International Portfolios and the U.S. Current Account

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Abstract

This paper presents a dynamic three-country endowment model, with both traded and non-traded goods. The main innovation of the model is the introduction of evolutive dynamic preferences that explain consumption patterns and international portfolio composition. The model departs from a home-biased state of the world that decreases through time creating a rebalancing effect on the international portfolio, raising the demand for the high-income country assets. The model sheds light on the behavior of the current accounts and the bilateral current accounts of the three countries. Results of the simulations are presented and the significant implications on what at is the current account sustainability of the U.S. and its future path are analyzed.

During the late nineties and the first part of this decade there has been an extensive discussion of the causes and consequences of the U.S. current account deficit. Figure 1 describes the U.S. current account defiict from 1980 to 2009. According to the IMF's World Economic Outlook data (April 2009), the U.S. current account deficit has been growing since the early nineties. In 2006 the deficit reached its peak of around 6% of the GDP (around $800 billions). The trend reverses in 2007 and following the downturn of 2008 the deficit has subsided even more. The persistent negative trend of the deficit during the last decade shows a possible disconnection between the current account and business cycles. This idea is suppported by the fact that during this period the U.S. economy has experienced three recessions without having a significant effect on the current account deficit. One would expect that during downturns in the U.S. economy the current account deficit should have subsided.

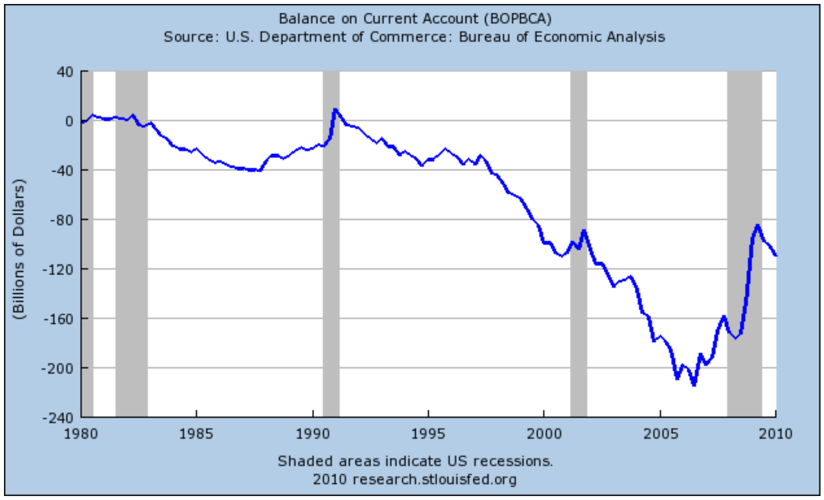


Figure 1

This figure presents the behavior of the US current account in absolute terms

Previous literature regarding the current account deficit of the U.S. has addresed the need for equilibrated fiscal policy and exchange rate adjustment.

Reduction of U.S. fiscal deficit has been one of the policy recommendations to reduce the current account imbalance of the U.S. but as Mussa (2007) state "it is also important to emphasize that U.S. fiscal consolidation is not the be-all and end-all of policies to address the U.S. external deficit." Furthermore "the fact is that the U.S. current account deficit disappeared between 1987 and 1991 as the fiscal deficit expanded to a postwar peak (as a share of U.S. GDP). Then the current account deficit widened to a new record of over 4 percent of U.S. GDP in 2000 as the fiscal deficit moved from large deficit to significant surplus".

Another important stream of literature explores the behavior of the international portfolio and its implications for the U.S.current account. The increase in the U.S. demand for foreign goods and the increase in the foreign demand for U.S. assets have been thought to drive the U.S. account deficit (Blanchard et.al, 2005). Macroeconomic imbalances reflect mostly private saving and investment decisions, and fiscal deficits often play a marginal role (Blanchard, 2007). At the same time, it is accepted that large current account imbalances could be corrected by a depreciation of the dollar, and that this depreciation would have an effect on the trade patterns (Blanchard, Giavazzi and Sa, 2005)

Sustained rise in the U.S. current account deficit, decline of long run real interest rates and the rise in the share of U.S. assets in global portfolio could be rationalized as an equilibrium outcome of two observed forces: the potential growth differentials among different regions of the world and, also to the heterogeneity in these regions' capacity to generate financial assets from real investments(Caballero et al., 2006). Thus, while it is generally accepted that the international portfolio composition has an effect on the current account deficit, the dynamic behavior between the two is not clear yet.

In this paper I use a dynamic extension of the three-country model by Obstfeld and Rogoff (2005) with imperfect substitute assets to explain wether the portfolio growth and portfolio rebalancing effects combined could have affected the behavior of the current account deficit of the U.S. in the last three decades. With the model I try to answer the question of why large U.S. current account deficits have not had a large effect on the exchange rate? And probably the most important question, how long does this situation could hold?.

The predictions of the model regarding international portfolio composition adjust to the observed pattern of the US current account during the last decades. The model also gives insight into the behavior of the bilateral current account of the three countries involved.

The remainder of this paper is organized as follows. Section I summarize previous work on current account deficit, portfolio theory and home bias in portfolios. Section II describes the model proposed in the paper. Section III presents the simulations and empirical analysis of the model, and Section IV concludes the paper.

1 Literature review

1.1 Current Account

Previous literature regarding the current account deficit of the U.S. has addresed the need for equilibrated fiscal policy and exchange rate adjustment.

Reduction of U.S. fiscal deficit has been one of the policy recommendations to reduce the current account imbalance of the U.S. but as Mussa (2007) state "it is also important to emphasize that U.S. fiscal consolidation is not the be-all and end-all of policies to address the U.S. external deficit." Furthermore "the fact is that the U.S. current account deficit disappeared between 1987 and 1991 as the fiscal deficit expanded to a postwar peak (as a share of U.S. GDP). Then the current account deficit widened to a new record of over 4 percent of U.S. GDP in 2000 as the fiscal deficit moved from large deficit to significant surplus".

Kim and Roubini (2007) collect literature arguing in favour of budget deficit, current account deficit and real exchange rate appreciation "General equilibrium endowment economy models of small open economy with optimizing individuals and no capital account restrictions (Sachs, 1982, for a one-good model and Frenkel and Razin, 1996, for both one and two-goods models), standard Keynesian models such as the Mundell-Fleming model and its rational expectations variants such as Dornbusch (1976), and calibrated international real business cycle models with investment such as Baxter (1995), Kollmann (1998), and Erceg, Guerrieri, and Gust (2005) tend to provide such predictions in most cases"

This paper is in the spirit of Caballero, Farhi and Gourinchas (2006) which states that “sustained rise in the U.S. current account deficit, decline of long run real interest rates and the rise in the share of U.S. assets in global portfolio” could be rationalize “as an equilibrium outcome of two observed forces: a) potential growth differentials among different regions of the world and, b) heterogeneity in these regions’ capacity to generate financial assets from real investments”.

Blanchard (2007) comments that macroeconomic imbalances “reflect mostly private saving and investment decisions, and fiscal deficits often play a marginal role; and the deficits are financed mostly through equity, FDI, and own-currency bonds rather than through bank lending.” This argument suggests that the imbalances come from private decisions, therefore subject to macroeconomic modeling.

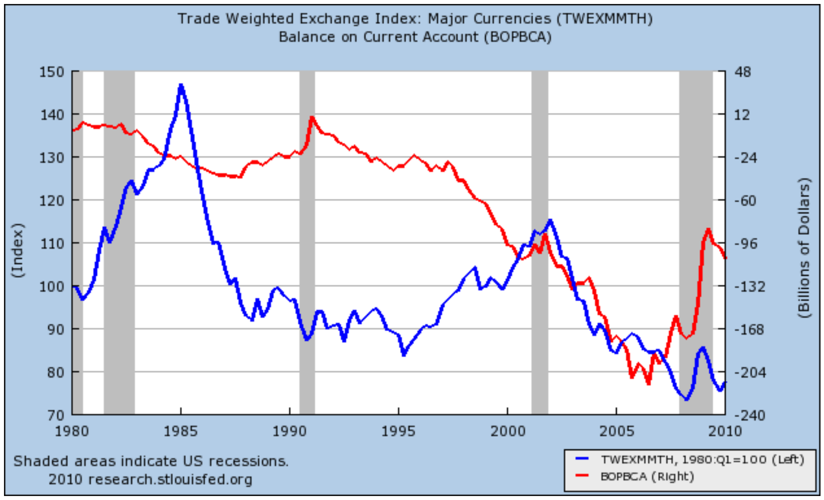
Obstfeld and Rogoff (2005) note that their "baseline simulation, in which Asia’s, Europe’s, and the United States’ current accounts all go to zero, implies that the dollar needs to depreciate in real effective terms by 33 percent". The authors also simulate a softer adjustment, "a halving of the U.S. deficit, with counterpart surplus reductions shared by Asia and Europe in the same proportions as in the first simulation (arguably a more likely scenario over the short term) of complete current account adjustment, would lead to a depreciation of the real effective dollar of 17 percent."

The large current account imbalances could be corrected by a depreciation of the dollar, but this depreciation will have an effect on the trade patterns as argue by Blanchard, Giavazzi and Sa (2005) “A large fall in the dollar is not by itself a catastrophy for the United States. By itself, it leads to higher demand and higher output, and it offers the opportunity to reduce budget deficits without triggering a recession. The danger is much more serious for Japan and Western Europe.”

Figure 2

This figure presents the behavior of the US Dollar

The lack of exchange rate adjustment comes as a surprise for many authors that have predicted large depreciations. “Our practical conclusions are that substantially more depreciation is to come, surely against the yen and the renminbi, and probably against the euro.” As noted by Blanchard, Giavazzi and Sa (2005). Most of the previous papers have focused on the real exchange rate but as Edwards (2005) puts it “the actual adjustment will depend on the pass through coefficient, as well as on exchange rate policies followed by some important U.S. trade partners, including China, Japan and other Asian countries.” He also state in his 2007 paper that “a realignment of global growth - with Japan and the Euro Zone growing faster, and the U.S. moderating its growth - would only make a modest contribution towards the resolution of global imbalances”. To actually reduce the global imbalances “a reduction in China’s (very) large surplus will be needed if global imbalances are to be resolved.”



1.2 Portfolio Theory and Home Bias in Portfolios

Another important stream of literature explores the behavior of the international portfolio and its implications for the U.S.current account. The increase in the U.S. demand for foreign goods and the increase in the foreign demand for U.S. assets have been tought to drive the U.S. account deficit (Blanchard et.al, 2005). In a world with free capital markets, capital should ideally flow from richer to poorer countries. Nevertheless "the classical Heckscher-Ohlin-Mundell paradigm states that trade and capital mobility are substitutes, in the sense that trade integration reduces the incentives for capital to flow to capital scarce countries. it has been demonstrated that in a world with heterogeneous financial development, the classic conclusion does not hold. In particular, in less financially developed economies trade and capital mobility are complements and not substitutes (Antras and Caballero, 2007).

International portfolio diversification has increase in recent years; however portfolios remain greatly biased towards domestic assets. There seems to be a correlation between home bias in consumption and home bias in portfolio as mentioned by Obstfeld and Rogoff (2000). Home bias in consumption can be explained in part by the existence of non traded goods and the assumption that country's residents have relative preference towards the traded good that is produced at home (Obstfeld and Rogoff, 2005). Some authors have link bilateral trade and openness to assets holdings. Portes and Rey (2005) use gravity model to cross-border equity flows and found that distance, which proxies' information asymmetries, is a surprisingly very large barrier to cross-border asset trade. Whereas Aviat and Coeurdacier (2005) show that there exist "complementarity between bilateral trade in goods and bilateral asset holdings"; where the "distance affects asset holdings mainly through its impact on trade in goods".

They prove “that a 10% increase in bilateral trade raises bilateral asset holdings by 6% to 7%.” The system they estimated shows that distance affects asset holdings mainly through its impact on trade in goods".

According to Coeurdacier, Kollamann and Martin (2008), there are “two familiar explanations of equity home bias: transaction costs that impede international diversification, and terms of trade responses to supply shocks that provide risk sharing”. Coeurdacier (2008) in his model shows that “the larger the home bias in consumption, the larger the home bias in portfolios” this explains the empirical relation between openness to trade and international diversification in portfolios.

Collard, Dellas Diba, and Stockman (2007) demonstrate “that international trade in goods is the main determinant of international equity portfolios”. Their model entail “that investors can achieve full international risk diversification if the share of wealth invested in foreign equity matches their country’s degree of openness (the imports to GDP share)”.

Baxter, Jermann and King (1995) argue that the presence of non traded goods do not explain the presence of home bias in portfolio under free international trade of financial assets. Home bias arraises only when there is a very low degree of substitution between traded and non trade goods. Baxter and Jermann (1997) argue that hedging for human capital risk should reduce even more the home bias in portfolio. Also Michaelides (2001) state that “optimal portfolios are internationally diversified while positive correlation between domestic stock market returns and permanent labour income shocks can worsen the home equity bias puzzle”. But he also argue that “small costs associated with investing abroad” is “sufficient to either deter households from participating in a foreign market or generate a substantial bias for home equities.”

The paper by Milesi-Ferretti (2003) makes a review of the main factors that could affect the level of home bias in portfolio. Such as:

- Liberalization of the capital account can increase the level of international assets hold.

- Openness in good markets can reduce portfolio home bias by increasing the willingness to conduct international financial transactions.

- Higher income per capita is associated with lower risk aversion. Given that international transactions are perceived as riskier than domestic transactions, this may also raise international assets hold.

- Higher income per capita can reduce home bias in portfolio if participation in foreign assets markets involves fixed costs.

- Underdeveloped domestic financial sector will push domestic agents to invest on foreign markets.

- Local financial intermediaries that distribute international assets increase agent exposure to foreign financial markets, increasing the desire for international diversification.

The change in the composition of the portfolio is due to an increase in the wealth of the rest of the World, especially in low-income countries. As noted by Caballero, Farhi and Gourinchas (2006) “The importance of US assets in global portfolios has increased throughout the period and now amounts to over

17 percent of the rest of the world's financial wealth, which is equivalent to 43 percent oftheir annual output." Contrary to what Ventura (2001) argue that the U.S. deficit is caused "by the spectacular increase in U.S. wealth experienced in the 1990s" , it is possible to argue that the portfolio growth effect and the portfolio rebalancing effect of the rest of the World does have an effect on the current account deficit of the U.S.

All the above literature leads me to conclude that portfolio theory can be helpful explaining current account deficits without significant exchange rate adjustments.

Previous literature has treated home bias in consumption as an exogenous ad-hoc feature of the model. Nevertheless, this shouldn't be the case. Assuming path dependent preferences, it is possible to show, that home bias arises from costly trade and from the fact that the level of international integration in the goods markets is incomplete. The degree of home bias that appear in the data today, will probably disappear, and the portfolio reallocation at international level will have important implications for stability of international finance and is likely to produce important macroeconomic imbalances as the one experimented by the U.S. in the last decade.

In the next section I present a dynamic three-country endowment model, with both traded and non-traded goods. The main innovation of the model is the introduction of evolutive dynamic preferences that explain consumption patterns and international portfolio composition. The model departs from a home-biased state of the world that decreases through time creating a rebalancing effect on the international portfolio, raising the demand for the high-income country assets. The model sheds light on the behavior of the current accounts and the bilateral current accounts of the three countries.

2 The Model

Before developing the model, it is usefull to build a framework for the recent evolution of the world economy in the last decades.

During the last two decades there has been an accelerated increase in the income and wealth of poor -and to least extent medium income- countries. Poor countries have also experienced the development of a middle class within their population. At the same time. growth of exports and imports have outpaced GDP growth, implying a greater diversification of the consumption bundle of the average consumer in the world. This latter change in the consumption bundle has a corresponding effect on the portfolio composition as Obstefeld and Rogoff (2000) argue.

Simulteanously, the increased financial markets integration has allowed the average investor to access (directly or indirectly) the international financial markets. U.S. assets are attractive because the are characterized by high liquidity; low risk; traded in an efficient and transparent financial market and within a stable economy (Lane and Milesi-Ferretti, 2003) .

As people become richer, their portfolio composition tends to change towards a more diversified one. This probably comes from the fact that gains from diversification increases, and or the marginal cost of doing it decreases. This can be interpreted as a fix cost per transaction. As the volume of the transaction increases, the marginal cost will decrease.

Under the circumstance of a relative increase in the share of U.S. assets in the world's portfolio, the recent U.S. current account deficit and its sustainability has to be review. The main reason for this is the expectation that the standard correction channel, meaning the exchange rate, will not correct these large deficits. This has two possible implications, one that a large depreciation of the dollar will not reduce the current account deficit, but instead could create a large distortion in the international markets, making artificially cheaper the U.S. assets and therefore increasing the demand for them, it will also mean higher inflation (because of the pass-through) in the U.S. without closing the gap of the current account. And second, large current account deficits in the U.S. could be seen has a situation that could last more than previously thought.

The "run" towards US assets that has helped finance large current account deficits could come to an abrupt end for three reasons: Sudden stop of income growth in developing countries; a swift in preferences towards another economy; loss of confidence in the US financial system.

The large accumulation of foreign reserves by developing countries, (especially Asian countries) is related to the rebalancing of the desired portfolio composition towards more diversification, that would help secure the purchase power. Excess foreign reserves may in some cases be a reflection of incomplete diversification of the portfolio of the representative agent.

In the following subsections I develop a model with imperfect substitute assets that explains how as income grows, home bias in consuption falls, generating also a fall in the home bias portfolio. The latter fall of the home bias portfolio leads me to the conclusion that international portfolio rebalancing finances the current account deficit in the U.S.

2.1 Core Model

I start my model from the a reduced version of the three-country endowment model as in Obstfeld and Rogoff (2005). In the first part I reduce the number of countries to two. Then the third country is added.

I then introduce evolving dynamic preferences in which the home and U.S. bias changes when the relative income changes. The model is thus a two-country model, with non-traded goods and three traded goods, with an infinitely living representative agent, preferences across countries are not symmetric but mirror like. The model also assumes imperfect substitutability of assets.The equilibrium terms of trade and the relative price of traded and non-traded goods (and thus both bilateral and effective real exchange rates) are determined endogenously

Another assumption is that countries differ in the initial level of income, and there is no population growth in any of the countries. Where by the income difference it is possible to differentiate the countries into high, middle and low income. For simplicity purposes I use a representative agent for each country[1].

For notational purposes the three countries will be named as follows: i, and j. In the three-country model I will add a country k. Countries will not be equal, one country is associated to the high-income country; j is the low-income country. I normalize the income of each representative agent with respect to that of country i, which in the paper is associated to the high-income country (U.S.). Thus,

$$\frac{Y^i}{Y^i} = y^i = 1, \ \frac{Y^j}{Y^i} = y^j \tag{1}$$

Where by assumption we have that $y^j < y^i = 1$

2.1.1 Preferences

Following the common practice in open economy litearture, individual preferences are CES as follows:

$$U\left(C_{m,t}\right) = \frac{\left(C_{m,t}\right)^{1-\varphi}}{1-\varphi} \quad for\ m = i, j, k \tag{2}$$

Where φ is the intertemporal elasticity of subsititution[2]. Note that the intertemporal elasticiy of subtitution is equal across countries.

Consumers in the three countries allocate their spending between traded and non trade goods as follows:

$$C_{m,t} = \left[\gamma^{\frac{1}{\theta}} \left(C^T_{m,t}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C^N_{m,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \quad form = i, j, k \tag{3}$$

Parameter θ is the (constant) elasticity of substitution between traded and nontraded goods. The proportion of consumption of traded (γ) and non-traded $(1-\gamma)$ goods remains constant throughout the model. I assume that the consumption of traded and non-traded goods is independent from the level of income of the agent. Preferences over traded goods differ accross countries and according to the level of income in each of them, arising an heterogeneity among countries that hasn't been previously studied.

Following Obstfeld and Rogoff (2005), the utility of consumption of traded goods is:

[1] Without changing the basic results of the model, the model can also be generalized to a model with heterogeneous agents, where each country has a distribution of agents who consume different baskets of traded goods according to their relative income.

[2] Note that when $\varphi = 1$ the individual's preferences become a logarithmic function. Thus $U\left(C_{m,t}\right) = \log\left(C_{m,t}\right)$

$$C_{i,t}^{T}=\left[\left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}}\left(c_{ii,t}\right)^{\frac{\eta-1}{\eta}}+\left(1_t^i-\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}}\left(c_{ij,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

Parameter η is the (constant) elasticity of substitution between domestically produced traded goods and imports from either foreign country.Coefficient $\widetilde{\alpha}_t^i$, is not constant. This coefficient depend on the relative income of each country, and their underlying coefficient α. The functions of these underlying coefficients are[3]:

$$\widetilde{\alpha}_t^i=\frac{\alpha}{\alpha+(1-\alpha)\left(y_t^i\right)^{2\rho}} \tag{5}$$

Where $1>\alpha>0$ [4]; $\rho>0$ is the velocity of adjustment to the long run value. In simpler terms, the higher the value of ρ, the slower the adjustment of the coefficient to its long run value.

The changing coeficient $\widetilde{\alpha}_t^i$ can be interpreted as if the foreign traded good is a form superior good. So when income rises the desired level of consumption of the boundle of foreign traded goods increase, more than the level of income. This increased desire to consume foreign traded goods is limited by the underlying coefficient α.

Table 1 summarizes the diversification pattern of the consumption bundles according to the level of normalized income. If the value of the normalized income is close to zero, the representative agent will devote most ofher income associated with traded goods in home-produced ones. This means a complete home-bias, and is equivalent to autarky.On the other hand, if the value of the normalized income is close to one, the representative agent will diversify her consumption matching the values of α.

The model still applies when there is no ad hoc home bias in the coefficient α, it ensures that for low levels of normalized income the value of $\widetilde{\alpha}$ is almost one. Furthermore the model still gives logical weights for values of y superior to one.

Table 1: Consumption bundles according to the level income

This table summarizes the diversification pattern of the consumption bundles according to the level of normalized income.$y^i$is defined as$\frac{Y^i}{Y^i}$

$y_i\to 0$	$\widetilde{\alpha}=1$	$1-\widetilde{\alpha}=0$
$y_i\to 1$	$\widetilde{\alpha}=\alpha$	$1-\widetilde{\alpha}=\alpha$

For better understanding of the model, Table 2 shows the distribution of the coefficients of the preferences for each country.It is possible to observe that for the case of the high-income country. the value of $\widetilde{\alpha}$ is equal to the value of α .

[3] Appendix A addresses the mathematical derivation of the function of $\widetilde{\alpha}$ and $\widetilde{\beta}$.

[4] Note that if $\widetilde{\beta}=1$ or if $\widetilde{\alpha}=\widetilde{\beta}$, the model transforms itself into a two-country model, in which at low levels of income there will be home bias, and as income grows the bias will be toward the high-income country (US).

Table 2: Distribution of coefficients across countries

Goods/Country	i	j
i	$\widetilde{\alpha}_i$	$1-\widetilde{\alpha}_i$
j	$1-\widetilde{\alpha}_j$	$\widetilde{\alpha}_j$

Where $\widetilde{\alpha}$ represent the desired proportion of spending in locally produced traded good.

2.1.2 Demands for traded and non-traded goods and the CBPI

Optimizing the utility function of equation (2) subject to the budget constraint, I obtain the demand for the traded and non traded goods.

$$\max_{\{C_{i,t}^T, C_{i,t}^N\}} \left[\gamma^{\frac{1}{\theta}} \left(C_{i,t}^T\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C_{i,t}^N\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{6}$$

$$\text{s.t. } C_{i,t}^T + p_{i,t} C_{i,t}^N \leq Z_{i,t}. \tag{7}$$

Where $p_{i,t} = \frac{P_{i,t}^N}{P_{i,t}^T}$

The first order conditions for this problem are:

$$C_{i,t}^T: \frac{\theta}{\theta-1}\left[\gamma^{\frac{1}{\theta}} \left(C_{i,t}^T\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C_{i,t}^N\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{-1}{\theta-1}} \frac{\theta-1}{\theta}\gamma^{\frac{1}{\theta}} \left(C_{i,t}^T\right)^{\frac{-1}{\theta}} - \lambda = 0 \text{ , and} \tag{8}$$

$$C_{i,t}^N: \frac{\theta}{\theta-1}\left[\gamma^{\frac{1}{\theta}} \left(C_{i,t}^T\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C_{i,t}^N\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{-1}{\theta-1}} \frac{\theta-1}{\theta}(1-\gamma)^{\frac{1}{\theta}} \left(C_{i,t}^N\right)^{\frac{-1}{\theta}} - \lambda p_{i,t} = 0 \tag{9}$$

Which implies that

$$\gamma^{\frac{1}{\theta}} \widetilde{C}_{i,t} \left(C_{i,t}^T\right)^{\frac{-1}{\theta}} = \lambda, \text{ and} \tag{10}$$

$$(1-\gamma)^{\frac{1}{\theta}} \widetilde{C}_{i,t} \left(C_{i,t}^N\right)^{\frac{-1}{\theta}} = \lambda p_{i,t} \tag{11}$$

Combining the latter conditions I obtain the relationship between consumption of traded and non-traded goods and the relative price as follows:

$$(1-\gamma)^{\frac{1}{\theta}}\,\widetilde{C}_{i,t}\left(C^N_{i,t}\right)^{\frac{-1}{\theta}} = \gamma^{\frac{1}{\theta}}\widetilde{C}_{i,t}\left(C^T_{i,t}\right)^{\frac{-1}{\theta}}\,p_{i,t} \tag{12}$$

Where $\widetilde{C}_{i,t} = \left[\gamma^{\frac{1}{\theta}}\left(C^T_{i,t}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}}\left(C^N_{i,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{-1}{\theta-1}}$.

Thus I obtain the following relative demand of non-traded goods with respect to the traded ones:

$$\left(\frac{1-\gamma}{\gamma}\right)^{\frac{1}{\theta}}\left(\frac{C^N_{i,t}}{C^T_{i,t}}\right)^{\frac{-1}{\theta}} = p_{i,t} \tag{13}$$

$$\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{C^N_{i,t}}{C^T_{i,t}}\right) = p_{i,t}^{-\theta} \tag{14}$$

Substituting the consumption of traded goods ($C^T_{i,t}$) in the budget constraint defined in equation (9), I obtain the demand for non-traded goods in country i:

$$\left(\frac{\gamma}{1-\gamma}\right)\left(\frac{C^N_{i,t}}{Z_{i,t}-p_{i,t}C^N_{i,t}}\right) = p_{i,t}^{-\theta} \tag{15}$$

$$\iff \gamma C^N_{i,t} = (1-\gamma)\,p_{i,t}^{-\theta}Z_{i,t} - (1-\gamma)\,p_{i,t}^{1-\theta}C^N_{i,t} \tag{16}$$

$$\iff C^N_{i,t} = \frac{(1-\gamma)\,p_{i,t}^{-\theta}}{\gamma+(1-\gamma)\,p_{i,t}^{1-\theta}}Z_{i,t} \tag{17}$$

Using again the budget constraiant of equation (9), I now obtain the demand for traded goods in country i as:

$$C^T_{i,t} = \left(1-\frac{(1-\gamma)\,p_{i,t}^{1-\theta}}{\gamma+(1-\gamma)\,p_{i,t}^{1-\theta}}\right)Z_{i,t} \tag{18}$$

$$C_{i,t}^{T} = \frac{\gamma}{\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}} Z_{i,t} \tag{19}$$

Consumer Based Price Index

I obtain the consumer-based price index as:

$$P_t = CBPI_t \tag{20}$$

$$\left[\gamma^{\frac{1}{\theta}} \left(C_{i,t}^{T}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C_{i,t}^{N}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} = 1 \tag{21}$$

$$\left[\gamma^{\frac{1}{\theta}} \left(\frac{\gamma}{\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}} P_t\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(\frac{(1-\gamma)\, p_{i,t}^{1-\theta}}{\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}} P_t\right)^{\frac{\theta-1}{\theta}}\right] = 1 \tag{22}$$

$$\left[\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}\right] P_t^{\frac{\theta-1}{\theta}} = \left(\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}\right)^{\frac{\theta-1}{\theta}} \tag{23}$$

Which implies that

$$P_t = \left(\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{24}$$

2.1.3 Demands for each traded good and the corresponding CPBI

In this case $Z_{i,t}^{T}$ is the available income intended for the consumption of traded goods. Thus, departing from equation (4) and optimizing the preferences for traded goods of the individual, subject to the budget constraint.

$$\max_{\{c_{ii,t},c_{ij,t},c_{ik,t}\}} \left[\left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ii,t})^{\frac{\eta-1}{\eta}} + \left(1-\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ij,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{25}$$

$$\text{s.t. } P_{ii,t}c_{ii,t} + P_{ij,t}c_{ij,t} \leq Z_{i,t}^{T} \tag{26}$$

The first order conditions of this problem are:

$$c_{ii,t} : \left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ii,t})^{\frac{-1}{\eta}} \widetilde{C_{i,t}^{T}} = \lambda P_{ii,t} \tag{27}$$

$$c_{ij,t} : \left(1-\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ij,t})^{\frac{-1}{\eta}} \widetilde{C_{i,t}^{T}} = \lambda P_{ij,t} \tag{28}$$

Where $\widetilde{C_{i,t}^{T}} = \left[\left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ii,t})^{\frac{\eta-1}{\eta}} + \left(1-\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ij,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{-1}{\eta-1}}$

Combining the first order conditions of $c_{ii,t}$and $c_{ij,t}$:

$$\frac{\left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ii,t})^{\frac{-1}{\eta}} \widetilde{C_{i,t}^{T}}}{P_{ii,t}} = \frac{\left(1-\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}} (c_{ij,t})^{\frac{-1}{\eta}} \widetilde{C_{i,t}^{T}}}{P_{ij,t}} \tag{29}$$

$$\left(\frac{c_{ii,t}}{c_{ij,t}}\right)^{\frac{-1}{\eta}} = \left(\frac{1-\widetilde{\alpha}_t^i}{\widetilde{\alpha}_t^i}\right)^{\frac{1}{\eta}} \frac{P_{ii,t}}{P_{ij,t}} \tag{30}$$

$$c_{ii,t} = \frac{\widetilde{\alpha}_t^i}{1-\widetilde{\alpha}_t^i} \left(\frac{P_{ij,t}}{P_{ii,t}}\right)^{\eta} c_{ij,t} \tag{31}$$

or

$$c_{ij,t} = \frac{1-\widetilde{\alpha}_t^i}{\widetilde{\alpha}_t^i} \left(\frac{P_{ii,t}}{P_{ij,t}}\right)^{\eta} c_{ii,t} \tag{32}$$

I define the terms of trade of country i with respect to countries j and k.as:

$$\tau_{ij,t} = \frac{P_{ii,t}}{P_{ij,t}} \tag{33}$$

Using the budget constraint of the traded goods optimization (equation 28), I obtain the demand of country i for good j:

$$P_{ii,t}\frac{\widetilde{\alpha}_t^i}{1-\widetilde{\alpha}_t^i}\left(\frac{P_{ij,t}}{P_{ii,t}}\right)^{\eta} c_{ij,t} + P_{ij,t}c_{ij,t} = Z_{i,t}^T \tag{34}$$

$$\left(1+\frac{\widetilde{\alpha}_t^i}{1-\widetilde{\alpha}_t^i}\left(\frac{P_{ij,t}}{P_{ii,t}}\right)^{\eta-1}\right) P_{ij,t}c_{ij,t} = Z_{i,t}^T \tag{35}$$

$$c_{ij,t} = \left(1+\frac{\widetilde{\alpha}_t^i}{1-\widetilde{\alpha}_t^i}\left(\frac{P_{ij,t}}{P_{ii,t}}\right)^{\eta-1}\right)^{-1}\frac{Z_{i,t}^T}{P_{ij,t}} \tag{36}$$

Expresing the relative prices as terms of trade, the demand of country i for good j:

$$c_{ij,t} = \left(1+\frac{\widetilde{\alpha}_t^i}{1-\widetilde{\alpha}_t^i}\left(\tau_{ji,t}\right)^{\eta-1}\right)^{-1}\frac{Z_{i,t}^T}{P_{ij,t}} \tag{37}$$

Once more using the budget constraint (equation 28) it is possible to obtain the demand of country i for good i expresing the relative prices as terms of trade:

$$c_{ii,t} = \left(1+\frac{1-\widetilde{\alpha}_t^i}{\widetilde{\alpha}_t^i}\left(\tau_{ij,t}\right)^{\eta-1}\right)^{-1}\frac{Z_{i,t}^T}{P_{ii,t}} \tag{38}$$

To obtain the Consumer Based Price Index of the traded goods, I do the same exercise as in the general CBPI (equation 22):

$$\begin{aligned} 1 \quad = \quad & \left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}}\left(\left(1+\frac{1-\widetilde{\alpha}_t^i}{\widetilde{\alpha}_t^i}\left(\tau_{ij,t}\right)^{\eta-1}\right)^{-1}\frac{P_{i,t}^T}{P_{ii,t}}\right)^{\frac{\eta-1}{\eta}} \\ & +\left(1-\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}}\left(\left(1+\frac{\widetilde{\alpha}_t^i}{1-\widetilde{\alpha}_t^i}\left(\tau_{ji,t}\right)^{\eta-1}\right)^{-1}\frac{P_{i,t}^T}{P_{ij,t}}\right)^{\frac{\eta-1}{\eta}} \end{aligned}$$

Which implies that

$$P_{i,t}^{T}=\left[\widetilde{\alpha}_{t}^{i}\left(P_{ii,t}\right)^{1-\eta}+\left(1-\widetilde{\alpha}_{t}^{i}\right)\left(P_{ij,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{39}$$

To obtain the final demands of each of the two traded goods remember that $Z_{i,t}^{T}=P_{i,t}^{T}C_{i,t}^{T}$ and that $C_{i,t}^{T}=\left(1-\frac{(1-\gamma)p_{i,t}^{1-\theta}}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right)Z_{i,t}$ and $Z_{i,t}=P_{i,t}Y_{i,t}$. Where $Y_{i,t}$ is the available income.

Demand for traded good produced in i and consumed in i :

$$c_{ii,t}=\frac{\frac{P_{i,t}^{T}}{P_{ii,t}}\left[\frac{\gamma}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right]P_{i,t}Y_{i,t}}{1+\frac{1-\widetilde{\alpha}_{t}^{i}}{\widetilde{\alpha}_{t}^{i}}\left(\tau_{ij,t}\right)^{\eta-1}} \tag{40}$$

Demand for traded good produced in j and consumed in i :

$$c_{ij,t}=\frac{\frac{P_{i,t}^{T}}{P_{ij,t}}\left[\frac{\gamma}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right]P_{i,t}Y_{i,t}}{1+\frac{\widetilde{\alpha}_{t}^{i}}{1-\widetilde{\alpha}_{t}^{i}}\left(\tau_{ji,t}\right)^{\eta-1}} \tag{41}$$

2.2 Real Exchange Rate

The real exchange rate is defined as the price of foreign goods relative to domestic goods

$$rx_{i,t}=\frac{P_{i,t}}{P_{t}^{*}} \tag{42}$$

Again, the international prices, are a weighted average of prices of each traded good, and is represented as follows:.

$$P_{t}^{*}=P_{ij,t} \tag{43}$$

The domestic prices are obtained as the CBPI of equiation (26)

$$P_{i,t} = \left(\gamma \left(P_{i,t}^{T}\right)^{1-\theta} + (1-\gamma)\left(P_{i,t}^{N}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{44}$$

Where the domestic traded-good price is:.

$$P_{i,t}^{T} = \widetilde{\alpha}_t^i \left(P_{ii,t}\right)^{1-\eta} + \left(1-\widetilde{\alpha}_t^i\right)\left(P_{ij,t}\right)^{1-\eta} \tag{45}$$

Bilateral real exchange rate

Combining the price index of traded and non trade good it is possible to obtain a bilateral real exchange rate as follows for countries i and j.

$$rx_{ij,t} = \frac{P_{j,t}^{T}}{P_{i,t}^{T}} \frac{\left(\gamma + (1-\gamma)\, p_{j,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{\left(\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \tag{46}$$

$$rx_{ij,t} = \frac{\left[\widetilde{\alpha}_t^j \left(P_{jj,t}\right)^{1-\eta} + \left(1-\widetilde{\alpha}_t^j\right)\left(P_{jk,t}\right)^{1-\eta}\right]}{\left[\widetilde{\alpha}_t^i \left(P_{ii,t}\right)^{1-\eta} + \left(1-\widetilde{\alpha}_t^i\right)\left(P_{ij,t}\right)^{1-\eta}\right]} \frac{\left(\gamma + (1-\gamma)\, p_{j,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{\left(\gamma + (1-\gamma)\, p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \tag{47}$$

Thus changes in bilateral exchange rates replicate movements in relative prices of traded and non traded goods and changes in the relative prices of imported and exported goods.

2.3 Portfolio Choice

The present document describe the portfolio decision process of the representative agent in the model of the paper “International Portfolios and the U.S. Current Account”.

2.3.1 Saving Decision

Each period the representative agent has to decide the amount he would consume of the endowment and how much he will save.

The representative agent budget constraint is the following.

$$P_{i,t}C_{i,t} - (Y_{i,t} + S_{i,t-1}) = S_{i,t} \tag{48}$$

Where $P_{i,t}$ is the consumer based price index, $C_{i,t}$ is the total consumption, $Y_{i,t}$ is the endowment in period t, and $S_{i,t}$ is total savings in period t.

The amount of savings are given by the Euler Equation.

$$\frac{\partial U_{c,t}}{\partial U_{c,t+1}} = \beta \tag{49}$$

The question is how does the agent will diversify his portfolio across home and foreign assets to minimize his future exposure to price volatility.

$$S_{i,t} = (\lambda_{ii,t} + \lambda_{ij,t})\, S_{i,t} \tag{50}$$

Where $\lambda_{ii,t}$ represents the proportion of the savings portfolio that will be held on country i assets. Where the constraint $\lambda_{ii,t} = 1 - \lambda_{ij,t}$ applies.

The question of the portfolio composition will be analized in the next section.

2.3.2 Desired Portfolio Composition

The hypothesis regarding the portfolio composition, is that he will choose a portfolio composition that matches his future consumption bundle. The agent is risk averse, therefore her optimal position is the one in wich changes in future prices do not affect her purchasing power.

At time t, the agent will have already choose $S_{i,t}$. But he knows how his next period $(t+1)$ budget constraint will look like.

$$S_{i,t} + E_t\{P_{i,t+1}Y_{i,t+1}\} = E_t\{P_{i,t+1}C_{i,t+1} + S_{i,t+1}\} \tag{51}$$

If the agent position himself at time $t+1$, the budget constraint will be as follows:

$$\begin{aligned}(P_{i,t+1}\lambda_{ii,t} + P_{j,t+1}\lambda_{ij,t})\,S_{i,t} + P_{i,t+1}Y_{i,t+1} = \\ P_{i,t+1}C_{ii,t+1} + P_{j,t+1}C_{ij,t+1} + S_{i,t+1}\end{aligned} \tag{52}$$

Where $\lambda_{ii,t}, \lambda_{ij,t}$ and $S_{i,t}$ where already chosen on period t.

In order to simplify the analysis I divided everything by the income (endowment) of period $t+1$ ($Y_{i,t+1}$). By doing this I obtain the following expression:

$$\begin{aligned}(P_{i,t+1}\lambda_{ii,t} + P_{j,t+1}\lambda_{ij,t})\,\tfrac{S_{i,t}}{Y_{i,t+1}} + P_{i,t+1} = \\ P_{i,t+1}\tfrac{C_{ii,t+1}}{Y_{i,t+1}} + P_{j,t+1}\tfrac{C_{ij,t+1}}{Y_{i,t+1}} + \tfrac{S_{i,t+1}}{Y_{i,t+1}}\end{aligned} \tag{53}$$

$$\begin{aligned}(P_{i,t+1}\lambda_{ii,t} + P_{j,t+1}\lambda_{ij,t})\,s_{i,t} + P_{i,t+1} = \\ P_{i,t+1}c_{ii,t+1} + P_{j,t+1}c_{ij,t+1} + P_{i,t+1}s_{i,t+1}\end{aligned} \tag{54}$$

Where the small letters represent the proportion relative to the income (endowment).

Furthermore dividing everything by price of good i at period $t+1$.

$$\begin{aligned}\left(\tfrac{P_{i,t+1}}{P_{i,t+1}}\lambda_{ii,t} + \tfrac{P_{j,t+1}}{P_{i,t+1}}\lambda_{ij,t}+\right) s_{i,t} + \tfrac{P_{i,t+1}}{P_{i,t+1}} = \\ \tfrac{P_{i,t+1}}{P_{i,t+1}}c_{ii,t+1} + \tfrac{P_{j,t+1}}{P_{i,t+1}}c_{ij,t+1} + \tfrac{P_{i,t+1}}{P_{i,t+1}}s_{i,t+1}\end{aligned} \tag{55}$$

$$\begin{aligned}(\lambda_{ii,t} + p_{ij,t+1}\lambda_{ij,t})\,s_{i,t} + 1 = \\ c_{ii,t+1} + p_{ij,t+1}c_{ij,t+1} + s_{i,t+1}\end{aligned} \tag{56}$$

The expected returns of the portfolio

$$E_t[R_{t+1}] = E_t[\lambda_{ii,t} + p_{ij,t+1}\lambda_{ij,t}] = E_t\left[\sum_{m=i,j} p_{im,t+1}\lambda_{im,t}\right] \tag{57}$$

2.3.3 Portfolio Variance

The decision by the agent on the portfolio composition is determined by her desired to mantain the purchase power in the future given her expected consumption. This mean that there should be a correlation equal to one, between the consumption boundle composition and the portafolio composition.

To this reduced form of the budget constraint now I will apply the variance (VAR) function. Assuming that the covariances are zero.

$$\begin{gathered} VAR\left[\left(\lambda_{ii,t} + p_{ij,t+1}\lambda_{ij,t}\right)s_{i,t} + 1\right] = \\ VAR\left[c_{ii,t+1} + p_{ij,t+1}c_{ij,t+1} + s_{i,t+1}\right] \end{gathered} \tag{58}$$

$$\begin{gathered} VAR\left(\lambda_{ii,t}s_{i,t}\right) + VAR\left(p_{ij,t+1}\lambda_{ij,t}s_{i,t}\right) + VAR\left(1\right) = \\ VAR\left(c_{ii,t+1}\right) + VAR\left(p_{ij,t+1}c_{ij,t+1}\right) + VAR\left(s_{i,t+1}\right) \end{gathered} \tag{59}$$

Assumptions:

$$VAR\left(\lambda_{ii,t}s_{i,t}\right) = 0,\ VAR\left(1\right) = 0 \tag{60}$$

Noting that $\lambda_{ij,t}$, $s_{i,t}$ is already given at time $t+1$. And $c_{ij,t+1}$, is decided on by the representative agent, therefore their variance is equal to zero.

Simplifying, I obtain:

$$\left(\lambda_{ij,t}s_{i,t}\right)^2 VAR\left(p_{ij,t}\right) = \left(c_{ij,t+1}\right)^2 VAR\left(p_{ij,t}\right) \tag{61}$$

$$\left[\left(\lambda_{ij,t}s_{i,t}\right)^2 - \left(c_{ij,t+1}\right)^2\right] VAR\left(p_{ij,t}\right) = 0 \tag{62}$$

This last equation has only one possible equilibrium given that $VAR\left(p_{ij,t}\right)$ will always be a positive number and greater than zero.

Equilibrium The equilibrium is where $(\lambda_{ij,t}s_{i,t})^2 - (c_{ij,t+1})^2 = 0$.This equilibrium implies the following conditions:

$$(\lambda_{ij,t}s_{i,t})^2 - (c_{ij,t+1})^2 = 0 \qquad \Rightarrow \lambda_{ij,t}s_{i,t} = c_{ij,t+1} \qquad \Rightarrow \lambda_{ij,t} = \frac{c_{ij,t+1}}{s_{i,t}} \quad (63)$$

Using the equation above the value of $\lambda_{ii,t}$ is obtain residually $\lambda_{ii,t} = 1 - \lambda_{ij,t}$

The agent uses backward induction, he knows that at time $t+1$, the optimal portfolio will be the one above. Therefore at time t he will choose $\lambda_{ii,t}$ and $\lambda_{ij,t}$ assuming rational expectations:

$$\lambda_{ij,t} = \frac{E_t\left[c_{ij,t+1}\right]}{s_{i,t}} \quad (64)$$

$$\lambda_{ii,t} = 1 - \lambda_{ij,t} = 1 - \frac{E_t\left[c_{ij,t+1}\right]}{s_{i,t}} \quad (65)$$

In order to obtain the values for the lambdas, the agent knows the expected demands for goods.

2.3.4 Expected Future Demand of Goods

The expected future demand for each good is obtain from the preferences:

$$E_t\left[c_{ij,t+1}\right] = E_t\left[\frac{\frac{P^T_{i,t+1}}{P_{ij,t+1}}\left[\frac{\gamma}{\gamma+(1-\gamma)p^{1-\theta}_{i,t+1}}\right]P_{i,t+1}Y_{i,t+1}}{1+\frac{\widetilde{\alpha}^i_{t+1}}{1-\widetilde{\alpha}^i_{t+1}}\left(\tau_{ji,t+1}\right)^{\eta-1}}\right] \quad (66)$$

The expected demand for the goods depend on the expected values on prices and relative prices: $E_t\left[P^T_{i,t+1}\right]$; $E_t\left[P_{ij,t+1}\right]$; $E_t\left[p_{i,t+1}\right]$; $E_t\left[P_{i,t+1}\right]$; and $E_t\left[\tau_{ji,t+1}\right]$.

In order to simplify the notation, let Π_{t+1} be the vector of prices and relative prices at time $t+1$, and $E_t[\Pi_{t+1}]$ is the expectation at time t of the vector of prices at time $t+1$.

The other expected values needed to obtain the expected demand are the coefficients $E_t\left[\widetilde{\alpha}^i_{t+1}\right]$ and $E_t\left[\widetilde{\beta}^i_{t+1}\right]$ that depend also on the expected value of the future endowment. $E_t[Y_{i,t+1}]$ and one the long-run values α and β.

Note that $E_t\left[\widetilde{\alpha}^i_{t+1}\right] = E_t[f_\alpha(\alpha, Y_{i,t+1})]$.

If we assume that the agent has an expectation on $E_t[Y_{i,t+1}, \Pi_{t+1}]$, he could choose a portfolio accordingly.

2.3.5 Expected Prices

The expectation on prices $E_t[\Pi_{t+1}]$ can be inferred if the agent not only has an expectation on his future endowment, but also on the future endowment of the representative agents of the other country.

$$E_t[\Pi_{t+1}] = E_t[\pi(Y_{i,t+1}, Y_{j,t+1})] \tag{67}$$

A simple assumption about the law of motion of the three endowments, could help us understand the expectation of the endowments in each country.

$$E_t[Y_{i,t+1}] = E_t[Y_{i,t} + \varepsilon_{i,t}] \tag{68}$$

$$E_t[Y_{j,t+1}] = E_t[Y_{j,t} + \varepsilon_{j,t}] \tag{69}$$

Where $\varepsilon_m = \xi_m + \epsilon_t$; also $\xi_{j,t} > \xi_{i,t} = 0$, $\sigma_{\xi_m} = 0$, and $\epsilon \sim N(0, \sigma^2_\epsilon)$; for $m = i, j, k$.

The condition $\xi_j > \xi_i = 0$ simply states that the expected endowment of the rich country (i) is equal to its previews one $E_t[Y_{i,t+1}] = E_t[Y_{i,t} + \varepsilon_{i,t}] = Y_{i,t}$. And the expected endowments for country j : $E_t[Y_{j,t+1}] = E_t[Y_{j,t} + \varepsilon_{k,t}] = Y_{j,t} + \xi_j$.

To wrap up, we have that under this conditions:

$$E_t [Y_{i,t+1}] = Y_{i,t} \tag{70}$$

$$E_t [Y_{j,t+1}] = Y_{j,t} + \xi_j \tag{71}$$

Plugging this expectations into the demands for each of the goods by each of the representative agents in each country it is possible to infer the vector of prices $E_t [\Pi_{t+1}]$ at time $t + 1$

2.3.6 Growth

Defining the expected endowment growth as g_m for $m = i, j, k$ we have that:

$$E_t [g_{i,t+1}] = E_t [Y_{i,t+1} - Y_{i,t}] = 0 \tag{72}$$

$$E_t [g_{j,t+1}] = E_t [Y_{j,t+1} - Y_{j,t}] = \xi_j \tag{73}$$

Which means that $E_t [g_{i,t+1}] < E_t [g_{j,t+1}]$

2.4 Demand for Assets

From the previous section it is possible to infer the demands for assets of each county are:

$$D_{i,t} = \lambda^N_{i,t} S_{i,t} + \lambda_{ii,t} S_{i,t} + \lambda_{ji,t} S_{j,t} \tag{74}$$

$$D_{j,t} = \lambda^N_{j,t} S_{i,t} + \lambda_{jj,t} S_{i,t} + \lambda_{ij,t} S_{i,t} \tag{75}$$

Where $S_{i,t}$ is the total savings the country i.

In this specification de global demand for country i assets grow when income growth. But the key assumption, is that the proportion demanded by country j for country i assets grow as its relative income growth.

$$\frac{\partial \lambda_{ji,t}}{\partial y_{j,t}} > 0 \tag{76}$$

The proportion demanded stabilizes to the steady state level when each country reaches the income of the high-income country.

2.5 Current Account

The straight forward definition of a bilateral current accunt is the change in the net demand for the country's assets. From this definition it is possible to extract from the model the bilateral current accounts of each country.

Defining the bilateral current account of each country as:

$$CA_{ij,t} = \Delta \left[\lambda_{ij,t} S_{i,t} \right] - \Delta \left[\lambda_{ji,t} S_{j,t} \right] \tag{77}$$

$$CA_{ji,t} = -CA_{ij,t} \tag{78}$$

Where Δ is the time-difference operator.

Stock market clearing requires:

$$\lambda_{i,t}^{N} + \lambda_{ii,t} + \lambda_{ji,t} = 1 \tag{79}$$

$$\lambda_{j,t}^{N} + \lambda_{jj,t} + \lambda_{ij,t} = 1 \tag{80}$$

$$\lambda_{k,t}^{N} + \lambda_{kk,t} + \lambda_{ik,t} = 1 \tag{81}$$

For equilibrium, the global current account must be equal to zero -by definition,. Thus

$$CA_{i,t} + CA_{j,t} = 0 \tag{82}$$

The model, eventhoug is algebraically complex gives an intuitive and simple explanation of the interaction between the international portfolio composition, and therefore demand for international assets on the current account behavior.

2.6 Three-Country Model

For this extension the three countries will be named as follows: i, j and k[5]. Where i is associated to the high-income country (U.S.); j is the middle-income country (Europe) and k is the low-income country (China). I normalize the income of each representative agent with respect to that of country i, which in the paper is associated to the high-income country (U.S.). Thus,

$$\frac{Y^i}{Y^i} = y^i = 1,\ \frac{Y^j}{Y^i} = y^j,\ \frac{Y^k}{Y^i} = y^k \tag{83}$$

2.6.1 Preferences

Preferences are equal to the two-country model.

$$U\left(C_{m,t}\right) = \frac{\left(C_{m,t}\right)^{1-\varphi}}{1-\varphi} \quad for\ m = i, j, k \tag{84}$$

Again consumers in the three countries allocate their spending between traded and non trade goods as follows:

$$C_{m,t} = \left[\gamma^{\frac{1}{\theta}}\left(C_{m,t}^T\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}}\left(C_{m,t}^N\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \quad form = i, j, k \tag{85}$$

Following Obstfeld and Rogoff (2005), the utility of consumption of traded goods is:

$$C_{i,t}^T = \left[\left(\widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}}\left(c_{ii,t}\right)^{\frac{\eta-1}{\eta}} + \left(\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i\right)^{\frac{1}{\eta}}\left(c_{ij,t}\right)^{\frac{\eta-1}{\eta}} + \left(1-\widetilde{\beta}_t^i\right)^{\frac{1}{\eta}}\left(c_{ik,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{86}$$

Parameter η is the (constant) elasticity of substitution between domestically produced traded goods and imports from either foreign country.Coefficients $\widetilde{\alpha}_t^i$, $\widetilde{\beta}_t^i$ are not constant. These coefficients depend on the relative income of each country, and their underlying coefficients α and β. The functions of these underlying coefficients are[6]:

$$\widetilde{\alpha}_t^i = \frac{\alpha}{\alpha + (1-\alpha)\left(y_t^i\right)^{2\rho}} \tag{87}$$

$$\left(\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i\right) = \frac{\beta - \alpha}{\alpha + (1-\alpha)\left(y_t^i\right)^{2\rho}} \tag{88}$$

[5] Where by assumption we have that $y^k < y^j < y^i = 1$

[6] Appendix A addresses the mathematical derivation of the function of $\widetilde{\alpha}$ and $\widetilde{\beta}$.

$$\left(1-\widetilde{\beta}_t^i\right)=1-\frac{\beta}{\alpha+(1-\alpha)\left(y_t^i\right)^{2\rho}} \tag{89}$$

Where $1>\beta>\alpha>0$ [7]

Table 1 summarizes the diversification pattern of the consumption bundles according to the level of normalized income. If the value of the normalized income is close to zero, the representative agent will devote most ofher income associated with traded goods in home-produced ones. This means a complete home-bias, and is equivalent to autarky.On the other hand, if the value of the normalized income is close to one, the representative agent will diversify her consumption matching the values of α and β

The model still applies when there is no ad hoc home bias in the coefficient α, it ensures that for low levels of normalized income the value of $\widetilde{\alpha}$ is almost one. Furthermore the model still gives logical weights for values of y superior to one.

Table 1: Consumption bundles according to the level income

This table summarizes the diversification pattern of the consumption bundles according to the level of normalized income.$y^i$is defined as$\frac{Y^i}{Y^i}$

$y_i \to 0$	$\widetilde{\alpha}=1$	$\widetilde{\beta}-\widetilde{\alpha}=0$	$1-\widetilde{\beta}=0$
$y_i \to 1$	$\widetilde{\alpha}=\alpha$	$\widetilde{\beta}-\widetilde{\alpha}=\beta-\alpha$	$1-\widetilde{\beta}=1-\beta$

For better understanding of the model, Table 2 shows the distribution of the coefficients of the preferences for each country.It is possible to observe that for the case of the high-income country (U.S.). the value of $\widetilde{\alpha}$ is equal to the value of α .

Table 2: Distribution of coefficients across countries

Goods/Country	i	j	k
i	$\widetilde{\alpha}_i$	$1-\widetilde{\beta}_i$	$\widetilde{\beta}_i-\widetilde{\alpha}_i$
j	$1-\widetilde{\beta}_j$	$\widetilde{\alpha}_j$	$\widetilde{\beta}_j-\widetilde{\alpha}_j$
k	$1-\widetilde{\beta}_k$	$\widetilde{\beta}_k-\widetilde{\alpha}_k$	$\widetilde{\alpha}_k$

Where $\widetilde{\alpha}$ represent the desired proportion of spending in locally produced traded good.

[7]Note that if $\widetilde{\beta}=1$ or if $\widetilde{\alpha}=\widetilde{\beta}$, the model transforms itself into a two-country model, in which at low levels of income there will be home bias, and as income grows the bias will be toward the high-income country (US).

2.6.2 Demands for each traded good and the corresponding CPBI

Define the terms of trade of country i with respect to countries j and k.as:

$$\tau_{ij,t} = \frac{P_{ii,t}}{P_{ij,t}}, \tau_{ik,t} = \frac{P_{ii,t}}{P_{ik,t}}, \tau_{jk,t} = \frac{P_{ij,t}}{P_{ik,t}} \tag{90}$$

Expresing the relative prices as terms of trade, the demand of country i for good j:

$$c_{ij,t} = \left(1 + \frac{\widetilde{\alpha}_t^i}{\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i} (\tau_{ji,t})^{\eta-1} + \frac{1 - \widetilde{\beta}_t^i}{\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i} (\tau_{jk,t})^{\eta-1}\right)^{-1} \frac{Z_{i,t}^T}{P_{ij,t}} \tag{91}$$

Expresing the relative prices as terms of trade the demand of country i for country k is:

$$c_{ik,t} = \left(1 + \frac{\widetilde{\alpha}_t^i}{1 - \widetilde{\beta}_t^i} (\tau_{ki,t})^{\eta-1} + \frac{\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i}{1 - \widetilde{\beta}_t^i} (\tau_{kj,t})^{\eta-1}\right)^{-1} \frac{Z_{i,t}^T}{P_{ik,t}} \tag{92}$$

Once more using the budget constraint (equation 28) it is possible to obtain the demand of country i for good i expresing the relative prices as terms of trade:

$$c_{ii,t} = \left(1 + \frac{\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i}{\widetilde{\alpha}_t^i} (\tau_{ij,t})^{\eta-1} + \frac{1 - \widetilde{\beta}_t^i}{\widetilde{\alpha}_t^i} (\tau_{ik,t})^{\eta-1}\right)^{-1} \frac{Z_{i,t}^T}{P_{ii,t}} \tag{93}$$

The Consumer Based Price Index of the traded goods, I do the same exercise as in the general CBPI.

$$P_{i,t}^T = \left[\widetilde{\alpha}_t^i (P_{ii,t})^{1-\eta} + \left(\widetilde{\beta}_t^i - \widetilde{\alpha}_t^i\right) (P_{ij,t})^{1-\eta} + \left(1 - \widetilde{\beta}_t^i\right) (P_{ik,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{94}$$

To obtain the final demands of each of the three traded goods remember that $Z_{i,t}^T = P_{i,t}^T C_{i,t}^T$ and that $C_{i,t}^T = \left(1 - \frac{(1-\gamma)p_{i,t}^{1-\theta}}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right) Z_{i,t}$ and $Z_{i,t} = P_{i,t} Y_{i,t}$. Where $Y_{i,t}$ is the available income.

Demand for traded good produced in i and consumed in i :

$$c_{ii,t} = \frac{\frac{P^T_{i,t}}{P_{ii,t}}\left[\frac{\gamma}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right] P_{i,t}Y_{i,t}}{1+\frac{\widetilde{\beta}^i_t-\widetilde{\alpha}^i_t}{\widetilde{\alpha}^i_t}\left(\tau_{ij,t}\right)^{\eta-1}+\frac{1-\widetilde{\beta}^i_t}{\widetilde{\alpha}^i_t}\left(\tau_{ik,t}\right)^{\eta-1}} \tag{95}$$

Demand for traded good produced in j and consumed in i :

$$c_{ij,t} = \frac{\frac{P^T_{i,t}}{P_{ij,t}}\left[\frac{\gamma}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right] P_{i,t}Y_{i,t}}{1+\frac{\widetilde{\alpha}^i_t}{\widetilde{\beta}^i_t-\widetilde{\alpha}^i_t}\left(\tau_{ji,t}\right)^{\eta-1}+\frac{1-\widetilde{\beta}^i_t}{\widetilde{\beta}^i_t-\widetilde{\alpha}^i_t}\left(\tau_{jk,t}\right)^{\eta-1}} \tag{96}$$

Demand for traded good produced in k and consumed in i :

$$c_{ik,t} = \frac{\frac{P^T_{i,t}}{P_{ik,t}}\left[\frac{\gamma}{\gamma+(1-\gamma)p_{i,t}^{1-\theta}}\right] P_{i,t}Y_{i,t}}{1+\frac{\widetilde{\alpha}^i_t}{1-\widetilde{\beta}^i_t}\left(\tau_{ki,t}\right)^{\eta-1}+\frac{\widetilde{\beta}^i_t-\widetilde{\alpha}^i_t}{1-\widetilde{\beta}^i_t}\left(\tau_{kj,t}\right)^{\eta-1}} \tag{97}$$

2.7 Real Exchange Rate

The real exchange rate is defined as the price of foreign goods relative to domestic goods

$$rx_{i,t} = \frac{P_{i,t}}{P^*_t} \tag{98}$$

Again, the international prices, are a weighted average of prices of each traded good, and is represented as follows:.

$$P^*_t = \left[\left(\frac{\widetilde{\beta}^i_t-\widetilde{\alpha}^i_t}{1-\widetilde{\alpha}^i_t}\right)P_{ij,t}+\left(\frac{1-\widetilde{\beta}^i_t}{1-\widetilde{\alpha}^i_t}\right)P_{ik,t}\right] \tag{99}$$

The domestic prices are obtained as the CBPI of equiation (26)

$$P_{i,t} = \left(\gamma\left(P^T_{i,t}\right)^{1-\theta}+(1-\gamma)\left(P^N_{i,t}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{100}$$

Where the domestic traded-good price is:.

$$P_{i,t}^{T}=\left[\widetilde{\alpha}_t^{i}\left(P_{ii,t}\right)^{1-\eta}+\left(\widetilde{\beta}_t^{i}-\widetilde{\alpha}_t^{i}\right)\left(P_{ij,t}\right)^{1-\eta}+\left(1-\widetilde{\beta}_t^{i}\right)\left(P_{ik,t}\right)^{1-\eta}\right] \tag{101}$$

Bilateral real exchange rate

Combining the price index of traded and non trade good it is possible to obtain a bilateral real exchange rate as follows for countries i and j.

$$rx_{ij,t}=\frac{P_{j,t}^{T}}{P_{i,t}^{T}}\frac{\left(\gamma+(1-\gamma)\,p_{j,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{\left(\gamma+(1-\gamma)\,p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \tag{102}$$

$$rx_{ij,t}=\frac{\left[\widetilde{\alpha}_t^{j}\left(P_{jj,t}\right)^{1-\eta}+\left(\widetilde{\beta}_t^{j}-\widetilde{\alpha}_t^{j}\right)\left(P_{jk,t}\right)^{1-\eta}+\left(1-\widetilde{\beta}_t^{j}\right)\left(P_{ji,t}\right)^{1-\eta}\right]}{\left[\widetilde{\alpha}_t^{i}\left(P_{ii,t}\right)^{1-\eta}+\left(\widetilde{\beta}_t^{i}-\widetilde{\alpha}_t^{i}\right)\left(P_{ij,t}\right)^{1-\eta}+\left(1-\widetilde{\beta}_t^{i}\right)\left(P_{ik,t}\right)^{1-\eta}\right]}\frac{\left(\gamma+(1-\gamma)\,p_{j,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{\left(\gamma+(1-\gamma)\,p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \tag{103}$$

For countries i and k:

$$rx_{ik,t}=\frac{P_{k,t}^{T}}{P_{i,t}^{T}}\frac{\left(\gamma+(1-\gamma)\,p_{k,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{\left(\gamma+(1-\gamma)\,p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \tag{104}$$

$$rx_{ik,t}=\frac{\left[\widetilde{\alpha}_t^{k}\left(P_{kk,t}\right)^{1-\eta}+\left(\widetilde{\beta}_t^{k}-\widetilde{\alpha}_t^{k}\right)\left(P_{kj,t}\right)^{1-\eta}+\left(1-\widetilde{\beta}_t^{k}\right)\left(P_{ki,t}\right)^{1-\eta}\right]}{\left[\widetilde{\alpha}_t^{i}\left(P_{ii,t}\right)^{1-\eta}+\left(\widetilde{\beta}_t^{i}-\widetilde{\alpha}_t^{i}\right)\left(P_{ij,t}\right)^{1-\eta}+\left(1-\widetilde{\beta}_t^{i}\right)\left(P_{ik,t}\right)^{1-\eta}\right]}\frac{\left(\gamma+(1-\gamma)\,p_{k,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{\left(\gamma+(1-\gamma)\,p_{i,t}^{1-\theta}\right)^{\frac{1}{1-\theta}}} \tag{105}$$

And for countries j and k:

$$rx_{jk,t} = \frac{P^T_{k,t}}{P^T_{j,t}} \frac{\left(\gamma + (1-\gamma)\, p^{1-\theta}_{k,t}\right)^{\frac{1}{1-\theta}}}{\left(\gamma + (1-\gamma)\, p^{1-\theta}_{j,t}\right)^{\frac{1}{1-\theta}}} \tag{106}$$

$$rx_{ij,t} = \frac{\left[\widetilde{\alpha}^k_t \left(P_{kk,t}\right)^{1-\eta} + \left(\widetilde{\beta}^k_t - \widetilde{\alpha}^k_t\right)\left(P_{kj,t}\right)^{1-\eta} + \left(1-\widetilde{\beta}^k_t\right)\left(P_{ki,t}\right)^{1-\eta}\right]}{\left[\widetilde{\alpha}^j_t \left(P_{jj,t}\right)^{1-\eta} + \left(\widetilde{\beta}^j_t - \widetilde{\alpha}^j_t\right)\left(P_{jk,t}\right)^{1-\eta} + \left(1-\widetilde{\beta}^j_t\right)\left(P_{ji,t}\right)^{1-\eta}\right]} \frac{\left(\gamma + (1-\gamma)\, p^{1-\theta}_{k,t}\right)^{\frac{1}{1-\theta}}}{\left(\gamma + (1-\gamma)\, p^{1-\theta}_{j,t}\right)^{\frac{1}{1-\theta}}} \tag{107}$$

Thus changes in bilateral exchange rates replicate movements in relative prices of traded and non traded goods and changes in the relative prices of imported and exported goods.

2.8 Portfolio Choice

2.8.1 Desired Portfolio Composition

The hypothesis regarding the portfolio composition, is that he will choose a portfolio composition that matches his future consumption bundle.

At time t, the agent will have already choose $S_{i,t}$. But he knows how his next period $(t+1)$ budget constraint will look like.

$$S_{i,t} + E_t\left\{P_{i,t+1}Y_{i,t+1}\right\} = E_t\left\{P_{i,t+1}C_{i,t+1} + S_{i,t+1}\right\} \tag{108}$$

If the agent position himself at time $t+1$, the budget constraint will be as follows:

$$\begin{aligned}\left(P_{i,t+1}\lambda_{ii,t} + P_{j,t+1}\lambda_{ij,t} + P_{k,t+1}\lambda_{ik,t}\right) S_{i,t} + P_{i,t+1}Y_{i,t+1} = \\ P_{i,t+1}C_{ii,t+1} + P_{j,t+1}C_{ij,t+1} + P_{k,t+1}C_{ik,t+1} + S_{i,t+1}\end{aligned} \tag{109}$$

Where $\lambda_{ii,t}, \lambda_{ij,t}, \lambda_{ik,t}$ and $S_{i,t}$ where already chosen on period t.

In order to simplify the analysis I divided everything by the income (endowment) of period $t+1$ $(Y_{i,t+1})$. By doing this I obtain the following expression:

$$
\begin{aligned}
&(P_{i,t+1}\lambda_{ii,t} + P_{j,t+1}\lambda_{ij,t} + P_{k,t+1}\lambda_{ik,t})\,\frac{S_{i,t}}{Y_{i,t+1}} + P_{i,t+1} = \\
&P_{i,t+1}\frac{C_{ii,t+1}}{Y_{i,t+1}} + P_{j,t+1}\frac{C_{ij,t+1}}{Y_{i,t+1}} + P_{k,t+1}\frac{C_{ik,t+1}}{Y_{i,t+1}} + P_{i,t+1}\frac{S_{i,t+1}}{Y_{i,t+1}}
\end{aligned} \tag{110}
$$

$$
\begin{aligned}
&(P_{i,t+1}\lambda_{ii,t} + P_{j,t+1}\lambda_{ij,t} + P_{k,t+1}\lambda_{ik,t})\,s_{i,t} + P_{i,t+1} = \\
&P_{i,t+1}c_{ii,t+1} + P_{j,t+1}c_{ij,t+1} + P_{k,t+1}c_{ik,t+1} + P_{i,t+1}s_{i,t+1}
\end{aligned} \tag{111}
$$

Where the small letters represent the proportion relative to the income (endowment).

Furthermore dividing everything by price of good i at period $t+1$.

$$
\begin{aligned}
&\left(\frac{P_{i,t+1}}{P_{i,t+1}}\lambda_{ii,t} + \frac{P_{j,t+1}}{P_{i,t+1}}\lambda_{ij,t} + \frac{P_{k,t+1}}{P_{i,t+1}}\lambda_{ik,t}\right) s_{i,t} + \frac{P_{i,t+1}}{P_{i,t+1}} = \\
&\frac{P_{i,t+1}}{P_{i,t+1}}c_{ii,t+1} + \frac{P_{j,t+1}}{P_{i,t+1}}c_{ij,t+1} + \frac{P_{k,t+1}}{P_{i,t+1}}c_{ik,t+1} + \frac{P_{i,t+1}}{P_{i,t+1}}s_{i,t+1}
\end{aligned} \tag{112}
$$

$$
\begin{aligned}
&(\lambda_{ii,t} + p_{ij,t+1}\lambda_{ij,t} + p_{ik,t+1}\lambda_{ik,t})\,s_{i,t} + 1 = \\
&c_{ii,t+1} + p_{ij,t+1}c_{ij,t+1} + p_{ik,t+1}c_{ik,t+1} + s_{i,t+1}
\end{aligned} \tag{113}
$$

The expected returns of the portfolio

$$
E_t\left[R_{t+1}\right] = E_t\left[\lambda_{ii,t} + p_{ij,t+1}\lambda_{ij,t} + p_{ik,t+1}\lambda_{ik,t}\right] = E_t\left[\sum_{m=i,j,k} p_{im,t+1}\lambda_{im,t}\right] \tag{114}
$$

2.8.2 Portfolio Variance

To this reduced form of the budget constraint now I will apply the variance (VAR) function. Assuming that the covariances are zero.

$$
\begin{aligned}
&VAR\left[(\lambda_{ii,t} + p_{ij,t+1}\lambda_{ij,t} + p_{ik,t+1}\lambda_{ik,t})\,s_{i,t} + 1\right] = \\
&VAR\left[c_{ii,t+1} + p_{ij,t+1}c_{ij,t+1} + p_{ik,t+1}c_{ik,t+1} + s_{i,t+1}\right]
\end{aligned} \tag{115}
$$

$$VAR\left(\lambda_{ii,t}s_{i,t}\right)+VAR\left(p_{ij,t+1}\lambda_{ij,t}s_{i,t}\right)+VAR\left(p_{ik,t+1}\lambda_{ik,t}s_{i,t}\right)+VAR\left(1\right)= \\ VAR\left(c_{ii,t+1}\right)+VAR\left(p_{ij,t+1}c_{ij,t+1}\right)+VAR\left(p_{ik,t+1}c_{ik,t+1}\right)+VAR\left(s_{i,t+1}\right) \tag{116}$$

Assumptions:

$$VAR\left(\lambda_{ii,t}s_{i,t}\right)=0,\ VAR\left(1\right)=0 \tag{117}$$

Noting that $\lambda_{ij,t}$, $\lambda_{ik,t}$, $s_{i,t}$ is already given at time $t+1$. And $c_{ij,t+1}$, $c_{ik,t+1}$ are decided on by the representative agent, therefore their variance is equal to zero.

Simplifying, I obtain:

$$\begin{gathered}\left(\lambda_{ij,t}s_{i,t}\right)^2 VAR\left(p_{ij,t}\right)+\left(\lambda_{ik,t}s_{i,t}\right)^2 VAR\left(p_{ik,t}\right)= \\ \left(c_{ij,t+1}\right)^2 VAR\left(p_{ij,t}\right)+\left(c_{ik,t+1}\right)^2 VAR\left(p_{ik,t}\right)\end{gathered} \tag{118}$$

$$\left[\left(\lambda_{ij,t}s_{i,t}\right)^2-\left(c_{ij,t+1}\right)^2\right]VAR\left(p_{ij,t}\right)+\left[\left(\lambda_{ik,t}s_{i,t}\right)^2-\left(c_{ik,t+1}\right)^2\right]VAR\left(p_{ik,t}\right)=0 \tag{119}$$

$$\left[\left(\lambda_{ij,t}s_{i,t}\right)^2-\left(c_{ij,t+1}\right)^2\right]VAR\left(p_{ij,t}\right)=-\left[\left(\lambda_{ik,t}s_{i,t}\right)^2-\left(c_{ik,t+1}\right)^2\right]VAR\left(p_{ik,t}\right) \tag{120}$$

This last equation has multiple equilibriums.

2.8.3 Equilibrium Analysis

The following subsection is a short equilibrium analysis of the equation in the previous section. Where I identify every possible equilibrium.

If we assume that $VAR(p_{ij,t}) > 0$, $VAR(p_{ik,t}) > 0$; and $VAR(p_{ij,t}) \lesseqgtr VAR(p_{ik,t})$

$(\lambda_{ij,t}s_{i,t})^2 - (c_{ij,t+1})^2 \lesseqgtr 0$ and $(\lambda_{ik,t}s_{i,t})^2 - (c_{ik,t+1})^2 \lesseqgtr 0$

The possible equilibriums are:

$(\lambda_{ik,t}s_{i,t})^2 - (c_{ik,t+1})^2$ $(\lambda_{ij,t}s_{i,t})^2 - (c_{ij,t+1})^2$	> 0	< 0	$= 0$
> 0	$\nexists$ Equi.	Could $\exists$ Equi.	$\nexists$ Equi.
< 0	Could $\exists$ Equi.	$\nexists$ Equi.	$\nexists$ Equi.
$= 0$	$\nexists$ Equi.	$\nexists$ Equi.	$\exists$ Equilibrium

Equilibrium The equilibrium I am interested is the one where $(\lambda_{ij,t}s_{i,t})^2 - (c_{ij,t+1})^2 = 0$ and $(\lambda_{ik,t}s_{i,t})^2 - (c_{ik,t+1})^2 = 0$:

This equilibrium implies the following conditions:

$$(\lambda_{ij,t}s_{i,t})^2 - (c_{ij,t+1})^2 = 0 \qquad \Rightarrow \lambda_{ij,t}s_{i,t} = c_{ij,t+1} \qquad \Rightarrow \lambda_{ij,t} = \frac{c_{ij,t+1}}{s_{i,t}} \tag{121}$$

$$(\lambda_{ik,t}s_{i,t})^2 - (c_{ik,t+1})^2 = 0 \qquad \Rightarrow \lambda_{ik,t}s_{i,t} = c_{ik,t+1} \qquad \Rightarrow \lambda_{ik,t} = \frac{c_{ik,t+1}}{s_{i,t}} \tag{122}$$

$\lambda_{ii,t}$ is obtain residually $\lambda_{ii,t} = 1 - \lambda_{ij,t} - \lambda_{ik,t}$

The agent uses backward induction, he knows that at time $t+1$, the optimal portfolio will be the one above. Therefore at time t he will choose $\lambda_{ii,t}$, $\lambda_{ij,t}$, $\lambda_{ik,t}$ assuming rational expectations:

$$\lambda_{ij,t} = \frac{E_t\left[c_{ij,t+1}\right]}{s_{i,t}} \tag{123}$$

$$\lambda_{ik,t} = \frac{E_t\left[c_{ik,t+1}\right]}{s_{i,t}} \tag{124}$$

$$\lambda_{ii,t} = 1 - \lambda_{ij,t} - \lambda_{ik,t} = 1 - \frac{E_t\left[c_{ij,t+1}\right]}{s_{i,t}} - \frac{E_t\left[c_{ik,t+1}\right]}{s_{i,t}} \tag{125}$$

In order to obtain the values for the lambdas, the agent knows the expected demands for goods.

2.8.4 Expected Future Demand of Goods

The expected future demand for each good is obtain from the preferences:

$$E_t\left[c_{ij,t+1}\right] = E_t\left[\frac{\frac{P^T_{i,t+1}}{P_{ij,t+1}}\left[\frac{\gamma}{\gamma+(1-\gamma)p^{1-\theta}_{i,t+1}}\right]P_{i,t+1}Y_{i,t+1}}{1+\frac{\widetilde{\alpha}^i_{t+1}}{\widetilde{\beta}^i_{t+1}-\widetilde{\alpha}^i_{t+1}}\left(\tau_{ji,t+1}\right)^{\eta-1}+\frac{1-\widetilde{\beta}^i_{t+1}}{\widetilde{\beta}^i_{t}-\widetilde{\alpha}^i_{t+1}}\left(\tau_{jk,t+1}\right)^{\eta-1}}\right] \tag{126}$$

$$E_t\left[c_{ik,t+1}\right] = E_t\left[\frac{\frac{P^T_{i,t+1}}{P_{ik,t+1}}\left[\frac{\gamma}{\gamma+(1-\gamma)p^{1-\theta}_{i,t+1}}\right]P_{i,t+1}Y_{i,t+1}}{1+\frac{\widetilde{\alpha}^i_{t+1}}{1-\widetilde{\beta}^i_{t+1}}\left(\tau_{ki,t+1}\right)^{\eta-1}+\frac{\widetilde{\beta}^i_{t+1}-\widetilde{\alpha}^i_{t+1}}{1-\widetilde{\beta}^i_{t+1}}\left(\tau_{kj,t+1}\right)^{\eta-1}}\right] \tag{127}$$

The expected demand for the goods depend on the expected values on prices and relative prices: $E_t\left[P^T_{i,t+1}\right]$; $E_t\left[P_{ij,t+1}\right]$; $E_t\left[p_{i,t+1}\right]$; $E_t\left[P_{i,t+1}\right]$; $E_t\left[\tau_{ji,t+1}\right]$; $E_t\left[\tau_{jk,t+1}\right]$; $E_t\left[P_{ik,t+1}\right]$; $E_t\left[\tau_{ki,t+1}\right]$; and $E_t\left[\tau_{kj,t+1}\right]$.

In order to simplify the notation, let Π_{t+1} be the vector of prices and relative prices at time $t+1$, and $E_t\left[\Pi_{t+1}\right]$ is the expectation at time t of the vector of prices at time $t+1$.

The other expected values needed to obtain the expected demand are the coefficients $E_t\left[\widetilde{\alpha}^i_{t+1}\right]$ and $E_t\left[\widetilde{\beta}^i_{t+1}\right]$ that depend also on the expected value of the future endowment. $E_t\left[Y_{i,t+1}\right]$ and one the long-run values α and β.

Note that $E_t\left[\widetilde{\alpha}_{t+1}^{i}\right] = E_t\left[f_{\alpha}\left(\alpha, Y_{i,t+1}\right)\right]$, and $E_t\left[\widetilde{\beta}_{t+1}^{i}\right] = E_t\left[f_{\beta}\left(\beta, Y_{i,t+1}\right)\right]$

If we assume that the agent has an expectation on $E_t\left[Y_{i,t+1}, \Pi_{t+1}\right]$, he could choose a portfolio accordingly.

2.8.5 Expected Prices

The expectation on prices $E_t\left[\Pi_{t+1}\right]$ can be inferred if the agent not only has an expectation on his future endowment, but also on the future endowment of the representative agents of the other two countries.

$$E_t\left[\Pi_{t+1}\right] = E_t\left[\pi\left(Y_{i,t+1}, Y_{j,t+1}, Y_{k,t+1}\right)\right] \tag{128}$$

A simple assumption about the law of motion of the three endowments, could help us understand the expectation of the endowments in each country.

$$E_t\left[Y_{i,t+1}\right] = E_t\left[Y_{i,t} + \varepsilon_{i,t}\right] \tag{129}$$

$$E_t\left[Y_{j,t+1}\right] = E_t\left[Y_{j,t} + \varepsilon_{k,t}\right] \tag{130}$$

$$E_t\left[Y_{k,t+1}\right] = E_t\left[Y_{k,t} + \varepsilon_{k,t}\right] \tag{131}$$

Where $\varepsilon_m = \xi_m + \epsilon_t$; also $\xi_{k,t} > \xi_{j,t} > \xi_{i,t} = 0$, $\sigma_{\xi_m} = 0$, and $\epsilon \sim N(0, \sigma_{\epsilon}^2)$; for $m = i, j, k$.

The condition $\xi_k > \xi_j > \xi_i = 0$ simply states that the expected endowment of the rich country (i) is equal to its previews one $E_t\left[Y_{i,t+1}\right] = E_t\left[Y_{i,t} + \varepsilon_{i,t}\right] = Y_{i,t}$. And the expected endowments for countries j and k are respectively: $E_t\left[Y_{j,t+1}\right] = E_t\left[Y_{j,t} + \varepsilon_{k,t}\right] = Y_{j,t} + \xi_j$ and $E_t\left[Y_{k,t+1}\right] = E_t\left[Y_{k,t} + \varepsilon_{k,t}\right] = Y_{k,t} + \xi_k$

To wrap up, we have that under this conditions:

$$E_t\left[Y_{i,t+1}\right] = Y_{i,t} \tag{132}$$

$$E_t\left[Y_{j,t+1}\right] = Y_{j,t} + \xi_j \tag{133}$$

$$E_t\left[Y_{k,t+1}\right] = Y_{k,t} + \xi_k \tag{134}$$

Plugging this expectations into the demands for each of the goods by each of the representative agents in each country it is possible to infer the vector of prices $E_t\left[\Pi_{t+1}\right]$ at time $t+1$

2.8.6 Growth

Defining the expected endowment growth as g_m for $m = i, j, k$ we have that:

$$E_t\left[g_{i,t+1}\right] = E_t\left[Y_{i,t+1} - Y_{i,t}\right] = 0 \tag{135}$$

$$E_t\left[g_{j,t+1}\right] = E_t\left[Y_{j,t+1} - Y_{j,t}\right] = \xi_j \tag{136}$$

$$E_t\left[g_{k,t+1}\right] = E_t\left[Y_{k,t+1} - Y_{k,t}\right] = \xi_k \tag{137}$$

Which means that $E_t\left[g_{i,t+1}\right] < E_t\left[g_{j,t+1}\right] < E_t\left[g_{k,t+1}\right]$.

2.9 Demand for Assets

From the previous section it is possible to infer the demands for assets of each county are:

$$D_{i,t} = \lambda_{i,t}^{N} S_{i,t} + \lambda_{ii,t} S_{i,t} + \lambda_{ji,t} S_{j,t} + \lambda_{ki,t} S_{k,t} \tag{138}$$

$$D_{j,t} = \lambda_{j,t}^{N} S_{i,t} + \lambda_{jj,t} S_{i,t} + \lambda_{ij,t} S_{i,t} + \lambda_{kj,t} S_{k,t} \tag{139}$$

$$D_{k,t} = \lambda_{k,t}^{N} S_{i,t} + \lambda_{kk,t} S_{i,t} + \lambda_{ik,t} S_{i,t} + \lambda_{jk,t} S_{k,t} \tag{140}$$

Where $S_{i,t}$ is the total savings the country i.

In this specification de global demand for country i assets grow when income growth. But the key assumption, is that the proportion demanded by countries j and k for country i assets grow as their relative income growth.

$$\frac{\partial \lambda_{ji,t}}{\partial y_{j,t}} > 0 \text{ and } \frac{\partial \lambda_{ki,t}}{\partial y_{k,t}} > 0 \tag{141}$$

But not only that, it is possible to make inference about the order of magnitude:

$$0 < \frac{\partial \lambda_{ji,t}}{\partial y_{j,t}} < \frac{\partial \lambda_{ki,t}}{\partial y_{k,t}} \tag{142}$$

The proportion demanded stabilizes to the steady state level when each country reaches the income of the high-income country.

2.10 Current Account

The straight forward definition of a bilateral current accunt is the change in the net demand for the country's assets. From this definition it is possible to extract from the model the bilateral current accounts of each country.

Defining the bilateral current account of each country as:

$$CA_{ij,t} = \Delta\left[\lambda_{ij,t} S_{i,t}\right] - \Delta\left[\lambda_{ji,t} S_{j,t}\right] \tag{143}$$

$$CA_{ik,t} = \Delta\left[\lambda_{ik,t} S_{i,t}\right] - \Delta\left[\lambda_{ki,t} S_{k,t}\right] \tag{144}$$

$$CA_{jk,t} = \Delta\left[\lambda_{jk,t} S_{j,t}\right] - \Delta\left[\lambda_{kj,t} S_{k,t}\right] \tag{145}$$

$$CA_{ji,t} = -CA_{ij,t} \tag{146}$$

$$CA_{ki,t} = -CA_{ik,t} \tag{147}$$

$$CA_{kj,t} = -CA_{jk,t} \tag{148}$$

Where Δ is the time-difference operator.

The multilateral current account for every country is:

$$CA_{i,t} = CA_{ij,t} + CA_{ik,t} \tag{149}$$

$$CA_{j,t} = CA_{ji,t} + CA_{jk,t} \tag{150}$$

$$CA_{k,t} = CA_{ki,t} + CA_{kj,t} \tag{151}$$

Stock market clearing requires:

$$\lambda_{i,t}^{N} + \lambda_{ii,t} + \lambda_{ji,t} + \lambda_{ki,t} = 1 \tag{152}$$

$$\lambda_{j,t}^{N} + \lambda_{jj,t} + \lambda_{ij,t} + \lambda_{kj,t} = 1 \tag{153}$$

$$\lambda_{k,t}^{N} + \lambda_{kk,t} + \lambda_{ik,t} + \lambda_{jk,t} = 1 \tag{154}$$

For equilibrium, the global current account must be equal to zero -by definition,. Thus

$$CA_{i,t} + CA_{j,t} + CA_{k,t} = 0 \tag{155}$$

The model, eventhoug is algebraically complex gives an intuitive and simple explanation of the interaction between the international portfolio composition, and therefore demand for international assets on the current account behavior.

3 Simulations and empirical analysis

3.1 Simulation of the theoretical model

To test the model I performed a simulation using MATLAB. The coefficients for the baseline model are presented in Table 3.

Table 3: Coefficients of Simulation

This table presents the results of the coefficients obtained from the baseline model.

Coefficient / Country	i	j	k
α	0.4	0.4	0.4
β	0.5	0.5	0.5
γ	0.5	0.5	0.5
η	1.5	1.5	1.5
ψ	1.1	1.1	1.1
θ	1.1	1.1	1.1
ρ	1.1	1.1	1.1
Productivity gains	0.0	0.01	0.017
Velocity of convergence	1.0	0.6	0.7
A_o	1.0	0.7	0.3
K	1.0	1.0	1.0
L	1.0	1.0	1.0
χ	0.5	0.5	0.5
t	200		

Figure 4 illustrates the evolution of income of the high, middle and low-income countries (y^i, y^j and y^k respectively):

Fig 4. Evolution of the Income

Figure 5 shows the evolution of the coefficients of preferences in the three countries. As expected the coefficients remain constant for country i. And they converge to the equilibrium value for the j and k countries.

Fig 5. Evolution of the preferences

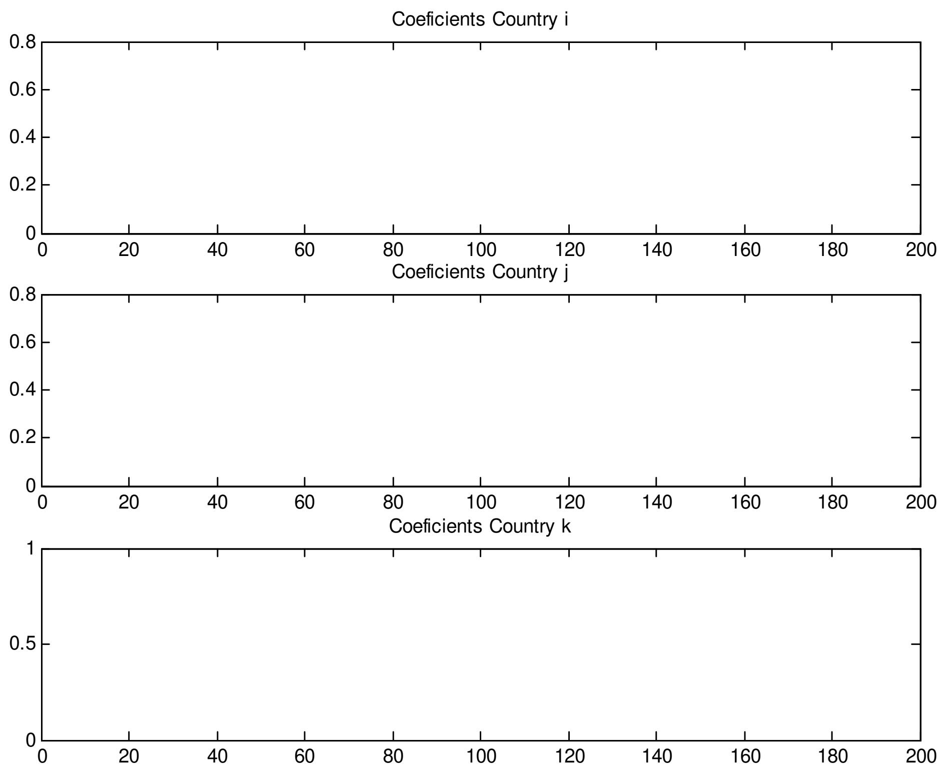
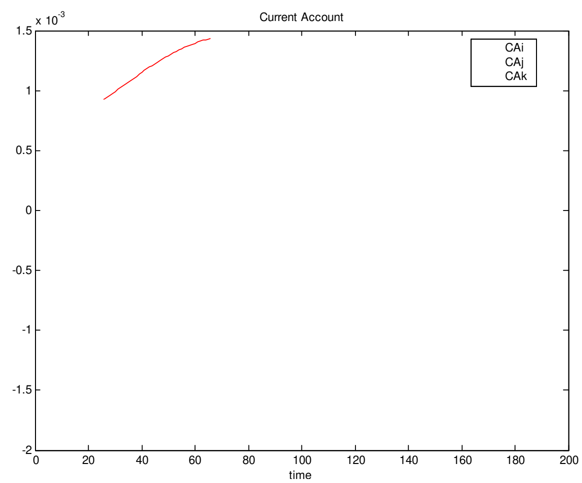


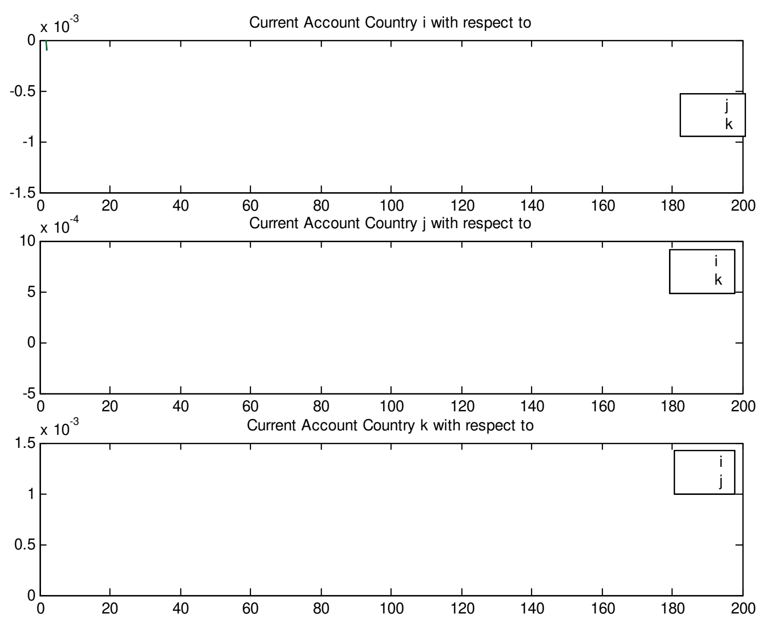
Figure 6 shows the results obtained from the modelation of the evolution of the multilateral current account for each country. The current account for country i is negative throughout the sample. It becomes more negative in the middle of the sample, and as the income of the other countries converge it becomes less and less negative until converges to zero. Countries j and k have surplus in their current account throughout the sample. But note that country k (the low-income country) has greater surpluses than country j (the middle-income country). Both converge to zero as their income catch-up with the high-income country.

Fig 6. Multilateral Current Account



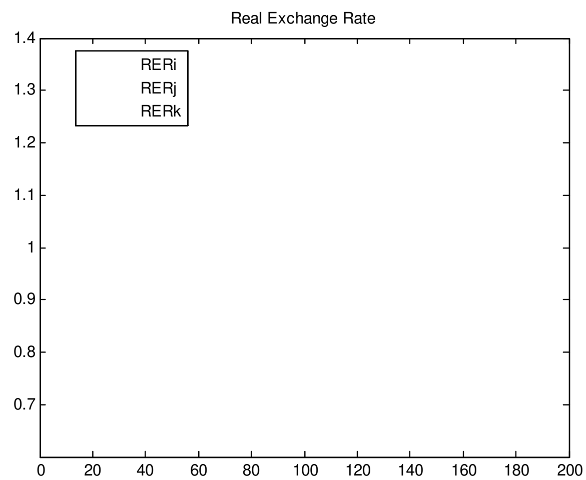
We can anhance our analysis by looking at the behavior of the bilateral current accounts in Figure 7.

Fig 7. Bilateral Current Account



The model described the evolution of the real exchange rate for both countries as shown in Figure 8.

Fig 8. Multilateral Real Exchange Rate



The next section presents the empirical estimation of the model.

3.2 Empirical estimation of the model

I performed the empirical analysis of the model collecting publicly available data from Penn World Tables, the Federal Reserve of St. Louis and the World Economic Outlook (IMF).

3.2.1 Change in the consumption bundle

The first part of the analysis tests the change in the consumption bundle of the representative agent. Equation XX proofs that consumption diversification as income increases.

$$\log\left(\frac{X_t^i + M_t^i}{Y_t^i}\right) = \alpha + \beta_1 \left(\frac{X_{t-1}^i + M_{t-1}^i}{Y_{t-1}^i}\right) + \beta_2 \log(Y_t^i) + \beta_1 \log\left(\frac{Y_t^i / Pop_t^i}{Y_t^{USA} / Pop_t^{USA}}\right) + + \beta_2 Pop_t^i + \mu_t^i \tag{156}$$

I use the measurement of openness of the economy as an indicator of the consumption bundle diversification. I then regress this variable using panel data it with respect to its lagged value and the lagged GDP (lcgdp1), controlling for the size of the economy and population. I found a positive and significant relationship between openness and GDP (0.01). This result tells that the richer the country, the more diversified its consumption bundle will be. The following table shows the results obtained from the regression:

Table 4: Regression output from equation (XX)

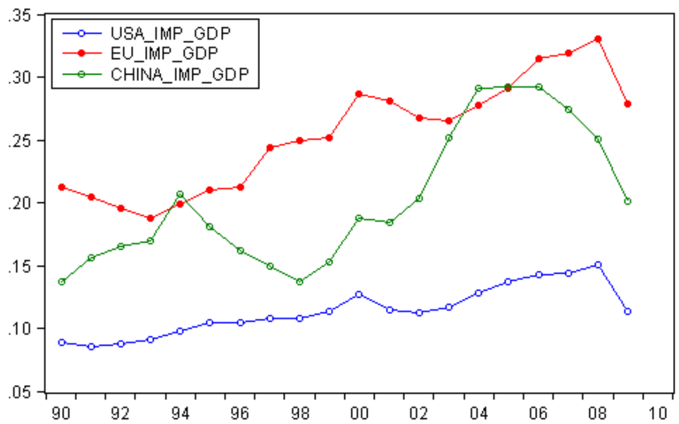
Variable	Coefficient	t	P>|t|
lopenk1	0.9676577	370.60	0.000
lcgdp1	0.0097674	5.77	0.000
y	-0.0001623	-2.38	0.017
lpop	-0.003088	-3.19	0.001
_cons	0.0986463	5.77	0.000
N			7146
Prob>F			0.0000
R-Squared			0.9702

One robustness test that I performed was spliting my sample in two (before and after 1980). The effect of GDP in Openness increases dramatically for the sample AFTER/BEFORE 1980. This result means that part of the effect that income has on the diversification of the consumption bundle has been stronger in the last 28 years.

Table 5: Regression output from equation (XX) for observations after1980

Variable	Coefficient	t	P>|t|
lopenk1	0.9574378	222.21	0.000
lcgdp1	0.0226943	5.77	0.000
y	-0.0004997	-3.31	0.001
lpop	-0.0029669	-2.14	0.032
_cons	0.0415058	1.23	0.220
N			3991
Prob>F			0.0000
R-Squared			0.9530

Table 6: Regression output from equation (XX)for observations before1980



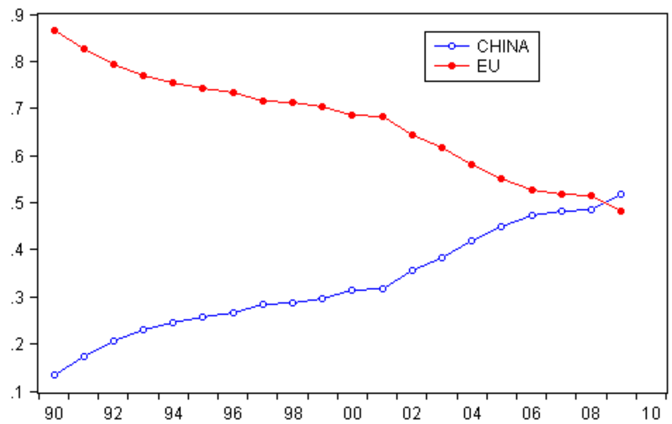
Variable	Coefficient	t	P>|t|
lopenk1	0.9734565	321.54	0.000
lcgdp1	0.0037851	1.20	0.230
y	-0.0000624	-0.62	0.535
lpop	-0.004979	-3.71	0.000
_cons	0.132374	4.93	0.000
N			3000
Prob>F			0.0000
R-Squared			0.9826

Another way to measure the diversification of the consumption bundle is to graph total imports for the selected countries as percentage of their income. Figure 9 presents the results of this test.

Figure 9:Total imports relative to the income for the U.S., Europe and China

The three series have a positive trend. The high and middle-income countries (U.S. and Europe) now import a little more than 15% of their income. While the low-income country (China) has followed a faster pace. Chinese imports now represent around 40% of the country's income.

Figure 10: Relative Imports of the U.S. by Origin



From the above figure it is possible to observe the increasing importance of imports from China. In 1980 almost all imports to the U.S. came from Europe, but by 2006 the total imports where almost equally divided by Europe and China.

Figure11: Relative Imports of Europe by Origin

The story repeats itself for the figure regarding the imports of Europe. Again, at the beginning almost all imports came from the U.S. and only a small proportion of them came from China. But in 2006 the predominant part of the European imports came from China.

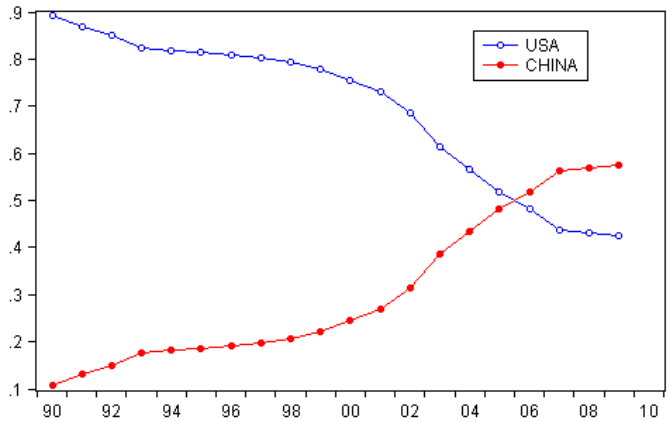
Figure 12: Relative Imports of China by Origin

Relative importance of imports to China has been quite stable during the period. Maybe the only interesting point is the fact that China imports more from Europe than from the U.S.

3.2.2 Calculation of the Demand for International Assets

The model stated that the demand of assets depends on the changes on income, income distribution and the preference's coefficients. Thus,

$$\triangle DA_t = f\left(\triangle y_t^i, \triangle \gamma_t^i, \triangle(\alpha, \beta)\right) \tag{157}$$


USA
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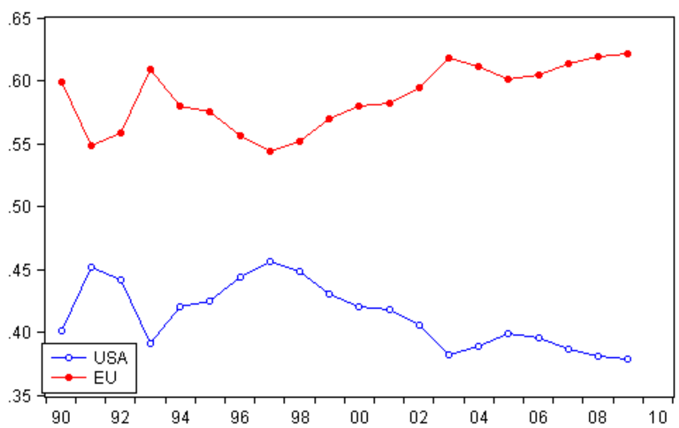
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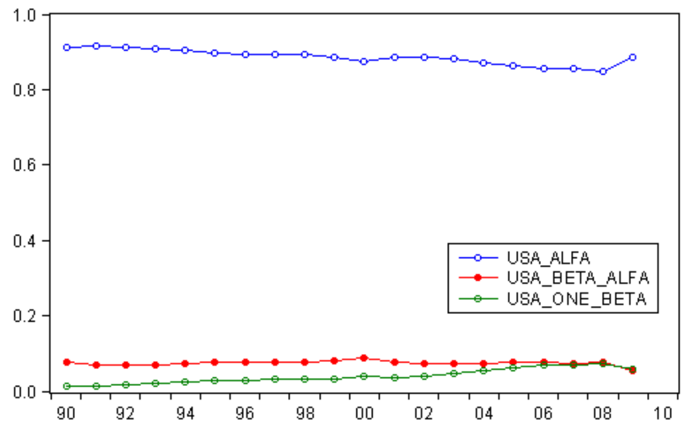


Change in the Estimated coefficients $\triangle(\alpha, \beta)$
The coefficients where estimated according with the following formulas:

$$\alpha_t^i = 1 - \frac{M_t^i}{Y_t^i} \tag{158}$$

$$\beta_t^i - \alpha_t^i = \frac{M_t^i}{Y_t^i} * \frac{M_t^{i,k}}{M_t^i} \tag{159}$$

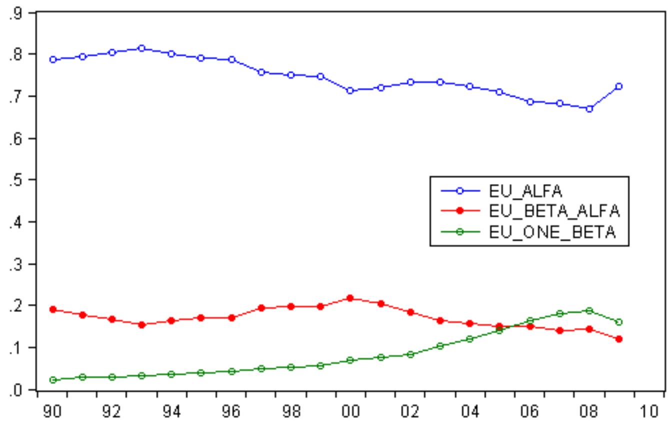
$$1 - \beta_t^i = \frac{M_t^i}{Y_t^i} * \frac{M_t^{i,j}}{M_t^i} \tag{160}$$

Where Y_t^i is the total level of income; $M_t^{i,j}$ is total imports to country i from country j; $M_t^{i,k}$ is total imports to country i from country k; and M_t^i is total imports to country i.

Figure 13: Behavior of Consumption of the U.S.

Home bias in consumption has gone down a little bit in the U.S. But this change is not so apparent. The consumption of Chinese goods has increase from almost null to around 8%.

Figure 14: Behavior of Consumption of Europe



Again home bias in consumption for Europe has been reduced, but only around 10%. Again the relative importance of China has increase.

Figure 15: Behavior of Consumption of China

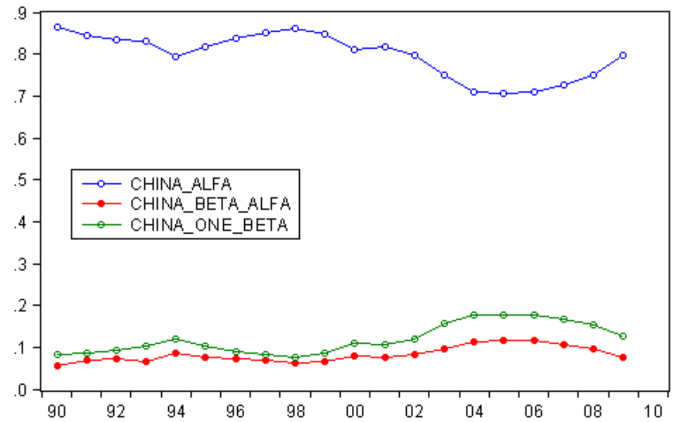
Home bias in consumption for China has been dramatically reduced, if we compare it with the home bias of the U.S. or Europe.

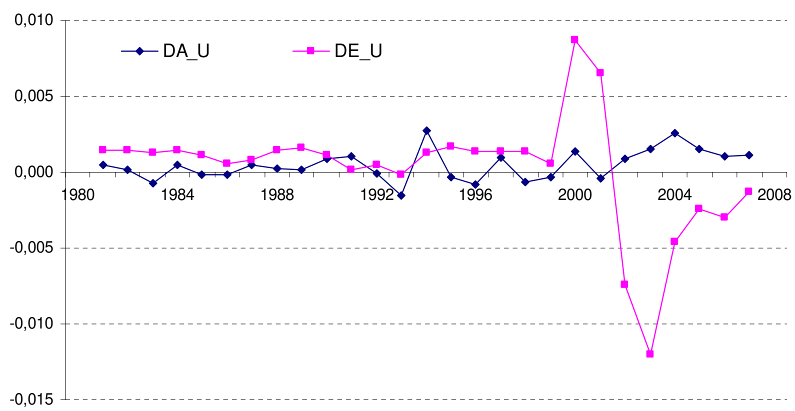
Estimated Demand of U.S. assets by origin

$$DA_T^{A,U} = y_t^A * dy_t^A * \triangle(\beta_t^A - \alpha_t^A) \tag{161}$$

$$DA_T^{E,U} = y_t^E * dy_t^E * \triangle(\beta_t^E - \alpha_t^E) \tag{162}$$

Figure 16: Estimated Demand of U.S. assets by origin



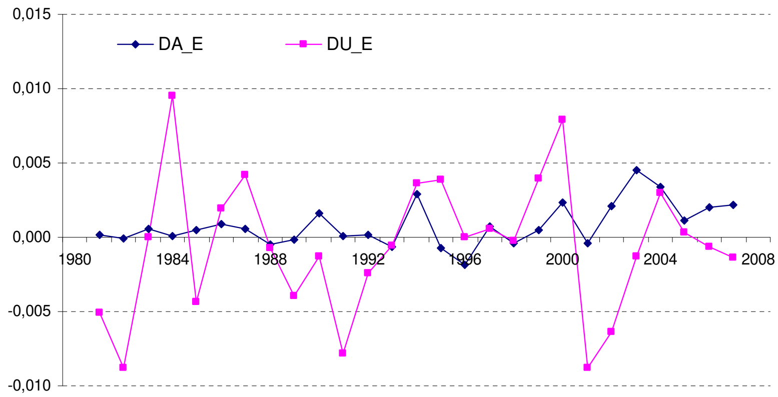


Demand for European assets by origin:

$$DA_T^{A,E} = y_t^A * dy_t^A * \triangle(1 - \beta_t^A) \tag{163}$$

$$DA_T^{U,E} = y_t^U * dy_t^U * \triangle(\beta_t^U - \alpha_t^U) \tag{164}$$

Figure 17: Estimated Demand Europe assets by origin

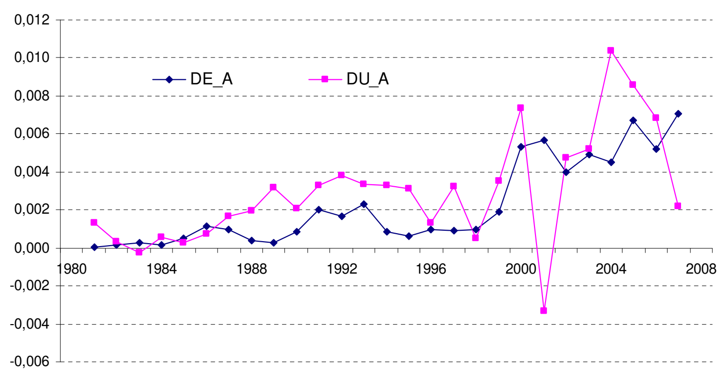


Demand for Chinese assets by origin:

$$DA_T^{E,A} = y_t^E * dy_t^E * \triangle(1 - \beta_t^E) \quad (165)$$

$$DA_T^{U,A} = y_t^U * dy_t^U * \triangle(1 - \beta_t^U) \quad (166)$$

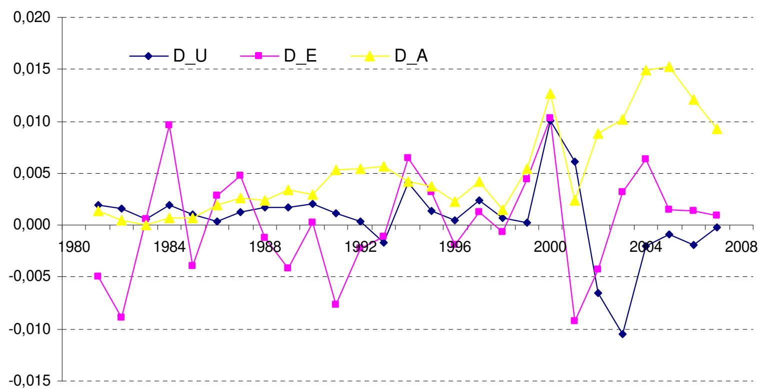
Figure 18: Estimated Demand China assets by origin



Total demands are estimated simply by adding the corresponding demands for each country.

$$DA_T^i = DA_T^{k,i} + DA_T^{j,i} \quad (167)$$

Figure 19: Total Demand of Assets for each country



3.2.3 Regressions of the Current Account

In this section I introduce the estimated demand for assets as an explanatory variable of the current account. Using a yearly sample from 1980 to 2007. With data from the IFS of the International Monetary Fund.

Remember that according to the model:

$$CA_{ij,t} = \Delta\left[\lambda_{ij,t}W_{i,t}\right] - \Delta\left[\lambda_{ji,t}W_{j,t}\right] \tag{168}$$

$$CA_{ik,t} = \Delta\left[\lambda_{ik,t}W_{i,t}\right] - \Delta\left[\lambda_{ki,t}W_{k,t}\right] \tag{169}$$

$$CA_{jk,t} = \Delta\left[\lambda_{jk,t}W_{j,t}\right] - \Delta\left[\lambda_{kj,t}W_{k,t}\right] \tag{170}$$

Therefore the sign for the change in the demand for foreign assets is expected to be negative.

I use a simple regression, where the current account as percentage of the GDP depends on the lagged value of the dependent variable, the real exchange rate and the change in the estimated demand.

$$\frac{CA_t^i}{Y_t^i} = \alpha + \beta_1 \frac{CA_{t-1}^i}{Y_{t-1}^i} + \beta_2 \Delta DA_t^{j,i} + \beta_3 \Delta DA_t^{k,i} + \beta_4 RER_t^i + \mu_t^i \tag{171}$$

The results for the current account of the U.S. are shown in the next table.

Table 7: Regression output from equation (XX)
Sample from1980 to 2007

Variable	Coefficient	t	P>|t|
$\frac{CA^i_{t-1}}{Y^i_{t-1}}$	0.9704065	17.70	0.000
$\Delta DA_t^{EUROPE,USA}$	-3.828573	-1.76	0.093
$\Delta DA_t^{CHINA,USA}$	-5.008142	-1.00	0.328
RER^i_t	-0.0006923	-1.66	0.111
Prob>F			0.0000
R-Squared			0.9648

Table 8: Regression output from equation (XX)
Sample from1980 to 2007

Variable	Coefficient	t	P>|t|
$\frac{CA^i_{t-1}}{Y^i_{t-1}}$	0.9707246	17.92	0.000
ΔDA_t^{TOTAL}	-4.081381	-1.92	0.068
RER^i_t	-0.0006915	-1.69	0.104
Prob>F			0.0000
R-Squared			0.9648

The main result from these two regressions is that the effect of the estimated demand for U.S. assets has indeed an effect on the current account of the U.S. Which makes the theoretical model more trustworthy. The effect has the expected negative sign; which means that if the demand for assets increases there is going to be deterioration in the current account. And the effect is statistically significant at 10%.

Using the same specification for the cases of Europe and China, the results are shown in the next two tables.

Table 9: Regression output from equation (XX)
Sample from1980 to 2007

Variable	Coefficient	t	P>|t|
$\frac{CA^i_{t-1}}{Y^i_{t-1}}$	0.8385842	4.18	0.000
$\Delta DA_t^{USA,CHINA}$	-8.522072	-1.36	0.188
$\Delta DA_t^{EUROPE,CHINA}$	-3.365956	-0.09	0.932
RER^i_t	0.0011201	1.02	0.320
Prob>F			0.0001
R-Squared			0.6291

Again the coefficients of the demands for Chinese assets by U.S. and Europe have the expected sign. Except that in this case none of the coefficients are statistically significant. This particular result even if it is not very encouraging at least provides some support to the theoretical model.

Table 10: Regression output from equation (XX)
Sample from1980 to 2007

Variable	Coefficient	t	P>|t|
$\frac{CA^i_{t-1}}{Y^i_{t-1}}$	0.4914303	2.36	0.100
$\Delta DA_t^{USA,EUROPE}$	7.703498	2.44	0.092
$\Delta DA_t^{CHINA,EUROPE}$	-3.53362	-0.35	0.751
RER^i_t	0.0003197	0.60	0.593
Prob>F			1.1768
R-Squared			0.6671

Finally, the estimation using European data does not provide information, since the sample is too short to obtain any results.

An important extension for these estimations is the use of quarterly data, but the quarterly data for all the required variables is not available.

4 Discussion and conclusions

A sudden change in the desired portfolio composition of the low-income country, towards a third country (say Europe in the model), will have a direct positive effect on the bilateral current account of the U.S., but this could be partly compensated by an raise of the current account deficit of Europe with the U.S. Given the three-country model, the effect of changes in the portfolio composition from one country to another will be mitigated by the redistribution of savings. In this model the imbalance originate from the growth of the low-income country, and would not disappear due to changes in the portfolio composition but will redistributed in a more balance way. In this case there is going to be significant effect on exchange rates, not only for a depreciation of the dollar, but also for an appreciation of the euro. This model argues that the macroeconomic imbalance come from a saving glut, that originates from the economic growth of the low-income country and the financial integration that allows for a more diversified portfolio, and is not related to a particular characteristic of the country that receive the savings flow.

An abrupt fall in income growth from the poor or middle-income countries will also have cause a reduction in the current account deficit of the U.S., without a rebalancing of the saving glut, because it has disappeared due to the stop in growth. But in this case the effect on exchange rates will not be significant.

Finally if income not only stop growing but it reduces, it is possible to argue that there is going to be a reverse in the decline of home bias. This reverse will have a far greater effect than the previous two scenarios, since it will eliminate the current account deficit, but it will have a significant effect on the exchange rate.

Under this model the policy recommendation of how to treat the saving glut in the rich-country (U.S.) would be a greater diversification of its international portfolio, channeling savings to one country to another to redistribute the imbalance in a more equilibrated way.

Even though the interest rate of the base country (U.S.) is the lowest, the demands for its assets by the other countries will growth, as long as their income growth.

As much as a 25% of the current account deficit of the U.S. can be financed by the increase in the demand for its assets. Depending on the coefficients and the initial relative income of the other countries.

The results and analysis in the present paper have an important repercussion in what is the current account sustainability of the U.S.

The model also projects a current account surplus in the middle-income country, and an initial current account surplus for the low-income country, but it will transform in a deficit in the late periods.

An important insight is that the model helps explain why low-income countries that are growing fast have current account surpluses instead of deficit.

The push in demand for U.S. assets would hold the exchange rate more or less constant, even as the current account deficit is mounting.

5 Appendix A

5.1 Home bias in consumption

One of the main contributions of this paper is the introduction of dynamic preferences of traded goods consumption, dependent on the relative income of the representative agent. I use the relative income as a proxy for other variables that can affect the consumption bundle of the individual such as exposure to publicity, inherited preferences from parents, peer pressure, credit availability and at country level such as trade costs, tariffs, distance, openness. Individuals prefer consumption smoothing therefore including path dependency (a lagged coefficient) helps soft the evolution of the coefficient. The $\alpha_t$coefficient would thus be:

$$\alpha_t = \psi \alpha_{t-1} + BZ_t \qquad 0 < \psi < 1$$

Where Z_t are the variables above mentioned, and B is the vector of coefficients.

Assuming that income could be used as a proxy for Z_t ($BZ_t \approx f(y_t)$) then:

$$\alpha_t = \psi \alpha_{t-1} + f(y_t)$$

Furthermore, if income follows an autoregressive process $y_t = \upsilon y_{t-1}$, with $\psi < \upsilon$, and $f(y_t)$ is a homogenous function of degree one. I obtain that

$$\alpha_t = \frac{\upsilon}{\upsilon - \psi} f(y_t)$$

The coefficient of traded goods consumption preference depends only on the present level of income.

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