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In a recent article, Ederington (1979) examined the hedging performance of financial futures markets using a portfolio model derived from the hedging theories of Stein (1961) and Johnson (1960). His article concluded that GNMA futures were more effective than T-Bill futures in reducing price change risk. Moreover, in the short term, the performance of T-Bill futures in reducing risk was extremely poor. The purpose of this article is to determine whether these results are due to a misspecification of the model and to test whether the hedging effectiveness of the T-Bill futures market has changed after three years of trading. A portfolio model of hedging effectiveness is formulated to account for the constant yield price accumulation over time on Treasury bills as distinguished from price changes due to instantaneous changes in yield. We test the T-Bill futures market using the portfolio model and conclude that the market provides very good opportunities for hedging, provided that the spot position is comprised of Treasury bills deliverable against the futures contract.

THE THEORY OF HEDGING EFFECTIVENESS

While there is some disagreement as to the exact motivation of those who use the futures market for hedging,[1] we assume here that hedgers are solely interested in

[1]For a detailed discussion of the reasons for hedging, see Holbrook Working (1953).

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minimizing the risks due to interest rate fluctuations. The issue of the hedging effectiveness of Treasury-bill futures is not a trivial question. To have an economic justification, futures markets must be used either for price discovery or for hedging. Unlike most other commodities, Treasury bills have an enormous existing secondary market which may be used to determine implied forward interest rates, so it is doubtful that the T-Bill futures market serves any price discovery function. Thus, if Ederington's results were to hold, meaning that T-Bill futures are poor devices for hedging, then the entire economic justification for their existence would be called into question.

For Treasury bills, it is important to distinguish between interest rate changes and price changes since Treasury bills are discount instruments which do not have coupon payments and do not bear interest. Instead, an investor in bills earns a return based on the difference between the discounted price and the full face value redemption price paid by the Treasury at maturity or the market price if sold prior to maturity. As a result, if interest rates remain constant, the cash price of a T-Bill will increase because of the change in the remaining term to maturity. However, if futures interest rates remain constant, then the price of a futures contract will not change. Therefore, the hedger who owns Treasury bills will want to protect against instantaneous price fluctuations other than those caused by a change in the term to maturity.

We consider the T-Bill futures contract, which is listed on the International Money Market of the Chicago Mercantile Exchange. Contracts currently trade for delivery in March, June, September, and December, and trading terminates on the second business day following the 3-month Treasury-bill auction of the third week of the delivery month. Upon expiration, each contract calls for delivery of $1 million of 90-, 91-, or 92-day T-Bills.[2] It is important to note, however, that upon the expiration of each contract, there is an outstanding 6-month bill with 3 months remaining to maturity. These outstanding 6-month bills are also deliverable against futures contracts. The Treasury issues 52-week, 6-month, and 3-month bills. The 6-month bill auctioned 3 months prior to the 3-month bill is perfectly interchangeable with the 3-month bill. Since traders may arbitrage between the outstanding 6-month bill and the corresponding futures contract, it is this bill which is the appropriate one to use to test the hedging effectiveness of the market when hedging 0–3 months before the date of delivery.

When one wishes to hedge against interest rate fluctuations, the futures contract corresponding to the deliverable spot market bill is the best hedging instrument. For periods when a perfect arbitrage situation does not exist, one might consider the futures contract as a hedge against price movements in the proxy 52-week bill. At the end of the desired hedging period, the investor can reverse his position.

A MODEL OF HEDGING EFFECTIVENESS

Following Ederington (1979) the return on any commodity held in the cash market which is totally unhedged is

$$U = X_s(P_s^2 - P_s^1) \tag{1}$$

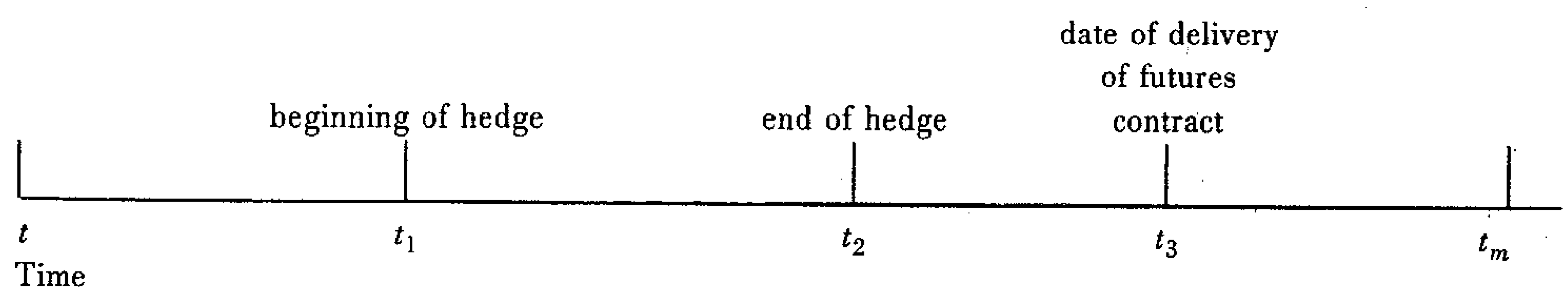
[2]For more information on the details of this market, see, for example, *Treasury Bill Futures: Opportunities in Interest Rates* which is available from the Chicago Mercantile Exchange. Also see Burger, Lang, and Rasche (1977).

where P_s^2 is the spot price at t_2, P_s^1 is the spot price at t_1, X_s is the quantity of commodity held (to be purchased).

If the holder of the commodity wishes to protect himself against unanticipated price changes from t_1 to t_2 by hedging a proportion of his spot position, then his return in both markets at the end of the hedging period is

$$R = X_s(P_s^2 - P_s^1) + X_f(P_f^2 - P_f^1) \tag{2}$$

where P_f^2 is the futures price at t_2, P_f^1 is the futures price at t_1, X_f is the quantity of futures sold (purchased). For simplification, we adopt the following notation: Time is assumed to flow from left to right; t represents time, t_1 is the beginning of the hedging period, t_2 is the end of hedge, t_3 is the date of delivery of the futures contract, t_m is the maturity of the deliverable security. From t_3 to t_m is 90 days. DM_1 is the period from t_1 to t_m. DM_2 is the period from t_2 to t_m.



In the case of Treasury bills, P_s^2 and P_f^2 must be redefined to separate out the factors which determine the randomness of Treasury-bill cash and futures prices.[3]

$$\begin{aligned}
P_s^2 &= [1 - (\mathrm{DM}_2 \times r_s^2/360)] \\
P_f^2 &= [1 - (90 \times r_f^2/360)] \\
P_s^1 &= [1 - (\mathrm{DM}_1 \times r_s^1/360)] \\
P_f^1 &= [1 - (90 \times r_f^1/360)]
\end{aligned}$$

where DM_1 is the days from t_1 to date of maturity of the T-Bill being hedged, DM_2 is the days from t_2 to date of maturity of the T-Bill being hedged, r_s^1 is the spot interest rate at t_1 of the bill being hedged, r_s^2 is the spot interest rate at t_2 of the bill being hedged, r_f^1 is the futures interest rate at t_1, r_f^2 is the futures interest rate at t_2.

Substituting into eq. (2) and rearranging terms:

$$\begin{aligned}
R &= X_s\{[1 - (\mathrm{DM}_2 \times r_s^2/360)] - [1 - (\mathrm{DM}_1 \times r_s^1/360)]\} \\
&\quad + X_f\{[1 - (90 \times r_f^2/360] - [1 - (90 \times r_f{}^1/360)]\} \\
&= X_s\left(\frac{\mathrm{DM}_1 \times r_s^1 - \mathrm{DM}_2 \times r_s^2}{360}\right) + X_f[(r_f^1 - r_f^2)/4] \qquad (3)
\end{aligned}$$

[3]Prices are normalized. A million dollar contract is represented as $1.00.

One of the factors determining the value of the above return is the change in the spot price due to holding the bill for the period of the hedge.[4] Removing this effect results in

$$H = X_s[\mathrm{DM}_2(r_s^1 - r_s^2)/360] + X_f[(r_f^1 - r_f^2)/4] \tag{4}$$

The return on a hedged position of Treasury bills is given by eq. (4). Traditional hedging theories hold that the cash market position X_s will equal the futures position X_f. According to the portfolio theory of hedging, the risk that the cash and futures price changes may not be equal must be considered in order to determine the optimal proportion of cash bills to be hedged.[5] Since the hedger has only an expectation of the values of P_s^2 and P_f^2, H is a random variable. The most commonly used measure of uncertainty, the variance of the hedged return, is a measure of the risk involved in hedging.

$$\mathrm{Var}(H) = (\mathrm{DM}_2/360)^2 \times X_s^2 \sigma_{r_s}^2 + X_f^2 \sigma_{rf}^2/16 + (\mathrm{DM}_2/720) X_f X_s \sigma_{r_s r_f} \tag{5}$$

where

$$\begin{aligned} \sigma_{r_s}^2 &= \mathrm{Var}(r_s^1 - r_s^2) \\ \sigma_{rf}^2 &= \mathrm{Var}(r_f^1 - r_f^2) \\ \sigma_{r_s r_f} &= \mathrm{Cov}(r_s^1 - r_s^2)(r_f^1 - r_f^2) \end{aligned}$$

Let $b = -X_f/X_s$ represent the proportion of the spot position to be hedged. Then

$$\mathrm{Var}(H) = X_s^2 [(\mathrm{DM}_2/360)^2 \sigma_{r_s}^2 + b^2 \sigma_{rf}^2/16 - (\mathrm{DM}_2/720) b\, \sigma_{r_s r_f})] \tag{6}$$

Unlike traditional hedging theories where $b = 1$, we are interested in determining the value of b, b^*, which will minimize the variance or risk.

$$\partial \mathrm{Var}(H)/\partial b = X_s^2 (b \sigma_{rf}^2/8 - \mathrm{DM}_2 \sigma_{r_s r_f}/720) \tag{7}$$

Setting the above equal to zero and solving for the optimal hedging proportion,

$$b^* = \mathrm{DM}_2 \sigma_{r_s r_f}/90 \sigma_{rf}^2 \tag{8}$$

The optimal hedged proportion b^* is a function of the term to maturity, the covariance between changes in spot rates and futures rates, and the variance of the change in futures rates. The greater the term to maturity remaining at the end of the holding period, and the greater the covariance between spot and futures rate changes, the greater will be the proportion of the spot position necessary to hedge.

[4]Constant yield price accumulation is equal to

$$[1 - (\mathrm{DM}_2 \times r_s^1/360)] - [1 - (\mathrm{DM}_1 \times r_s^1/360)]$$

[5]Unequal changes in cash and futures prices is referred to as basis risk. The basis is the difference between futures prices and cash prices. For a perfect hedge, the change in the basis should be zero. For an explanation of why the basis changes over time, see Working (1948, 1949). Among the factors influencing the basis for Treasury bills are borrowing costs and the term structure of interest rates. See Poole (1978) and Rendleman and Carabini (1979).

Term to maturity is positively related to the hedging proportion because an equal change in interest rates causes a greater change in the spot price than in the futures price if the term to maturity remaining on the spot bill is greater than 90 days.[6] The greater the variance of future rates the smaller the proportion to be hedged.

Following Ederington (1979) a measure of hedging effectiveness is the percent reduction in the variance of the return when the cash minimizing hedge is chosen

$$1 - \frac{\mathrm{Var}(H^*)}{\mathrm{Var}(U)} = \frac{\sigma^2{}_{r_s r_f}}{\sigma^2{}_{r_s}\sigma^2{}_{r_f}} = \rho^2 \tag{9}$$

where $\mathrm{Var}(H^*)$ is the minimum variance obtained by substituting b^* from eq. (8) into eq. (6) and $\mathrm{Var}(U) = (\mathrm{DM}_2/360)^2 \times X_s^2 \sigma_{r_s}^2$. The coefficient of determination, r^2, obtained from the following equation is an estimate of ρ^2:

$$r_f^2 - r_f^1 = \alpha + \beta(r_s^2 - r_s^1) + \mu \tag{10}$$

Ederington estimated ρ^2 from the equation:

$$P_f^2 - P_f^1 = \gamma + \alpha(P_s^2 - P_s^1) + \nu \tag{11}$$

We use interest rate changes as opposed to price changes to estimate hedging effectiveness to take into account the constant yield accumulation of the price of a Treasury bill. Observing price changes is acceptable if price quotations are used which exclude the effect of term of maturity.[7]

Ederington tested eq. (11), using 2-week and 4-week holding periods, for the months of trading from March 1976 to December 1977, using the constant 90-day Treasury bill for his cash market prices. He obtained unadjusted R^2 values which ranged from 0.27 to 0.14 in the biweekly hedging period. The effectiveness of the market, as estimated from his model, increased when a 4-week hedging period was used. R^2 values in this case ranged from 0.74 to 0.37. These findings represent a very poor hedging record and indicate that the relationship between cash and futures prices is quite loose and unstable. In contrast, his findings for other commodities produced values of R^2 consistently close to one.

Our model indicates that when testing the effectiveness of hedging Treasury bills, one must take into account the term to maturity on the deliverable bill and adjust for the constant yield price accumulation. Therefore, we use the deliverable 180-day bill when the hedging instrument is the nearby futures con-

[6] For example, for a 2-week hedge, the following would hold:

If $r_f^1 = 9.00$ percent, $r_s^1 = 9.05$ percent, $\mathrm{DM}_1 = 140$ and $\mathrm{DM}_2 = 126$, and $\sigma_{r_s r_f}/\sigma_{r_f}^2 = 1$ indicating a riskless hedge, then if both spot rates and futures rates rose by 10 basis points (0.10 percent), the spot market price would rise from \$964,806 to \$967,975 (million dollar security). However, if the spot interest rate had remained constant the price would have risen to \$968,325. The holder of the security loses \$350 as the result of an interest rate change in the spot market. During the same period, a short in the futures market of the same amount would result in a gain of \$250 as the futures price fell from \$977,500 to \$977,250. A proper hedge would be to short $350/250 = 1.4$ futures contracts, or $\mathrm{DM}_2/90 = 126/90 = 1.4$.

[7] In the case of using spot prices of 90-day Treasury bills, an adjustment needs to be made. Values of b^* for a 2-week hedge would have to be multiplied by $(76/90) = 0.84$ and values of b^* for a 4-week hedge would have to be multiplied by $(62/90) = 0.69$.

tract. When the hedging instrument is a futures contract 3–9 months in the future, we use the 52-week bill which has a maturity date closest to the maturity date of the bill deliverable against the futures contract.

RESULTS

The results of our specification of the hedging model are given in Tables I–III. They are obtained by regressing rates as specified in eq. (10). Table I shows the results for 22 months of trading from March 1976 to December 1977 and Table II contains the results for 34 months of trading from March 1976 to December 1978. When the deliverable bill is used instead of the constant maturity 3-month bill and adjustment is made for price accumulation, the results are altered substantially.

Values of R^2 are vastly improved in all hedging periods. However, some unexplained variance remains indicating that basis risk exists. As expected, the hedge with the nearby contract is the most effective, since in this period futures rates must converge to actual spot rates. The 4-week hedge involves less risk than the 2-week hedge. The longer hedge allows the market more time to smooth out short-run price fluctuations. Finally, the estimated values of β increase as more distant futures contracts are used as hedging instruments. This is consistent with the fact that the basis—the difference between the cash price and the futures price—consistently narrowed over time for each of the hedging periods we studied. Table II contains the results of testing hedging effectiveness for the period March 1976–December 1978. The R^2 values do not differ markedly from those obtained in testing hedging effectiveness in the first two years of trading

Table I
REGRESSION RESULTS FOR TREASURY BILLS: CASH MARKET DATA ARE DELIVERABLE BILLS (22 MONTHS OF TRADING)

Hedging Period	Hedging Instrument	Estimated β[a]	R^2
2-week rate changes	Nearby futures contract (n = 41)	0.955 (0.087)	0.755
	3–6-Months futures contract (n = 41)	1.166 (0.128)	0.679
	6–9-Months futures contract (n = 41)	1.284 (0.149)	0.654
4-week rate changes	Nearby futures contract (n = 21)	1.029 (0.086)	0.883
	3–6-Months futures contract (n = 21)	1.191 (0.114)	0.852
	6–9-Months futures contract (n = 21)	1.338 (0.140)	0.827

[a]Standard error in parentheses.

Table II
REGRESSION RESULTS FOR TREASURY BILLS: CASH MARKET DATA ARE *DELIVERABLE BILLS* (34 MONTHS OF TRADING)

Hedging Period	Hedging Instrument	Estimated β[a]	R^2
2-week rate changes	Nearby futures contract (n = 66)	0.991 (0.051)	0.855
	3–6-Months futures contract (n = 67)	1.118 (0.079)	0.754
	6–9-Months futures contract (n = 64)	1.107 (0.100)	0.664
4-week rate changes	Nearby futures contract (n = 33)	1.040 (0.065)	0.893
	3–6-Months futures contract (n = 33)	1.183 (0.086)	0.859
	6–9-Months futures contract (n = 31)	1.257 (0.114)	0.807

[a]Standard error in parentheses.

Table III
OPTIMAL HEDGING PROPORTIONS[a] (34 MONTHS OF TRADING)

Nearby Futures Contract			3-6-Months Futures Contract			6-9-Months Futures Contract		
DM_2	$\hat{b}^*$ 2-Week Hedge	$\hat{b}^*$ 4-Week Hedge	DM_2	$\hat{b}^*$ 2-Week Hedge	$\hat{b}^*$ 4-Week Hedge	DM_2	$\hat{b}^*$ 2-Week Hedge	$\hat{b}^*$ 4-Week Hedge
90	0.862	0.859	181	1.358	1.460	272	1.813	1.940
97	0.929	0.926	188	1.410	1.517	279	1.860	1.990
104	0.996	0.993	195	1.463	1.573	286	1.907	2.040
111	1.063	1.059	202	1.515	1.623	293	1.953	2.090
118	1.130	1.126	209	1.568	1.686	300	2.000	2.140
125	1.197	1.193	216	1.620	1.742	307	2.047	2.190
132	1.264	1.260	223	1.673	1.799	314	2.093	2.240
139	1.331	1.327	230	1.725	1.855	321	2.140	2.290
146	1.398	1.394	237	1.778	1.912	328	2.187	2.340
153	1.465	1.460	244	1.830	1.969	335	2.233	2.390
160	1.532	1.527	251	1.883	2.025	342	2.280	2.440
167	1.600	1.594	258	1.935	2.081	349	2.327	2.490
174	1.667	1.661	265	1.998	2.138	356	2.373	2.540

[a]$\hat{b}^* = DM_2/90 \times$ estimated $\sigma_{r_s r_f} / \sigma^2_{r_f}$.

(Table I), indicating that the hedging effectiveness of the market remained fairly constant over the entire period from March 1976 to December 1978.

The estimated values of b^* for selected values of DM_2 are given in Table III. Because we adjust for constant yield price accumulation, these values are not directly comparable to Ederington's results. Within each period, as the days until maturity increase, the hedging proportion increases to adjust for the greater effect which a change in interest rates has on spot prices. A perfect hedge, if there were no unexplained variance, would be approximately two-to-one for a 180-day cash bill with a 90-day futures contract ($DM_2/90$). Our results indicate that a risk minimizer should not be 100 percent hedged since there exists unexplained variance in the hedged position as indicated by the estimated R^2's which are less than one. The lower R^2 values associated with a delivery date in 3–6 months and in 6–9 months are reflected in the fact that the optimal hedging proportion decreases when crossing from one hedging period to another.

Ederington states that part of the reason for his poor results might be due to the fact that the Federal Reserve System uses the cash T-Bill market to conduct open market operations and that the T-Bill rate is closely related to the federal funds rate. This is usually true only for very short-term bills. Thus, to the extent that this may be relevant, it does not apply to his findings since he uses constant 3-month bill rates. Our findings indicate that Treasury-bill futures may be used as effective hedging instruments, but that an optimal hedge consists of a less than fully hedged position.

CONCLUSIONS

In this article a model of hedging effectiveness was developed which accounts for characteristics of the Treasury-bill futures market which are unique to that market. By studying interest rate changes rather than price changes, we avoided the problem of constant yield price accumulation over time. Our measurement of the optimal hedging proportion is also an improvement because it adjusts for the common practice of closing out futures contracts before the date of delivery.

We conclude that one way that rational, risk-averse hedgers may use the futures markets is by buying the Treasury bill which is deliverable against the nearby futures contract, or in the case of a more distant hedge, by purchasing the proxy 52-week bill. Treasury-bill futures may be used as hedging instruments if one takes into account the unexplained variance in the hedged position. However, no final judgment on the usefulness of T-Bill futures as a hedging instrument is made since they are still a relatively new financial instrument. Nonetheless, they seem to compare favorably with most other futures markets. Finally, Ederington's application of the Johnson and Stein portfolio theory of hedging to financial futures is an imaginative and valuable contribution to the measuring and understanding of the usefulness of futures contracts as hedging instruments.

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