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# The Macroeconomics of Immigration<sup>\*†</sup>

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## **Abstract**

Immigration has been a significant part of US population growth over recent decades, with the number of “foreign born to non-US nationals” rising from approximately 10 million in 1970 to nearly 40 million or 12.9% of the US total population in 2010. In this paper, using a VAR with sign restriction identification, we find that unexpected increases in the working population lead to temporary reductions in GDP per capita and consumption per capita as would be predicted by the standard neoclassical growth model. However they do not lead to increases in non-residential investment or short run decreases in real wages as would also be predicted. The paper shows how a neoclassical growth model with a CES production function where migrant labor and capital are complements to skilled domestic labor and substitutes to each other can produce responses closer to those in the VAR. The paper thus provide support for the microeconomic studies on the impacts of immigration which found that immigrant labor is complementary to, rather than a substitute for, most native labor.

Keywords: Macroeconomics, Immigration

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\*We thank

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# 1 Introduction

While there have been a lot of recent studies on the microeconomic impacts of immigration there has been less attention focussed on the implications of immigration for the macroeconomy. According to US Census Bureau and Current Population Survey (CPS) data, immigration has been a significant part of the US population growth over recent decades. In 1970 about 9.6 million (4.7%) of the US total population was foreign born to non-US nationals, by 2010 this number had risen to nearly 40 million or 12.9% of the US total population. In this paper we examine the effect of shocks to working population on the macroeconomy using the techniques of macroeconomic time series analysis. The analysis shows that, consistent with the standard neoclassical growth model, GDP per capita and consumption per capita temporarily fall in response to a positive shock to the working population. However non-residential investment per capita does not rise and real wages do not fall in the short run following an unexpected increase in the working population and as would also be predicted by the standard growth model.

The paper shows that a neoclassical growth model with a CES production function where migrant labor is a substitute for capital but a complement to skilled domestic labor can produce responses to an immigration shock much closer to those of the VAR. In particular it can produce responses where investment falls in response to an immigration shock and where the wage response of most agents is initially positive due to the complementarity of immigrant labor with most domestic labor. Thus the VAR results and the macroeconomic growth model both lend support to the findings of the microeconomic literature that immigrant labor is a much closer substitute for native unskilled labor than native skilled labor, see for example Ottaviano and Peri (2012) for the US Economy and Manacorda, Manning and Wadsworth (2012) for the UK economy.

One feature of the empirical analysis is that it takes account of the fact that increases in the working population differ from other macroeconomic variables in that much of its movement can be predicted years ahead. Birth and mortality data are publicly available and so a large proportion of the changes in the working population can be anticipated 16 years ahead of time. As the work of Ramey (2011) and Auerbach and Gorodnichenko (2012) detail, correcting for anticipated changes in the variables of a VAR is necessary to remove potential biases from the analysis. We therefore correct for such anticipated changes in population in the data and find, very intuitively, that unanticipated changes in the working population correspond quite closely with immigration

levels. We therefore interpret shocks to unanticipated changes in the working population as immigration shocks.

The analysis and results of the paper are of interest for two distinct reasons. Firstly the key state variable in balanced growth models is the capital labor ratio and while the literature has paid a lot of attention to the determinants of individual labor supply<sup>1</sup> much less attention has been given to the determinants of the size of the working population, although see Doepke, Hazan and Maoz (2012) for a notable exception. This paper attempts to redress this imbalance by focussing on the macroeconomic effects of immigration which is one of the key determinants of changes in the labor force. Secondly there is a large microeconomic literature on the effects of immigration on the Labor market. One of the key puzzles of this literature was the finding that immigration has only a small effect on aggregate wages, with only the wages of the least skilled workers being adversely affected by immigration.<sup>2</sup> This paper, using a very different methodology and different, macroeconomic, data provides macroeconomic support for this analysis by also finding that immigration shock is not empirically associated with short run decreases in aggregate wage rates.

The paper is organized as follows. In section 2 we present and discuss the raw data. In Section 3 we present results from the VAR analysis and in Section 4 we discuss to what extent the standard macroeconomic growth model can be adapted to explain these results.

## 2 Trends in Population Growth and Immigration

This section presents the data we will be using below in our VAR analysis. One contribution of this section is to compute an unanticipated change in population variable by removing anticipated changes in the working population caused by publicly recorded changes in the birth and mortality rates. This is important and interesting for two reasons. Firstly because controlling for predictable changes in variables in a VARs is necessary to remove bias, as the work of Ramey (2011) and Auerbach and Gorodnichenko (2012) detail. Secondly when we do construct this series it corresponds quite closely to immigration level which is intuitive. Thus in the VAR section below we interpret the

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<sup>1</sup>For surveys of the literature see e.g. Uhlig (1999) or Christiano, Eichenbaum and Evans (2005)

<sup>2</sup>See e.g. Dustmann, Frattini and Preston (2008), as well as the introductions to Ottaviano and Peri (2012) and Manacorda, Manning and Wadsworth (2012) for surveys of this literature.



Figure 1: Live Births 16 years previously are a major and predictable influence on the Working Population

shocks to unanticipated population as immigration shocks.

Figure 1 plots the changes in the rate of growth of the working population and the number of live births 16 years previously. It shows that they are highly related, although not perfectly correlated. VAR analysis assumes that the errors in the VAR are orthogonal to information contained in the past values of the variables in the VAR, see for example Canova (2007). Thus it is necessary to remove this predictable element from the population series, see for example the discussion in Ramey (2011) and Auerbach and Gorodnichenko (2012). We do this by constructing an unanticipated change in population series,  $WPop_t^U$ , to correct for these predictable effects, using the following formula

$$WPop_t^U = WPop_t - WPop_t^A$$

where  $WPop_t^A = (1 - \delta_{t-1}^{65} - mort_{t-1}^{16-64})WPop_{t-1} + (1 - mort_{t-16}^{1-15})Births_{t-16}$

where the  $WPop_t$  is the series for working population in the US is taken from Cociuba, Prescott and Ueberfeldt (2009).  $WPop_t^A$  is the anticipated working population in

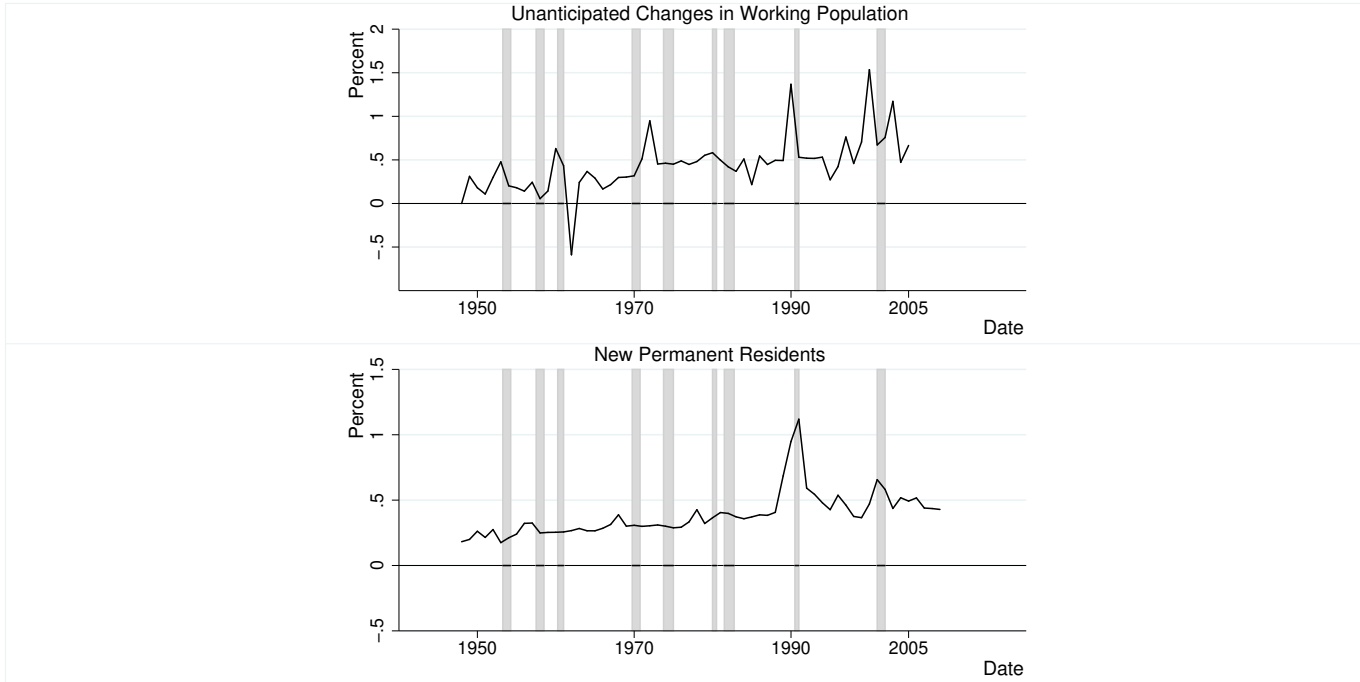


Figure 2: Unanticipated changes in Population and Census Department Figures on New Permanent Residents

time which is equal to the previous year's working population minus an estimate of the proportion aged 64 who will retire,  $\delta_{t-1}^{65}$ , and an estimate of the mortality rate of the working population plus the births from 16 years previously also adjusted for mortality. The data used is all freely and publicly available on the internet. That for mortality rates and birth rates are taken from 5 yearly samples from the CDC/NCHS *National Vital Statistics* and data on the age distribution is taken from decennial census data. Linear interpolation is used to generate annual numbers for mortality rates.

Figure 2 displays the time series for the constructed unanticipated changes in the working population and also the immigration series from the US census bureau which corresponds to the numbers of new permanent resident status individuals. Both series are plotted as a percentage of the working population. The series have a similar pattern in that they both show a gradual rise from the 1950's to the 1980's and then a large increase in the latter period of the 1980's and a second peak around 2000 although the size of this last peak does differ. The first large peak was caused by the Immigration Reform and Control Act of 1986 which allowed significant numbers of formerly temporary

workers to apply for permanent resident status after a period of three years.<sup>3</sup> The passing of the act also coincided with a period of high Mexican unemployment and so caused many temporary workers who would otherwise have returned to their country of origin to remain in the United States and become permanent residents. It is estimated that 2.3 million Mexicans took advantage of this possibility, see Durand, Massey and Parrado (1999). The gradual track to permanent residency also explains why the peak of the new permanent residents series occurs after that for the changes in working population series.

The series are also similar in scale. Over the sample period 1950-2005 the cumulative unanticipated changes in the working population is approximately 38.2 million with 17.8 million occurring since 1990. The corresponding numbers for the new permanent residents series are 31.9 million and 15.7 million. One should not expect a perfect correspondence between these two figures since one can attain new permanent resident status and not be part of the working population and vice versa. However the similarity between the two series is reassuring. Figure 2 also plots the NBER business cycle dates with the recessions shaded in gray. It is noticeable that the response of the unanticipated changes in working population is more volatile and reactive to recessions than the series for new permanent residents which is intuitive.

## 3 VAR Analysis

### 3.1 Description of the VAR

We use an 8 dimensional VAR with annual data from 1950 to 2005 for the following variables; GDP, private consumption, non-residential investment, residential investment, hours worked, real wages and the two immigration series, the numbers of new permanent residents and the constructed unanticipated population variable described above.<sup>4</sup> All variables are real and, with the exception of the wage series, expressed as per capita of

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<sup>3</sup>This is known as the Special Agricultural Workers provision.

<sup>4</sup>The series used the series for gross domestic product personal consumption expenditures, nonresidential fixed investment and Residential Fixed Investment taken from Bureau of Economic Analysis' NIPA table 1.1.5 all deflated by the GDP deflator from Table 1.13. The wage series is the Nonfarm Business Sector: Real Compensation Per Hour, series COMPRNFB, from the Bureau of Labor Statistics and the working population and hours worked series come from Cociuba, Prescott and Ueberfeldt (2009).

TABLE 1  
IDENTIFYING SIGN RESTRICTIONS

	GDP Cons	Non-Res Invest	Hours	Unanticipated Working Pop.
Business Cycle	+	+	+	
Population				+

This table shows the sign restrictions on the impulse responses for each identified shock. ‘Non-Res Inv’ stands for Non-Residential Investment. A ”+” means that the impulse response of the variable in question is restricted to be positive for two years following the shock, including the year of impact. A blank entry indicates, that no restrictions have been imposed.

the working population. The VAR has 2 lags, no constant or a time trend, and uses the logarithm for all variables except for the population variables where we have used the level.

The VAR in reduced form is given by

$$Y_t = \sum_{i=1}^2 B_i Y_{t-i} + u_t, t = 1, \dots, T, \quad E[u_t u_t'] = \Sigma$$

where  $Y_t$  are  $8 \times 1$  vectors, 2 is the lag length of the VAR,  $B_i$  are  $8 \times 8$  coefficient matrices and  $u_t$  is the one step ahead prediction error.

### 3.2 Identification

The problem of identification is to translate the one step ahead prediction errors,  $u_t$ , into economically meaningful, or ‘fundamental’, shocks,  $v_t$ . In this paper we identify shocks using the sign restriction approach of Uhlig (2005) and Mountford and Uhlig (2009). Identification in this methodology amounts to identifying a matrix  $A$ , such that  $u_t = Av_t$  and  $AA' = \Sigma$ . Each column of  $A$  represents the immediate impact, or impulse vector, of a one standard error innovation to a fundamental shocks. Each column is identified as the vector which minimizes a criterion function,  $\Psi(a)$ , based on the impulse responses of some of the variables in the VAR to a particular shock’s impulse vector,  $a$ . If we define  $r_{ja}(k)$  as the impulse response to the impulse vector  $a$  of the  $j$ th variable at



horizon  $k$  then the criterion function,  $\Psi(a)$ , is

$$\Psi(a) = \sum_{j \in J_{S,+}} \sum_{k=0}^1 f\left(-\frac{r_{ja}(k)}{s_j}\right) + \sum_{j \in J_{S,-}} \sum_{k=0}^1 f\left(\frac{r_{ja}(k)}{s_j}\right)$$

where  $f$  is the function  $f(x) = 100x$  if  $x \geq 0$  and  $f(x) = x$  if  $x \leq 0$ ,  $s_j$  is the standard error of variable  $j$ ,  $J_{S,+}$  is the index set of variables, for which identification of a given shock restricts the impulse response to be positive and  $J_{S,-}$  is the same for variables restricted by identification to be negative. Since we use annual data we only restrict the signs of the impulses for two periods i.e. for the two years after the shock. When multiple shocks are identified there is an additional constraint on the minimization that the identified shock be orthogonal to previously identified shocks, as detailed in Mountford and Uhlig (2009).

In this paper we use two identification schemes. We first only identify the unanticipated population/immigration shocks and then we identify two shocks, first a business cycle shock and then the unanticipated population/immigration shock. Table 3.2 provides a description of the identifying sign restrictions for these shocks. The advantage of the penalty function approach is that, by rewarding larger responses of the correct sign, it gives the shock identified first the greatest opportunity to explain the variation in the data. Thus when the unanticipated population/immigration shock is identified second it is restricted to explaining the variation in the data left over after the variation explained by the business cycle shock has been taken out. As well as a robustness exercise this identification scheme is interesting in its own right, as it should also pick up temporary variations in immigration which may be associated with business cycle fluctuations.

### 3.3 Empirical Results

The impulse responses for these fundamental shocks are shown in Figures 3 through 5, where we have plotted the impulse responses of all our 8 variables. The shocks are identified for each draw from the posterior and the 16th, 50th and 84th quantiles plotted, calculated at each horizon between 0 and 16 years after the shocks. The impulses restricted by the identifying sign restrictions are identified by the shaded area in the figures.

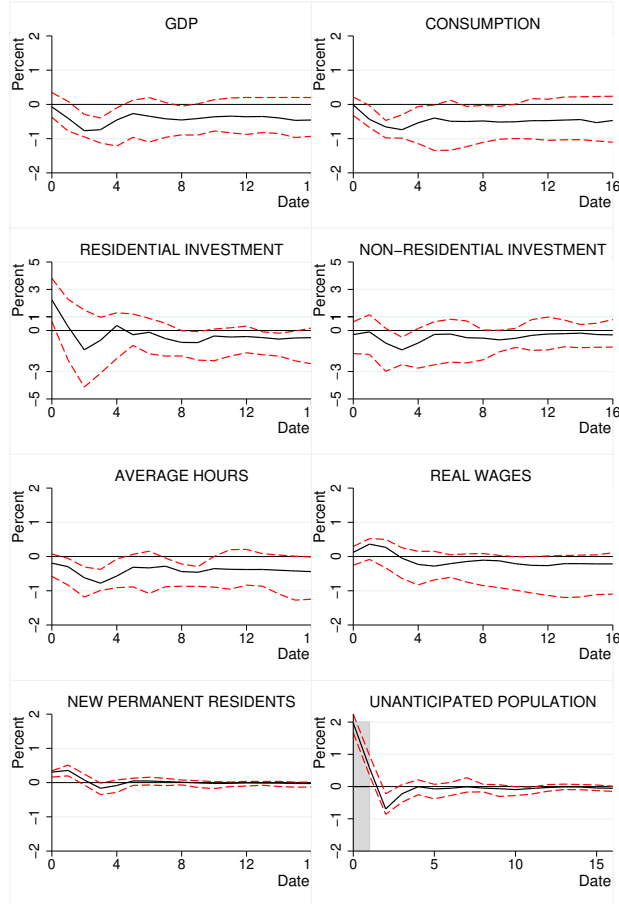


Figure 3: Impulse Responses to an Immigration Shock Ordered First.

### 3.3.1 The Immigration Shock Ordered First

The impulse responses of the Immigration shock, which is the shock to the unanticipated working population variable in the VAR, are plotted in Figure 3. They show that, as would be predicted by a standard growth model, output and consumption temporarily fall in response to the immigration shock. However, although there is an increase in residential investment on impact, there is not a positive response from non-residential investment which is the response predicted by a growth model after an unexpected increase in its labor force, see Figure 8 below. Indeed the median response of non-residential investment is always negative. With respect to the labor market real wages do not change significantly with the median response being initially positive before coming negative while average hours worked falls. Again this is not the pattern of responses

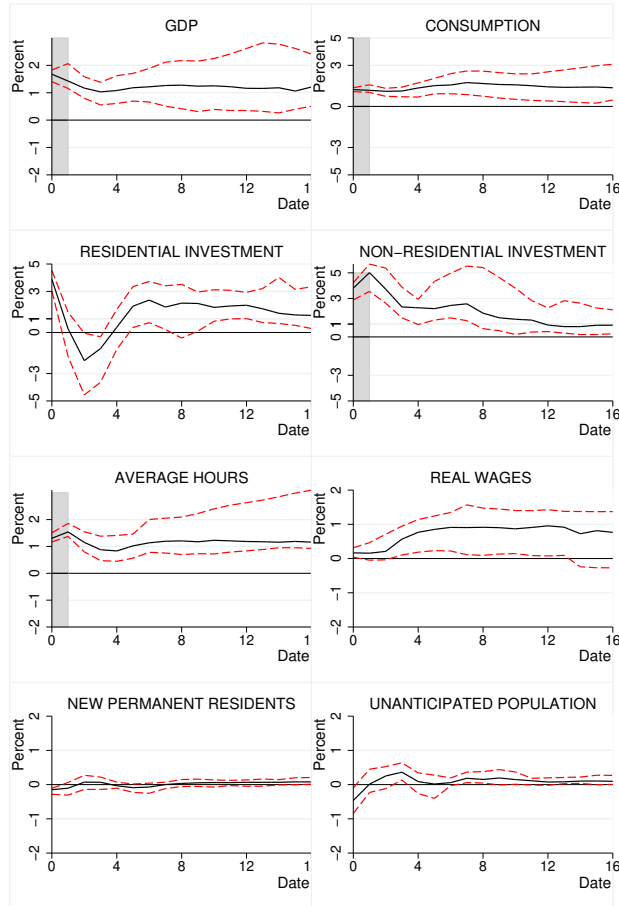


Figure 4: Impulse Responses to a Business Cycle Shock Ordered First.

that would be predicted by a standard growth model where wages fall on impact after an unexpected increase in its labor force. Finally note that the response of the new permanent residents to the immigration shock is intuitive. It is much smoother than the responses of the unanticipated population variable which is intuitive and consistent with the view that the unanticipated working population variable will contain more temporary immigrants than the new permanent residents series.

### 3.3.2 The Immigration Shock Ordered Second

In this section we present the impulse responses of the immigration shock when it is identified second after a business cycle shock. This is important to do for two reasons. Firstly because identification methods are never definitive and so there is always a suspicion that

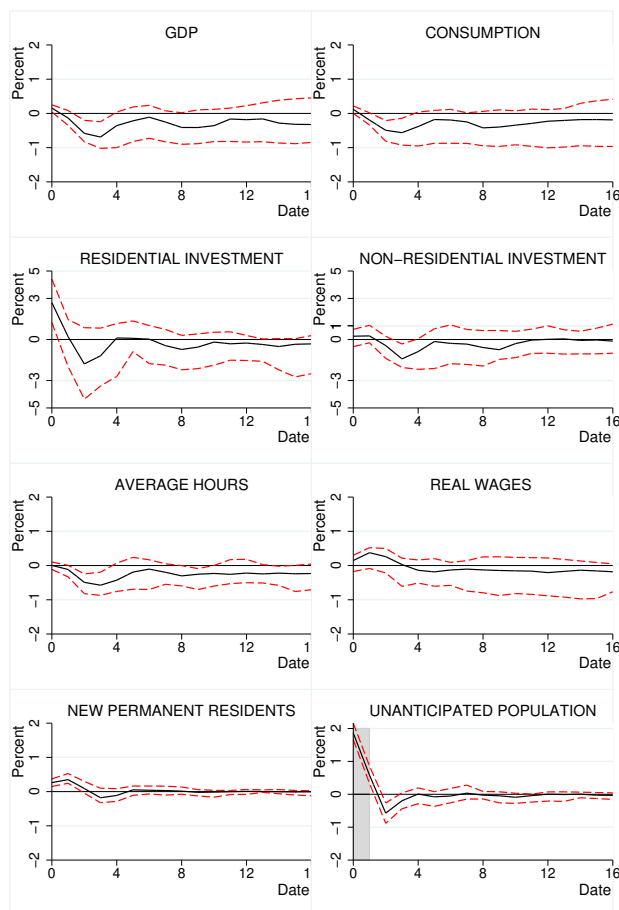


Figure 5: Impulse Responses to an Immigration Shock Ordered Second After a Business Cycle Shock

the variation attributed to one identified shock may actually be due to another shock. In macroeconomics the business cycle shock is commonly felt to be an important source of variation and so as a robustness check it is interesting to see whether the responses to the immigration shock change significantly once the business cycle variation is accounted for<sup>5</sup>. Secondly, it is often thought immigration reacts to the business cycle and that while the stage of the business cycle should not matter for permanent immigrants temporary migrants may be affected by the state of the business cycle.

Figure 4 displays the responses to the Business Cycle shock. These responses of the non-population variables are as expected and very similar to those in Mountford and Uhlig (2009) The responses of all the macro variables are positive and persistent. The

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<sup>5</sup>See Mountford and Uhlig (2009) for more discussion of this

population variables both show cyclical variation in response to the business cycle shock although with a lag. The immigration response is negative on impact before rising and becoming significantly positive after three years after the shock. The new permanent residents series shows the same pattern but is a much smaller response and so insignificant for all horizons after impact.

Figure 5 shows the impulse responses of the Immigration shock identified after the business cycle shock. What is striking is how similar the responses are to those in Figure 3. This means that the restriction to be orthogonal to the business cycle shock hardly binds at all which is consistent with the most of the variation in immigration not being influenced by the business cycle. The main differences between Figures 5 and 3 are that the error bands around consumption are tighter and the responses of real wages appears less negative in Figures 5 .

### **3.3.3 Adding Labour Share**

In Figures 6 and 7 we substitute a Labor share variable in to the VAR in place of the real wage rate. The variable is the Nonfarm Business Sector: Labor Share, series PRS85006173, from the Bureau of Labor Statistics. This is interesting as a robustness check and also because an increase in the ability of immigrant labor may reduce the bargaining power of labor and so reduce labor share, see Kiguchi (2013). These Figures do indeed show evidence that over the medium term labor share does decline in response to an immigration shock. Thus the medium term responses of wages and labor share do seem to differ from their short term effects. This is a large issue see for example the work of Duenhaupt (2011), Heathcote, Perri and Violante (2010) and Piketty and Saez (2003, 2006) and which is beyond the scope of this paper, but it is certainly an exciting area for further research.

## **4 A Growth Model with Immigration Shocks**

The evidence discussed in Section 3 suggest that while increases in immigration are associated with temporary decreases in output and consumption per capita, as would be predicted by an exogenous shock to population in the standard neoclassical growth model, they are not associated with increases in non-residential investment which would also be expected in this case. To explain these results we use the findings of the recent labor economics literature which suggest that migrant labor is not a substitute for much

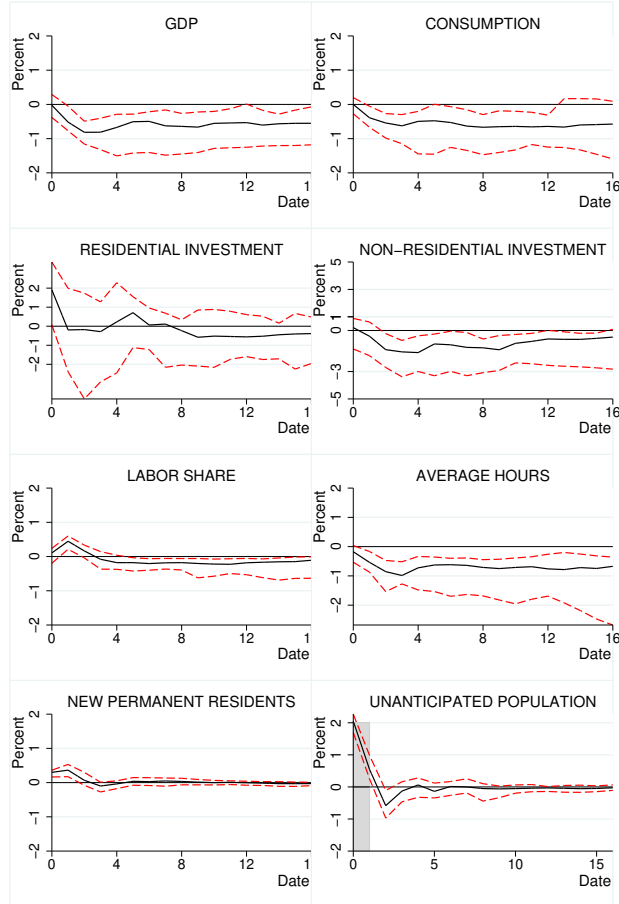


Figure 6: Impulse Responses to an Immigration Shock Ordered First With Labor Share.

of the domestic population, but a complement. For recent evidence on this see Ottaviano and Peri (2012) for the US and Manacorda, Manning and Wadsworth (2012), for the UK. The intuition for these results is that migrants, perhaps because of poorer communication skills, tend to undertake unskilled work even when they possess skills themselves and this influx of unskilled labor allows business to expand without needing to invest in new machinery. The assumption that physical capital is complementary to skilled labor is well accepted and while the assumption that physical capital is a substitute for unskilled labor is less common it has support in the literature, see Cahuc and Zylberberg (2004).

We therefore adapt the standard neoclassical growth to allow for two types of labor, unskilled and skilled and a household which is growing in size though time. To deal with this added complexity we will follow the literature and assume perfect risk

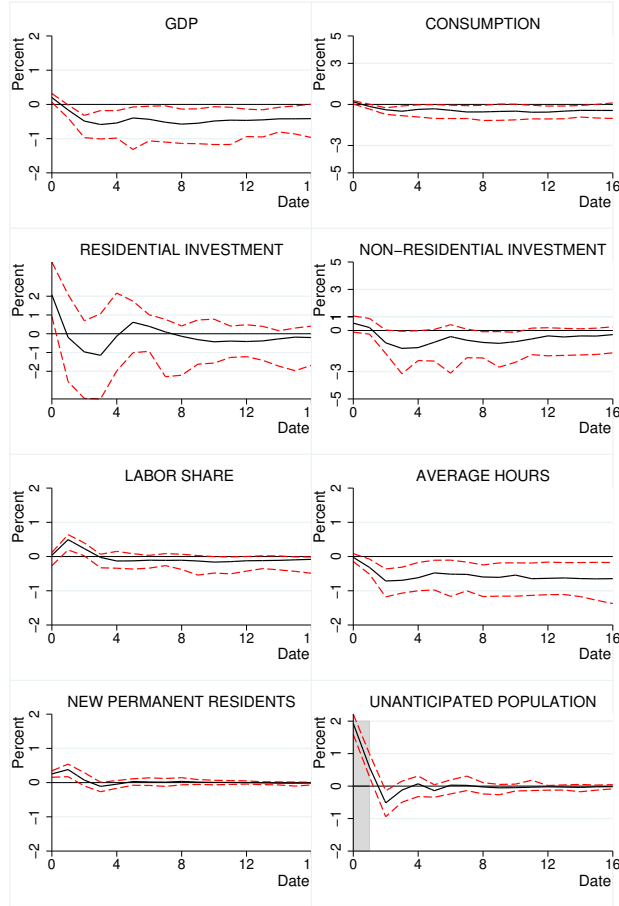


Figure 7: Impulse Responses to an Immigration Shock Ordered Second after a Business Cycle Shock with Labor Share.

sharing within the household, see e.g. Galí (2011) or Brückner and Pappa (2012). Intuitively one can think of the household as a composite representative agent made up of a certain proportion of skilled and unskilled labor which can only be supplied together. This is a simplifying assumption that allows the model to be solved in a standard way. There are clearly possible extensions of the model, such as allowing Household members to differ in some dimension, as in Galí (2011) and Brückner and Pappa (2012), but this is not necessary for the purpose of this paper.

We follow the discrete time balanced growth model of Uhlig (2010) with the addition of stochastic population growth and two types of labor. We will use lower case letters to denote per capita terms. A representative Household's utility function,  $U$ , has

the following form

$$U = E \left[ \sum_{t=0}^{\infty} \beta^t N_t u_t \right]$$

where  $N_t$  is the number of agents in each household,  $\beta$ , the discount factor, and where  $u_t$  is given by

$$u_t = \frac{(c_t \Phi(l_t))^{1-\eta} - 1}{1 - \eta}$$

with  $c_t$  denoting per capita consumption,  $l_t$  denoting per capita labor supplied,  $1/\eta > 0$  the intertemporal elasticity of substitution, and  $\Phi(l_t)$  a strictly positive, decreasing, concave and thrice differentiable function. We will assume that  $\Phi(l_t)$  is such that there is a constant Frisch Elasticity of labor supply with respect to the wage rate, see Trabandt and Uhlig (2011). As mentioned above we will assume that the representative household is a composite of both types of labor, skilled and unskilled, which it can only supply together so that  $l_{s,t} = l_{u,t} = l_t$  where  $l_{s,t}$  and  $l_{u,t}$  is the labor supply of skilled and unskilled household member's respectively. The proportion of skilled labor in the representative household's composite labor is the same proportion as in the economy,  $\lambda_t^s$ , where  $\lambda_t^s = N_{s,t}/N_t$  and  $N_t = N_{s,t} + N_{u,t}$ . The representative agent's labor supply decision is to choose  $l_t$  subject to the weighted average wage,  $w_t = (w_{s,t}\lambda_t^s + w_{u,t}\lambda_t^u)$ . The Household's budget constraint is in each period therefore

$$c_t N_t + x_t N_t = w_t l_t N_t + r_t k_t N_t$$

where  $x_t$  is investment per person,  $k_t$  is the capital per person,  $w_t$  is the wage rate and  $r_t$  is the capital rental rate. Capital accumulates via investment thus

$$k_{t+1} N_{t+1} = (1 - \delta) k_t N_t + x_t N_t$$

Production takes place under perfect competition and constant returns to scale according to a CES production function. We use the standardized function form as in Cantore, Ferroni, and León-Ledesma (2011),

$$y_t = \left[ \alpha \left( \frac{k_t}{\bar{k}} \right)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) \left( \frac{A_t l_t \lambda_t^u}{\lambda^u \bar{l}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\theta\sigma}{\sigma-1}} \left( \frac{A_t l_t \lambda_t^s}{\lambda^s \bar{l}} \right)^{1-\theta}$$

where  $y_t = Y_t/N_t$ , and we assume that the level of technology,  $A_t$ , augments both skilled and unskilled labor. The parameters  $\sigma$  and  $\alpha$  denote the degree of substitutability



between capital and unskilled labor, and the capital intensity in production respectively. Factor prices are determined by factors marginal products so that

$$\begin{aligned}
r_t &= \frac{y_t}{k_t} \frac{\theta \alpha \left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha \left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{A_t l_t \lambda_t^u}{\lambda^u l}\right)^{\frac{\sigma-1}{\sigma}}\right]} \\
w_{u,t} &= \theta \frac{y_t}{l_t \lambda_t^u} \frac{(1-\alpha) \left(\frac{A_t l_t \lambda_t^u}{\lambda^u l}\right)^{\frac{\sigma-1}{\sigma}}}{\left[\alpha \left(\frac{k_t}{k}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{A_t l_t \lambda_t^u}{\lambda^u l}\right)^{\frac{\sigma-1}{\sigma}}\right]} \\
w_{s,t} &= (1-\theta) \frac{y_t}{l_t \lambda_t^s}
\end{aligned}$$

The stochastic processes driving technological progress and population growth are

$$\begin{aligned}
\frac{A_t}{A_{t-1}} &= \zeta_t^A, \\
\frac{N_t}{N_{t-1}} &= \zeta_t^N
\end{aligned}$$

where  $\zeta_t^A$  and  $\zeta_t^N$  are both stationary stochastic processes with mean  $\bar{\zeta}^A$  and  $\bar{\zeta}^N$  respectively.

Finally the market clearing/feasibility constraint is given by

$$c_t + x_t = y_t.$$

## 4.1 Modeling Immigration Shocks

We model an immigration shock as a shock which leads to an increase in the proportion of unskilled labor in the economy,  $\lambda_t^u$ , as well an increase in the working population. We use a broad definition of skills and set  $\bar{\lambda}^s = 0.9$  which is justified by the fact that 90% of young people graduate from high school in the US, although clearly alternative interpretations and calibrations are possible. Given this if at the steady state the working population increases by  $a\%$  and all of this increase is in unskilled workers then the new share of unskilled workers will rise to  $\lambda_t^{u,new} = (100\bar{\lambda}^u + a)/(100 + a)$  and so  $\widehat{\lambda}_t^u = (100a(1 - \bar{\lambda}^u))/(100 + a)\bar{\lambda}^u$ <sup>6</sup>. Thus an immigration shock in our model is a simultaneous shock to both population,  $\zeta_t^N$ , and also to  $\lambda_t^u$ . The proportion of unskilled amongst immigrants can be varied so that the size of the response of  $\widehat{\lambda}_t^u$  also varies. This is discussed below.

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<sup>6</sup>Thus a 1% increase in population will increase the share of unskilled agents from 0.10 to  $11/101 = 0.1089$  which is a percentage increase of  $(100 \times 0.9)/(101 \times 0.1) = 90/10.1 = 8.9\%$ .

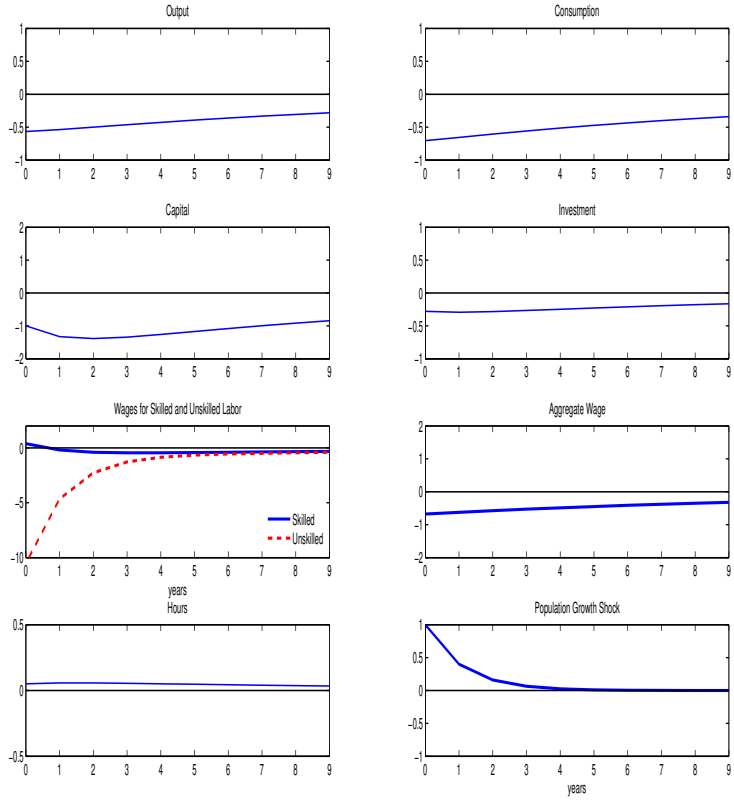


Figure 8: Impulse Responses to an Immigration Shock – a shock where population rises and the proportion of unskilled workers in the labor force also rises.

## 4.2 Log-Linearizing around the Balanced Growth Path

In order to be able to log-linearize around the steady state, we need to detrend variables on the balanced growth path. Following Uhlig (2010) we will denote log-deviations by hats so that  $\hat{c}_t = \log(\tilde{c}_t) - \log(\bar{c}) = (\tilde{c}_t - \bar{c})/\bar{c}$  where  $\tilde{c}_t \equiv c_t/A_t$  and  $\bar{c}$  is the steady state of  $\tilde{c}_t$ . Thus noting that  $\hat{\lambda}_t^u = -(\bar{\lambda}^s/\bar{\lambda}^u)\hat{\lambda}_t^s$  and that  $w_t = (w_{s,t}\lambda_t^s + w_{u,t}\lambda_t^u)$  and so  $\hat{w}_t = \eta^s(\hat{w}_{s,t} + \hat{\lambda}_t^s) + (1 - \eta^s)(\hat{w}_{u,t} - \hat{\lambda}_t^s)$  where  $\eta^s = \overline{w_s\lambda^s}/\bar{w}$  and  $\bar{w} = \overline{w_s\lambda^s} + \overline{w_u\lambda^u}$ , we can write the log linearized equations of the model as in Table 4.2.

TABLE 2  
LOG-LINEARIZED EQUATIONS OF THE MODEL

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$\begin{aligned} \widehat{y}_t &= \theta[\alpha\widehat{k}_t + (1-\alpha)(\widehat{l}_t + \widehat{\lambda}_t^u)] + (1-\theta)(\widehat{l}_t + \widehat{\lambda}_t^s) \\ \widehat{w}_{u,t} &= \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}_t^u) - \alpha\left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t + \alpha\left(\frac{\sigma-1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}_t^u) \\ \widehat{r}_t &= \widehat{y}_t - \widehat{k}_t + (1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\widehat{k}_t - (1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)(\widehat{l}_t + \widehat{\lambda}_t^u) \\ \widehat{k}_{t+1} &= \frac{(1-\delta)}{\zeta^N \zeta^A} \widehat{k}_t + \left[1 - \frac{(1-\delta)}{\zeta^N \zeta^A}\right] \widehat{x}_t - \widehat{\zeta}_{t+1}^A - \widehat{\zeta}_{t+1}^N \\ \widehat{w}_t &= \eta^s(\widehat{w}_{s,t} + \widehat{\lambda}_t^s) + (1-\eta^s)(\widehat{w}_{u,t} + \widehat{\lambda}_t^u) \\ \widehat{R}_t &= (1-\beta(1-\delta)(\zeta^A)^{-\eta})\widehat{r}_t \\ \widehat{\zeta}_t^A &= \rho^A \widehat{\zeta}_{t-1}^A + \varepsilon_{A,t} \end{aligned}$	$\begin{aligned} \widehat{y}_t &= (\bar{c}/\bar{y})\widehat{c}_t + (\bar{x}/\bar{y})\widehat{x}_t \\ \widehat{w}_{s,t} &= \widehat{y}_t - (\widehat{l}_t + \widehat{\lambda}_t^s) \\ \widehat{w}_t &= \frac{1}{\omega_S} \widehat{l}_t + \widehat{c}_t \\ \widehat{\lambda}_t &= -\eta\widehat{c}_t - (1-\eta)\kappa\widehat{l}_t \\ 0 &= E_t[\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \eta\widehat{\zeta}_t^A + \widehat{R}_{t+1}] \\ \widehat{\lambda}_t^u &= -(\bar{\lambda}^s/\bar{\lambda}^u)\widehat{\lambda}_t^s \\ \widehat{\zeta}_t^N &= \rho^N \widehat{\zeta}_{t-1}^N + \varepsilon_{N,t} \end{aligned}$
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This table shows the equations of the log-linearized version of the model

#### 4.2.1 Choosing Parameters

On the balanced growth path  $l_{t+1} = l_t = \bar{l}$  and  $c_{t+1} = \bar{\zeta}^A c_t$ . Hence the first order conditions imply that  $1 = E_t[(\bar{\zeta}^A)^{-\eta}\beta(1 + \bar{r} - \delta)]$

$$\bar{r} = \frac{(\bar{\zeta}^A)^\eta}{\beta} - (1 - \delta).$$

Following Uhlig (2010) and Trabandt and Uhlig (2011) we set the expected annual rate of growth of technology to be  $\bar{\zeta}^A = 1.02$ , the depreciation rate to be  $\delta = 0.07$ , and the intertemporal elasticity of substitution is set at 0.5 hence  $\eta = 2$ . The discount rate  $\beta = 0.998$  hence  $\bar{R} \equiv (1 + \bar{r} - \delta) = (\bar{\zeta}^A)^\eta/\beta = 1.0404/0.998 = 1.042$ . From above we know that  $\bar{r} = \theta\alpha\bar{y}/\bar{k}$  and so given calibrated value for  $\theta$ , and  $\alpha$  and given that  $\bar{r} = \frac{(\bar{\zeta}^A)^\eta}{\beta} - (1 - \delta)$  we will have an expression for  $\bar{y}/\bar{k}$ . Thus if  $\theta = 0.4$  and  $\alpha = 0.9$  then  $\bar{y}/\bar{k} = 0.112/0.36 = 0.312$  hence  $\bar{k}/\bar{y} = 3.20$ . The capital accumulation equation in the steady state gives  $\bar{x}/\bar{y} = (\bar{k}/\bar{y})[\bar{\zeta}^N \bar{\zeta}^A - (1 - \delta)] = 3.20[1.012 \times 1.02 - 0.93] = 0.327$  which is similar to that in Trabandt and Uhlig (2011) and which implies that  $\bar{c}/\bar{y} = 0.673$ .

#### 4.3 Impulse Responses

We discuss the impulse responses from two kinds a shock. In Figure 8 we present what we call an immigration shock where both the rate of population growth and the proportion of unskilled workers in the economy have a positive shock in the manner described in

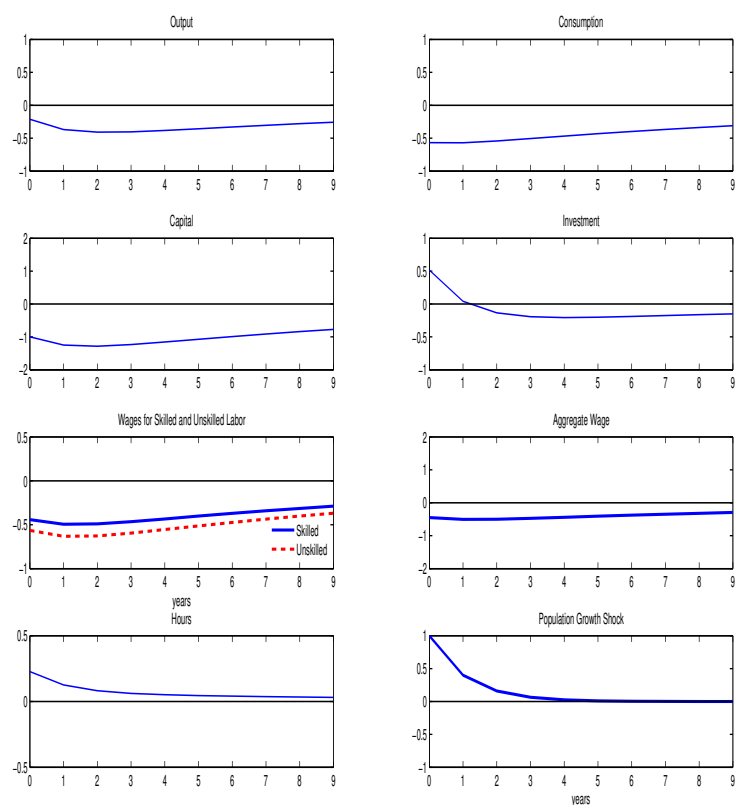


Figure 9: Impulse Responses to a pure Population Shock – a shock where population rises and the proportion of skilled and unskilled workers is unchanged.

section 4.1. In Figure 9 by way of contrast we present the impulse of a pure population shock which is a positive shock to the rate of population growth with no change in the proportion of unskilled workers in the economy.

The immigration shock in Figure 8 has many features in common with the VAR impulses responses in Figures 3 and 5. Notably GDP per capita and Consumption per capita both decline before recovering back towards their balanced growth paths. This is also the case for the pure population shock as shown in Figure 9. However the similarity does not carry over to the response of investment where in the pure population shock, in Figure 9 investment rises immediately in response to the population shock as the economy wide capital to labor ratio falls. In contrast in response to an immigration

shock, investment falls as the increased in unskilled labor substitutes for capital in the production function. The response of unskilled wage rates also differs greatly between the two cases with unskilled wages falling sharply in response to an immigration shock while skilled wages initially rise in response to the immigration shock before falling slightly.

The responses to the immigration shock in Figure 8 are much closer to the VAR responses of Figure 3 and Figure 5 than those of 9. However they are clearly not a perfect fit. The most notable discrepancy is that aggregate wages still fall in response to an immigration shock. It is interesting to note however that the skilled wage does initially rise in response to an immigration shock before falling which is indeed the qualitative response of the wage variable in the VAR. Note that in our calibration 90% of labor is skilled labor and so if the wage variable in the VAR - Nonfarm Business Sector wage rate - has a greater skill component than the economy as a whole, or if immigrants wages do not make it onto the official wage data then the equivalent variable to the VAR in this model would indeed be the skilled wage responses. However we do not model an informal sector in this paper and so again we leave this as a potential fruitful avenue for further research.

## Conclusion

The paper has presented macroeconomic evidence on the effects of immigration on the macroeconomy. It has shown empirically that immigration shocks are not associated rises in non-residential investment or short run reductions in average wages. It has shown how a standard growth model with a CES production function where migrant labor and capital are complements to skilled domestic labor can produce responses closer to those of the VAR than a skill-neutral shock to the working population. Thus using a very different empirical and theoretical methodology, as well as macroeconomic data, this paper has provided empirical support for the microeconomic finding that immigrant labor is complementary to, rather than a substitute for, most native labor.

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