Inflation Skewness and Price Indexation

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April 2013
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Abstract

One of the two price indexation schemes in the staggered price DSGE models is the indexation to the average inflation. In this essay we show that using average of inflation as index multiplier may lead to the deviation from the optimal price for intermediate good producer. Although there is no problem with this indexation method as far as the inflation distribution is symmetric, when we have a skewed inflation (as we have in the U.S. economy and most of the G7 countries) indexation to average inflation does not reflect the profit maximizer firm’s decision making process. After showing the deficiencies of this method we introduce the Median of inflation distribution ($\text{Med}(\pi)$) as a measure, explain it’s advantage and support our claim by comparing the simulated inflation and the real data. Our results suggest that using $\text{Med}(\pi)$ as index multiplier in the Calvo price setting procedure of intermediate good producer, helps us to reproduce an inflation distribution, similar to the real one.

Keywords: DSGE; Inflation Skewness; Non-Linearity; Calvo Pricing.

JEL classification: E5,E3.

*We are grateful to Dr. M.R Jahan-Parvar for valuable comments. Newer versions of this paper are found at http://myweb.ecu.edu/student/firouzinaeimp12/

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1 Introduction

One of the commonly accepted approaches to look at the staggered price framework in the DSGE literature, is to assume that intermediate good producers use the Calvo (1983) pricing procedure. By the definition of Calvo pricing, in each period firms can re-optimize their prices only by the probability $1 - \alpha$. Consequently, if we allow firms to index their price in each period, they should consider the price level in the upcoming periods. Indexation may take place using two different schemes. Indexation to the lagged price which is suggested by Christiano et al. (2005) and indexation to the long run inflation which can be seen in the works like Yun (1996), Ascari and Ropele (2009) and Schmitt-Groh and Uribe (2004). The trend or long run inflation is evaluated by Schmitt-Groh and Uribe (2004) for the period of 1969 to 1998. It is supposed that the firms are using the trend inflation as the indexation multiplier since it helps them to maximize their overall multi period profit. This, in fact is a good assumption as long as we have symmetrically distributed inflation. When this assumption is violated, the shape of the profit function plays an important role in the price setting decision.

it should be considered that indexation to the long run inflation is no longer the best behaviour of firms in the price setting problem.

![Figure 1: Profit function of intermediate good producer is highly non-linear and asymmetric with respect to real price. If we plot profit, we have $\frac{\Pi_t(i)}{P_t}$ on the horizontal axis, where $p_t(i)$ is the firm’s price which is set in the period $t$ and $P_t$ is the index price of the whole economy at the current period. On the vertical axis we have the $profit_t$.](image)

Analyzing the post world war II data, shows that the inflation is positively skewed in almost all countries and particularly in U.S. [Aizenman and Hausmann (1994)]. This skewness is calculated

\[ \text{Intermediate Good Producer Firms} \]

Since the profit function of the intermediate good producer in the Neo-Keynesian framework is highly non-linear and asymmetric with respect to the real price\(^2\) and the skewed inflation, using the expected inflation\((\bar{\pi})\) as the index multiplier\((\tilde{\pi})\) is no longer optimal for intermediate good producers since the probability of surprising by inflation rate less than expected inflation is more than half. Note that skewness implies that \(Med(\pi) < \bar{\pi}\). And it means in each period firm’s indexed price deviates more and more from the optimal price. It is depicted in the Figure 2.

![Figure 2: The indexed real price in the period \(t + 1\) is \(\frac{\pi_{t+1}}{\tilde{\pi}_{t+1}}\) and since \(Prob[\pi_{t+1} < \bar{\pi}] > Prob[\pi_{t+1} > \bar{\pi}]\) it is more likely that the real price deviates in each period to the right. So if the \(\rho\) is the probability of the deviation of the same length to the left, \(1 - \rho > \frac{1}{2}\).](image)

In this paper we want to show the deficiencies of using \(\bar{\pi}\) as index multiplier and introduce the \(Med(\pi)\) as the new multiplier. In part 2 we investigate the price setting behaviour of intermediate good producer under the alternative scenarios. Part 3 is dedicated to explaining the benefits of deploying the \(Med(\pi)\) as index multiplier. The simple DSGE model and the calibration are described in part 4 and 5 respectively. Part 6 includes the analyzing of the results and at last part 7 is the conclusion part.

\(^2\)One of the sources of this nonlinearity is the elasticity of substitution between intermediate goods in the CES production function of the final goods. This elasticity of substitution, usually calibrated around 11 in the U.S. economy.
2 Profit Maximization and Uncertainty

2.1 Deterministic Optimization

If the firm have access to all the information and there is no uncertainty about the inflation in the next period, the firm simply sets the price where the maximum level of profit is gained in the current period and in each forthcoming periods multiplies this price by the certain level of inflation which is known, to reach the optimum prices in the next periods too. since both the \( p_t(i) \) and \( P_t \) are multiplied by the same rate (inflation), there is no change in the real price. in the figure below \( A \) is the deterministic price and it will be same in the next period and the firm will gain the maximum profit in each period.

![Deterministic optimal price (A) is shown in the figure.](image)

2.2 Optimization under uncertainty

If there is uncertainty about the price in the next period and the firm can not change it’s price in the next period by the probability of \( \alpha \), this uncertainty deviates the price from the deterministic price. We can define two effects here. one of them is the effects which can be routed in the variance of inflation distribution and another one in the skewness of it.

2.2.1 Variance effect

If the inflation of the next period (\( \pi_{t+1} \)) is uncertain and it is distributed normally, we can assign a distribution and like any other distribution we can check that what is the standard error (\( \sigma \)) of this distribution. the variation of inflation around it’s mean (\( \bar{\pi} \)) makes the expected profit maximizer firms to select a price in which they can reach the maximum of some alternative scenarios. Since of
the asymmetries that we have in the profit function the optimum price differs from the deterministic one. the deviation depends on the shape of the profit function. analytically we can say that, since the slope of the profit function is higher in the left hand side of the deterministic maximum - to avoid the loss from unexpected inflation - the firm chooses the optimum price in the right hand side of the deterministic optimum price. Note that the price in the next price, in the uncertain situation will be \( \frac{p_t^i(i)}{P_t} - \bar{\pi}_{t+1} \) and since of the difference between \( \bar{\pi} \) and \( \pi_{t+1} \) the price will differs from \( \frac{p_t^i(i)}{P_t} \).

And it means that if firm chooses the deterministic maximum and there is chance the real price will be deviated to the left or right by the same amount, since of the asymmetric profit function, the deviation to the left (\( \pi_{t+1} - \bar{\pi} > 0 \)) harms the firm much more than the deviation to the right. It is why the firm chooses the optimum price bigger than the deterministic optimum price.

![Figure 4: The price here is set assuming that the inflation is normally (symmetrically) distributed. As a result only the variance is important. This price (B) is more than the deterministic price (A).](image)

### 2.2.2 skewness effect

The mechanism by which the firm is allowed to index it’s price by expected inflation (\( \bar{\pi} \)) first suggested by Yun (1996) and developed by Ascari and Ropele (2009) and had not been seen in the Calvo’s work. If we consider the problem for \( n \) periods we will have 

\[
\frac{p_t^i(i), \pi^n}{P_{t+n}} = \frac{p_t^i(i), \pi^n}{P_t, \pi_{t+1}, \pi_{t+2}, \pi_{t+3}, \ldots, \pi_{t+n}}.
\]

Since we know the \( P_t \), the only source of uncertainty is the forthcoming inflations. It is important to suppose that the distribution of inflation will be same through the time. If the distribution of inflation is skewed the other problem will be aroused and needs to be considered by the firm. to address this problem we need to explain the meaning of skewness. if the distribution is positively skewed we will have \( Mode < Med < Mean \). By \( Med \) we mean the point in our data which separates the data into two equally block of information. Also, for example if the stochastic...
variable is inflation, $Med(\pi_{t+n})$ is the inflation rate which the chance of occurring some inflation more than that is 50 percent and the chance of facing some inflation below that is 50 percent as well. Since the Mean of the positively skewed distribution is located in the right hand side of the $Med$ ($Med < Mean$) we can expect that the chance of occurrence of some inflation more than the $\pi$ is less than 50 percent (say 30) and the chance of getting some inflation below the mean is more than 50 percent (say 70).

This chances we talked about leads to the problem that in the case of infinite horizons, since of the skewness we can expect that in the 70 percent of times the actual inflation is less than the predicted one ($\pi$) and since we multiplied the current period’s optimal price by $\pi$ in each period, the real price which was defined as $P_t^*\pi_{t+n}$ will increase in 70 percent of times (because the denominator is multiplied by the number which is less than the number which numerator is multiplied by) and increase in 30 percent of times. It leads in the long run to the zero profit for the firms. This situation is depicted in Figure 5.

![Figure 5: Since the unexpected inflation $\pi_{t+j} - \pi$ is more likely to be less than zero, the real price will increase through the time and the Profit_{t+j} converges to zero](image)

### 3 Multiplying by $Med$ as a Solution

In the Yun (1996) and Ascari and Ropele (2009) the index multiplier assumed to be the expectation of the inflation ($\pi$). When we think about the reasons they have picked this multiplier we understand that it is kind of assumption that works when we have symmetric distribution for $\pi_{t+n}$. Since in this situation we have $Med(\pi_{t+n}) = E[(\pi_{t+n})] = Mode((\pi_{t+n}))$. If we consider the fact that inflation is positively skewed in the U.S. and the most G7 countries, the $\pi$ is no longer the proper index multiplier for the firms. One good candidate in this situation is $Med(\pi_{t+n})$. Using $Med$ as an index
multiplier leads to the stability of the real price around the optimum price which is set in the current (first) period. Although the Mean is better measure of center tendency of distribution with respect to the fact that it is weighted average, since the deviation from the expected inflation is important here and not the inflation itself, this advantage of Mean has no application in the maximization problem and hence it seems that using the $\text{Med}(\pi_{t+n})$ is more useful. Using $\text{Med}(\pi t + n)$ sets the profit in each period around the maximum level and hence the aggregate profit will be bigger. So it is more plausible for the intermediate good producer to use $\text{Med}(\pi_t)$ as an index multiplier instead of the $\bar{\pi}$. This imply that this model is nearer to the behaviour of the profit maximizer firm.

4 Model

4.1 Household

Here we use the simple Neo-Keynesian model as it can be found in Ascari and Ropele (2009) so the utility function for the Ricardian household which we deploy here is:

$$U(C, M/P, N) = \frac{C^{1-\sigma_c}}{1 - \sigma_c} - \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1 - \sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1 + \sigma_n} \quad (1)$$

Here $U$ stands for the household’s utility, $C_t$ consumption in period $t$, $M_t$ the money stock which households hold, and $N_t$ Labor force which is provided by household. In the above equation $\sigma_c, \sigma_n$ and $\sigma_m$ represent respectively, the inversed Intertemporal elasticity of substitution of Consumption, Labor and Money.

Considering the constraint on household budget we have:

$$P_tC_t + M_t + B_t \leq W_tN_t + M_{t-1} + (1 + i_{t-1})B_{t-1} + F_t + TR_t \quad (2)$$

In this equation $W_t$ is the wage in time $t$, $B_t$ is the holding of the Bonds and $i_t$ is the interest rate in the time $t$. $F_t$ represents the profit from the firms which household gains in each period as shareholder and $TR$ is the transfers from the government to the household.

Maximizing equation 2 in the infinite horizon framework, constrained to the Budget constraint
leads to the First order Conditions as follows:

$$\max_{C_t, M_t, N_t, B_t} \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma_c} - 1}{1 - \sigma} - \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1 - \sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1 + \sigma_n} \right)$$ (3)

Subject to:

$$P_tC_t + M_t + B_t \leq W_tN_t + M_{t-1} + (1 + \delta_{t-1})B_{t-1} + F_t + TR_t$$ (4)

Solving the 4 considering the 5, we get the FOCs:

$$\chi_m \frac{(M_t/P_t)^{-\sigma_m} - 1}{1 - \sigma_m} = \frac{i_t}{1 + i_t}$$ (6)

and the Euler equation:

$$1 = \beta E_t \left\{ \frac{C_{t+1}^{1-\sigma_c}}{C_t^{1-\sigma_c}} \left( 1 + i_t \right) \frac{P_t}{P_{t+1}} \right\}$$ (7)

### 4.2 Final Good Producer

Suppose that Final good producers are producing in perfectly competitive market production function, using all the intermediate goods is defined by:

$$Y_t = \left( \int_0^1 Y_t(i) \left( \frac{\theta + \delta - 1}{\delta - 1} \right) \frac{\delta i}{\delta - 1} \right)^{\frac{\theta}{\delta - 1}}$$ (8)

Since the final good producer is in the competitive market, it is price taker. Also it chooses the quantity to maximize it’s profit. By maximizing the profit function with respect to the quantity we get the demand for the factors which themselves are the production of the intermediate good producers. This demand function will be as follow:

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t$$ (9)

### 4.3 Intermediate Good Producer

The framework of the problem is defined here by Monopolistic Competitor firms, Prices are sticky(We have Staggered price procedure) and it is determined by the procedure which first was
introduced to the literature by Calvo (1983). The technology which the firm uses is $Y_t(i) = N_t(i)$ which means that the firm can produce the good $i$ by deploying the exact same amount of Labor and hence the technology function is supposed to take the most simplest form which is possible. The intermediate good producer faces the demand function $Y_t(i) = (P_t(i)/P_t)^\theta.Y_t$.

Using the Calvo pricing procedure a firm can re-optimize its price by the probability of $1 - \alpha$. Although there is no limitation in Calvo’s formalism, it has not been usual to consider the trend inflation in this framework before Ascari and Ropele (2009). In their work which was based on the Calvo’s formalism, they introduced the trend inflation ($\bar{\pi}$) as an index multiplier, which means that firms use $\bar{\pi}$ to index their prices and for now, we use their formalism too ($\bar{\pi} = \tilde{\pi}$).

Total share of the Firms = \begin{cases} 
\text{Share of the Firms re-optimize their price} & \alpha \\
\text{Share of the Firms which index their price} & 1 - \alpha 
\end{cases}

Total share of firms they index = \begin{cases} 
\text{Share of the firms they index using trend inflation} & \epsilon \\
\text{Share of the firms they keep last optimized price} & 1 - \epsilon 
\end{cases}

Knowing the Total real cost function as $TC_r^r(Y_t(i)) = w.Y_t(i)$ we can put them both into the profit function and Maximize it with respect to the price as below:

$$\max_{p_t^r(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}[p_t^r(i)\bar{\pi}(i)Y_{t+j} - TC_r^r(Y_{t+j}(i))]$$ (10)

s.t.

$$Y_{t+j}(i) = \left(\frac{p_t^r(i)\bar{\pi}(i)}{P_{t+j}}\right)^{-\theta}Y_{t+j}$$ (11)

Note that $\bar{\pi} = \pi$ in the work of Ascari and Ropele (2009). So the FOC will be:

$$p_t^r(i) = \frac{\theta}{1 - \theta} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}[p_{t+j}^\theta Y_{t+j}MC_{t+j}(i)\bar{\pi}^{-\theta}] - \theta \pi]}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j}[p_{t+j}^{\theta-1}Y_{t+j}\pi(1-\theta)\epsilon]}$$ (12)

Now that we have the First Order Conditions, we are ready to run the model using the standard software packages like Dynare\textsuperscript{3}. But before that we investigate the different scenarios under which we can detect different profit maximization behaviour based on the cost benefit analysis.

\textsuperscript{3}we use Dynare 4.3.2
5 Calibration

Following Ascari and Ropele (2009), frictionless or desired markup is 10 percent in product market. The calibration is for the quarterly data, so by setting $\alpha = .75$ The prices re-optimize approximately each year (after four periods). The steady state value of labor ($N$) and Consumption ($C$) which is used in our model set at 1. The calibrated parameters, their values and definitions can be found in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_n$</td>
<td>Intertemporal rate of substitution of Labor</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Intertemporal rate of substitution of Consumption</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective rate of time preference</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>probability of not re-optimizing</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between the intermediate goods</td>
<td>11</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Coefficient of $i$ in the Taylor rule</td>
<td>1.2</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Coefficient of $\pi$ in the Taylor rule</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Coefficient of $c$ in the Taylor rule</td>
<td>0.5</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>degree of indexation</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega_\pi$</td>
<td>coefficient of AR(1) for the cost push shock</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1: The values are calibrated for the united states postwar data

Like Schmitt-Groh and Uribe. (2004) and Ascari and Ropele (2009) we consider the cost push shock here. The process which describes the evolution of the cost push shock assumed to be $AR(1)$ as follows:

$$z_{\pi_t} = \omega_\pi.z_{\pi_t} + u_t \quad (13)$$

In the above formula $u_t$ is assumed to be an $i.i.d$ process with mean zero and standard error one. The nominal interest rate is %2.2 at steady state and marginal cost which is interpreted to real wage in this model is 0.9.

6 Analysis

Using the postwar inflation data in the U.S. economy we know that the mean of the annualized inflation rate is 3.308 and $Med(\pi) = 2.634$. Schmitt-Groh and Uribe. (2004) use the mean of the
annualized deflation growth rate in the United States between 1960 to 1998. Since we want the result to be comparable with other researches, we use the growth rate of GDP deflator for the United States between 1960 to 1998 too. The descriptive statistics which are reached considering the mentioned period can be seen in Table 2:

<table>
<thead>
<tr>
<th>Descriptive Stat</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Mean</td>
<td>3.96</td>
</tr>
<tr>
<td>Median</td>
<td>3.20</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>2.562</td>
</tr>
<tr>
<td>Skewness</td>
<td>.9861</td>
</tr>
<tr>
<td>Geometric Mean</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Table 2: The Annualized descriptive statistics of the United States between 1960 to 1998. There is a little difference between geometric mean and Med in the data.

6.1 Simulating the Inflation Distribution

Here we analyze the changes which are reached by using Med(\(\pi\)) instead of \(\bar{\pi}\) as the index multiplier (\(\tilde{\pi}\)). One of the parameters which can affect the intermediate good producer’s profit is \(\alpha\). \(\alpha\) shows the probability that firm will not be allowed to re-optimize its price in each upcoming period. So by the bigger \(\alpha\), it is more vital for the firm to predict the inflation of the forthcoming periods accurately and set its price knowing that it might change by changes in the realized inflation rate. In other words, it is more likely that firm can not adjust its price in the forthcoming periods and the firm should put more weight on the fact that index multiplier should be selected by the firm such that minimizes the loss from deviation from optimum price. Table 3 addresses the changes in \(\alpha\) and the results from simulating our model.

When \(\alpha\) increases the probability of re-optimizing the price in the next period decreases. So the effect of using Med(\(\pi\)) instead of \(\bar{\pi}\) can be more important, since by bigger \(\alpha\) it is more important to set the index multiplier accurately. Remember that \(\bar{\pi}\) causes the deviation from the optimal price in each period while we have skewed distribution for inflation.

By having bigger \(\alpha\) any deviation from the optimal index multiplier has bigger penalty since the probability of re-optimization of price in each period, is smaller now. Comparing the Mean and the Standard Deviations in Table 3 which reports the simulation results and Table 2 which indicates
the characteristics of real data, it appears that using $Med(\pi)^4$ can produce (simulate) the inflation distribution which is closer to the real distribution of inflation in first and second moments rather than using the Mean of the inflation. The difference between simulated mean of the inflation which is produced by $\bar{\pi}$ and the simulated mean of inflation which is simulated using the $Med(\pi)$, is about 1 percent which is considerably big. this difference is bigger when $\alpha$ is bigger which emphasizes what we argued before that, by increasing the $\alpha$ it is more important to use the index multiplier which causes no systematic deviation in price setting behavior.

The other important parameter is $\epsilon$ which is the share of intermediate good producers which index their price when they can’t re-optimize it. As a result $1 - \epsilon$ of the firms which the don’t re-optimize, keep their price as it was in the last re-optimization period. The changes in $\epsilon$ and its effects on the first and second moments of inflation distribution are shown in Table 4

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Simulated Mean $= \bar{\pi}$</th>
<th>Simulated Mean $= Med(\pi)$</th>
<th>Simulated S.D $= \bar{\pi}$</th>
<th>Simulated S.D $= Med(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.5$</td>
<td>4.84</td>
<td>4</td>
<td>.56</td>
<td>1.76</td>
</tr>
<tr>
<td>$\alpha = 0.75$</td>
<td>4.92</td>
<td>4.04</td>
<td>.84</td>
<td>.88</td>
</tr>
<tr>
<td>$\alpha = 0.8$</td>
<td>5</td>
<td>4.08</td>
<td>.64</td>
<td>.68</td>
</tr>
</tbody>
</table>

Table 3: Simulation mean and Standard Deviation of inflation based on 6 scenarios that each one is the combination of using $\bar{\pi}$ or $Med(\pi)$ and one of three values suggested for $\alpha$. here we set $\varphi_i = 1.2$, $\varphi_c = 1.5$, $\varphi_c = 0.5$ and $\epsilon = 0.5$

The results in Table 4 support our guess even more strongly. The Changes in $\epsilon$ have the inverse effect on the importance of accuracy in selecting the index multiplier($\bar{\pi}$). So bigger the $\epsilon$ is, we see the results from using the $Med(\pi)$ and $\bar{\pi}$ are closer. When $\epsilon = 1$ it means full indexation, so all the

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It is also helpful to argue about the Geometric mean which here is exactly same as $Med(\pi)$. Indeed Geometric Mean has the same characteristics which we mentioned about the Median.
firms which they can’t re-optimize their price, should index under this assumption. Note that if they don’t index their price, they deviate from the optimal price even more, while they keep leave their price unchanged. As a result when \( \epsilon = 1 \) the difference between two state of using \( Med(\pi) \) and \( \bar{\pi} \) is \( \%84 \). Still the \( Med(\pi) \) has a big advantage on \( \bar{\pi} \) since its first and second moments are closer to the first and the second moments of real inflation data. When \( \epsilon = 0.5 \) which is more common in the literature, the difference is about \( \%0.9 \). This difference is bigger when \( \epsilon = 0 \) and is equal to \( \%1.08 \).

So by increasing the \( \epsilon \) the difference between the simulated mean which is produced using \( Med(\pi) \) and the mean produced by \( \bar{\pi} \) is decreased. The reason as we discussed is that by higher \( \epsilon \) all the firms which can’t re-optimize in this period, index their prices, which causes higher profit\(^5\) for them comparing with the state in which they can not index and should keep their price unchanged.

7 Conclusion

As we suggested in the first part of the paper, using \( Med(\pi) \) as an index multiplier can maximize the firm’s benefit through the time.

We assume that inflation is not normally distributed and this assumption is based on the fact that inflation is positively skewed in almost all G7 countries. This asymmetry in distribution of inflation with the another asymmetry which is inherited in intermediate good producer’s profit function leads to the unavailability of firms in keeping the price in its optimal level in upcoming periods(when they index their prices in each period by \( \bar{\pi} \)). Using the \( Med(\pi) \) eliminates the systematic error in the model and is necessary for keeping the assumption that intermediate good producers are profit maximizer. Using the \( Med(\pi) \) can minimize the error in inflation forecast of intermediate good producers and since firms are profit maximizers, they should avoid the loss from unaccurate forecasts.

Using the simple Neo-Keynesian DSGE model we showed that multiplying the price in each period, by \( Med(\pi) \) instead of \( \bar{\pi} \) gets the considerable better results and in our model decreases the difference between the realized price of the intermediate good producer and the optimum price.

Another result is in the simulating the inflation distribution. Using the \( Med(\pi) \) in the staggered

\(^5\)bigger \( \epsilon \) leads to higher profit only if we have non-zero long run inflation
price model as the index multiplier (The value which in each period is multiplied to the firm’s price to get the new price without re-optimization) produces the inflation distribution which is closer to the real distribution of inflation in first and second moments comparing with the simulation which uses the $\bar{\pi}$. 
References


8 Appendix

Figure 6: Response to the unit cost push shock under the standard Taylor Rule with $\varphi_i = 1.2$, $\varphi_c = .5$ and $\varphi_p = 1.5$