Market Risk Measurement: Key Rate Duration as an asset allocation instrument

Zeballos, David

Central Bank of Bolivia

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David E. Zeballos Coria♦♦

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Abstract

Currently, the financial institutions are exposed to different types of risks, including the market, credit and operational risks. Consequently, there has increased the need for new financial and analytical instruments for the risk management.

Among the traditional ones we have the duration, which measures the bond price sensitivity to changes of interest rates. Nevertheless, it has two disadvantages: it assumes parallel changes in the yield curve and it is inaccurate if we consider large percentage changes. In this sense, a tool that allows correcting these disadvantages is The Key Rate Durations. The present work tries to provide an additional tool to the investment analysis, so the economic agents can adopt better decisions.

JEL: G32, G12

Key words: Market risk, Duration, Key Rate Duration

♦ The views expressed in this paper are those of the author and do not necessarily represent those of the Central Bank of Bolivia. This paper was presented at the 4th Bolivian Economists' Annual Meeting, August, 25-26, 2011.

♦♦ Senior Analyst at the Central Bank of Bolivia. E-mail: dzeballos@bcb.gob.bo, zeballosdavid@gmail.com.

1 Introduction

In the financial literature, risk is defined as the volatility of unexpected outcomes, generally the value of assets or liabilities of interest. Financial Institutions are exposed to various types of risks, which can be broadly classified into financial and nonfinancial risks. The financial risks include: the market risk, the credit risk and operational risk.

In the last five decades, the theory and the practice of risk management have developed enormously, it has developed to the point where risk management is now regarded as a distinct sub-field of the theory of finance and is one of the more intensely discussed topic not just for the finance agents or regulatory entities but also for specialists in the academic field.

One factor behind the rapid development of risk management was the high level of instability in the economic environment within which firms operated. A volatile environment exposes firms to greater financial risk, and therefore provides an incentive for firms to find new and better ways of managing this risk.

One of the most traditional used tools is the duration. Duration is given as the weighted-average time to maturity of the cash flows, where the weights are defined as the present values of the cash flows divided by the bond price. The duration assumes that the yield curve experiences infinitesimal and parallel shifts. However, normally we can expect that shorter maturity rates are more volatile than the longer maturity rates, so the assumption of parallel yield curve shifts is obviously false.

Recently a new class of models called the key rate durations has become popular among practitioners. Similar to the duration models, key rate durations can manage interest rate risk exposure arising from arbitrary nonparallel shifts in the term structure of interest rates. They hedge against the changes in a finite number of key interest rates that proxy for the shape changes in the entire term structure.

The key rate duration model describes the shifts in the term structure as a discrete vector representing the changes in the key zero-coupon rates of various maturities. Key rate durations are then defined as the sensitivity of the portfolio value to the given key rates at different points along the term structure.

Therefore, this paper considers estimation of the key rate durations of a given portfolio and its corresponding benchmark and compares it with the traditional duration method, and finally analyzes its usefulness to the asset allocation problem.

The document proceeds as follows: Section II discusses some theoretical issues on Risk Management and analyzes in detail various approaches to financial risk measurement. The results of the empirical work using a given portfolio data are presented in Section IV. Finally, Section V concludes.

2 Measurement of Market Risk

2.1 Duration

One method traditionally used by financial institutions for measuring interest-rate risk is duration analysis. The Macaulay duration (McD) of a bond (or any other fixed-income security) can be defined as the weighted average term to maturity of the bond's cash flows, where the weights are the present values of each cash flow relative to the present value of all cash flows (C):

$$McD = \sum_{t=1}^{T} \frac{t * \left({C_t}/{(1+y)^t} \right)}{\Sigma \left({C_t}/{(1+y)^t} \right)}$$

Duration is a measure of the approximate sensitivity of a bond's value to rate changes[1]. More specifically, it is the approximate percentage change in value for a 100 basis point change in rates. The following formula is used to approximate the percentage price change for a given change in yield and a given duration:

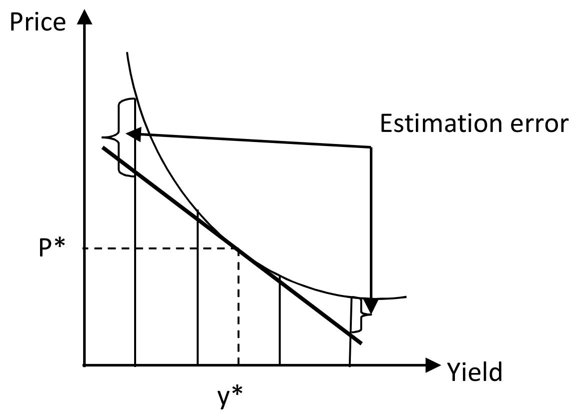
$$Approximate\ percentage\ price\ change = -duration * \Delta y * 100$$

Graphically, the shape of the price/yield relationship for a bond (option-free) is convex (Figure N° 1). In this graph, we also see a tangent line that touches the curve at the point where the yield is equal to y* and the price is equal to p*. The tangent line can be used to estimate the new price if

[1] Fabozzi (2005)

the yield changes. Recall that duration tells us the approximate percentage price change, therefore, this estimation is on the tangent line. Notice that for a small change in yield, the tangent line does not depart much from the price/yield relationship, hence, the tangent line does a good job of estimating the new price. Nevertheless, the error in the estimate gets larger the further one moves from the initial yield, the estimate is less accurate the more convex the bond.

Figure N°1



Also note that regardless of the magnitude of the yield change, the tangent line always underestimates what the new price will be for a bond.

We have two different measures of duration:

a) Modified duration, measure in which it is assumed that yield changes do not change the expected cash flows (treasury bills)

   Modified duration can be written as:

$$MD = \frac{McD}{(1 + yield/k)}$$

b) Effective duration, measure in which yield changes may change the expected cash flow (callable bonds)

A portfolio's duration can be obtained by calculating the weighted average of the duration of the bonds in the portfolio. The weight is the proportion of the portfolio that a security comprises. Mathematically, a portfolio's duration can be calculated as follows:

$$D = w_1 * D_1 + w_2 * D_2 + \cdots + w_n * D_n$$

Where: D: Portfolio's duration

w_i: proportion of security i in the total portfolio

D_i: individual duration of security i

Likewise, the spread duration for a portfolio or a bond index is computed as a market-weighted average of the spread duration for each sector.

Some portfolio managers look at exposure of a portfolio or a benchmark index to an issue or to a sector simply in terms of the market value percentage of that issue or sector in the portfolio. A better measure of exposure to an individual issue or sector is its *contribution to portfolio duration or contribution to benchmark index duration.* This is found by multiplying the percentage of the market value of the portfolio represented by the individual issue or sector by the duration of the individual issue or sector:

$$Contribution\ to\ portfolio\ duration = w_j * D_j\ of\ issue\ or\ sector$$

Duration is often used for immunization of a stream of future cash flows. Christensen (1999) describes the immunization in the following manner: an investor has a liability at a given future date (the horizon) and wishes to construct a portfolio such that, regardless of a rise or a fall in the interest rate, the value of the portfolio at the horizon will be at least as large as the liability. This can be achieved if the portfolio is selected so that (a) the value equals the present value of the liability calculated by the current term structure of interest rates, and (b) its duration equals the length of the horizon.

If a parallel shift in the term structure occurs immediately after this, the value of the portfolio and the present value of the liability are changed. However, the changes occur so that the value of the portfolio remains at least as large as the present value of the liability, whether the interest rate rises or falls. If no further unexpected changes in the term structure occur during the horizon, the value of the portfolio at the horizon will therefore be at least as large as the liability. The investor can hedge against new changes in the interest rate by constantly rebalancing the portfolio so that its duration is always equal to the duration of the liability.

2.2 Convexity

The duration measure indicates that regardless of whether interest rates increase or decrease, the approximate percentage price change is the same. Moreover, we saw that the duration is only a good approximation of the percentage price change for small changes in yield.

The reason for this is that duration is in fact a first (linear) approximation for a small change in yield. The approximation can be improved by using a second approximation. This approximation is referred to as “convexity”. The *convexity measure* of a security can be used to approximate the change in price that is not explained by duration.

The convexity measure of a bond is approximated using the following formula:

$$Convexity\ measure = \frac{1}{2} * \frac{d^2P}{dy^2} * \frac{1}{P}$$

Given the convexity measure, the approximate percentage price change adjustment due to the bond’s convexity is:

$$Convexity\ adjustment = convexity\ measure * (\Delta y)^2 * 100$$

The approximate percentage price change based on duration and the convexity adjustment is found by adding the two estimates:

$$Total\ estimated\ \%\ price\ change = Estimated\ \Delta\ using\ duration + Convexity\ adjustment$$

2.3 Disadvantages

Even the Duration is the most commonly measure used to measure the interest rate risk exposure, it presents two limitations:

i. According to the formula of the Macaulay Duration (McD), we assume that all cash flows are discounted at the same discount rate, which means that we are assuming that the yield curve is flat and all shifts are parallel. However, if a portfolio has bonds with different maturities, the duration measure my not provide a good estimate for unequal changes in interest rates of different maturities.

ii. The accuracy of the duration diminishes if we consider large changes in the yield curve. In this case we have to consider a second order approximation, the convexity (Figure N° 1).

2.4 Key Rate Duration

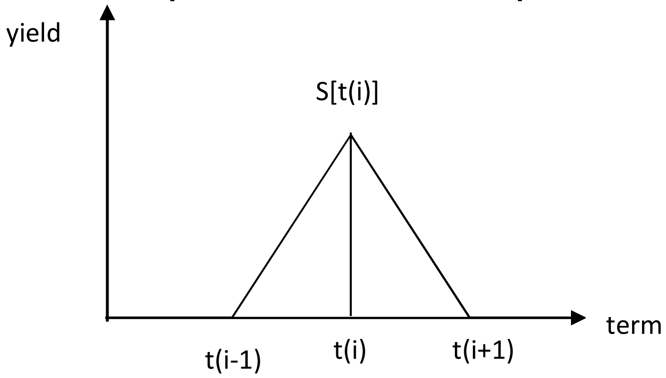
2.4.1 Definition

Effective duration is a standard measure of the interest rate risk exposure of a bond or a portfolio, which has come to have many applications in managing interest rate risk. Its main assumption is that the spot yield curve shift is parallel. Parallel shifts of the spot curve can capture much of the nature of term structure movements, although the returns of two securities with the same effective duration can be significantly different if the yield curve undergoes non-parallel shifts, such as steepness or curvature.

In this sense Ho (1999) introduced a new measure of interest rate risk exposure called "key rate duration". Key Rate Durations (hereafter KRD) is a vector representing the price sensitivity of a security to each key rate change; the market practice is to choose as key rates changes the yield curve movements. The sum of the key rate durations is identical to the effective duration.

Figure N°2

Linear Interpolation of a shift of the spot curve



We denote these key rates by t(i) where i=1,…,m; and S[t(i)] as the shift at each key rate. Then the yield curve shift along the maturity range S(t) can be approximated by linear interpolation of the shifts of each key rate. Note that the linear interpolation is used to model the change of the yield curve, not the yields curve itself. Figure N° 2 depicts the linear interpolation as an approximation to the shift of the spot curve.

2.4.2 Basic Key rate shift

We say that the first key rate shifts x basis points if the first key rate (and all the spot rates with shorter terms than the first key rate) shifts x basis points, and the shifts decline linearly with the increase in term to zero at the second key rate, then remain zero for any maturity beyond the second key rate. That is to say, the first basic yield curve shift requires no spot curve shift beyond the second key rate, and the shift peaks at the first key rate.

Other key rates shifts are defined similarly. In general, the ith key rate shift is defined to be zero shift for maturities shorter than the (i-1)th key rate and longer than the (i+1)th key rate. The shift between the (i-1)th key rate term and the (i+1)th key rate term is defined by the triangle with the peak at the ith key rate. These key rate shifts are depicted in Figure N°3.

Formally, let $s(t,t_i)$ be the ith basic key rate shift of term t, with the level of shift being $\Delta y(t)$. Then,

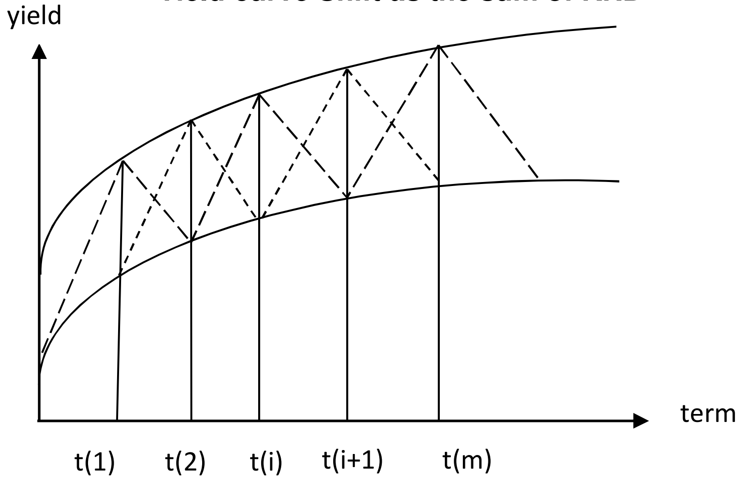
$$s(t,t_1) = \begin{cases} \Delta y(t_1) & t < t_1 \\ \Delta y(t_i)\dfrac{t_2 - t}{t_2 - t_1} & t_1 \le t \le t_2 \\ 0 & t > t_2 \end{cases}$$

$$s(t,t_i) = \begin{cases} \Delta y(t_i) & t < t_{i-1} \\ \Delta y(t_i)\dfrac{t_2 - t_{i-1}}{t_i - t_{i-1}} & t_{i-1} \le t \le t_i \\ \Delta y(t_i)\dfrac{t_{i+1} - t}{t_{i+1} - t_i} & t_i \le t \le t_{i+1} \\ 0 & t > t_{i+1} \end{cases}$$

$$s(t,t_m) = \begin{cases} 0 & t < t_{m-1} \\ \Delta y(t_m)\dfrac{t - t_{m-1}}{t_m - t_{m-1}} & t_{m-1} \le t \le t_m \\ \Delta y(t_m) & t > t_m \end{cases}$$

Figure N°3

Yield curve Shift as the sum of KRD



It is clear that any linear approximation of the yield curve shift Δy(t) can be represented by the sum of all these basic key rate shifts. Thus:

$$\Delta y(t) = s(t, t_1) + s(t, t_2) + \cdots + s(t, t_m)$$

2.4.3 Key Rate Durations

Now say that there is an infinitesimal shift in one specific key rate. This may induce a price change in the security. The price sensitivity to each key rate change is the key rate duration. Formally:

$$\frac{\Delta P_i}{P} = -KRD_i * \Delta y(t_i)$$

Where KDR_i is the ith key rate duration, in a continuous time we got:

$$KRD_i = -\frac{1}{P}\frac{\partial P}{\partial y(t_i)}$$

Given that each infinitesimal change in a key rate contributes to the proportional change in price, and the shift of the yield curve can be approximated by the sum of all the basic key rate shifts, we have:

$$\frac{\Delta P}{P} = -\sum KRD_i * \Delta y(t_i)$$

Key rate durations are in fact a linear decomposition of effective duration. Let the effective duration of a bond be KRD_i , then we have:

$$D = \sum KRD_i$$

2.4.4 Evaluation

Key rate durations have several advantages over existing measures:

i. They can identify the price sensitivity of a bond to each segment of the spot yield curve. Effective duration is the total risk exposure, and the key rate durations are the component parts of the effective duration. As is the case with effective duration, key rate durations aggregate linearly, so that portfolio analysis is straightforward

ii. They recognize that the yield curve movement is driven by multiple market factors. The validity of the key rate durations does not depend on any equilibrium model of the yield curve movement. Key rate durations are applicable over a broad range of arbitrary yield curve movements

iii. It is easy to use key rate durations to create a replicating portfolio of a bond with embedded options using zero-coupon bonds. Thus the cash flow of the replicating portfolio correctly represents the instantaneous expected cash flow of the option

However, there are some areas of concern regarding to the use of the KRD:

i. The choice of the key rates is subjective. A natural election will be to employ as key rates the most common used treasury rates in the markets. Nevertheless, the portfolio manager can reduce (increase) this group according to the structure of his portfolio.

ii. The movements in the yield curve are not independent. For example: changes in the 8-year and 10-year rates in the same direction; but a change in the 9-year rate in the opposite way is possible in the KRD model, but it is unlikely that this happen in the reality.

iii. There is a lost of accuracy due to the fact that the KRD do not use the past yield curves changes to estimate the interest rate risk.

3 Empirical application

In this section, we proceed to compute the key rate durations of a portfolio composed by sovereign, mainly US treasuries bills, and non-sovereign securities. The total return of the portfolio is the weighted sum of the individual returns, where the weights are the percentage participation of each security in the total portfolio.

As it was mentioned before, the traditional technique to measure the exposition of the portfolio to the variations of the interest rates was using the duration. The portfolio managers divide the portfolio into several pools according to the time to maturity in order to have an approximation of the exposition to the risk of change in specific sectors of the curve.

However, these pools have some disadvantages:

i. The decisions with respect to the pools (number and length) are too arbitrarily

ii. We can observe a great variation from one day to the next one, produce by the fact that one security or a group of securities change from one to another pool

iii. It is not an accurate measure of the exposition to a specific sector of the curve

The application of the KRD model is very straightforward. First, we choose the key rates; in the case of the portfolio A, which establishes a maximum individual duration of 2 years, we choose five key rates:

$$KRD = [KRD_{1m} \quad KRD_{3m} \quad KRD_{6m} \quad KRD_{1y} \quad KRD_{2y}]$$

Furthermore:

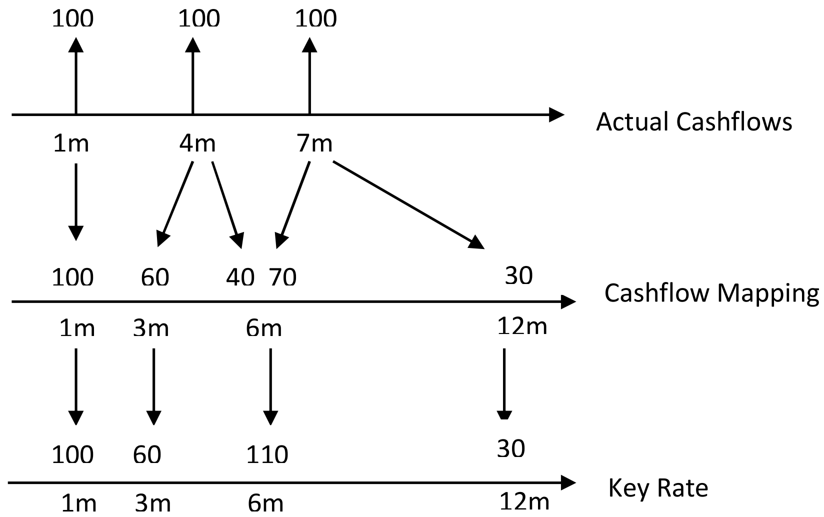
$$Portfolio's\ Duration = \sum_{i=1}^{5} KRD_i$$

However, the choice of the key rates is made under the assumption that the cash flows of all the securities coincide with these key rates. Since it is an assumption that is not possible in all the cases, an essential second-step is to express the securities' cash flows in terms of the limited number of rates, this procedure is known as mapping.

The method selected is the one followed by RiskMetrics: Cash Flow Map. According to this method, fixed income securities can be easily represented as cash flows given their standard future stream of payments. In practice, this is equivalent to decomposing a bond into a stream of zero-coupon instruments. To map the cash flows, we use the key rate closest to the maturity and redistribute the actual cash flows as shown in the example of Figure N°4.

Figure N°4

Mapping actual cash flows onto key rates



The methodology for mapping cash flows is detailed in Appendix A.

The third step is to calculate the key rate duration following the formula:

$$KRD_i = -\frac{1}{P}\frac{\partial P}{\partial y(t_i)}$$

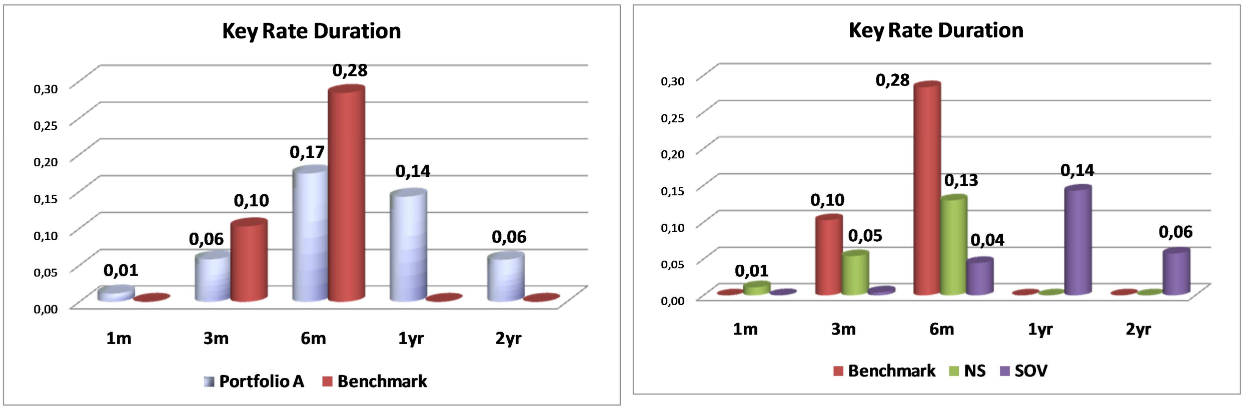
The results for the portfolio and the benchmark are presented in the Figure N° 5. As we can observe in Figure N°5 (a), the portfolio has the l arger expositions in the sector of 6 months and 1 year, which means that it is more sensible to changes in the 6-month and 1-year rates of the yield curve, comparing to the others sectors. Regarding the Benchmark, as it was expected, the larger exposure is in the sector of 6 months; however, there is a considerable sensibility to the 3-month rate change, something that is not taken into account by the traditional measure of duration. Therefore, the portfolio is expecting a flattening of the yield curve[2].

[2] A Flattening of the yield curve indicates that the yield spread between the yield on the long-term and a short-term Treasury has decreased (Fabozz, 2005).

If we compare this expectation with the actual change in the yield curve (Figure N° 7), which remained practically unchanged in the sector of 1 and 2 years; but it moved upward in the sector of 1, 3 and 6 months; we can see that the benchmark was affected negatively, reflecting it in the daily return, with a portfolio's daily return over the benchmark's return of 4 bp.

Figure N° 5

(a) **(b)**



In Figure N° 5 (b), we have the same analysis, but now we decompose the portfolio into Sovereign and Non Sovereign securities. This Figure shows us that, the longer positions (6 months – 2 years) were taken in the sector of Sovereigns, expecting the flattening of the yield curve. Regarding the shorter positions (0 – 6 months), they were composed entirely of Non Sovereigns, reflecting two characteristics: (i) the expectation of increasing short-term rates, and (ii) the proximity to maturity of these instruments.

Additionally, in Figure N° 6, we present the analys is of KRD differentiating between Sovereign and Non Sovereign securities. In both cases, the analysis confirms the results found above. Moreover, in Figure N° 6 (b), we observe that the N on Sovereign securities with a duration between 1 and 2 years have a more sensitivity to the change of 1-year rate than the change of 2-year rate. Once more, the analysis of the duration would not take into account this sensitivity and would have allocated the whole effect to the change in the 2-year rate.

Figure N°6

(a) (b)

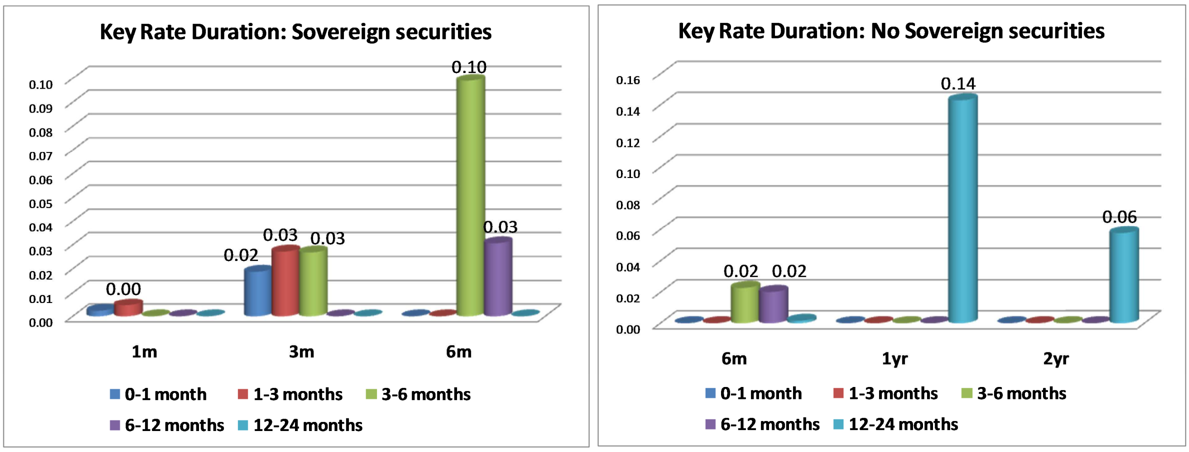
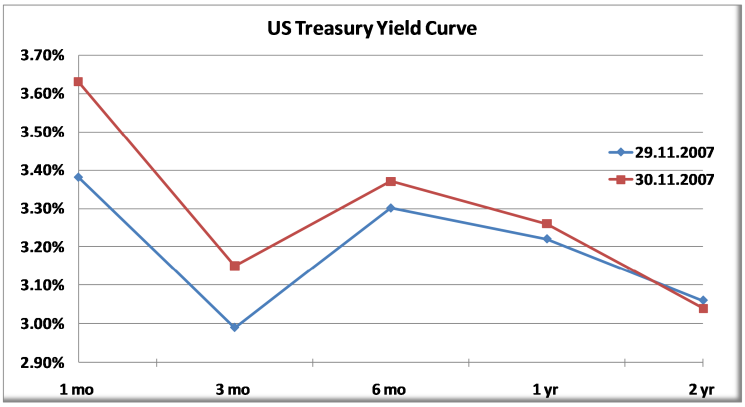


Figure N°7



4. Concluding remarks

One of the most traditional used tools in the asset allocation process is the duration. The duration model assumes that the yield curve experiences infinitesimal and parallel shifts; however, we know that shorter maturity rates are more volatile than the longer maturity rates, so the assumption of parallel yield curve shifts is obviously false.

In this sense, recently a new class of models called the key rate durations has become popular among investors. Similar to the duration models, key rate durations can manage interest rate risk exposure arising from arbitrary nonparallel shifts in the term structure of interest rates. The duration model hedges against the shape changes in the term structure of interest rates, while the key rate durations hedges against the changes in a finite number of key interest rates that proxy for the shape changes in the entire term structure.

Key rate durations are then defined as the sensitivity of the portfolio value to the given key rates at different points along the term structure. These duration measures can be used in decomposing portfolio returns and identifying interest rate risk exposure. Therefore, key rate durations offer a better approach than the traditional duration to measure the interest rate risk, making easier to portfolio managers the asset allocation process.

However, some authors consider that key rate models have three limitations: (i) the choice of the key rates is arbitrary, (ii) the unrealistic shapes of the individual key rate shifts, and (iii) loss of efficiency by not modeling the history of term structure movements.

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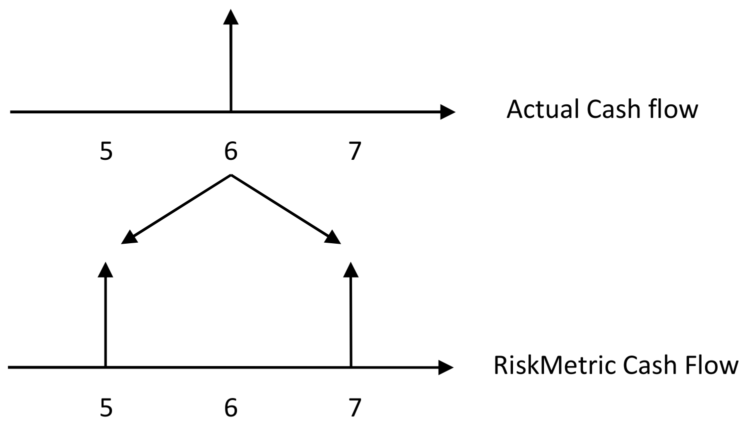
6. Appendices

Appendix A: RiskMetrics Mapping methodology

For allocating actual cash flows to RiskMetrics vertices, RiskMetrics proposes a methodology that is based on the variance (σ^2) of financial returns. The advantage of working with the variance is that it is a risk measure closely associated with one of the ways RiskMetrics computes VaR, namely the simple VaR method as opposed to the delta-gamma or Monte Carlo methods.

In order to facilitate the necessary mapping, the RiskMetrics data sets provide users with volatilities on, and correlations across many instruments in 33 markets. For example, in the US government bond market, RiskMetrics data sets provide volatilities and correlations on the 2-, 3-, 4-, 5-, 7-, 9-, 10-, 15-, 20-, and 30-year zero coupon bonds.

Consider the following example:



We denote the allocations to the 5- and 7-year vertices by α and (1- α), respectively. The procedure presented below is not restricted to fixed income instruments, but applies to all future cash flows.

1. Calculate the actual cash flow's interpolated yield:
   We obtain the 6-year yield, y_6, from a linear interpolation of the 5- and 7-year yields provided in the RiskMetrics data sets. Using the following equation,

$$y_6 = \hat{a}y_5 + (1 - \hat{a})y_7 \qquad 0 \leq \hat{a} \leq 1$$

Where y_6: interpolated 6-year zero yield

$\hat{a}$: linear weighting coefficient,

y_5: 5-year zero yield

y_7: 7-year zero yield

If an actual cash flow vertex is not equidistant between the two RiskMetrics vertices, then the greater of the two values, â and (1-â), is assigned to the closer RiskMetrics vertex.

2. Determine the actual cash flow's present value:
   From the 6-year zero yield, y_6, we determine the present value, P_6, of the cash flow occurring at the 6-year vertex. (In general, P_i denotes the present value of a cash flow occurring in i years.)
3. Calculate the standard deviation of the price return on the actual cash flow:
   We obtain the standard deviation, σ_6, of the return on the 6-year zero coupon bond, by a linear interpolation of the standard deviations of the 5- and 7-year price returns, i.e., σ_5 and σ_7, respectively.
   Note that σ_5 and σ_7 are provided in the RiskMetrics data sets as the VaR statistics $1.65*\sigma_5$ and $1.65*\sigma_7$ respectively. Hence, $1.65*\sigma_6$ is the interpolated VaR. To obtain σ_6, we use the following equation:

$$\sigma_6 = \hat{a}\sigma_5 + (1-\hat{a})\sigma_7 \qquad 0 \leq \hat{a} \leq 1$$

Where $\hat{a}$: linear weighting coefficient,

σ_5: standard deviation of 5-year return

σ_7: standard deviation of 7-year return

4. Compute the allocation, α and (1-α), from the following equation:

$$Variance(r_{6yr}) = Variance[\alpha r_{5yr} + (1-\alpha) r_{7yr}]$$

$$\sigma_6^2 = \alpha^2\sigma_5^2 + 2\alpha(1-\alpha)\rho_{5,7}\sigma_5\sigma_7 + (1-\alpha)^2\sigma_7^2$$

where $\rho_{5,7}$, is the correlation between the 5- and 7- year returns. (Note that $\rho_{5,7}$ is provided in the correlation matrix in RiskMetrics data sets). This equation can be written in the quadratic form:

$$a\alpha^2 + b\alpha + c = 0$$

Where $a = \sigma_5^2 + \sigma_7^2 - 2\rho_{5,7}\sigma_5\sigma_7$

$b = 2\rho_{5,7}\sigma_5\sigma_7 - 2\sigma_7^2$

$c = \sigma_7^2 - \sigma_6^2$

The solution to α is given by

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. Distribute the actual cash flow onto the RiskMetrics vertices:
   Split the actual cash flow at year 6 into two components, α and (1-α), where you allocate α to the 5-year RiskMetrics vertex and (1-α) to the 7-year RiskMetrics vertex.