Was there a "Greenspan conundrum" in the Euro area ?

Lamé, Gildas

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Gildas Lamé†

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Abstract

This paper implements an affine term structure model that accommodates "unspanned" macro risks for the Euro area, i.e. distinct from yield-curve risks. I use a Near-Cointegrated VAR-like approach to obtain a better estimation of the historical dynamics of the pricing factors, thus providing more accurate estimates of the term premium incorporated into the Eurozone's sovereign yield curve. I then look for notable episodes of the monetary cycle where long yields display a puzzling behavior vis-à-vis the short rate in contrast with the Expectation Hypothesis. The Euro-area bond market appears to have gone through its own "Greenspan conundrum". At least three "conundra" episodes can be singled out in the Eurozone between January 1999 and August 2008. The term premium substantially contributed to these odd phenomena.

JEL classification : C51; E43; E44; E47; E52; G12

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†Macroeconomic Research Division, INSEE-CREST, 15 boulevard Gabriel Péri 92245 Malakoff Cedex, FRANCE. Email address : gildas.lame@insee.fr

1 Introduction

In February 2005 in a speech before Congress, former Federal Reserve chairman Alan Greenspan noted that the 10Y treasury yield failed to increase significantly so far despite the 150-bp increase in the federal funds rate. This behavior was puzzling under the prevailing term structure theory called the "Expectation Hypothesis" as long rates should have also increased mechanically. While Greenspan mentioned several possible explanations for the phenomenon such as the global savings glut, the origin of this "conundrum" was left without any relevant answers at that moment. In a later monetary policy testimony in July 2005, Greenspan emphasized that yields can be divided into two components : the first one reflecting short-rate expectations and a second term a risk compensation. Greenspan suggested the prominent role of this second component in the relatively stable levels of long-term interest rates. Previous studies suggest this risk premium in the US is time-varying and substantial, thus complicating the transmission of monetary policy as it blurs the relationship between short-term interest rates controlled by central bankers and the long-term ones.

As the sovereign yield curve matters for businesses and households through the interest-rate channel and putting aside the current problems due to the sovereign debt crisis, one central question naturally follows : in parallel with the American conundrum, were there any periods before the crisis when long rates didn't seem to be responsive to rate hikes in the Eurozone ? That is, was there also a "Greenspan conundrum" in the Euro area ? If the answer is affirmative, was the term premium behind it ? These questions are deemed essential if one assumes the Expectation Hypothesis should hold. Affine term structure models represent one way to answer this question. Naturally, macroeconomic factors ought to play a significant role in the determination of short-rate expectations and risk premia. Therefore, a dynamic term structure model which includes not only the standard "level", "slope" and "curvature" factors but also macroeconomic factors is welcomed. Furthermore, Eurozone data favor a model which accomodates unspanned macro risks, i.e. risks that impact bond investment decisions separately from information about the shape of the yield curve.

In this paper, I implement a simple and parcimonious dynamic term structure model initially developed by Joslin & Al. (2010) based on a vector of pricing factors which includes the first three principal components of yields and two macroeconomic factors (an economic activity indicator and inflation) for the Euro area. Their model has the interesting feature of accomodating unspanned macro risks, feature that should be taken into account in a model for the Euro area as economic activity and inflation are not "spanned" by the yield curve. The usual estimation of such affine term structure model is done with a two-step procedure which consists in a first phase in the estimation of the historical dynamics of the pricing factors and then the risk-neutral dynamics while taking advantage of the cross-section of yields. I improve the first step by using an estimation technique inspired by Jardet & Al. (2009). By taking into account unit-root constraints, cointegration relationships among state variables and by minimizing the long-term forecast errors of the state variables, the implemented methodology provides better estimates of long-term expectations of the short rate and thus more accurate long-horizon term premium. Focusing on the 5Y maturity for the Eurozone, I find that the 5Y yield term premium has been hovering around 1% and represent on average over the period 21% of the 5Y bond yield. All in all, under the framework of the Expectation Hypothesis, the Euro area also went through its own "Greenspan conundrum" with the 5Y yield behaving counter-intuitively. I distinguish three notable "conundra" episodes from 1999 to 2008. Similar to past US analyses, two of them took place during the monetary policy tightenings decided by the ECB. The third one deserves particular attention as it took place at the same time as the US bond market's conundrum between June 2004 and December 2005. The estimated affine term term structure model uncovers the dominant role of the term premium in these "euro-conundra".

2 Related literature

Several papers developed yield curve models with no macroeconomic component such as the popular factor models of Duffie & Kan (1996) or Dai & Singleton (2000), in which the set of yields is explained by a few latent factors. Joslin & Al. (2011, henceforth JSZ) among others develop an affine term structure model with only observable factors. Several important papers modeled the joint dynamics of the macroeconomy and interest rates such as Ang & Piazzesi (2003). In addition to three latent factors, they also include two macroeconomic variables extracted from the PCA on a set of inflation-related measures and on another set reflecting real activity. But the majority of these macro-finance models make the implicit assumption that macroeconomic variables are actually risk factors determined by yields. On the contrary, Joslin & Al. (2010, henceforth JPS) introduced affine term structure models with observable yields and macro factors that accomodate unspanned macro risks.

Another important issue being dealt with in the literature is the high persistence displayed by interest rates. With relatively short samples (Euro area for example), estimating correctly the historical dynamics is not straightforward and often leads to errors. Modeling it with a standard VAR would often lead to flat long-term expectations of the short rate. Kim & Orphanides (2012) managed to circumvent that problem by including survey data on long-term rates expectations so that their model-implied expectations match those of the market. Another possibility is to estimate the dynamics by properly taking into account their persistance like Jardet & Al. (2013, henceforth JMP). They make use of averaging estimators which combine estimates resulting from a standard unconstrained VAR and those obtained with a constrained one which takes into account cointegration relationships.

The "Greenspan conundrum" in the US has been extensively studied by the literature and many contributions focused on the term premium which estimation has been very challenging. Several papers in the literature, such as Bernanke & Al. (2004), Cochrane & Piazzesi (2005), Kim & Wright (2005) have attempted to obtain a precise estimation of the US term premium mainly in order to have a better understanding of the conundrum. Unfortunately, estimates computed in the literature are quite different from one another but its long-term declining trend is at least one common qualitative feature they all share.

Kim & Wright (2005) found that a declining term premium is the key factor behind the puzzling behavior of long-term interest rates. Rudebusch & Al. (2007) compared several term premium's estimates and noted the decreasing trend during the 2004-2005 period. Rudebusch & Al. (2006) underlined the significant role of "out-of-model" variables during the conundrum such as the volatility of long-term treasury yields, foreign official purchase of Treasury bonds etc. In contrast, Thornton (2012) views the conundrum as evidence of the severed link between the short and long-term rates in the US. Assuming the term structure to be anchored by the long-term rate, which in turn depends on macroeconomic fundamentals such as productivity, is more relevant than the famous "Expectation Hypothesis". He suggests change in the use of the Federal Funds rate by the FED was behind the conundrum as the long-term rate still depends on fundamentals while the short rate is essentially impacted by monetary policy considerations.

3 A term structure model with macro factors

3.1 Term premium

Financial theory states that the term structure of interest rates is governed by what is usually called the "Expectation Hypothesis" (EH). According to this hypothesis, the expected return an investor expects from holding a long-term bond until maturity is the same as the expected return one gets when rolling over a series of short-term bonds. Equivalently, the long-term yield is equal to the average expected short-term yield. Unfortunately, with risk-averse investors, this hypothesis is unlikely to hold, given that a compensation may be required by them in order to hold such bond. The term premium refers exactly to this compensation for bearing the risk of variation in the riskless rate. In this paper, I will only consider the following term premium :

$$Yield\ premium : YTP_t^n = y_t^n - \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t\left(r_{t+i}\right) = y_t^n - Exp_t^n \tag{1}$$

Here r_t denotes the short rate ie the one-period yield y_t^1, y_t^n the yield of a n-period zero-coupon bond and $Exp_t^n = \frac{1}{n}\sum_{i=0}^{n-1}\mathbb{E}_t\left(r_{t+i}\right)$, the expected average path of the short rate over the next n periods.

3.2 The general setup

Following Ang & Piazzesi's (2003), I implement here a standard discrete-time affine term structure model which also incorporates unspanned macro risks as developed in JPS.

Let P_t be a N_1-dimensional vector of pricing factors, M_t a N_2-dimensional vector of macro factors ($N = N_1 + N_2$) and $Z_t = [P_t, M_t]$. Suppose the short rate satisfies the following equation :

$$r_t = \rho_0 + \rho_1 Z_t \tag{2}$$

The state factors Z_t follow a first order Gaussian VAR under the probability measure $\mathbb{P}$:

$$\Delta Z_t = K_{0Z}^{\mathbb{P}} + K_{1Z}^{\mathbb{P}} Z_{t-1} + \Sigma_Z \varepsilon_{Z_t}^{\mathbb{P}} \tag{3}$$

where $\varepsilon_{Z_t}^{\mathbb{P}} = \varepsilon_t \sim N(0, I_N)$ and Σ_Z is a non-singular $N \times N$ matrix[1].

Under the assumption of complete markets and no arbitrage, there exists a risk-neutral probability measure that is equivalent to the physical measure. Under this measure, the state vectors follow an alternative law of motion :

$$\Delta Z_t = K_{0Z}^{\mathbb{Q}} + K_{1Z}^{\mathbb{Q}} Z_{t-1} + \Sigma_Z \varepsilon_{Z_t}^{\mathbb{Q}} \tag{4}$$

with $\left(K_{0Z}^{\mathbb{Q}}, K_{1Z}^{\mathbb{Q}}\right)$ both linearly related to $\left(K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}\right)$ by the market prices of risk. (see Appendix A for further details). Under the risk-neutral measure, states of the world in which investor's marginal utility is high are in fact overweighted compared to the situation in the physical world.

Within a risk-neutral pricing framework, the price of a zero-coupon bond can be written simply as :

$$p_t^n = \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{i=0}^{n} r_{t+i} \right) \right] \tag{5}$$

Bond prices are actually exponential affine functions of the state variables. More precisely, bond prices are given by :

$$p_t^n = \exp\left(\overline{A_n} + \overline{B}_n' Z_t \right) \tag{6}$$

where the coefficients $\overline{A_n}$ is a scalar and $\overline{B_n}$ is a $N \times 1$ vector for a given maturity. The continuously-compounded bond yield y_t^n is then:

$$y_t^n = A_n + B_n' Z_t \tag{7}$$

with $A_n = -\overline{A_n}/n$ and $B_n = -\overline{B}_n/n$.

[1] I choose in the paper to only consider a VAR(1)-based affine term structure model for parcimony. This setting makes the estimation of the model easier and faster.

3.3 A model with unspanned macro risks

In the present affine term structure model, I allow the yield curve to respond to macroeconomic shocks and vice-versa.

Given equation (7), both P_t and the macro factors determine the model-implied bond yields. Such models are said to contain "spanned macro risks" (see JPS and Ludvigson & Ng (2009) for example). With the bond yields as given, one would be able to solve for the factors using (7) and would conclude that M_t is "spanned" by bond yields[2]. If this model correctly reflects reality, then projections of macro factors on the P_t should yield excellent adjusted R^2, which is rarely the case with empirical data. Another problem with including macro factors isthe subsequent decrease in pricing power. Absence of additional pricing power to the model is also possible. The three native yield-curve factors P_t are actually sufficient to replicate observed bond yields.

Everything above suggests excluding macro factors but the compensations required by investors for bearing real-interest risk does necessarily depend on macroeconomic conditions. Including "unspanned macro risks" is one simple solution to reconcile these two contradicting stances. Restrictions can be imposed such that yields have a zero loading on macro factors in equation (7). However, it does not mean yields cannot have any forecasting power on the macro variables and vice-versa.

Suppose the last N_2 elements of ρ_1 and the upper-right $N_1 \times N_2$ block in $K_{1Z}^{\mathbb{Q}}$ are set equal to 0. Then equation (2) and (4)[3] can be reduced to

$$r_t = \rho_0 + \rho_{1P} P_t \tag{8}$$

$$\Delta P_t = K_{0P}^{\mathbb{Q}} + K_{1P}^{\mathbb{Q}} P_{t-1} + \Sigma_P \varepsilon_{P_t}^{\mathbb{Q}} \tag{9}$$

with ρ_{1P} and $K_{0P}^{\mathbb{Q}}$ two N_1-dimensional vectors, $K_{1P}^{\mathbb{Q}}$ a $N_1 \times N_1$ matrix, $\Sigma_P \Sigma_P'$ the $N_1 \times N_1$ upper-left block of $\Sigma_Z \Sigma_Z'$ and $\varepsilon_{P_t}^{\mathbb{Q}} \sim N\left(0, I_{N_1}\right)$.

Those restrictions also imply that the last N_2 elements of B_n are equal to 0. Thus, equation (7) is equivalent to

$$y_t^n = \widetilde{A_n} + \widetilde{B_n}' P_t \tag{10}$$

where $\widetilde{A_n}$ and $\widetilde{B_n}$ are deduced with recursive equations (see Appendix A).

In such framework, changes in the macro factors do not impact the current yield curve and they are not needed to fit the cross-section of bond yields at time t according to equation (10). With unspanned macro factors, only the risks associated with P_t are priced by the model. However, even if macro risks are not explicitly present in the risk-neutral dynamics, they still have a significant impact through the historical dynamics (3). Indeed, they provide additional information on the

[2] The macro variables could be described through the following equation : $M_t = a_0 + a_1' P_t$

[3] In this paper, I will only model the risk-neutral dynamics without modeling the market prices of risk which would create a direct link between the historical and risk-neutral dynamics.

Yields	1M	6M	1Y	2Y	3Y	5Y	7Y	10Y
Mean	0.032	0.032	0.033	0.035	0.036	0.039	0.042	0.044
SD	0.009	0.009	0.009	0.008	0.008	0.007	0.007	0.006
Skewness	0.264	0.253	0.183	0.132	0.143	0.177	0.137	-0.001
Kurtosis	1.844	1.964	1.935	1.955	2.007	2.114	2.181	2.180
Min	0.020	0.019	0.019	0.020	0.022	0.025	0.028	0.031
Max	0.049	0.051	0.052	0.053	0.053	0.053	0.055	0.057

Table 1: Summary statistics on Euro Area monthly zero-coupon bond yields. Period : 1999M1-2008M8

future path of P_t, which are linked to the short rate. Therefore, as the term premium is impacted by short rate expectations, it is now actually linked to the macroeconomic situation[4].

4 The data

4.1 Yield data

I use data on monthly zero-coupon bond yields of maturities 6, 12, 24, 36, 60, 84 and 120 months from January 1999 to August 2008. The short-term rate used throughout the paper is the 1-month OIS rate rather than the 1-month Euribor in which non-negligible liquidity and credit risk premia are priced. Until September 2004, I use the German sovereign yield curve [5]as representative of the Euro area risk-free interest rates. From October 2004 to August 2008, zero-coupon bond yields provided by the ECB for the Eurozone AAA countries are used in this paper. All yield data are end-of-month. Some of the sovereign yields are plotted in figure 1.

According to JSZ, P_t can be rotated to become principal components of bond yields. A PCA shows that the first three principal components of bond yields explains 99.9% of the cross-sectional variation. I choose to use the first $N_1 = 3$ PCs of bond yields[6], which are usually interpreted as the level, slope and curvature of the yield curve as the yield pricing factors P_t.

Table 1 reports some descriptive statistics of the various bond yields used in the sample.

[4]Like JSP, we also suppose that the inclusion of spanned or unspanned macro factors in the affine term structure model is independent of the issue of bond yields' or macro factors' measurement errors.

[5]provided by the Bundesbank

[6]Like others in the literature, we rescale the principal components obtained from the PCA. we denote $l_{j,i}$ the loading on yield i in the construction of PC_j, the PCs have been rescaled so that: (1) $\sum_{i=1}^{8} l_{1,i}/8 = 1$, (2) $l_{2,10Y} - l_{2,6M} = 1$ and (3) $l_{3,10Y} - 2l_{3,2Y} + l_{3,6M} = 1$. This way, the PCs are on a similar scale. All the variables we will be using take on values in $[-3\%, 8\%]$.

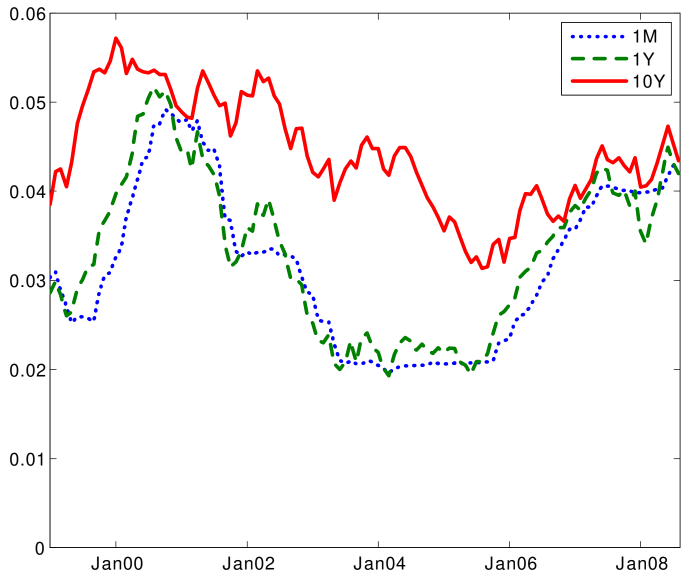

Figure 1: Eurozone historical zero-coupon bond yields for three different maturities

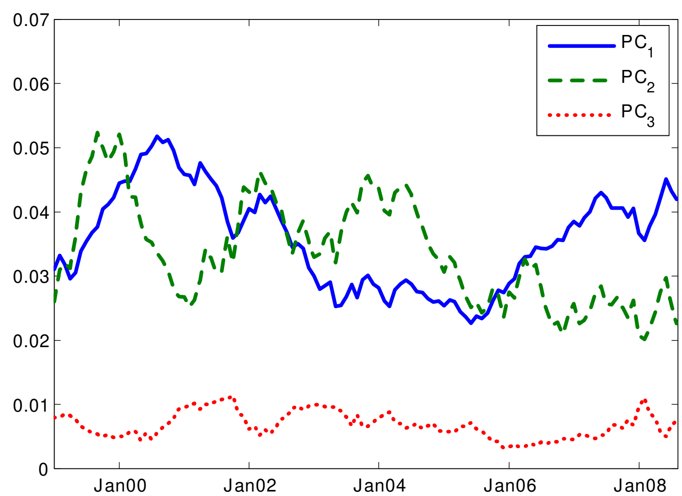

Figure 2: Historical series of the first three principal components of bond yields from 1999 to Aug 2008

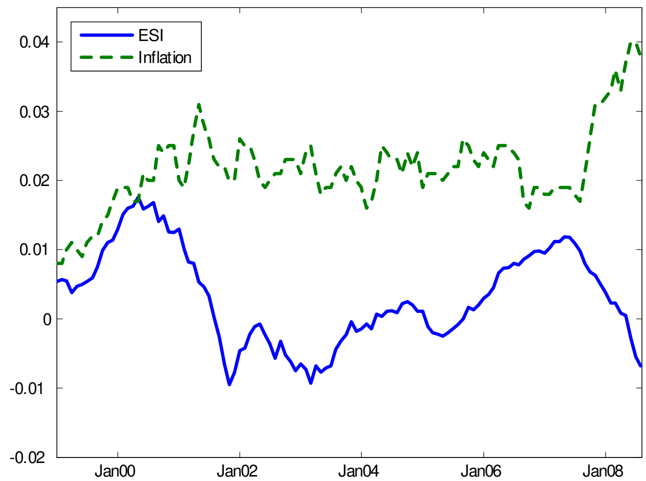

Figure 3: Time series of Euro-area inflation and of the Economic Sentiment Indicator from 1999 to Aug 2008

4.2 Macro variables

I use two macro variables in the model. The first one is the Economic Sentiment Indicator for the Euro area (rescaled)[7], published every month by the European Commission, which is to capture real activity. The second one is the Euro-Area monthly year-on-year inflation (HICP, overall index). Figure 3 plots the two variables.

To assess the need for a model that accomodates "unspanned" macro risks, we can check how

[7] The ESI is issued following harmonized surveys by the European Commission for different sectors of the economy in the European Union. Industry (manufacturing), services, retail trade and construction sectors, as well as consumers contribute to the indicator. The raw indicator is rescaled so that the variable take on values in $[-3\%, 8\%]$

	Activity (Act)	Inflation (Inf)
Mean	0.003	0.021
SD	0.007	0.006
Skew	0.180	0.578
Kurtosis	2.122	4.637
Min	-0.010	0.008
Max	0.018	0.04

Table 2: Summary statistics on Euro area macroeconomic factors. Period : 1999M1-2008M8

well the macro factors are explained by the PCs. With the present data sample, the projection of real activity on the first three PCs of yields gives an adjusted R^2 of 55% and the projection of inflation 15%. Thus almost 45% of the variation in activity and 85% of inflation do not stem from the yields' PCs.

Projections of changes in activity and inflation onto changes in the three PCs give even smaller adjusted R^2 (21% and 1% respectively). All in all, accomodating unspanned macro risks in the Gaussian term structure model is welcomed.

5 Estimation

5.1 First approach

The methodology used for the estimation of the model closely follows JPS but without the reparameterization detailed in JSZ, which reduces the number of parameters estimated and allows for faster computation. Nevertheless, I choose to stick to a standard procedure which will be detailed below. The parameters to be estimated are included in the following equations under the risk-neutral measure :

$$r_t = \rho_{0P} + \rho_{1P} P_t$$

$$\Delta P_t = K_{0P}^{\mathbb{Q}} + K_{1P}^{\mathbb{Q}} P_{t-1} + \Sigma_P \varepsilon_{P_t}^{\mathbb{Q}}$$

And under the physical measure:

$$\Delta Z_t = K_{0Z}^{\mathbb{P}} + K_{1Z}^{\mathbb{P}} Z_{t-1} + \Sigma_Z \varepsilon_{Z_t}^{\mathbb{P}}$$

In the model, the Z_t are priced without errors ($Z_t = Z_{t,o}$) whereas the zero-coupon bond yields equal their model-implied counterparts plus mean zero, normally distributed errors. As JPS relevantly remarked, the absence of constraints linking the physical and risk-neutral measures allows me to separate the time-series properties of Z_t in the physical world from the cross-sectional constraints imposed by no-arbitrage. The conditional likelihood function (under $\mathbb{P}$) of the observed data $\left(y_{t,o}^n\right)$ can be written as :

$$f\left(y_{t,o}^n \mid y_{t-1,o}^n, Z_{t-1}; \Theta\right) = f(y_{t,o}^n \mid Z_t; \rho_{0P}, \rho_{1P}, K_{0P}^{\mathbb{Q}}, K_{1P}^{\mathbb{Q}}, \Sigma_P) * f(Z_t \mid Z_{t-1}; K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}, \Sigma_Z) \quad (11)$$

As mentioned earlier, I suppose the yields on zero-coupon bonds $y_{t,o}^n$ equal their model-implied values $y_{t,m}^n = \widetilde{A_n} + \widetilde{B_n}' P_t$ plus mean zero, i.i.d. and normally distributed errors $\eta_t = y_{t,o}^n - y_{t,m}^n$, which entails :

$$f(y_{t,o}^n \mid Z_t; \rho_{0P}, \rho_{1P}, K_{0P}^{\mathbb{Q}}, K_{1P}^{\mathbb{Q}}, \Sigma_P) = (2\pi)^{-(J-N)/2} \left|\Sigma_\eta\right|^{-1} \times \exp\left(-\frac{1}{2} \left\|\Sigma_\eta^{-1} \times (\eta_t)\right\|^2\right) \tag{12}$$

Using the assumption under which Z_t is conditionally Gaussian, the second term can be expressed as :

$$f(Z_t \mid Z_{t-1}; K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}, \Sigma_Z) = (2\pi)^{-N/2} \left|\Sigma_Z\right|^{-1} \times \exp\left(-\frac{1}{2} \left\|\Sigma_Z^{-1} \times (Z_t - \mathbb{E}_{t-1}[Z_t])\right\|^2\right) \tag{13}$$

where $\mathbb{E}_{t-1}[Z_t] = K_{0Z}^{\mathbb{P}} + \left(I + K_{1Z}^{\mathbb{P}}\right) Z_{t-1}$, J is the total number of yield maturities and where for a vector x, $\|x\|^2$ denotes the euclidean norm squared $\sum x_i^2$.

The (conditional on $t = 0$) log-likelihood function is then the sum:

$$L = \sum_{t=1}^{T} \left[\log\left[f(y_{t,o}^n \mid Z_t; \rho_{0P}, \rho_{1P}, K_{0P}^{\mathbb{Q}}, K_{1P}^{\mathbb{Q}}, \Sigma_P)\right] + \log\left[f(Z_t \mid Z_{t-1}; K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}, \Sigma_Z)\right]\right] \tag{14}$$

Parameters in equation (3) can be estimated from time series without considering cross-sectional restrictions. If Z_t is priced perfectly by the model ($Z_{t,o} = Z_t$), JPS proved that the ML estimates of $\left(K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}\right)$ are actually given by the OLS estimation of the $VAR(1)$ process Z_t[8] and are independent from (Σ_P, Σ_Z). The remaining parameters of the model $(\rho_{0P}, \rho_{1P}, K_{0P}^{\mathbb{Q}}, K_{1P}^{\mathbb{Q}}, \Sigma_Z)$ are then estimated by maximum log-likelihood, assuming the observed bond yields are measured with a i.i.d Gaussian error (see Appendix C.2). I choose here not to take into account the internal-consistency constraint which requires model-implied yields to reproduce the PCs. Nevertheless, I check that the constraint actually holds *ex-post*[9].

5.2 Near-cointegrated VAR (NCVAR)

Interest rates are well-known to be highly persistent and given the short range of data at my disposal on the Eurozone yield curve, bias can easily arise in the estimation of the historical dynamics of interest rates. Because of high persistence in the data, I face what the literature usually calls the "discontinuity problem", which is the huge difference between predictions (especially long-run forecasts of the short rate) based on unconstrained VAR models and those taking into account unit-roots and cointegration relationships.

The approach I use to solve these issues is largely drawn from JMP and introduces "Near-Cointegrated VARs" (NCVAR) to get better estimations of long-run short rate expectations, using averaging estimators. I call "CVAR" the model under the historical measure estimated under a VECM framework.

[8]Actually, though additional lags should be considered in light of standard lag selection procedures, our sample is too limited in size. Nevertheless, if I consider a VAR(2) process for Z_t, the estimation shows most coefficients of Z_{t-2} are found to be not significantly different from 0. Thus, my VAR(1)-based model is still preferable.

[9]I find a RMSE of 1.7 bps which means the internal constraint holds *ex-post*.

5.2.1 Unit roots and VECM model

Standard unit-root tests (Appendix B) reveal that the first PC, which is a proxy for the level of the yield curve, is closer to a $I(1)$ process or at least very persistent, as well as PC_2 and Inf. Results are more mixed for PC_3 and the activity factor but given KPSS superior power to the ADF test, PC_3 and Act are closer to stationarity. In the end, choosing a simple VAR to model the historical dynamics of Z_t will most likely lead to significant estimation bias, given the high persistence and potential cointegration relationships among the five state variables.

The historical dynamics of the factors which is described through equation (3) can be directly interpreted as a vector error correction model (VECM). I determine the rank r of matrix $K_{1Z}^{\mathbb{P}}$ with a Johansen cointegration test using a trace and maximum eigenvalue test. r actually represents the number of cointegrating relationships among the state variables.

By choosing to write the VECM with equation (3), I actually make with the unrestricted constant term $K_{0Z}^{\mathbb{P}}$ the implicit hypothesis that there's a linear trend in Z_t or/and an intercept in the cointegrating component[10]. Equation (3) can be rewritten as:

$$\Delta Z_t = \alpha \left(\beta' Z_{t-1} + c_0\right) + \Sigma_Z \varepsilon_{Z_t}^{\mathbb{P}} \tag{15}$$

or

$$\Delta Z_t = \alpha \left(\beta' Z_{t-1} + c_0\right) + c_1 + \Sigma_Z \varepsilon_{Z_t}^{\mathbb{P}} \tag{16}$$

with the decomposition $K_{0Z}^{\mathbb{P}} = \alpha c_0$ ($K_{0Z}^{\mathbb{P}} = \alpha c_0 + c_1$ respectively)[11]. Under the restricted specification (15), the trace and eigenvalue tests both point to the same rank of cointegration. Both tests[12] accept the presence of $r = 2$ cointegrating relations. Therefore, I can write $K_{1Z}^{\mathbb{P}} = \alpha\beta'$ where α is a (5×2) adjustement coefficient vector and β a (5×2) cointegrating vector. The cointegration analysis was based on the model with a restricted constant so I still have to test the hypothesis $H_0 : K_{0Z}^{\mathbb{P}} = \alpha c_0$ against its alternative $H_a : K_{0Z}^{\mathbb{P}} = \alpha c_0 + c_1$ using a $\chi^2(3)$-distributed likelihood ratio statistic (see Johansen (1995)) to confirm that specification (15) is the most appropriate one.

[10] Critical values of the Johansen test actually depend on the assumptions made concerning the cointegrating relations and the VECM which are :

- absence or presence of an intercept/trend in the cointegrating relations
- absence or presence of an intercept in the VECM (which is equivalent to a linear trend in the data. I choose to neglect quadratic trends).

[11] Actually, though additional lags should be considered in light of standard lag selection procedures, our sample is too limited in size. Nevertheless, if I consider a VECM for Z_t that also includes ΔZ_{t-2}, the estimation shows most coefficients of this term are found to be not significantly different from 0. Thus, my simple VECM is still preferable.

[12] See Appendix B

The test confirms specification (15) and therefore tells us that there's no drift in the common trend[13].

Estimates of the different VECM parameters are provided in Appendix C.3.

5.2.2 Averaging estimators

Averaging estimators were first proposed by Hansen (2009). The idea consists in combining two different kinds of estimators. Firstly, I estimate the parameter θ_{UNC} of the unconstrained VAR with one lag representing the historical dynamics of the factors by OLS. In a second step, I proceed with the estimation of a one-lag VECM of the state variables (therefore imposing unit-root constraints), which gives the parameter vector θ_{CON}. The averaging estimator specifying the Near-cointegrated VAR is then defined as :

$$\theta_{NCVAR} = \theta_{NCVAR}(\lambda) = \lambda\theta_{UNC} + (1-\lambda)\,\theta_{CON} \tag{17}$$

with $\lambda \in [0,1]$ a parameter used to minimize a chosen criterion.

Given that the short rate will depend on the yields' PCs as stated in equation (8), I chose to focus on minimizing the forecast error (RMSFE) when predicting the PCs. As the objective is to provide a more precise estimation of the term premium, I could have actually minimized the error in forecasting the short-rate or Exp_t^n, given the definition of YTP_t^n in the paper which heavily relies on the precision of these forecasts. JMP based their criterion on $\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\left(r_t + ... + r_{t+h-1}\right)\right)\right]$, thus having at disposal more points for the computation of the criterion. Both alternative approaches would have been more relevant but computationally more intensive in the present framework as λ, ρ_0, ρ_1 and the parameters governing the historical/neutral dynamics of the state variables would have to be estimated together. So the present approach can benefit from the two-step estimation speed.

So for a forecast horizon h, the parameter $\lambda(h)$ is selected through the following minimization program :

$$\lambda^*(h) = \arg\min_{\lambda\in[0,1]} \sum_i \left[\sum_t \left(P_{i,t+h} - \mathbb{E}_t^{implied}\left[P_{i,t+h}\right]\right)^2\right] \tag{18}$$

where $P_{i,t+h}$ is the observed realization of the PC_i for each date t and horizon h whereas $\mathbb{E}_t^{implied}\left[P_{i,t+h}\right]$ is the model-implied prediction of the PC_i. The criterion is just (up to a factor) the standard $TMSFE$ (Trace Mean Square Forecast Error).

[13] The likelihood ratio statistic is $LR = -T\sum_{j=3}^{5}\log\left[\left(1-\widetilde{\lambda}_j\right)/\left(1-\lambda_j\right)\right]$ where $\left(\lambda_j, \widetilde{\lambda}_j\right)$ are the smallest eigenvalues associated to the maximum likelihood estimation of the unrestricted and restricted model respectively. We find $LR = 0.830$ which is lower than $\chi^2_{0.01}(3)$=11.35

Like a conventional out-of-sample forecasting exercise, I first estimate θ_{UNC} and θ_{CON} over the period 1999M1-2002M08[14] and compute $\widehat{P}_{t+h}$ with $t = 2002M08$. For each later date t, I re-estimate θ_{UNC} and θ_{CON} over the expanded window and compute the model-implied forecast value of the PCs. This methodology replicates the typical behavior of an investor that incorporates new information over time. The out-of-sample forecasts are performed for $t \in [2002M09, 2008M08 - h]$.

In the end, as I'm interesed in long-term risk premium, h is set equal to 60 months given the limited time length of the data and the optimization yields $\lambda = 0.3042$ with the Trace Root Mean Square Forecast Error (TRMSFE) being equal to 82 bps[15], while for the VAR-based model $TRMSFE_{VAR} = 109$ bps and $TRMSFE_{CVAR} = 158$ bps for the CVAR-based one. This estimated value for λ implies θ_{NCVAR} is something closer to θ_{CON} than the VAR-based estimator (see Appendix C).

6 Results

6.1 Parameters estimates

Tables in Appendix C.4 give the estimated parameters of the term structure model based on the previously described NCVAR method. $\left(\rho_0, \rho_{1P}, K_{0P}^{\mathbb{Q}}, K_{1P}^{\mathbb{Q}}, K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{Q}}, \Sigma_Z\right)$ are initiated at the values obtained from the estimations of the associated standard VARs. Maximization of the log-likelihood and computation of the asymptotic standard errors (for the short-rate and risk-neutral parameters) are computed using a quasi-Newton algorithm as available in the Matlab software. Estimates for $\left(\rho_0, \rho_{1P}, K_{0P}^{\mathbb{Q}}, K_{1P}^{\mathbb{Q}}\right)$ are highly significant because the estimation takes advantage from the large cross-sectional information on yields. In the end, the root mean square fitting error of yields is extremely low (around 1 bps), indicating that the first three PCs are able to account for almost all cross-sectional variation in yields thus proving that the specification of the $\mathbb{Q}$-dynamics of bond yields reflected in equation (4) and (10) is appropriate for Eurozone data. Observed and model-implied yields almost coincide. For instance, the difference between the observed and model-implied 5Y bond yield never exceeds 7 bps.

6.2 Estimation of the term premium

I attempt now to provide an estimate of the Eurozone term premium for the 5Y horizon as the averaging estimator was optimally chosen for this maturity[16]. Figure 4 first compares different

[14] I am clearly aware that the initial window is very narrow for an estimation of the historical dynamics but the relative short existence of the Eurozone leaves me with no other choice than using this short time span so that I can at least consider 5-year-ahead expectations of the PCs for the estimation strategy.

[15] The estimate of λ stays robust after slight changes to the initial time window (see Appendix for details).

[16] Past US studies focused on the 10-year maturity. Unfortunately, due to limitations on the data, I can only consider the 5-year horizon.

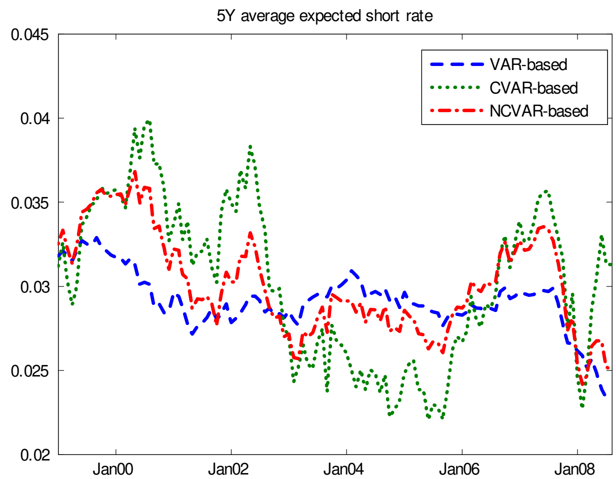

Figure 4: Expected average path of the short rate over a 5-year horizon estimated with the VAR-based (blue dashed line), the NCVAR-based (red dash-dotted line) and the CVAR-based models (green dotted-line).

estimates for the model-implied 5-year expected path of the short rate $Exp_t^{5Y} = \frac{1}{60} \sum_{i=0}^{60-1} \mathbb{E}_t (r_{t+i})$. As mentioned earlier, using a simple VAR-based term structure model would lead to a rather flat 5Y average expected short rate path while the one based on the CVAR model is much more volatile.

Figure 5 shows the 5Y term premium obtained with the model based on a VAR, CVAR and NVCAR processes. The figure typically illustrates once again the differences between the three methodologies with the VAR-based premium being much more volatile than the others for instance.

On average over the whole sample, the 5Y term premium in the model is estimated to represent 21% of the 5Y yield. It has therefore the potential to disturb the conduct of monetary policy in the Euro area.

7 Was there a bond yield conundrum in the Euro area ?

7.1 A first look

Under the asssumption of the Expectation Hypothesis, long rates should be responsive to any changes in the short rate. What triggered the debate around the Greenspan conundrum was the muted response of long rates after the successive rate hikes decided by the FED between 2004 and 2006. Thus, in this parallel analysis, I check whether or not the Euro area also experienced the same thing during its monetary tightening episodes. Figure 6 shows how the 5Y rate evolved throughout the

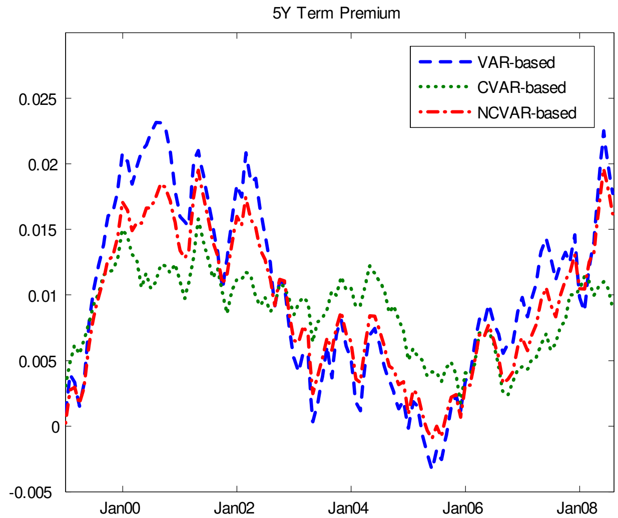

Figure 5: 5-year yield term premium estimated with the VAR-based (blue dashed line), the NCVAR-based (red dash-dotted line) and the CVAR-based models (green dotted-line).

sample's period compared with its two components (Exp_t^{5Y} and YTP_t^{5Y}) and r_t as estimated with the NCVAR model. At first sight, during the first episode of tightening, from November 1999 to March 2000, the short-term interest rate rose while the 5Y interest rate stayed around 5.20% with a stagnant Exp_t^{5Y} and a volatile YTP_t^{5Y} in the background.

What happened during the second episode is slightly different. A first phase can be distinguished with both yield components moving hand in hand in the same direction following the tightening. The second phase (June 2007 - January 2008) witnesses another puzzling phenomenon : the short rate is stable while the model-implied 5Y yield falls from 4.41% to 3.60%. The expectation component's puzzling behavior and its subsequent drop exceeded the term premium's growth which was not high enough to compensate for the fall of the former.

Apart from these periods discussed above from which a parallel has been drawn with past analyses on the US bond market, figure 6 reveals an intriguing event. From June 2004 to December 2005, while the US bond market was experiencing its "Greenspan conundrum", the Euro area was also going through its own "euro-conundrum" simultaneously. The short rate was stable during that period but the 5Y bond yield fell dramatically from 3.70% to 2.94% while the short rate was unchanged. Turning to the sub-components, the term premium was apparently the major contributor to this significant fall.

Under the framework of the EH and mirroring past US analyses, we saw that the Euro area experienced at least three notable phases of puzzling behaviors which we can dub "euro-conundra" :

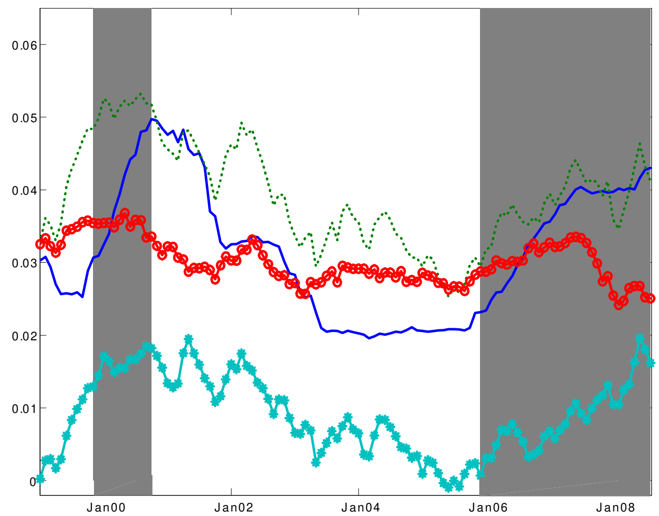

Figure 6: Evolution of the model-implied short rate (solid line), y_t^{5Y} (model-implied, dotted line), Exp_t^{5Y} (model-implied, solid line with circle markers) and YTP_t^{5Y} (model-implied, solid line with star markers) over the Eurozone's previous monetary tightening episodes (shaded in grey).

two of them displaying odd responses from bond yields after rate hikes in a similar fashion to the US experience and a third one which took place simultaneously with the Greenspan conundrum. In all these episodes, the model's term premium apparently played a significant role, which I'll properly disentangle below.

7.2 Contribution analysis

Figure 7 plots the contributions of both the expectation (Exp_t^{5Y}) and the term premium component (YTP_t^n) of the 5Y bond yield during the first phase we described earlier. The figure confirms the dominant contribution of the 5Y term premium at first to the puzzling behavior of the associated bond yield as the ECB was raising the short rate. Similar to what was found by the literature in the US, this quite unusual movements of the 5Y yield we witnessed at the beginning of the monetary tightening was primarily driven by the term premium according to the model.

Turning to the second tightening episode in Euro-area history, figure 8 plots again the contributions of both components associated to the 5Y bond yield. As suspected earlier, the fall of the long rate is actually due to the investors' long-term expectations of the path of r_t. As the financial crisis began to slowly spread to the Eurozone, investors believed the ECB could not hold very long their strong tightening monetary policy. Thus, investors changed their long-term expectations of

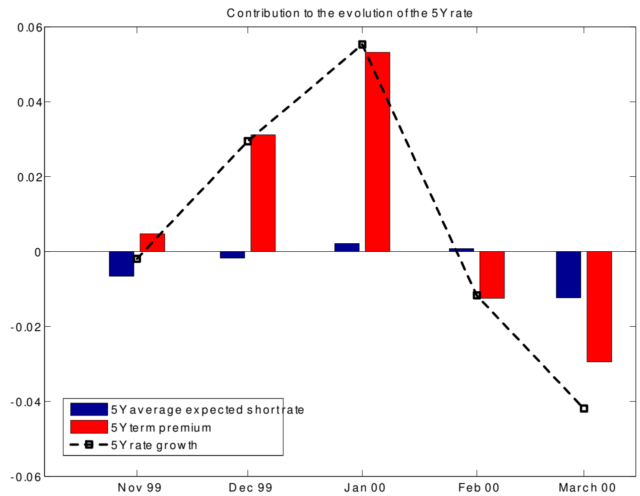

Figure 7: Contribution of Exp_t^{5Y} (in blue, left bar) and of YTP_t^{5Y} (in red, right bar) to the evolution of the 5-year bond yield (black dashed line) from November 1999 to March 2000

the short rate's path and this change actually mainly contributed to the fall of y_t^{5Y}.

The most interesting period in the Euro-area bond market is probably the one when the "Greenspan conundrum" actually took place at the same time in the US bond market. In figure 9, the orientation and length of the red bars show the significant impact of the 5Y term premium on the dramatic fall of the associated bond yield during that period even though the monetary policy rate was flat all that time.

Under the standard framework of the EH, a substantial and time-varying term premium is responsible for the puzzling behavior of long-term rates according to the model.

8 Conclusion

Central banks attempt to influence the movements of the sovereign yield curve. Unfortunately, the task is not without difficulties. The Expectation Hypothesis emphasizes the decisive role of short rate expectations in determining long-term interest rates. Under this framework, deviations from the hypothesis primarily stem from investors' expectations, who then demand a risk premium.

In this paper, I estimate an arbitrage-free Gaussian term structure model for the Euro area which allows for macro risks to be priced distinctly from the yield curve. Indeed, the state factors of the model include macroeconomic variables which are not entirely spanned by bond yields. I also adopt a relevant estimation approach which yields better term premium estimates than a conventional unconstrained VAR model by using averaging estimators. The estimated term structure is consistent

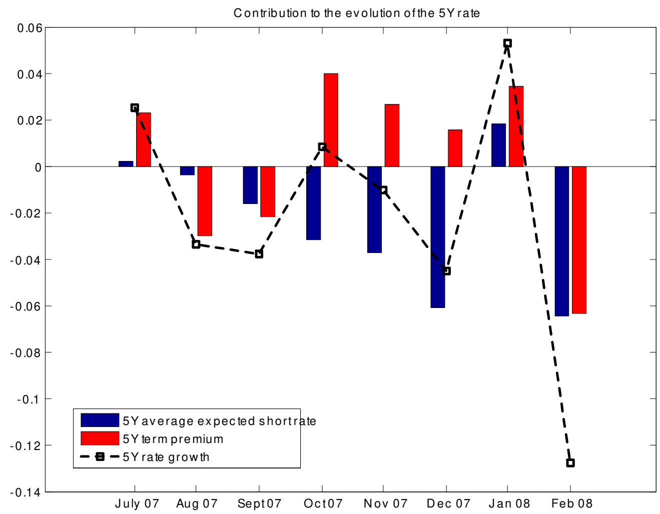

Figure 8: Contribution of Exp_t^{5Y} (in blue, left bar)and of YTP_t^{5Y} (in red, right bar) to the evolution of the 5-year bond yield (black dashed line) from June 2007 to January 2008

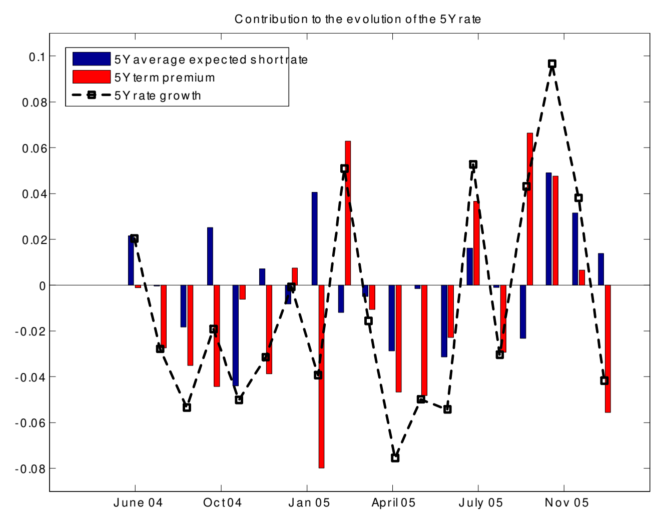

Figure 9: Contribution of Exp_t^{5Y} (in blue, left bar) and of YTP_t^{5Y} (in red, right bar) to the evolution of the 5-year bond yield (black dashed line) from June 2004 to December 2005

with the euro area, as unspanned macro risks are taken into account in line with the observed data. Moreover, the econometric methodology used provides more accurate estimates of long-horizon term premium.

In parallel with past studies on the US bond market, the present analysis shows that the Eurozone went through its own "Greenspan conundra". In contrast with what would have been predicted with the Expectation Hypothesis, long-term interest rates didn't follow the policy rate while the ECB tightened in 2000H1. In 2007-2008, they fell unexpectedly. The most interesting feature is the simultaneous "Greenspan conundrum" in 2004-2005 in the US and Eurozone bond market. The estimated affine term structure model emphasizes the major contribution of the long term premium to the "conundra" in 2000 and 2004-2005, which is similar to the US case.

All in all, resorting to term premia to explain the deviations from the Expectation Hypothesis found in the data might be the proof of the shortcomings of this widespread term structure theory. As in Thornton (2012), a term structure anchored on long rates might be a more relevant alternative to the prevailing Expectation Hypothesis.

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A Appendix: The framework of the term structure model

A.1 Bond pricing

It can be shown that yields linearly depend on Z_t with $p_t^n = \exp\left(\overline{A_n} + \overline{B}_n' Z_t\right)$ (6) with $\left(\overline{A_n}, \overline{B_n}\right)$ both satisfying the following recursive equations :

$$\overline{A_{n+1}} = \overline{A_n} + \overline{B}_n'(K_{0Z}^{\mathbb{P}} - \Sigma_Z \lambda_0) + \frac{1}{2}\overline{B}_n' \Sigma_Z \Sigma_Z' \overline{B}_n - \rho_0 \tag{19}$$

$$\overline{B_{n+1}} = (I + K_{1Z}^{\mathbb{P}} - \Sigma_Z \lambda_1)' \overline{B_n} - \rho_1 \tag{20}$$

The initial conditions are $\overline{A_1} = -\rho_0$ and $\overline{B_1} = -\rho_1$. (λ_0, λ_1) are the market prices of risk.

When $\lambda_0 = \lambda_1 = 0$, investors are then supposed to be risk-neutral. In fact, risk-averse investors actually value any bonds the same way as risk-neutral investors do if the latter thought that the state vectors follow an alternative law of motion under a different probability measure $\mathbb{Q}$:

$$\Delta Z_t = K_{0Z}^{\mathbb{Q}} + K_{1Z}^{\mathbb{Q}} Z_{t-1} + \Sigma_Z \varepsilon_{Z_t}^{\mathbb{Q}} \tag{21}$$

where $K_{0Z}^{\mathbb{Q}} = K_{0Z}^{\mathbb{P}} - \Sigma_Z \lambda_0$ and $K_{1Z}^{\mathbb{Q}} = K_{1Z}^{\mathbb{P}} - \Sigma_Z \lambda_1$.

Equation (3) is commonly refered to the physical/historical risk representation and (4) as the risk-neutral representation of the law of motion for the state vector ($\mathbb{P}$ and $\mathbb{Q}$ respectively). Notice that both laws are identical to each other when $\lambda_0 = \lambda_1 = 0$, which is equivalent to the hypothesis of risk-neutral investors.

To estimate the model, one can either specifify the set of parameters as $\left(\rho_0, \rho_1, K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}, \lambda_0, \lambda_1, \Sigma_Z\right)$ or in terms of $\left(\rho_0, \rho_1, K_{0Z}^{\mathbb{P}}, K_{1Z}^{\mathbb{P}}, K_{0Z}^{\mathbb{Q}}, K_{1Z}^{\mathbb{Q}}, \Sigma_Z\right)$. With the second parameterization, one needs to specify the factors' dynamics under the historical and risk-neutral measure in the model's assumptions. Following standard risk-neutral asset pricing, the price of any zero-coupon bond can then also be written as :

$$\begin{aligned} p_t^n &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{i=0}^{n} r_{t+i}\right)\right] \\ &= \exp\left(\overline{A_n} + \overline{B}_n' Z_t\right) \end{aligned} \tag{22}$$

with $\overline{A_n}$ and $\overline{B_n}$ following the usual recursive equations :

$$\overline{A_{n+1}} = \overline{A_n} + \overline{B}_n' K_{0Z}^{\mathbb{Q}} + \frac{1}{2}\overline{B}_n' \Sigma_Z \Sigma_Z' \overline{B}_n - \rho_0 \tag{23}$$

$$\overline{B_{n+1}} = (I + K_{1Z}^{\mathbb{Q}})' \overline{B_n} - \rho_1 \tag{24}$$

A.2 A model with unspanned macro factors

In the modified framework, the continuously-compounded bond yield is now related to the pricing factors through $\widetilde{A_n}$ and $\widetilde{B_n}$ which are obtained with the following recursive equations :

$$\overline{A_{n+1}} = \overline{A_n} + \overline{B}_n'(K_{0P}^{\mathbb{Q}}) + \frac{1}{2}\overline{B}_n'\Sigma_P\Sigma_P'\overline{B}_n - \rho_0 \tag{25}$$

$$\overline{B_{n+1}} = (I + K_{1P}^{\mathbb{Q}})'\overline{B_n} - \rho_1 \tag{26}$$

with $\overline{A_1} = -\rho_0$, $\overline{B_1} = -\rho_1$, $\widetilde{A_n} = -\overline{A_n}/n$ and $\widetilde{B_n} = -\overline{B}_n/n$.

B Appendix: Unit-root tests and the VECM

B.1 Unit-root tests

	Order	ADF	KPSS	ERS	ADF (1st diff)	KPSS (1st diff)	ERS (1st diff)
PC_1	1	-1.341	0.317	8.385	-8.347***	0.176	1.133***
PC_2	1	-1.718	0.653††	6.183	-9.105***	0.121	1.726***
PC_3	0/1	-2.357	0.322	2.920**	-10.714***	0.055	0.446***
Act	0/1	-1.686	0.203	3.555*	-4.581***	0.110	0.941***
Inf	0/1	-1.840	0.539††	21.318	-9.673***	0.127	0.475981***

Table 3: Order of integration of the state variables. ADF, KPSS and ERS unit-root tests are performed and the associated t-stat are listed. *(** and ***) indicates that the null hypothesis of non-stationarity (ADF and ERS) is rejected at 10% (5% and 1% respectively). †(†† et †††) indicates that the null hypothesis of stationarity (KPSS) is rejected at 10% (5% and 1% respectively).

B.2 Johansen tests

r	Eigenvalue	Max-Eigen statistic	0.05 critical value	Prob.**	Trace statistic	0.05 critical value	Prob.**
None	0.549	91.472	33.877	0.000	153.112	69.819	0.000
At most 1	0.281	37.959	27.584	0.002	61.640	47.856	0.002
At most 2	0.141	17.517	21.132	0.149	23.681	29.797	0.214
At most 3	0.045	5.248	14.268	0.710	6.164	15.495	0.676
At most 4	0.008	0.916	3.842	0.339	0.916	3.841	0.339

Table 4: Unrestricted Cointegration Rank Test for the variables $(PC_1, PC_2, PC_3, Act, Inf)$. The Maximum eigenvalue and trace test are used to determine the rank r. * denotes rejection of the hypothesis at the 0.05 level. ** denotes MacKinnon-Haug-Michelis (1999) p-values

C Appendix: Parameter estimates

C.1 Robustness of the λ parameter estimate

The initial estimation window used in the paper is $[1999M01, t]$ with $t = 2002M08$. For t varying from $t = 2002M06$ to $2002M10$, Table 5 below shows the value of the λ parameter is still close to our chosen estimate in the paper.

t	2002M06	2002M07	2002M08	2002M9	2002M10
$\lambda\,(60M)$	0.2926	0.2962	0.3042	0.3105	0.3230
$TRMFSE$ (in bps)	82.27	82.22	82.00	82.73	83.54

Table 5: λ estimate with different initial estimation window

C.2 The VAR-based model

ρ_0		ρ_{1P}	
	PC_1	PC_2	PC_3
-0,0009	1.0593	-0.3177	0.9008
(0.0000)	(0.0002)	(0.0002)	(0.0007)

Table 6: Short rate equation parameters for the VAR-based model. Asymptotic standard errors in parentheses

	$K_{0P}^{\mathbb{Q}}$		$K_{1P}^{\mathbb{Q}}$	
		PC_1	PC_2	PC_3
PC_1	0,0003	0.0038	0.0237	-0.1820
	(0.0000)	(0.0000)	(0.0000)	(0.0002)
PC_2	-0,0007	-0.0322	-0.0186	0.5184
	(0.0000)	(0.0000)	(0.0000)	(0.0006)
PC_3	0,0005	0.0116	-0.0020	-0.1815
	(0.0000)	(0.0000)	(0.0000)	(0.0001)

Table 7: Risk-neutral dynamic parameters for the VAR-based model. Asymptotic standard errors in parentheses.

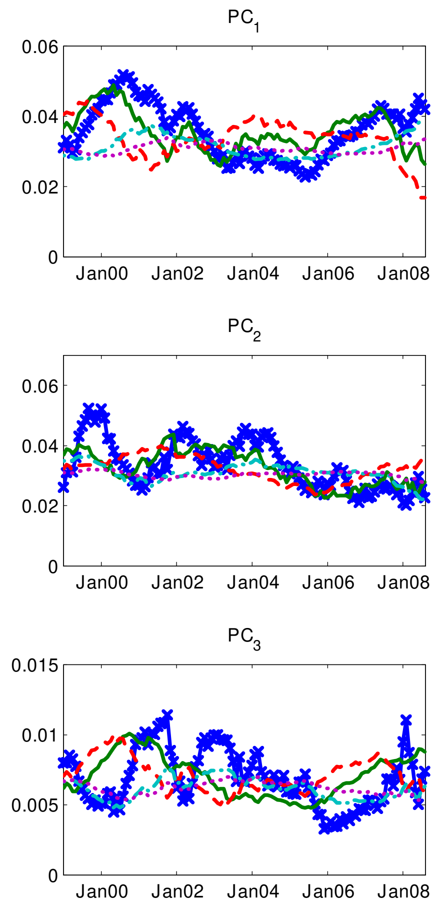

Figure 10: Forecasts of the PCs (solid line with cross markers) under the VAR framework at various horizons : 1Y (solid line), 2Y (dashed line), 5Y (dash-dotted line) and 10Y (dotted line).

	$K_{0Z}^{\mathbb{P}}$			$K_{1Z}^{\mathbb{P}}$		
		PC_1	PC_2	PC_3	Act	Inf
PC_1	0,0025	-0.0525	0.0053	-0.1013	0.0594	-0.0094
	(0.0010)	(0.0318)	(0.0173)	(0.0914)	(0.0431)	(0.0363)
PC_2	0,0054	0.0075	-0.1059	0.2753	-0.0790	-0.1726
	(0.0020)	(0.0459)	(0.0319)	(0.1385)	(0.0654)	(0.0525)
PC_3	-0,0005	0.0393	-0.0031	-0.1167	-0.0163	0.0019
	(0.0005)	(0.0179)	(0.0085)	(0.0490)	(0.0251)	(0.0192)
Act	0,0025	-0.0418	0.0441	-0.2506	-0.0148	-0.0369
	(0.0008)	(0.0288)	(0.0120)	(0.0764)	(0.0400)	(0.0249)
Inf	0,0022	0.0496	-0.0382	-0.0119	-0.0317	-0.1037
	(0.0016)	(0.0446)	(0.0256)	(0.1391)	(0.0579)	(0.0428)

Table 8: Historical dynamic parameters for the VAR-based model. Asymptotic standard errors in parentheses.

C.3 The CVAR-based model

ρ_0 (10^{-4})	ρ_{1P}		
	PC_1	PC_2	PC_3
-6.0824	1.0911	-0.3814	0.8731
(0.0076)	(0.0000)	(0.0000)	(0.0003)

Table 9: Short rate equation parameters for the CVAR-based model. Standard errors in parentheses

	$K_{0P}^{\mathbb{Q}}$ (10^{-4})	$K_{1P}^{\mathbb{Q}}$		
		PC_1	PC_2	PC_3
PC_1	1.3192	0.0010	0.0261	-0.1182
	(0.0008)	(0.0000)	(0.0000)	(0.0000)
PC_2	-2.9365	-0.0301	-0.0183	0.3089
	(0.0022)	(0.0000)	(0.0000)	(0.0000)
PC_3	2.4777	0.0120	0.0007	-0.1081
	(0.0003)	(0.0000)	(0.0000)	(0.0000)

Table 10: Risk-neutral dynamic parameters for the CVAR-based model. Standard errors in parentheses.

	α		β		c_0
PC_1	-0.011	0.022	1	0	-0.088
	(0.007)	(0.009)	.	.	(0.030)
PC_2	0.004	-0.035	0	1	-0.021
	(0.011)	(0.015)	.	.	(0.017)
PC_3	0.003	0.004	-5.975	-9.888	
	(0.004)	(0.005)	(2.893)	(1.598)	
Act	-0.036	0.047	4.161	2.688	
	(0.005)	(0.007)	(0.871)	(0.481)	
Inf	0.003	-0.004	4.118	2.368	
	(0.010)	0.013	(0.941)	(0.520)	

Table 11: Restricted normalized cointegrating parameters β, adjustment coefficients α and intercept terms

	$K_{0Z}^{\mathbb{P}} = \alpha \times c_0$
PC_1	0.0005
	(0.0004)
PC_2	0.0004
	(0.0007)
PC_3	-0.0003
	(0.0002)
Act	0.0021
	(0.0003)
Inf	-0.0001
	(0.0006)

Table 12: Historical dynamic parameters.

	$K^{\mathbb{P}}_{1Z} = \alpha\beta^{'}$				
	PC_1	PC_2	PC_3	Act	Inf
PC_1	-0.011	0.022	-0.154	0.013	0.006
	(0.007)	(0.009)	(0.060)	(0.014)	(0.014)
PC_2	0.004	-0.035	0.324	-0.078	-0.067
	(0.011)	(0.015)	(0.099)	(0.023)	(0.023)
PC_3	0.003	0.004	-0.058	0.023	0.022
	(0.004)	(0.005)	(0.032)	(0.007)	(0.007)
Act	-0.036	0.047	-0.256	-0.021	-0.035
	(0.005)	(0.007)	(0.045)	(0.011)	(0.011)
Inf	0.003	-0.004	0.024	0.000	0.001
	(0.010)	(0.013)	(0.087)	(0.020)	(0.020)

Table 13: Historical dynamic's parameters for the CVAR-based model. Standard errors in parenthesis.

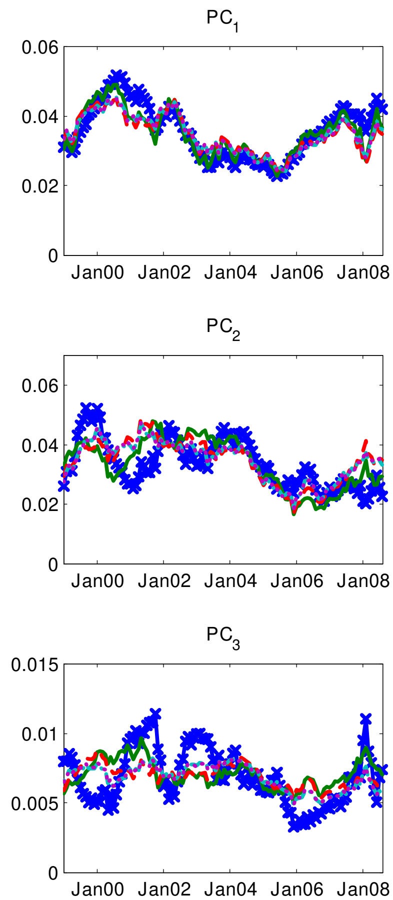

Figure 11: Forecasts of the PCs (solid line with cross markers) under the CVAR framework at various horizons : 1Y (solid line), 2Y (dashed line), 5Y (dash-dotted line) and 10Y (dotted line).

C.4 The NCVAR-based model

ρ_0 (10^{-4})	ρ_{1P}		
	PC_1	PC_2	PC_3
-6.0888	1.0911	-0.3814	0.8734
(0.0175)	(0.0000)	(0.0000)	(0.0001)

Table 14: NCVAR short rate equation parameters. Standard errors in parentheses.

	$K_{0P}^{\mathbb{Q}}$ (10^{-4})	$K_{1P}^{\mathbb{Q}}$		
		PC_1	PC_2	PC_3
PC_1	1.3189	0.0010	0.0261	-0.1182
	(0.0028)	(0.0000)	(0.0000)	(0.0000)
PC_2	-2.9469	-0.0301	-0.0183	0.3090
	(0.0070)	(0.0000)	(0.0000)	(0.0000)
PC_3	2.4789	0.0120	0.0007	-0.1082
	(0.0035)	(0.0000)	(0.0000)	(0.0000)

Table 15: NCVAR risk-neutral dynamic parameters. Standard errors in parentheses.

	$K_{0Z}^{\mathbb{P}}$	$K_{1Z}^{\mathbb{P}}$				
		PC_1	PC_2	PC_3	Activity	Inflation
PC_1	0.0011	-0.0239	0.0172	-0.1380	0.0272	0.0016
	(0.0004)	(0.0108)	(0.0082)	(0.0502)	(0.0163)	(0.0147)
PC_2	0.0019	0.0050	-0.0566	0.3090	-0.0784	-0.0992
	(0.0008)	(0.0159)	(0.0143)	(0.0807)	(0.0255)	(0.0226)
PC_3	-0.0004	0.0140	0.0019	-0.0757	0.0112	0.0158
	(0.0002)	(0.0061)	(0.0043)	(0.0268)	(0.0091)	(0.0076)
Act	0.0022	-0.0376	0.0464	-0.2544	-0.0191	-0.0354
	(0.0003)	(0.0094)	(0.0061)	(0.0390)	(0.0144)	(0.0108)
Inf	0.0006	0.0170	-0.0145	0.0132	-0.0095	-0.0306
	(0.0006)	(0.0152)	(0.0119)	(0.0739)	(0.0224)	(0.0191)

Table 16: NCVAR historical dynamic parameters estimated using the averaging estimator. Standard errors in parentheses.