are based on the idea that the weight of satisfaction from leisure is higher for leisure lovers than for work lovers in the setting of utility functions.}

\footnote{Insiders who make high efforts will also face the same situation. But being insiders, they can also choose exerting low efforts on work and still have a chance of getting high wage. Therefore, this disutility for insiders is smaller than for outsiders. In order to simplify the model, we assume that this disutility for insiders is zero.}
This disutility is assumed to increase with $Q$; that is, $\gamma = \gamma(Q) > 0$ with $\gamma'(Q) > 0$ and $\gamma(0) = 0$. Table 3 gives the payoffs to an outsider agent:

### Table 3: Payoff matrix for an outsider agent

<table>
<thead>
<tr>
<th>Work lover</th>
<th>Leisure lover</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td>$w_1 - \gamma(Q)$</td>
</tr>
<tr>
<td>$w_1$</td>
<td>$w_2 - \gamma(Q)$</td>
</tr>
<tr>
<td>$e$</td>
<td>$w_1$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>$w_2$</td>
</tr>
</tbody>
</table>

Table 2 indicates that work-loving insiders always prefer exerting high efforts on the job. In addition, we make the following assumptions so that work-loving outsiders also prefer exerting high efforts while leisure-loving insider and outsider prefer making low efforts.$^{12}$

$$\mu > \bar{w}_1 - w_1 > \bar{w}_2 - w_2 - \gamma(Q) > 0,$$

(1)

and

$$\alpha(Q) \left( \bar{w}_2 - w_2 \right) > \gamma(Q).$$

(2)

To prevent leisure-loving insiders from revealing their trait, we also assume that leisure-loving insiders prefer to work in industry $P_1$ to $P_2$. That is

$$\bar{w}_1 - w_1 > \alpha(Q) \left( \bar{w}_2 - w_2 \right).$$

When a firm offers $P_1$, leisure-loving insiders will accept it without revealing their types.

### 2.3. Transmission of preferences

Children are naive at birth in the sense of having no well-defined preferences before the cultural transmission takes place and depending on parents' exertion of socialization effort (direct vertical socialization) and the peer effect of neighborhood (oblique socialization) they adopt a particular preference. A crucial assumption of the model is that parents are altruist towards their children and want to maximize their child's welfare when deciding how much

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$^{12}$ These assumptions imply that $(1 - \alpha(Q)) \left( \bar{w}_2 - w_2 \right) < \min \{ \mu, \bar{w}_2 - w_2 - \gamma(Q) \}$. 
socialization effort to put onto their children. Given that parents are ignorant about the future best outcome for their child, they evaluate their child's future utility through their own payoff matrix (imperfect empathy).

Let $\tau^i = [0, 1]$ be the socialization effort made by a parent of type belonging to insider group ($k = I$) or outsider group ($k = O$) with trait $i \in \{W, L\}$. With a success probability equal to the parental effort, $\tau^i$, the child adopts his or her parent's preference of trait but with probability $(1 - \tau^i)$, the child adopts the preference of the other trait getting matched randomly from the population. Since the caste-based reservation of jobs segregates the society into insider and outsider groups, the children are assumed to acquire only the trait of the peers from the same group. The fractions of the work-loving agents among the insider and outsider populations, at generation $t$, are respectively denoted by $q_{I_t}$ and $q_{O_t}$. The distribution of preferences within a group ($q_{k_t}, k = I, O$) is endogenously determined by the socialization decision made by the parents of that group.

Let $P_{kt}^{ij}$, $i, j = W, L$, be the probability that a parent of type $i$ belonging to group $k$ has a child adopting a preference of type $j$. Since there is a continuum of agents, by the Law of Large Numbers $P_{kt}^{ij}$ will also denote the fraction of children with a type $i$ parent and becoming a type $j$ person. The mechanism of 'working attitude' preference transmission within a group is then characterized by the following transition probabilities:

$$P_{kt}^{WW} = \tau^W + (1 - \tau^W)q_{kt},$$

$$P_{kt}^{WL} = (1 - \tau^W)(1 - q_{kt}),$$

$$P_{kt}^{LW} = \tau^L + (1 - \tau^L)(1 - q_{kt}),$$

$$P_{kt}^{WW} = (1 - \tau^L)q_{kt}.$$ (2)

Given these transition probabilities the fraction of agents in the insider or outsider group with work-loving trait at period $t + 1$ is given by:

$$q_{k, t + 1} = \left[ q_{kt} P_{kt}^{WW} + (1 - q_{kt}) P_{kt}^{WW} \right].$$ (7)

---

13 See Al-Najjar (1995) and Sun (1998), for formal constructions of the Law of Large Numbers on the basis of a continuum of agents.

14 We express the probability of oblique transmission of preferences, for example high-productive trait, within a group, in terms of $q_{kt}$, rather than $q_{kt} n_{kt}$, since we assume a stationary population.
Substituting (3) and (6) into (7),

\[ q_{k_{t+1}} - q_{k_{t}} = q_{k_{t}}(1 - q_{k_{t}})\left(\frac{\tau_{W}^{k}}{\tau_{k_{t}}} - \frac{\tau_{L}^{k}}{\tau_{k_{t}}}\right), \]  

(8)

which are the equations in differences that characterize the dynamic of the distribution of preferences among the insider and outsider populations.

3. The agents' socialization effort choice

Socialization of children to a certain type of preference is costly for the parents. Let the cost of direct parental socialization effort take the following quadratic form: 

\[ C\left(\tau_{k_{t}}\right) = \left(\tau_{k_{t}}\right)^{2}/2\psi, \]

where \(\psi > 0\). Let \(V_{k_{t}}^{j}\) be the utility a parent of type \(i\) belonging to group \(k\) attributes to his or her child having preferences \(j\). Note that \(V_{k_{t}}^{j}\) depends on the parents' expectations on the strategy of the firm. Assuming perfect foresight, parents of either group know the firm's optimal strategy at period \(t + 1\), \(\sigma_{t+1}^{*} \).  

Given a strategy expectation parents in group \(k\) choose the socialization effort \(\tau_{k_{t}}^{i}\) that maximizes:

\[ \left[ P_{k_{t}}^{ui}\left(\tau_{k_{t}}^{i}, q_{k_{t}}\right)V_{k_{t}}^{ui}\left(\sigma_{t+1}\right) + P_{k_{t}}^{ui}\left(\tau_{k_{t}}^{i}, q_{k_{t}}\right)\right] V_{k_{t}}^{ui}\left(\sigma_{t+1}\right) = C\left(\tau_{k_{t}}^{i}\right). \]  

(9)

According to the imperfect empathy notion a parent (of type \(i\)) uses his or her own payoff matrix in evaluation of \(V_{k_{t}}^{j}\). Since the work-loving (leisure-loving) agents prefer exerting high (low) efforts, therefore, \(V_{k_{t}}^{ui} > V_{k_{t}}^{ui}\) always.

Maximization of (9) with respect to \(\tau_{k_{t}}^{i}\) yields the first-order condition:

\[ \frac{dP_{k_{t}}^{ui}\left(\tau_{k_{t}}^{i}, q_{k_{t}}\right)}{d\tau_{k_{t}}^{i}} V_{k_{t}}^{ui}\left(\sigma_{t+1}\right) + \frac{dP_{k_{t}}^{ui}\left(\tau_{k_{t}}^{i}, q_{k_{t}}\right)}{d\tau_{k_{t}}^{i}} \frac{\tau_{k_{t}}^{i}}{\psi} \]  

(10)

Substituting (3) – (6), we get the optimal effort levels:

\[ \tau_{k_{t}}^{W}\left(q_{k_{t}}, \sigma_{t+1}\right) = \psi \Delta V_{k_{t}}^{W}\left(\sigma_{t+1}\right)(1 - q_{k_{t}}), \]  

(11)

\[ \tau_{k_{t}}^{L}\left(q_{k_{t}}, \sigma_{t+1}\right) = \psi \Delta V_{k_{t}}^{L}\left(\sigma_{t+1}\right)q_{k_{t}}. \]  

(12)

\[ ^{15} \text{In the next section we will see that} \sigma_{t+1} \text{ depends on the distribution of preferences in the insider and outsider population,} \ q_{k_{t+1}}. \text{ Assuming perfect foresight, equivalent to rational expectation in this deterministic framework, parents' expectation formation at period} \ t \text{ on the distribution of preferences in their own group in the next period} \ t + 1 \text{ is:} \ q_{k_{t+1}} = q_{k_{t+1}}. \]
Here the difference \( \Delta V'_k(\sigma_{t+1}) \equiv V'_k(\sigma_{t+1}) - V''_k(\sigma_{t+1}) \) represents the perceived utility gains by a parent transmitting his or her preference to his or her child, given a strategy expectation. In order to guarantee interior solutions \( \tilde{\tau}^i_{kt} \in (0, 1) \) of the socialization problem, assume that the parameter \( \psi \) must be small enough so that \( 1/\psi > \max \Delta V'_k(\sigma_{t+1}) \).

Let us now analyze how, for a given strategy expectation \( \sigma_{t+1} \), the optimal socialization effort of parents depends on \( q_{kt} \). Differentiation of (11) and (12) with respect to \( q_{kt} \) yields:

\[
\frac{d\tilde{\tau}^w_{kt}(q_{kt}, \sigma_{t+1})}{dq_{kt}} = -\psi \Delta V'_k(\sigma_{t+1}) < 0, \tag{13}
\]

\[
\frac{d\tilde{\tau}^L_{kt}(q_{kt}, \sigma_{t+1})}{dq_{kt}} = \psi \Delta V'_k(\sigma_{t+1}) > 0. \tag{14}
\]

That is, the higher the proportion of the work-loving individuals among the insider (outsider) population, the better children are socialized to the work-loving trait by the social environment, inducing the work-loving parents to exert less effort on their children's socialization. Therefore, the work-loving insider (outsider) parents' socialization effort, \( \tilde{\tau}^w_{kt}(q_{kt}, \sigma_{t+1}) \), is decreasing in the fraction of the work-loving individuals in their own population, \( q_{kt} \), as revealed by (13).

Symmetrically, leisure-loving insider (outsider) parents' effort, \( \tilde{\tau}^L_{kt}(q_{kt}, \sigma_{t+1}) \), depends negatively on the fraction of leisure-loving trait, \( 1-q_{kt} \), hence positively on \( q_{kt} \). That means, the larger the proportion of work-loving preferences in the population, the greater is the effort exerted by the leisure-loving parents in order to offset the pressure of the environment, as they want to transmit their own preferences to their children. Hence for a given strategy expectation, vertical cultural transmission and oblique cultural transmission are substitutes.

4. The firms’ optimal strategy

The other determinant of the optimal socialization effort of parents is their expectations about the firms’ optimal strategy which we will analyze in this section. At each period \( t \) the firms have to decide which industry is to delegate to the insider and outsider agents with whom they are matched.
The firms’ aim is to maximize the expected payoffs when they know the proportions of work-loving insiders \((q,in_i)\) and outsiders \((q,ont_o)\) in the population but not the type of a particular agent in either group. Firms have the following two strategies: offering \(P_1\) to everyone within a group \((\sigma^f)\) and offering \(P_2\) to everyone within a group \((\sigma^d)\).

The firms prefer strategy \(\sigma^f\) to \(\sigma^d\) if for the insider group,
\[
q_{i1}n_i(\pi^h_i - \pi^h_2) + (1 - q_{i1})n_i(\pi^l_i - (1 - \alpha(Q))\pi^l_2) \geq 0,
\]
or equivalently,
\[
q_{i1} \geq \frac{(1 - \alpha(Q))\pi^l_2 - \pi^l_1}{\pi^h_1 - \pi^h_2 + (1 - \alpha(Q))\pi^l_2 - \pi^l_1} = \tilde{q}_i(Q),
\]
and for the outsider group,
\[
q_{o1}n_o(\pi^h_1 - \pi^h_2) + (1 - q_{o1})n_o(\pi^l_1 - \pi^l_2) \geq 0,
\]
or equivalently,
\[
q_{o1} \geq \frac{\pi^l_2 - \pi^l_1}{\pi^h_1 - \pi^h_2 + \pi^l_2 - \pi^l_1} = \tilde{q}_o.
\]

That is, the conditions for adopting the \(\sigma^f\) strategy for the insiders and outsiders are that the fractions of work-loving agents in the insider and outsider populations, at generation \(t\), are higher than the critical values \(\tilde{q}_i(Q)\) and \(\tilde{q}_o\) respectively. Notice that \(\tilde{q}_i(Q) < \tilde{q}_o\) since \(\alpha(Q) \in (0,1)\). Furthermore, an increase in \(Q\) reduces firms’ profits of hiring insiders in industry \(P_1\) (the \(\sigma^d\) strategy). Therefore, firms will switch to the \(\sigma^f\) strategy at a lower value of \(\tilde{q}_i(Q)\).

By (15) and (16), the firms’ optimal set of strategies can be written as
\[
\Omega(q_{i1}, q_{o1}) = \{\sigma(q_{i1}), \sigma(q_{o1})\} = \begin{cases} 
\{\sigma^d, \sigma^d\} = \sigma^D & \text{if} \; q_{ki} < \tilde{q}_i(Q) \\
\{\sigma^f, \sigma^d\} = \sigma^{fd} & \text{if} \; \tilde{q}_i(Q) \leq q_{ki} < \tilde{q}_o \\
\{\sigma^f, \sigma^f\} = \sigma^f & \text{if} \; q_{ki} \geq \tilde{q}_o
\end{cases}
\]

If the fractions of work-loving agents in the insider and outsider populations are lower than the critical value of the insider group \((q_{ki} < \tilde{q}_i)\) or higher than the critical value of the outsider group \((q_{ki} \geq \tilde{q}_o)\) then it is optimal for the firm to adopt respectively \(\sigma^D\) and \(\sigma^f\).
strategy. However, when the fraction of work-loving agents in either group's population exceeds the insider group's critical value but is below the outsider group's critical value ($\tilde{q}_I < q_{k,t} < \tilde{q}_O$), then the optimal strategy set for firms is the mixed strategy set, $\sigma^M = \{\sigma^f, \sigma^d\}$, i.e., offering $\sigma^f$ to the insiders and $\sigma^d$ to the outsiders.

5. The steady states

In this section we will analyze the pattern of the distribution of preferences in the long run, as characterized by the firm's optimal strategy under the assumption of rational expectations. The dynamics of the distribution of preferences for work-loving trait among the insider and outsider populations are derived by substituting the optimal socialization effort, $\hat{\tau}^i_k(q_{k,t}, \sigma_{t+1})$ from (11) and (12) into (8):

$$q_{k,t+1} - q_{k,t} = \psi_k (1 - q_{k,t}) \left[ \Delta V^w_k (\sigma_{t+1}) (1 - q_{k,t}) - \Delta V^i_k (\sigma_{t+1}) q_{k,t} \right]. \quad (18)$$

It is useful to know how (18) behaves under a stationary strategy expectation, i.e., if $\Delta V^w_k (\sigma_{t+1}) = \Delta V^w_k (\hat{\sigma})$ and $\Delta V^i_k (\sigma_{t+1}) = \Delta V^i_k (\hat{\sigma})$, where $\sigma_{t+1} = \hat{\sigma}$ for all $t$. The dynamics (18) has three steady states: (i) $q_k = 0$, (ii) $q_k = 1$ and (iii) $q_k = q_k^* \in (0,1)$, where

$$q_k^* = \frac{\Delta V^w_k (\hat{\sigma})}{\Delta V^w_k (\hat{\sigma}) + \Delta V^i_k (\hat{\sigma})} \quad (19)$$

with $\hat{\tau}^w_k (q_k^*, \hat{\sigma}) = \hat{\tau}^i_k (q_k^*, \hat{\sigma})$.

The degenerate steady states, $q_k = 0$ and $q_k = 1$, are locally unstable. By (13), if work-loving parents within a group $k$ are in a minority (that is, $q_{k,t}$ is very close to 0), they produce higher socialization effort in order to offset the counter effect of environment. In this context, $\hat{\tau}^w_k$ exceeds $\hat{\tau}^i_k$ and work-loving preferences tend to expand among next generations preventing their disappearance from the society. Similar argument applies for the distribution of leisure-loving preferences when $q_{k,t}$ is very close to 1. The interior rest point $q_k^* \in (0,1)$ characterizing the heterogeneous distribution of preferences is, however, globally stable (as shown later). The process of convergence and the stability of steady state depend on the agents' payoff structure under the firms' different strategies.

We have shown that the exertion of socialization effort by a parent in $t$ depends on his or her expectation about the firm's optimal strategy in the future. As we have shown in (17)
that the firm’s optimal strategy depends on the distribution of preferences in the populations of insider and outsider agents, we consider the following three possibilities.

**Case I:** Assume that parents’ expectation at period $t$ on the distribution of preferences in the next period $t+1$ is: $q_{t+1}^E < \tilde{q}_{t}(Q)$. Therefore, both the insider and outsider parents expect that firms will adopt $\sigma^d$ in the future, $\{\sigma^d_{t+1} = \{\sigma^d_{t+1} \}$. Then according to the imperfect empathy notion, a parent (of type $i$) using his or her own utility function evaluates the payoffs of his or her child having preferences $j$, $V_i^d(\sigma^d)$, as follows:

$$V_i^{wW}(\sigma^d) = \bar{w}_2; \quad V_i^{wL}(\sigma^d) = \alpha(Q)\bar{w}_2 + (1-\alpha(Q))w_2; \quad V_i^{lW}(\sigma^d) = \bar{w}_2 - \mu;$$

$$V_o^{wW}(\sigma^d) = \bar{w}_2 - \gamma(Q); \quad V_o^{wL}(\sigma^d) = \bar{w}_2; \quad V_o^{lW}(\sigma^d) = \bar{w}_2 - \mu - \gamma(Q).$$

Therefore, the relative gains for insider and outsider parents of transmitting own preferences to their child are given by

$$\Delta V_i^{wW} = V_i^{wW}(\sigma^d) - V_i^{wL}(\sigma^d) = (1-\alpha(Q))\bar{w}_2 - \bar{w}_2;$$

$$\Delta V_i^{lL} = V_i^{lL}(\sigma^d) - V_i^{lW}(\sigma^d) = \mu - (1-\alpha(Q))\bar{w}_2 - \bar{w}_2;$$

$$\Delta V_o^{wW} = \bar{w}_2 - \bar{w}_2 - \gamma(Q);$$

$$\Delta V_o^{lL} = \mu + \gamma(Q) - \bar{w}_2 - \bar{w}_2.$$
chance of receiving high wage rate for insiders who make low efforts, as well as, increases the disutility for outsiders making high efforts but working with more insiders making low efforts. By similar argument, the leisure-loving parents will have higher incentive to make their children like them.

**Case II:** If \( \bar{q}_i(Q) \leq q_{k,t+1}^E < \bar{q}_O \), parents expect that the firm will adopt the mixed strategy in the future, i.e., \( \{\sigma^f\}_{z=t+1}^\infty = \{\sigma^M_{z=t+1}^M\}_{z=t+1}^\infty \), and offer \( \sigma^f \) to the insiders and \( \sigma^d \) to the outsiders. Parents’ expected payoffs of their children are:

\[
\begin{align*}
V_{i}^{\text{WW}}(\sigma^f) &= \tilde{w}_1 - \gamma(Q); & V_{i}^{\text{WL}}(\sigma^f) &= \tilde{w}_1; & V_{i}^{\text{LW}}(\sigma^f) &= -\tilde{w}_1 - \mu; \\
V_{O}^{\text{WW}}(\sigma^d) &= \tilde{w}_2; & V_{O}^{\text{WL}}(\sigma^d) &= \tilde{w}_2; & V_{O}^{\text{LW}}(\sigma^d) &= \tilde{w}_2 - \mu - \gamma(Q).
\end{align*}
\]

Therefore, the relative gains for the insider and outsider parents are:

\[
\begin{align*}
\Delta V_{i}^{\text{W}}(\sigma^f) &= \tilde{w}_1 - \gamma(Q); & \Delta V_{i}^{\text{L}}(\sigma^f) &= \mu - (\tilde{w}_1 - \gamma(Q)); \\
\Delta V_{O}^{\text{W}}(\sigma^d) &= \tilde{w}_2 - \gamma(Q); & \Delta V_{O}^{\text{L}}(\sigma^d) &= \mu + \gamma(Q) - (\tilde{w}_2 - \gamma(Q)).
\end{align*}
\]

Note that with the expectation of the mixed strategy profile in the future, \( \tilde{z}_O^{\text{W}}(q_{k,t}, \{\sigma^d_{z=t+1}^d\}) \) is decreasing in \( Q \) and \( \tilde{z}_O^{\text{L}}(q_{k,t}, \{\sigma^d_{z=t+1}^d\}) \) is increasing in \( Q \) while \( \tilde{z}_I^{\text{W}}(q_{k,t}, \{\sigma^f_{z=t+1}^f\}) \) and \( \tilde{z}_I^{\text{L}}(q_{k,t}, \{\sigma^f_{z=t+1}^f\}) \) are not affected by \( Q \).

**Case III:** If \( q_{k,t+1}^E \geq \bar{q}_O \), both the insider and outsider parents expect that the firm will adopt \( \sigma^f \) in the future, i.e., \( \{\sigma^f\}_{z=t+1}^\infty = \{\sigma^f_{z=t+1}^f\} \). Parents’ expected payoffs of their children are:

\[
\begin{align*}
V_{k}^{\text{WW}}(\sigma^f) &= \tilde{w}_1; & V_{k}^{\text{WL}}(\sigma^f) &= \tilde{w}_1; & V_{k}^{\text{LW}}(\sigma^f) &= \tilde{w}_1 - \mu.
\end{align*}
\]

Therefore, the perceived net utility gains for insider and outsider parents of transmitting their own preferences are given by

\[
\begin{align*}
\Delta V_{k}^{\text{W}}(\sigma^f) &= \tilde{w}_1 - \gamma(Q); & \Delta V_{k}^{\text{L}}(\sigma^f) &= \mu - (\tilde{w}_1 - \gamma(Q));
\end{align*}
\]

Note that with the expectation of the \( \sigma^f \) strategy in the future, the socialization efforts exerted by the two types of parents among the insiders and outsiders are unaffected by \( Q \).
Using (19) we are now ready to derive the preference dynamics for both the insider and the outsider group in the above three possibilities under the perfect foresight assumption, i.e., when \( q_{k,+1}^e = q_{k,+1} \). If \( q_{k,+1} < \bar{q}_I(Q) \), the firm offers \( \sigma^d \) to both insiders and outsiders. Then we have:

\[
q_{I,+1} = q_{I,I}[1 + \psi(1-q_{I,I})(1-\alpha(Q))(w_2 - w_3) - \mu q_I];
\]

\[
q_{O,+1} = q_{O,I}[1 + \psi(1-q_{O,I})(w_2 - w_3 - \gamma(Q) - \mu q_{O,I})];
\]

If \( q_{k,+1} \geq \bar{q}_I \), the firm offers \( \sigma^f \) to both insiders and outsiders. Then we have:

\[
q_{k,+1} = q_{k,I}[1 + \psi(1-q_{k,I})(w_1 - w_3 - \mu q_{k,I})], \quad k = I, O.
\]

If \( \bar{q}_I(Q) \leq q_{k,+1} < \bar{q}_O \), the firm offers \( \sigma^f \) to insiders and \( \sigma^d \) to outsiders. This implies that the preference dynamics for the insider agents is given by (C) and that for the outsider agents is given by (B). Notice that there are discontinuities in \( q_{I,+1} = \bar{q}_I(Q) \) and \( q_{O,+1} = \bar{q}_O \). The dynamics (A), (B) and (C) will alternatively be denoted as \( F_A(\cdot) \), \( F_B(\cdot) \) and \( F_C(\cdot) \) respectively. Under our assumptions of (1) and (2), dynamics (C) lies above dynamics (B) which is above dynamics (A).

Let \( q_{I}(Q) = q_{I}^{*}(\sigma^d) = \frac{(1-\alpha(Q))(w_2 - w_3)}{\mu} \), \( q_{O}(Q) = q_{O}^{*}(\sigma^d) = \frac{w_2 - w_3 - \gamma(Q)}{\mu} \) and \( \bar{q} = q_{k}^{*}(\sigma^f) = \frac{w_1 - w_3}{\mu} \) denote respectively the stable steady states characterizing the heterogeneous preference of \( F_A(\cdot) \), \( F_B(\cdot) \) and \( F_C(\cdot) \). As shown in Appendix C, there are three steady states of the dynamics (A), (B) and (C), where 0 and 1 are unstable steady states and \( q_{k}^{*} (q_{I}, q_{O} \) and \( \bar{q} \) ) is a stable steady state.

Note that the order of three stable steady states is \( q_{I}(Q) < q_{O}(Q) < \bar{q} \). We refer to \( q_{I} \) and \( q_{O} \) as the inefficient steady states for the insider and outsider populations respectively and \( \bar{q} \) as the efficient steady state. In the efficient equilibrium outcome, the proportions of work-loving insider and outsider agents are high and the low-caste agents do not have an access to the reserved quota since firms adopt the \( \sigma^f \) strategy; whereas in the inefficient
equilibrium outcome, the respective proportions are low and the low-caste agents benefit from the implementation of the reservation policy.

**Lemma 1:** Comparing the socialization efforts exerted by the two types of parents, we have:

1. $\tau_k^W \left( q_{k_t}, \left\{ \sigma^f \right\}_{t+1} \right) \geq \tau_k^L \left( q_{k_t}, \left\{ \sigma^f \right\}_{t+1} \right)$ when $q_{k_t} \leq q$;

2. $\tau_I^W \left( q_{I_t}, \left\{ \sigma^d \right\}_{t+1} \right) \geq \tau_I^L \left( q_{I_t}, \left\{ \sigma^d \right\}_{t+1} \right)$ when $q_{I_t} \leq q_I$  

and $\tau_O^W \left( q_{O_t}, \left\{ \sigma^d \right\}_{t+1} \right) \geq \tau_O^L \left( q_{O_t}, \left\{ \sigma^d \right\}_{t+1} \right)$ when $q_{O_t} \leq q_O$.

**Proof:** See Appendix B.

The quantum of reservation ($Q$) will affect $\alpha(Q)$ and $\gamma(Q)$, which will further affect the critical value $\tilde{q}_I(Q)$ as well as the steady states $q_{O}(Q)$ and $q_{I}(Q)$. The order of the critical values is $\tilde{q}_I(Q) < \tilde{q}_O$, whereas the order of the steady states is $q_{I}(Q) < q_{O}(Q) < \tilde{q}$.

6. **Dynamics**

Depending on the values of parameters and the level of quantum of reservation, the economy will have different long-run performances. There are three possible situations for both insiders and outsiders: (1) $\tilde{q}_k < q_k < \tilde{q}$; (2) $q_k < \tilde{q} < \tilde{q}_k$; and (3) $q_k < \tilde{q}_k < \tilde{q}$, $k = I, O$. Note that $\tilde{q}_O$ and $\tilde{q}$ are independent of $Q$ while $\tilde{q}_I(Q)$, $q_I(Q)$ and $q_O(Q)$ decrease with an increase in $Q$. In the following, we discuss each situation separately.

6.1. $\tilde{q}_k < q_k < \tilde{q}$

We start our analysis by considering the situation of a low threshold value ($\tilde{q}_k < q_k < \tilde{q}$). The low values of $\tilde{q}_I$ and $\tilde{q}_O$ can be results of low profits for firms in industry $P_2$ ($\pi^2_1$ or $\pi^b_2$) or high profits in industry $P_1$ ($\pi^1_1$ or $\pi^b_1$). Assume initially, most insider (outsider) agents have leisure-loving preferences (that is, $q_{I_o}$ ($q_{O_o}$) close to 0). The insider (outsider) parents expect the $\sigma^d$ strategy for the next generation. Nevertheless, as parents try to transmit their own preferences and the work-loving parents within a group $k$ are in a minority, the socialization
efforts of this type of parents are high in order to offset the counter effect of environment on their children. The opposite applies for the leisure-loving parents because oblique transmission is a substitute of vertical transmission. This results in an expansion of work-loving preferences over next generations among the group \( k \) population.

In order to describe the dynamics of \( q_{\mu} \), the situation of \( \tilde{q}_k < q_{\mu} < \tilde{q} \) for both insiders and outsiders is depicted in Figure 1. Especially, we assume that the level of reserved jobs is low enough such that \( \alpha(Q) < 1 - \frac{\mu}{w_2 - w_1} \left( \frac{\pi^1_2 - \pi^1_1}{\pi^1_h - \pi^1_2 + \pi^1_1 - \pi^1_1} \right) \), thereby leading to the situation of \( \tilde{q}_O < q_{I} \). This gives the order of threshold values and steady states as \( q_i < \tilde{q}_I < q_{O} < \tilde{q}_O < q \). The phase diagram in Figure 1 describes the intergenerational evolution of preferences in the insider and outsider populations and convergence of the distribution of preferences to the efficient steady state, \( \tilde{q} \), when \( \tilde{q}_O < q_{I} \).

In Figure 1, \( q_i' \) is the value that yields \( q_{i_{I+I}} = \tilde{q}_I \) with dynamics (A) and \( q_{C_i}' \) is the value that yields \( q_{O_{1+I}} = \tilde{q}_O \) with dynamics (C) (namely \( F_A(q_i') = \tilde{q}_I \) and \( F_C(q_{C_i}') = \tilde{q}_I \)). Similarly, \( q_{B}' \) and \( q_{C_i}' \) are the values that yield \( q_{O_{1+I}} = \tilde{q}_O \) with dynamics (B) and (C) respectively (namely \( F_B(q_{B}') = \tilde{q}_O \) and \( F_C(q_{C_i}') = \tilde{q}_O \)). For any particular value of the parameters, \( q_i' > q_{C_i}' \) and \( q_{B}' > q_{C_i}' \) always.

<Figure 1 is inserted about here>

It follows from inspection of the above figure that from any \( q_{I0} \in (0, q_{C_i}') \) and \( q_{O0} \in (0, q_{C_i}') \), a unique \( q_{Ii} \) path starts following dynamics (A) and a unique \( q_{Oi} \) path starts following dynamics (B). With a low threshold level for switching to implement the \( \sigma^f \) strategy, the expansion of work-loving agents among insider and outsider populations leads to a situation such that \( q_{Ii} \) and \( q_{Oi} \) reach the intervals \([q_{C_i}', q_{i}']\) and \([q_{C_i}', q_{B}']\) respectively, when both \( \sigma^d \) and \( \sigma^f \) strategies are possible. If the agents expect the \( \sigma^d \) strategy, then the dynamics (A) and the dynamics (B) will be followed and \( q_{Ii} \) and \( q_{Oi} \) will increase over time. Once when \( q_{Ii} \) and \( q_{Oi} \) are respectively higher than \( q_{A}' \) and \( q_{B}' \) then the agents expect that only \( \sigma^f \) strategy will be adopted. The shift in expectation from a \( \sigma^d \) strategy to a \( \sigma^f \) strategy removes leisure-loving insider agents' chance of having high wage rates when
making low efforts and resolves work-loving outsider agents' disutility, thereby leading both $q_{1t}$ and $q_{0t}$ to follow dynamics (C). Consequently, both the insider and outsider agents' preferences for work-loving trait converge to the efficient steady state $\bar{q}$ with high proportions of work-loving insider and outsider agents.

The convergence to the efficient steady state $\bar{q}$ is also achieved from any other initial condition. If $q_{10} \in (q_A', 1)$ and $q_{00} \in (q_B', 1)$, there is a unique and the same $q_{kt}$ path for both insiders and outsiders, following dynamics (C), results with $q_{kt}$ converging to $\bar{q}$. Therefore, we have the following proposition.

**PROPOSITION 2:** The economy will converge to the efficient equilibrium $\bar{q}$ for both insiders and outsiders if $\bar{q} < q_k < \bar{q}_k$, $k = I, O$.

**Proof:** See Appendix C.

**6.2.** $q_k < \bar{q} < \bar{q}_k$

We now turn to consider the situation of a high threshold value ($q_k < \bar{q} < \bar{q}_k$). The high value of $\bar{q}_I$ and $\bar{q}_O$ can be caused by high profits in industry $P_2$ ($\pi_2' \text{ or } \pi_2^b$) or low profits in industry $P_1$ ($\pi_1' \text{ or } \pi_1^b$). Begin with an initial situation where most insider (outsiders) agents have leisure-loving preferences (that is, $q_{0I}$ ($q_{0O}$) close to 0). Initially and similarly to the previous case, the work-loving parents in either group have more incentives than the leisure-loving parents to intensify their socialization effort ($\hat{\tau}_k^w > \hat{\tau}_k^L$), leading to increases in $q_{kt}$, and $q_{Ot}$. With the increase in $q_{kt}$, the difference in efforts between the work-loving and leisure-loving parents diminishes. Before reaching the threshold level for the adoption of the $\sigma^f$ strategy by firms, the socialization efforts of both types of insider and outsider parents under the $\sigma^d$ strategy are equalized, and the economy is trapped in the steady state, $q_f$ (for insiders) and $q_o$ (for outsiders), with a relatively high proportion of leisure-loving insider and outsider agents.
The situation of $q_k < q < \tilde{q}$ for both insiders and outsiders is given in Figure 2. Especially, we assume that the level of reserved jobs is low enough such that

$$\alpha(Q) < 1 - \frac{1}{\pi^2} \left[ \pi_1' + \frac{(w_1 - w_1^b)(\pi_1^b - \pi_1^h)}{\mu - w_1 + w_1} \right],$$

thereby leading to a situation of $\tilde{q} < \tilde{q}_f$. This gives the order of threshold values and steady states as $q_f < q_o < \tilde{q} < \tilde{q}_f < \tilde{q}_o$. The phase diagram in Figure 2 describes the process of convergence of the distribution of preferences to the inefficient steady states $q_f$ for the insiders and $q_o$ for the outsiders. The values $q_f'$, $q_o'$, $q_b'$ and $q_c'$ are defined as before.

<Figure 2 is inserted about here>

**PROPOSITION 3:** The distribution of preferences will converge to the inefficient steady states $q_f$ for the insiders and $q_o$ for the outsiders if $q < \tilde{q}_k$, $k = I, O$.

**Proof:** See Appendix D.

The rise in the proportion of work-loving agents among outsiders, $q_o'$, is relatively faster than that among insiders, $q_b$ (dynamics (B) lies above dynamics (A)), because of two opposite effects on socialization efforts under the anticipation of $\sigma^d$ strategy. The first one is that the perceived utility gained by the high effort making outsider parents is relatively lower than that by the insider parents as the former also perceive utility loss due to the possibility of working with low effort making insiders. The second effect is that the anticipation of $\sigma^d$ strategy enhances the incentives of leisure-loving insider parents to transmit their trait to their children. If the extent of disutility is low, the second effect will dominate the first effect, thereby inducing the steady state $q_f$ to reach earlier than $q_o$.

The convergence to the inefficient steady state $q_f$ and $q_o$ are attained from any initial situation. Even a large proportion of work-loving agents in the population to begin with would lead the economy to end up with the inefficient steady state in the long run, because then leisure-loving parents take relatively higher initiative than the work-loving parents to transmit their own preferences to their children ($\tilde{\tau}^I_w > \tilde{\tau}^W_w$) despite the $\sigma^f$ strategy taken by firms, thereby leading to a decrease in $q_k$. When the contraction of work-loving agents
among insider and outsider populations lead to a situation such that $q_{i,t}$ and $q_{o,t}$ reach the intervals $[q_{c_1}', q_{d_2}']$ and $[q_{c_1}', q_{y_2}']$ respectively, then the possible strategies are both $\sigma^f$ and $\sigma^d$. If the agents switch their expectation from the $\sigma^f$ strategy to the $\sigma^d$ strategy, it forms the expectation of getting a high wage rate for the low effort making insiders and a high utility loss for outsiders who exert high efforts. The combined effect of cultural substitution and a switch in firms’ strategy leads to a situation such that $q_{i,t+1} < \tilde{q}_i$ and $q_{o,t+1} < \tilde{q}_o$, which self-confirms the insider and outsider agents' expectations. Consequently, a dynamics is generated that moves the economy towards the inefficient steady state $\underline{q}_i$ (for insiders) and $\underline{q}_o$ (for outsiders).

6.3 $\underline{q}_k < \tilde{q}_k < \overline{q}$

Finally, we consider the situation where the threshold value is in the middle between inefficient equilibrium and efficient equilibrium ($\underline{q}_k < \tilde{q}_k < \overline{q}$). The situation of $\underline{q}_k < \tilde{q}_k < \overline{q}$ for both insiders and outsiders is depicted in Figure 3. Especially, we assume the order of threshold values and steady states follows $\underline{q}_i < \underline{q}_o < \tilde{q}_i < \tilde{q}_o < \overline{q}$.

<Figure 3 is inserted about here>

The convergence of preferences to the efficient equilibrium, $\overline{q}$ or to the inefficient equilibrium, $\underline{q}_i$ (for insiders) and $\underline{q}_o$ (for outsiders) is summarized in the next two propositions. The long-run situation for the insider agents is given in the proposition 4.

**Proposition 4:** If $q_{i,o} < \tilde{q}_i < \overline{q}$, then

(i) for all $q_{i,o} \in [q_{c_1}', q_{d_2}']$, there are two perfect foresight paths converging to $\underline{q}_i$ and $\overline{q}$ respectively.

(ii) for all $q_{i,o} \in (0, q_{c_1}' )$, there is convergence to $\underline{q}_i$ if $q_{i,o} < q_{c_1}'$ and there are two perfect foresight paths converging to $\underline{q}_i$ and $\overline{q}$ respectively if $q_{i,o} > q_{c_1}'$.

(iii) for all $q_{i,o} \in (q_{d_2}', 1)$, there is convergence to $\overline{q}$ if $q_{i,o} < \overline{q}$ and there are two perfect foresight paths converging to $\underline{q}_i$ and $\overline{q}$ respectively if $q_{i,o} > \overline{q}$. 

21
Proof: See Appendix E.

The long-run situation for the outsider agents is given in the Proposition 5.

**Proposition 5:** If $q_0 < \bar{q}_0 < q$, then

(i) for all $q_{00} \in [q'_c, q'_b]$, there are two perfect foresight paths converging to $q_{00}$ and $\bar{q}$ respectively.

(ii) for all $q_{00} \in (0, q'_{c_2})$, there is convergence to $q_{00}$ if $q_{00} < q'_{c_2}$ and there are two perfect foresight paths converging to $q_{00}$ and $\bar{q}$ respectively if $q_{00} > q'_{c_2}$.

(iii) for all $q_{00} \in (q'_b, 1)$, there is convergence to $\bar{q}$ if $q'_b < \bar{q}$ and there are two perfect foresight paths converging to $q_{00}$ and $\bar{q}$ respectively if $q'_b > \bar{q}$.

Proof: Proof of proposition 5 follows similar argument as in proposition 4.

Let us start with a condition when $q_{10} \in [q'_{c_1}, q'_{d_1}]$ and $q_{00} \in [q'_{c_2}, q'_{b}]$, the path of the distribution of preferences may lead to a convergence to the efficient equilibrium ($\bar{q}$) or the inefficient equilibrium ($q_{00}$ and $q_{10}$ for the insiders and outsiders respectively), depending on the insider and outsider parents’ expectation about future firms’ strategies.

When the initial state is $q_{10} \in (0, q'_{c_1})$ and $q_{00} \in (0, q'_{c_2})$, the insider and outsider parents believe that the today's $\sigma^d$ strategy will be followed by the firm in the future if $q_{10} < q'_{c_1}$ and $q_{00} < q'_{c_2}$, then the economy will get trapped in the inefficient equilibriums, $q_{10}$ (in case of insiders) and $q_{00}$ (in case of outsiders). However, if $q_{10} > q'_{c_1}$ and $q_{00} > q'_{c_2}$, the insider and outsider parents may expect a switch in the firm's strategy for the next generations once when $q_{10} \in [q'_{c_1}, q'_{d_1}]$ and $q_{00} \in [q'_{c_2}, q'_{b}]$. Then there will be two paths converging to the inefficient and efficient equilibria respectively.

On the other hand, when the initial state is $q_{10} \in (q'_{d_1}, 1)$ and $q_{00} \in (q'_{b}, 1)$, the insider and outsider parents will expect a $\sigma^f$ strategy adopted in the future if $q'_{d_1} < \bar{q}$ and $q'_{b} < \bar{q}$ and the economy will converge to the efficient equilibrium. However, if $q'_{d_1} > \bar{q}$ and $q'_{b} > \bar{q}$, the insider and outsider parents may expect a switch in the firm's strategy for the next
generations once when $q_{it} \in [q_{c_t}^{e}, q_{d}^{e}]$ and $q_{ot} \in [q_{c_o}^{e}, q_{d}^{e}]$. Under this situation, there will be two paths converging to the inefficient and efficient equilibria respectively.

### 6.4 Effects of the reservation policy

First note that $\bar{q}_o$ and $\bar{q}$ are independent of the quantum of reserved jobs. Therefore, changes in the degree of reservation policy will affect the ‘working attitude’ preference dynamics of outsiders by affecting the value of $q_o$. If the parameter values indicate that the threshold value for outsiders is sufficiently high, such that $\bar{q} < \bar{q}_o$, then as discussed in Section 6.2, the ‘working attitude’ preference dynamics of outsiders will converge to the inefficient equilibrium, regardless of the quantum of reserved positions in the industry.

However, if the parameter values define a lower threshold value for outsiders, such that $\bar{q}_o < \bar{q}$, then the degree of reservation policy will affect the long-run distribution of preferences of outsiders. Note that a decrease in the level of reserved jobs raises $q_o$. If the decrease in the level of reserved jobs is large enough to turn the case of $q_o < \bar{q}_o$ (see Section 6.2) into the case of $\bar{q}_o < q_o$ (see Section 6.1), then reducing the degree of reservation policy is one way to push the dynamics of outsiders to converge to the efficient equilibrium. This is because a reduction in the level of reserved jobs lowers the disutility of outsiders who make high efforts in industry with job reservation. With a lower level of reserved jobs, work-loving outsider parents will be more willing to transmit their trait to their children when expecting the firms to adopt the $\sigma^d$ strategy, thereby leading to a higher steady state of $q_o$. With a sufficiently high $q_o$, firms will switch to adopt the $\sigma^f$ strategy before $q_{ot}$ reaches $q_o$; therefore, $q_{ot}$ will converge to the efficient equilibrium. However, with firms adopting the $\sigma^f$ strategy in the long run the possibility of the low-caste agents benefitting from the reservation policy is also discarded.

For insiders, the impact of the reservation policy on their long-run distribution of preferences is more complicated since changes in $Q$ will affect both $\bar{q}_i$ and $q_i$. For firms, a lower level of $Q$ increases the profits of hiring insiders in industry $P_2$, which in turn raises

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16 Note that changes in the level of reserved jobs will not affect the ‘working attitude’ preference dynamics of outsiders if $\gamma = 0$. 

23
the threshold value of $\bar{q}_i$ for them to switch to adopt the $\sigma^i$ strategy. For leisure-loving insider parents, a lower level of $Q$ reduces the incentive to transmit their trait to their children, inducing a higher value of $q_i$. If initially, the level of $Q$ is so low that the threshold value is higher than the efficient steady state (that is, $\bar{q} < \bar{q}_i$), then a sufficiently large increase in $Q$ leads to a situation such that the threshold value is lower than the efficient steady state (that is, $\bar{q}_i < \bar{q}$), which may prevent $q_i$ from being trapped in the inefficient equilibrium. However, this increase in the level of reserved jobs cannot guarantee the convergence of $q_i$ to the efficient equilibrium since both $\bar{q}_i$ and $\bar{q}_i$ will decrease with an increase in $Q$.

7. Conclusions

This paper proposes a dynamic model of ‘working attitude’ preference formation and examines the effects of the compensatory-discrimination policy. The cultural attitude towards ‘work efforts’ preference evolves endogenously and leads to a heterogeneous distribution of preferences, since both the work-loving and leisure-loving insider and outsider agents choose positive socialization efforts in order to prevent disappearance of their trait from the population of their own group. We show that the lower the proportion of a given type in the population, the higher is the socialization effort and vice-versa. Besides being theoretically more satisfactory than a model with exogenous ‘working attitude’ preference, this makes it possible to study long-run efficiency outcome of affirmative action.

The present model has found that the economy is characterized by two steady states: an efficient equilibrium and an inefficient equilibrium. The driving force in attaining one of these equilibria is the parental socialization effort which depends on profits for firms, the distribution of preferences in the insider and outsider populations as well as on agents’ expectations about future policies. The economy will converge to the efficient (inefficient) equilibrium with larger (smaller) fractions of work-loving insiders and outsiders if profits in industry with job reservation are low (high) or profits in industry without job reservation are high (low). Changes in the degree of the reservation policy will affect the ‘working attitude’ preference dynamics of insiders and outsiders differently.

This study also indicates that the implementation of the compensatory-discrimination policy will cause a trade-off between benefits gained by the low-caste agents and economic
growth, latter being positively correlated to the fraction of work-loving agents. With the convergence to the inefficient equilibrium, this policy may be an effective step towards empowerment of backward castes, but it may be harmful from the standpoint of the economy as a whole since it changes agents’ working attitudes and causes loss in firms’ profits. With the convergence to the efficient equilibrium, the low-caste agents will not benefit from the implementation of the compensatory-discrimination policy in the long run since workers are all allocated to the industry without job reservations, although they can enjoy the benefit brought by this policy in the short run.
Fig. 1: Convergence to the efficient steady state, $\overline{q}$, when $\overline{q}_o < \overline{q}_I$. 

$q_{k,t+1}$

$q_k$
Fig. 2: Convergence to the inefficient steady states, $q_l$ (for insiders) and $q_o$ (for outsiders), when $\bar{q} < \bar{q}_l$. 
Fig. 3: Convergence either to the *efficient* steady state, $\bar{q}$, or to the *inefficient* steady states, $q_i$ (for insiders) and $q_o$ (for outsiders), when $q_i < q_o < \tilde{q}_i < \tilde{q}_o < \bar{q}$.
References


Appendix A. Proof of Proposition 1

Substituting $\Delta V_k^W(\{\sigma^d_{z-1}\})$ into (11) and (12) we compute $\tau^W_k(q_{kt}, \{\sigma^d_{z-1}\})$ and then differentiation with respect to $Q$ yields

\[
\frac{d\tau^W_k(q_{kt}, \{\sigma^d_{z-1}\})}{dQ} = -\psi \alpha' (Q)(w'_2 - w'_2)(1 - q_{1t}) < 0;
\]

\[
\frac{d\tau^L_k(q_{kt}, \{\sigma^d_{z-1}\})}{dQ} = \psi \alpha' (Q)(w'_2 - w'_2) q_{1t} > 0;
\]

\[
\frac{d\tau^W_0(q_{ot}, \{\sigma^d_{z-1}\})}{dQ} = -\psi \gamma' (Q)(1 - q_{ot}) < 0;
\]

\[
\frac{d\tau^L_0(q_{ot}, \{\sigma^d_{z-1}\})}{dQ} = \psi \gamma' (Q) q_{ot} > 0.
\]

Q.E.D.

Appendix B. Proof of Lemma 1

From (11) and (12), we can derive that

\[
\tau^W_k(q_{kt}, \{\sigma^d_{z-1}\}) > \tau^L_k(q_{kt}, \{\sigma^d_{z-1}\})
\]

if

\[
q_{kt} < \frac{\Delta V_k^W(\{\sigma^d_{z-1}\})}{\Delta V_k^W(\{\sigma^d_{z-1}\}) + \Delta V_k^L(\{\sigma^d_{z-1}\})}.
\]

(B.1)

After computing the R.H.S. of (B.1) for the insider and outsider agents under the expected strategy profiles $\{\sigma^F_{t+1}\}, \{\sigma^M_{t+1}\}$ and $\{\sigma^d_{t+1}\}$ and comparing these values with $\bar{q}_i$, $\underline{q}_o$ and $\bar{q}_o$, we can get the results as stated in Lemma 1.

Q.E.D.

Appendix C. Proof of Proposition 2

Given that $\bar{q}_i < \bar{q}_o < \underline{q}_i < q_{o} < \bar{q}$. If $\bar{q}_i < \underline{q}_i < \bar{q}$, then $\underline{q}_i$ cannot be a rest point of the two-branch dynamics for the insiders (dynamics (A) and (C)), because for all $q_{1t} > \bar{q}_i$, the relevant preference dynamics is (C). Also when $\bar{q}_o < \underline{q}_o < \bar{q}$, $\underline{q}_o$ cannot be a rest point of
the two-branch dynamics for the outsiders (dynamics (B) and (C)), because for all $q_{ot} > \tilde{q}_o$,
dynamics (C) holds. The dynamics (C) has three rest points: $q_k = 0$, $q_k = 1$ and $q_k = \tilde{q}$.

We show first that $q_k = 0$ and $q_k = 1$ are locally unstable. Denote $q_{k,t+1}$ as $F(q_k)$ in
(18). Then differentiation yields
\[
F'(q_k) = \frac{dq_{k,t+1}}{dq_k} = 1 + (1 - 2q_k)\psi \left[\Delta V^w_k(\cdot)(1 - q_k) - \Delta V^l_k(\cdot)q_k\right] - q_k(1 - q_k)\psi \left[\Delta V^w_k(\cdot) + \Delta V^l_k(\cdot)\right]
\]
(C.1)
Since $\psi > 0$, $\Delta V^w_k(\sigma^f) > 0$ and $\Delta V^l_k(\sigma^f) > 0$, we obtain the following results when
evaluating the derivative (C.1) at $q_k = 0$ and $q_k = 1$:
\[
F'(0) = \frac{dq_{k,t+1}}{dq_k} \bigg|_{q_k = 0, q_k = 1} = 1 + \psi \Delta V^w_k(\cdot) > 1, \quad \text{(C.2)}
\]
\[
F'(1) = \frac{dq_{k,t+1}}{dq_k} \bigg|_{q_k = 0, q_k = 1} = 1 + \psi \Delta V^l_k(\cdot) > 1, \quad \text{(C.3)}
\]
Therefore, $q_k = 0$ and $q_k = 1$ are locally unstable.

We further differentiate (C.1) with respect to $q_k$ and obtain:
\[
F''(q_k) = -2\psi \left[\Delta V^w_k(\cdot)(1 - q_k) - \Delta V^l_k(\cdot)q_k\right] - 2\psi(1 - 2q_k)\left[\Delta V^w_k(\cdot) + \Delta V^l_k(\cdot)\right]. \quad \text{(C.4)}
\]
The dynamics $F(q_k)$ has a turning point ($\hat{q}_k$) when $F''(q_k) = 0$, where
\[
\hat{q}_k = \frac{1}{3}\left(1 + \frac{\Delta V^w_k(\hat{\sigma})}{\Delta V^w_k(\hat{\sigma}) + \Delta V^l_k(\hat{\sigma})}\right) = \frac{1}{3}(1 + q_k^*).
\]
(C.5)
If $q_k > \hat{q}_k$, then $F''(q_k) > 0$ implying that $F(q_k)$ is convex when $\hat{q}_k < q_k < 1$. On the other hand, if $q_k < \hat{q}_k$, then $F''(q_k) < 0$ which implies that $F(q_k)$ is concave when
$0 < q_k < \hat{q}_k$. Also note that the dynamics (C) lies above dynamics (B) which lies above
dynamics (A).

We now turn to prove the global stability of $\tilde{q}$. Define $q'_{A} < \tilde{q}_l$, such that $F_A(q'_A) = \tilde{q}_l$
and $q'_{C_l} < \tilde{q}_l$ such that $F_C(q'_{C_l}) = \tilde{q}_l$. Also define $q'_b < \tilde{q}_o$, such that $F_b(q'_b) = \tilde{q}_o$ and
\( q_{c_1}' < \tilde{q}_o \) such that \( F_c(q_{c_1}') = \tilde{q}_o \). For any particular value of the parameters, \( q_A' > q_{c_1}' \) and \( q_b' > q_{c_2}' \) always. The existence and uniqueness of \( q_A', q_{c_1}', q_b' \) and \( q_{c_2}' \) are shown below.

We first show that \( \forall q_{10} \in (0, 1) \) and \( \forall q_{00} \in (0, 1) \) there is a perfect foresight path of distribution of preferences among insiders and outsiders that converge to the steady state \( \tilde{q} \):

(a) Assume \( q_{1t} < q_{c_1}' \) and \( q_{0t} < q_{c_2}' \). If the insider parents expect \( q_{1t} < q_{1, t+1}^E < \tilde{q}_1 \) and the outsider parents expect \( q_{0t} < q_{o, t+1}^E < \tilde{q}_o \), the relevant preference dynamics for the insiders is (A) and that for the outsiders is (B). Then by lemma 1, we have \( \tau_k^w(q_{kt}, \sigma^d) > \tau_k^l(q_{kt}, \sigma^d) \). Therefore, \( q_{k, t+1}^E = q_{k, t+1} > q_{kt} \) and the expectations are self-confirmed.

(b) Assume \( q_A' > q_{1t} \geq q_{c_1}' \) and \( q_b' > q_{0t} \geq q_{c_2}' \). If the insider parents expect \( q_{1t} < \tilde{q}_1 < q_{1, t+1}^E \) and the outsider parents expect \( q_{0t} < \tilde{q}_o < q_{0, t+1}^E \), there will be a switch in the strategy with the firm adopting a \( \sigma' \) strategy for both the insiders and outsiders. Then, by lemma 1, we have \( \tau_k^w(q_{kt}, \sigma^d) > \tau_k^l(q_{kt}, \sigma^d) \). Therefore, \( q_{k, t+1}^E = q_{k, t+1} \geq \tilde{q}_k > q_{kt} \) and the expectations are fulfilled.

(c) Assume \( q_A' < q_{1t} < \tilde{q} \) and \( q_b' < q_{0t} < q \). If the insider and outsider parents expect \( q_{kt} < q_{k, t+1}^E < \tilde{q} \), the relevant preference dynamics for both of them is (C). Then \( \tau_k^w(q_{kt}, \{f\}_{kt}^E) > \tau_k^l(q_{kt}, \{f\}_{kt}^E) \). Therefore, \( q_{k, t+1}^E = q_{k, t+1} > q_{kt} \) and the expectations are fulfilled.

(d) Assume \( q_{kt} > \tilde{q} \). If the insider and outsider parents expect \( q_{kt} > q_{k, t+1}^E > \tilde{q} \), the relevant preference dynamics for them is (C). Then by lemma 1, \( \tau_k^w(q_{kt}, \{f\}_{kt}^E) < \tau_k^l(q_{kt}, \{f\}_{kt}^E) \). Therefore, \( q_{k, t+1}^E = q_{k, t+1} < q_{kt} \) and the expectations are self-confirmed.

Notice that by lemma 1, \( \tau_k^w(q_{kt}, \{f\}_{kt}^E) \geq \tau_k^l(q_{kt}, \{f\}_{kt}^E) \) when \( q_{kt} \leq \tilde{q} \), implying that \( q_{k, t+1} \geq q_{kt} \) when \( q_{kt} \leq \tilde{q} \). Hence there exists a steady state \( \tilde{q} \in (0, 1) \).

Evaluating the derivative (C.1) at the steady state \( q_k = \tilde{q} \), we obtain
\[ F'(\bar{q}) = \left. \frac{dq_{k,t+1}}{dq_{k,t}} \right|_{q_{k,t} = \bar{q}, q_{k,t+1} = \bar{q}} = 1 - \bar{q}(1 - \bar{q})\psi \left[\Delta V^w_k(\bar{q}) + \Delta V^l_k(\bar{q})\right]. \tag{C.6} \]

Note that the second term in (C.1) vanishes as \( \tau^w_k(\bar{q}, \sigma^f_{l+1}) = \tau^l_k(\bar{q}, \sigma^f_{l+1}) \) by lemma 1.

Let \( \bar{q}^w_k(\bar{q}) = \bar{q}^l_k(\bar{q}) = \bar{\tau} \). Then by (11) and (12), we have \( \psi \Delta V^w_k(\bar{q}) = \frac{\bar{\tau}}{1 - \bar{q}} \) and \( \psi \Delta V^l_k(\bar{q}) = \frac{\bar{\tau}}{\bar{q}} \). Substituting these values into (C.6), we obtain \( F'(\bar{q}) = 1 - \bar{\tau} \). As \( \bar{\tau} \in (0, 1) \), then \( F'(\bar{q}) \in (0, 1) \). Given that the function \( F(q_{i,0}) \) is a polynomial of third degree and that \( F'(0) > 1 \), \( F'(1) > 1 \) and \( F'(\bar{q}) \in (0, 1) \), there are two possibilities to get global stability of \( \bar{q} \). \(^{17} \) A sufficient condition for global stability of \( \bar{q} \) is that \( F_{c}(q_{i,0}) > 0 \) and \( F_{l}(q_{i,0}) > 0 \) for all \( q_{i,0} \in (0, 1) \); and \( F_{c}(q_{o,0}) > 0 \) and \( F_{l}(q_{o,0}) > 0 \) for all \( q_{o,0} \in (0, 1) \). The above sufficient conditions in case of insiders and outsiders hold when \( C(\tau) \) is convex enough, in particular,

\[ C^*(\tau) = 1/\psi \geq \left( w_1 - w_i \right) \left[ 1 - \frac{w_1}{\mu} \right] \] for both the insiders and outsiders.

Finally we turn back to the existence and uniqueness of \( q'_{i} \), \( q'_{c} \), \( q'_{b} \) and \( q'_{c} \). Notice that \( F_{d}(q_{i,0}) > 0 \) for all \( q_{i,0} \in (0, 1) \), \( F_{d}(q_{o,0}) > 0 \), and \( F_{d}(0) = 0 \), implying that there exists a unique \( q'_{i} \in (0, q_{i,0}) \), such that \( F_{d}(q'_{i}) = q_{i} \). A similar argument applies for \( q'_{c} \). Further, notice that \( F_{b}(q_{o,0}) > 0 \) for all \( q_{o,0} \in (0, 1) \), \( F_{b}(q_{o,0}) > q_{o} \), and \( F_{b}(0) = 0 \), implying that there exists a unique \( q'_{b} \in (0, q_{o,0}) \), such that \( F_{b}(q'_{b}) = q_{o} \). A similar argument applies for \( q'_{c} \).

Q.E.D.

**Appendix D. Proof of Proposition 3**

Given that \( q_{i} < q_{o} < \bar{q} < \bar{q}_{i} < \bar{q}_{o} \). If \( q_{i} < \bar{q} < q_{i} \), then \( \bar{q} \) cannot be a rest point of the two-branch dynamics for the insiders (dynamics (A) and (C)), because for all \( q_{i} < \bar{q}_{i} \), the relevant preference dynamics is (A), which has three rest points: \( q_{i} = 0 \), \( q_{i} = 1 \) and \( q_{i} = q_{i} \). Further, if \( q_{o} < q < q_{o} \), then \( \bar{q} \) cannot be a rest point of the two-branch dynamics

\(^{17} \) See Olcina and Penarrubia (2004) and Escrèche et al. (2004).
for the outsiders (dynamics (B) and (C)), because for all \( q_{O_t} < \tilde{q}_O \), the relevant preference dynamics is (C), which has three rest points: \( q_O = 0 \), \( q_O = 1 \) and \( q_O = \tilde{q}_O \). Following the same arguments as in the proof of proposition 2, we can show that \( \dot{q}_k = 0 \) and \( q_k = 1 \) are locally unstable, because \( \psi > 0 \) and \( \Delta V^w_k(\sigma_z) > 0 \) and \( \Delta V^L_k(\sigma_z) > 0 \) always, \( \sigma_z \in \{\sigma^f, \sigma^d\} \).

We now turn to prove the global stability of \( q_j \) and \( q_O \). Define \( q_j' > \tilde{q}_j \), such that \( F_A(q_j') = \tilde{q}_j \) and \( q_j' > \tilde{q}_j \); such that \( F_C(q_j') = \tilde{q}_j \). Also define \( q_O' > \tilde{q}_O \) and \( q_O' > \tilde{q}_O \), such that \( F_B(q_O') = \tilde{q}_O \) and \( F_C(q_O') = \tilde{q}_O \). In general, for any particular value of the parameters, \( q_j' > q_j \), and \( q_O' > q_j \). The existence and uniqueness of \( q_j' \), \( q_j' \), \( q_O' \) and \( q_O' \) are shown below.

We first show that \( \forall q_{I0} \in (0, 1) \) there is a perfect foresight path of preferences among insiders that converges to the steady state \( q_j \) and \( \forall q_{O0} \in (0, 1) \) there is a perfect foresight path of preferences among outsiders that converges to the steady state \( q_O \).

(a) Assume \( q_{I_t} > q_A' \) and \( q_{O_t} > q_B' \). If the insider parents expect \( q_{I_t} > q_{I,t+1}^E > \tilde{q}_I \) and the outsider parents expect \( q_{O_t} > q^E_{O,t+1} > \tilde{q}_O \), dynamics (C) holds. Then, by lemma 1, we have \( \tau^w_k(q_{I_t}, \sigma^f) < \tau^L_k(q_{I_t}, \sigma^f) \). Therefore, \( q_{k,t} > q^E_{k,t+1} = q_{k,t+1} > \tilde{q}_k \), and the expectations are fulfilled.

(b) Assume \( q_{C_1} < q_{I_t} \leq q_A' \) and \( q_{C_2} < q_{O_t} \leq q_B' \). If the insider parents expect \( q_{I,t+1}^E < \tilde{q}_I < q_{I_t} \) and the outsider parents expect \( q_{O,t+1}^E < \tilde{q}_O < q_{O_t} \), there will be a switch in the strategy with the firm adopting a \( \sigma^d \) strategy for both the insiders and outsiders. By lemma 1, we have \( \tau^w_k(q_{k,t}, \sigma^d) < \tau^L_k(q_{k,t}, \sigma^d) \). Therefore, \( q^E_{k,t+1} = q_{k,t+1} < q_{k,t} \), and the expectations are self-confirmed.

(c) Assume \( q_{C_1} > q_{I_t} > q_j \) and \( q_{C_2} > q_{O_t} > q_O \). If the insider and outsider parents expect \( q_{I_t} > q_{I,t+1}^E > q_j \) and \( q_{O_t} > q_{O,t+1}^E > q_O \), the relevant preference dynamics for the insiders is (A) and that for the outsiders is (B). Then \( \tau^w_k(q_{k,t}, \sigma^d) < \tau^L_k(q_{k,t}, \sigma^d) \). Therefore, \( q^E_{k,t+1} = q_{k,t+1} < q_{k,t} \), and the expectations are self-confirmed.
(d) Assume \( q_{i_t} < q_j \) and \( q_{o_t} < q_o \). If the insider and outsider parents expect \( q_{i_t} < q_{i,t+1}^E < q_j \) and \( q_{o_t} < q_{o,t+1}^E < q_o \), the relevant preference dynamics for the insiders is (A) and for the outsiders is (B). Then we have \( \tau^w_k(q_{i,t}, \sigma^d) \geq \tau^l_k(q_{i,t}, \sigma^d) \).

Therefore, \( q_{k,t+1}^E = q_{k,t+1} \geq q_{i,t} \) and the expectations are self-confirmed.

Following the same argument as in proposition 2, we can show that \( F'(q_j) \in (0, 1) \) and \( F'(q_o) \in (0, 1) \). Given that the function \( F(q_j) \) is a polynomial of third degree and that \( F''(0) > 1, \ F''(1) > 1 \) and \( F'(q_j) \in (0, 1) \), \( F'(q_o) \in (0, 1) \) there exist global stability. A sufficient condition for global stability of \( q_j \) is that \( F'_c(q_{i,t}) > 0 \) and \( F'_c(q_{o,t}) > 0 \) for all \( q_{i,t} \in (0, 1) \); and that of \( q_o \) is that \( F'_c(q_{o,t}) > 0 \) and \( F'_b(q_{o,t}) > 0 \) for all \( q_{o,t} \in (0, 1) \). The above sufficient conditions in case of insiders and outsiders hold when \( C(\tau) \) is convex enough, in particular, \( C^*(\tau) = 1/\psi \geq (1 - \alpha(Q)) \left( \frac{w_2 - w_2}{w_2 - w_2} \right) \left( 1 - \frac{(1 - \alpha(Q))(w_2 - w_2)}{\mu} \right) \) for the insiders and \( C^*(\tau) = 1/\psi \geq \left( \frac{w_2 - w_2 - \gamma(Q)}{w_2 - w_2 - \gamma(Q)} \right) \left( 1 - \frac{w_2 - w_2 - \gamma(Q)}{\mu} \right) \) for the outsiders.

Also notice that \( F'_c(q_{i,t}) > 0 \) for all \( q_{i,t} \in (0, 1) \), \( F'_c(q_{i,t}) < q_{i,t} \) and \( F'_c(1) = 1 \), implying that there exists a unique \( q_{i,t}^* \in (q_{i,t}, 1) \), such that \( F'_c(q_{i,t}^*) = q_{i,t} \). A similar argument applies for \( q_{o,t}^* \). Further, notice that \( F'_b(q_{o,t}) > 0 \) for all \( q_{o,t} \in (0, 1) \), \( F'_b(q_{o,t}) < q_{o,t} \) and \( F'_b(1) = 1 \), implying that there exists a unique \( q_{o,t}^* \in (q_{o,t}, 1) \), such that \( F'_b(q_{o,t}^*) = q_{o,t} \). A similar argument applies for \( q_{o,t}^* \).

Q.E.D.

**Appendix E. Proof of Proposition 4**

To prove part (i) of the proposition we first construct the two perfect foresight paths of preferences which exist for any \( q_{i_0} \in [q_{C_1}^*, q_{A_1}^*] \). For any \( q_{i_1} \in [q_{C_1}^*, q_{A_1}^*] \) and the insider parents expecting \( \sigma^d \), lemma 1 indicates that \( \tau^w_k(q_{i,t}, \sigma^d) \geq \tau^l_k(q_{i,t}, \sigma^d) \) for \( q_{i,t} \leq q_j \). Thus, \( q_{i,t+1}^E = q_{i,t+1} \geq q_{i,t} \) if \( q_{i,t} \leq q_j \). The perfect foresight path will converge to \( q_j \). On the
other hand, if the insider parents expecting $\sigma^f$, lemma 1 indicates that $\tau_1^w(q_{it}, \sigma^f) \geq \tau_1^t(q_{it}, \sigma^f)$ for $q_{it} \leq \bar{q}$. Thus, $q_{i,t+1}^E = q_{i,t+1} \geq q_{it}$ if $q_{it} \leq \bar{q}$. The perfect foresight path will converge to $\bar{q}$.

We now prove part (ii) of the proposition. First, if $q_i < q_i^C$, then for all $q_{i0} \in (0, q_i^C)$, the insider parents expect $\sigma^d$. Lemma 1 indicates that $\tau_1^w(q_{it}, \sigma^d) \geq \tau_1^t(q_{it}, \sigma^d)$ for $q_{it} \leq q_i$. Thus, $q_{i,t+1}^E = q_{i,t+1} \geq q_{it}$ if $q_{it} \leq q_i$. The perfect foresight path will converge to $q_i$. Second, if $q_i > q_i^C$, there are two perfect foresight paths. In the first case, the insider parents expect $\sigma^d$ and the path will converge to $q_i$. In the second case, initially the insider parents expect $\sigma^d$. But once when the dynamics reaches $q_{it} \in [q_i^C, q_i')$, the insider parents expect a switch of the strategy to $\sigma^f$ and the path will converge to $\bar{q}$.

A similar argument proves part (iii) of the proposition.

Q.E.D.