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17 March 2013

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Abstract

Since the seminal work by Engle (1982), the autoregressive conditional heteroscedasticity (ARCH) model has been an important tool for estimating the time-varying volatility as a measure of risk. Numerous extensions of this model have been put forward in the literature. The current paper offers an alternative approach for dealing with asymmetry in the underlying volatility model. Unlike previous papers that have dealt with asymmetry, this paper suggests to explicitly separate the positive shocks from the negative ones in the ARCH modeling approach. A test statistic is suggested for testing the null hypothesis of no asymmetric ARCH effects. In case the null hypothesis is rejected, the model can be estimated by using the maximum likelihood method. The suggested asymmetric volatility approach is applied to modeling separately the potential time-varying volatility in markets that are rising or falling by using the changes in the world market stock price index.

Keywords: GARCH; Asymmetry; Modelling volatility; Hypothesis testing, World stock price index.

Running title: A New Asymmetric GARCH Model
JEL classification: C32, C12, G10.

1. Introduction

It has been known for decades in the literature that the distribution of returns in financial markets has fat tails compared to the normal distribution (see Mandelbrot, 1963). The volatility of many financial variables seems to be characterized by a time-varying structure. Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) model, which is being extensively used for modeling the underlying time-varying volatility. Numerous generalizations of this model have been put forward by, among others, Bollerslev (1986), Nelson (1991), Bollerslev, Chou and Kroner (1992), Engle and Kroner (1995) and Francq and Zakoian (2012).[1] Given the increasingly globalized character of financial markets and the recent turmoil in these markets, attempts aimed at knowing more about the intrinsic properties of the underlying volatility, as a measure of risk that influences significantly the behavior of economic agents, is a task worth undertaking. One issue that seems to be particularly pertinent within this context is the potential asymmetry that can characterize the time-varying volatility. This issue has been raised by, among others, Nelson (1991) who has suggested the exponential generalized ARCH model that allows for asymmetry by including indicator variables in the time-varying volatility process. Another approach is the asymmetric power ARCH model that is suggested by Ding et. al. (1993). One additional approach to deal with asymmetry is the threshold ARCH model based on the work by Petruccelli and Woolford (1984). Zakoian (1994) has shown how thresholds can be used for allowing some form of asymmetry in an ARCH model.

This potential asymmetric property of a volatility measure is an important issue in practice because of the fact that the behavior of most financial time series is usually characterized by an asymmetric structure. There are several logical reasons in addition to the psychological ones behind this prevailing phenomenon. By nature, people tend to react more to the negative news than to the positive ones. This is especially the case in the financial markets. As an example, 5% profit has different consequences than 5% loss for a business activity. Reacting to the 5% profit is easy in terms of expending and hiring more employees. However, the

[1] For a collection of important papers on ARCH models see Engle (1995). A review of literature on ARCH modeling is provided by Engle (2002). For a review of the software for the estimation of ARCH models the interested reader is referred to Brooks (1997). In addition, Poon and Granger (2003) provide a review on forecasting volatility in different financial markets. For an application of the asymmetric volatility model based on the existing methods see also Apergis and Miller (2005). Furthermore, Francq and Zakoian (2010) have published recently an interesting book on GARCH modeling with applications in finance.

reaction to the 5% loss is not as simple. It is not as easy to fire employees as hiring them because of the legal restrictions as well as the moral considerations. In addition, the company might be reluctant to lay off the potential employees that might not be easily replaced when things get better. In fact, there are also natural limitations that can underlie the asymmetric behavior in finance. For example, there is no upper limit on the increase of a stock price. Theoretically speaking, the stock price can increase beyond any limit but there is, however, a lower limit on the stock price decrease. After all, the minimum value that a stock price can undertake is zero since negative stock prices do not exist. As a consequence, a given magnitude of the price increase must have different repercussions, in the absolute terms, than the same amount of price decrease. There are also additional theoretical reasons for possible asymmetric behavior. For instance, it is well-known since the late sixties that markets with asymmetric information prevail. If markets with asymmetric information exist then the asymmetric volatility can also exist. For all these reasons, this paper argues that it is important to allow for an asymmetric structure in the ARCH modeling. Furthermore, allowing for asymmetry might be extra useful information for investors for dealing with the underlying risk. It is especially during the periods in which the markets are under stress that the investors require precise calculation of the underlying risk.

The main objective of the current paper is to provide an alternative approach that, unlike the existing methods in the literature, explicitly deals with the potential asymmetric property in a ARCH model by fitting separate models to the positive and negative changes of the underlying variable. That is, it is shown how the variable of interest can be decomposed into the positive and negative components. Then tests for ARCH effects in each component can be implemented. If ARCH effects prevail in any component, then a separate ARCH model can be fitted for that particular component. The procedure that is developed in this paper can deal with both stationary and integrated variables of the first degree. To the best knowledge, this approach is the first of its kind. An application to the returns of the world stock market index is also provided.

The organization of the rest of the paper is the following. Section 2 presents the new asymmetric ARCH model for a stationary process. It also outlines how this asymmetric model can be tested and estimated. Section 3 provides an application to the modeling of the potential asymmetric time-varying volatility of the world stock market index. The last section concludes the paper. An appendix is also provided at the end of the paper to show how an integrated variable with one unit root and with potential deterministic parts can be transformed into positive and negative cumulative components.

2. The Asymmetric ARCH Model

As mentioned previously, whether or not the volatility of any quantity measured across time is time-varying has crucial repercussions in practice. This is especially the case in the financial markets. The Engle's ARCH model or some form of its modification is commonly utilized for this purpose. An important issue to take into account within this context is the issue of potential asymmetric structure that can prevail for several logical reasons as outlined previously. This issue has been dealt with in the existing literature to some extent. However, in the exiting literature, to the best knowledge, the impact of positive changes is not totally separated from the impact of negative changes. In the current paper we suggest achieving this by transforming the underlying variable into positive and negative components and fit a generalized ARCH (i.e. GARCH) model to each of the components separately. Consider the time series P_t that is assumed to be a stationary process.[2] The positive and negative decompositions of this variable can be obtained by the following expressions:

$$X_t^+ = \max(\Delta P_t, 0) \tag{1}$$

and

$$X_t^- = \min(\Delta P_t, 0) \tag{2}$$

for $t = 1, 2, ..., T$. The symbol Δ represents the first difference operator. Note that, we have $\Delta P_t = X_t^+ + X_t^-$, per definition. The test for ARCH effects in the positive component can be conducted by running the following regression model:

$$\varepsilon_t^{2+} = b_0^+ + \sum_{i=1}^{p+} b_i^+ \varepsilon_{t-i}^{2+} + u_t^+ \tag{3}$$

Where u_t^+ is a white noise process and ε_t^+ is the error term from the following regression:

[2] It is also possible to conduct a similar analysis for the integrated variables. However, the solution will be different in such cases. See the Appendix, at the end of the paper, for decomposing an integrated variable into its positive and negative cumulative components.

$$X_t^{+} = a^{+} + \sum_{i=1}^{k+} \gamma_i^{+} X_{t-i}^{+} + \varepsilon_t^{+}$$

The error term ε_t^{+} is assumed to be distributed as $\varepsilon_t^{+} \sim \mathrm{N}\left(0, \sigma_t^{2+}\right)$. The null hypothesis of no ARCH effects of order p^{+} for the positive component is defined as

$H_0 : b_1^{+} = b_2^{+} = \cdots = b_p^{+} = 0$.

This hypothesis can be tested by estimating the following test statistic, which is based on the Lagrange Multiplier (*LM*) procedure:

$LM^{+} = T \times R^{2+}$. Where R^{2+} is the coefficient of determination in the unrestricted model (3). This test statistic is asymptotically chi-square distributed with p^{+} degrees of freedom. The null hypothesis is rejected at a given significance level if the p-value of the test statistic is less than the underlying significance level.

Similarly, the test for ARCH effects in the negative component can be tested as the following. First, run the following regression model:

$$\varepsilon_t^{2-} = b_0^{-} + \sum_{i=1}^{p-} b_i^{-} \varepsilon_{t-i}^{2-} + u_t^{-} \tag{4}$$

Where u_t^{-} is a white noise process and ε_t^{-} is the error term from the regression below

$$X_t^{-} = a^{-} + \sum_{i=1}^{k-} \gamma_i^{-} X_{t-i}^{-} + \varepsilon_t^{-}$$

The error term ε_t^{-} is assumed to be distributed as $\varepsilon_t^{-} \sim \mathrm{N}\left(0, \sigma_t^{2-}\right)$. The null hypothesis of no ARCH(p^{-}) in the negative component is defined as

$H_0 : b_1^{-} = b_2^{-} = \cdots = b_p^{-} = 0$.

This hypothesis can be tested by estimating the following test statistic:

$LM^{-} = T \times R^{2-}$. Where R^{2-} is the coefficient of determination in the unrestricted model (4). This test statistic has also an asymptotic chi-square distribution with p^{-} degrees of freedom. This statistic is similar to the one that Engle (1982) introduced originally. Since ε_t^{+} and ε_t^{-} both satisfy the same conditions as ε_t, then the suggested test statistics LM^{+} and

LM^- must satisfy the same asymptotic properties as the original *LM* statistic, based on the results of the proofs provided by Engle (1982). The same is true regarding the maximum likelihood approach for estimating the underlying parameters for the positive and negative components respectively.

If the null hypothesis is rejected, then the following two GARCH(p, q) models can be estimated.[3] For the positive component, the following model can be estimated jointly by using the maximum likelihood method:

$$\begin{aligned} X_t^+ &= a^+ + \sum_{i=1}^{k+} \gamma_i^+ X_{t-i}^+ + \varepsilon_t^+ \\ \sigma_t^{2+} &= c^+ + \sum_{j=1}^{p+} \beta_j^+ \varepsilon_{t-1}^{2+} + \sum_{j=1}^{q+} \lambda_j^+ \sigma_{t-j}^{2+} \end{aligned} \tag{5}$$

Where σ_t^{2+} is the conditional variance of ε_t^+. The next model can be estimated jointly for the negative component case:

$$\begin{aligned} X_t^- &= a^- + \sum_{i=1}^{k-} \gamma_i^- X_{t-i}^- + \varepsilon_t^- \\ \sigma_t^{2-} &= c^+ + \sum_{j=1}^{p-} \beta_j^- \varepsilon_{t-1}^{2-} + \sum_{j=1}^{q-} \lambda_j^- \sigma_{t-j}^{2-} \end{aligned}, \tag{6}$$

Where σ_t^{2-} is the conditional variance of ε_t^-. The lag orders in each case can be determined by minimizing an information criterion.

[3] The initial values are assumed to exist. It is also possible to consider the GARCH in mean or the integrated GARCH processes for positive and negative components.

3. An Application

The suggested method is used to test whether or not the time-varying volatility of the returns of the world market stock price index is asymmetric. The returns, which are based on the Morgan Stanley Capital International (MSCI) world stock price index in the US dollar, are used during the period 04-01-2005 to 09-03-2010 on weakly basis. The index covers 1600 stocks worldwide.

The estimation procedure is as follows. First, we decomposed the variable into the positive and negative components by using equations (1) and (2). These decompositions were conducted via an algorithm written in Gauss, which is available on request. Next, we fitted an autoregressive model to each component and tested the null hypothesis of no ARCH(1) for the particular component. The null hypothesis could not be rejected for the positive component but it could be strongly rejected for the negative component. Based on this finding, we fitted an GARCH(1, 1) model for the negative component. We also tested the null hypothesis that the sum of the estimated parameters in the volatility model is equal to one. This null could not be rejected. Therefore, an integrated GARCH(1, 1) was estimated for the negative component. The results of the estimations are presented in Table 1. As these results reveal, the time-varying volatility is indeed very persistent for the negative changes of the world market index. Each estimated parameter is also statistically significant strongly.

Table 1: The estimation results for the integrated GARCH model for negative components.

Parameter	Estimated value	P-value
γ_1^-	0.165413	0.0002
β_1^-	0.173814	<0.0001
λ^-	0.826186	<0.0001

Note: An integrated GARCH was estimated because the null hypothesis that the sum of beta and lambda being equal to one could not be rejected at the conventional significance levels.

4. Conclusions

Conditional autoregressive heteroskedasticity models are useful tools for measuring time-varying risk, especially in the financial markets. An important issue within this context is to account for the potential asymmetry that can prevail because people tend to react more to negative shocks than to the positive ones. The main objective of this paper is to provide an alternative approach that explicitly deals with the asymmetric property in the GARCH model. It is shown how the underlying variable can be decomposed into positive and negative components. These components provide the possibility to fit a specific GARCH model to the positive and negative changes of the underlying variable respectively. This is an important issue because the behavior of most time series variables can be characterized by an asymmetric structure for several logical reasons. The suggested method is applied to modeling the asymmetric time-varying volatility of the returns of the world market stock price index. The results show that the null hypothesis of no ARCH cannot be rejected for the positive component. However, the null hypothesis of no ARCH for the negative component is strongly rejected. A GARCH model is fitted for the negative component and the underlying parameters are estimated by the maximum likelihood method. The results show that the time-varying volatility for the negative changes in the world stock market is very persistent. This conclusion is based on the fact that the null hypothesis that the sum of the volatility parameters is equal to one could not be rejected. Thus, the time-varying volatility for the negative changes of the world stock price index is an integrated process. This means that any shock to the underlying time-varying volatility for the negative changes of the stock price will have a permanent impact. However, the volatility for the positive changes seems to be constant.

Future applications of the suggested method will reveal whether or not individual stock markets or other financial assets possess similar asymmetric volatility. Since volatility as a measure of risk is an important input in many financial models, such as portfolio analysis, hedging, option evaluation and the value at risk calculations, it will be useful information for the investors to find out whether or not the underlying volatility is asymmetric. Our conjecture is that the precision of these underlying estimations can improve if the potential asymmetric structure of the time-varying volatility is dealt with in the way it is suggested in this paper.

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Appendix

In this appendix we show how an integrated variable of the first degree, i.e. an $I(1)$ process, can be decomposed into positive and negative components. It should be mentioned that this $I(1)$ variable can also contain deterministic trend parts. Assume that the variable of interest, P_t, is generated by the following process:

$$P_t = a + bt + P_{t-1} + \varepsilon_t, \tag{A1}$$

here a and b are constants, and t is the time trend. The error term ε_t is assumed to be a white noise process. Via the recursive approach, the solution to this process is the following:

$$P_t = at + \frac{t(t+1)}{2}b + P_0 + \sum_{i=1}^{t} \varepsilon_i, \tag{A2}$$

for $t = 1, 2, \ldots T$. The constant P_0 is the initial value. Positive and negative changes are defined as $\varepsilon_i^+ := \max(\varepsilon_i, 0)$, and $\varepsilon_i^- := \min(\varepsilon_i, 0)$. This results in the following equation:

$$P_t = a + bt + P_{t-1} + \varepsilon_t = at + \frac{t(t+1)}{2}b + P_0 + \sum_{i=1}^{t} \varepsilon_i^+ + \sum_{i=1}^{t} \varepsilon_i^-, \tag{A3}$$

Consequently, the positive and negative changes are defined in the following cumulative format:

$$P_t^+ := \frac{at + \left[\frac{t(t+1)}{2}\right]b + P_0}{2} + \sum_{i=1}^{t} \varepsilon_i^+, \tag{A4}$$

and

$$P_t^- := \frac{at + \left[\frac{t(t+1)}{2}\right]b + P_0}{2} + \sum_{i=1}^{t} \varepsilon_i^-. \tag{A5}$$

Note that these decompositions ensure that $P_t = P_t^+ + P_t^-$ holds. The definitions in equations (A4) and (A5) can be used to test whether or not there are ARCH effects in the cumulative positive and negative components of the underlying integrated variable. For a proof of these results see Hatemi-J and El-Khatib (2013). A code written in Gauss to implement these decompositions is available from the author on request.