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Demand Uncertainty and Capacity Utilization in Airlines*

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Abstract

This paper studies the relationship between demand uncertainty—the key source of excess capacity—and capacity utilization in the U.S. airline industry. We present a simple theoretical model that predicts that lower demand realizations are associated with higher demand volatility. This prediction is strongly supported by the results of estimating a panel GARCH framework that pools unique data on capacity utilization across different flights and over various departure dates. A one unit increase in the standard deviation of unexpected demand decreases capacity utilization by 21 percentage points. The estimation controls for unobserved time-invariant specific characteristics as well as for systematic demand fluctuations.

Keywords: Demand uncertainty; capacity utilization; airlines; panel GARCH; GARCH-in-mean

JEL Classification: C33; L93

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1. Introduction

Sellers that need to decide production levels before demand is realized are likely to finish the selling season with unsold inventories. This is a typical problem in industries such as airlines, automobile rentals, hotels, hospitals, restaurants, theaters, fashion apparel, and sporting events. These industries are characterized by having a highly volatile demand, capacity—or inventory—is fixed or can only be modified at a relatively high marginal cost, and excess capacity that expires once the selling season is over. Unsold inventories are, of course, an inefficient allocation of resources. Based on data from the Bureau of Transportation and Statistics, 19.8% of U.S. domestic flights' seating capacity was empty in 2009. Dana and Orlov (2009) estimate that for the U.S. airline industry, a 6.7% increase in capacity utilization—the ratio of inventories sold to total inventory levels—translates into \$2.7 billion in cost savings each year.^{1,2} It is easy to understand that the key source behind excess capacity is demand uncertainty; without demand uncertainty, airlines would simply choose the level of capacity for a particular flight to match perfectly the level of its demand. Borenstein and Rose (2007) explain that large volatility in airlines' profits comes mainly from large volatility in demand. Hence, demand uncertainty and its effect on capacity utilization are particularly important issues in light of the recent turmoil in the industry (see Berry and Jia, 2010).

Despite its importance, there is relatively little empirical work on capacity utilization, mostly due to the difficulty in many industries of coming with an empirical measure.³ For the

¹ For hospitals, Gaynor and Anderson (1995) estimated that increasing the occupancy rate from 65 percent to 76 percent reduced costs by 9.5 percent.

² Capacity utilization is important for other industries as well. Kim (1999) argues that it is an important issue in economic analysis, while Schultze (1963) explains that it serves as a productivity measurement and can be used as an indicator of the strength of aggregate demand.

³ Nelson (1989) discusses practical problems in measuring capacity utilization and offers suggestions for estimating theoretical measures, while Shapiro (1993) describes how to estimate the capital utilization of an industry as a whole using the survey data of individual plants. Kim (1999) argues that conventional capacity utilization measures (e.g.,

airlines, however, the measure of capacity utilization is relatively straightforward. Potential capacity is directly observed as the total number of available seats on a scheduled flight, whereas the utilized capacity equals the amount of seats sold. While monthly data for calculating capacity utilization is available from the Bureau of Transportation Statistics T-100 database, these data are perhaps too aggregate to capture demand uncertainty. In this paper we follow the recent work in Escobari (2009, 2012) and observe day-to-day fluctuations in capacity utilization across different flights and over various departure dates, which are more appropriate to capture demand uncertainty.

Our work is motivated by a body of literature on capacity utilization. Hubbard (2003) examines the extent to which the use of on-board computers, which reduces demand uncertainty, raises capacity utilization and thus productivity in the trucking industry. Similarly, Dana and Orlov (2009) show that capacity utilization increases when the proportion of informed consumers in a market is larger. Deneckere and Peck (2012) present a multiple-period price posting model that predicts no underutilized capacity because in the last period sellers set prices to clear the market. Underutilized capacity, however, is possible in stochastic peak load pricing models; if demand realizations are known only after firms set capacity and prices, idle capacity can still exist during off-peak times (see e.g., Brown and Johnson, 1969; and Carlton, 1977). Underutilized capacity is also possible in Prescott's (1975) competitive model, where capacity is costly and there are price commitments. Dana (1999) extends this model to a monopoly and imperfect competition.

Our findings are also related to the literature on irreversible investment and excess capacity. Pindyck (1988) finds that in a market with volatile demand, firms should hold less

Nelson, 1989) appear to be biased and proposes a measure that incorporates information about production and demand.

capacity than if future demand is known otherwise. Gabszewicz and Poddar (1997) also find that excess capacity exists in oligopolistic markets when demand is uncertain. Alderighi (2010) extends the work of Belobaba (1989) to present theory and simulations that suggest a negative relationship between demand uncertainty and capacity utilization. However, Bell and Campa (1997) study the chemical processing industry and find that volatility in product demand has no effect on capacity utilization.

Against the above background, this paper reexamines—theoretically and empirically—the relationship between demand uncertainty and capacity utilization. Our theory builds on the market competition model developed by Prescott (1975). We show that if prices are set in advance based on a distribution of demand uncertainty, then higher demand uncertainty is associated with a lower average demand realization and thus lower average capacity utilization. Our empirical work takes advantage of a unique panel data set from the U.S. airline industry. Demand uncertainty is assumed to follow a GARCH (Generalized Autoregressive Conditional Heteroskedasticity) framework, where we further extend the conventional GARCH model to the panel regression framework.⁴ As shown in Cermeño and Grier (2006) and Lee (2010), there is substantial efficiency gain in the estimation of the conditional variance and covariance processes in the GARCH model when the estimation also incorporates interdependence across different flights within each panel.

In line with the theoretical prediction, our empirical results specifically indicate that a one-unit increase in the standard deviation of unexpected demand is associated with a 21 percentage point decrease in capacity utilization. This result is robust to cumulative ticket sales data at different points prior to the departure date as well as different sets of control variables.

⁴ The GARCH modeling approach is widely used in the financial economic literature to measure market uncertainty with conditional volatility over time.

Besides controlling for unobserved time-invariant flight-number-, route-, and carrier-specific characteristics, the estimation controls for systematic demand fluctuations associated with the different days of the week and major holidays.

The organization of the paper is as follows. In Section 2 we develop a simple theoretical model to illustrate the link between demand uncertainty and capacity utilization. Section 3 describes the data. The empirical model and estimation methods are outlined in Section 4. Section 5 presents the estimation results. Finally, Section 6 concludes.

2. Demand Uncertainty and Average Capacity Utilization

This section presents a simple theoretical model based on Prescott (1975) to understand the link between demand uncertainty and capacity utilization. Reflecting some key features of airline markets, this model explains price dispersion and underutilized capacity in perfect competition where there exists demand uncertainty and firms decide output in advance (i.e., capacity is costly). We begin by providing motivation for the existence of an upward schedule of prices, as largely documented in the airline industry (see, e.g. Bilotkach et al., 2010; Escobari and Gan, 2007; Mantin and Koo, 2009). To this end, we follow Prescott (1975) and Dana (1999) and derive a price schedule by assuming that prices are set in advance based on the aggregate demand uncertainty distribution. Next, we use this price schedule to show how the mean of the distribution of demand realizations is lower when the demand uncertainty is higher.

2.1. Price Schedule and Demand Realizations

Consider a competitive model in which sellers that offer airline seats take the distribution of prices and quantities as given. There is aggregate demand uncertainty in the form of $H+1$

demand states denoted by $h = \{0, 1, 2, \dots, H\}$. We use $\{\rho_0, \rho_1, \dots, \rho_H\}$ to denote the probability associated with each of the demand states. Let $DEMAND_h$ be the number of consumers who buy plane tickets at demand state h . We assume that demand states are ordered, meaning that consumers who buy tickets at demand state h will also buy tickets at a higher-numbered demand state, i.e., $DEMAND_{h+1} \geq DEMAND_h$. Hence, the probability that at least $DEMAND_h$ consumers buy tickets is obtained by adding the probabilities of all higher-numbered demand states, $\sum_{\kappa=h}^H \rho_\kappa$. Of course, $\sum_{h=0}^H \rho_h = 1$.

As in Prescott (1975) and Dana (1999), airlines face a unit cost of capacity equal to λ for all seats on a particular flight, whether they are sold or not. In equilibrium and under the assumption of a competitive market, the expected (economic) profit is zero. Then, the model predicts dispersed prices given by:

$$p_\omega = \frac{\lambda}{\sum_{h=\omega}^H \rho_h} \quad \text{for } \omega = \{0, 1, 2, \dots, h\}, \quad (1)$$

over the range $\lambda \leq p_\omega \leq \theta$, where θ is the highest reservation value for a given seat. There are $\omega = \{0, 1, 2, \dots, h\}$ different batches of consumers who buy tickets at demand state h , and each of the batches pays a different price as given by equation (1). This is the widely used Prescott (1975) spot market equilibrium (see also Eden, 1990; Dana, 1998; and Dana, 1999).

The intuition behind the dispersed prices in equation (1) is simple. Consider the following example in which the unit cost of capacity is $\lambda = 1$ and there are two equally likely demand states. During low demand, only one consumer buys a ticket; during high demand, two consumers buy tickets. The first consumer buys in both demand states; hence, she buys with probability 1 and pays a price of \$1. The second consumer buys only during the high demand

state, which occurs only half of the times, thus she pays \$2. Notice that in both demand states the expected profit is equal to the unit cost of capacity, hence complying with the zero expected profit condition.

Even though the above setting is a one-period model because sellers are not allowed to update their prices during the selling season, it can have an interesting dynamic interpretation.⁵ Different batches ω can be thought of as arriving sequentially and because airline seats are homogeneous, consumers always prefer the cheapest remaining ticket. Then the next batch of consumers arrives and buys at the next available lowest price. There is price dispersion across consumers of different batches and those consumers who arrive in latter batches pay higher prices.

Given the price schedule in equation (1), we now derive the corresponding equilibrium demand realizations.⁶ Suppose that airplane seats are homogeneous, and let consumers within each batch ω have reservation values that are uniformly distributed $[0, \theta]$. Therefore, the number of seats sold for each of the batches can be written as:

$$DEMAND_{\omega} - DEMAND_{\omega-1} = \left(1 - \frac{P_{\omega}}{\theta}\right) \quad \text{for } \omega = \{1, 2, \dots, h\}, \quad (2)$$

where $DEMAND_0 = 0$. Hence, the realized aggregate demand at state h is obtained by summing across all batches in h :

$$DEMAND_h = \sum_{\omega=1}^h \left(1 - \frac{P_{\omega}}{\theta}\right). \quad (3)$$

⁵ The dynamic interpretation is in line with Hazledine (2010) and Kutlu (2012), although these papers work under demand certainty. Deneckere and Peck (2012) present a generalization of Prescott's (1975) one-period model to allow sellers to change prices over different periods.

⁶ Notice that we keep track of two distributions that capture demand uncertainty. The first is the distribution of demand states h and the second is the distribution of demand realizations, $DEMAND_h$.

2.2. Link between Demand Uncertainty and Capacity Utilization

As in Prescott (1975) and Dana (1999), one key characteristic of this model is that airlines set the schedule of prices based on the distribution of demand uncertainty, and those prices remain fixed throughout the selling period. Now, to see the predictions of this model for the link between volatility in demand realizations and average demand realizations, we first derive the price schedule using equation (1) for a given distribution of demand uncertainty. By keeping prices fixed *a priori*, we will then use equation (3) to show the effects of a change in the mean of the distribution of demand uncertainty on both the mean and variance of the demand realizations.

Suppose the demand uncertainty that a flight faces when deriving its price schedule follows a discrete uniform distribution with $H = 20$, i.e., $h = \{0,1,2,\dots,20\}$. Hence, $\rho_h = 1/(H + 1)$. Furthermore, let $\lambda = 1$ and $\theta = 10$. Using the price schedule derived from equation (1), we fix the mean of h at 10 and present in Table 1 the means and standard deviations of the demand realizations $DEMAND_h$ for different standard deviations of the distribution of demand uncertainty. The results show that a higher volatility in the realizations of demand, as measured by standard deviation of $DEMAND_h$, is associated with lower average demand realizations.

[Table 1, about here]

[Figure 1, about here]

The intuition behind this negative relationship is illustrated in Figure 1. Based on the previous example the fourth quadrant plots two different distributions of h i.e., $h = \{2,\dots,18\}$ and

$h = \{5, \dots, 15\}$. The solid line in the first quadrant is the $DEMAND_h$ schedule used in Table 1, which maps the two distributions of h into the distributions of $DEMAND_h$ presented in the second quadrant. Along with the distributional assumption of h , equation (1) generates a nondecreasing convex schedule of prices, which translates into a nondecreasing concave $DEMAND_h$ function. Hence, a larger volatility in the distribution of demand states causes the last batches of consumers that arrive at higher demand states to face relatively higher prices. Because individual consumers have their own downward sloping demand schedules, these higher prices translate into lower ticket sales and hence a lower mean in the demand realizations. This can be appreciated in the second quadrant at higher demand realizations, where the frequencies get closer together.⁷ Notice that while the derivation of the $DEMAND_h$ schedule draws on the particular models found in Prescott (1975) and Dana (1999), our main conclusion does not need to rely on the specifics of these models or the functional form of the demand. Figure 1 shows that similar settings that result in a nondecreasing concave function for $DEMAND_h$ can have the same empirical implication.

Notice that if we abstract prices from the analysis, a simplified setup can also illustrate the negative link between demand uncertainty and capacity utilization. Assume that two distributions of demand states share the same mean but one distribution has a larger variance. If the plane capacity contains the two distributions entirely, then the mean is the same for both. However, this is not the case if an aircraft's limited capacity does not fully contain those

⁷ At the highest demand state when $h = \{2, \dots, 18\}$, the last batch of consumers faces a price larger than θ and so does not buy any tickets. This explains why the highest demand realization of $DEMAND_h = 13.59$ is twice as likely—during the two highest demand states of $h = 17$ and $h = 18$.

distributions so that truncations occur. As a result of truncation, the observed (conditional) mean will be smaller for the distribution with higher variance.⁸

3. Data

For empirical analysis, we collected U.S. airlines' realized demand data from the popular online travel agency expedia.com. Following Escobari (2009, 2012), we looked up data on the map of seats on each aircraft and counted the total number of seats in the aircraft (total aircraft's capacity) and the number of seats sold up to 15 days, 8 days and 1 day prior to departure, respectively. Because overbookings are usually a small fraction of ticket sales, we assume that our measure is proportional to bookings.⁹ For the production of nonperishable goods, inventories can be used to absorb demand shocks that can lead to deviations between production and sales. In the case of perishable goods such as airline seats, however, cumulative ticket sales are a measure of realized demand, so that unsold inventories are a measure of idle capacity. Realized demand, which is capacity utilization for a specific flight, is calculated as the ratio of occupied seats to the total number of available seats on an aircraft.¹⁰

We collected three sets of panel data by the number of days prior to departure. Each data set is a panel that pools seat inventories of 20 flights ($N=20$) across a fixed period of 126 days ($T=126$). More specifically, the first set measures seat inventories at one day prior to departure for the 20 specific flights over 126 consecutive days between Tuesday, June 2 and Monday, October 5, 2009. Correspondingly, the second dataset consists of inventories at 8 days prior to

⁸ We thank an anonymous referee for raising this point. In the empirical work below, the effects of truncated conditional means will be taken into consideration.

⁹ Seats protected for later purchases (usually labeled as preferred or prime seats) are counted as available seats. This is consistent with serial nesting of booking classes. In this case, for booking classes within the same cabin seats from a higher booking class (e.g., prime seats) are ready to be released into a lower booking class if needed (e.g., in an expected off-peak flight), see Escobari (2012, p. 719).

departure for the same 20 flights and over 126 consecutive days between Tuesday, June 9, and Monday, October 12, 2009. The third consists of corresponding data at 15 days to departure for flights departing from Tuesday, June 16, to Monday, October 19, 2009. Accordingly, each dataset contains a total of 2,520 observations, where each cross-sectional unit is a non-stop, one-way flight-number from a carrier on a particular domestic route in the U.S. Each flight-number (e.g. American Airlines Flight 637 from Miami, FL to New Orleans, LA) is offered every day with the same aircraft size. A route is defined as a pair of departure and destination airports, and the carriers with flights in the data sample are Alaska, American, Delta, United and US Airways. In model estimation, the panel structure of the data will allow us to control for unobserved time-invariant flight-number-, carrier-, and route-specific characteristics that may affect demand realizations. Time invariant characteristics include the distance between airports, the aircraft type, and the unit cost of capacity λ .

[Table 2, about here]

Table 2 displays some descriptive statistics for the airline data across the panel of 20 flights over the different sample periods of 126 days. The 20 flights had an average capacity of 103 seats. The smallest aircraft carried a capacity of 50 seats, and the largest aircraft carried a capacity of 166 seats. The columns in the panel of utilized capacity show the statistics for the proportions of seats sold to total seats in the aircraft at 15-days, 8-days, and 1-day prior to flights departures. An average of 74% of seats were sold 15 days prior to departure, compared to 82% for 8 days and 89% for one day prior to departure. The dispersion of utilized capacity across

¹⁰ Bilotkach et al. (2011) use similar information on seat capacity availability to see how yield management affects a flight's load factors.

flights, as measured by the standard deviation, ranges from 0.18 in the 15-days-to-departure panel to 0.14 in the 1-day-to-departure panel.

4. Empirical Model

In this section, we present the empirical model for estimating the relationship between demand uncertainty and capacity utilization in the airline industry. Realized demand for air travel is measured by cumulative ticket sales for a particular flight. A flight's capacity utilization is the ratio of purchased seats to the total number of seats in the aircraft. Given the panel nature of our dataset and our focus on demand uncertainty, we consider GARCH-type models that also take into account interdependence across flights. For a cross-section of N flight-numbers, T departure dates and a fixed number of days to departure, the conditional mean equation for air travel realized demand ($DEMAND_{it}$) can be expressed as a dynamic panel model with fixed effects:

$$DEMAND_{it} = \sum_{k=1}^K \alpha_k DEMAND_{i,t-k} + \mathbf{x}_{it} \boldsymbol{\beta} + \mu_i + \varepsilon_{it}, \quad i = 1, \dots, N; t = 1, \dots, T, \quad (4)$$

where the subscript i refers to a specific flight-number and the subscript t refers to a given departure date. Notice that the definition of the variable $DEMAND_{it}$ in this section is analogous to $DEMAND_h$ in the theoretical model of Section 2. The subscript h is replaced by the subscripts i and t because, for simplicity, the theoretical section presents a single period model, while the empirical model in this section identifies demand uncertainty through different demand realizations $DEMAND_{it}$ across flights i and over time t . The term \mathbf{x}_{it} is a vector of exogenous variables with coefficients captured by the vector $\boldsymbol{\beta}$. The term μ_i captures possible time-

invariant effects associated with the given routes, carriers, airports and flights; and ε_{it} is a disturbance term with the following conditional moments:

$$E[\varepsilon_{it}\varepsilon_{js}] = 0 \quad \text{for } i \neq j \text{ and } t \neq s, \quad (5)$$

$$E[\varepsilon_{it}\varepsilon_{js}] = 0 \quad \text{for } i = j \text{ and } t \neq s, \quad (6)$$

$$E[\varepsilon_{it}\varepsilon_{js}] = \sigma_{ij,t}^2 \quad \text{for } i \neq j \text{ and } t = s, \quad (7)$$

$$E[\varepsilon_{it}\varepsilon_{js}] = \sigma_{it}^2 \quad \text{for } i = j \text{ and } t = s. \quad (8)$$

The first condition assumes no non-contemporaneous cross-sectional correlation, and the second condition assumes no autocorrelation. The third and fourth assumptions define the general conditions of the conditional variance-covariance process.

Demand uncertainty is captured by conditional volatility in the disturbance term in the condition mean equation (4). Due to its popularity and parsimony, the conditional variance and covariance processes of ε_{it} are assumed to follow a GARCH(1,1) process:

$$\sigma_{it}^2 = \phi_i + \gamma\sigma_{i,t-1}^2 + \delta\varepsilon_{i,t-1}^2, \quad i = 1, \dots, N, \quad (9)$$

$$\sigma_{ij,t} = \varphi_{ij} + \eta\sigma_{ij,t-1} + \rho\varepsilon_{i,t-1}\varepsilon_{j,t-1}, \quad i \neq j \quad (10)$$

Using matrix notation, equation (4) can be written as:

$$DEMAND_t = \mathbf{Z}_t\boldsymbol{\theta} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad (11)$$

where $DEMAND_t$ and $\boldsymbol{\varepsilon}_t$ are $N \times 1$ vectors, $\boldsymbol{\mu}$ is the corresponding $N \times 1$ vector of individual-specific effects, s and $\mathbf{Z}_t = [DEMAND_{t-1} \dots \mathbf{x}_t]$ is a matrix with their corresponding coefficients in $\boldsymbol{\theta} = [\alpha_k \dots \beta]'$. The disturbance term has a multivariate normal distribution $N(\mathbf{0}, \boldsymbol{\Omega})$.

Because the disturbance term $\boldsymbol{\varepsilon}_t$ is conditional heteroskedastic and cross-sectionally correlated, the least-squares estimator for this model is no longer efficient even though it is still consistent. Alternatively, Cermeño and Grier (2006) and Lee (2010) suggest the application of the maximum-likelihood (ML) method, which maximizes the following log-likelihood function:

$$L = -\frac{1}{2} \left\{ NT \log(2\pi) + \sum_{t=1}^T \log |\mathbf{\Omega}_t| + \sum_{t=1}^T [(DEMAND_t - \mathbf{Z}_t \boldsymbol{\theta} - \boldsymbol{\mu})' \times \mathbf{\Omega}_t^{-1} (DEMAND_t - \mathbf{Z}_t \boldsymbol{\theta} - \boldsymbol{\mu})] \right\}. \quad (12)$$

However, there is yet another issue in the estimation. Because capacity utilization is constrained to be less than 100%, the disturbance term $\boldsymbol{\varepsilon}_t$ has a truncated normal distribution. As a result, estimation with the log-likelihood function of equation (12) would result in biased coefficient estimates. To estimate the dependent variable that is truncated from above, we adopt Wooldridge's (1999) quasi-conditional maximum likelihood (QCML) method, which essentially augments the log-likelihood function with a condition that depends on the truncation.

5. Estimation Results

Our empirical work begins with specifying a baseline model for estimating realized demand for airline tickets. For each of the alternative 1-day, 8-days and 15-days-in-advance tickets, the conditional mean equation is expressed as an AR(7), meaning that 7 autoregressive lagged values of the dependent variables are included in \mathbf{Z}_t . This model specification is determined in light of the Bayesian Information Criterion, which suggests a rather long lag structure. The particular autoregressive model specification is also in line with the number of days within a week. As pointed out above, realized demand— $DEMAND_{it}$ in equation (4)—is measured as the ratio of occupied seats to the total number of available seats.

[Table 3, about here]

Table 3 presents the estimation results for the AR(7) model of airline ticket demand estimated with OLS along with heteroskedasticity and autocorrelation-consistent (HAC) standard

errors. The different columns show the individual regression results of 1-day-, 8-days-, and 15-days-in-advance tickets for the 20 particular flight-numbers in the sample. Except for the second lag, most coefficient estimates are statistically significant. The positive coefficients for the first and seventh lags—which are the largest—imply that demand is positively correlated with the demand the previous day and the demand the same way of the week from the week before. The negative coefficient for the second lag is only significant at a 10% level for the 1-day-in-advance specification. As discussed below, the statistical significance of this lag disappears once we include the GARCH process in the model. The R^2 statistics indicate that the three regressions explain 50% to 65% of variations in the measures of realized demand.

[Table 4, about here]

Given the OLS regression results for the AR(7) specification of the conditional mean equation, Table 4 reports diagnostic statistics for testing serial correlation. The Ljung-Box Q -statistics and partial correlations are computed for both the residuals and squared residuals in orders up to 7 autoregressive lags. In the case of residuals, most partial correlations are not statistically significant. The only exceptions are the estimates for the seventh lag. The significant estimates reflect correlation between ticket sales during the same day of the week. The negative estimates may reflect airlines' increased efforts in reducing any idle capacity observed in the past. There is scant evidence of serial correlation in the residuals, meaning that the condition in (6) is satisfied. However, the partial correlations for squared residuals suggest a rather high-order ARCH process. These statistics support the application of a GARCH-type model.

[Table 5, about here]

Next, we evaluate flight-specific effects in the variance and covariance equations by applying likelihood-ratio (LR) tests based on the log-likelihood values of the panel GARCH(1,1) model estimated separately with and without individual effects. The complete model is captured by equations (4) through (10). The conditional mean equation is the AR(7) as described above. Table 5 shows the LR statistics for testing individual effects in the variance and covariance equations. All test statistics are statistically significant, supporting the presence of flight-specific effects for the 1-day-, 8-days- and 15-days-in-advance tickets.

Motivated by the test results in Table 5, we report in Table 6 the estimates for the panel GARCH(1,1) model with individual effects in the variance and covariance equations. Again, the results are displayed for the 1-day-, 8-days- and 15-days-in-advance tickets alternatively. For all three datasets, the log-likelihood values of the QCML estimation are appreciably higher than their OLS counterparts shown in Table 3, even though the coefficient estimates in the conditional mean equation are quite similar. For the 1-day-in-advance cumulative ticket sales, the estimated coefficients on the autoregressive terms in the conditional variance and covariance equations are 0.60 and 0.54, respectively. These estimates indicate that demand volatility in individual flights and their comovements across flights follow moderately persistent GARCH processes. By comparison, the measure of persistence in demand volatility is higher at 0.73 for the 8-days-in-advance realized demand, but lower at 0.42 for the 15-days-in-advance realized demand. For the covariance equation, the corresponding measure of persistence is higher for both 8-days- and 15-days-in-advance data.

In the variance equation, the estimate for the lagged squared disturbance term, $\varepsilon_{i,t-1}^2$, is the highest (0.93) for seats sold one day prior to departure. This highlights the greater impact of a shock to market demand on a flight's utilized capacity one day prior to departure than 8 or 15 days prior to departure. Similarly, the estimate for the second term in the covariance equation, $\varepsilon_{i,t-1} \varepsilon_{j,t-1}$, is statistically significant only in the case of the 1-day-in-advance tickets. The negative estimate indicates that a shock to one flight reduces its covariance, or interdependence, with another flight.

[Table 6, about here]

To explore the possible association between realized demand and demand uncertainty in air travel, we augment the conditional mean equation with the conditional standard deviation of shocks to the dependent variable (σ_{it}), which captures demand uncertainty. This term is equivalent to the standard deviation of the demand realizations, $DEMAND_{it}$, presented in the third column of Table 1 from the theoretical model in Section 2. Extending Engle et al.'s (1987) model to a panel setting, we add σ_{it} as an additional explanatory variable in the conditional mean equation (4). The resulting regression model is regarded as a GARCH-in-mean process.

The first column of Table 7 shows the estimation results for the panel GARCH-in-mean model for the 1-day-in-advance realized demand. The coefficient estimate for the conditional standard deviation term enters with a negative sign and it is statistically significant at the 1% level. This estimated coefficient indicates that, all else being equal, a one unit increase in the standard deviation of unexpected demand decreases capacity utilization by 21 percentage points. While the inclusion of the GARCH term in the conditional mean equation does not noticeably

affect any of the estimates previously reported in Table 6, this GARCH-in-mean specification is preferable given its higher log-likelihood value over the basic GARCH parameterization.

Similarly, the regression results for the 8-days-in-advance tickets (second column) and 15-days-in-advance tickets (third column) reaffirm a negative correlation between conditional volatility and mean realizations in demand for airline tickets. In comparison with the estimate for the 1-day-in-advance tickets, the absolute size of the coefficient estimate is about half as large for the 8-days-in-advance tickets, but rather similar for the 15-days-in-advance tickets. While it is intuitive to argue for lower demand uncertainty and higher capacity utilization during a date closer to the flight departure, our theoretical model does not have any predictions on how the link between these two variables changes as the departure date nears. The point estimates suggest non-monotonicity in the effect. The differences in the point estimates across columns could be a result of consumer heterogeneity at different points prior to departure.

[Table 7, about here]

Given the above findings, we further carry out some sensitivity analysis. In particular, it is well known that air travel demand is typically higher during weekends and holidays (e.g., Escobari, 2009). To evaluate whether our findings are robust to the presence of the day of the week and holiday effects, we also estimate the panel GARCH-in-mean model along with some day-dependent dummy variables. The first four dummy variables take the value of 1 for flights departing on a Tuesday, Wednesday, Thursday and Friday, respectively, and the value of 0 otherwise. These variables control for unobserved effects associated with the specific day of the week in comparison with Monday. Another dummy variable is *WEEKEND*, which takes the

value of 1 for Saturdays and Sundays. The final dummy variable, *HOLIDAYS*, takes the value of 1 for the days before and after the Independence Day and the Labor Day.

Table 8 shows the results for the three datasets estimated with the addition of those dummy variables within the panel GARCH-in-mean framework. All the dummy variables enter with the expected signs and they are also statistically meaningful. More specifically, the coefficient estimates suggest that airline demand is relatively lower on flights departing on Tuesdays in comparison with Mondays, but higher during weekends and national holidays. The estimates are positive for Thursday and Friday in the cases of the 8-days- and 15-days-in-advance tickets, but not the 1-day-in-advance tickets. These results suggest that the day of the week matters only for travelers who purchase airline tickets well in advance.

[Table 8, about here]

Despite the consideration of weekday and holiday effects, the estimates on the coefficient of the conditional volatility variable (σ_{it}) reaffirm our previous finding about the relationship between realized demand and demand uncertainty. Their quantitative estimates are largely unaffected by the inclusion of additional control variables. Overall, the results in Table 8 lend strong support to the robustness of our main conclusion.

One dimension that we control in our empirical framework is the effect that days to departure may have on capacity utilization and demand volatility. This is important because as Table 2 suggests, capacity utilization is higher and demand volatility is lower when it is closer to departure; hence, the correlation between capacity utilization and demand uncertainty can be driven by days to departure. Such identification in this paper comes from observing demand

realizations across different flights and departure dates, keeping days to departure fixed at 1, 8, or 15 days. This strategy, however, does not consider ticket prices. This would be a concern if the observed demand realizations are correlated with the prices of the tickets that have been sold for the same flight during previous dates. If that is the case, then our estimates may be biased due to the omitted price variable. However, two possible conditions about prices can exist conceptually. First, a seller may lower prices to boost sales, suggesting a negative correlation between prices and demand realizations. Second, the seller may only want to lower prices if sales are falling short, which suggests a positive correlation. Thus, it is not clear whether we should expect a positive or a negative sign for the price variable that enters the regression models. Moreover, there are various prices for each level of capacity utilization and ultimately the correlation between previous prices and capacity utilization depends on the sequences of prices and sales as the departure date nears. This in turn depends on the degree of price flexibility and how airlines use advance sales to learn about the aggregate demand. Such issues are beyond the scope of this study.¹¹ Notice that while we do not have ticket prices in equation (4), including flight-number fixed effects allows for systematic price differences across flight-numbers. Moreover, the day-dependent and holiday dummies control for price differences across different days of the week and holidays.¹²

Other variables that can potentially affect capacity utilization are, for example, managerial capacity, whether the flight departs or lands in the carrier's hub, and the size of the

¹¹ Escobari (2012) empirically studies the dynamics of prices and inventories as the departure date nears.

¹² An alternative specification that included contemporaneous posted prices showed that estimates for the key variable σ_{it} remain close to those reported in Table 8. Because of the potential endogeneity of posted prices we have included the ticket price variable in an IV model for the conditional mean equation using a sequential procedure (rather than the simultaneous estimation), in which the GARCH-in-mean term is included along with the ticket price variable in the second step. The instruments include the lagged values of the explanatory variables. We do not report those results partly due to a lack of theoretical motivation for such a specification. In addition, Deneckere and Peck (2012) suggest that airlines post prices based on beginning-of-period cumulative bookings and not really cumulative bookings as a function of posted prices.

aircraft. These features can be regarded as time-invariant and thus are controlled for with the flight-number fixed effects. Furthermore, capacity decisions are usually made months in advance and modifying the size of the aircraft comes at a relatively high marginal cost. We did not observe any change in aircraft size for the same flight-number in our sample.

Because perishable inventories such as airline seats are an inefficient allocation of existing resources, our empirical findings have significant implications for the airlines. Dana and Orlov (2009) estimate that a 6.7% increase in capacity utilization in the airline industry translates into a \$2.7 billion in cost savings each year. Against the backdrop of the tremendous turmoil in the U.S. airline industry in recent years, with bankruptcies and decreased profits among major airlines (see Berry and Jia, 2010), Borenstein and Rose (2007) explain that large volatility in airlines' profits comes from large volatility in airline ticket demand. Our results provide a better understanding of the airline industry performance by documenting the effect of demand volatility on capacity utilization.

6. Conclusion

This paper contributes to the existing literature by exploring both theoretically and empirically the effect of demand uncertainty on capacity utilization in the airline industry. Unlike other industries, some unique characteristics of airlines make this an ideal place for examining this relationship: Capacity is set in advance when there is uncertainty about the demand, and unsold inventories perish once a plane leaves the gate. In our simple theoretical model, airlines set dispersed prices in advance based on a distribution of demand states. The main empirical implication is that a larger variance in demand realizations is associated with lower average capacity utilization rates.

Our empirical work focuses on testing the theoretical prediction about the link between demand uncertainty and capacity utilization. The analysis has benefited from the collection of unique panel datasets, which allowed us to observe fluctuations in capacity utilization levels over a large number of departure dates and across different flights. Another contribution of our empirical work stems from the estimation of the data with GARCH-type models under the panel setting rather than the conventional time-series setting.

We collected data panels with flight-level seat inventories at three points prior to the departure date covering a total of 140 departure days. The data are used to estimate GARCH-in-mean models that allow for fixed-effects as well as time-varying conditional variance-covariance processes. In line with our theoretical prediction, the empirical results indicate a negative link between demand uncertainty and capacity utilization. More specifically, a one unit increase in the standard deviation of unexpected demand for a particular flight is associated with a 21 percentage point decrease in its capacity utilization. The estimate for this key relationship is robust to cumulative ticket sales data at different points prior to departure. This empirical relationship has also been found to be robust to the presence of various control variables, including systematic demand fluctuations over days of the week and holidays, as well as unobserved flight-, carrier-, and route-specific characteristics.

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Figure 1. Demand States h and Demand Realizations $DEMAND_h$

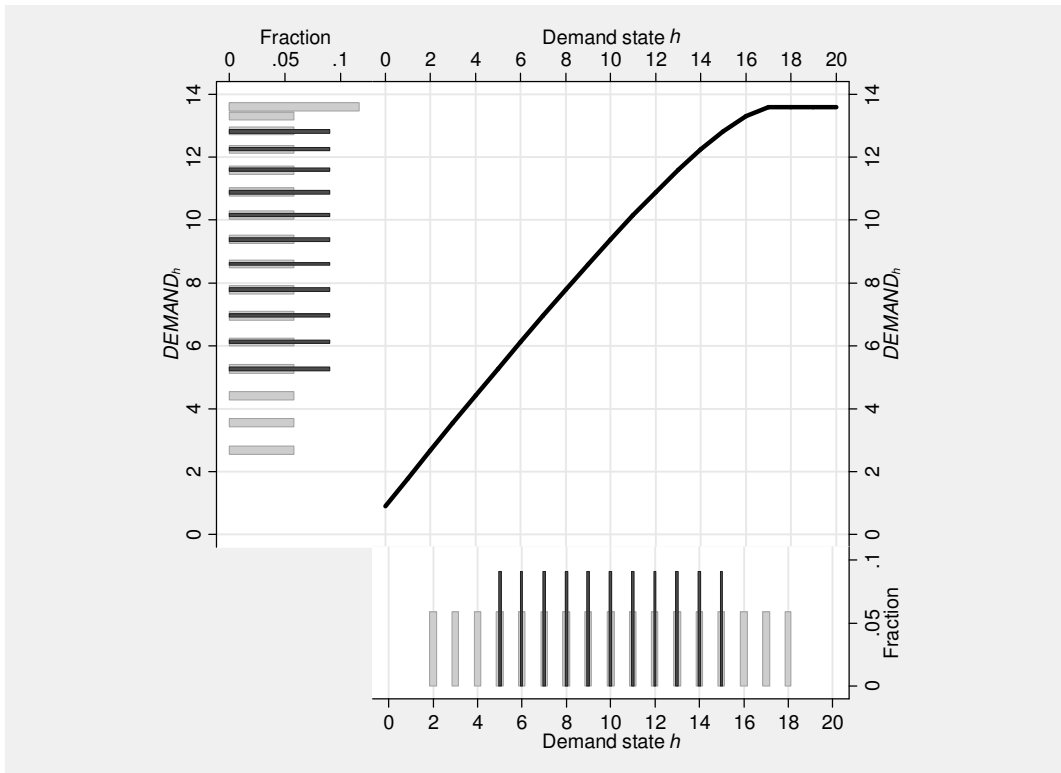


Table 1: Demand Uncertainty and Mean Demand Realizations.

h	Std. Dev. of h	Std. Dev. of $DEMAND_h$	Mean of $DEMAND_h$
$\{0, \dots, 20\}$	5.774	4.305	8.662
$\{1, \dots, 19\}$	5.196	3.901	8.858
$\{2, \dots, 18\}$	4.619	3.568	8.996
$\{3, \dots, 17\}$	4.041	3.214	9.111
$\{4, \dots, 16\}$	3.464	2.826	9.194
$\{5, \dots, 15\}$	2.887	2.414	9.256

Price schedule derived with $h = \{0, 1, 2, \dots, 20\}$.

Table 2: Data Descriptive Statistics.

	Capacity (seats)	Utilized Capacity		
		At 1 Day	At 8 Days	At 15 Days
Mean	103	0.89	0.82	0.74
Std. dev.	39.52	0.14	0.16	0.18
Minimum	50	0.20	0.17	0.14
Maximum	166	1.00	1.00	1.00

Table 3: OLS Estimation Results.

	1 Day	8 Days	15 Days
<i>Mean Equation:</i>			
<i>Intercept</i>	0.09 *** (0.02)	0.08 *** (0.02)	0.07 *** (0.02)
<i>DEMAND</i> _{i,t-1}	0.39 *** (0.03)	0.33 *** (0.03)	0.30 *** (0.02)
<i>DEMAND</i> _{i,t-2}	-0.06 (0.03)	-0.03 (0.03)	-0.01 (0.03)
<i>DEMAND</i> _{i,t-3}	0.08 ** (0.04)	0.06 ** (0.03)	0.05 ** (0.02)
<i>DEMAND</i> _{i,t-4}	0.05 (0.03)	0.06 ** (0.03)	0.001 (0.02)
<i>DEMAND</i> _{i,t-5}	0.12 *** (0.03)	0.08 ** (0.03)	0.08 ** (0.03)
<i>DEMAND</i> _{i,t-6}	0.09 ** (0.03)	0.10 *** (0.03)	0.13 *** (0.03)
<i>DEMAND</i> _{i,t-7}	0.27 *** (0.03)	0.31 *** (0.03)	0.32 *** (0.03)
σ^2	0.08	0.12	0.13
Log-likelihood	2707.14	1738.93	1546.45
R ²	0.65	0.51	0.50

HAC standard errors are in parentheses. The number of observations is 2520.

* denotes statistical significance at the 10% level.

** denotes statistical significance at the 5% level.

*** denotes statistical significance at the 1% level.

Table 4: Autocorrelation Diagnostics.

Lag	Partial Correlation					
	Residuals			Squared Residuals		
	1 Day	8 Days	15 Days	1 Day	8 Days	15 Days
1	-0.01	0.00	-0.02	0.25 ***	0.23 ***	0.22 ***
2	0.01	0.01	-0.01	0.28 ***	0.25 ***	0.19 ***
3	0.02	0.02	0.03	0.23 ***	0.17 **	0.18 **
4	0.02	0.03	0.03	0.18	0.19 ***	0.15 ***
5	0.01	0.01	0.01	0.19 ***	0.19 ***	0.18 ***
6	-0.02	-0.03	-0.02	0.21 ***	0.21 ***	0.23 ***
7	-0.04 *	-0.06 *	-0.07 *	0.20 ***	0.22 ***	0.23 ***
Q(7)	7.14	13.17	19.83	832.28 ***	737.99 ***	657.98 ***

* denotes statistical significance at the 10% level.
** denotes statistical significance at the 5% level.
*** denotes statistical significance at the 1% level.

Table 5. LR Tests for Individual Effects

	<u>1 Day</u>	<u>8 Days</u>	<u>15 Days</u>
Variance Equation (9)	72.01 ***	128.72 ***	265.79 ***
Covariance Equation (10)	720.46 ***	421.81 ***	434.61 ***

*** denotes statistical significance at the 1% level.

Table 6: Panel GARCH Estimation Results.

	1 Day	8 Days	15 Days
<u>Mean Equation:</u>			
<i>Intercept</i>	0.09 (0.02)	0.08 *** (0.02)	0.07 *** (0.02)
<i>DEMAND</i> _{i,t-1}	0.39 *** (0.03)	0.32 *** (0.03)	0.30 *** (0.02)
<i>DEMAND</i> _{i,t-2}	-0.06 (0.03)	-0.03 (0.03)	-0.01 (0.03)
<i>DEMAND</i> _{i,t-3}	0.08 ** (0.04)	0.05 ** (0.03)	0.05 * (0.02)
<i>DEMAND</i> _{i,t-4}	0.05 (0.03)	0.06 * (0.03)	0.001 (0.02)
<i>DEMAND</i> _{i,t-5}	0.12 *** (0.03)	0.08 ** (0.034)	0.08 ** (0.03)
<i>DEMAND</i> _{i,t-6}	0.09 ** (0.03)	0.09 ** (0.03)	0.13 *** (0.03)
<i>DEMAND</i> _{i,t-7}	0.27 *** (0.03)	0.31 *** (0.03)	0.32 *** (0.03)
<u>Variance Equation:</u>			
$\sigma_{i,t-1}^2$	0.60 *** (0.06)	0.73 *** (0.03)	0.42 *** (0.11)
$\varepsilon_{i,t-1}^2$	0.93 *** (0.11)	0.46 ** (0.06)	0.82 ** (0.18)
<u>Covariance Equation:</u>			
$\sigma_{ij,t-1}$	0.54 *** (0.02)	0.83 *** (0.02)	0.47 *** (0.02)
$\varepsilon_{i,t-1} \varepsilon_{j,t-1}$	-0.02 *** (0.01)	0.02 (0.01)	-0.01 (0.01)
Log-likelihood	2738.53	1772.23	1581.27

Standard errors are in parentheses. The number of observations is 2520.

* denotes statistical significance at the 10% level.

** denotes statistical significance at the 5% level.

*** denotes statistical significance at the 1% level.

Table 7: Panel GARCH-in-Mean Estimation Results.

	1 Day	8 Days	15 Days
<u>Mean Equation:</u>			
<i>Intercept</i>	0.09 *** (0.02)	0.15 *** (0.04)	0.16 *** (0.03)
<i>DEMAND</i> _{i,t-1}	0.36 *** (0.03)	0.32 *** (0.03)	0.29 *** (0.02)
<i>DEMAND</i> _{i,t-2}	-0.07 (0.04)	-0.04 (0.03)	-0.02 (0.03)
<i>DEMAND</i> _{i,t-3}	0.06 ** (0.03)	0.04 ** (0.03)	0.04 * (0.02)
<i>DEMAND</i> _{i,t-4}	0.03 (0.03)	0.05 * (0.03)	-0.001 (0.02)
<i>DEMAND</i> _{i,t-5}	0.10 *** (0.03)	0.07 ** (0.03)	0.07 ** (0.03)
<i>DEMAND</i> _{i,t-6}	0.07 ** (0.03)	0.10 ** (0.03)	0.13 *** (0.03)
<i>DEMAND</i> _{i,t-7}	0.25 *** (0.03)	0.31 *** (0.03)	0.31 *** (0.03)
σ_{it}	-0.21 *** (0.04)	-0.10 *** (0.04)	-0.14 ** (0.04)
<u>Variance Equation:</u>			
$\sigma_{i,t-1}^2$	0.58 *** (0.06)	0.73 *** (0.03)	0.76 *** (0.04)
$\varepsilon_{i,t-1}^2$	0.37 *** (0.04)	0.46 ** (0.06)	0.82 ** (0.18)
<u>Covariance Equation:</u>			
$\sigma_{ij,t-1}$	0.53 *** (0.02)	0.82 *** (0.02)	0.47 *** (0.02)
$\varepsilon_{i,t-1} \varepsilon_{j,t-1}$	-0.02 ** (0.01)	0.02 (0.01)	-0.01 (0.01)
Log-likelihood	2745.60	1784.43	1592.11

Standard errors are in parentheses. The number of observations is 2520.

* denotes statistical significance at the 10% level.

** denotes statistical significance at the 5% level.

*** denotes statistical significance at the 1% level.

Table 8: Panel GARCH-in-Mean (with Controls) Estimation Results.

	1 Day	8 Days	15 Days
<u>Mean Equation:</u>			
<i>Intercept</i>	0.09 *** (0.03)	0.09 *** (0.04)	0.15 *** (0.03)
<i>DEMAND</i> _{i,t-1}	0.39 *** (0.03)	0.32 *** (0.03)	0.27 *** (0.02)
<i>DEMAND</i> _{i,t-2}	-0.05 (0.03)	-0.03 (0.03)	-0.001 (0.03)
<i>DEMAND</i> _{i,t-3}	0.09 *** (0.03)	0.04 ** (0.02)	0.07 *** (0.02)
<i>DEMAND</i> _{i,t-4}	0.06 (0.03)	0.05 * (0.03)	0.03 (0.02)
<i>DEMAND</i> _{i,t-5}	0.19 *** (0.03)	0.07 ** (0.03)	0.08 ** (0.03)
<i>DEMAND</i> _{i,t-6}	0.08 ** (0.03)	0.10 * (0.03)	0.10 *** (0.03)
<i>DEMAND</i> _{i,t-7}	0.24 *** (0.03)	0.31 *** (0.03)	0.26 *** (0.04)
σ_{it}	-0.20 *** (0.04)	-0.11 ** (0.03)	-0.15 ** (0.04)
<i>TUESDAY</i>	-0.02 ** (0.01)	-0.03 *** (0.01)	-0.01 * (0.005)
<i>WEDNESDAY</i>	-0.02 ** (0.01)	-0.01 (0.01)	-0.01 (0.01)
<i>THURSDAY</i>	-0.01 (0.01)	0.03 *** (0.01)	0.04 *** (0.01)
<i>FRIDAY</i>	0.01 ** (0.006)	0.05 *** (0.01)	0.06 *** (0.01)
<i>WEEKEND</i>	0.01 *** (0.005)	0.03 *** (0.01)	0.04 *** (0.01)
<i>HOLIDAYS</i>	0.03 ** (0.01)	0.05 ** (0.03)	0.07 ** (0.03)
<u>Variance Equation:</u>			
$\sigma_{i,t-1}^2$	0.59 *** (0.06)	0.73 *** (0.03)	0.43 *** (0.11)
$\varepsilon_{i,t-1}^2$	0.37 *** (0.04)	0.46 ** (0.06)	0.86 ** (0.19)
<u>Covariance Equation:</u>			
$\sigma_{ij,t-1}$	0.58 *** (0.06)	0.83 *** (0.02)	0.76 *** (0.04)
$\varepsilon_{i,t-1} \varepsilon_{j,t-1}$	-0.04 *** (0.01)	0.02 (0.01)	-0.01 (0.01)
Log-likelihood	2801.60	1824.21	1681.96

Standard errors are in parentheses. The number of observations is 2520.

* denotes statistical significance at the 10% level.

** denotes statistical significance at the 5% level.

*** denotes statistical significance at the 1% level.