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On smoothing macroeconomic time series using HP and modified HP filter

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Abstract

In business cycle research, smoothing data is an essential step in that it can influence the extent to which model-generated moments stand up to their empirical counterparts. To demonstrate this idea, we compare the results of McDermott's (1997) modified HP-filter with the conventional HP-filter on the properties of simulated and actual macroeconomic series. Our simulations suggest that the modified HP-filter proxies better the true cyclical series. This is true for temporally aggregated data as well. Furthermore, we find that although the autoregressive properties of the smoothed observed series are immune to smoothing procedures, the multivariate analysis is not. As a result, we recommend and hence provide series-, country- and frequency specific smoothing parameters.

JEL Classification: C32, C43, E32

Key Words: Business Cycles; Cross Country Comparisons; Smoothing Parameter; Time Aggregation

I. Introduction[1]

Our prior view is that a 5 percent cyclical component is moderately large, as is one-eight of 1 percent change in the growth rate in a quarter. This led us to select $\sqrt{\lambda} = 5/(1/8) = 40\ (\lambda = 1600)$ *as a value for smoothing parameter (*Hodrick and Prescott, 1997, p. 4*)*

Business cycle research studies the cyclical component of relevant macroeconomic time series. This requires selecting a detrending method. Whilst other methods exist, the Hodrick-Prescott filter (HP filter hereafter) remains a popular choice and the conventional wisdom has become to fix the value of the smoothing parameter, λ, at 1600 (100) for quarterly (annual) frequency data following Hodrick and Prescott's (1997) view. Indeed, the term 'Hodrick-Prescott filter' reveals no less than 44,800 hits on various search engines and is cited in more than 4527 papers.[2]

Despite its popularity, the practice of fixing λ=*1600 (100) for quarterly (annual)* frequency across series and countries remains a contentious issue. This is because the determination of the smoothing parameter of a given series relies on the underlying behavior of economic agents from where the

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[2] Search carried out on 20 March 2013.

dynamical properties originate. This issue is revisited afresh in this paper with noteworthy implications for business cycle research.

The literature offers two alternatives for selecting λ. The first is based on Hodrick and Prescott (1997), Cooley and Ohnain (1991), Backus and Kehoe (1992), Correia, Neves and Rebelo (1992), and Baxter and King (1999). These studies recommend fixing the smoothing parameter to isolate the cyclical component of economic time series[3] and mainly focus on developed economies with quarterly data. Ravn and Ulhig (2002), while still proposing a fixed lambda across countries, find that HP filter should adjust to the frequency of data. They suggested a value of 6.25 for annual and 1600 for quarterly data.

The second is based on Agénor et al. (1998), McDermott (1997) and Marcet and Ravn (2003) which emphasize using a more country specific approach. While working on quarterly industrial output along with other variables for a set of 12 countries, the former two studies show that the estimated values of lambda is closer to the traditional value of 1600 for only one series. However, Marcet and Ravn (2003) argue that fixing the lambda across the countries may be inappropriate when there are important cross country differences in the persistence of the cyclical component. Their study of 8 countries finds that in the presence of higher persistence in the cyclical component, fixing lambda for decomposing the permanent and cyclical component and using the conventional HP-filter approach inaccurately assigns a large fraction of economic swings to the trend.

We contribute to this literature in several ways. First, we conduct a simulation study designed to compare the modified HP filter approach with that of HP filter to evaluate which one produces a closer approximation of given permanent (cyclical) components of an artificial macroeconomic time series. Second, we estimate the smoothing parameter using the modified HP filter approach in McDermott (1997) for three core macroeconomic time series of real income, investment and private consumption, and their cyclical components thereof for 93 countries using annual data and for 25 countries for which we could find quarterly data from a single source and compare those with the corresponding cyclical components based upon fixed values of lambda conventionally used in the literature. Third, we examine the sensitivity of standard deviations, degree of persistence of the 'estimated' cyclical components to the choice of smoothing method. Fourth, we compare the impact of the choice of smoothing parameter on the unconditional correlation between the cyclical components of the real income- real investment and real income- real consumption pairs. Fifth, the analysis is done for the largest set of countries to the authors' knowledge. As business cycle research variant become common place across countries where there exist severe data constraints, the extent of comparisons offered in this paper sheds an important light on the implications of the choice of smoothing parameters.

Few results deserve highlighting. First, in a simulation study designed for macroeconomic time series type data, including the temporally aggregated one, we find the approach endogenously estimating lambdas produces lower mean square errors (of the given and estimated permanent as well as cyclical components) compared to those of fixing lambdas. Second, in an empirical study based on quarterly dataset, the net differences for persistence and unconditional means of cyclical components emanating from a fixed λ=1600 and of those extracted using endogenously-estimated lambdas are statistically negligible. Third, for annual datasets the net differences in correlation coefficients are statistically significant for 1/3 of the countries in all income-wise group heads in our sample implying exercising caution against using fixing lambda at the level of country as well as series.

[3] Except Backus and Kehoe (1992), all suggested fixing lambda for annual data- which is not necessarily 100.

The remainder of the paper is organized as follows. The next Section revisits both the original HP filter and its modified version. Section 3, provides the setup and the results of our simulation. In Sections 4 and 5 we discuss the results of when the two filters are applied to a large set of countries and observed macro series with different frequencies. A final Section presents concluding remarks.

II. The HP and Modified HP Filters

A brief review of the conventional HP filter

Hodrick-Prescott (1997) method decomposes a seasonally adjusted time series into a permanent (long-term) and a cyclical (short-term) component so that

$$y_t = g_t + c_t\ , \quad t = 1,\ 2\ ,3\ , \dots , T \tag{1}$$

where y_t, g_t and c_t are a given time series (in logs), trend, and cyclical components respectively. The method essentially computes a stochastic series (c_t) by minimizing the sum of squared deviations of the original time series (y_t) from its trend (g_t), essentially the goodness of fit, subject to the constraint that the squared sum of dynamic differences of the permanent component, a measure of the degree of smoothness, is not too large. Therefore, the optimization problem is

$$\min_{g_t} \left[\sum_{t=1}^{T}(y_t - g_t)^2\right]$$

Subject to

$$\sum_{t=1}^{T}(\Delta^2 \mathrm{g_t})^2 = \sum_{t=1}^{T}[(\mathrm{g_{t+2}} - \mathrm{g_{t+1}}) - (\mathrm{g_{t+1}} - \mathrm{g_t})]^2 = \nu$$

where Δ^2 is the second-order differences of the trend and ν is a known constant. The standard method to solve this problem assumes that $\nu = 0$ so that using the Lagrange multiplier we get

$$\min_{g_t} \left[\sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \sum_{t=1}^{T}[(\mathrm{g_{t+2}} - \mathrm{g_{t+1}}) - (\mathrm{g_{t+1}} - \mathrm{g_t})]^2\right] \tag{2}$$

In this optimization problem, there is a trade-off between the goodness of fit and the degree of smoothness that depends on the value of λ. The conditional expectation of $\mathrm{g_t}$ solves (2) where λ is the ratio of variances of the cyclical and change in growth of the permanent series.

Assuming a fixed value of λ (1600 for quarterly- and 100 for annual- frequency) the solution to the minimization problem in (2) for, $\mathbf{g_t}$, is

$$\hat{g}_t = [I + \lambda A]^{-1} y_t = B y_t \tag{3}$$ [4]

Where $A = K'K$ where $K = \{k_{ij}\}$ is a (T-2) ×T matrix with elements are given below

[4] Technical details are available with the corresponding author upon request.

$$k_{ij} = \begin{cases} 1 & if\ j = i\ or\ j = i + 2, \\ -2 & if\ \ j = i + 1, \\ 0 & otherwise \end{cases}$$

Using this procedure on post war quarterly US GDP data and the value of smoothing parameter selected by Hodrick and Prescott (1997) fixed at 1600. It has now been convention for smoothing quarterly macroeconomic series across economies and across series.

The Modified HP-filter

As the variances of the underlying series are different across countries (See Figure 1), the anticipation that the ratio of the variances of the cyclical component and that of changes in growth of the trend is same across nations and series may be misplaced. Furthermore, business cycle fluctuations may have moderated in developed world (Stock and Watson 2003), business cycles in emerging economies exhibit large volatility (Aguiar and Gopinath, 2007). In such a situation, each country/series should have a customized smoothing parameter to extract the cyclical component.

To address this issue, the modified HP filter approach was developed in McDermott (1997) based on the cross-validation method (also known as the leave-out procedure) from Craven and Wahba's (1979). Here the emphasis is on selecting the optimal value for the smoothing parameter 'λ' instead. The basic idea to use the HP filter method but exclude a single data point at a time and then choose a λ that provides a spline series replicating best the missing data point.

To explain, *let* $g^{k}_{T,\lambda}$ denote the spline obtained from (3) on the basis of the leave-out procedure which implies. using an arbitrary λ and all the data points but leaving out the k^{th} point. Now, the ability of $g^{k}_{T,\lambda}$, the predicted value from the spline to replicate the left out k^{th} data point, say y_k, determines the fitness of the chosen λ. Note that each time a data point is left out, a new spline is computed from (3) and also assuming a λ. In practice, this is implemented using the mean square sum of the predicted and the left out data point, known as a cross-validation function, for a given λ so that

$$\mathrm{CV} \mid \lambda = \frac{\sum_{k=1}^{T}(y_k - g^{k}_{T,\lambda}(t_k))^2}{T} \tag{4}$$

The λ that minimizes cross-validation function is our optimal smoothing parameter. This minimization is a fairly tedious task as an array of cross-validation functions have to be obtained and compared for different values of λ. However, Craven and Wahba (1979) show that the overall solution for this complex minimization problem can be simplified by replacing y_k with $g^{k}_{T,\lambda}(t_k)$ in (2) leading to a generalized version of cross validation functions so that

$$GCV(\lambda) = T^{-1}\sum_{k=1}^{T}\frac{(y_k - g_{t,k}(\lambda))^2}{(1-\frac{1}{T}trA(\lambda))^2} = T^{-1}\sum_{k=1}^{T}(y_k - g_{t,k}(\lambda))^2 * (1 - \frac{1}{T}trB(\lambda))^{-2}$$

$$\cong T^{-1}\sum_{k=1}^{T}(y_k - g_{t,k}(\lambda))^2 * (1 + \frac{2}{T}trB(\lambda)) \tag{5}$$

where $g_{t,k}(\lambda) = \sum_{s=1}^{T} b_{ks}(\lambda)\, y_s$ and $B(\lambda)$ is a weighting matrix.

To compute the trace of $B(\lambda)$, one may use singular value decomposition or the approximation of Silverman (1984) giving us

$$GCV(\lambda) = T^{-1}(1 + \frac{2T}{\lambda}) \sum_{k=1}^{T} (y_k - g_{t,k}(\lambda))^2 \quad (6)$$

To recap the steps to obtain the new λ: . First, we estimate $g_{t,k}(\lambda)$ applying the leave-out method using Eq. (3) and an arbitrary value for λ. Seecond, we estimate $GCV(\lambda)$ from Eq. (6). Iterating values for λ>0, we obtain different estimates of (6) and λ that gives the minimum value of the objective function (6) is chosen as the optimal smoothing parameter.

Before we move to show the implications of the choice of smoothing parameter, another modification of HP filter which we do not pursue but deserves mention is that of Marcet and Ravn (2003) in which the sum of squared deviations of the time series (y_t) from its permanent component (g_t) are minimized by assuming a ceiling for the ratio of variability of the changes in the growth of the trend $\sum_{t=1}^{T}[(\mathrm{g}_{\mathrm{t+2}} - \mathrm{g}_{\mathrm{t+1}}) - (\mathrm{g}_{\mathrm{t+1}} - \mathrm{g}_{\mathrm{t}})]^2$ and the variability of the cyclical component $\sum_{t=1}^{T}\{y_t - g_t\}^2$. They impose this ceiling based on an anchor country, which in their case is the US. This approach has the following concerns: (i) the choice of an anchor country is subjective and (ii) the assumption of a similar ceiling for a set of countries with differing under dynamics is quitionable..

In the following Section we conduct simulation so as to determine how well the HP filter and its modified version produce smoothed (detrended) series replicate the actual trend (cyclical) components which are known to the researcher.

III. Simulation

Our simulation is based on a Monte Carlo experiment with the following experimental design. Based on the framework of Hodrick and Prescott (1997) that $y_t = g_t + c_t$, $t = 1, 2, 3, \dots, T$ as discussed in Eq. (1), and following Watson (1986), and Guay and St. Amant (2005) we use a data generating process DGP given in Eq. (7), for trend (g_t) and cyclical (c_t) component that generate artificial quarterly data as :

$$g_t = drift + trend_t + g_{t-1} + \varepsilon_t \text{ and } c_t = \emptyset_1 c_{t-1} + \emptyset_2 c_{t-2} + \xi_t \quad (7)$$

Where $\varepsilon_t \sim NIID(0, \sigma_\varepsilon^2)$ and $\xi_t \sim NIID(0, \sigma_\xi^2)$.

These data generating processes are chosen on the observation that for most macroeconomic series, the trend component is a random walk with a drift, which can be linear or non-linear. In our design we choose relatively general specification but within the context of macroeconomic time series[5] where trend and cyclical components (see Table 1) satisfy conditions pertaining to unit root and the stationarity of the trend and cyclical components [$\emptyset_1 + \emptyset_2 < 1$ and $|\emptyset_2| < 1$] respectively. Furthermore, we also vary the ratio of the standard deviations of the disturbances ($\sigma_\varepsilon / \sigma_\xi$) to change

[5] Macroeconomic time series are often represented as a sum of an unobserved permanent component (containing a unit root) and an unobserved (stationary) cyclical component (Guay and St. Amant, 2005).

the relative importance of the trend and cyclical component because business cycle fluctuations may be 'moderate' in developed countries (Stock and Watson, 2003) compared to emerging economies (Aguiar and Gopinath, 2007). By considering all the possibilities for the ratio[6] of the standard deviations of ε_t and ξ_t (to be greater than, equal to, and less than unity) we allow predominance of trend component over the cyclical one and vice versa.

We then generate 200 observations from equation (7) based on relevant parameter values given in Table 1. This length is assumed to represent 50 years worth of quarterly data. Since most of the macroeconomic data are heavily time aggregated (see Aadland, 2005), we also introduce time aggregation[7] to convert high frequency (quarterly) data to low frequency (annual) data. For this part of simulation our time dimension reduces to one-fourth i.e. 50.

Once the series has been generated, we extract cyclical component by using a) the modified HP filter where the (endogenous) value of lambda is based on the estimation process described above; and b) the HP filter where the (exogenous) value of lambda is 1600 for data imagined as being generated at quarterly frequency and is 100 for temporally aggregated (as annual) data. We repeated this experiment 1000 times.

An ideal filter would extract the permanent (cyclical) component in such a manner that the mean square error (MSE) of the extracted component and the one generated by (7) would be zero. To assess and compare the performance of the modified HP filter and HP filter, we compare the MSE of the permanent (cyclical) component extracted by the two filters.

In Table 1 we show the results of the performance of the modified HP filter and the HP filter by comparing the mean square errors. Modified HP filter dominantly out-performs the HP filter in our simulation study; including for the time aggregated data.

In the next Section, we apply the conventional and modified HP filters to observed data of a large set of countries and then analyze afresh various important univariate and multivariate features of cyclical series obtained by using these filters. But first we describe these data.

IV. Data

The empirical evaluation of a RBC model typically requires matching model moments with the relevant detrended macro series. The common practice is to compare the autoregressive coefficients and unconditional correlations of relevant series. These steps determine the fit of the model. Here we focus the extent to which autoregressive coefficients and their unconditional correlations are affected by the choice of the filtering method.

[6] Values of this ratio (at 10, 1, and 0.5) are taken from Guay and St. Amant (2005).

[7] There are three possibilities for converting the high frequency (quarterly) data to low frequency (annual) data. It may be systematic like in case of consumer price index (stock type data where we take end points), or by summing as in case of consumption (flow type data where we usually sum) or by averaging like in case of growth rate (rates type data where we usually average)).

To do so, we use quarterly (seasonally adjusted) and annual series of real GDP, real private consumption and real investment from International Financial Statistics database. The data are then transformed in logarithms. The data span for each country is mixed. Indeed for annual frequency, 93 countries have the relevant data series with some countries going as far back as 1950 while others only 1990. For quarterly frequency, data availability is scarcer. All in all 25 countries have the relevant quarterly data from a single source. Most series end at 2010 and our shortest data span is twenty years. The sample periods and the country lists for annual and quarterly series are reported in Table 2.

V. Empirical Results

In Table 2, we report the values of lambdas estimated from the modified HP filter from Eq. 6 for 93 (25) countries' annual (quarterly) real income, consumption and investment series. Three observations deserve highlighting. First, across countries and series for both annual and quarterly datasets there is an important level difference in the smoothing parameters. Second, for quarterly data the estimated cross country and series range for λ is 229-4898. This range is not too dissimilar from Agenor et al (1998)'s range of 380-5100. However, their study was limited to 12 countries. Third, for the annual data the range is 11-6566. Fourth, relatively few countries fall close to the conventionally used λ=100 for annual and λ=1600 for quarterly series. Given the level differences in λs, it is important to establish the extent to which these impact the empirical moments of series; a task we turn to next.

To do so, first we obtain (for 93 countries) the first order autoregressive coefficients, standard errors, and the unconditional correlation of relevant series using the two filtering methods. For the first filtering method, detrending is based on using the modified HP filter with endogenously determined λs per country per series. For the second method, the conventional wisdom is applied by fixing λ=100 (1600) for annual (quarterly) frequency for all series and countries. Each set contains data on real income, real consumption and real investment. Second we use the detrended series to obtain the AR1 coefficient and unconditional correlations and carry out coefficient equality tests (see Paternoster et al. (1998)). Although we carry out multiple tests, we only report the results of Fisher Z-test for correlation.

In Tables 3 and 4 we present a summary of our results by country based on their income brackets. The former table is devoted to feature individual series and while the latter Table presents differences in cross correlations. We discuss each in turn.

In terms of the individual detrended series there are three noticeable observations in Table 3. First, we find that on average net differences of standard deviations of detrended components of same series but from the two methods (where λ is first endogenous, then exogenous) are positive across countries and frequencies; i.e. detrended series using endogenous λs are more volatile. This result is in sync with the observation of Marcet and Ravn (2003) that too much variability is assigned to the permanent component when the cyclical component is extracted using the conventional HP filter approach. Second, the AR1 coefficients based on detrended series obtained from conventional method of fixing λs tends to give results that are biased downwards. Third, a statistical comparison (only Z-tests for coefficient equality are reported) of AR1 coefficients of same series but originating from the two methods reveal that coefficients are not too different from one another across country groups and data frequencies. Therefore, in term of levels of persistence[8] of our detrended macroeconomic series the choice of the λ appears immaterial - a result also found in our simulations.

Next, we turn to comparison of unconditional correlation in Table 4 from the two sets of data. For this purpose we apply the correlation-equality Fisher Z test of Bundick (1975)[9]. There are three important noteworthy findings. First, the point estimates of annual correlation coefficients between the cyclical components (extracted by modified HP filter) of the income-consumption and income-investment pairs are marginally higher as evidenced by the positive averages (across countries) of net of method-wise correlation coefficients. However, the opposite is true for quarterly correlations where the averages of net of method-wise correlation coefficients are negative. Second, although point estimate difference between pair-wise correlations emanating from our datasets is small, some of these differences appear to be statistically strong. Indeed, for a bigger set of countries within each country group the differences are empirically valid. This result is stronger for annual rather than quarterly correlation coefficients where about 1/3[rd] of countries in each income group reveal statistically different pair-wise correlations.

Thus, an important lesson to draw from our study is that the choice of λ is relevant. Thus, for business cycle research it is worthwhile to examine results using detrended series from endogenously determined λs.

VI. Concluding Remarks

As the use of business cycle knowledge becomes commonplace, a basic question on the choice of smoothing parameter for detrending macro series arises. In a simulation study we find that modified HP filter of McDermott (1997) performs better than Hodrick and Prescott (1997) filter in producing the generated permanent (cyclical) component under various definitions of permanent and cyclical components relevant to macroeconomic time series. We find the same result for the time aggregated cases as well.

In an empirical assessment, where we estimated λs for three core macroeconomic series of 93 countries with annual data and 25 countries with quarterly data using modified HP filter approach, we find that smoothing parameters differs across countries and frequency of data substantially. We do not find statistical differences in the AR1 coefficients of cyclical series either extracted using the modified HP filter or the traditional one that relies on fixing λs. A similar pattern is observed for pair-wise correlations of macroeconomic series generated from the two detrending methods and quarterly data. However, we find that the method of detrending tends to make more of a difference for annual series and multivariate analysis; though in this paper the multivariate analysis is restricted to unconditional correlation coefficients.

[8] We also experimented with the autoregressive coefficients up to order 5 and our conclusions do not change.
[9] See Yu and Dunn (1982).

References:

Aaland, David, (2005), Detrending Time-aggregated data, *Economic Letters*, 89, 287-93.

Agenor, P. R., McDermott, C. J. and Prasad, S. S. (1999), Macroeconomic Fluctuations in Developing Countries: Some Stylized Facts, *International Monetary Fund Working Paper*, 35.

Aguiar, M. and G. Gopinath (2007), "Emerging Market Business Cycle: The Trend is the Cycle," Journal of Political Economy, Volume 115, pp 69-102

Backus, D. K. and Kehoe, P. J. (1992), International Evidence on the Historical Properties of Business Cycles, *American Economic Review*, 82, 864-88.

Baxter, M. and King, R. G. (1999), Measuring Business Cycles: Approximate Band-Pass Filters For Economic Time Series, *Review of Economics and Statistics*, 81, 575–93.

Bundick, L. E. (1975), Comparison of Tests of Equality of Dependent Correlation Coefficients, Ph.D. dissertation, University of California, Los Angles, USA.

Burns, A. F. and Mitchell, W. C. (1946), Measuring Business Cycles: New York, National Bureau of Economic Research

Cooley, T. F. and Ohanian, L. E. (1991), The Cyclical Behavior of Prices, *Journal of Monetary Economics,* 28, 25-60.

Correia, I., Neves, J. C. and Rebelo, S. (1992), Business Cycles in Portugal: Theory and Evidence, in J. Amaral, D. Lucena, and A. Mello (eds.) The Portuguese Economy Towards 1992, Springer, 1-64.

Guay, A. and St-Amant, P.(2005), Do the Hodrick-Prescott and Baxter-King Filters Provide a Good Approximation of Business Cycles, Annals of Economics and Statistics, No.77,pp 133-155.

Hodrick, R. J. and Prescott, E. C. (1997), Postwar U.S. Business Cycles: An Empirical Investigation, *Journal of Money, Credit and Banking*, 29, 1-16.

King, R. G. and Rebelo, S. T. (1993), Low Frequency Filtering and Real Business Cycles, *Journal of Economic Dynamics and Control*, 17, 1-2, 207-231.

McDermott, C. J.(1997), An Automatic Method for Choosing the Smoothing Parameter in the HP Filter," unpublished, International Monetary Fund (June 1997).

Marcet, A. and Ravn, M. O. (2003), The HP-Filter in Cross Countries Comparisons, CEPR Working Paper No. 4244.

Paternoster, R., Brame, R., Mazerolle, P., and Piquero, A. (1998), Using the Correct Statistical Test for the Equality of Regression Coefficients, *Criminology*, 36, 4, 711-898.

Pedersen, T. M. (2001), The Hodrick-Prescott Filter, the Slutzky Effect, and the Distortionary Effect of Filters, *Journal of Economic Dynamics and Control*, 25, 1081-1101.

Ravn, M. O. and Uhlig, H. (2002), On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations, *The Review of Economics and Statistics*, 84, 2, 371-376.

Silverman, B. W. (1984), A Fast and Efficient Cross- Validation Method for Smoothing Parameter Choice in Spline Regression, *Journal of the American Statistical Association*, 79, 387, 584-589.

Stock, J. H., and M. W. Watson (2003), Understanding Changes in International Business Cycle Dynamics, Working Paper No. w9859 (July), NBER, Cambridge

Yu, M. C. and Dunn, O. J. (1982), Robust Tests for the Equality of two Correlation Coefficients: A Monte Carlo Study, *Educational and Psychological Measurement*, 42, 4, 987-1004.

Watson, M.W.(1986), Univariate Detrending Methods with Stochastic Trends, Journal of monetary economics, No. 18, pp 49-75

Appendix

Tables 1

Simulation Results of Performance Comparison of modified HP filter and HP filter

Scenario	Standard deviation ratio ($\sigma_\varepsilon / \sigma_\xi$)	AR coefficient		Percent of times when modified HP filter outperforms HP filter							
		First ($\emptyset_1$)	Second ($\emptyset_2$)	(Generated as) Quarterly		Time Aggregated (Annual)					
						Systematically		By Summing		By Averaging	
				Linear trend	Non-linear trend	Linear trend	Non-linear trend	Linear trend	Non-linear trend	Linear trend	Non-linear trend
1	10	0.9	0.01	100	100	100	100	100	100	100	100
2	10	1.2	-0.25	100	100	100	100	100	100	100	100
3	10	1.2	-0.4	100	100	100	100	100	100	100	100
4	10	1.2	-0.55	100	100	100	100	100	100	100	100
5	10	1.2	-0.75	100	100	100	100	100	100	100	100
6	5	0.9	0.01	100	100	100	100	100	100	100	100
7	5	1.2	-0.25	100	100	100	100	100	100	100	100
8	5	1.2	-0.4	100	100	100	100	100	100	100	100
9	5	1.2	-0.55	100	100	100	100	100	100	100	100
10	5	1.2	-0.75	100	100	100	100	100	100	100	100
11	1	0.9	0.01	100	100	100	100	100	100	100	100
12	1	1.2	-0.25	90	100	100	100	100	100	100	100
13	1	1.2	-0.4	100	100	100	100	100	100	100	100
14	1	1.2	-0.55	100	100	100	100	100	100	100	100
15	1	1.2	-0.75	100	100	100	100	100	100	100	100
16	0.5	0.9	0.01	90	100	100	100	100	100	100	100
17	0.5	1.2	-0.25	69	100	100	100	100	100	100	100
18	0.5	1.2	-0.4	100	100	100	100	100	100	100	100
19	0.5	1.2	-0.55	100	100	100	100	100	100	100	100
20	0.5	1.2	-0.75	100	100	100	100	100	100	100	100
21	0.01	0.9	0.01	100	100	100	100	100	100	100	100
22	0.01	1.2	-0.25	100	100	100	100	100	100	100	100
23	0.01	1.2	-0.4	100	100	100	100	100	100	100	100
24	0.01	1.2	-0.55	100	100	100	100	100	100	100	100
25	0.01	1.2	-0.75	100	100	100	100	100	100	100	100
26	10	0.8	0	100	100	100	100	100	100	100	100
27	5	0.8	0	100	100	100	100	100	100	100	100
28	1	0.8	0	100	100	100	100	100	100	100	100
29	0.5	0.8	0	100	100	100	100	100	100	100	100
30	0.01	0.8	0	100	100	100	100	100	100	100	100

Table 2

Smoothing Parameter based upon Modified HP Filter for Annual and Quarterly Data Series

Income Group	Country	Frequency	Time Period		Real GDP λ	Real Consumption λ	Real Investment λ
			Start	End			
High Income	Australia	Annual	1959	2010	363	278	412
		Quarterly	Q1-1959	Q4-2010	1398	3306	1051
	Austria	Annual	1964	2010	177	249	183
		Quarterly	Q1-1964	Q4-2010	921	2665	2074
	Bahrain	Annual	1975	2009	948	133	260
	Belgium	Annual	1953	2010	207	223	515
		Quarterly	Q1-1980	Q4-2010	669	885	816
	Canada	Annual	1950	2010	540	1280	860
		Quarterly	Q1-1957	Q4-2010	867	1054	506
	Denmark	Annual	1966	2010	203	620	798
		Quarterly	Q1-1977	Q4-2010	654	637	452
	Finland	Annual	1960	2010	504	279	566
		Quarterly	Q1-1970	Q4-2010	421	1885	571
	France	Annual	1959	2010	188	222	287
		Quarterly	Q1-1970	Q4-2010	476	818	283
	Germany	Annual	1960	2010	192	215	382
		Quarterly	Q1-1960	Q4-2010	1035	1040	489
	Greece	Annual	1958	2010	351	614	1482
	Hungary	Annual	1970	2010	90	59	97
	Iceland	Annual	1960	2010	593	745	666
	Ireland	Annual	1950	2010	91	411	140
	Israel	Annual	1981	2010	1079	1526	109
		Quarterly	Q1-1980	Q4-2010	2802	3216	892
	Italy	Annual	1960	2010	222	235	310
		Quarterly	Q1-1980	Q4-2010	602	863	627
	Japan	Annual	1955	2010	347	404	290
		Quarterly	Q1-1957	Q4-2010	1092	2633	630
	Korea	Annual	1953	2010	317	443	482
		Quarterly	Q1-1960	Q4-2010	2357	3576	1623
	Luxembourg	Annual	1985	2010	136	98	213
	Malta	Annual	1954	2007	101	91	674
	Netherlands	Annual	1980	2010	112	102	156
		Quarterly	Q1-1977	Q4-2010	414	602	568
	New Zealand	Annual	1954	2010	462	584	951
		Quarterly	Q2-1987	Q4-2010	356	416	311
	Norway	Annual	1966	2010	173	675	525
		Quarterly	Q1-1966	Q4-2010	1365	830	1146
	Poland	Annual	1981	2010	1493	501	1632
	Portugal	Annual	1977	2010	225	1059	442
	Qatar	Annual	1980	2010	152	87	305
	Singapore	Annual	1960	2009	393	205	420
	Spain	Annual	1956	2010	345	484	227
		Quarterly	Q1-1970	Q4-2010	1527	3346	1491
	Sweden	Annual	1950	2010	320	402	589
		Quarterly	Q1-1980	Q4:2010	656	834	284
	Switzerland	Annual	1950	2010	335	250	461
		Quarterly	Q1-1970	Q4-2010	503	1605	334
	Trin & Tobago	Annual	1966	2001	28	142	177
	U.K.	Annual	1950	2010	479	343	946
		Quarterly	Q1-1957	Q4-2010	1040	1229	1013
	U.S.	Annual	1950	2009	491	332	1754
		Quarterly	Q1-1957	Q1-2010	941	841	488
Upper Middle Income	Anguilla	Annual	1984	2009	210	4592	484
	Antigua	Annual	1977	2010	294	495	577
	Argentina	Annual	1989	2010	52	166	168
	Chile	Annual	1974	2010	601	342	302
	Costa Rica	Annual	1961	2010	560	343	653
	Dominican Rep.	Annual	1962	2010	232	385	705
	Ecuador	Annual	1965	2010	199	408	220
	Grenada	Annual	1975	2010	188	214	190
	Iran	Annual	1966	2007	178	736	243
	Jamaica	Annual	1960	2009	130	219	205
	Jordan	Annual	1976	2007	40	49	683
	Malaysia	Annual	1970	2010	193	1687	606
		Quarterly	Q1-1991	Q4-2010	674	592	463
	Mauritius	Annual	1953	2010	984	964	1613
	Mexico	Annual	1978	2010	592	589	5656
		Quarterly	Q1-1981	Q4-2010	2671	2461	760

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	Country	Frequency	Time Period		Real GDP λ	Real Consumption λ	Real Investment λ
			Start	End			
Upper Middle Income	Montserrat	Annual	1977	2009	74	379	511
	Panama	Annual	1950	2009	336	517	517
	Peru	Annual	1989	2010	96	118	40
		Quarterly	Q3-1989	Q4-2010	1753	4898	704
	South Africa	Annual	1950	2010	299	722	289
		Quarterly	Q1-1960	Q4-2010	1316	1127	402
	St Lucia	Annual	1977	2010	179	1483	952
	St Vincent/Grens	Annual	1975	2010	321	6566	1392
	Thailand	Annual	1950	2010	217	350	496
	Tunisia	Annual	1961	2009	220	323	324
	Uruguay	Annual	1975	2010	184	185	184
	Venezuela	Annual	1977	2010	1624	390	451
Lower Middle Income	Belize	Annual	1979	2008	298	161	894
	Bhutan	Annual	1980	2009	566	1026	166
	Bolivia	Annual	1984	2010	621	290	306
	Cameroon	Annual	1969	2006	11	73	56
	Egypt	Annual	1982	2009	367	273	390
	El Salvador	Annual	1990	2010	139	544	161
	Fiji	Annual	1968	2005	115	108	121
	Guatemala	Annual	1951	2010	98	220	399
	Honduras	Annual	1950	2010	512	252	599
	India	Annual	1960	2010	479	671	401
	Indonesia	Annual	1978	2010	147	288	78
	Lesotho	Annual	1980	2008	303	295	193
	Mongolia	Annual	1980	2005	51	168	185
	Morocco	Annual	1964	2009	413	198	365
	Nigeria	Annual	1969	2003	75	217	173
	Pakistan	Annual	1960	2010	104	240	481
	Paraguay	Annual	1962	2010	165	141	475
	Philippines	Annual	1958	2010	217	335	463
		Quarterly	Q1-1981	Q4-2010	229	621	329
	Senegal	Annual	1960	2009	690	1254	5162
	Sri Lanka	Annual	1950	2010	307	313	1478
	Swaziland	Annual	1977	2006	143	344	1174
	Vietnam	Annual	1990	2009	151	138	67
	Zambia	Annual	1972	2008	319	363	335
Lower Income	Bangladesh	Annual	1973	2010	144	303	369
	Benin	Annual	1970	2006	651	239	519
	Burundi	Annual	1970	2010	86	253	149
	Haiti	Annual	1966	2007	211	381	285
	Kenya	Annual	1967	2010	411	374	295
	Madagascar	Annual	1980	2010	452	527	456
	Myanmar	Annual	1976	2003	51	77	76
	Nepal	Annual	1975	2010	247	458	977
	Niger	Annual	1986	2009	165	340	157
	Rwanda	Annual	1968	2010	720	611	410
	Sierra Leone	Annual	1971	2008	76	635	230
	Togo	Annual	1970	2004	1079	493	217
	Uganda	Annual	1983	2008	162	2565	420
	Zimbabwe	Annual	1980	2004	97	311	97

Table 3

Net AR(1) Coefficients

Country Group→	High Income			Upper Middle Income			Lower Middle Income			Lower Income		
Series[1]	*Y*	*C*	*I*	*Y*	*C*	*I*	*Y*	*C*	*I*	*Y*	*C*	*I*
Annual Data[2]												
Number of Countries	32			24			23			14		
Average of $(\beta_i^e - \beta^f)$[3]	0.08	0.09	0.07	0.06	0.10	0.07	0.03	0.08	0.06	0.05	0.11	0.08
Average of $(\sigma_i^e - \sigma^f)$[?]	0.35	0.38	1.41	0.90	1.43	2.33	0.33	0.78	1.57	0.32	0.85	1.41
Countries *not* passing Z-test at 10% for H^0 : $\beta_i^e - \beta^f = 0$	0	0	0	0	0	0	1	0	0	0	0	0
Quarterly Data[4]												
Number of Countries	20			4			1			0		
Average of $(\beta_i^e - \beta^f)$	-0.05	-0.03	-0.08	0.00	0.00	-0.10	-0.15	-0.10	-0.21	-	-	-
Average of $(\sigma_i^e - \sigma^f)$	-0.16	-0.06	-0.91	-.04	-.06	-1.26	-0.96	-0.26	-2.82			
Countries *not* passing Z-test at 10% for H^0: $\beta_i^e - \beta^f = 0$[5]	1	0	6	0	0	1	1	0	1	-	-	-

Notes: 1. Y, C and I denote detrended income, consumption and investment series. 2. Annual frequency predetermined λ^f =100. 3. The average of the net difference in the AR1 coefficients where λ^e and λ^{fix} denote endogenous and fixed values of the smoothing parameter. Quarterly frequency λ^f = 1600. 4. The average of the net difference of the standard deviation (%) of detrended series 5. AR1 coefficient equality tests.

Table 4

Net Unconditional Correlations

Country Group→	High Income		Upper Middle Income		Lower Middle Income		Lower Income	
Pairs[1]	Y-C	Y-I	Y-C	Y-I	Y-C	Y-I	Y-C	Y-I
Annual Data[2]								
Number of Countries	32		24		23		14	
Average of $(\rho_i^e - \rho^f)$[3]	0.005	0.011	0.013	0.016	0.011	0.018	0.006	0.004
Countries *not* passing Z-test at 10% for H^0: $\rho_i^e - \rho^f = 0$	10	8	6	4	7	9	4	2
Quarterly Data[4]								
Number of Countries	20		4		1		0	
Average of $(\rho_i^e - \rho^f)$	-0.021	-0.036	-0.010	-0.008	-0.049	-0.12	-	-
Countries *not* passing Z-test at 10% for H^0 $\rho_i^e - \rho^f = 0$[5]	3	2	0	2	0	0	-	-

Notes: 1. Y-C and Y-I denote unconditional correlations of individually detrended income-consumption and income-investment pairs. 2. Annual frequency predetermined λ^f =100. 3. The average of net of the correlation coefficients $(\rho_i^e - \rho^f)$ where the correlation coefficient ϱ^e: and ϱ^f denote are obtained from endogenous and fixed values of the smoothing parameter separately. 4. Quarterly frequency λ^f = 1600. 5. Correlation equality tests.

Figure 1
Scatter Plot of Means and Standard Deviations of (real PPP) GDP