Can Price Discrimination be Bad for Firms and Good for All Consumers? A Theoretical Analysis of Cross-Market Price Constraints with Entry and Product Differentiation

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Can Price Discrimination be Bad for Firms and Good for All Consumers? A Theoretical Analysis of Cross-Market Price Constraints with Entry and Product Differentiation

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Abstract

The article examines a differentiated-products duopoly model where the firms make entry decisions to two markets and then choose prices. The effects of product differentiation and entry costs are analyzed in two games: with and without price discrimination between the markets. Allowing price discrimination encourages more entry and tends to reduce prices and profits and to increase consumer welfare in both markets. The model suggests that firms might be better off if they agree not to price discriminate between different markets. It also suggests that when the market is not a natural monopoly, regulators should consider the effects of universal service requirements on entry before adopting them, because entry might be discouraged by such requirements, leading to less competitive markets.

1. Introduction

Many firms serve more than one market, either because they sell more than one product, or because they operate in more than one geographic location (or both). While in most cases the firm is free to choose different prices in its different markets, sometimes it is unable or unwilling to price discriminate between its markets. There are several potential reasons why the firm might be unable to price discriminate between markets. One reason is regulation: in telecommunications, railroad transportation and postal services, for example, the government often imposes a universal service requirement that includes a condition that the price in the rural market cannot exceed that in the urban market (Anton et al., 2002).

A second source of cross-market price constraints is anti-dumping provisions in international trade, which involve a comparison of prices across countries (see Prusa, 1994). A third reason why firms may not be able to price discriminate is arbitrage. This is particularly relevant when the various markets represent different geographic locations rather than different products. If the price difference is high enough, consumers may purchase in a market other than their geographic location. Alternatively, other firms may take advantage of the arbitrage opportunity, purchasing in the cheap market and selling in the more expensive one. Competition between the arbitraging firms will then drive the higher price down until it gets to the lower price (plus transportation costs), and the producer will also have to lower its price in the expensive market in order to sell.

Finally, the firms may themselves act to tie their hands and eliminate their ability to price discriminate between markets if they benefit from doing so (the model indeed suggests they do). For example, if national advertisement for a Big Mac promises to sell it for $2.99, McDonald's stores in various locations must all choose this price; the firm can thus self-impose a constraint not to price discriminate among different locations by advertising in national channels. Firms can also commit not to price discriminate by including most-favored-customer clauses in their contracts. If a firm offers such clauses to all its customers, it cannot price discriminate at all because all its customers will be entitled to get the lowest price.

How does the existence of cross-market price constraints affect the equilibrium? A few articles address this question. Leontief (1940) was the first to develop the theory of multi-market monopoly. More recently, Armstrong and Vickers (1991) showed that consumers prefer uniform pricing when regulation caps the monopoly's average revenue. More closely related to the current article are the studies that look on the effects of cross-market price constraints in oligopoly. DeGraba (1987), for example, analyzes the effects of most-favored-customer clauses in the sales contracts of a national firm on its competition with local firms. Armstrong and Vickers (1993) analyze a model in which an incumbent firm faces entry in one of its two markets, and show that banning price discrimination tends to encourage more entry and to reduce prices. Anton et al. (2002) analyze a model in which two firms who serve an urban market bid for entry to a rural market (the winner is the firm that is willing to accept a lower subsidy), under the constraint that price in the rural market cannot exceed the equilibrium price in the urban market.[1]

The current article also analyzes a duopoly with cross-market price constraints, but it introduces several important changes from previous models. It allows both firms to enter both markets, and entry to both markets is simultaneous, and is not subsidized by a regulatory agency. The article also analyzes the effects of product differentiation and entry costs on equilibrium entry and pricing. In addition, the article compares the equilibrium with and without price discrimination, and reaches the opposite conclusion to that of Armstrong and Vickers (1993): banning price discrimination in the model presented here actually tends to discourage entry and to increase equilibrium prices. The idea that price discrimination can intensify competition and lower prices in imperfect competition appeared also in Corts (1998). In Corts' model, however, this is the result of asymmetries in the demand functions of the two consumer groups, while here this result comes from the effect that allowing price discrimination has on entry decisions (Corts does not consider entry).

This comparison between the equilibria with and without the ability of firms to price discriminate is of great importance for both governmental regulators and for the firms themselves. Regulators are interested in this question because they want to know how requiring firms to charge the same price in urban and rural areas affects prices and consumer welfare in the different markets. Firms are interested in this question because they can affect their own ability to price discriminate between markets, as was discussed above.

[1] Additional related theoretical contributions include Bulow et al. (1985), Lal and Matutes (1989), Bernheim and Whinston (1990) and Phillips and Mason (1996).

While a monopoly is better off having fewer constraints, and therefore better off being able to price discriminate, with oligopoly firms are sometimes better off being constrained because their rivals are also constrained and because of the strategic effects of the constraints. This turns out to be the case here: firms are better off not being able to price discriminate. Interestingly, while the constraint imposed is on pricing, firms prefer the game without price discrimination not because of its effect on prices but because of the equilibrium entry decisions. In contrast, consumers in both markets prefer the game with price discrimination. This implies that if the market is not a natural monopoly, the common wisdom that suggests that imposing equal prices in two markets (e.g. rural and urban) benefits at least one of them is not necessarily correct once we account for the effects of this restriction on entry. This conclusion has to be taken cautiously, however, especially when the markets are not symmetric; the last section discusses this point in more detail.

2. The model

Two firms that sell differentiated products compete in prices, and competition takes place potentially over two markets, denoted by A and B. There is no substitutability between the two markets, so the firms compete only in the markets to which both entered. These two markets may be an urban area and a rural area, or day hours and night hours, or two distinct geographic locations and so on. The extent of product differentiation between the firms is the same in both markets. The game unfolds in two stages: in the first stage, each firm decides to which markets to enter: 0 (none), A, B, or AB (both). In the second stage, the firms choose prices. I first assume that each firm can choose only one price, that is, it cannot price discriminate between the markets to which it entered in the first stage. I later relax this assumption and compare the results with and without the assumption.

Each market entered entails a fixed cost of entry, F, and the marginal cost of production is zero. The inverse demand in each market takes the form:[2]

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[2] The two markets in the model are symmetric. It could be an interesting exercise to examine asymmetric markets as well, but in this model the analysis turns out to be too complex to be traceable if we add parameters to capture the asymmetry between the markets (e.g. in the fixed cost of entry, the demand function, and the marginal cost). Since

$$p_i^s = 10 - q_i^s - dq_j^s \qquad i = 1, 2;\ j = 2, 1;\ s = A, B.$$

The parameter $d \in [0, 1)$ measures the extent of product differentiation: when d approaches one, an increase in q_j or in q_i have almost the same effect on p_i, meaning that the two firms sell very close substitutes. When d is zero, q_j has no effect on p_i, meaning that the goods sold by the two firms are not substitutes at all. Intermediate values of d represent an intermediate extent of product differentiation; notice, however, that a *higher* value of d corresponds to *less* product differentiation.[3]

A nice characteristic of this inverse demand function is that it applies to both a monopoly and a duopoly in the market. It is immediate to see that if firm i is a monopoly in market s, the resulting demand is $q_i^s = 10 - p_i$. If both firms enter a market s, combining the inverse demand functions and solving for quantities in terms of prices yield the demand functions:

$$q_i^s = \max\{[10(1-d) + dp_j - p_i]/(1-d^2), 0\} \quad i = 1, 2;\ j = 2, 1;\ s = A, B.$$

3. The equilibria in the second stage

To find the subgame-perfect Nash equilibrium of the game, I proceed by solving the game backwards. There are a few cases to analyze in the second stage, depending on the entry decisions made in the first stage:

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the main results do not depend on a knife-edge case, however, they should be qualitatively similar also with asymmetric markets as long as the asymmetry is not too big.

[3] The demand function is a specific form of the linear function $p_i^s = \alpha - \beta q_i^s - dq_j^s$. In order to simplify the analysis and the presentation of the results, I wanted to minimize the number of varying parameters, so I substituted $\alpha = 10$ and $\beta = 1$. The results of the model are qualitatively similar with other linear demand specifications (i.e. different values of α and β) because (1) the value of α changes the profits in the market in a similar way to a change in the fixed cost F; and (2) the value of β changes the profits in the market and the extent of product differentiation, but these two effects are already captured by letting F and d vary. Using a general demand function without assuming a specific functional form is not possible because we need specific values of the profits following each pair of entry decisions in order to find the equilibrium in the first stage (i.e. the optimal entry decisions).

Case 1: Each firm operates at most in one market, and no market is served by both firms

This case occurs when one of the following occurs: (1) both firms do not enter at all; (2) one firm enters one market and the other firm does not enter at all; or (3) each firm enters a different market. Since each active firm serves only one market and it does not face competition in this market, each active firm chooses the monopoly price, $p^m = 5$ (m for "monopoly"). The quantity sold in each active market is the monopolistic quantity, $q^m = 5$, and the firm's profits are $\pi^m = 25 - F$.

Case 2: One firm is a monopoly in both markets

Since the markets are symmetric, the optimal price is again $p^m = 5$, resulting in quantity of 5 in each market and in total profits of $2\pi^m = 50 - 2F$ for the active firm.

Case 3: Both firms enter either the same market or both markets

Proposition 1: The equilibrium in Case 3 is characterized by $p_1 = p_2 = 10(1 - d)/(2 - d) \equiv p^c$, and $q_1^A = q_2^A = q_1^B = q_2^B = 10/(2 - d)(1 + d) \equiv q^c$.

Profits per firm are $\pi_1 = \pi_2 = 200(1 - d)/(2 - d)^2(1 + d) \equiv \pi^c$ if each firm enters both markets, and $\pi^c/2 = 100(1 - d)/(2 - d)^2(1 + d)$ if each firm enters only one market (the same one).

Proof: Combining the solutions to the profit maximization problems of the two firms yields these results immediately and is omitted to conserve space.

Case 4: One firm enters both markets and the other firm enters only one market

For notational convenience, let us assume that firm 1 enters both markets and firm 2 enters market A only.

Proposition 2: There is no equilibrium in which the firm that enters both markets finds it optimal to sell eventually only in one market (so in any equilibrium of Case 4, $q_1^A > 0$).

Proof: see Appendix.

Proposition 3: When the goods are very close substitutes ($d > 0.8263$), there is no pure-strategy equilibrium in the game, unless we assume that a firm that enters a market must choose a price that results in a strictly positive quantity in that market. When $d \leq 0.8263$ (or when $d > 0.8263$ but we make the above assumption), equilibrium prices, quantities, and profits are given by:

$p_1^* = 10(-3d^2 - d + 4)/(8 - 5d^2)$,

$p_2^* = 10(d^3 - 3d^2 - 2d + 4)/(8 - 5d^2)$,

$q_1^A = 10(d^4 + 2d^3 - 4d^2 - 3d + 4)/(5d^4 - 13d^2 + 8)$,

$q_1^B = 10(-2d^2 + d + 4)/(8 - 5d^2)$,

$q_2^A = 10(d^3 - 3d^2 - 2d + 4)/(5d^4 - 13d^2 + 8)$,

$\pi_1 = (q_1^A + q_1^B)p_1^* - 2F \equiv \pi^b$ (b for "big" firm), and

$\pi_2 = q_2^A p_2^* - F \equiv \pi^s$ (s for "small" firm).

Proof: see Appendix.

Since there is no pure-strategy equilibrium in Case 4 when $d > 0.8263$, we cannot attach payoffs to the firms when the entry decisions are (AB, A) or similar ones and we cannot find the equilibrium in the two-stage game. There are two ways to proceed. One is to assume that $d \leq 0.8263$, implying that the goods of the two firms are not very close substitutes. Alternatively, we can assume that once a firm declares that it enters a certain market, it must choose a price that results in a strictly positive quantity in this market. A possible justification for imposing such restriction (in addition to the technical reason) is that the firm harms its reputation (with respect to both competitors and customers) when it enters a market but then chooses such high prices that it does not sell at all.

Comparing the prices that result from the various entry decisions and analyzing them numerically show that for all $d \in (0, 1)$ we have the following relationship: $p^m = 5 > p_1^* > p_2^* > p^c$, where p_1^* and p_2^* are the prices of the big and small firm, respectively (assume that firm 1 chooses AB and firm 2 chooses A). The intuition for this result is as follows: the derivative of firm 1's profits in its monopolistic market with respect to p_1 when evaluated at $p_1 = 5$ is zero (since p^m maximizes profits in a monopolistic market), while in the shared market (B) it is negative (since the optimal price against p_2 is smaller than p^m), so the big firm chooses a price p_1^* that is smaller than p^m. The reason that $p_1^* > p_2^*$ is that the big firm has a cross-market consideration that the small firm does not have. When the big firm reduces its price in market A, it must also reduce it in market B, and since its price is already below p^m, this reduces its profits from market B. As a result, the big firm is less willing to cut its price than the small firm, and p_1^*

$> p_2^*$. Finally, to explain why $p_2^* > p^c$, suppose that we have firm 1 in both markets and firm 2 in market A and now firm 2 enters also market B. Firm 1 no longer protects market B by choosing a high price, because this market is not monopolistic anymore, so it reduces its price. This implies $p_1^* > p_c$. The intuition why also $p_2^* > p_c$ is that when $d > 0$, prices in duopolistic markets are strategic complements: the optimal price for each firm is increasing in the price of its rival (this can be seen easily in the systems of equations in the proofs of Proposition 1 and Proposition 3). Consequently, the price reduction of firm 1 causes firm 2 to reduce its price once firm 2 enters market B, resulting in $p_c < p_2^*$.

4. The equilibria in the two-stage game

The payoff table for the various entry decisions in the first stage is depicted in Table 1.

Table 1: Payoff matrix when price discrimination is not allowed

Firm 2 Firm 1	0	A	B	AB
0	0, 0	0, 25 – F	0, 25 – F	0, 50 – 2F
A	25 – F, 0	$0.5\pi^c$, $0.5\pi^c$	25 – F, 25 – F	π^s, π^b
B	25 – F, 0	25 – F, 25 – F	$0.5\pi^c$, $0.5\pi^c$	π^s, π^b
AB	50 – 2F, 0	π^b, π^s	π^b, π^s	π^c, π^c

The first column and row depict the markets entered by firms 1 and 2, respectively. The first payoff in each cell is that of firm 1.

The notation in what follows is similar to that in the table: for example, (A, AB) means that firm 1 enters market A and firm 2 enters both markets. Clearly, the table is symmetric. I assume $F < 25$ (the monopoly profit net of the fixed cost of entry is positive). Since the monopoly profit is the maximum that can be earned in each market, it follows immediately that $\pi^c \leq 50 - 2F$, $\pi^b \leq 50 - 2F$, and $\pi^s \leq 25 - F$ (the inequalities hold with equality only when $d = 0$).

We can see that the candidates for an equilibrium in the two-stage game are of four types:

(1) Each firm enters a different market: (A, B) or (B, A). I call this type the "niches equilibrium."
(2) One firm enters both markets and the other firm enters just one market: (A, AB); (B, AB); (AB, A); (AB, B). I denote this type as a "big & small equilibrium" (the big firm is the one that enters both markets).
(3) Both firms enter both markets (AB, AB). This is the "competitive equilibrium."
(4) One firm enters both markets and the other firm remains inactive, the "winner-takes-all equilibrium": (AB, 0) or (0, AB).

4.1. Conditions for each type of equilibrium to occur

(1) Niches equilibrium, e.g. (A, B): three conditions have to be simultaneously met for (1) to be a Nash equilibrium: (1a) $25 - F \geq 0$, which is satisfied by the assumption $F < 25$; (1b) $25 - F \geq 0.5\pi^c$, which is satisfied since $\pi^c \leq 50 - 2F$; and (1c) $25 - F \geq \pi^b$.
Notice that $\pi^b|_{d = 0} = 50 - 2F$, because a firm that enters both markets when the competitor has a totally differentiated product earns monopoly profits in each of the markets. In addition, π^b is decreasing in d, so we can expect condition (1c) to be satisfied for values of d above a certain threshold.[4] The numerical analysis shows that this is indeed the case, where the critical value of d above which (A, B) is a Nash equilibrium is decreasing in F.

(2) Big & small equilibrium, e.g. (AB, A): all the following conditions have to be met simultaneously for (2) to be a Nash equilibrium: (2a) $\pi^b \geq 0$; (2b) $\pi^b \geq 0.5\pi^c$; (2c) $\pi^b \geq 25 - F$; (2d) $\pi^s \geq \pi^c$; and (2e) $\pi^s \geq 0$. Conditions (2a), (2b) and (2c) ensure that the firm that enters both markets cannot gain from deviation; conditions (2d) and (2e) ensure that the other firm has no profitable deviation. Whenever (2c) is true, (2a) and (2b) are also true, so conditions (2c), (2d) and (2e) are necessary and sufficient for (2) to be an equilibrium.

[4] $\partial\pi^b/\partial d = \{(p_2 - 10)(1 - d^2) + 2d[(10 - p_1) - d(10 - p_2)]\}/(1 - d^2)$, where firm 1 is the "big" firm. The denominator is positive; the numerator is equal to $2d(10 - p_1) + (p_2 - 10)(1 + d^2)$. The first term is positive while the second is negative, and $1 + d^2 > 2d$ (since $1 + d^2 - 2d = (1 - d)^2 > 0$). Since $p_2^* < p_1^*$ (this can be easily seen from Proposition 3), the negative term is bigger in absolute value than the positive term, implying that $\partial\pi^b/\partial d < 0$.

(3) Competitive equilibrium (AB, AB): the conditions for (3) to be an equilibrium are: (3a) $\pi^c \geq 0$; and (3b) $\pi^c \geq \pi^s$.

(4) Winner-takes-all equilibrium, e.g. (AB, 0): the active firm cannot do any better, but the inactive firm has no profitable deviation only if: (4a) $\pi^s \leq 0$; and (4b) $\pi^c \leq 0$.
Table 2 describes how the above conditions translate into conditions about the value of d given different value of F.

Table 2 specifies how conditions (1c), (2c), (2d), (2e), (3a), (3b), (4a) and (4b) translate into conditions about the value of d, for different F-values.[5]

4.2. The equilibria for different values of d and F

Table 3 and Figure 1 present the equilibria for different values of F and d, under the assumption that a firm that enters a market must choose a price that results in a strictly positive quantity. The alternative assumption, that $d \leq 0.8263$, gives the exact same equilibria, except that the equilibria for values of d above 0.8263 are irrelevant anymore.

[5] Some of the conditions are just the counterparts of others, but presenting them all makes it easier to see the conditions for each type of equilibrium.

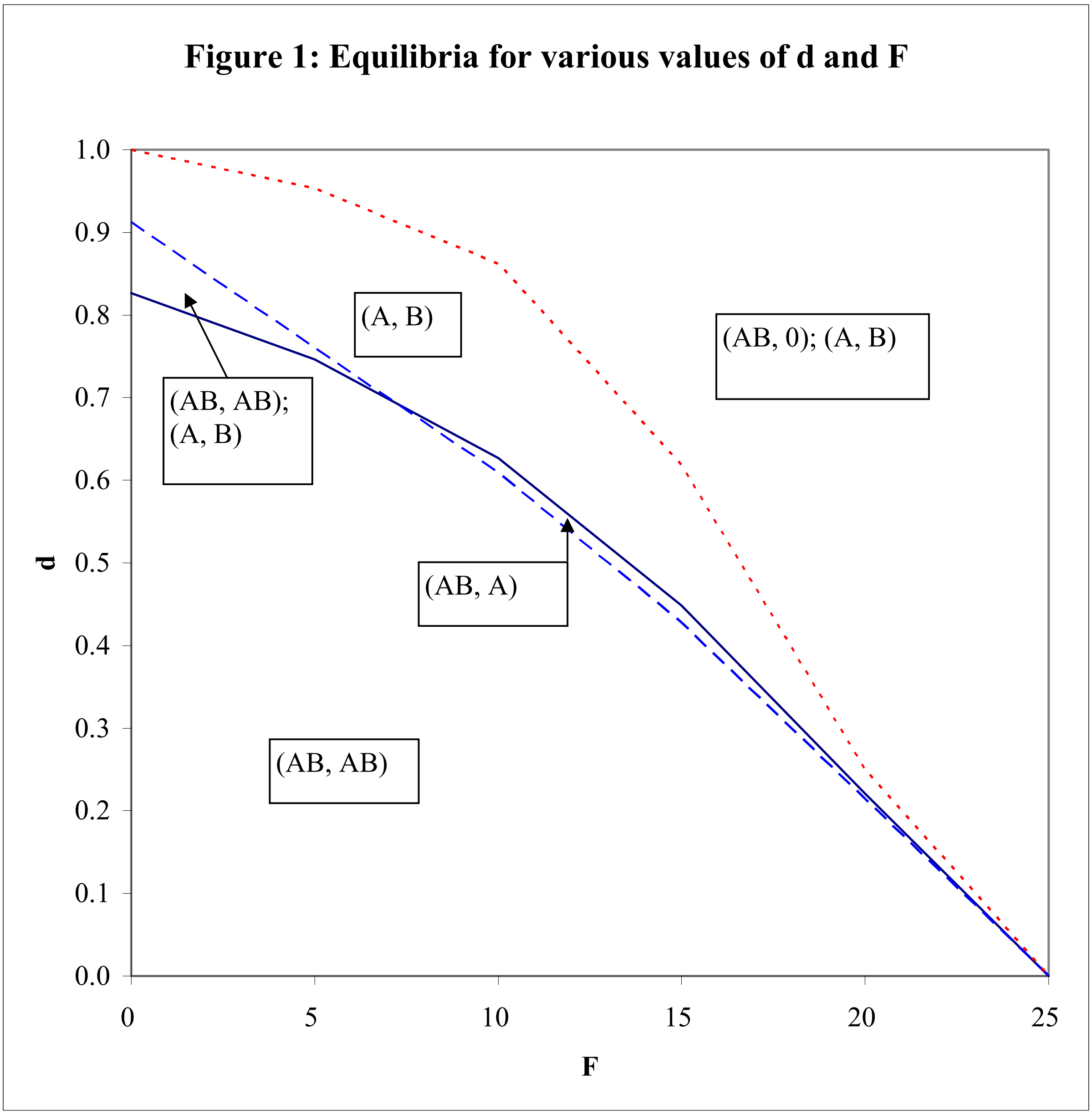
Table 2: Conditions for different types of equilibria

	Niches equilibrium E.g. (A, B)	Big & small equilibrium E.g. (AB, A)			Competitive equilibrium (AB, AB)		Winner-takes-all equilibrium E.g. (AB, 0)	
F	**1c**:$\pi^b \leq 25-F$	**2c**:$\pi^b \geq 25-F$	**2d**:$\pi^s \geq \pi^c$	**2e**:$\pi^s \geq 0$	**3a**:$\pi^c \geq 0$	**3b**:$\pi^s \leq \pi^c$	**4a**:$\pi^s \leq 0$	**4b**:$\pi^c \leq 0$
0	d ≥ 0.8263	d ≤ 0.8263	d ≥ 0.9121	d < 1	d < 1	d ≤ 0.9121	N/A	N/A
1	d ≥ 0.8127	d ≤ 0.8127	d ≥ 0.8756	d ≤ 0.9925	d ≤ 0.9794	d ≤ 0.8756	d ≥ 0.9925	d ≥ 0.9794
5	d ≥ 0.7463	d ≤ 0.7463	d ≥ 0.7597	d ≤ 0.9533	d ≤ 0.8824	d ≤ 0.7597	d ≥ 0.9533	d ≥ 0.8824
10	d ≥ 0.6267	d ≤ 0.6267	d ≥ 0.6091	d ≤ 0.8624	d ≤ 0.7175	d ≤ 0.6091	d ≥ 0.8624	d ≥ 0.7175
15	d ≥ 0.4487	d ≤ 0.4487	d ≥ 0.4276	d ≤ 0.6178	d ≤ 0.4906	d ≤ 0.4276	d ≥ 0.6178	d ≥ 0.4906
20	d ≥ 0.2209	d ≤ 0.2209	d ≥ 0.2143	d ≤ 0.2495	d ≤ 0.2291	d ≤ 0.2143	d ≥ 0.2495	d ≥ 0.2291
24	d ≥ 0.0410	d ≤ 0.0410	d ≥ 0.0408	d ≤ 0.0417	d ≤ 0.0412	d ≤ 0.0408	d ≥ 0.0417	d ≥ 0.0412
25	d ≥ 0	d = 0	d ≥ 0	d = 0	d = 0	d = 0	d ≥ 0	d ≥ 0

Table 3: Equilibria for different values of d and F

F	Niches equilibrium E.g. (A, B)	Big & small equilibrium E.g. (AB, A)	Competitive equilibrium (AB, AB)	Winner-takes-all equilibrium E.g. (AB, 0)
0	[0.8263, 1)	None	[0, 0.9121]	None
1	[0.8127, 1)	None	[0, 0.8756]	[0.9925, 1)
5	[0.7463, 1)	None	[0, 0.7597]	[0.9533, 1)
10	[0.6267, 1)	[0.6091, 0.6267]	[0, 0.6091]	[0.8624, 1)
15	[0.4487, 1)	[0.4276, 0.4487]	[0, 0.4276]	[0.6178, 1)
20	[0.2209, 1)	[0.2143, 0.2209]	[0, 0.2143]	[0.2495, 1)
24	[0.0410, 1)	[0.0408, 0.0410]	[0, 0.0408]	[0.0417, 1)
25	[0, 1)	0	0	[0, 1)

The cells in the table present the values of d for which there exists an equilibrium of the type written in the first row, for the value of F that appears in the left column.



As the table and figure show, several types of equilibria exist, depending on the values of d and F. For example, when F = 5, values of d below 0.7463 yield the competitive equilibrium; there are both niches and competitive equilibria when 0.7463 ≤ d ≤ 0.7597; when 0.7597 < d <

0.9533, the resulting equilibrium is of the niches type; and when $0.9533 \leq d < 1$ both the niches and the winner-takes-all equilibria exist. The main results regarding the entry decisions in equilibrium can be summarized as follows:

When d is small, competition is soft due to significant product differentiation, so each firm enters both markets in equilibrium. When the products become close substitutes (i.e. d increases) and competition becomes more intense, each firm finds its own niche (by entering a different market) in order to avoid head-to-head competition. When F is large or the products are close substitutes, however, in addition to the niches equilibrium there is a winner-takes-all equilibrium. The higher F is, the more differentiated the products can be and still sustain the winner-takes-all equilibrium. In addition, the higher is the fixed cost of entry to a market, the more likely are the firms to find different niches and not to compete: the competitive equilibrium (AB, AB) is replaced by the niches equilibrium, (A, B) or (B, A), beginning at lower values of d.

It is interesting to notice that for some values of d and F there is more than one type of equilibrium for the game. In addition, even though the game is completely symmetric, there are values of F and d for which there are asymmetric equilibria with different entry decisions and different profits for the two firms. The big & small equilibrium, however, occurs only for a very narrow range of d-values.

Another interesting point is that while prices and profits are weakly decreasing in d given a pair of entry decisions, the equilibrium profits and prices are not. For example, consider the case $F = 5$. As we increase d from zero, equilibrium prices and profits decrease. When we reach d-values in the range of [0.7463, 0.7597], there are both the competitive and the niches equilibria, so equilibrium prices and profits are not unique. When we increase d further, however, the niches equilibrium becomes the unique equilibrium, and results in higher prices and profits than those in the competitive equilibrium for values of d above a certain threshold. In the niches equilibrium each firm enjoys monopoly profits in one market, and therefore prices and profits do not change as d increases further.[6] For example, with $F = 5$, the niches equilibrium

[6] When $d \geq 0.9533$ the winner-takes-all equilibrium also becomes possible in addition to the niches equilibrium. The profits and prices of the niches equilibrium remain the same and the niches equilibrium remains an equilibrium of the game, though it is not the unique equilibrium anymore. Under the winner-takes-all equilibrium prices are still the

takes over the competitive equilibrium around d = 0.76. Profits in the niches equilibrium for d = 0.76 are similar to those in the competitive equilibrium at d = 0.49, and therefore equilibrium profits for $d \in [0.76, 1)$ are higher than those for $d \in [0.49, 0.76]$. Prices in the niches equilibrium are the monopoly prices and therefore are higher than those of the competitive equilibrium for all d > 0. The analysis for other values of F is similar.

5. Price discrimination is bad for firms and good for consumers

So far we did not allow the firms to price discriminate between the markets. How does the equilibrium change if we allow price discrimination? This question is of great importance to both governmental regulators and the firms. To answer this question, let us analyze the same model but now allow the firms to price discriminate between the two markets. Consider the equilibrium in the second stage. When neither firm enters more than one market, the ability to charge different prices is irrelevant and therefore the equilibrium is unchanged ($p^m = 5$ is the price in all active markets). When one firms enters both markets and the other firm remains inactive, the active firm chooses the same price ($p^m = 5$) in both markets because the markets are symmetric, so the equilibrium is the same as before. When both firms enter markets A and B, they both choose p^c in both markets because of the symmetry between the markets and the equilibrium is as before.

The only case in which this game differs from the previous one is when one firm enters both markets and the other enters only one market. Now, the big firm charges $p^m = 5$ in the market in which it is a monopoly, and both firms charge p^c in the other market. Notice that now the condition $d \leq 0.8263$ is irrelevant, since the large firm never wants to sell zero quantity in a market it entered once price discrimination is allowed. The payoff matrix for different entry decisions is presented in Table 4.

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monopoly prices and profits are the monopoly profits, except that one firm makes the entire profits rather than the two firms splitting them as in the niches equilibrium.

Table 4: Payoff matrix when price discrimination is allowed

Firm 2 Firm 1	**0**	**A**	**B**	**AB**
0	0, 0	0, 25 – F	0, 25 – F	0, 50 – 2F
A	25 – F, 0	$0.5\pi^c$, $0.5\pi^c$	25 – F, 25 – F	$0.5\pi^c$, 25 – F + $0.5\pi^c$
B	25 – F, 0	25 – F, 25 – F	$0.5\pi^c$, $0.5\pi^c$	$0.5\pi^c$, 25 – F + $0.5\pi^c$
AB	50 – 2F, 0	25 – F + $0.5\pi^c$, $0.5\pi^c$	25 – F + $0.5\pi^c$, $0.5\pi^c$	π^c, π^c

The first column and row depict the markets entered by firms 1 and 2, respectively. The first payoff in each cell is that of firm 1.

It is easy to see that whenever $\pi^c > 0$, the unique equilibrium is the competitive equilibrium, (AB, AB). When $\pi^c < 0$, the equilibria have a monopoly in both markets, either as a niches equilibrium or as a winner-takes-all equilibrium. When π^c is exactly equal to zero, all four types of equilibria are possible (niches, big & small, competitive, and winner-takes-all). Notice that now the only condition for the competitive equilibrium to occur is $\pi^c \geq 0$ (condition (3a) in the previous section) whereas previously we also needed condition (3b), $\pi^s \leq \pi^c$. Moreover, the set of d-values that satisfy condition (3b) is a proper subset of the set of d-values that satisfy condition (3a), for a given F. This implies that the competitive equilibrium is more likely to occur (it occurs under a broader range of parameter values) when price discrimination is allowed.

Except for the knife-edge case of $\pi^c = 0$, the big & small equilibrium cannot occur now. The intuition why this equilibrium is virtually eliminated is the following (assume that firm 1 enters markets A and B, and firm 2 enters only A): without price discrimination, firm 1 chooses a price that is above p^c because this price also applies to market B where firm 1 would like the price to be as close to $p^m > p^c$ as possible. Firm 2 therefore benefits from a softer competitor in market A[7]; by entering market B, firm 2 eliminates the incentive of firm 1 to price higher than p^c, and therefore firm 2 faces tougher competition and reduced profits in market A. This incentive of firm 2 to avoid entry to market B is eliminated once price discrimination is allowed, because now the price in market A is p^c anyway. Consequently, if it is profitable for firm 2 to be in market A when firm 1 enters both A and B, then firm 2 is better off entering both A and B, because this doubles its profits.

For simplicity, since the big & small equilibrium is virtually eliminated now, and since even in the previous game it occurred only for a very narrow region of parameter values, let us concentrate on the other forms of equilibria. When $\pi^c < 0$ the equilibrium is either the niches equilibrium or the winner-takes-all equilibrium, so prices in both markets are p^m, each market is served by only one firm, total quantity in each market is $q^m = 5$, and the sum of the profits of the two firms is $50 - 2F$ (these characteristics of the niches and winner-takes-all equilibria also hold when price discrimination is not allowed). Whether these profits go to only one firm or to both firms equally is not of much interest for our purposes now, so it does not matter much whether the equilibrium is of the niches type or the winner-takes-all type. In the previous game, however, there was a large region in which only the niches equilibrium exists, and whenever the winner-takes-all equilibrium exists so does the niches equilibrium, so it is more convenient to take the niches equilibrium when both equilibria exist. Consequently, we limit attention to the symmetric equilibria, of the competitive and the niches type.

Let us denote by d_p the critical d-value above which the competitive equilibrium does not exist anymore in the current game (p for price-discrimination), and similarly, this critical value in the previous game as d_n (n for no-price-discrimination). Since the d-values that satisfy condition (3b) are a proper subset of the values that satisfy condition (3a), it follows that $d_p > d_n$. For example, with $F = 5$ we have $d_p = 0.8824$ and $d_n = 0.7597$, and with $F = 15$ we have $d_p = 0.4906$ and $d_n = 0.4276$. Since the competitive equilibrium occurs when $d \in [0, d_g]$ ($g = p$ or n depending on the game), this means that the competitive equilibrium is more likely to occur when price discrimination is allowed.

[7] The result that a firm becomes a softer competitor when it enters two markets and its rival enters only one is analogous to the results in the models of Armstrong and Vickers (1993) and Anton et al. (2002).

Conditional on the entry decisions being the same, in the symmetric equilibria the firms choose the same prices and obtain the same profits in both games (with and without price discrimination). The only thing that matters to the firms is therefore the type of equilibria. For low values of d and F the firms prefer the competitive equilibrium to the niches equilibrium, because it enables them to extract profits in two markets rather than one, competition is not intense, and fixed costs of entry are low. When F or d increase, however, the firms start to derive more profits in the niches equilibrium than in the competitive equilibrium because the former saves costs of entry to an additional market and avoids head-to-head competition. The question is, for the values of d and F for which price discrimination results in the competitive equilibrium and no-price-discrimination results in the niches equilibrium, what do the firms prefer?

The answer is that the firms prefer the niches equilibrium in this case, meaning that they prefer the game without price discrimination. The reason is as follows: the values of d and F for which the equilibria in the two games differ are those for which $0 \leq \pi^c < \pi^s$ (this can be seen from conditions (3a) and (3b)). Since $\pi^s < 25 - F$, it follows that $\pi^c < 25 - F$ in these cases; therefore, the firms prefer the niches equilibrium (with profits of $25 - F$ for each firm) to the competitive equilibrium.

Table 5: Equilibrium prices, quantities and profits in both games (for F = 5)

	No price discrimination			Price Discrimination allowed		
d	Price	Total quantity	Profit (each firm)	Price	Total quantity	Profit (each firm)
0.00	5.00	10.00	40.00	5.00	10.00	40.00
0.04	4.90	9.81	38.06	4.90	9.81	38.06
0.08	4.79	9.65	36.22	4.79	9.65	36.22
0.12	4.68	9.50	34.46	4.68	9.50	34.46
0.16	4.57	9.37	32.78	4.57	9.37	32.78
0.20	4.44	9.26	31.15	4.44	9.26	31.15
0.24	4.32	9.16	29.57	4.32	9.16	29.57
0.28	4.19	9.08	28.03	4.19	9.08	28.03
0.32	4.05	9.02	26.50	4.05	9.02	26.50
0.36	3.90	8.97	24.99	3.90	8.97	24.99
0.40	3.75	8.93	23.48	3.75	8.93	23.48
0.44	3.59	8.90	21.96	3.59	8.90	21.96
0.48	3.42	8.89	20.41	3.42	8.89	20.41
0.52	3.24	8.89	18.83	3.24	8.89	18.83
0.56	3.06	8.90	17.20	3.06	8.90	17.20
0.60	2.86	8.93	15.51	2.86	8.93	15.51
0.64	2.65	8.97	13.74	2.65	8.97	13.74
0.68	2.42	9.02	11.86	2.42	9.02	11.86
0.72	2.19	9.08	9.87	2.19	9.08	9.87
0.76	5.00	5.00	20.00	1.94	9.16	7.74
0.80	5.00	5.00	20.00	1.67	9.26	5.43
0.84	5.00	5.00	20.00	1.38	9.37	2.92
0.88	5.00	5.00	20.00	1.07	9.50	0.18
0.92	5.00	5.00	20.00	5.00	5.00	20.00
0.96	5.00	5.00	20.00	5.00	5.00	20.00
0.9999	5.00	5.00	20.00	5.00	5.00	20.00

Table 5 presents the equilibrium price, profits per firm, and total quantity (the sum over the two firms) in each market when F = 5.[8] We can see that for low values of d the two games yield the same equilibrium (the competitive equilibrium). For some higher values of d, however, price discrimination results in the competitive equilibrium while no-price-discrimination yields the niches equilibrium; as argued above, for these d-values, profits (and prices) are higher without price discrimination. Finally, when d is very large, the niches equilibrium takes over also with price discrimination and once again the two games have the same equilibrium.

Interestingly, while the constraint relaxed in the second game is on pricing, firms prefer the game without price discrimination not because of its effect on prices but because of the equilibrium entry decisions.[9] Notice that this preference was derived when comparing the symmetric equilibria in both games, and in these equilibria the prices in both games are the same. The preference for the game without price discrimination is therefore only a result of the firms preferring the niches equilibrium to the competitive equilibrium, that is, a preference for the entry decisions that this game yields. Allowing price discrimination encourages more entry, and thus acts to increase competition and reduce prices, benefiting consumers and hurting the firms.

While the firms prefer not to be able to price discriminate, consumers prefer to allow price discrimination because they prefer the competitive equilibrium to the niches equilibrium, since the former results in lower prices and higher total quantity. Notice that the consumers in both markets are better off with price discrimination. The implications of this result for regulation policy and firm strategy are discussed in the next section.

6. Discussion and conclusion

The result that firms are worse off and all consumers better off when price discrimination between the markets is possible (whenever the equilibria with and without price discrimination differ) is of importance for both firms and governmental regulators. This result implies that firms

[8] When F changes, profits change and the equilibrium for different d-values may change; conditional on the type of equilibrium, however, prices and quantities do not depend on the value of F.

[9] Depending on the values of d and F, this preference can be either weak or strict. The same applies to the discussion in the rest of this and the next paragraphs.

may be better off if all of them commit to avoid price discrimination between markets. Since the actual interaction between firms takes place over multiple periods, it is easy to construct models of repeated interaction where the firms initially agree not to price discriminate between the markets, and if one firm ever deviates and charges different prices in its markets, other firms retaliate by price discriminating in all future periods, leading to less favorable equilibria for all firms. With discount factors above a certain threshold, the equilibrium in the repeated game will then be that firms do not price discriminate. Interestingly, in the model presented here the ability to deviate will not even be profitable, because once firms believe that other firms are not price discriminating, the equilibria are virtually always symmetric, and in these equilibria no firm gains from price discriminating.

A simple way to commit not to price discriminate is to advertise prices in broad channels that apply to more than one market, such as national TV or magazines. If Gap advertises in national magazines that a certain jacket costs $99.90, this commits the company to charge the same price in every location.

Many firms in various industries, such as fast-food restaurants and other chain restaurants, retail stores, and automobile manufacturers, often charge the same prices in various locations that differ significantly in terms of both their demand functions and costs (e.g. rent in an urban area is much more expensive than in a rural area). Since there is no regulatory constraint to charge the same price in different locations in these cases, the prevalence of uniform pricing is puzzling. While there are probably alternative explanations for this puzzle, the model provides a possible explanation why firms prefer not to price discriminate.

Another implication of the model is for regulation. Regulators often require firms in certain industries (such as railroad transportation, telecommunications and postal services) to charge the same price in urban and rural areas (this is known as universal service requirements), while the firm would rather charge a higher price in the rural areas. The presumption is that this requirement benefits the rural market that would otherwise have to pay a higher price. If the market in question is not a natural monopoly, the model suggests that the regulator should consider the effects of disallowing price discrimination on entry (and consequently competition) before adopting a certain policy, and that cross-market price constraints may discourage entry. However, it remains a question for future research whether both consumer groups can benefit from allowing price discrimination when significant asymmetries exist between the markets (as is usually the case when universal service requirements are imposed). When the asymmetry in the markets is big, the firms would probably find it optimal to charge a much higher price in the rural market. While entry considerations could still play a role, they might be dominated by the asymmetry in the markets, and consequently disallowing price discrimination would benefit consumers in the rural market (though it will probably hurt consumers in the urban market). Until further research suggests to what extent the results here are robust to asymmetries between the markets, one should not infer from the model that universal service requirements hurt the rural consumers.

Appendix

Proof of Proposition 2: Clearly firm 1 sells a positive quantity in the market in which it is a monopoly, market B. To show that it also sells positive quantity in the other market, assume by contradiction that there is an equilibrium in which firm 1 finds it optimal not to sell in market A (i.e. firm 1 chooses p_1 high enough, denoted by p_{1h}, so that $10(1 - d) + dp_2 - p_{1h} \leq 0$). Then it must be the case that $p_{1h} = p^m = 5$, otherwise firm 1 can deviate and choose $p_1 = 5$, doing no worse in market A (since $q_1^A (p_{1h}) = 0$, the firm's profits from market A cannot be any lower), and strictly better in market B (this follows from strict concavity of profits from market B in p_1). To proceed, recall that $q_i^s = \max\{[10(1 - d) + dp_j - p_i]/(1 - d^2), 0\}$. Since $p_{1h} = 5$ and $q_1^A = 0$, it follows that $[10(1 - d) + dp_2 - 5] \leq 0$. The effect of decreasing p_1 (to something slightly less than 5) depends on whether this inequality binds or not. We therefore need to consider two cases:

<u>Case 4A</u>: If $10(1 - d) + dp_2 - 5 = 0$ (so $p_2 = 10 - 5/d$), then any increase in p_2 or a decrease in p_1 yields $q_1^A > 0$. However, an increase in p_1 or a decrease in p_2 leaves q_1^A unchanged ($q_1^A = 0$), so we are in a point of discontinuity of the first derivatives of q_1^A (and therefore of π_1) with respect to p_1 and p_2 and we have to be specific whether we consider the derivatives from the right or from the left. <u>For a decrease in p_1</u> the total profits of firm 1 in markets A and B are:

$$\pi_1 (p_1 \leq 5; p_2 = 10 - 5/d) = p_1[10(1 - d) + dp_2 - p_1]/(1 - d^2) + p_1(10 - p_1).$$

We then get $\partial\pi_1/\partial p_{1\,(\text{from the left})} (p_1 = 5) = [10(1 - d) + dp_2 - 2p_1]/(1 - d^2) + 10 - 2p_1 = -5/(1 - d^2)$, where the second equality follows from $p_1 = 5$ and $10(1 - d) + dp_2 - p_1 = 0$. Since $-5/(1 - d^2) < 0$, it follows that firm 1 can increase its profit by decreasing p_1, so this cannot be an equilibrium.

Case 4B: If $q_1^A = 0$ and Case 4A does not occur, it must be that $10(1 - d) + dp_2 - p_1 < 0$, which implies (using $p_1 = 5$, which still holds) $p_2 < 10 - 5/d < 5$. In this case, for a small change in p_2 (in either direction), firm 2 remains a monopoly in market A, and therefore for any $p_2 < 10 - 5/d$ we have $\pi_2 = p_2(10 - p_2)$ and $\partial\pi_2/\partial p_2 = 10 - 2p_2 > 0$. This means that firm 2 can increase its profits by increasing its price a little, so this cannot be an equilibrium either. It follows that no equilibrium with $q_1^A = 0$ exists (when firm 1 enters markets A and B, and firm 2 enters market A only). Q.E.D.

Proof of Proposition 3: From Proposition 2 it follows that in any equilibrium of Case 4, p_1 is chosen such that $q_1^A > 0$. Conditional on $q_1^A > 0$, firm 2's profits are given by: $\pi_2 = q_2^A p_2 - F = [10(1 - d) + dp_1 - p_2]p_2/(1 - d^2) - F$. Firm 1's profits are $\pi_1 = (q_1^A + q_1^B)p_1 - 2F = \{[10(1 - d) + dp_2 - p_1]/(1 - d^2) + 10 - p_1\}p_1 - 2F$. It is easy to verify that the second-order conditions are satisfied, and therefore the candidates to be equilibrium prices are given by the relevant first-order conditions. Solving them and substituting into the demand functions yield:

$p_1^* = 10(-3d^2 - d + 4)/(8 - 5d^2)$,

$p_2^* = 10(d^3 - 3d^2 - 2d + 4)/(8 - 5d^2)$,

$q_1^A = 10(d^4 + 2d^3 - 4d^2 - 3d + 4)/(5d^4 - 13d^2 + 8)$,

$q_1^B = 10(-2d^2 + d + 4)/(8 - 5d^2)$, and

$q_2^A = 10(d^3 - 3d^2 - 2d + 4)/(5d^4 - 13d^2 + 8)$.

So far we have just proved that if there is a pure-strategy equilibrium in Case 4, it must involve these prices, but not that these prices are in fact an equilibrium. The reason that solving the first-order conditions does not yet yield an equilibrium is that their derivation was based on the implicit assumption that $q_1^A > 0$; now we have to consider how incorporating the possibility of $q_1^A = 0$ changes the analysis. For p_1^* and p_2^* to constitute an equilibrium, we have to verify whether q_1^A that results from p_1^* and p_2^* is strictly positive, and consider whether charging a price that results in $q_1^A = 0$ is better than p_i^* for either firm.[10]

[10] Firm 1 can make $q_1^A = 0$ by charging a high price; Firm 2 can make $q_1^A = 0$ by charging a low price in some cases (when such a low price is still above zero). Notice that Proposition 2 suggested that $q_1^A = 0$ cannot be part of an equilibrium, but it does not imply that charging price that results in $q_1^A = 0$ cannot be a profitable deviation for one of the firms.

Step 1: Do the candidate equilibrium prices result in $q_1^A > 0$?

We obtained that $q_1^A = 10(d^4 + 2d^3 - 4d^2 - 3d + 4)/(5d^4 - 13d^2 + 8)$. The denominator is equal to $(8 - 5d^2)(1 - d^2)$ and is therefore strictly positive. The numerator is equal to $10[(1 - d^2)(-d^2 - 2d - 3) - d + 7]$. Notice that $d^2 + 2d + 3 < 6$, so the term in brackets is strictly positive and therefore $q_1^A > 0$.

Step 2: Is firm 2 better off charging a price that results in $q_1^A = 0$?

The derivation of p_2^* implies that it is optimal conditional on $q_2^A = [10(1 - d) + dp_1 - p_2]/(1 - d^2)$. In some cases, however, firm 2 may become a monopoly by charging p_2 low enough, and then its demand becomes $q_2^A = 10 - p_2$.[11] The derivative of q_2^A with respect to p_2 when both firms sell positive quantities is $-1/(1 - d^2)$ and when only firm 2 sells positive quantity we get $\partial q_2^A/\partial p_2 = -1$. In addition, q_2^A is continuous in p_2.[12] So when firm 2 contemplates the effect of reducing p_2, it either does so accurately, or it overestimates the increase in the quantity it will sell (this occurs when the reduction in p_2 results in firm 2 becoming a monopoly). Thus, firm 2 weakly overestimates the benefits from reducing its price. Therefore, understanding the true nature of its demand could only lead to a higher price of firm 2, but any price higher than p_2^* also results in $q_1^A > 0$, so firm 2 is not better off charging a price that leads to $q_1^A = 0$.

Step 3: Is firm 1 better off charging a price that results in $q_1^A = 0$?

The best firm 1 can do conditional on $q_1^A = 0$ is to charge the monopoly price $p^m = 5$, to obtain the maximum revenues in market B, since it has no revenues in market A anyway when

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[11] These cases occur when $[10(1 - d) + dp_2 - p_1] = 0$ can be obtained with $p_2 > 0$ given p_1^*, implying $[p_1^* - 10(1 - d)]/d > 0$. Substituting for p_1^* and rearranging shows that this happens when $-5d^3 + 2d^2 + 7d - 4 > 0$, which implies $d \in (0.6434, 1)$. So when $d < 0.6434$ firm 2 cannot be a monopoly, and when $d > 0.6434$ we have to show that firm 2 is better off charging p_2^* and being a duopoly than reducing its price so much that it becomes a monopoly.

[12] The only p_2 in which q_2^A may be discontinuous is the p_2 such that for prices above it firm 2 is not a monopoly and for prices below it firm 2 becomes a monopoly. This p_2 is given by $p_2 = [p_1 - 10(1 - d)]/d$, or equivalently, $p_1 = [10(1-d) + dp_2]$. Substituting this p_1 into $q_2^A = [10(1 - d) + dp_1 - p_2]/(1 - d^2)$ yields $q_2^A = 10 - p_2$, which shows that indeed q_2^A is continuous in p_2 (it has a kink but not a discontinuity in $p_2 = [p_1 - 10(1 - d)]/d$).

$q_1^A = 0$.[13] Firm 1's profits are then 25 – 2F. When firm 1 charges p_1^* its profit is given by $(q_1^A + q_1^B)p_1^* - 2F$. Substituting for q_1^A, q_1^B and p_1^* and solving shows that firm 1 wants to deviate from p_1^* to p^m whenever $d > 0.8263$. Therefore, for $d > 0.8263$, p_1^* and p_2^* do not constitute an equilibrium. But $p_1 = p^m$ and p_2^* do not constitute an equilibrium either, because p_2^* is no longer optimal against p^m. By proposition 2, there is no pure-strategy equilibrium with $q_1^A = 0$, but the only candidate equilibrium with $q_1^A > 0$ fails as well. Therefore, when $d > 0.8263$ there is no pure-strategy equilibrium in the game, and when $d \leq 0.8263$ the equilibrium is given by p_1^*, p_2^*, and the resulting quantities q_1^A, q_1^B and q_2^A (see above).[14] In addition, if we assume that a firm that enters a market must choose a price that results in a strictly positive quantity in that market, the problem of firm 1 deviating from p_1^* to prices that result in $q_1^A = 0$ is eliminated and the analysis then suggests that p_1^*, p_2^*, q_1^A, q_1^B and q_2^A are then the equilibrium prices and quantities. Q.E.D.

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[13] If at the monopoly price $p^m = 5$ we have $q_1^A(p_1 = p^m, p_2^*) > 0$ then p_1^* yields higher profits than p^m since p_1^* is the solution to the first-order condition and p^m is implicitly considered as an alternative. Therefore we need to consider explicitly the alternative of charging p^m only when $q_1^A(p_1 = p^m, p_2^*) = 0$.

[14] Analyzing mixed-strategy equilibria in this framework complicates the analysis significantly and in addition is of very little relevance. Actual firms do not randomize their prices. A common justification for mixed-strategy equilibrium is that a mixed strategy of firm 1 represents not randomization by firm 1 but rather the uncertainty of firm 2 with respect to the action that firm 1 takes. This justification, however, is irrelevant in the context discussed here: since consumers can observe both prices, it is unreasonable to assume that firms cannot observe each other's prices.

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