The silence that precedes hypocrisy: a formal model of the spiral of silence theory

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Abstract

This paper exposes a formal model of the spiral of silence theory. It is based on game theory. The game consists on three players: players 1 and two have two strategies, to speak (s) or not ($\sim s$); the third player is Nature, which decides whether players 1 and 2 agree in their opinion or not. If players $1, 2$ speak and agree, they receive a payoff b. If they speak and disagree, they receive a payoff $-c$, for $b, c > 0$. The Nash equilibrium is in mixed strategies and each player chooses the profile $(s, c\,(b+c)^{-1}\,;$ $\sim\, s, 1 - c\,(b+c)^{-1})$. To analyse what happens when there are many players and interactions, I have run some simulations where players can update their beliefs about the opinion climate. The time when the spiral of silence process begins decays exponentially with the initial beliefs.

Key words: public opinion, opinion dynamics, spiral of silence, hidden vote, social simulation, agent-based modeling, game theory.

1 Introduction

Public opinion dynamics has been a hot topic in social simulation in last years. Agent-based models have found a place where to put into practice and apply their supposed advantages. How a population may reach a consensus about a topic, how two opinions may fight until one of them wins the battle to the other and how this evolves over time are emergent phenomena, global patterns that are the output of micro patterns. This is one of the strong points of ABMs (Bankes 2002; Berry, Kiel, and Elliott 2002; Bonabeau 2002; Macy and Willer 2002). So public opinion dynamics seem some kind of region to conquest by agent-based modelers.

Some models put agents in a lattice where agents can only interact with their four neighbours. For example, Sznajd-Weron and Sznajd (2000)'s propose a model in which, whenever two neighbours agree, they convince the rest of their neighbours. Otherwise, they can not do it.

A special case is that of "bounded confidence" models (BC), also known as the "consensus literature". One of its maximum exponents is that of Weisbuch,

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Deffuant, Amblard, and Nadal (2001). On it, a set of agents has opinions drawn from a uniform distribution in $(0, 1)$. At each iteration, two agents are picked up at random. If the difference of their opinions is below a confidence level, their opinions close to each other; otherwise, nothing happens. Another exponent is that of Hegselmann and Krause (2002). However, here agents take into account more than one opinion whenever it is close enough to their own. Later versions of BC models may be found in Deffuant, Amblard, and Weisbuch (2004) and Martin, Nadal, and Weisbuch (2004).

Nevertheless, these models show some faults. Sznajd-Weron and Sznajd's work is no more than another application of a physical model to social sciences. BC models deal with a very broad question, like how heterogeneous agents may reach a consensus. In any case, all of them consider agents as physical particles, agents are not truly agents: that is, an entity with goals (d'Inverno and Luck 2004, p. 24). And finally, none of them take into account public opinion literature, perhaps because they do not bother, they do not know or their purpose is very different.

My aim in this article is to begin to build a bridge between public opinion literature and social simulation. I think that social simulation may be a good tool to study the classical problems that have guided the debates in the public opinion field. In Section 2 I put on the table one of these debates. Section 3 shows some data that in some sense motivates my work. Then, Section 4 explains a formal model than incorporates some postulates already exposed in Section 2 and solves the model by a simulation. Results appear in Section 3 and Section 6 ends the article.

2 A theory looking for a formalization

One of the debates in the public opinion field has revolved around the spiral of silence theory (SST), by Noelle-Neumann (1974). The theory posits a relation between opinion climate and public opinion. and rests on five hypothesis:

1. Society threatens deviant individuals with isolation.
2. Individuals fear isolation.
3. This fear of isolation makes individuals scan continuously the opinion climate.
4. The perceived climate of opinion influences their behaviour in public, specially in speaking out.
5. The previous four axioms lead to a spiral of silence. Those who think their opinion is the majority tend to speak out; otherwise, they remain silent. Thus, the majority opinion seems to be more supported than it is in reality; the minority one seems to be less supported than it is. This feedbacks the perceptions of the climate of opinion. At last stage, the minority opinion reduces to a small core group or disappears (Noelle-Neumann 1995, 259–260).

From its publication, a lot of empirical research has tried to test the theory or at least some of its postulates, mainly the fourth (Glynn and McLeod 1984;

Oshagan 1996; Glynn, Hayes, and Shanahan 1997; Scheufele 1999; Neuwirth 2000; Hayes, Shanahan, and Glynn 2001; Willnat, Lee, and Detenber 2002; Kim, Han, Shanahan, and Berdayes 2003). The fifth postulate has received much less attention (Katz and Baldassare 1992; Katz and Baldassare 1994; Noelle-Neumann 1995). In all of this literature, one can find evidence for and against the SST. When it is against, the vagueness of the theory makes easy for authors to excuse the fault with different arguments *ad hoc*, generally related to the possibility that not all of conditions required by the theory to work are given.

Which conditions? SST says nothing about how social sanctions are created and maintained. They constitute a public good, and its existence posits a second-order collective action problem (Oliver 1980). If someone in the majority executes the sanctions, the rest benefit of it without incurring in the cost of taking time and effort to punish the minority. If everybody knows it, why does some member be the loser and create a system of social sanctions? Just wait for other to the work. If everybody think so, nobody in the majority do nothing, there are no social sanctions, no fear of isolation, no spiral of silence. How does the majority solve this collective action problem? SST says nothing about the exact type of relation between opinion climate and public opinion. SST says nothing about when a spiral of silence process starts and finishes, if it does it. Why in some situations the process ends with a set of hardliners and why in others it does not. Which conditions yield the first outcome and which the second. How sensible is the process to a small variation of some of its parameters so that it finishes with one of these two equilibria.

The theory needs to be formalized to derive strong and precise predictions, to know under what conditions it works, if any. The content of a theory is given by its domain of validity. When this is vague, its content is vague. And a vague theory says everything or nothing, a vague theory says what you want it says.

3 Empirical motivation

Before going into the model, let's see some evidence that spiral of silence processes happen in the real world. To do so, let's look for where vote hides in surveys. I want to compare actual vote to recalled vote to the right wing party (PP) and left wing party (PSOE) in Spain. In Figure 1 we can see a comparison between actual vote and recalled vote to both PP and PSOE from January 1984 to July 2005. There are four measures per year in January, April, July, and October to yield a time series of 87 points. When data were not available for some of these four months, I have taken an average of the adjacent months, and if even it were impossible, the nearest measure.

Hidden vote has not affected in the same way both parties: it has been more pronounced for the right. It is also true that its incidence has been reduced during those years where the right has ruled the country. In the same way, we note that hidden vote has begun to affect the left when it was in the opposition. However, even when the PP was in government, only after getting the absolute majority in the parliament the recalled vote was higher than the actual vote. While the PSOE was in government in the 80s, the recalled vote always was higher than the actual vote. It may be due to people in some way associate PP with the past dictatorship. Not in vain, a minister of the dictatorship was the leader of the PP during the 80s.

An analysis depends on the glasses one uses. Some data can be studied from difference perspectives or scales. Here I use wavelet or multirresolution analysis to get a deep look in the data. Wavelets are some special functions that represent data by decomposing it in different scales. What is a scale? It is the inverse of a frequency. Thus, highest scales correspond to lowest frequencies and vice versa. For example, in a time series noise travels along lowest scales or highest frequencies. And the highest scales give information about the trend of a time series. Wavelets are like the glasses of a microscope: the information you analyse depends on the precision of your glasses. Those interested in wavelet analysis can find more information in Resnikoff and Raymond O. Wells (1998).

Now, let's turn to our case. Figure 2 shows the euclidean distance between actual and recalled vote to PP and PSOE over time. I have decomposed data in six scales, each of them corresponding to six frequencies. Here it seems that from 1996 the hidden vote has affected both parties in the same way. For more help, let's see Figure 3. It plots the difference between the outputs of the previous Figure, that is, it shows to which extent the hidden vote has affected both parties in the same way. And we see that, in every scale, this is so from the 1996 elections to the present: the influence is the same. In scales 1 and 2, the difference drops below 10^{-5} from April 1996 to October 2003, when it raises again. May be the debate about the presence of Spanish troops in Iraq has something to tell. In scales 3 and 4, the difference drops below 10^{-5} from April 1996 to the last measure. Moreover, in scales 5 and 6 the differences already drops below 10^{-5} from April 1986. Remember that higher scales show the trend, and in some sense, scales 5 and 6 were anticipating in 1986 what was going to happen ten years later.

The conclusion is that there is no correspondence between actual and recalled vote. People lie. In the case of PSOE, when it has ruled, more people affirm having voted left; when in opposition, even its voters do not admit being so. In the case of PP, when in government, there are more voters willing to admit having voted right than when PP has been in opposition. Even in some moments after getting the absolute majority in parliament, some nonvoters "remember" having voted right. Those who feel siding the minority tend to remain silent and, sometimes, lie and "remember" what can not be remembered since it never happened.

4 Model

4.1 Analysis

First things first. And when modeling, one of the first things is keep the model simple. Although it seems that people in the minority may opt to lie instead of only to keep quiet, a model with two strategies (to speak or not to speak) is a good starting point. Later, I can always introduce the possibility that players may choose whether they want to reveal their true opinion or lie.

By the moment, the model says so: we have three players, Nature, P1 and P2. Both P1 and P2 have two strategies, to speak (s) or not to speak $(\sim s)$. By the contrary, Nature chooses whether P1 and P2 agree in their opinion (a) or not $(\sim a)$. P1 and P2 receive a payoff b whenever they speak and they agree;

whereas receive a payoff of $-c$ whenever they speak and disagree, for $b, c > 0$. If they keep quiet, they always receive a payoff of 0. In its normal form, P1 chooses rows; P2, columns; and Nature, matrices. If P1 and P2 knew for sure that they were playing in matrix 1, the Nash equilibrium would be (s, s).

a			
		P2	
		s	$\sim s$
P1	s	b, b	$b, 0$
	$\sim s$	$0, b$	$0, 0$

(1)

$\sim a$			
		P2	
		s	$\sim s$
P1	s	$-c, -c$	$-c, 0$
	$\sim s$	$0, -c$	$0, 0$

(2)

If they knew for sure they were playing in matrix 2, the Nash equilibrium would be $(\sim s, \sim s)$. But they know nothing. So they can only look for an equilibrium in mixed strategies. Let q their belief about Nature choosing a and $1-q$ about choosing $\sim a$, and let p their belief about the other player choosing s and $1-p$ about choosing $\sim s$. Thus, their expected utilities are:

$$\begin{aligned} EU(s) &= q(pb + (1-p)b) + (1-q)(-pc - c(1-p)) \quad (3) \\ EU(\sim s) &= 0 \quad (4) \end{aligned}$$

and they are indifferent between s and $\sim s$ when $q = c(b+c)^{-1}$. Hence, the Nash equilibrium is that both players play the choose the profile $(s, c(b+c)^{-1};$ $\sim s, 1 - c(b+c)^{-1})$.

This is so in a one-shot interaction, but we want players make many interactions among them. We want that they update their belief q. To do so, let n be the number of people holding opinion A and m those holding B. Let $\sigma \equiv \text{signum}(n-m)$ and define two states of the world $\theta \equiv e$ when $\sigma = 1$ and $\theta' \equiv e^{-1}$ when $\sigma = -1$. Now, take the expectation of these two states:

$$\tilde{\theta} = z\theta + (1-z)\theta' \quad (5)$$

where z is the probability that one sides with the majority and, thus, $\tilde{\theta}$ is an increasing function of it. So, let define $q \equiv \hbar\tilde{\theta}$. Here the reader can appreciate why I have defined $\theta \equiv e^{\sigma}$ instead of $\theta \equiv \sigma$: it is a sufficient condition for q to be positive when $z \in [0,1]$, although it is not the only one. Players only can update their belief z when the meet another player who decides to speak. In that case, they update z by Bayes' rule:

$$z_{t+1} = \frac{z_t \hbar e}{z_t \hbar e + (1-z_t)\hbar e^{-1}}, \quad (6)$$

when they meet someone who agrees with them and

$$z_{t+1} = \frac{z_t(1-\hbar e)}{z_t(1-\hbar e) + (1-z_t)(1-\hbar e^{-1})}, \quad (7)$$

when they meet someone who disagrees with them. Although I have no formal proof of it yet, I conjecture that $z, q \in [0,1]$ if $\hbar \in \left(0, e^{-1}\right]$. I also do not know if the game with N-players can be solved analytically and they play it again and again among them. Maybe it is, but I have run some simulations instead.

4.2 Simulation

Here I have a simulated a world with agents who can only take two positions on a topic: 0 or 1. The user sets up the number of agents and encounters, the initial belief (which is the same for all of the agents), the value of $\hbar$, the proportion of agents holding opinion 1, and the payoffs b, c for speaking and remaining silent respectively. In all the simulations here reported, I have fixed these values: 100 agents, 2000 iterations, $b, c = 1$. I have varied the other variables to see their effect, one at each time. So I have:

1. A set of runs with changing beliefs: I have run nine simulations varying beliefs from 0.1 to 0.9 with increases of 0.1 each one. Here I have fixed $\hbar$ to 0.15 and the proportion of agents holding 1 to 0.9.

2. A set of runs with changing $\hbar$: I have run seven simulations varying $\hbar$ from 0.05 to 0.35 with increases of 0.05 each one. Here I have fixed the initial belief to 0.9 and the proportion of agents holding 1 to 0.9.

3. A set of runs with changing proportions: I have run five simulations varying the proportion of those holding opinion 1 from 0.1 to 0.9 with increases of 0.1. Here I have fixed the initial belief to 0.9 and $\hbar$ to 0.15.

I have followed an object-oriented design for the program. In the last page of this paper, the reader can see a class diagram describing classes of objects and their relations. The program has been written in Matlab, since this environment allows object-oriented programming, comes with many functions for analysing and plotting results and the output can be transferred easily to an statistical program like R. The folders with the classes to run the program are available under request.

The sequence of the program is as follows: at each iteration two agents are picked up at random and the play the one shot game. If their belief q satisfies $q > \frac{c}{b+c}$, they speak; otherwise, they keep quiet. When $q = \frac{c}{b+c}$, they are indifferent, so any strategy is good. Instead of fixing a strategy to solve these cases, I have preferred that they choose at random between both strategies with probability 0.5. Only when they decide to speak, they reveal their opinions and the other agent can update her beliefs; otherwise, they do not update their beliefs, since they cannot. In any case, they received their payoffs of the one shot game, the program picks up other two agents and the same story goes.

5 Results

Let's see Figures 4 and 5. On the one hand, when the initial belief lies in $[0.5, 0.9]$, the mean belief goes up and then down. As the initial belief becomes smaller, the maximum is also smaller and it takes more iterations to reach it, although it seems that the slope gets higher. At the same time, the drop becomes slower, even though it takes less iterations to reach an equilibrium, which it is always the same: 0.87, the percentage of those holding 1, the majority position. From the time when beliefs reach their maximum, both beliefs and strategies go down with more or less the same slope. On the other hand, when the initial belief is smaller than 0.5, the mean belief does not change: they are stationary. The mean strategy goes down up to reach zero with the same slope.

One interesting prediction is when the spiral of silence process begins. What do I mean by the beginning of the process? Well, take the number of people defending publicly the less supported opinion, in this case 0. Since I know, because I have fixed, the number of people holding that opinion, I can perform a binomial test to check whether both proportions coincide. Figure 8 plots the evolution of these p-values over time for the nine simulations with varying initial belief. As the initial belief goes down, the starting point of the spiral of silence gets smaller. I can state even more: Figure 9 shows that it decays exponentially as the initial belief increases.

With $\hbar$ happens the contrary that with initial beliefs: as $\hbar$ decreases, beliefs reach higher maximums, decay slower and it takes more iterations to reach an equilibrium (Figure 6. But, as before, strategies wait for beliefs to reach its maximum to begin to go down with the same slope. The equilibrium is the same too: the percentage of those holding the majority position. I have performed some binomial tests to see when the spiral of silence begins. Figure 10 confirms that the effect of $\hbar$ is the opposite to that of the initial belief. As $\hbar$ goes down, the starting point of the spiral of silence gets higher. Indeed, it grows exponentially with $\hbar$ as Figure 11 shows.

Finally, let's study what happens with proportions in Figure 7. As the number of supporters of the two positions comes closer, people tend more and more to speak. When the proportions are equal, the mean strategy is to speak and the mean belief is 1, "I side the majority position for sure".

In general, I would say that the effect of both the initial belief and $\hbar$ is to modify the time when an equilibrium is reached. The higher the initial belief about you siding the majority, the sooner an equilibrium is reached. The higher $\hbar$ is, the later an equilibrium is reached. In both cases, the equilibrium consists in that only those in the majority choose to speak and only those in the minority choose to remain silent. If we vary the proportions of supporters, as they get closer, everybody speaks.

6 Conclusion

I have exposed a formal model of the spiral of silence. First, I have studied a single interaction between two players who must decide whether or not to speak and reveal their opinion about some topic. Whenever they speak and agree, they receive a payoff $b > 0$; whenever they speak and disagree, they receive a payoff $-c$, for $c > 0$; otherwise, they receive a payoff of zero. The mixed strategy Nash equilibrium is that both players choose the profile $\left(s, c\left(b+c\right)^{-1}; \sim s, 1-c\left(b+c\right)^{-1}\right)$.

But, what happens when there are many players and, from time to time, they encounter with each other? I have run some simulations to get some predictions. In this case, players can update their belief about they siding the majority position by Bayes' rule. The higher the initial belief of players about they siding the majority, the sooner an equilibrium is reached where neither beliefs nor strategies change any more. Moreover, the first iteration when the minority shuts up at a 0.05 significance level decays exponentially with the initial beliefs.

In the model, there is also an special constant $\hbar$ which shows the contrary effects that initial beliefs. The higher $\hbar$ is, the later an stationary state is reached. And the first iteration when minority keep quiet at a 0.05 significance level grows exponentially with $\hbar$.

In both cases, if the number of supporters of the majority is 0.70 or higher, only they speak, while the minority remains silent. Under this limit, everybody speaks.

Finally, open queries. Consider the possibility that players can lie. If they remain silent, others may interpret it as a signal of holding the minority position and receive a cost for not cooperating with the majority in the same way as if they had revealed their opinion. In such a case, remaining silent is not enough, players should be allowed the possibility of lying. This is the situation which Kuran (1995) calls spiral of prudence.

Where do social sanctions come from? As I have written some pages ago, social sanctions for not supporting the majority position may be consider as a second-order collective action problem. For example, Kuran argues that some people in the minority may lie and, even more, contribute to the sanctions to those who do not support publicly the majority. Social sanctions are crucial for the spiral of silence or prudence to work. But nobody has explained how a majority can produce this collective good and, even more, how a majority can make some people of the opposition also contribute to this good. So we have a group, a majority, which must solve a collective action problem, the use of sanctions, to impose its view to the rest of the society. But how can be possible that people in the minority also contribute to the public good called "social sanctions"?

Locke talks about the law of opinion on his *Essay concerning human understanding*. For something to be a law, whether it is self-enforcing like a Nash equilibrium or someone or something enforces it. To understand public opinion we must understand how the system of social sanctions is created and supported. A collective action problem is waiting for us to solve it.

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A Figures

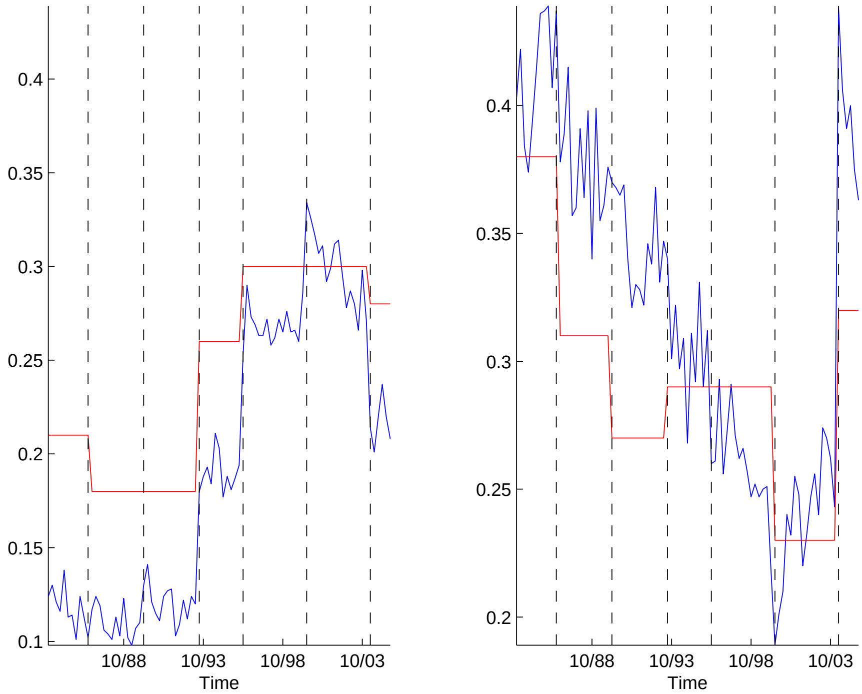

Figure 1: Actual vote (red line) and recalled vote (blue line) from 1984 to the present. The left panel represents the data for PP and the right one for PSOE. The vertical dashed lines represent the first measurement after an election.

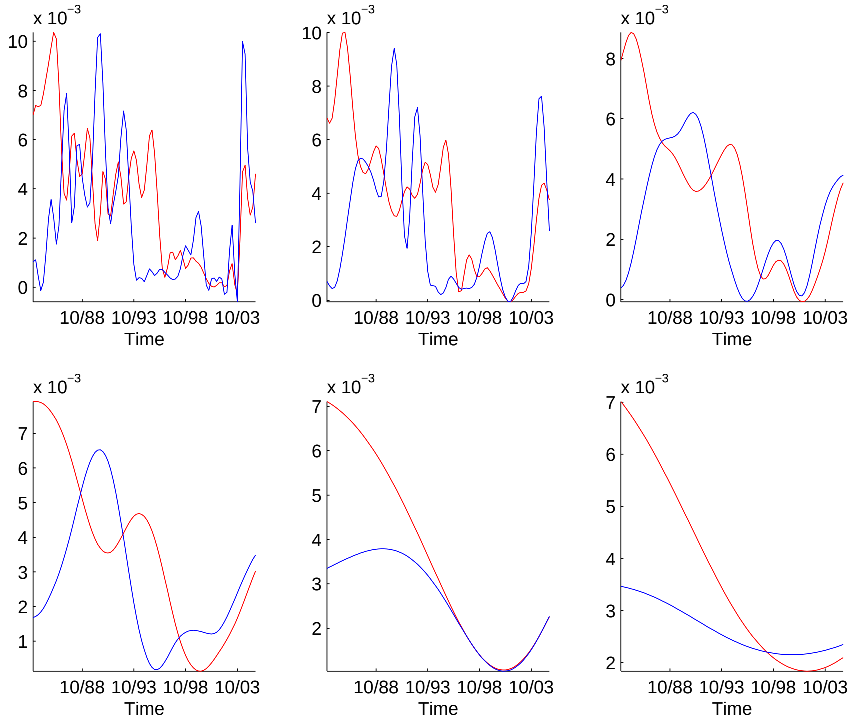

Figure 2: Euclidean distance between actual and recalled vote for PP (blue line) and PSOE (red line) as a function of time. The panels show the wavelet decomposition of the original data in six scales, from the lowest to the highest if one reads the figure from left to right, up to down. Highest scales correspond to lowest frequencies, so the last scales correspond to the trend in the difference between actual and recalled vote.

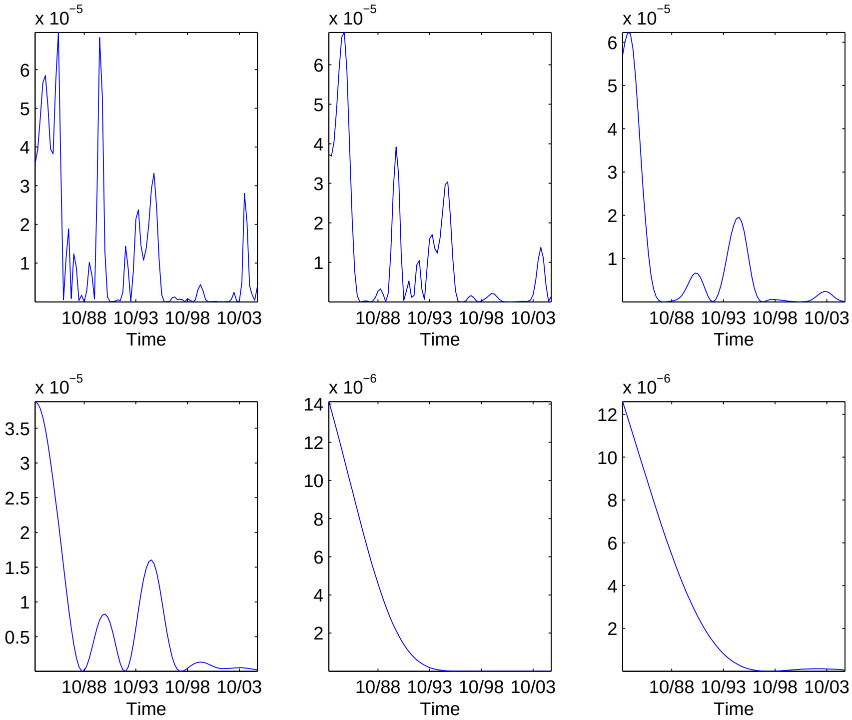

Figure 3: Difference of the distance between actual and recalled vote between PP and PSOE as a function of time. That is, the difference between the red and blue lines of Figure 2. The panels show the wavelet decomposition of the original data in six scales, from the lowest to the highest if one reads the figure from left to right, up to down. The figure shows how the hidden vote have affected in the same way to both parties in last years.

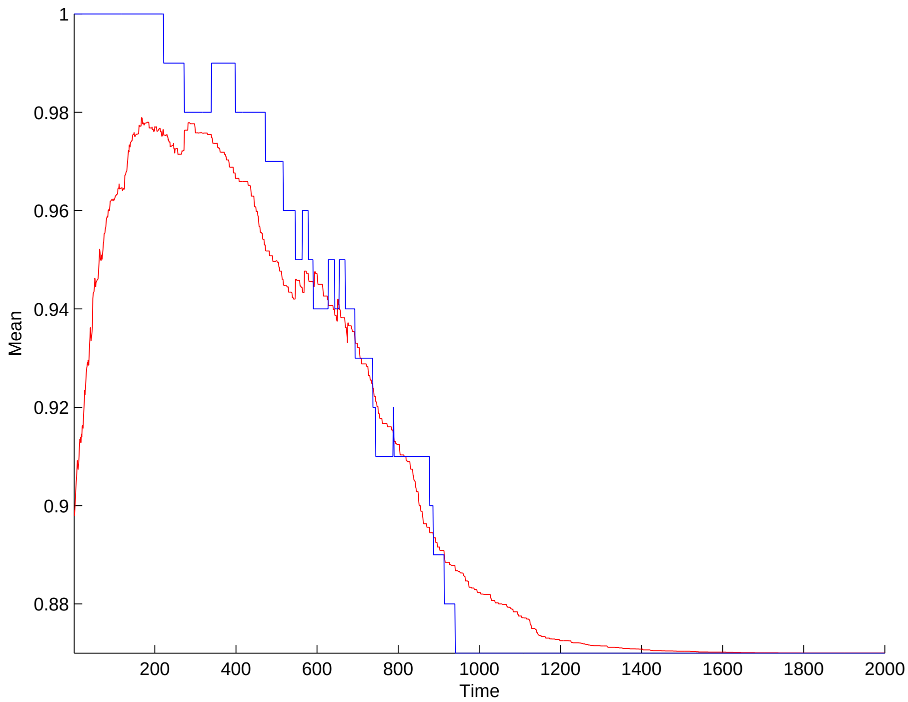

Figure 4: Mean beliefs (red line) and strategies (blue line) from a simulation where the initial belief is $0.9, \hbar = 0.15$ and the proportion of those holding 1 is 0.9.

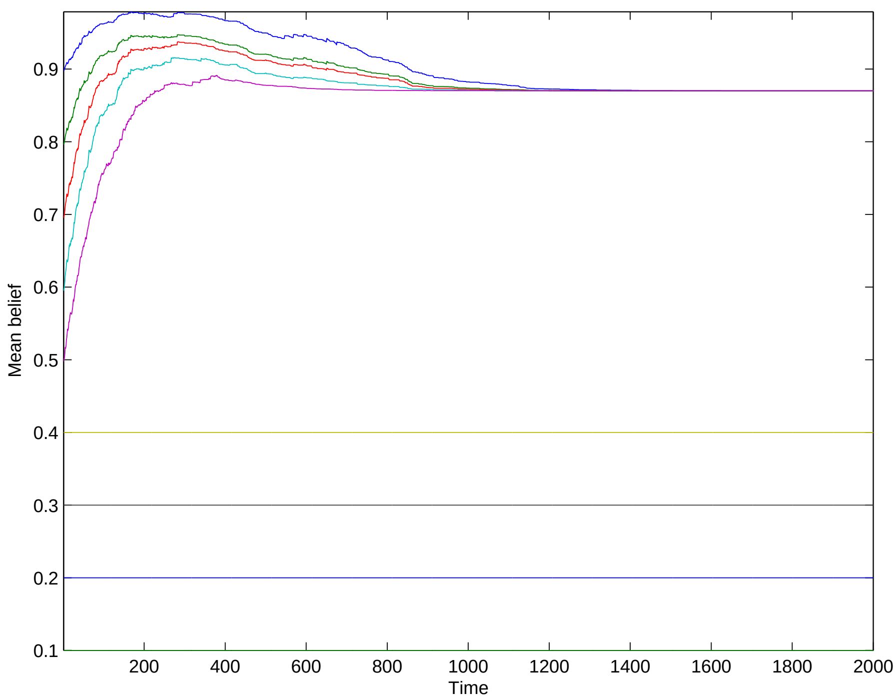

Figure 5: Mean beliefs for nine simulations with varying initial belief from 0.9 (the top blue line) to 0.1 (the bottom green line) with decreases of 0.1.

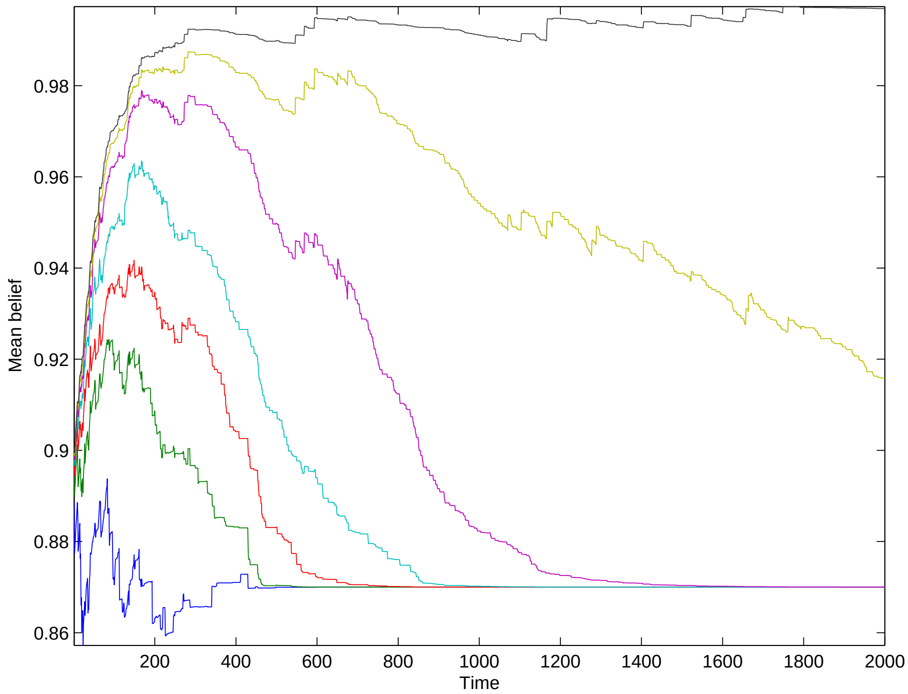

Figure 6: Mean beliefs for seven simulations with varying $\hbar$ from 0.35 (the bottom blue line) to 0.05 (the top line) with increases of 0.05.

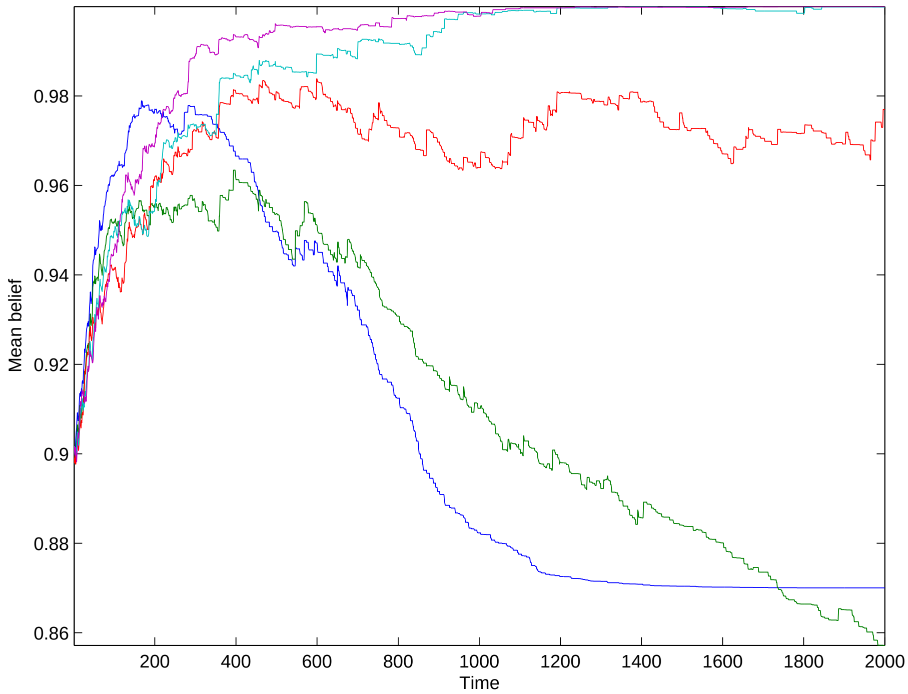

Figure 7: Mean beliefs for five simulations with varying proportion of those holding 1 from 0.9 (the blue line) to 0.5 (the violet line) with decreases of 0.1.

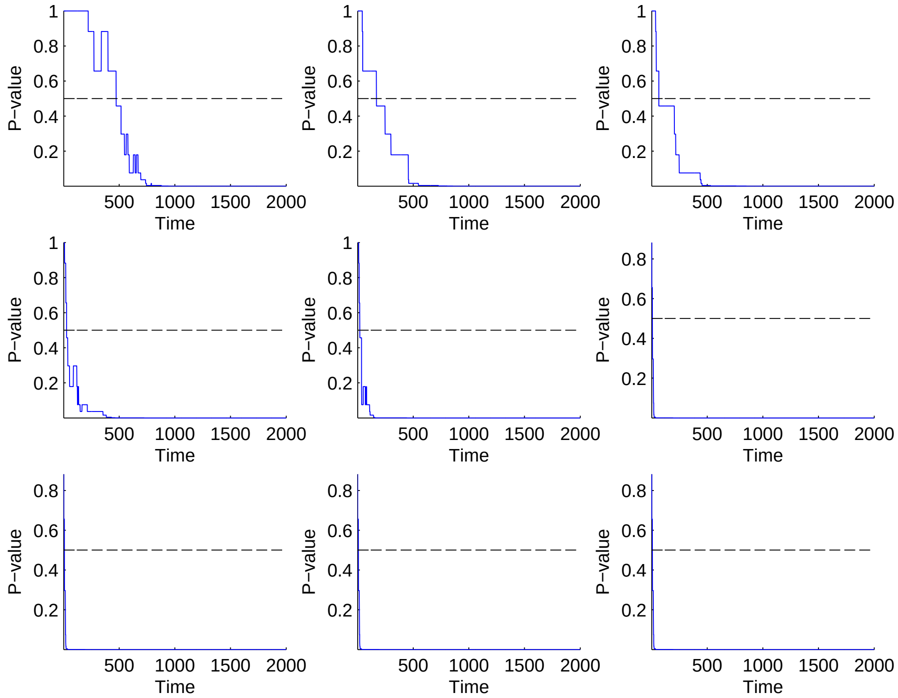

Figure 8: P-values of a binomial test when the initial belief decreases from 0.9 to 0.1 at decrements of 0.1. The figure is read from left to right, from up to down. The upper-left panel represents the results for the simulation when the initial belief is 0.9. The null hypothesis is that the number of people defending publicly an opinion of 0 is the same as the number of those actually holding it. The alternative hypothesis is that these proportions are different. I have performed a binomial test for each iteration and here I plot the p-values of these tests against their corresponding iteration. The dashed line is located at 0.05 as a reference.

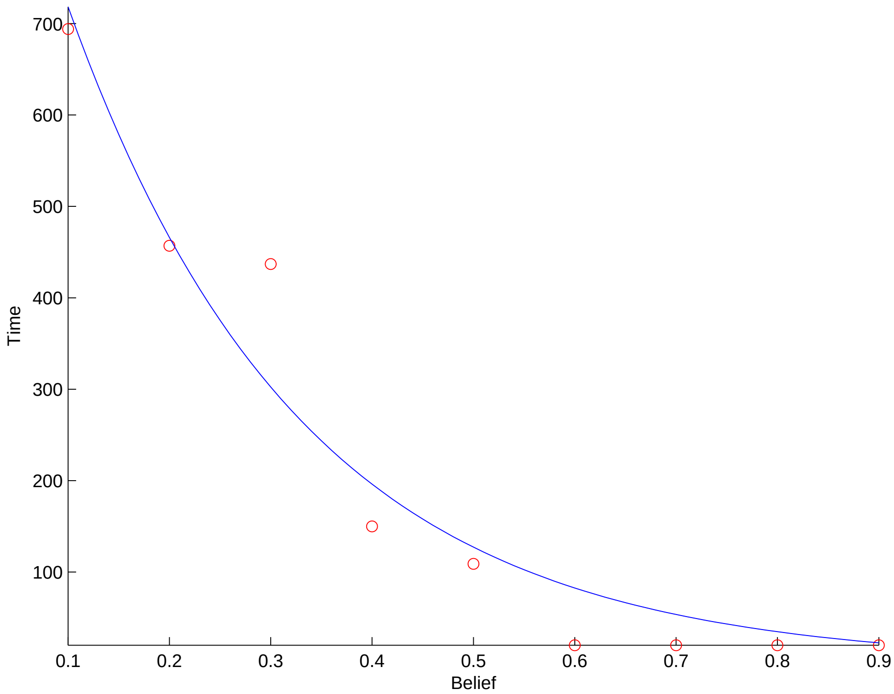

Figure 9: Iteration when first p-value under 0.05 occurs against initial belief. From data used in Figure 8, I have taken the first time when a p-value drops below a 0.05 significance level and plotted it against the initial belief used in its respective simulation. Points represent the actual p-value and the solid line, a fit of the curve ce^{-at}, where t is the initial belief. I have used nonlinear least squares to estimate the parameters $c = 1\,107.336$ ($\hat{s} = 133.781$) and $a = 4.327$ ($\hat{s} = 0.5902$).

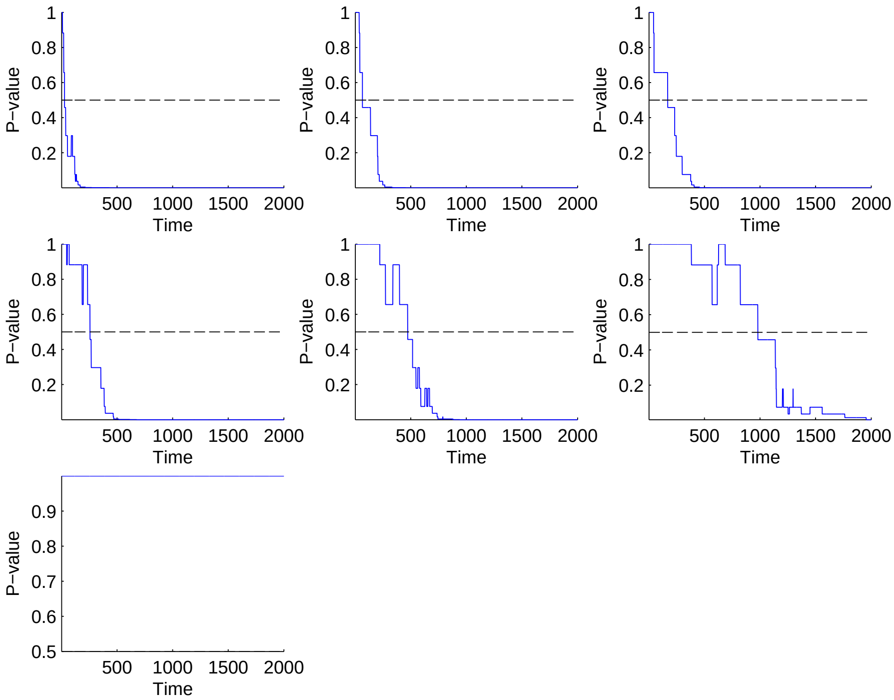

Figure 10: P-values of a binomial test when $\hbar$ decreases from 0.35 to 0.05 at decrements of 0.05. The figure is read from left to right, from up to down. The upper-left panel represents the results for the simulation when $\hbar = 0.35$. The null hypothesis is that the number of people defending publicly an opinion of 0 is the same as the number of those actually holding it. The alternative hypothesis is that these proportions are different. I have performed a binomial test for each iteration and here I plot the p-values of these tests against their corresponding iteration. The dashed line is located at 0.05 as a reference.

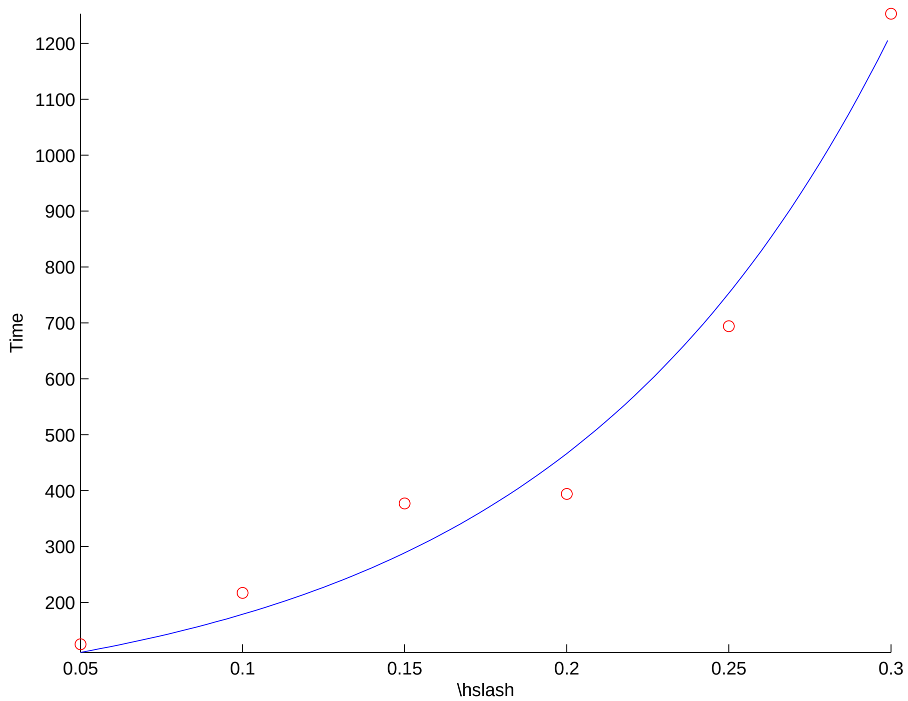

Figure 11: Iteration when first p-value under 0.05 occurs against $\hbar$. From data used in Figure 10, I have taken the first time when a p-value drops below a 0.05 significance level and plotted it against the initial belief used in its respective simulation. Points represent the actual value and the solid line, a fit of the curve ce^{-at}, where t is $\hbar$. I have used nonlinear least squares to estimate the parameters $c = 68.467$ ($\hat{s} = 17.939$) and $a = 9.592$ ($\hat{s} = 0.956$).

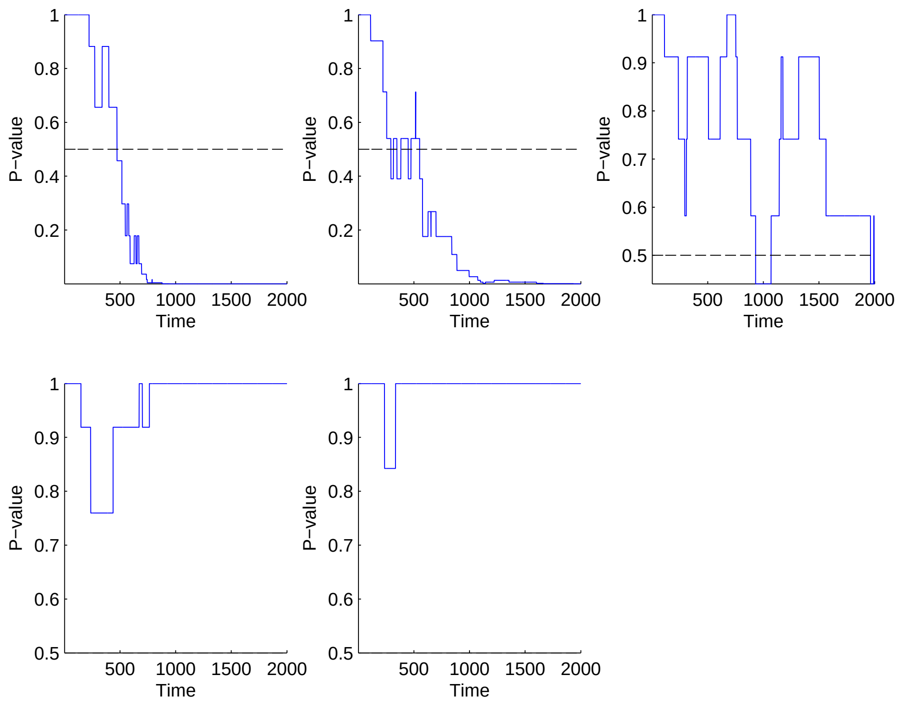

Figure 12: P-values of a binomial test when the proportion of those holding opinion changes from 0.9 to 0.5 at decrements of 0.1. The figure is read from left to right, from up to down. The upper-left panel represents the results for the simulation when the proportion is 0.9. The null hypothesis is that the number of people defending publicly an opinion of 0 is the same as the number of those actually holding it. The alternative hypothesis is that these proportions are different. I have performed a binomial test for each iteration and here I plot the p-values of these tests against their corresponding iteration. The dashed line is located at 0.05 as a reference.