# Money flexibility and optimal consumption-leisure choice under price dispersion

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## Introduction

The synthesis of the G.Sigler's rule of the optimal search with the classical individual labor supply model enlarges the understanding of the phenomenon of money flexibility. The constraints of the search model makes the Lagrangian multiplier equal to the marginal utility of the wage rate and establish the correspondence between the purchase price elasticity of the marginal utility of consumption expenditures, the wage rate elasticity of the marginal utility of money, and the wage rate elasticity of purchase prices. This correspondence can review the "leisure model" of behavior as well as the Veblen effect. The phenomenon of the sunk costs sensitivity also becomes more understandable.

JEL Classification: D11, D83.

## The Lagrangian multiplier in the search model

The optimal consumption-leisure choice under price dispersion can be presented by the utility function *(U(Q,H)* which is constrained by the equality of marginal values of search to its marginal benefit, or

$$w\frac{\partial L}{\partial S} = Q\frac{\partial P}{\partial S} \quad (1)$$

where

$w$ – the wage rate; $Q$ – the quantity purchased; $L$ – the labor time: $S$ - the time of search.

$\partial L/\partial S$ – propensity to search ($\partial L/\partial S<0$)

$\partial P/\partial S$ – price reduction during the search ($\partial P/\partial S<0$)

We take the value of the wage rate and the value of the price reduction as constant values. The constant $\partial P/\partial S$ value means that when we concludes the search at the certain satisficing price level we have spent time $S$ on the search and we have got the price reduction $\Delta P$ in our willingness to pay. At this price level we determine how much we should buy for the period of time until the next purchase. The chosen quantity $Q$ determines the labor time $L$ we need to restore our cash-in-the-pocket or money balances (Here, we can bypass the monetary theory by the simple appeal to the H.Leibenstein's understanding of the static analysis, who presented the static situation as one in which the order of events was of no significance (Leibenstein 1950, p.187), or by the come-back to old days when debts were recorded by shopkeepers in notebooks). And for the chosen time horizon of our consumption-leisure choice $T$ (a day, couple of days, a week, a year) we finally determine the leisure time $H = T - S - L$ we will need to consume the chosen quantity $Q$. So, the time of search $S$ diminishes the labor income $wL(S)$, required for the purchase. However, multiplied by the amount of search $S=dS$, the value $\partial wL/\partial S<0$ gives us the total loss in labor income during the search with regard to the labor income reserved for the purchase $wL_0$.

So, we can formulate the following problem:

$$\max U(Q,H) \quad subject\ to \quad w - Q\frac{\partial P/\partial S}{\partial L/\partial S} \quad (2)$$

This search model of the optimal consumption-leisure choice has been already presented with the help of the Cobb-Douglas utility function $U(Q,H) = Q^{-\partial L/\partial S}H^{\partial H/\partial S}$. That intuition was supported by the fact that the marginal utility of substitution of leisure for consumption in the search model corresponds to the $MRS$ value for the utility function with constant elasticity of substitution $\sigma = 1$, because from the equation $L+S+H=T$ we have $(\partial L/\partial S+\partial H/\partial S) = -1$:

$$MRS(HforQ) = -\frac{dQ}{dH} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$\frac{dQ}{dH} = \frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = \frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = \frac{QT}{(H-T)}\frac{1}{T} = \frac{Q}{H}\frac{H}{(H-T)} = \frac{Q}{H}\frac{(H-T+T)}{(H-T)}; \quad (3)$$

$$\frac{dQ}{dH} = \frac{Q}{H}(1+\frac{T}{H-T}) = \frac{Q}{H}(1+\frac{1}{\partial L/\partial S}) = \frac{Q}{H}(\frac{\partial L/\partial S+1}{\partial L/\partial S});$$

$$\partial L/\partial S = -\alpha \Rightarrow \frac{dQ}{dH} = -\frac{Q}{H}(\frac{1-\alpha}{\alpha})$$

The only difference between the search model and the classical model of the individual labor supply is the extraction of the time of search as productive time from the time of leisure and its analytical addition, as we are going to see in the set of Equations (4), to the time of labor.

Usually the search cuts not only the labor time but also the leisure time ($\partial H/\partial S<0$). The following set of inequalities ($-1<\partial L/\partial S<0;\ \partial H/\partial S<0$) describes the “common model” of consumer behavior. And this “common model” can be presented with the help of the Archimedes’ principle. The search displaces in the time horizon of the consumption-leisure choice both the labor time and the leisure time, like ice displaces both whiskey and soda in the glass. The Archimedes’ principle enables the understanding of the relationship between the labor and the search, or the function $L= L(S)$:

$$\begin{aligned}
&L(S) = T - H(S) - S;\\
&\partial L / \partial S = -\partial H / \partial S - 1;\\
&a)\quad dH(S) = dS\frac{\partial H}{\partial S} = -dS\frac{H}{T}\\
&\qquad\qquad\qquad \Downarrow\\
&b)\quad \frac{\partial L}{\partial S} = -\frac{\partial H}{\partial S} - 1 = \frac{H}{T} - 1 = \frac{H-T}{T} = -\frac{L+S}{T} \quad (4)\\
&c)\quad \frac{\partial L}{\partial S} = -\frac{L+S}{T} \Rightarrow \partial^2 L / \partial S^2 = -\frac{\partial L / \partial S + 1}{T} < 0\\
&d)\quad \frac{\partial L}{\partial S} = \frac{H-T}{T} \Rightarrow \partial^2 L / \partial S \partial H = 1/T;
\end{aligned}$$

Using the Archimedes' principle, we can reconsider the value of the time horizon of the consumption-leisure choice. In the search model the value of the time horizon equals not to the chronological period but to the period of the product lifecycle, i.e., to the moment of the next purchase of the same item. (Equation 4d) The greater time horizon increases not only the required labor time but also the leisure time, required by consumption. So, the increase in the value of time horizon decreases the absolute value of ***propensity to search*** **$|\partial L/\partial S|$.**

The set of Equations (4) simplifies the presentation of the values of marginal utilities, derived form the solution of the utility maximization problem (2), and it explains step-by-step derivation of the *MRS(H for Q)* in the Equation (3):

$$MU_Q = \lambda\frac{\partial P / \partial S}{\partial L / \partial S} = \lambda\frac{w}{Q} \quad (5a)$$

$$MU_H = -\lambda Q\frac{\partial P / \partial S}{(\partial L / \partial S)^2}\partial^2 L / \partial S \partial H = -\lambda\frac{w}{\partial L / \partial S}\partial^2 L / \partial S \partial H = \lambda\frac{w}{L+S}\frac{T}{T} = \lambda\frac{w}{L+S} \quad (5b)$$

Now, if we optimize utility $U^*$ with regard to the wage rate $w$ with the help of the chain rule, we get

$$\begin{aligned}
&\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda\left[\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} + \frac{w}{L+S}\frac{\partial H}{\partial w}\right]\\
&\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} = \frac{\partial P / \partial S}{\partial L / \partial S}\frac{1}{\partial P / \partial S}\left(\frac{\partial L}{\partial S} + w\frac{\partial(\partial L / \partial S)}{\partial w}\right) = 1 + \frac{w}{\partial L / \partial S}\frac{\partial(\partial L / \partial S)}{\partial w} = 1 + e_{\partial L/\partial S, w}\\
&\frac{w}{L+S}\frac{\partial H}{\partial w} = -\frac{w}{L+S}\frac{\partial(L+S)}{\partial w} = -\frac{w}{(L+S)/T}\frac{\partial(L+S)/T}{\partial w} = -\frac{w}{\partial L / \partial S}\frac{\partial(\partial L / \partial S)}{\partial w} = -e_{\partial L/\partial S, w} \quad (6)\\
&\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda\left[\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} + \frac{w}{L+S}\frac{\partial H}{\partial w}\right] = \lambda\left[1 + e_{\partial L/\partial S, w} - e_{\partial L/\partial S, w}\right] = \lambda
\end{aligned}$$

Then, we can re-arrange the equation (1) with the help of the Equation (4b) in the following manner:

$$- T\partial P/\partial S = w(L+S) = P_0 \qquad (7)$$

The Equation (7) gives us the value of ***the price equivalent of the potential labor income w(L+S),*** which could be earned by the consumer if he has spent all time of search on working.

The value of the price equivalent of the potential labor income simplifies the presentation of the value of the marginal utility of consumption. Now we can write for consumption and leisure

$$\lambda = \frac{\partial U^*}{\partial w};$$

$$MU_Q = \lambda \frac{\partial P/\partial S}{\partial L/\partial S} = \lambda T \frac{\partial P/\partial S}{L+S} = \lambda \frac{P_0}{L+S} = \frac{\partial U^*}{\partial w}\frac{P_0}{T-H}; \quad (8)$$

$$MU_H = \lambda \frac{w}{L+S} = \frac{\partial U^*}{\partial w}\frac{w}{T-H}$$

However, if we repeat step by the step the same utility optimization procedure within the individual labor supply model, we get almost the same result for the classical consumption-leisure choice (Baxley and Moorhouse 1984):

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda P \frac{\partial Q}{\partial w} + \lambda w \frac{\partial H}{\partial w} = \lambda \left[ P\frac{\partial Q}{\partial w} + w\frac{\partial H}{\partial w} \right]$$

$$wT = PQ + wH \Rightarrow T = P\frac{\partial Q}{\partial w} + w\frac{\partial H}{\partial w} + H \Rightarrow T - H = P\frac{\partial Q}{\partial w} + w\frac{\partial H}{\partial w} \quad (9)$$

$$\lambda = \frac{\partial U^*}{\partial w}\frac{1}{T-H}$$

and, finally

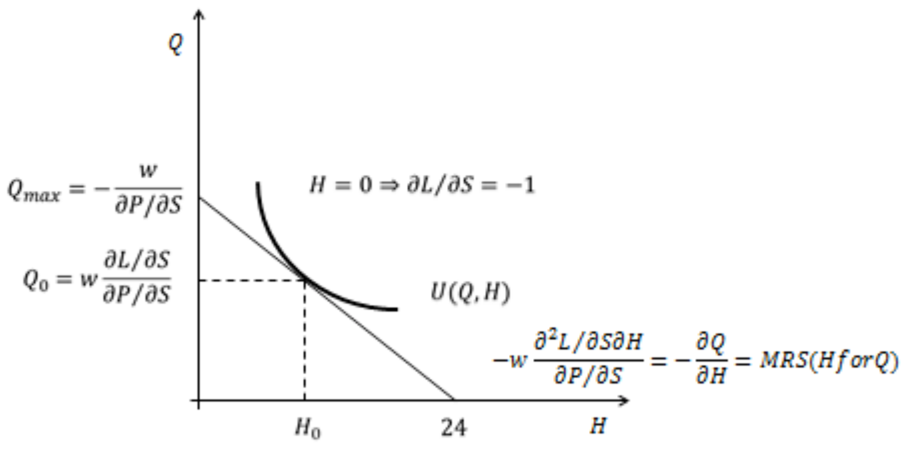
$$\lambda = \frac{\partial U^*}{\partial w}\frac{1}{T-H};$$

$$MU_Q = \lambda P = P\frac{\partial U^*}{\partial w}\frac{1}{T-H}; \ (10)$$

$$MU_H = \lambda w = w\frac{\partial U^*}{\partial w}\frac{1}{T-H}$$

So, the graphical presentation, given with unusual attributes, simply represents the specific form of the graphical resolution of the optimal consumption-leisure choice for $T$ = *24 hours* (Fig1):

Fig.1



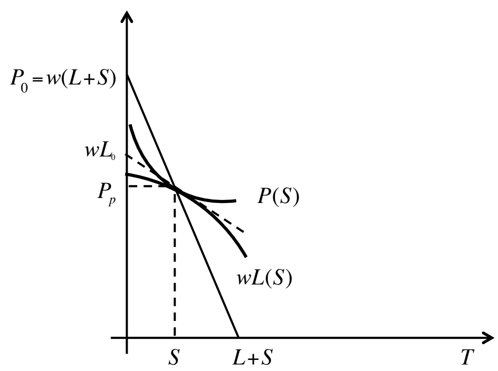

There are two important differences between two models. First, the value of the Lagrangian multiplier $\lambda$ in the search model is exactly equal to the marginal utility of the wage rate, or $MU_w=\lambda$, while in the classical model we need to adjust it to the allocation of time between labor and leisure. Second, in the classical model the marginal utility of consumption is determined by the equilibrium purchase price $P_e$, while the marginal utility of consumption in the search model is determined by the price equivalent of the potential labor income $P_0=w(L+S)$.

Now we cannot definitely answer to the question whether, when search costs are positive, the price equivalent of potential labor income is the equilibrium price, but, if we re-arrange the Equation 3, we can simplify the presentation of the marginal rate of substitution of leisure for consumption:

$$\frac{\partial U/\partial H}{\partial U/\partial Q}=MRS(HforQ)=-\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{\partial P/\partial S}\frac{1}{T}=\frac{w}{P_0} \quad (11)$$

The analysis of the satisficing decision procedure (Malakhov 2012b) can clarify the concept of the price equivalent of the potential labor income (Fig.2):

Fig.2




Here, the purchase price $P_P$ is equal to the labor income $wL$. So, the value of the labor income, reserved for the purchase before we start to search, $wL_0=wL+dwL(S)$, can represent our willingness to pay. And we can say that the ***equilibrium value of price reduction*** $\partial P/\partial S$ simply represents the rate of the decrease in the willingness to pay during the search.

It is more difficult to determine the price equivalent of the potential labor income. It looks like a high-order willingness to pay with regard to the reservation level or the reservation price. P.Diamond used that approach when he described the behavior of shoppers with high willingness to pay (Diamond 1987). On the other hand, the Equation (11) initiates the other P.Diamond's consideration, that with positive search costs the equilibrium price is equal to the monopoly price (Diamond 1971). However, the idea of the monopoly price seems not to be an appropriate explanation of the price equivalent of the potential labor income, because if a monopoly sets the price at the level of the potential labor income, consumers will increase their labor supply by the time of search, and this increase in labor supply will decrease the wage rate. The price equivalent of the potential labor income at lower wage rate will become unattainable.

We should keep in mind that this value is to some extent virtual, which can be determined only *ex post*, when we conclude the search. This *ex ante – ex post* relationship between the reserved labor income and the potential labor income provokes the idea that here we have the WTP-WTA relationship and the potential labor income represents the price that compensates not only monetary losses, which are equal to the purchase price $P_P=wL$, but also losses in the time of search, calculated *ex post* on the base of the wage rate, or $wS$. However, this consideration needs the answer to the question whether the willingness to accept is an appropriate equivalent of the equilibrium price when search costs are positive.

Finally, it is easy to demonstrate that all above-presented equations are valid for the case when the search costs are equal to zero, or $S=0$. Here, all properties of the search model become equivalent to the corresponding values of the classical individual labor supply model. The utility function takes the usual $U(Q,H)=Q^{L/T}H^{H/T}$ form. The propensity to search becomes equal to the $(-L/T)$ ratio, the *MRS (H for Q)* becomes equal to the $Q/L$ ratio, and the value of the marginal utility of consumption $MU_Q$ becomes equal to the classical $\lambda P_P$ value, where $\lambda$ is equal to the $(\partial U^*/\partial w)/(T-H)$ ratio. Indeed, when the search costs are equal to zero, the purchase price is equal to the potential labor income and the willingness to pay becomes equal to the willingness to accept.

**Money flexibility and the Veblen effect**

The equation $MU_w=\lambda$ tells us that the marginal utility of labor income per unit of time is equal to the marginal utility of an extra unit of money. However, the search process produces the non-labor income through the reduction in prices of purchases.

We can analyze the marginal utility of price reduction. However, for a better understanding and for illustrative purposes, we will use the absolute value of price reduction $|\partial P/\partial S|$, keeping in mind that due to $\partial^2 P/\partial S^2 >0$, the increase in the absolute value $|\partial P/\partial S|$ means the increase in price. The substitution of the value $\partial P/\partial S$ by the absolute value $|\partial P/\partial S|$ as well as the substitution of the value $\partial L/\partial S$ by the absolute value $|\partial L/\partial S|$ does not matter. However, somebody can use real values, keeping in mind corresponding changes in marginal values.

$$\begin{aligned}\frac{\partial U^*}{\partial|\partial P/\partial S|} &= \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{\partial U}{\partial H}\frac{\partial H}{\partial|\partial P/\partial S|}=\lambda\left[\frac{w}{Q}\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{w}{L+S}\frac{\partial H}{\partial|\partial P/\partial S|}\right]\\ &=\lambda\frac{w}{\partial P/\partial S}\left[\frac{|\partial P/\partial S|}{Q}\frac{\partial Q}{\partial|\partial P/\partial S|}+\frac{|\partial P/\partial S|}{L+S}\frac{\partial(T-(L+S))}{\partial|\partial P/\partial S|}\right]\\ &=\lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|}-\frac{|\partial P/\partial S|}{(L+S)/T}\frac{\partial(L+S)/T)}{\partial|\partial P/\partial S|}\right]\\ &=\lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|}-e_{|\partial L/\partial S|,|\partial P/\partial S|}\right]\end{aligned} \tag{13}$$

The key equation (1) of the search model can transform this result. If we present the key equation (1) in the elasticity form, we get for the constant wage rate

$$Q\left|\frac{\partial P}{\partial S}\right| = w\left|\frac{\partial L}{\partial S}\right| \Rightarrow e_{Q,|\partial P/\partial S|} + 1 = e_{|\partial L/\partial S|,|\partial P/\partial S|} \qquad (14)$$
$$e_{Q,|\partial P/\partial S|} - e_{|\partial L/\partial S|,|\partial P/\partial S|} = -1$$

This elasticity form of the key equation (1) gives us the value of the marginal utility of price reduction

$$\frac{\partial U^*}{\partial |\partial P/\partial S|} = \lambda \frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|} - e_{|\partial L/\partial S|,|\partial P/\partial S|}\right] = -\lambda \frac{w}{|\partial P/\partial S|} \quad (15)$$

We see, that the increase in the absolute value of price reduction $|\partial P/\partial S|$ decreases the utility and the decrease in the absolute value of price reduction increases the total utility. The negative value of the marginal utility of price reduction $MU_{|\partial P/\partial S|}$ needs some methodological comments. The search produces the non-labor income and the greater absolute value of price reduction increases money balances. In this sense, the marginal utility of price reduction is positive. But the assumption of the decreasing efficiency of search, or $\partial^2P/\partial S^2>0$, establishes the direct correspondence between the value of the purchase price and the value of price reduction. The greater absolute value of the price reduction corresponds to the greater purchase price. (The case, when consumers meet big discounts and thereby find low purchase prices, is examined with regard to the reduction in the time horizon of consumption-leisure choice or shelf lives, like it happens with pork sausages, for example. (Malakhov 2012b). Then, in order to avoid the confusion, we will regard the value $MU_{|\partial P/\partial S|}$ as the marginal utility of the price reduction at a certain price level of purchase, and, because the increase in price decreases the utility and $MU_{|\partial P/\partial S|} <0$, we will use the notion of ***the marginal disutility of price reduction***.

Its unusual form raises the question whether we have the same value of marginal utility of an extra unit of money when we compare labor income with non-labor income. However, it is easy to show that *pecunia non olet* and the value of the Lagrangian multiplier, or the extra utility of an extra unit of money, is the same for labor income as well as for non-labor income.

The Equation (15) is very useful. It uncovers the anatomy of the search process. First of all, we can see that different levels of wage rate result in different values of the marginal disutility of price reduction. Moreover, their relationship is reciprocal. The greater wage rate results in the greater marginal disutility of price reduction. This is a real phenomenon because when the value of price reduction represents the rate of the decrease of the purchase price with respect to the willingness to pay, the increase in the absolute level of price reduction $|\partial P/\partial S|$ first of all means the increase in the willingness to pay as the result of the increase in the wage rate.

The increase in the willingness to pay happens due to the increase in the opportunity costs of search, i.e., in the increase of the wage rate. Trying to save time, individuals begin to buy in nearby stores at high prices.[1] However, they can keep their willingness to pay constant and they can continue to buy in a distant supermarket. If the value $|\partial P/\partial S|$ is constant, we expect that the marginal disutility of price reduction $MU_{|\partial P/\partial S|}$ is also constant.

In order to verify this assumption, let's analyze the total change in the marginal disutility of price reduction with regard to the change in the wage rate

$$\frac{\partial MU_{|\partial P/\partial S|}}{\partial w} = -\left[\frac{\partial \lambda}{\partial w}\frac{w}{|\partial P/\partial S|} + \lambda \frac{\partial\left(\frac{w}{|\partial P/\partial S|}\right)}{\partial w}\right] = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (16)$$

We can see, that the marginal disutility $MU_{|\partial P/\partial S|}$ of consumers who continues to buy in the same place at the same price after the increase in their wage rates ($e_{|\partial P/\partial S|.w} = 0$) stays constant only if the marginal utility of the wage rate is unit elastic, or $e_{\lambda,w} = -1$. If it is not, consumers will face either the increase or the decrease in the marginal disutility of price reduction, may be, in the form of *the irritation from shopping* with respect to their income levels.

Here, we can address to the original R.Frisch's analysis of the phenomenon of money flexibility (Frisch 1959). It can give us some ideas about feelings of consumers with different income levels when they meet each other in the same supermarket. Things look like consumers with lower wage rate elasticity of the marginal utility of income $e_{\lambda,w}$, i.e., "the better-off part of the population" get greater disutility in supermarkets than "the median part of the population", because the increase in wage rates of the "median part" considerably reduces the disutility of shopping to its minimal negative values.

From here we can continue to follow the Frisch's methodology. When the second cross derivatives of utility are equal, or $\partial MU_{|\partial P/\partial S|}/\partial w = \partial MU_w/\partial|\partial P/\partial S|$ and when $MU_w=\lambda$, we can simply re-write the Equation (16) in the following form

$$\frac{\partial MU_{|\partial P/\partial S|}}{\partial w} = \frac{\partial \lambda}{\partial |\partial P/\partial S|} = -\left[\frac{\partial \lambda}{\partial w}\frac{w}{|\partial P/\partial S|} + \lambda \frac{\partial\left(\frac{w}{|\partial P/\partial S|}\right)}{\partial w}\right] = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (17)$$

Then, we make one more step and we get the general relationship of the marginal utilities of wage rate and of consumption expenditures under the search process.

$$\begin{aligned}
&\frac{\partial \lambda}{\partial |\partial P/\partial S|} = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \\
&\frac{\partial \lambda}{\partial |\partial P/\partial S|}\frac{|\partial P/\partial S|}{\lambda} = -\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (18) \\
&e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w} = 0
\end{aligned}$$

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[1] Here we simply follow the Salop-Stiglitz's assumption, that only high-search-cost individuals make purchases in high-price stores (Stiglitz 1979, p. 141 )

Really, the value $e_{\lambda,|\partial P/\partial S|}$ seems to be an appropriate representation of the concept of *the marginal utility of money expenditures* carefully derived by M.Blaug from the analysis of the Marshallian "Principles" (Blaug 1997, pp.322-323). Really, even when second cross derivatives are equal, they have different economic sense. From here we can leave the concept of the marginal disutility of price reduction and begin to use the concept of the price reduction elasticity of the marginal utility of money expenditures $e_{\lambda,|\partial P/\partial S|}$.

We can see that for the unit elastic marginal utility of income ($e_{\lambda,w} = -1$) the choice of the high-price store after the increase in the wage rate makes the elasticity of the marginal utility of money expenditures, here the value $e_{\lambda,|\partial P/\partial S|}$, positive, or

$$e_{\lambda,|\partial P/\partial S|} = e_{|\partial P/\partial S|,w} \quad (19)$$

However, again different fluctuations of the wage rate elasticity of the marginal elasticity of income produce different combinations of all above-presented values.

Here we come to the real variety of combinatorics because every value from the Equation (18) represents the pair of the other independent variables. It is very difficult to describe all possible changes in the Equation (18). But it is worth looking at some of them.

First of all, we should take into consideration the proportional change in the marginal utility of an item to be bought and the change in willingness to pay. This proportional change happens, for example, when we make the trade-off between the quality and the price. Again, the shift in the $|\partial P/\partial S|$ value means here the choice of high-price store with quality items. However, the assumption of the proportional change in the marginal utility and in the willingness to pay results in the constant value of the marginal utility of consumption expenditures. And its price reduction elasticity $e_{\lambda,|\partial P/\partial S|}$ becomes equal to zero.

The Equation (18) takes the following form

$$e_{\lambda,w} = e_{|\partial P/\partial S|,w} - 1 \quad (20)$$

It means, that the inappropriate ambitious choice of a store, where the wage rate elasticity of price reduction is greater than one, or $e_{|\partial P/\partial S|.w} > 1$, makes the wage rate elasticity of the marginal utility of income positive.

There are two possible explanations to this phenomenon. First and the most evident explanation is the risk-seeking behavior, or $\partial\lambda/\partial w>0$. We can find many examples of such risky decisions in the real estate sector.

However, not all luxury items are risky. The Chateau Lafite Rothschild 1995 from Pauillac and the Opus XA from Arturo Fuente bought to celebrate new position can hardly be regarded as investments in risky assets. There is another consideration. We can ask ourselves whether a dollar spent on those items has really increased the utility, or, more definitely whether the marginal utility of the increase in the wage rate has been positive.

Before we reject these seemingly absurd questions, let's analyze changes in marginal utilities of both consumption and leisure under this assumption.

If we come back to our previous results we can see that

$$MU_Q = \lambda \frac{\partial P / \partial S}{\partial L / \partial S} \quad (21a)$$

$$MU_H = -\lambda \frac{w}{\partial L / \partial S} \partial^2 L / \partial S \partial H \quad (21b)$$

If we take the inequality $w<|\partial P/\partial S|$, we part with the "common model" of behavior and we come to the "leisure model" of behavior, because the $w<|\partial P/\partial S|$ relationship changes the allocation of time. The absolute value of the propensity to search becomes greater than one, or $|\partial L/\partial S|>1$ and $\partial L/\partial S<-1$. But now the relationship between leisure and search becomes positive, or $\partial H/\partial S>0$. And this change means that the increase in the absolute value of the propensity to search $|\partial L/\partial S|$, i.e., its decrease in real terms, is followed now by the increase in leisure time, or $\partial^2 L/\partial S\partial H<0$.

Let's come back to the Equations (21). This result doesn't change the marginal utility of consumption, but it dramatically changes the marginal utility of leisure. The latter becomes ***negative*** and the leisure becomes "bad".

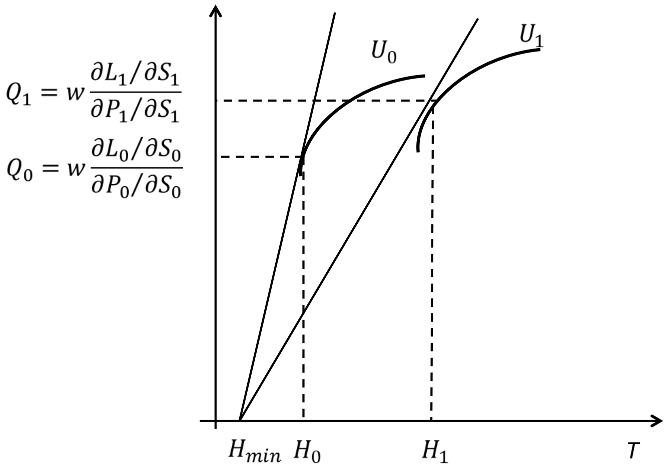
That conclusion had been done in the previous analysis of the Veblen effect (Malakhov 2012c). Things looked like we wasted time at parties after the purchase of a suit. However, that consideration was dialectically incomplete. We told about it, when it was discussed that either the purchase of a suit made parties "reasonable", or parties made "reasonable' the purchase of a suit.

If we presuppose that the value of the Lagrangian multiplier becomes negative, we change the representation of marginal utilities of both consumption and leisure. Now the consumption becomes "bad", whereas the marginal utility of leisure again becomes positive. From here we can increase the total utility only with the increase in leisure time. The party organized in order to celebrate the new position increases the total utility, where negative marginal utilities of the Chateau Lafite Rothschild 1995 from Pauillac and of the Opus XA from Arturo Fuente become invisible.

Moreover, the increase in leisure time, required to restore the positive utility of such a "luxury" consumption-leisure choice, automatically increases the absolute value of the equilibrium price reduction $|\partial P/\partial S|$, i.e., the purchase price, and the Veblen effect takes place (Fig.3):[2]

Fig.3

[2] Here we should keep in mind that usually there is a certain physical or psychological minimum level of leisure time where $\partial L/\partial S=-1$.




It is very easy to follow all these consideration with the help of the value of the *MRS (H for Q),* derived earlier and repeated here in illustrative purposes:

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (22)$$

In addition, this consideration can enlarge our understanding of the phenomenon of the sunk costs sensitivity and it can also explain dramatic increase in leisure time during last decades (Aguiar and Hurst 2007). Coming back to the key equation of the model (1) we can see that the "leisure model" of behavior ($\partial L/\partial S<-1; \partial H/\partial S>0; \partial^2 L/\partial S\partial H<0$) can be produced not only by luxury items with corresponding high absolute values of the equilibrium price reduction $|\partial P/\partial S|$. We can also see that the value of the lower wage rate can produce the same effect even on almost perfect markets with low absolute values of the equilibrium price reduction, but with very important quantity to be purchased, or $w<|Q\partial P/\partial S|$.

It seems that the "leisure model" of behavior is the result of large money balances, when the marginal usefulness of money becomes negative (Friedman 1969). However, the phenomenon of the positive wage rate elasticity of the marginal utility of money needs more detailed analysis.

**Conclusion**

When market environment progressively relaxes budget constraints, the development of Stigler's rule of the equality of marginal costs of search with its marginal benefit becomes more and more important in the understanding of consumer behavior (Stigler 1961). The general relationship between elasticities of the marginal utility of money (18) can be developed in two ways. First, we can use the key equation of the search model (1) in order to show the relationship between the marginal utility of money and income elasiticities of both consumption and leisure for the "common model" of behavior in the following form:

$$
\begin{aligned}
& e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w} = 0 \\
& 1 - e_{|\partial P/\partial S|,w} = e_{Q,w} - e_{|\partial L/\partial S|,w} = e_{Q,w} + \frac{H}{L+S} e_{H,w} \qquad (23) \\
& e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} + e_{Q,w} + \frac{H}{L+S} e_{H,w} = 0
\end{aligned}
$$

Second, if we presuppose that the price reduction itself is unit elastic, or $e_{|\partial P/\partial S|,P} = 1$, we can re-write the Equation (18) in the following form:

$$e_{\lambda,P} + e_{\lambda,w} = e_{P,w} - 1 \quad (24)$$

And this equation can give us the better understanding of many economic phenomena, money illusion, for example.

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