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Abstract

This paper studies the optimal design of delegation rule in a three-tier principal-intermediary-agent hierarchy. In this hierarchy, monetary transfer is not feasible, delegation is made sequentially, and all players are strategic. We characterize the optimal delegation mechanism. It is shown that the single-interval delegation *a la* Holmstrom is optimal only when the intermediary is moderately biased. Otherwise, as responses to the distortion caused by a biased intermediary, the optimal delegation set may involve a hole. Thus, multi-interval delegation set would arise when subordinates have opposing biases. This result sheds some light on policy threshold effects: "slight" changes in the underlying state cause a jump in the policy responses.

Key words: Delegation, Intermediary, Hierarchies

JEL classification codes: D73, D78, D86

1 Introduction

This paper investigates the optimal design of delegation rule in a three-tier principal-intermediary-agent hierarchy. In this environment, all players are strategic, the principal cannot directly contract with the informed agent, and contingent monetary transfers are not feasible. For instance, the current policy-maker need to affect the behavior of future executives by restricting the choice set of the future policy-maker, whose interest might differ due to political turnover or shocks on preferences, etc.; delegation can be made only among certain parties within a multi-tier government; corporate headquarters need to command product-line managers via division managers. Following Tirole (1986) and McAfee and McMillan (1995), we will take the multi-tier hierarchy as granted, and highlight the implications of hierarchies on the optimal design of delegation rule.

It is well known that when a principal could delegate decision-making authority directly to an agent, the optimal delegation set is a single interval of decisions (Holmstrom 1977; Melumad and Shibano 1991; Alonso and Matouschek 2008, hereafter, AM). Examples include budgeting on a manager who is biased toward invest too much on a project, price caps on a monopolist, etc. In this paper, we provide a characterization of the optimal delegation mechanism in multi-tier hierarchies. It is shown that ceiling strategy is optimal only when the intermediary is moderately biased. The optimal set of permissible decisions may be a finite union of intervals. In other words, some modest policy choices are deliberately discarded by the principal, while extreme decisions are reserved. The "hole" in delegation set arises as a principal's control device to limit the possible distortion in downstream delegation from an intermediary, whose preference opposes to the agent's, and gives rise to policy threshold effects : "slight" changes in the underlying state cause a jump in the policy responses.

In this hierarchy, only the agent could observe the state of the world and undertake the decision. There are preference misalignment among all players due to different intrinsic preferences, compensation package, or personal career concerns, etc.. Delegation is modeled as a sequential game. The principal knows the biases of all subordinates, and offers a set of permissible decisions to an uninformed intermediary. The latter then exerts control by specifying a set of options from which an agent may choose.[1] In other words,

[1] If we allow the intermediary to choose ordering the informed agent after hearing from him, i.e., cheap talk, then, as Dessein (2002) shows, under the standard specification, delegation always dominates informative communication. Thus, cheap talk between the intermediary and the agent cannot be a part of the Subgame Perfect Nash Equilibrium outcome of this sequential delegation game. If the agent communicate with the interme-

an intermediary could further shrink, but is unable to expand, the agent's choice set. The optimal delegation mechanism is an equilibrium outcome of this game. To make the model tractable, we adopt the standard quadratic utility specification as in Crawford and Sobel (1982, hereafter, CS).[2]

There are at least two different stories justifying the relevance of our sequential delegation game. First, a principal has limited commitment power in that she can only promise to delegate to the immediate subordinate, and put restrictions on the choice set. Thus, an intermediary plays a role in delegation. This is related to the study of cheap talk game with partial commitment power in Alonso and Matousheck (2007) and Kolotilin et al (2012). Alternatively, if it is costly for a principal to verify an informed agent's decisions *ex post* due to time constraint or limited attentions, she may grant the agent with the discretion within a specified menu of actions, and hire an intermediary who specializes in monitoring the agent. This intermediary could overrule his decision within this choice set, e.g., take more strict stance in supervision; but is unable to approve any actions beyond his choice set, e.g., it may be easy to observe over-spending. The principal could be the legislature or a company board, the agent could be investment banks, or a CEO, and the intermediary may be the Securities and Exchange Commission, or an external audit. As long as both the principal and the intermediary could make some commitment, the spirit of this story is consistent with our delegation game. In this story, an agent essentially is delegated by two asymmetric principals who move sequentially. This interpretation is also related to the work of Goltsman and Pavlov (2011) that studies the cheap talk game with multiple receivers.

Whenever the subordinates have *like biases*, and the intermediary is moderately biased, i.e., the intermediary is less restrictive than the principal, then it is optimal for the principal to use ceiling strategy. The existence of an intermediary will not add any more distortion.[3]

However, when the subordinates have *opposing biases*, i.e., the intermediary is more restrictive than the principal, he wants to impose extra restrictions on the agent's choice set. The principal and intermediary's disagreement on delegation derives from their different ideal caps on the agent's choice

diary, but the latter could commit to rubber-stamp the agent's recommendations within a specified set, then it is equivalent to that the intermediary delegates a set of decisions to the agent, and the latter undertakes his preferred options within it.

[2] Quadratic utilities are convenient for obtaining first-order conditions, but the intuition for results does not depend on this. For more general payoff functions and priors, the optimal delegation mechanism may still contain a hole.

[3] By Goltsman et al. (2009), interval delegation can attain the best outcome of an universal mechanism.

set. Thus, attempting to use single-interval delegation alone will result in the flat (or unresponsive in terms of AM) agent's response for the realizations of the extreme state. It is optimal to exclude some moderate actions from the intermediary's choice set, in order to make the agent's decision-making partially responsive to the extreme state. By doing this, the principal can limit the additional distortion from the biased intermediary. The essence is similar to Melumad and Shibano (1991) and AM, though here the unresponsiveness arises endogenously from the *derived conflict of interest*. This oppositely biased intermediary not only shrinks the agent's choice set, but also reduces the responsiveness for the realizations of the moderate and high state. The restricted optimal delegation set involves an interval of delegated decisions and a discrete option. This sheds light on policy overreaction, in the sense that we observe big differences in policy responses with respect to "small" changes in the underlying state. For instance, small changes in the extent of law violation would get quite different penalties.

There is a large body of literature on delegation. Dessein (2002) and Ambrus et al. (2011) investigate delegation via an uninformed intermediary, and suggest that under some conditions indirect delegation dominates direct principal-agent delegation. However, they don't allow the intermediary to make delegation decision, and by assuming noncontractible actions, Dessein (2002) treats delegation as an "All-or-Nothing" choice, e.g., the principal cannot restrict the subordinate's choice set. Holmstrom (1977) uses mechanism-design approach to study delegation under exogenous information structure, and establishes the optimality of interval delegation. Goltsman et al. (2009) further demonstrate that, by Holmstrom's interval delegation, a principal can implement the optimal universal mechanism (Myerson, 1982). Melumad and Shibano (1991), Martimort and Semenov (2006), AM, Mylovanov (2008), and Kovac and Mylovanov (2009) also characterize the conditions for the optimality of interval delegation.

Tirole (1986) investigates a simple three-tier principal-supervisor-agent hierarchy. In his work, a supervisor holds private information about the type of agents, and the focus is the collusion between subordinates. He establishes the equivalence between coalition-proof contract and giving ownership to a supervisor, who subcontracts with a downstream agent. Prendergast (2002) uses this framework to study customer complaint management mechanism. Sequential delegation is a kind of subcontract, but differing from these works, monetary transfer is not allowed in this paper. Therefore, the equivalence fails and subcontract can implement the optimal delegation outcome only under certain conditions.

Some works have established the possibility of jump-discontinuity delegation rule. Melumad and Shibano (1991) show that when the principal and the agent have disparate sensitivities on preferred actions, it will be optimal to have an one-jump discontinuity decision rule. AM use a complicate numerical example to illustrate that the principal may remove some intermediate options to encourage the agent to be more responsive, i.e., the slope of the agent's bias matters. In a two-dimension setting, Armstrong and Vickers (2010) show that the optimal permission set may forbid some desirable projects. On the other hand, Szalay (2005) demonstrates that to motivate the agent to exert more efforts in information acquisition, it can be optimal to preclude some compromised decisions. We extend these results to three-player sequential delegation game, where the information structure is exogenous, and the relative decision-making responsiveness to the state is endogenized. In this work, a deliberately designed hole is the necessary incentive cost, and acts as a device to control (imperfectly) an overly-restrictive intermediary (supervisor), instead of a biased informed agent.

This paper is organized as follows. In the following section we lay out the basic model, and investigate the benchmark case of direct delegation, which serves as the efficiency criterion. Section 3 highlights the sequential delegation, and characterizes the optimal delegation set, and demonstrates that the holes in delegation set arise under opposing biases. Some extensions are also discussed. Section 4 concludes.

2 Model

A hierarchy is composed of three players: a principal (she, denote as player P), an intermediary, and an agent (he, denote as M and A, respectively). The utility of each player is of "quadratic loss function" in line with the classical CS specification:[4]

$$U(\theta, y, b_i) = -(\theta - y + b_i)^2, i = P, M, A \tag{1}$$

Thus, their payoffs depend on the true state $\theta \in \Theta \equiv [0, 1]$, the action undertaken $y \in Y \equiv [0, 1]$, and their biases b_i. Each player wants to respond to the true state, but they have different ideal responses, e.g., i's ideal action is $\theta + b_i$. Without loss of generality, we normalize $b_P = 0$ and use b_M, b_A to measure how nearly the subordinates' interest coincide with that of the principal. In most analysis we assume $b_A \geq 0$ and $b_M \leq b_A$,[5] e.g., the agent

[4] Quadratic utilities are quite standard and extensively used in the literature, e.g., Krishna and Morgan (2001), Goltsman et al. (2009), Mylovanov (2008), Kovac and Mylovanov (2009).

[5] In Section 3.3 we will discuss the more complicated composition of preferences, including overly biased intermediary. It will show that the main insights still hold.

wants to exaggerate the need of fund, and the intermediary may be inclined to him or prefer to take a stance more strict than the principal. All of these are common knowledge among parties. We use $U_i(\theta, y)$ to refer the payoff of player i.

P has the right to make decision, and only A would be informed about the true state and choose the decision. But, he could not communicate directly with P, neither P can allocate the authority directly to A. In other words, M has the full control of the flow of information and commands between P and A. We assume *ex ante* both P and M have the uniform prior over Θ.

Now we specify the timing of this delegation game. P can only make delegation decision one step downwardly. However, since M is uninformed, he will further grant the informed agent with a menu of decisions. The timing of this game is as follows:

1. P grants M with a closed set of permissible decisions $Y_M \subseteq Y$.

2. M allows A to select any action from a closed set $Y_A \subseteq Y_M$.

3. A privately observes the realizations of the state θ, and chooses a decision $y \in Y_A$.

In this sequential game, Y_M and Y_A are the control variables of P and M, respectively. Any set of delegated decisions delivered to A has to be subject to the incentives of M.

It is noteworthy that this game admits multiple equilibria. Actually, there are infinite number of equilibrium since there are infinite many redundant decisions. These redundant decisions might be those that would never be undertaken by A in stage 3, even if they are granted. For example, if $b_A \in Y_A$, then including or excluding any subset of $[0, b_A)$ does not affect A's choice, since for A the action b_A strictly dominates any choices lower than it. Alternatively, redundant decisions might come from M's delegating behavior in stage 2. In equilibrium it might be of his interest to preclude some decisions from Y_A. Hence, if P add to Y_M some decisions that would never be delegated by M, this would not affect Y_A. Therefore, in either way, expanding Y_M or Y_A by adding arbitrary redundant decisions will not affect P's expected payoff.

Therefore, we will concentrate on the *implemented delegation set*, the minimal set of all delegation sets attaining the same equilibrium outcome. With a little abuse of terminology, we use Y_A to refer the set of *implementable* decisions that might be undertaken by A in equilibrium. For instance, the implemented delegation set would only assign one action within the segment of action $[0, b_A]$. Any $y < \sup\{y \,|Y_A \cap [0, b_A]\}$ is not an element of the implemented delegation set. We also ignore M's exclusion of redundant decisions in stage 2, thus in this equilibrium it should be $Y_A = Y_M$. Any other equilibrium delegation sets are payoff-equivalent to the implemented delegation set.

2.1 Benchmark: direct delegation

We start from the benchmark case where the principal can contract with any subordinates, e.g., she could costlessly verify the agent's choice, or she has complete commitment power. Because the intermediary has no additional information, the principal would bypass him and directly delegate to the informed agent. Goltsman et al. (2009) establish that the second-best optimal outcome of an universal mechanism can be attained by the interval delegation *a la* Holmstrom (1977). Thus, this delegation outcome will serve as the *efficiency criterion.*

Lemma 1 *If P can directly delegate to the agent, then, the efficient delegation set Y^* is $[b_A, 1-b_A]$ if $0 \leq b_A \leq \frac{1}{2}$, and the implemented actions are*

$$y(\theta) = \begin{cases} \theta + b_A, \textit{ if } \theta \in [0, 1-2b_A] \\ 1 - b_A, \textit{ otherwise} \end{cases} \tag{2}$$

If $b_A > \frac{1}{2}$, the efficient delegation set consists of only P's ex ante optimal action $\frac{1}{2}$.
Proof. *See Theorem 1 in Goltsman et al. (2009).* ■

The efficient delegation set of action is a single interval $Y^* = [b_A, 1-b_A]$, which is determined by the preference misalignment between P and A. When A is upwardly biased, he has the incentive to make decisions that are too large from P's perspective. As a response, P would remove the extremely high actions by imposing a ceiling $(1-b_A)$. Optimal delegation seeks the balance between the loss of control and the gain of efficiency in decision-making. In the low state, the latter effect outweighs, thus A is allowed to act according to his interest. In the high state, the former dominates. Consequently, P keeps *de facto* control by imposing a ceiling.

Lemma 2 specifies the (local) properties of an optimal delegation set on any subset of Y .

Lemma 2 *In the direct P—–A delegation, suppose that the set of actions of P is restricted to an interval, $Y' = [y_1, y_2] \subseteq Y$, then an optimal delegation set on Y' is an interval of decisions, or one decision, or no decision.*
Proof. *See the Appendix A.* ■

Lemma 2 states that on any interval of Y, the optimal delegation set assigns a connected set of decisions on it. In other words, the implemented choice $y(\theta)$ will be continuous on the corresponding segment of state $[\max\{y_1 - b_A, 0\}, \max\{y_2, 0\}]$. Melumad and Shibano (1991) and AM have expressed the similar result, based on slightly different assumptions of the model. While Lemma 1 demonstrates that given an unrestricted set of options, interval delegation with a binding cap attains the efficient outcome, Lemma 2 implies that if the choice set available to P is an interval, then it is impossible that P delegates a union of more than two intervals (points) of choices to A. This lemma will significantly simplify our analysis on M's delegation behavior. In particular, the optimal delegation set under direct interactions would be a single interval or a point.

3 Indirect Delegation

Now we turn to indirect delegation, where the principal has to delegate to the intermediary and leave him with the discretion in sub-delegation. In other words, the principal could not verify the agent's choice, or she has limited commitment power in that P could not commit to delegate directly to A. M's bias will influence A's choice set. We would use $\Delta = b_A - b_M$ to represent the divergence of preference between A and M.

In the second stage, M acts as if a sub-principal within the choice set Y_M, and delegates based on Δ. In the first stage, P designs the delegation set as the best response to her anticipated M's downstream delegating behavior. Thus, both Δ and b_A affect the optimal delegation set. As the first step, Lemma 3 studies the intermediary's delegating behavior in the second stage by specifying the agent's highest available alternative $\overline{y_A} = \max\{y : y \in Y_A\}$,

conditional on that Y_M is an interval.

Lemma 3 *If Y_M is an interval of decisions, then* $\overline{y_A} = \min\{1 - \Delta + b_M, \sup\{y \,|y \in Y_M\}\}$
Proof. *See the Appendix A.* ■

This lemma illustrates the impact of preference misalignment between P and M. If M is delegated with extensive discretionary authority, unless his preference coincides with the principal's, i.e., $b_M = 0$, he is inclined to impose his ideal cap different from P's ideal cap $1 - b_A$. Actually, the difference between the ideal caps of P and M represents a kind of *derived conflict of interest*, which roots into their different inclination to restrict the agent's discretion.

Section 3.1 demonstrates that whenever the subordinates are of like biases, and M is moderately biased, the single-interval delegation set is still optimal. Section 3.2 points out that ceiling strategy is not optimal when the subordinates are of opposing biases. The main reason is that since M would like to impose a tighter bound on the agent's decisions than would be ex-ante optimal for the principal, A's decision-making is not responsive for high values of the state. Lemma 4 suggests that when an interval of intermediate decisions is exogenously removed, the optimal delegation set on this restricted action space will be an union of an interval of options and an isolated option. Then, Proposition 2 and 3 demonstrate that a principal can optimally exclude some moderate decisions to make A's decision-making partially responsive to high realizations of the state. Proposition 4 states the optimal delegation rule in this three-tier hierarchy. These results give rise to observed policy threshold effects. Section 3.3 discusses some extensions, including other preferences composition, more complicate chains, etc..

3.1 Like biases

We first examine the situation that M is moderately biased, i.e., $b_A \geq b_M \geq 0$. Thus, $\Delta \in [0, b_A]$, both subordinates are upwardly biased, but M is less biased than A. Since $1-\Delta+b_M \geq 1-b_A$, M always prefer to expanding the discretion of the agent. However, by Lemma 3 P could also eliminate this loss by imposing a binding cap $1-b_A$ on Y_M. Thus, her best outcome in (2) would be attained in the hierarchy, as the following proposition shows.

Proposition 1 *If M and A have like biases and M is less biased, i.e., $b_A \geq b_M \geq 0$, then the optimal delegation rule of P is to impose a ceiling* $\sup\{y\,|y \in Y_M\} = 1-b_A$*, and the efficient delegation set Y^* can be implemented.*
Proof. *See the Appendix A.* ■

This demonstrates that interval delegation is still optimal in a three-tier hierarchy with moderately biased intermediary. In equilibrium, P imposes a ceiling on M's choice set Y_M, M then delivers all the remaining options, and A finally picks up his preferred decision from Y_A. Besides, this proposition says that the bias of M is irrelevant to the payoff of P as long as it lies between P's and A's preferences. Therefore, there is no efficiency loss from adding a moderate intermediary.

3.2 Opposing biases

If A and M have opposing biases, i.e., $b_A \geq 0 \geq b_M$ and $\Delta \in \left[0, \frac{1}{2}\right]$, then we have $\Delta > b_A$. A downwardly biased intermediary is interested in imposing a tighter ceiling $(1-\Delta+b_M)$ on Y_A than the efficient ceiling $(1-b_A)$. Thus, Lemma 3 says that it is impossible to attain Y^* since $\overline{y_A} \leq 1-b_A$. From the perspective of P, the superiors have the conflict over delegating the decisions within $[1-\Delta+b_M, 1-b_A]$. Within this segment, P wants to give A full discretion, while M wants to restrict A. The conflict is derived from the difference in ideal caps. Thus, the more restrictive intermediary incurs additional distortion on the principal. Remark 1 constructs an example to illustrate that interval delegation sets are no longer optimal.

Remark 1 *When A and M have opposing biases, i.e., $b_A \geq 0 \geq b_M$, interval delegation is not optimal for P.*
Proof. *See the Appendix A.* ■

Therefore, we should look at the optimal (restricted) delegation rule of M to A, if the set of choices available to M is a given arbitrary subset of $[0,1]$, e.g., a finite union of intervals. However, Remark 2 suggests that we could concentrate on the union of two intervals of actions.

Remark 2 *If $b_A \geq 0 \geq b_M$, under the optimal delegation rule, A's implementable action $y(\theta)$ is continuous for $\theta \in [0, 1-\Delta+b_M-b_A]$ and $\theta \in (1-\Delta+b_M-b_A, 1]$.*
Proof. *See the Appendix A.* ■

Therefore, the optimal delegation rule of P prescribes that the intersections of Y_A and the low and high segment of actions, respectively, are connected sets. Hence, it is not in the interest of P that Y_M, as well as Y_A, contain more than two intervals of implementable decisions. It turns out to be sufficient to consider a union of two intervals as the candidates for optimal delegation rules.

However, the connectedness property in Lemma 2 could not be extended to the full space of decisions Y. If the principal attempts to use the single interval delegation set *a la* Holmstrom, the upper-bound of Y_A shrinks, and the agent's decision-making would be unresponsive for high realizations of the state $\theta \in (1-\Delta+b_M-b_A, 1]$. Therefore, by Remark 2 we could concentrate on Y_M as a union of two intervals, in which some modest actions around M's ideal ceiling might be removed.

How to implement a decision (partially) responsive to high values of the state? In other words, how will an intermediary react to the removal of decisions. Lemma 4 suggests that in a two-layer hierarchy, when the principal is restricted in that her ideal cap becomes unavailable due to exogenous reasons, she will delegate a union of an interval of low actions and a single high option.

Lemma 4 *In the direct P—–A delegation, P would like to delegate at most one action higher than $1-b_A$. If an interval of intermediate actions $(x, 1-b_A)$ is removed from the available decisions, then the restricted optimal delegation set is the union of an interval of low actions and a single high point. Formally, $Y_A = [b_A, x] \cup \{z^*\}$, in which $z^* = 1 - b_A$.*
Proof. *see the Appendix A.* ■

Under this restricted optimal delegation rule, A undertakes his preferred choice $\theta + b_A$ whenever the realizations of state θ lies within $[0, x - b_A]$. For $\theta \in [x - b_A, \frac{x+z}{2} - b_A]$, he undertakes the action x, while for $\theta \in [\frac{x+z}{2} - b_A, 1]$, he chooses the decision z^*.

From this lemma we know that any restricted optimal delegation set should contain the interval $[b_A, x]$, where the loss from control is outweighed by the gain in responsive decision-making. Furthermore, the principal will discover it is optimal to add only one action greater than $1 - b_A$, the upper bound of the set of removed decisions. The reason is: a restricted principal will delegate some high actions to compensate the loss of responsiveness in the modest state. But, since in the range of the high state the agent wants to undertake decisions that are too far from the principal, she would restrict A's discretion by delegating a single option. This lemma provides the key insight on the shape of optimal delegation set in three-tier hierarchies with oppositely biased subordinates.

P and M have the mutual interest in choosing the options corresponding to the low state, and the conflict of interest occurs only in the ideal response to the high state. Because in the second stage M acts as if a principal within Y_M, by Lemma 4, if P wants the decision-making to be more responsive to the high state, she might need to preclude M's ideal cap $1 - \Delta + b_M$ from Y_M. Consequently, P's delegation problem becomes designing a "hole" in Y_A. Formally, P chooses the cap x of the interval of low actions and the discrete decision z in Y_A, subject to that M has the incentive to deliver it to A. The principal's delegation problem can be stated as:

$$\max_{x,z} - \int_0^{x-b_A} b_A^2 d\theta - \int_{x-b_A}^{\frac{x+z}{2}-b_A} (\theta - x)^2 \, d\theta - \int_{\frac{x+z}{2}-b_A}^{1} (\theta - z)^2 \, d\theta \qquad (3)$$

subject to

$$b_A \leq x \leq 1 - \Delta + b_M \tag{4}$$

$$-\int_{x-b_A}^{\frac{x+z}{2}-b_A} (\theta - x + b_M)^2 \, d\theta - \int_{\frac{x+z}{2}-b_A}^{1} (\theta - z + b_M)^2 \, d\theta \geq -\int_{x-b_A}^{1} (\theta - x + b_M)^2 \, d\theta \tag{5}$$

(3) is the principal's objective function. P's expected utility is the sum of three parts: when the states are lower than $x - b_A$, the agent can implement his ideal options, so the distance between his action and P's ideal decision is a constant b_A. When the state lies in the intermediate region, i.e., $\theta \in \left[x - b_A, \frac{x+z}{2} - b_A\right]$, the agent strictly prefers to take decision x, and is indifferent between x and z when $\theta = \frac{x+z}{2} - b_A$. When the state falls into the high region, i.e., $\theta > \frac{x+z}{2} - b_A$, the agent always takes the highest available option z.

(4) is the boundary condition: the lower interval could not be too large, e.g., $x < 1 - \Delta + b_M$. Otherwise, M will only delegate a single interval $[b_A, 1 - \Delta + b_M]$. The constraint (5) is the incentive-compatible condition of M: it is also in his interest to deliver the union of the lower interval $[b_A, x]$ and the discrete high contingency z to A. These two inequalities represent the cost to incentivize M's delegating behavior, and ensure that it is M's best response to deliver Y_M to A. Lemma 5 below suggests that the optimal delegation set prescribes that the cap of the lower interval and the single high point should be symmetric around M's ideal ceiling.

Lemma 5 *Under the optimal delegation rule, z and x are symmetric around $1 - \Delta + b_M$.*
Proof. *Analyzing (5) shows that the choice of z and x should satisfy the jump condition:*

$$\frac{1}{2}(z + x) \leq 1 - \Delta + b_M \tag{6}$$

Because EU_P is strictly increasing with x whenever (4) holds, which means that once z is determined, P is always better off by increasing the discretion of M in the low state. Thus, (6) is always binding, and we can rewrite the incentive-compatibility constraint (5) as

$$x = 2\left(1 - b_A + 2b_M\right) - z$$

i.e., z and x are symmetric around $1 - b_A + 2b_M$. ■

Therefore, we could represent x and z in a more tractable way:

$$x = 1 - b_A + 2b_M - \lambda, \text{ and } z = 1 - b_A + 2b_M + \lambda$$

Thus, λ represents the *wedge* between P's choice variables and M's ideal cap.

Therefore, if we consider the outcome of this delegation game as a function from the true state to the set of decisions, then this function admits at most one discontinuity at $\theta = 1-\Delta+b_M-b_A$, and this discontinuity must be a jump discontinuity. Depend on whether the interval of low actions degenerates to a single decision, Proposition 2 and 3 characterize this restricted optimal delegation rule.

Proposition 2 *If the subordinates have opposing biases, and the bias of the intermediary is not too large, e.g.,* $\overline{b_M} \leq b_M \leq 0 \leq b_A$*, then the optimal implemented delegation set is*

$$Y_A = [b_A, 1 - b_A + 2b_M - \lambda^*] \cup \{1 - b_A + 2b_M + \lambda^*\} \tag{7}$$

The optimal wedge is

$$\lambda^* = -b_A + 2b_M + \sqrt{2\left(2b_M - b_A\right)^2 - b_A^2} \tag{8}$$

in which $\overline{b_M} = \frac{b_A - \sqrt{b_A^2 - b_A + \frac{1}{2}}}{2}$.

Proof. *If the optimal delegation set can contain a lower interval, then we substitute (6) into (3). The optimization problem is rewritten as*

$$\max_{\lambda} - b_A^2\left(1 - 2b_A + 2b_M - \lambda\right) - \int_{1-2b_A+2b_M-\lambda}^{1-2b_A+2b_M} \left[\theta - \left(1 - b_A + 2b_M - \lambda\right)\right]^2 d\theta$$
$$- \int_{1-2b_A+2b_M}^{1} \left[\theta - \left(1 - b_A + 2b_M + \lambda\right)\right]^2 d\theta$$

By the first-order condition, the optimal wedge is:

$$\lambda^* = -b_A + 2b_M + \sqrt{2\left(2b_M - b_A\right)^2 - b_A^2}$$

It is straightforward to verify that λ^* *is always non-negative.*[6] *Thus, the optimal discrete contingency* z^* *is:*

$$z^* = 1 - 2b_A + 4b_M + \sqrt{2\left(2b_M - b_A\right)^2 - b_A^2}$$

[6] $\lambda^* \geq 0 \Leftrightarrow \left(b_A - 2b_M\right)^2 \geq b_A^2$, which always hold for $b_M \leq 0 \leq b_A$.

Consequently, the optimal cap of the lower interval is

$$x^* = 1 - \sqrt{2\left(2b_M - b_A\right)^2 - b_A^2}$$

(3) and (4) also impose further boundary conditions on the range of b_A *and* b_M*, namely:*

$$x^* \geq b_A \text{ and } z^* \leq 1$$

These guarantee that the lower interval contains more than one decision, and the discrete high option is feasible.
Because

$$\begin{aligned} z^* &\leq 1 \Leftrightarrow 2b_A - 4b_M \geq \sqrt{2\left(2b_M - b_A\right)^2 - b_A^2} \\ &\Leftrightarrow 8b_M^2 - 8b_A b_M + 3b_A^2 \geq 0 \Leftrightarrow 8\left(b_M - \frac{b_A}{2}\right)^2 + b_A^2 \geq 0 \end{aligned}$$

we have that $z^* \leq 1$ *always holds.*
On the other hand,

$$\begin{aligned} x^* &\geq b_A \Leftrightarrow 1 - b_A \geq \sqrt{2\left(2b_M - b_A\right)^2 - b_A^2} \\ &\Leftrightarrow 8b_M^2 - 8b_A b_M - \left(1 - 2b_A\right) \leq 0 \end{aligned}$$

which requires that

$$\frac{b_A + \sqrt{b_A^2 - b_A + \frac{1}{2}}}{2} \geq b_M \geq \frac{b_A - \sqrt{b_A^2 - b_A + \frac{1}{2}}}{2}$$

Define $\overline{b_M} \equiv \frac{b_A - \sqrt{b_A^2 - b_A + \frac{1}{2}}}{2}$*, because* $\frac{b_A + \sqrt{b_A^2 - b_A + \frac{1}{2}}}{2} \geq 0$*, we have that when* $\overline{b_M} \leq b_M \leq 0$*,* $Y_A = Y_M = [0, x^*] \cup \{z^*\}$*.* ■

Figure 1 illustrates the shape of the optimal delegation set when the intermediary is more restrictive than the principal. In equilibrium, the principal delegates to the intermediary with the choice set Y_M characterized by (7), then the intermediary delivers this set to the agent, and lets the latter to chose his preferred options from this set.

Figure 1 here

Proposition 2 describes that when a downwardly biased intermediary is not too biased, the optimal delegation set contains an interval of low decisions. However, if the intermediary is further biased from the principal, then the optimal delegation rule of P may degenerate to a set of two discrete decisions, e.g., "accept" and "reject", as the following proposition illustrates.

Proposition 3 *If the intermediary is very downwardly biased, e.g.,* $\max\{\underline{b_M}, b_A - \frac{1}{2}\} \leq b_M \leq \overline{b_M}$*, then the optimal implemented delegation set contains two decisions, namely*

$$Y_A = \{1 - b_A + 2b_M - \lambda^*\} \cup \{1 - b_A + 2b_M + \lambda^*\} \tag{9}$$

The optimal wedge is

$$\lambda^* = \frac{1}{2} - 2\left(2b_A - 1\right) b_M + 4b_M^2 - b_A \tag{10}$$

in which $\underline{b_M} = \frac{b_A - \sqrt{b_A^2 + \frac{1}{2}}}{2}$.
If $b_M \leq \max\{\underline{b_M}, b_A - \frac{1}{2}\}$*, then* $Y_A = \{\frac{1}{2}\}$.

Therefore, when the subordinates have opposing biases, the multi-interval delegation set arises as the optimal one due to incentive costs, and serves as a device to control the intermediary's sub-delegation behavior. This exclusion of modest actions in effect maintains the effective use of information in the bottom state, and improves the responsiveness in the high state, on the expense of loss of the use of information in the modest state.

The jump discontinuity gives rise to *policy threshold effects*: small changes in the state around a certain threshold level has dramatic impacts on the policy response. With opposing biased subordinates, the threshold level is M's ideal cap $1 - b_A + 2b_M$. As Proposition 2 and 3 show, the variation of policy response will be 2λ.

In contrast with the case of like biased subordinates, a downwardly biased intermediary entails additional distortion on the principal. In particular, compare with the direct delegation, the agent not only has *less* decisions available, but also chooses *worse* responses for the realizations of the high state, as the following corollary demonstrates.

Corollary 1 *When the subordinates are of opposing biases, the high contingency* $z^* \leq 1 - b_A$*. And a less biased intermediary can increase the agent's discretionary authority.*
Proof. *See the Appendix A* ■

Under the direct delegation, the highest option available to A is $1-b_A$. When M is less restrictive than P, this decision is still implementable, as Proposition 1 shows. However, when M is more restrictive, the highest implementable decision z^* is lower than $1-b_A$. Thus, responsiveness to the high state is also partially sacrificed.

The less biased M is, the fewer modest options needed to be precluded. In other words, the greater preference misalignment among subordinates, the smaller the hole, and the larger policy threshold effects. It is noteworthy that the wedge λ disappears ($x^* = z^* = 1-b_A$) only if $b_M = 0$, i.e., P will delegate M with an interval of decisions only if he is neutral. Thus, interval delegation is a limit case in a multi-tier hierarchy. We summarize an equilibrium in the sequential delegation game here.

Summary 1 *In this principal-intermediary-agent delegation game, in equilibrium, the principal's strategies are:*

1. *If* $\Delta \in [0, b_A]$, *she delegates all options within* $[b_A, 1-b_A]$ *to the intermediary*
2. *If* $\Delta \in \left[b_A, \frac{1}{2}\right]$, *she specifies an unconnected choice set* Y_M *(as in (7) and (9)) from which the intermediary may choose;*
   For the intermediary, it is his best response to deliver all the remaining decisions in Y_M *to the agent. Then, the informed agent selects his ideal action within this choice set.*

Figure 2 illustrates the features of the optimal delegation rule in Summary 1, conditional on the subordinates' biases. When the combination of subordinates' biases falls into the upper shadow triangle, e.g., the intermediary is less restrictive than the principal, interval delegation is optimal. When the intermediary is more restrictive than the principal, but not too restrictive, the optimal implemented delegation set contains an interval of low decisions and a high single point. If the intermediary is quite restrictive, e.g., his bias lies into the lower shadow triangle, then it is optimal for the principal to delegate only two implementable options. Moreover, if the intermediary is too restrictive, then delegation becomes trivial in that the principal would allow the subordinates to undertake only one action.

Figure 2 here

3.3 Extensions

So far, we only consider the situation that the intermediary is moderately biased or downwardly biased, e.g., $\Delta \geq 0$. However, it is also possible that $\Delta < 0$, namely M is overly-biased. In Appendix B we show that a hole is still involved in the optimal delegation mechanism with an over-biased intermediary. As in Lemma 1 and 2, the optimal delegation rule prescribes ceiling the agent's choice set against the direction of bias. Thus, it is optimal for M to restrict Y_A by imposing a bottom, and P still caps A's discretion in the high state. Again, P removes decisions only in the direction where the derived conflict of interest between superiors exists. Since M wants to expand the agent's discretion in the high state, P still imposes a cap. On the other hand, because M has an additional ideal bottom when delegating to A, P can remove some options around this bottom from Y_M. Consequently, it is in M's best interest to delegate the remaining implementable decisions to A. The optimal delegation set is a union of a low single point and an interval of moderate decisions. In summary, when the intermediary is overly-biased, we still have the multi-interval delegation set, and the policy threshold effect occurs in the low state.

A chain is an abstract of hierarchies in the real world. If we have a multiple branch hierarchy, e.g., a tree, however, our analysis still applies as long as there is no payoff interdependence among different branches of subordinates. Then, the principal can take each branch as a chain, and design the optimal delegation rule conditional on the biases of the members on that branch. Thus, the restricted optimal delegation mechanism characterized in Proposition 2 and 3 still holds, and policy threshold effects remain.

The basic trade-off in a three-tier hierarchy continues in the optimal design of delegation rule in a chain with arbitrary depth. If all subordinates have like biases, and all intermediaries' biases are between the bottom-level agent and the principal, e.g., every intermediary is less restrictive than his immediate superior, then a principal can employ the approach described in Proposition 1. By imposing an appropriate cap conditional on b_A, hierarchies will have the same extent of responsibility delegated at the bottom level as that in a two-tier hierarchy. Thus, single-interval delegation is still optimal. Otherwise, a principal has to take into account each subordinate's direction of bias. Again, it is optimal for her to keep the subordinates' discretion in the range of mutual interests, and exclude the decisions around the thresholds in the direction of derived conflict of interest. Some intermediaries may want to impose extra restrictions on the low state, and some may prefer taking more strict stance for the realizations of the high state. However, as Lemma 4 shows, any (sub-)principal only wants to implement at most one action higher (lower) than her ideal cap (bottom). Moreover, there are at most two directions of conflict of interest on this one-dimension state space. Therefore, the optimal delegation set is a union of at most three interval of actions: a low single point, an interval of intermediate decisions possibly degenerating into a single point, and a high single point. In equilibrium, policy threshold effects become more salient since there are two thresholds now, and multiple intermediaries entail more distortion on the principal. In summary: when all intermediaries are moderately biased, interval delegation remains the optimal; otherwise, the optimal delegation set involves one or two holes, and policy threshold effects exist.

Therefore, when hierarchies become more complicated, the principal's loss weakly increases, the optimal delegation rule has more holes, and policy threshold effects are more salient. In effect, this doesn't require that the principal has very precise knowledge of the subordinates' biases. If she expects that *ex post*, the intermediaries' responses may be distorted towards the direction opposing to the agent's ideal action, she will still exclude some modest decisions against this distortion.

4 Conclusion

This paper contributes by providing a complete characterization of the optimal delegation rule in a multi-tier hierarchy. We show that as the response to the distortion caused by a biased intermediary, a principal may exclude some moderate options from the set of delegated decisions. The optimal delegation set contains an interval of decisions of the mutual interest among superiors, and precludes the options around a certain threshold, which is determined by the direction of derived conflict of interest among superiors. Thus, there may exist optimal delegation sets that are not connected.

Our paper doesn't touch the very interesting question on the design of optimal hierarchies. This has received considerable attentions in literatures which mainly aims at explaining the reasons for hierarchies, including bounded rationality or information processing costs (Geanakoplos and Milgrom, 1991; Radner, 1993), heterogenous knowledge (Garicano, 2000) or conflicts over hiring and promotion decisions (Friebel and Raith, 2004), etc.. However, as long as hierarchies are formed based on considerations beyond the strategic use of information, our results on the design of optimal delegation *within* hierarchies still hold.

In this paper, we only analyze delegation in hierarchies. It will be interesting to compare delegation with communication in hierarchies, e.g., the principal chooses decisions after hearing reports from the agent via a strategic intermediary. Previous studies establish that an opposing biased intermediary may improve communication efficiency (Ivanov, 2010, Ambrus et al. 2013). Investigating whether delegation always dominates communication even with opposing biased subordinates may provide new insights about the value of delegation. A more interesting direction will be combining skip-level communication with sequential delegation, e.g., the bottom-level agent has the opportunity to skip his immediate superiors and report directly to the top-level principal. This may significantly change the shape of implemented delegation set. We leave these promising research directions for future studies.

A Appendix A

PROOF OF LEMMA 2

Let's denote the intersection of the optimal delegation set and the interval Y' as Ψ. Suppose that Ψ is not a connected set. Then, there exists at least one interval of decisions $(t-\mu, t+\mu) \subset [y_1, y_2]$, in which $\mu \geq 0$, such that $t-\mu, t+\mu \in \Psi$ but $(t-\mu, t+\mu) \subsetneq \Psi$. It suffices to show that the principal has the incentive to add the segment of decisions $(t-\mu, t+\mu)$ to Ψ.

1. If $t+\mu < b_A$, then, the decisions within this interval are redundant, and we would not change the principal's expected payoff by adding the interval $(t-\mu, t+\mu)$ to Ψ.

2. If $t-\mu \geq b_A$, because $(t-\mu, t+\mu) \subsetneq \Psi$, A will chooses

$$y(\theta) = \begin{cases} t-\mu, \text{ if } \theta \in [t-\mu-b_A, t-b_A] \\ t+\mu, \text{ if } \theta \in [t-b_A, t+\mu-b_A] \end{cases}$$

We now show that by decreasing μ to some ω we increase P's expected payoff in contradiction to the assumed optimality of Ψ. This variation would not affect P's payoff on any segment of the state outside $[t-\mu-b_A, t+\mu-b_A]$, but create two new subsegments of the state $(t-\mu-b_A, t-\omega-b_A]$ and $[t+\omega-b_A, t+\mu-b_A)$, $\omega \in [0, \mu]$, in which A chooses $y = \theta + b_A$. P's expected payoff on $\theta \in (t-\mu-b_A, t+\mu-b_A)$ is, then,

$$-\int_{t-\mu-b_A}^{t-\omega-b_A} (b_A)^2 \, d\theta - \int_{t-\omega-b_A}^{t-b_A} [\theta-(t-\omega)]^2 \, d\theta - \int_{t-b_A}^{t+\omega-b_A} [\theta-(t+\omega)]^2 \, d\theta$$
$$-\int_{t+\omega-b_A}^{t+\mu-b_A} (b_A)^2 \, d\theta$$

The first derivative with respect to ω is $-2\omega^2$. This derivative is negative. Thus, given that $\omega \in [0, \mu]$, the optimal $\omega = 0$; that is, P's expected payoff increases by adding to Ψ the interval of actions previously precluded.

3. Finally, if $t - \mu < b_A \leq t + \mu$, we continue creating the variation stated above. We divide our discussion of the new subsegments into three cases:

   - If for the new subsegments we have $t + \omega < b_A$, then A chooses $y = \theta + b_A$ for the state $\theta \in [0, t + \mu - b_A]$. Hence, reducing ω further to zero would not affect P's expected payoff.
   - If the new segments are characterized by $t - \omega < b_A \leq t + \omega$, then A chooses $y = t - \omega$ for $\theta \in [0, \max\{t - b_A, 0\}]$, $y = t + \omega$ for $\theta \in [\max\{t - b_A, 0\}, t + \omega - b_A]$, and $y = \theta + b_A$ for $\theta \in [t + \omega - b_A, t + \mu - b_A)$. P's expected payoff on $\theta \in [0, t + \mu - b_A)$ is

   $$-\int_0^{\max\{t-b_A,0\}} [\theta - (t-\omega)]^2 \, d\theta - \int_{\max\{t-b_A,0\}}^{t+\omega-b_A} [\theta - (t+\omega)]^2 \, d\theta - \int_{t+\omega-b_A}^{t+\mu-b_A} (b_A)^2 \, d\theta$$

   The derivative with respect to ω is $-(t+\omega)^2 + b_A^2$ if $t - b_A < 0$, and $-2\omega^2 - b_A^2 + (t-\omega)^2$ otherwise. Both are negative. Thus, it is still optimal to set $\omega = 0$.
   - If the lower new subsegment has $t - \omega \geq b_A$, then P's expected payoff on $\theta \in [0, t + \mu - b_A)$ is

   $$-\int_0^{t-\omega-b_A} (b_A)^2 \, d\theta - \int_{t-\omega-b_A}^{t-b_A} [\theta - (t-\omega)]^2 \, d\theta - \int_{t-b_A}^{t+\omega-b_A} [\theta - (t+\omega)]^2 \, d\theta$$
   $$-\int_{t+\omega-b_A}^{t+\mu-b_A} (b_A)^2 \, d\theta$$

   And the derivative with respect to ω is $-2\omega^2$, which is negative. Again, it is optimal to further reduce ω to zero.

Therefore, the improvement in P's expected payoff by adding to Ψ the new subsegments $(t - \mu - b_A, t - \omega - b_A]$ and $[t + \omega - b_A, t + \mu - b_A)$ is in contradiction to the assumed optimality of Ψ. So Ψ has to be a connected set.

PROOF OF LEMMA 3

Since $Y_A \subset Y_M$, the highest action available to the agent, $\overline{y_A}$, can not exceed the highest action in Y_M. On the other hand, if the intermediary is delegated with an unrestricted set of decisions, e.g., $Y_M = Y$, then he acts as if a sub-principal within Y_M. By Lemma 1, he could attain his best payoff by imposing an ideal cap $\min\{1, 1 - \Delta + b_M\}$. Since $Y_M \subset Y \equiv [0, 1]$, we have the results.

PROOF OF PROPOSITION 1

By Lemma 2, we know that the optimal delegation set assigns an interval or a point on any interval of actions. Since $1-\Delta+b_M \geq 1-b_A$, by Lemma 3, if $[b_A, 1 - \Delta + b_M] \subset Y_M$, M could implement his best outcome. Therefore, when P imposes the cap $1 - b_A$ on M's choice set, by Lemma 3 M has the inclination to delegate all available decisions lower than $1 - b_A$ to A, exactly the same outcome as Y^* prescribed.

PROOF OF REMARK 1

We will prove it by constructing a contradiction. Suppose that the interval delegation is the optimal. By Lemma 3 we know that the highest available action to A, if Y_M is an interval, is $1 - \Delta + b_M$. Hence, interval delegation leads to $Y_A^I = [b_A, 1 - \Delta + b_M]$. P's highest expected payoff when delegating an interval of decisions thus is

$$EU_P^I = -\int_0^{1-\Delta+b_M-b_A} b_A^2 d\theta - \int_{1-\Delta+b_M-b_A}^{1} \left[\theta - (1 - \Delta + b_M)\right]^2 d\theta$$

Now we construct a variation of this delegation rule. The alternative is a union of two disconnected action sets

$$Y_A^\varepsilon = [b_A, 1 - \Delta + b_M - \varepsilon] \cup \{1 - \Delta + b_M + \varepsilon\}$$

in which $\varepsilon \geq 0$. The expected payoff to P is

$$\begin{aligned} EU_P^\varepsilon &= -\int_0^{1-\Delta+b_M-b_A-\varepsilon} b_A^2 d\theta - \int_{1-\Delta+b_M-b_A-\varepsilon}^{1-\Delta+b_M-b_A} \left[\theta - (1 - \Delta + b_M - \varepsilon)\right]^2 d\theta \\ &\quad -\int_{1-\Delta+b_M-b_A}^{1} \left[\theta - (1 - \Delta + b_M + \varepsilon)\right]^2 d\theta \end{aligned}$$

The first derivative of EU_P^ε with respect to ε is

$$\frac{\partial EU_P^\varepsilon}{\partial \varepsilon} = -b_A^2 - 2\epsilon + (\Delta - b_M - \varepsilon)^2$$

By definition $EU_P^\varepsilon = EU_P^I$ when $\epsilon = 0$. Let's consider the situation $b_M < 0$, then we have $\Delta - b_M > b_A$. Hence $\frac{\partial EU_P^\varepsilon}{\partial \varepsilon} > 0$ when $\varepsilon = 0$. This implies that there exists a $\eta > 0$ such that for $\varepsilon \in [0, \eta]$, we have $EU_P^\varepsilon > EU_P^I$. Therefore, compared with the new disconnected delegation set Y_A^ε, P's expected payoff is strictly lower under the interval delegation set Y_A^I.

Besides, this new delegation set Y_A^ε also satisfies the jump condition in (6). Hence, it is in M's interest to deliver Y_A^ε to A. Therefore, the higher expected payoff under Y_A^ε is in the contradiction to the assumed optimality of Y_A^I.

PROOF OF REMARK 2

We need to show that the optimal delegation set Y_A has to prescribe that $y(\theta)$ is continuous on both $[0, 1 - \Delta + b_M - b_A]$ and $(1 - \Delta + b_M - b_A, 1]$, respectively.

By Lemma 1, it is in the best interest of the downwardly biased M to cap A's discretion on his ideal ceiling $1 - \Delta + b_M$. Mathematically, we have the intersection $Y_M \cap [0, 1 - \Delta + b_M] = Y_A \cap [0, 1 - \Delta + b_M]$.

Moreover, suppose that in equilibrium $Y_A \cap [0, 1 - \Delta + b_M]$ is a finite union of intervals, e.g., A's choice $y(\theta)$ is discontinuous at some interior points $\theta \in [0, 1 - \Delta + b_M - b_A]$. By Lemma 2, we have that M's expected payoff increases by adding these discontinuities to Y_A. Since $1 - \Delta + b_M < 1 - b_A$, including these discontinuities is also in the best interest of P. The improvement by this variation is in contradiction to the assumed optimality of a disconnected delegation set on this range. Therefore, we have that the intersection $Y_A \cap [0, 1 - \Delta + b_M]$ is a connected set.

Similarly, for the set of decisions $(1 - \Delta + b_M, 1]$, M prefers to reduce A's discretion for the corresponding range of state. If the implemented delegation set Y_A may contain more than one intervals on the range $(1 - \Delta + b_M, 1]$, then it would be in P and M's mutual interest to include all the actions between the intervals into Y_A. Again, it is in contradiction to the assumed optimality of the original delegation set. The implementable choice $y(\theta)$ thus is continuous on the corresponding segment of state $(1 - \Delta + b_M - b_A, 1]$. Hence we reach the conclusion we want.

PROOF OF LEMMA 4

We divide the proof into four parts. Part 1 shows that if P's action set is constrained to interval $[0, x]$ with $x \leq 1 - b_A$, then the optimal delegation set has form $[b_A, y_2]$ on it. Part 2 shows that if P's action set is constrained to $[1 - b_A, 1]$, then the optimal delegation set assigns a singleton $\{z\}$ on it. Part

3 establishes that for action sets $[0, x]$ and $[1 - b_A, 1]$ the optimal delegation set have forms, respectively, $[b_A, x]$ and $\{z^*\}$, in which $z^* = 1 - b_A$. Finally, Part 4 demonstrates that delegating a union of the two dominates delegating only an interval. Hence, the optimal delegation set is also the union of an interval and a point on the union of the two action sets $[0, x] \cup [1 - b_A, 1]$.

1. First, by Lemma 2, we know that the intersection of the optimal delegation set and $[0, x]$ has to be a connected set. Thus, we can concentrate on the form of an interval or a point. Suppose the choice set on this segment is an interval of options $[y_1, y_2] \subset [0, x]$, and $y_1 \geq b_A$. We need to show that for any fixed higher endpoint y_2, it is optimal for P to reduce the implementable y_1 to b_A. Since the change in y_1 will not affect P's payoff for any state $\theta > y_2 - b_A$, we can focus on P's payoff on the interval of the state $[0, y_2 - b_A]$ alone. A will undertake y_1 for any state $\theta \leq y_1 - b_A$, and choose $\theta + b_A$ for $\theta \in (y_1 - b_A, y_2 - b_A]$. Hence, P's payoff on this interval is

$$-\int_0^{y_1 - b_A} (\theta - y_1)^2 \, d\theta - \int_{y_1 - b_A}^{y_2 - b_A} b_A^2 d\theta = \frac{b_A^3}{3} - \frac{y_1^3}{3} - b_A^2 (y_2 - y_1)$$

   The derivative with respect to y_1 shows that the maximum is attained when $y_1 = b_A$.

2. Then we look at the optimal delegation set on the choice set $[1 - b_A, 1]$. By Lemma 2, it has to be a connected set. We start from supposing it is an interval $[y_3, y_4]$, then we need to show that it is optimal for P to shrink this interval into a single point, namely, $y_3 = y_4$. The expected payoff of P for $\theta \in \left[\frac{y_2+y_3}{2} - b_A, 1\right]$ under the delegation set $[y_3, y_4]$ is

$$\begin{aligned} & -\int_{\frac{y_2+y_3}{2} - b_A}^{y_3 - b_A} (\theta - y_3)^2 \, d\theta - \int_{y_3 - b_A}^{y_4 - b_A} b_A^2 d\theta - \int_{y_4 - b_A}^{1} (\theta - y_4)^2 \, d\theta \\ = & -\frac{1}{3} \left(\frac{y_2 - y_3}{2} - b_A \right)^3 - b_A^2 (y_4 - y_3) - \frac{(1 - y_4)^3}{3} \end{aligned}$$

   The first derivative with respect to y_4 is $-b_A^2 + (1 - y_4)^2$, since $y_4 \geq y_3 \geq 1 - b_A$, it is negative. Therefore, P strictly benefits from shrinking the upper bound of the interval of the high decisions, she would delegate at most one action within the action set $[1 - b_A, 1]$.

3. Now we know that for action sets $[0, x]$ and $[1 - b_A, 1]$, the restricted optimal delegation set takes the forms, respectively, $[b_A, y_2]$ and $\{z\}$.

We need to show that $y_2 = x$ and look for the analytical form of z. P's expected payoff under the delegation set $[b_A, y_2] \cup \{z\}$ is

$$\begin{aligned} EU_P &= -b_A^2 (y_2 - b_A) - \frac{1}{3}[2\left(\frac{z - y_2}{2}\right)^3 + 6\left(\frac{z - y_2}{2}\right) b_A^2 \\ &\quad + b_A^3 + (1 - z)^3] \end{aligned}$$

Since $\frac{\partial EU_P}{\partial y_2} = \left(\frac{z-y_2}{2}\right)^2 \geq 0$, P always wants to increase y_2 till the higher endpoint x, so $y_2 = x$. Moreover, since

$$\frac{\partial EU_P}{\partial z} = -\left(\frac{z - y_2}{2}\right)^2 - b_A^2 + (1 - z)^2$$

and

$$\frac{\partial^2 EU_P}{\partial z^2} = \frac{3z + y_2 - 4}{2} < 0$$

we know that

$$\left.\frac{\partial EU_P}{\partial z}\right|_{z=1-b_A} = -\frac{(b_A + y_2 - 1)^2}{4} < 0$$

Thus, the solution z^* to the maximization problem with the constraint $z \geq 1 - b_A$ is $z^* = 1 - b_A$.

4. Step 3 has shown that $[b_A, x] \cup \{z^*\}$ is the optimal among all delegation sets with the form $[y_1, y_2] \cup [y_3, y_4]$. As the final step, we need to show that $[b_A, x] \cup \{z^*\}$ dominates any other possible delegation sets. Because of Remark 2, the optimal delegation sets which consists no more than two intervals of implementable delegated decisions. Therefore, we only need to show that P's expected payoff under the delegation set $[b_A, x] \cup \{z^*\}$ is higher than that under the single-interval delegation set $[0, x]$. P's expected payoff under the union of action sets is

$$EU_P^{[0,x]\cup\{z^*\}} = -b_A^2 (x - b_A) - \frac{1}{3}[2\left(\frac{1 - b_A - x}{2}\right)^3 + 6\left(\frac{1 - b_A - x}{2}\right) b_A^2 + 2b_A^3]$$

On the other hand, if the delegation set is $[0, x]$, P's expected payoff is

$$EU_P^{[0,x]} = -\int_0^{x-b_A} b_A^2 d\theta - \int_{x-b_A}^1 (\theta - x)^2 d\theta = -b_A^2 (x - b_A) - \frac{1}{3}\left[(1 - x)^3 + b_A^2\right]$$

Therefore, it suffices to show that $EU_P^{[0,x]\cup\{z^*\}} - EU_P^{[0,x]} \geq 0$ for any

$x \leq 1 - b_A$. In other words, we need to demonstrate that for any $x \leq 1 - b_A$, we have

$$2\left(\frac{1-b_A-x}{2}\right)^3 + 6\left(\frac{1-b_A-x}{2}\right)b_A^2 + b_A^3 - (1-x)^3 \leq 0$$

Because the first derivative of the LHS with respect to x is

$$-3\left[\left(\frac{1-b_A-x}{2}\right)^2 - (1-x)^2 + b_A^2\right]$$

Denote $x = 1 - b_A - \varepsilon$ where $\varepsilon \geq 0$, we have the terms in the bracket as $\left(\frac{\varepsilon}{2}\right)^2 - (b_A+\varepsilon)^2 + b_A^2$, which is negative. Hence, we have $\frac{\partial LHS}{\partial x} \geq 0$ for any $x \leq 1 - b_A$. Besides, when $x = 1 - b_A$, the LHS turns out to be 0, i.e., $EU_P^{[0,x]\cup\{z^*\}} = EU_P^{[0,x]}$. Therefore, we have

$$EU_P^{[0,x]\cup\{z^*\}} - EU_P^{[0,x]} \geq 0, \forall x \leq 1 - b_A$$

Hence, for P delegating the union of an interval and a point dominates interval delegation.

PROOF OF PROPOSITION 3

If $x^* \geq b_A$ fails, e.g., $b_M \leq \overline{b_M}$, then there is no interval of implementable decisions in Y_A. Consequently, the ideal decisions of A is not available even in the lowest state ($\theta = 0$). Hence, P picks up two decisions x and z, which corresponding to the low states and the high states, respectively, to maximize (3), which is modified as

$$\max_{x,z} - \int_0^{\frac{x+z}{2}-b_A} (\theta - x)^2 \, d\theta - \int_{\frac{x+z}{2}-b_A}^1 (\theta - z)^2 \, d\theta \tag{11}$$

To ensure that $\{x\} \cup \{z\}$ will be delivered to A, we need the incentive compatibility constraint:

$$\begin{aligned} & - \int_0^{\frac{x+z}{2}-b_A} (\theta - x + b_M)^2 \, d\theta - \int_{\frac{x+z}{2}-b_A}^1 (\theta - z + b_M)^2 \, d\theta \\ \geq \ & \max\{- \int_0^1 (\theta - x + b_M)^2 \, d\theta, - \int_0^1 (\theta - z + b_M)^2 \, d\theta\} \end{aligned}$$

This constraint says that M would not further shrink the two-action delegation set to a singleton. Tedious calculation shows that any x and z satisfying $2b_A \leq z + x \leq 2 - 2b_A + 4b_M$ will satisfy this constraint. Because the payoff in (11) is increasing with respect to x, P can choose any x and z satisfying the same jump condition (6) to maximize the new objective function (11). Hence, we can rewrite

$$x = 1 - b_A + 2b_M - \lambda \text{ and } z = 1 - b_A + 2b_M + \lambda$$

We substitute it into (11), and derive the first-order condition, then we have the optimal wedge

$$\lambda^* = \frac{1}{2} - 2\left(2b_A - 1\right) b_M + 4b_M^2 - b_A$$

We need that $\lambda^* \geq 0$. Furthermore, to guarantee that two-action delegation set is feasible, we need $0 \leq x \leq z \leq 1$.

First, we have

$$\begin{aligned}
\lambda^* &= \frac{1}{2} - 2\left(2b_A - 1\right) b_M + 4b_M^2 - b_A \\
&= 4\left[b_M^2 + \frac{\left(1 - 2b_A\right) b_M}{2} + \frac{\left(1 - 2b_A\right)^2}{16}\right] - \frac{\left(1 - 2b_A\right)^2}{4} + \frac{1 - 2b_A}{2} \\
&= 4\left[b_M + \frac{\left(1 - 2b_A\right)}{4}\right]^2 + \frac{1 - 2b_A}{2}\left(1 - \frac{1 - 2b_A}{2}\right) \geq 0
\end{aligned}$$

The last inequality uses the fact that $b_A \in \left[0, \frac{1}{2}\right]$.

Then, we check whether $x^* \geq 0$. Substitute λ^* into the expression of x, we find out that this needs

$$\frac{b_A - \sqrt{b_A^2 + \frac{1}{2}}}{2} \leq b_M \leq \frac{b_A + \sqrt{b_A^2 + \frac{1}{2}}}{2}$$

On the other hand, $z^* \leq 1$ requires that

$$\frac{b_A - 1 - \sqrt{b_A^2 + \frac{1}{2}}}{2} \leq b_M \leq \frac{b_A - 1 + \sqrt{b_A^2 + \frac{1}{2}}}{2}$$

Obviously, $\frac{b_A-1-\sqrt{b_A^2+\frac{1}{2}}}{2} \leq \frac{b_A-\sqrt{b_A^2+\frac{1}{2}}}{2}$, define $\underline{b_M} \equiv \frac{b_A-\sqrt{b_A^2+\frac{1}{2}}}{2}$, we need $b_M \geq \underline{b_M}$ to ensure that $x, z \in [0,1]$. Moreover, since

$$\overline{b_M} \leq \frac{b_A - 1 + \sqrt{b_A^2 + \frac{1}{2}}}{2} \leq \frac{b_A + \sqrt{b_A^2 + \frac{1}{2}}}{2}$$

always holds, we could conclude that when $b_M \in \left[\underline{b_M}, \overline{b_M}\right]$, two-action delegation set is feasible.

PROOF OF COROLLARY 1

1. Since $b_M \leq 0$, from (8), it's straightforward to see that $\frac{\partial\lambda}{\partial b_M} \leq 0$. Therefore, when the delegation set contains an interval of decisions, the bigger b_M, the smaller the wedge λ.

2. Then we turn to (10). Here $\frac{\partial\lambda}{\partial b_M} \leq 0$ if $b_M \leq -\frac{1-2b_A}{4}$. However, since $b_A \in \left[0, \frac{1}{2}\right]$, $-\frac{1-2b_A}{4} \geq \overline{b_M}$ always holds, we still have for $\underline{b_M} \leq b_M \leq \overline{b_M}$, a less biased M reduces the wedge.
   Therefore, we can conclude that when M is less downwardly biased, the scope of the agent's discretion widens.

B Appendix B

In this section we will briefly describe the optimal delegation mechanism when $b_M \geq b_A \geq 0$.

By Lemma 1, we know that an unrestricted overly-biased intermediary would like to impose a bottom $b_M - \Delta$ on the agent's choice set, while the principal would only impose a ceiling $1 - b_A$. Hence, there exists the derived conflict of interest in the high state $[1 - b_A, 1]$ and the low state $[0, 2(b_M - b_A)]$. Since M would like to expand A's discretion for the segment of the high state, P could still attain her ideal cap. However, because compared with P, M wants to take more strict stance in the segment of the low state, the decision-making becomes flat for the realizations of the low state. Therefore, we could still concentrate on the delegation set with the form $[0, 2b_M - b_A] \cup [x, 1]$. To demonstrate the optimality of a hole, we only need to prove that when the agent is downwardly biased, and some modest actions are ruled out, then it is optimal to include a low action in the delegation set.

As an analogue to Lemma 4, we establish the following lemma on the behavior of a restricted principal with a downwardly biased agent.

Lemma 6 *In P—–A direct delegation, if $b_A \leq 0$, and an interval of intermediate actions $(-b_A, x)$ is removed from the available decisions, then the restricted optimal delegation set is $\{z^*\} \cup [x, 1]$, in which $z^* = -b_A$.*

Problem 2 *Still, by Lemma 2 and Remark 2, the restricted optimal delegation set will be a union of two connected sets, e.g., the optimal delegation set admits at most one discontinuity. Thus, we assume that Y_A takes the form $[y_1, y_2] \cup [y_3, y_4]$, in which $y_2 \leq -b_A \leq x \leq y_3$. P's expected payoff would be*

$$\begin{aligned}
EU_P &= -\int_0^{y_1-b_A} (\theta - y_1)^2 \, d\theta - \int_{y_1-b_A}^{y_2-b_A} b_A^2 d\theta - \int_{y_2-b_A}^{\frac{y_2+y_3}{2}-b_A} (\theta - y_2)^2 \, d\theta \\
&\quad - \int_{\frac{y_2+y_3}{2}-b_A}^{y_3-b_A} (\theta - y_3)^2 \, d\theta - \int_{y_3-b_A}^{y_4-b_A} b_A^2 d\theta - \int_{y_4-b_A}^{1} (\theta - y_4)^2 \, d\theta \\
&= -\frac{1}{3}\left[(-b_A)^3 - (-y_1)^3\right] - b_A^2 (y_2 - y_1 + y_4 - y_3) - \frac{1}{3}\left[\left(\frac{y_3 - y_2}{2} - b_A\right)^3 - (-b_A)^3\right] \\
&\quad -\frac{1}{3}\left[(-b_A)^3 - \left(\frac{y_2 - y_3}{2} - b_A\right)^3\right] - \frac{1}{3}\left[(1 - y_4)^3 - (-b_A)^3\right]
\end{aligned}$$

Take the first derivatives of EU_P with respect to y_1, y_2, y_3, y_4, respectively, we have

$$\begin{aligned}
\frac{\partial EU_P}{\partial y_1} &= b_A^2 - y_1^2 \geq 0 \\
\frac{\partial EU_P}{\partial y_2} &= \left(\frac{y_3 - y_2}{2}\right)^2 \geq 0 \\
\frac{\partial EU_P}{\partial y_3} &= -\left(\frac{y_3 - y_2}{2}\right)^2 \leq 0 \\
\frac{\partial EU_P}{\partial y_4} &= (1 - y_4)^2 - b_A^2 \geq 0
\end{aligned}$$

Therefore, we have the optimal $y_1 = y_2 = z^* = -b_A$, and $y_3 = x$, $y_4 = 1$. The restricted optimal delegation set turns out to be a union of a low singleton and an interval of high options.

This lemma has implications for the sub-delegation behavior of the overly-biased intermediary, given that Y_M is a union of two intervals. It is easy to proceed to the principal's delegation problem with an overly-biased intermediary. Similar to Proposition 2, the principal would impose the low action z and the lower endpoint of the interval x. Besides, the principal would still impose the cap $1 - b_A$.

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Figure

Figure 1. Optimal delegation set with *opposing biased* subordinates

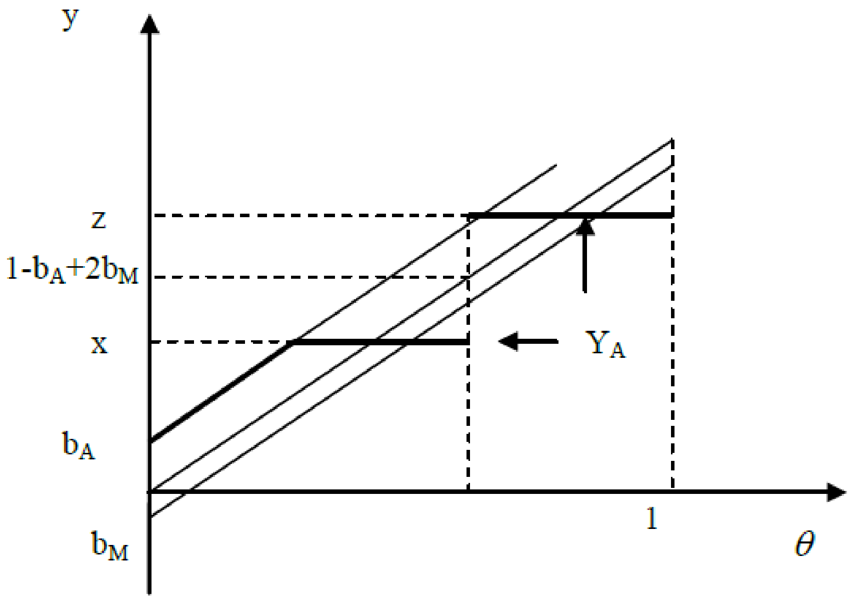

Figure 2 The features of optimal delegation set when $b_M < b_A$