Price Level vs. Nominal Income
Targeting: Aggregate Demand Shocks
and the Cost Channel of Monetary
Policy Transmission

Hamza Malik

March 2005

Online at http://mpra.ub.uni-muenchen.de/456/
Price Level vs. Nominal Income Targeting: Aggregate Demand Shocks and the Cost Channel of Monetary Policy Transmission

Hamza Ali Malik  
Department of Economics  
955 Oliver Road  
Lakehead University  
Thunder Bay, Ontario, P7B 5E1  
Canada  
hamza.malik@lakeheadu.ca  
Phone: (807) 343-8638
Price Level vs. Nominal Income Targeting: Aggregate Demand Shocks and the Cost Channel of Monetary Policy Transmission

Abstract

This paper incorporates both the traditional aggregate demand-interest rate channel and the cost channel of monetary policy in a baseline ‘new Keynesian’ model and study two targeting regimes --- price-level targeting and nominal income targeting. In light of empirical considerations, alternative specifications for the aggregate demand and aggregate supply side of the economy also considered. The main result is that the cost channel matters: in case of a moderate policy response and with the cost channel operating the volatility of real output decreases under both price-level and nominal income targeting, while it increases in case of an aggressive policy response. The paper also finds that nominal income targeting performs better than price level targeting in bringing down the volatility of real output in almost all the specifications of the macro models used in the analysis.

JEL Classification: E30, E32, E52

Keywords: the cost channel, price level targeting, nominal income targeting
1- Introduction

While there is broad understanding regarding the overall monetary policy strategy that central banks should primarily focus on policies that promote price stability in the economy and that a rule-based monetary policy is superior to discretion-based monetary policy actions, the channels through which monetary policy affects the real economy are not completely understood. The traditional interest rate channel operates by affecting the spending decisions of households and firms and thus works through the aggregate demand side of the model. An alternative view, often termed as the bank lending channel (or the cost channel), operates by affecting the cost of production of firms and thus the aggregate supply. Most of the literature has, so far, concentrated mainly on the traditional channel of monetary policy while assessing alternative targeting regimes. However, several researchers such as Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (1997) and Barth and Ramey (2001) have emphasized the cost channel as a powerful collaborator in the transmission of short run effects of monetary policy. By analyzing both the traditional and the cost channel of monetary policy in one unified framework, this paper is an attempt to bridge the gap between these two strands of literature.

Distinguishing the relative importance of the traditional and the cost channel is useful for various reasons. First, it improves our understanding of the link between the financial and real sectors of the economy. Second, it provides alternative indicators to help gauge the stance of monetary policy and thus increases its ability to offset particular types of adverse shocks. Third, a clear understanding of the transmission mechanism has the potential to give more information regarding the choice of intermediate targets. Informed by these observations, especially the last one, the objective of this paper is to assess the robustness of policy recommendations for a closed economy in the presence of the cost channel of monetary policy. In particular, I study two interest rate based monetary policy rules --- price-level targeting and nominal income targeting in a ‘new Keynesian model’ that incorporate both the traditional interest rate channel and the cost channel of monetary policy transmission mechanism.

\[\text{\textsuperscript{1}}\text{ For a detailed discussion see Kashyap and Stein (1994)}\]
The highlighting features of the model(s) used in this paper are as follows. First, a continuous-time modeling approach is used instead of the more conventional discrete-time approach. Apart from the advantage in terms of analytical simplicity, continuous-time models avoid the unappealing problems regarding the model properties being dependent on small changes in assumptions concerning information availability. I explain this point further while explaining the structure of the model below. Second, rather than deriving the optimal policy this paper makes use of the Taylor-type interest rate based monetary policy rules which have become quite popular in policy circles in recent years. More specifically, two such rules, price-level targeting and nominal income targeting are used in a continuous-time version of the ‘new Keynesian model’ that incorporate both the traditional interest rate channel and the cost channel of monetary policy transmission mechanism. Third, the paper also studies alternative specifications for the aggregate demand side of the economy --- the IS-type relationship, and for the aggregate supply side of the economy --- a Phillips curve type relationship that have been proposed recently in the literature in light of empirical considerations. Since the results in favour of or against a price level and nominal income target are very model specific, especially regarding the specification of the Phillips curve, therefore, this consideration adds robustness in assessing the role of the cost channel. Fourth, instead of just incorporating the nominal interest rate, the real interest rate is also considered in the Phillips curve relationship to represent the cost channel.\(^2\) Fifth, it is assumed that the two targeting regimes generate the same outcome regarding long-term inflation. Thus, the criterion for evaluating the performance of a monetary regime is its ability to minimize the volatility in real output in response to aggregate demand shocks.\(^3\)

The main result of the paper is that the cost channel matters in the sense that the volatility of real output decreases under both price-level and nominal income targeting when the cost channel is included.

\(^2\) In a recent paper Walsh and Ravenna (2006) assumed that firms needed to pay the hired workers before the receipt of the sales revenues. For this purpose, they borrowed from banks at the nominal interest rate \(i\). Thus, there was a payment lag involved and the relevant ‘cost of borrowing’ was represented by the nominal interest rate. On the other hand, following Mitchell (1984) and Myatt (1985), in this paper I also incorporate the assumption that firms borrow from banks to pay for the wage-bill before the production process begins. Thus, there is a production lag involved here and the relevant ‘cost of borrowing’ is represented by the real interest rate.

\(^3\) This point is discussed below while explaining the structure of the model.
in the model(s). However, this result holds only for moderate policy responses. With aggressive approach to policy, the volatility in output increases. Moreover, the inclusion of the cost channel does not say much on the choice between the two regimes. It appears that nominal income targeting performs better than price-level targeting in bringing down the volatility of real output in almost all the specifications of the macro models used in the analysis regardless of the cost channel.

1.1- Comparison of Price-level Targeting and Nominal Income Targeting

Although price-level targeting is quite similar to inflation targeting and it shares many of its benefits, the two regimes have a fundamental difference. If there is an unexpected increase in prices then according to price level targeting the monetary authority will attempt to tighten monetary policy so as to restore the price level back to the target in order to prevent the base drift in the price level. Under inflation targeting no action will be taken and the new level of prices would be maintained. Thus, price-level targeting offers the potential benefit of delivering greater certainty of the level of prices through time and may provide greater prospects for maintaining price stability in the longer run than under an inflation targeting regime. However, short-term price volatility (and thus output volatility) may be higher under price-level targeting because unexpected rises in the price level will be followed by attempted reductions in the price level.

The conventional literature (e.g., Fischer (1994) and Haldane and Salmon (1995)) focus on this alleged increased output-gap volatility under price-level targeting to argue against it. Kiley (1998) has also reached a similar conclusion using a new Keynesian Phillips curve. However, Dittmar, Gavin and Kydland (1999) and Svensson (1999a) have challenged this conventional wisdom and, employing a neo-classical Phillips curve, shown the price-level targeting to be preferred over inflation targeting. Svensson (1999a) finds that price-level targeting results not only in lower variability in the price-level but also delivers lower inflation variability in the presence of output persistence. More recently, Dittmar and Gavin (2000) and Vestin (2003) have confirmed this result using the new Keynesian Phillips curve by demonstrating that price-level targeting provides a better inflation-output-gap variability trade-off.
compared to inflation targeting with discretionary policy making regardless of the degree of importance of past levels of output for current output. Thus, the debate over the relative benefits of price-level targeting is far from being settled. As Mishkin (2000) has correctly pointed out, the results in favour of or against a price level target are very model specific, especially regarding the specification of the Phillips curve. In particular, the assumptions about private sector’s inflation expectations entering the Phillips curve, amount of persistence in the output gap and whether policy is conducted under a commitment rule or in a discretionary fashion play important roles in determining the desirability of price level targeting. In this paper, I add one more consideration; namely, the cost channel of monetary transmission.

Nominal income targeting is another desirable strategy for monetary policy as it shares many positive features of inflation targeting. But, the most attractive feature of nominal income targeting is that it is closely related to both real output and prices --- the two variables that central bank seem to care about most. In addition, nominal income targeting allows the monetary policy to adjust to offset disturbances to both aggregate demand and aggregate supply. For example, in case of an adverse demand shock (that would cause both real output and prices to go below target), policymakers would ease monetary policy that would return nominal income (the product of real income and prices) to target. Similarly, an adverse supply shock results in falling real output and rising price levels. This could pose a dilemma if central bank is pursuing price level targeting. Stabilizing the price level would mean further decline in real output. Nominal income targeting would help policy makers resolve the dilemma as it places equal emphasis on stability of both real output and price level.

Recently, several contributions in the literature have been made that study the stability properties of the nominal income-targeting regime. Two key papers in this regard are Ball (1999) and McCallum (1997). Using a backward looking macro model, Ball (1999) has forcefully argued that nominal income targets are not merely inefficient, but also disastrous: they imply that output and inflation have infinite

---

4 For an in-depth analysis of the conditions under which price level targeting would be preferred over inflation targeting see Barnett and Engineer (2000).
5 The case in favour of nominal income targeting has been well documented in Hall and Mankiw (1994).
6 The paper first came out in 1997 as a working paper of Reserve Bank of New Zealand, G-97/3.
variances. Svensson (1999b) replicates Ball’s instability result and suggest that it is the stylized fact that policy affects real output before inflation which Ball builds into his model that lies at the heart of the instability result. Challenging the negative assessment of nominal income targeting, McCallum (1997) has shown that Ball’s instability result is not robust; it critically depends on the specification of the Phillips curve relationship. Using a forward-looking model McCallum demonstrates that nominal income targeting does not generate instability. Using a Phillips curve with mixed expectations, Dennis (2001) has shown that nominal income targeting will not generate instability as long as inflation expectations contain some forward-looking component. More recently Rudebusch (2002), however, has shown that nominal income targeting performs poorly after taking into account of the range of model and data uncertainty that policy makers face.

It is evident from the above discussion that the case for or against price level targeting and nominal income targeting relies critically on how inflation expectations are formed in the Phillips curve or more generally on the specification of the model. For this reason, I evaluate the performance of price-level targeting and nominal income targeting in a series of macroeconomic models with different specifications for the IS and Phillips curve relationship. In addition, I also explore the implications of adding the supply side effects --- the cost channel --- of interest rates to each specification. It has been argued in the literature that such effects can be significant in evaluating the performance of monetary policy (e.g., Myatt and Scarth (2003)). These considerations provide an additional and comprehensive contribution to the ongoing debate between choosing an appropriate targeting regime. Thus, the analysis not only allows for a direct comparison between price-level and nominal income targeting in a range of macroeconomic models, but also highlights the importance of the transmission mechanism of monetary policy.

---

7 The issue of the importance of Phillips curve or the supply side of the economy for the performance of nominal income targeting is not new; it has been previously highlighted by Bean (1983) and West (1986).
2 - The Baseline Continuous-time ‘new Keynesian’ Model

The model is defined by equations (1) through (5). These equations define (respectively) the “new” IS relationship (aggregate demand), the “new” Phillips curve (aggregate supply), monetary policy, relationship between nominal and real interest rate, and the exogenous cycle in autonomous spending. The definition of variables and a more detailed description of the structure are given following the equations.

\[
\dot{y} = \alpha(r - \bar{r}) + \beta \dot{a} \quad (1)
\]
\[
\dot{p} = -\lambda(y - \bar{y}) + \psi(a - \bar{a}) - \kappa \gamma(r - \bar{r}) \quad (2)
\]
\[
i = \bar{i} + \Omega(p + \mu y) \quad (3)
\]
\[
r = i - \pi \quad (4)
\]
\[
a = \bar{a} + \delta \sin(t) \quad (5)
\]

All variables except the interest rates \(r\) and \(i\) and the time index \(t\) are the natural logarithms of the associated variable. Dots and bars above a variable denote (respectively) the time derivative, and the full-equilibrium value of that variable. All coefficients (the Greek letters) are positive. The variables are: \(a\) – autonomous spending, \(p\) – the general price level, \(r\) – the real interest rate, \(i\) - the nominal interest rate, and \(y\) – the level of real output.

Before discussing each equation in turn, I discuss the continuous-time specification. Discrete-time specifications are more common, but following this practice can involve model properties being dramatically dependent on small changes in assumptions concerning information availability. For example, consider the original “policy relevance” paper by Sargent and Wallace (1976). The central conclusion in this study does not emerge if it is assumed that the information available to agents when deciding how much to spend is the same as what is now usually assumed (that is, when the assumption involved in McCallum and Nelson (1999) is invoked). Also, if the McCallum and Nelson analysis (p. 309) is reworked with the information-availability assumption used by Sargent and Wallace, the entire undetermined coefficients solution procedure breaks down (with restrictions on structural, not reduced
form, coefficients being called for). A continuous-time specification precludes such unappealing problems from developing.

Equation (1) is the “new” IS relationship which states that the rate of change of real output depends positively on the real interest rate and on the rate of change of autonomous spending. The motivation for such a relationship can be appreciated by referring to a dynamic general equilibrium macro model with optimizing economic agents. I start with a log-linear approximation of the economy’s resource constraint: \( y = \alpha c + \beta a \), where ‘\( c \)’ is the log of consumption expenditure, ‘\( a \)’ is the log of the autonomous spending. The parameters ‘\( \alpha \)’ and ‘\( \beta \)’ are the steady-state ratios of household spending and autonomous spending to total real output respectively. The Ramsey model is used to model forward-looking domestic households. If the instantaneous utility function involves separable terms, log consumption and the square of labour supply, the first-order conditions are \( \ddot{c} = r - \bar{r} \), and (ignoring constants) \( n = w - p - c \). ‘\( n \)’ and ‘\( w \)’ denote the log of employment and the nominal wage. Equation (1) follows by taking the time derivative of the resource constraint and substituting in the Euler equation for consumption. The labour supply function is used below.

Equation (2) is the “new” Phillips curve that relates the rate of change of inflation to the output gap, autonomous-spending gap and the real rate of interest gap. This relationship essentially captures the supply side of the economy and can be derived by incorporating nominal price rigidities using Calvo’s (1983) model of sluggish price adjustment and imperfect competition a la Dixit and Stiglitz (1977) in a dynamic general equilibrium macro model. Many authors have shown that if we assume that firms minimize the undiscounted present value of the squared deviations between the log of marginal cost (\( mc \)) and price (\( p \)), optimal behaviour at the individual firm level leads to \( \ddot{p} = -[(1 - \tau)^2 / \tau](mc - p) \) at the aggregate level. \( (1 - \tau) \) is the fraction of firms that can change prices at each point in time. To represent this price-adjustment process in a format that resembles the traditional Phillips curve, I follow King

---

8 See, Lam and Scarth (2002).
9 For detailed derivation and discussion, see Clarida, Gali and Gertler (1999), McCallum and Nelson (1999), and Walsh (2003a)
(2000) and replace real marginal cost with the output gap (and any other term that emerges as relevant given that I have autonomous spending and supply-side effects of interest rate in the model). In order to incorporate the cost channel I assume that firms borrow from banks to pay for the wage-bill before the production process begins. Thus, there is a production lag involved here and the relevant ‘cost of borrowing’ is represented by the real interest rate, $r$. However, the results for ‘nominal’ interest rate as the relevant cost of borrowing are also considered. This assumption allows me to explicitly analyze the supply-side effects (the cost channel) of monetary policy. The cost channel makes firms’ marginal costs depend directly on the rate of interest. I assume a standard Cobb-Douglas production function of the form $Y = N^θ$. Thus, in log terms, $y = θn$ and the marginal product of labour, $MPL$, equals $θY/N$. Now, the marginal cost is defined as $MC = W(1 + r)/MPL$; we can (ignoring constants) approximate the log of real marginal cost by $mc - p = w - p + r - y + n$. Equation (2) is then derived in three more steps. Use the labour supply function, the production function and the resource constraint to eliminate $(w - p), n$ and $c$ by substitution; define units so that, in full equilibrium, all prices are unity (so that $mc - p = 0$); and substitute out the deviation of real marginal cost from its full-equilibrium value. The coefficients in (2) have the following interpretations: $λ = (1 - τ)^2((2/θ) + (1/α) - 1)/τ$, $ψ = (1 - τ)^2 β/ατ$, and $γ = (1 - τ)^2 / τ$. Thus parameters ‘$λ$’, ‘$γ$’ and ‘$ψ$’ are functions of “deep” parameters like the fraction of firms adjusting their prices, labour’s exponent in the production function, and ‘$α$’ and ‘$β$’. The parameter ‘$κ$’ is introduced to capture the cost channel of monetary transmission. By setting $κ = 0$, I can close this channel.

Equation (3) defines monetary policy and encompasses both price-level targeting, $μ = 0$ and nominal income targeting, $μ = 1$. According to this rule, the central bank adjusts the nominal interest rate above its steady-state value whenever either the price level is above its target (assumed to be zero), or the nominal income is above its target (also assumed to be zero). With this rule I also consider various degrees of ‘leaning against the wind’ in both cases. For example, $Ω = 1$ depicts the case when the central
bank conducts monetary policy in a ‘modest’ manner. On the other hand, \( \Omega \) approaching infinity would imply that the central bank responds in an ‘aggressive’ manner.

Equation (4) simply captures the relationship between nominal and real interest rate, while equation (5) depicts the anticipated ongoing cycles in exogenous spending defined by the sine curve. Since the focus of the paper is on the role of the cost channel in affecting the volatility of output under alternative monetary policy regimes, the simplest way to introduce fluctuations in output is to assume that these are caused by exogenous variations in the autonomous spending.

To understand the assumption that the criterion for evaluating the performance of a monetary regime is its ability to minimize the volatility in real output only and that the two targeting regimes generate the same outcome regarding long-term inflation, substitute equation (1) in (2) and take the time derivative. The resulting expression is:

\[
p = -\lambda \dot{y} - \psi \dot{\alpha} - \left( \frac{\kappa \gamma}{\alpha} \right) \ddot{y} + \left( \frac{\beta \kappa \gamma}{\alpha} \right) \ddot{\alpha}.
\]

Clearly, the policy parameter \( \mu \) does not affect the behavior of inflation (\( \dot{p} \)) over time.

Before analyzing the model and discussing the results I briefly talk about the parameter values that are used in calibrating the model(s) below. Consumption is 60% of the total output, that is, \( \alpha = 0.6 \). This implies that \( \beta = 0.4 \). The other summary coefficients for the baseline Phillips curve relationship can be calculated by referring to the corresponding values of the ‘deep’ parameters. For example, if labour’s exponent in the Cobb-Douglas production function is two-thirds (\( \theta = 0.67 \)) and the fraction of firms that are able to adjust their prices once a quarter is one-fourth (Gali and Gertler (1999)), then the corresponding annual values are: \( \lambda = 4 \times ((0.25)^2 \times (2 / 0.67) + (1/0.6) - 1)/0.75 = 1.21 \), \( \psi = 4 \times ((0.25)^2 \times (0.4)/(0.6)(0.75) = 0.22 \), and \( \gamma = 4 \times ((0.25)^2 / 0.75) = 0.33 \). The parameter \( \delta \) in equation (4) is taken as 1.

\[\text{In order to ensure that my results are not dependent on particular values of these parameters, I have considered a range of other parameter values as well. For example, if we assume that the fraction of firms with sticky prices is 0.85 rather than 0.75 than the values of all summary parameters change accordingly. In particular, they are: } \lambda = 0.40, \psi = 0.07 \text{ and } \gamma = 0.11. \text{ The results, reported in table 2, are not sensitive to these alternative values for various parameters.} \]
3- Solution Procedure

In this section I derive the reduced form for real output to see how the cost channel affects the amplitude of the cycle in \( y \), and to see the relative performance of price-level and nominal income targeting in this context. I explain this derivation in the baseline case only. The reader can use similar steps to verify the results that I report for other cases in the following sections.

Substitute equation (3) and (4) in equation (1) to get:

\[
\dot{y} = (\alpha \Delta) p + (\alpha \Delta \mu) y - \alpha \dot{p} + \beta \dot{a}
\]  

(6)

Substitute equation (3) and (4) in equation (2) and use equation (6) to eliminate \( p \):

\[
\ddot{p} = -\lambda (y - \bar{y}) + \psi (a - \bar{a}) - \left( \frac{\kappa \gamma}{\alpha} \right) \dot{y} + \left( \frac{\kappa \gamma \beta}{\alpha} \right) \dot{a}
\]

(7)

Take time derivative of equation (6), use equation (7) to eliminate \( \dot{p} \), take the time derivative of the resulting expression and again use equation (7) to eliminate \( \ddot{p} \). The result is:

\[
\dddot{y} = -\alpha \lambda \Delta (y - \bar{y}) + (\alpha \lambda - \kappa \gamma \Omega) \ddot{y} + (\alpha \Delta \mu + \kappa \gamma) \ddot{y} \\
+ \alpha \psi \Delta (a - \bar{a}) - (\alpha \psi - \kappa \gamma \beta \Omega) \dot{a} - (\kappa \gamma \beta) \dot{a} + (\beta \bar{a})
\]

(8)

Using the undetermined coefficient solution procedure as described in Chiang (1984), the solution for output can be written as:

\[
y = \bar{y} + B[\cos(t)] + C[\sin(t)]
\]

(9)

where \( B \) and \( C \) are arbitrary constants that must be related to the underlying parameters of the model. To solve for \( B \) and \( C \), first take the time derivatives of (9), \( \dot{y} = -B \sin(t) + C \cos(t) \), \( \ddot{y} = -B \cos(t) - C \sin(t) \), \( \dddot{y} = B \sin(t) - C \cos(t) \) along with the time derivatives of (5), \( \dot{a} = \delta \cos(t) \), \( \ddot{a} = -\delta \sin(t) \), \( \dddot{a} = -\delta \cos(t) \) and then substitute these results and equation (5) and (9) in equation (8).

The resulting coefficient-identifying restrictions are:

\[
B = \frac{1}{b_1} \left[ (\alpha \psi \Delta + \kappa \gamma \beta) - b_2 C \right]
\]

(10)
\[ C = \frac{1}{b_1^2 + b_2^2} \left[ b_1 (\beta + \alpha \psi - \kappa \gamma \Omega) + b_2 (\alpha \psi \Omega + \kappa \gamma \beta) \right] \tag{11} \]

where, \( b_1 = 1 + \alpha \lambda - \kappa \gamma \Omega \)

\[ b_2 = \alpha \Omega (\lambda + \mu) + \kappa \gamma \]

The amplitude of the cycles in real output that correspond to the ongoing cycles in autonomous spending can be examined by substituting the calibrated expressions for B and C in equation (9). The results are reported in table 1 and discussed in section 5 below.

4 - Sensitivity Test --- Endogenous Persistence

In this section I generalize the model to allow for “hybrid” IS and Phillips curve relationships. This extension is motivated by the fact that the simple versions of the “new” relationships do not fit actual data as well as we would like. For example, Fuhrer (1997, 2000) and Amato and Laubach (2003) have pointed out that the standard Ramsey type Euler Equation for consumption (which gives rise to an IS-type relationship) fails to capture the dynamics of the aggregate output. Similarly, it has been pointed out by many researchers that ‘the new Keynesian Phillips curve’ based on Calvo’s (1983) sticky price model generates inertia in the price level and not the inflation rate and that this is inconsistent with stylized facts on inflation dynamics. The empirical evidence (for example, Nelson (1998)) indicates that inflation responds sluggishly to economic shocks\(^{11}\). The ‘new Keynesian Phillips curve’ implies that inflation is determined by the current output gap and current expectations of future inflation. Inflation is, therefore, very flexible and responds immediately to monetary policy shocks and hence does not accord with stylized facts. Therefore, based on these considerations, many authors (Walsh (2003b), Amato and Laubach (2003), Jensen (2002)) are now using IS and Phillips curve relationships that involve a fraction of agents who optimize (just as we have assumed in the earlier sections of the paper) and a fraction of

\(^{11}\) Empirical research of Gali and Gertler (1999) and Fuhrer (1997) have generally found that when lagged inflation is added to the basic ‘new Keynesian Phillips curve’, its coefficient is statistically and economically significant.
agents who find it too expensive to optimize, so they follow a rule of thumb that is intended to approximate optimal behaviour. This second group of agents simply mimic what the optimizers do – with a one-period time lag (so that the behaviour of the optimizers can be observed). This set of assumptions introduces more inertia into the dynamic supply and demand relationships, and this is what makes the resulting “hybrid” relationships more consistent with the data. Since the debate over the relative benefits of price-level and nominal income targeting rests critically on the specification of the IS and Phillips curve type relationships, it is a worthwhile exercise to redo the analysis with these more general specifications for the model. Below I consider three representative versions that correspond to three cases: extension 1, extension 2, and extension 3.

4.1 – Extension 1

Following Walsh (2003b), Clarida, Gali, and Gertler (1999) and Fuhrer and Moore (1995), if the weight on the lagged output term (in the IS relationship) and lagged inflation term (in the Phillips curve) is assumed to be 0.5, then the hybrid model in the discrete-time can be written as:

\[ y_t = 0.5y_{t-1} + 0.5y_{t-1} - \alpha(r_t - \bar{r}) - \beta(a_{t+1} - a_t) \]
\[ \pi_t = 0.5\pi_{t+1} + 0.5\pi_{t-1} + \lambda(y_t - \bar{y}) - \psi(a_t - \bar{a}) + \kappa\gamma(r_t - \bar{r}) \]

Replacing first differences with time derivatives to return to continuous time, we get:

\[ \dot{y} = 2\alpha(r - \bar{r}) + 2\beta \dot{a} \]  
(12)
\[ \ddot{p} = -2\lambda(y - \bar{y}) + 2\psi(a - \bar{a}) - 2\kappa\gamma(r - \bar{r}) \]  
(13)

4.2 - Extension 2

This extension follows Jensen (2002). For household consumption behaviour (ignoring autonomous expenditure for initial explanation) we note that the fraction of optimizing agents follow the Ramsey rule: \( c_t = c_{t+1} - (r_t - \bar{r}) \), and the rule-of-thumb agents mimic what other agents did in the previous period: \( c_t = c_{t-1} \). Giving a one-half weight to each of these two decision rules, replacing first
differences with time derivative as we switch to continuous time (as above), and noting that $c = \gamma$ in this simple case, we arrive at the hybrid IS curve: $\ddot{y} = (r - \bar{r})$. The existence of autonomous spending leads to additional terms in the new hybrid IS relationship. Thus, the IS relationship actually used is:

$$\ddot{y} = (1 - \alpha)y + \alpha(r - \bar{r}) + \beta \dot{a} - \beta a$$

(14)

On the supply side (using $\pi$ to denote the inflation rate, ignoring autonomous spending and cost channel for simple exposition, and writing relationship in discrete time), the optimizers set prices according to: $\pi_t = \pi_{t-1} + \lambda (y_t - \bar{y})$, while the rule-of-thumb agents set prices according to: $\pi_t = \pi_{t-1}$. A hybrid Phillips relationship can be had by giving each of these component equations a weight of one half in an overall equation for $\pi$. After doing just that, and replacing first differences with time derivatives to return to continuous time, we have the hybrid Phillips curve: $\ddot{p} = -\lambda (y - \bar{y})$. The existence of autonomous spending and the cost channel leads to additional terms in the new hybrid Phillips curve. Thus, the Phillips curve actually used is:

$$\ddot{p} = -\lambda (y - \bar{y}) + \psi (a - \bar{a}) - \kappa \gamma (r - \bar{r})$$

(15)

### 4.3 - Extension 3

Extension 3 follows Amato and Laubach (2003) and Gali and Gertler (1999) to derive the hybrid versions of IS and Phillips curve equations. Since the derivation is slightly more tedious, I report just the final expressions to conserve space.

$$\dot{y} = -0.5 \dot{y} + 0.5 \alpha (r - \bar{r}) + \beta \dot{a} + 0.5 \beta \ddot{a}$$

(16)

$$\ddot{p} = \left(1 - \frac{2 \tau}{2 \tau}\right) \dot{p} - 0.5 \lambda (y - \bar{y}) + 0.5 \psi (a - \bar{a}) - 0.5 \kappa \gamma (r - \bar{r})$$

(17)

To solve the models in these extensions, I proceed through the same steps as outlined in section 3 above, but do not report the revised expressions for reduced-form parameters $B$ and $C$. The analysis is no more involved on conceptual grounds in the hybrid cases, but the actual derivation is quite tedious. (The
analogue of equation (8) is a fifth-order differential equation in these cases, so many time derivatives of the trial solution need to be substituted in.)

5 - Results

The results corresponding to baseline parameter values for all four models (the baseline model and three extensions) with and without the cost channel for both policies (price-level targeting and nominal income targeting) are reported in table 1. The table also reports both ‘modest’ and ‘aggressive’ policy reactions. In all cases the peaks and troughs in the cycle for output are almost exactly in phase with those for the cycle in autonomous spending.

The first main result is that in a baseline ‘new Keynesian’ model, nominal income targeting performs better as compared to price-level targeting in terms of reducing the volatility of real output in the face of ongoing demand shocks whether cost channel is operating or not or policy response is moderate or aggressive. Nominal income targeting allows the monetary policy to adjust to offset direct disturbances to aggregate demand and indirect disturbance to aggregate supply via the cost channel. For example, in case of an adverse demand shock (corresponding to the trough in the cycle for real output due to the downward cycle in autonomous spending) that would cause both real output and prices to go below target, policymakers would ease monetary policy that would push nominal income (the product of real income and prices) towards target and keep the volatility of nominal income to a minimum. In the presence of the cost channel, this ease in the monetary policy would lead to favorable supply side movements as well that would result in rising real output and falling price levels. This could pose a dilemma if central bank is pursuing price level targeting. The central bank would have to manipulate the aggregate demand by a large magnitude that would ensure the achievement of the original level of prices at the cost of an increased volatility in output. On the other hand, with nominal income targeting it would adjust aggregate demand just enough to reach a targeted level of nominal income with slightly lower level of prices and slightly higher level of output. The volatility in real output would definitely be lower
compared to the price-level targeting case. Since the metric used to evaluate the performance of a targeting regime is the minimization of real output volatility, nominal income targeting is preferred to price-level targeting.

Table 1: Output Effects of Ongoing Demand Cycles --- Baseline Parameters

<table>
<thead>
<tr>
<th>Amplitude of ongoing cycle in real output</th>
<th>Moderate Price Stickiness (prices fixed for four quarters on average)</th>
<th>Aggressive policy response $\Omega \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderate policy response $\Omega = 1$</td>
<td>Without cost channel, $\kappa = 0$</td>
<td>With cost channel, $\kappa = 1$</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>Price-level targeting, $\mu = 0$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Nominal Income targeting, $\mu = 1$</td>
<td>0.9</td>
</tr>
<tr>
<td>Extension # 1</td>
<td>Price-level targeting, $\mu = 0$</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Nominal Income targeting, $\mu = 1$</td>
<td>1.6</td>
</tr>
<tr>
<td>Extension # 2</td>
<td>Price-level targeting, $\mu = 0$</td>
<td>5.07</td>
</tr>
<tr>
<td></td>
<td>Nominal Income targeting, $\mu = 1$</td>
<td>2.33</td>
</tr>
<tr>
<td>Extension # 3</td>
<td>Price-level targeting, $\mu = 0$</td>
<td>2.37</td>
</tr>
<tr>
<td></td>
<td>Nominal Income targeting, $\mu = 1$</td>
<td>1.67</td>
</tr>
</tbody>
</table>

---

12 The absolute value(s) of the amplitude of the cycle in real output depends on parameter $\delta$ (which is taken as one for baseline calibration). To avoid the sensitivity of results to changes in an arbitrary parameter value and to ease comparison across different cases, I normalize the result for price-level targeting without the cost channel to a moderate policy response to 1. The results for all other cases are simply reported relative to this baseline case.
The result regarding the effects of the cost channel in the baseline case is quite interesting and somewhat surprising. In the case of ‘moderate policy response’ the volatility of real output goes down in the presence of the cost channel (in both targeting regimes), while it increases in the case of ‘aggressive policy response’. Conceptually, this result is quite similar to the one reported by Clarida, Gali and Gertler (1999) (Result 2, page 1673) for optimal inflation targeting. They call for a gradual convergence of inflation to its target over time and report that extreme inflation targeting --- adjusting policy to immediately reach an inflation target --- is optimal only if cost push inflation is absent and there is no concern for output deviations. Since in the model presented the cost channel is a policy-induced cost push factor affecting inflation and output volatility is the primary criterion to evaluate the effects of alternative targeting regimes, a parallel can be drawn between the observation made by Clarida, Gali and Gertler (1999) and the result reported above. More specifically, a moderate policy response would lead to an ‘optimal’ outcome if the cost channel is operating. Similar conflicting views, also noted by Clarida, Gali and Gertler (1999), regarding the optimal transition path to the target have emerged in the literature. Goodfriend and King (1997) favour aggressive policy response because cost push considerations are absent in their paradigm, while Ball (1999) and Svensson (1999b) consider moderate policy responses as optimal because cost push inflation considerations play an important role in their frameworks.

How robust are these results? Extension 1, 2 and 3 introduce endogenous inflation and output persistence by following various approaches employed in the literature. For extension 1 with a moderate policy response, price-level targeting performs slightly better than nominal income targeting in the absence of cost channel. However, this difference is rather small. In the presence of cost channel, nominal income targeting leads to slightly better outcome. Further, the volatility of output goes down in either targeting regime in the presence of cost channel. With an aggressive approach to policy, price-level targeting is rather dangerous as the volatility in real output approaches infinity whether the cost channel is operating or not. This result seems consistent with the observation made by Barnett and Engineer (2000): “….. price-level targeting is desirable only for a purely forward looking specification of the Phillips
curve”. Nominal income targeting, on the other hand, results in finite and small (compared to moderate policy response) volatility in real output and, as before, the cost channel increases this volatility.

For extension 2, although the amplitude in real output is rather large but the baseline model results carry over to this general case with one notable exception: with an aggressive policy response, price-level targeting implies infinite volatility in real output in the absence of cost channel and not when it is operating. Thus, introducing backward-looking features does not necessarily imply disastrous results for price-level targeting; the observation noted above by Barnett and Engineer (2000) cannot be generalized. The cost channel seems to provide stable solutions. This is an important result and needs to be researched further. Another somewhat interesting result to note is that the volatility in real output virtually goes to zero with nominal income targeting with the cost channel and an aggressive policy response. Moreover, the difference between price-level targeting and nominal income targeting is more pronounced. Extension 3 confirms the results for extension 2 and thus provides more support for them. As noted by Clarida, Gali and Gertler (1999), in the presence of endogenous persistence in addition to affecting the gap between the target variable and its target along the convergence path, policy also affects the rate of convergence and leads to less favorable trade-off between stabilizing inflation and output. Thus, a more aggressive response to deviation from the target is required because any gap not eliminated today will persist into the future. This means that we should expect increases in the volatility in real output, and indeed this is what we observe in all the extensions considered: the volatility in real output is larger for models with endogenous persistence compared to the baseline forward-looking models. Lastly, if we introduce endogenous persistence for only output, that is, re-specify the IS relationship and use the baseline forward-looking inflation dynamics ---the Phillips curve --- (results not reported), even then nominal income targeting results in smaller and stable outcomes compared to price-level targeting. This contradicts the claim made by McCallum (1997) that it is only the specification of the Phillips curve that is important: “…replacement of the Ball-Svensson Phillips curve with the mentioned alternative (new Keynesian Phillips curve) results in a model in which both output and inflation are dynamically stable under nominal income targeting whether or not the IS relationship is re-specified”.

18
Table 2: Output Effects of Ongoing Demand Cycles --- Alternative parameters

<table>
<thead>
<tr>
<th>Amplitude of ongoing cycle in real output</th>
<th>High Price Stickiness (prices fixed for six quarters on average)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moderate policy response Ω = 1</td>
</tr>
<tr>
<td></td>
<td>Aggressive policy response Ω → ∞</td>
</tr>
<tr>
<td>Without cost channel, κ = 0</td>
<td>With cost channel, κ = 1</td>
</tr>
<tr>
<td>Without cost channel, κ = 0</td>
<td>With cost channel, κ = 1</td>
</tr>
</tbody>
</table>

**Baseline Model**

<table>
<thead>
<tr>
<th>Price-level targeting, μ = 0</th>
<th>1</th>
<th>0.97</th>
<th>0.48</th>
<th>0.66</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Income targeting, μ = 1</td>
<td>0.85</td>
<td>0.80</td>
<td>0.14</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Extension # 1**

<table>
<thead>
<tr>
<th>Price-level targeting, μ = 0</th>
<th>11.66</th>
<th>2.88</th>
<th>infinity</th>
<th>infinity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Income targeting, μ = 1</td>
<td>1.8</td>
<td>1.37</td>
<td>0.26</td>
<td>0.57</td>
</tr>
</tbody>
</table>

**Extension # 2**

<table>
<thead>
<tr>
<th>Price-level targeting, μ = 0</th>
<th>1.69</th>
<th>1.37</th>
<th>infinity</th>
<th>infinity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Income targeting, μ = 1</td>
<td>1.06</td>
<td>0.94</td>
<td>0.23</td>
<td>0.009</td>
</tr>
</tbody>
</table>

**Extension # 3**

<table>
<thead>
<tr>
<th>Price-level targeting, μ = 0</th>
<th>1.89</th>
<th>1.77</th>
<th>infinity</th>
<th>45.51</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Income targeting, μ = 1</td>
<td>1.4</td>
<td>1.34</td>
<td>3.71</td>
<td>3.63</td>
</tr>
</tbody>
</table>

Of course, robustness across alternative parameter values is just as important as robustness across model specification. As far as alternative parameter values for the IS relationship (α and β) are concerned, the results reported above remain essentially the same, so I do not report them. For Phillips curve, the key parameter is the degree of price stickiness since ψ, λ, and γ are all sensitive to changes in its value. Table 2 reports the results for a high degree of price stickiness. Almost all results survive this sensitivity test with two exceptions. For extension 1 with moderate policy response, price-level targeting
performs significantly worse than nominal income targeting without the cost channel. For extension 2 with aggressive policy response, price-level targeting implies infinite volatility in output even in the presence of cost channel. Another sensitivity test considered is the inclusion of nominal interest rate (rather than real interest rate) in the Phillips curve to capture the cost channel. Since virtually all general conclusions survive this sensitivity testing, I do not include additional tables here.

What broad conclusions can we draw from the above discussion? First, nominal income targeting is superior to price-level targeting in all most all cases regardless of the cost channel and alternative degrees of policy responses. Second, output volatility goes down due to the cost channel in case of a moderate policy response, while it increases for aggressive policy. Third, in case of an aggressive response to policy for models with endogenous persistence, price-level targeting leads to infinite volatility in output without the cost channel. With cost channel it results in stable outcomes.

6- Concluding Remarks

This paper studied the main developments in the macroeconomic theory regarding the specifications for the aggregate demand and aggregate supply side of the model and the transmission mechanism of monetary policy in the context of two rule-based monetary policy regimes: price-level targeting and nominal income targeting. Comparing the results presented in the series of macroeconomic models indicate that analysing both the traditional and the cost channel of monetary policy in one unified framework has been worthwhile. They confirm the results of earlier theoretical and empirical research on the potency of supply side effects of monetary policy (the cost channel) in effecting the real economy. Moreover, the paper also finds strong support for a case in favour of nominal income targeting when compared with price-level targeting as it keeps the volatility of real output low. There is a growing literature that studies and compares the performance of these targeting regimes and a consensus has not been reached yet. Thus, the results of this paper can be considered as an addition to this debate. An important point in this regard is that the specification of both the demand side and the supply side of the model are crucial while analysing various monetary policy targeting regimes.
However, I agree with McCallum (1997) when he concluded while comparing the performance of inflation targeting and nominal income targeting: “This demonstration does not establish that nominal income targeting is preferable to inflation targeting or to other rules for monetary policy. To reach such a conclusion would require an extensive combination of theoretical and empirical analyses, conducted in a manner that gives due emphasis to the principle of robustness to model specification, plus attention to concerns involving policy transparency and communication with the public”. The point of this paper was not to attempt any such ambitious undertaking. However, the results can be considered as a small step in that direction.

References


