Effectiveness of securities with fuzzy probabilistic return

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6 July 2011

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EFFECTIVENESS OF SECURITIES WITH FUZZY PROBABILISTIC RETURN

The generalized fuzzy present value of a security is defined here as fuzzy valued utility of cash flow. The generalized fuzzy present value cannot depend on the value of future cash flow. There exists such a generalized fuzzy present value which is not a fuzzy present value in the sense given by Ward [35] or by Huang [14]. If the present value is a fuzzy number and the future value is a random variable, then the return rate is given as a probabilistic fuzzy subset on the real line. This kind of return rate is called a fuzzy probabilistic return. The main goal of this paper is to derive the family of effective securities with fuzzy probabilistic return. Achieving this goal requires the study of the basic parameters characterizing fuzzy probabilistic return. Therefore, fuzzy expected value and variance are determined for this case of return. These results are a starting point for constructing a three-dimensional image. The set of effective securities is introduced as the Pareto optimal set determined by the maximization of the expected return rate and minimization of the variance. Finally, the set of effective securities is distinguished as a fuzzy set. These results are obtained without the assumption that the distribution of future values is Gaussian.

Keywords: *behavioural present value, fuzzy present value, random future value, fuzzy probabilistic return, effective financial security.*

Introduction

Typically, an analysis of the properties of any security is conducted as an analysis of the properties of the return rate. Any return rate is an increasing function of the future value (FV) and a decreasing function of the present value (PV).

Using the classical approach from finance, PV is defined as a discounted cash flow. This cash flow may be a present or future one. Ward [35] defines a fuzzy PV as a discounted fuzzy cash flow. The fuzzy cash flow used here is interpreted as an imprecise forecast of future crisp cash flow. Ward's definition is generalized to the case of a fuzzy duration by Greenhut et al. [11]. Sheen [29] generalizes Ward's definition to the case of a fuzzy interest

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rate. Buckley [1,2], Gutierrez [12], Kuchta [19] and Lesage [22] discuss some problems connected with the application of fuzzy arithmetic to calculating fuzzy PV.

According to the uncertainty thesis expressed by Mises [24] and Kaplan et al. [16], each future cash flow is uncertain. In the crisp case, this uncertainty is usually modeled in such a way that cash flow is described as a random variable. Therefore, Huang [14] generalizes Ward's definition for the case when future cash flow is given as a fuzzy random variable in the sense given by Kwakernaak [20, 21]. A more general definition of a fuzzy PV is proposed by Tsao, who assumes that the future cash flow is a fuzzy probabilistic set in the sense given by Hiroto [13].

All the kinds of fuzzy PV defined above may be used for the determination of fuzzy net present value (NPV) defined as a sum of fuzzy PV.

In recent years the concept of cash flow utility has played an important part in behavioral finance research. This problem is discussed for example by Frederick et al. [33], Dacey et al. [6], Killeen [17], Zauberman et al. [37], Kontek [18] and Doyle [7]. PV is defined there as the utility of cash flow. Thus a generalized fuzzy PV is defined as a fuzzy valued utility of cash flow. This generalized fuzzy PV is more general than the definitions of a fuzzy PV proposed by Greenhut et al. [11] and Sheen [29].

In financial market theory the current market price of a security is interpreted as a crisp PV. Piasecki [27] considers the impact of chosen behavioural factors on PV. The formal model of behavioural PV is presented as a result of these considerations. The behavioural PV is dependent on the equilibrium price, current market price and degree of an investor's acceptance of risk. This implies that the present value can deviate from its observed market price under the influence of behavioural factors. The model obtained explains the mechanism of maintaining the balance between supply and demand in an efficient financial market. The states of the behavioural environment are defined imprecisely. Therefore, behavioural PV is described by a trapezoidal fuzzy number in the sense given by Campos et al. [3]. This means that behavioural PV is subject to imprecision risk. Behavioural PV is an example of a generalized fuzzy PV defined as above. On the other hand, in the general case, behavioural PV does not depend on the value of future cash flow. This means that there exists such a behavioural PV which is not a fuzzy PV in the sense given by Ward [35] or by Huang [14].

According to the thesis cited above as expressed by Mises [24] and Kaplan et al. [16], each FV is uncertain. Therefore, the security FV is usually presented as a random variable. The distribution of this random variable is a formal image of the uncertainty risk. Detailed information on uncertainty risk is gathered in this way. The papers [8], [30], [31], [32], [33] and [36] are examples of this knowledge.

Piasecki [27] noted that if PV is a fuzzy number and FV is a random variable, then the return rate is given as a probabilistic fuzzy subset on the real line. Thus this return rate will be called a fuzzy probabilistic return. Despite a careful preliminary survey of the literature, the author has not found any similar model of the return rate. When we apply a fuzzy probabilistic return to an assessment of the security based on a generalized fuzzy PV, then we can use, without any changes, all the rich empirical knowledge about the probability distributions of return rate which has been gathered. This fact expands the possibility of real applications. It is also a highly advantageous feature of the proposed model.

Investment in effective security (ES) is a standard goal of investors in normative theories of the financial markets. Therefore, the main goal of our considerations is to derive the family of ESs with fuzzy probabilistic return.

Achieving this goal requires the study of the basic parameters characterizing fuzzy probabilistic return. In Section 1 the fuzzy expected value and variance are determined for this case of return. The expected return rate is replaced there by the fuzzy return rate which also takes into account behavioural aspects of decision making in finance. However, such an increase in cognitive value has a price. This price is the introduction of imprecision risk.

Imprecision is composed of ambiguity and indistinctness. Moreover, fuzzy probabilistic return is at uncertainty risk. Hence, the three-dimensional image of risk described in Section 2.

When we use imprecise images of security, we cannot precisely indicate the recommended investment out of a set of alternatives. Each alternative is thus recommended to some extent. The investor shifts some of the responsibility to advisers or the forecasting tool applied. For this reason, the investor restricts his choice of investment decisions to alternatives recommended to the greatest degree. In this way, the investor minimizes his individual responsibility for financial decision making. It shows that the assessment of imprecision risk is relevant to the analysis of investment processes. This problem was widely discussed in [25].

In this paper we consider the case when each effective security is indicated as a recommended investment. In comparison with classical Markowitz theory, imprecision is a new aspect of risk assessment. We ask the question of whether such an extension of risk assessment is appropriate. The usefulness of taking imprecision into account in the study of risk is well justified by the following three arguments.

Firstly, it is always possible to reduce the uncertainty risk of forecasting by appropriately lowering the forecast precision.

Secondly, if we take into account the imprecision risk, we can reject investment alternatives which are attractive from the viewpoint of classical Markowitz theory, but, unfortunately, the information gathered about them is highly imprecise.

Thirdly, from the viewpoint of classical Markowitz theory and its implications, we witness many anomalies in financial practice. Seeing these paradoxes is the starting point for the development of behavioural finance.

In Section 3, a set of effective securities is defined as the Pareto optimal set determined by the maximization of the fuzzy expected return rate and the minimization of risk assessments. Two cases of risk management are taken into account. The minimization of uncertainty risk and the simultaneous minimization of uncertainty risk and imprecision risk are considered.

This article is addressed to two groups of readers. The results obtained may be of interest to financial market theorists and to practitioners constructing support systems for investment decisions.

1. Imprecise assessment of the return rate

Let us assume that the time horizon $t > 0$ of an investment is fixed. Thus, the security considered here is determined by two values:

- the anticipated future value (FV) $V_t \in \mathbb{R}^+$,
- the assessed present value (PV) $V_0 \in \mathbb{R}^+$.

The basic characteristics of benefits by ownership this instrument is the return rate r_t given by the identity

$$r_t = r(V_0, V_t). \tag{1}$$

In the general case, the function: $r: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is a decreasing function of PV and a increasing function of FV. This implies that for any FV V_t we can determine the inverse function $r_0^{-1}(\cdot, V_t): \mathbb{R} \to \mathbb{R}^+$. In the special case we have here:

- the simple return rate is given by

$$r_t = \frac{V_t - V_0}{V_0} = \frac{V_t}{V_0} - 1. \tag{2}$$

– the logarithmic return rate is given by

$$r_t = ln\frac{V_t}{V_0}. \tag{3}$$

The FV is at risk of uncertainty. Formal model of this uncertainty is presentation FV V_t as a random variable $\tilde{V}_t: \Omega = \{\omega\} \longrightarrow \mathbb{R}^+$. The set Ω is the set of elementary states of the financial market. In the classical approach to the problem of determining return rate, the PV of a security is identified with the observed market price $\check{C}$. Then return rate is a random variable which is at uncertainty risk. This random variable is determined by the identity

$$\tilde{r}_t(\omega) = r\left(\check{C}, \tilde{V}_t(\omega)\right). \tag{4}$$

In the practical analysis of financial markets, uncertainty risk is usually described by a probability distribution of return rates. We already have extensive knowledge on this subject. Let us assume that this probability distribution is given by the cumulative distribution function $F_r: \mathbb{R} \longrightarrow [0;1]$. Then, the probability distribution of FV is described by the cumulative distribution function $F_V: \mathbb{R}^+ \longrightarrow [0;1]$, given as follows

$$F_V(x) = F_r\left(r_t(\check{C}, x)\right). \tag{5}$$

Assessment of the security FV is based only on objective measurements. This means that the cumulative distribution function of the future value is independent of how the present value is determined.

It is shown in [27] that the security PV may be at imprecision risk. This imprecision risk was determined using behavioural premises. An imprecisely assessed PV is described as a generalized fuzzy PV, which is represented by its membership function $\mu: \mathbb{R}^+ \to [0;1]$. Then the return rate is at risk of coincidence uncertainty and imprecision. According to the Zadeh extension principle, for each fixed elementary state $\omega \epsilon \Omega$ of a financial market, the membership function $\rho(\cdot, \omega): \mathbb{R} \to [0;1]$ of the return rate is determined by the identity

$$\rho(r, \omega) = max\left\{\mu(y): y \epsilon \mathbb{R}^+, r = r\left(y, \widetilde{V}_t(\omega)\right)\right\} = \mu\left(r_0^{-1}\left(r, \widetilde{V}_t(\omega)\right)\right). \tag{6}$$

This means that considered return rate is represented by a fuzzy probabilistic set as defined by Hiroto [13]. For this reason, this return rate is called a fuzzy probabilistic return. In the special cases we have here:

– for the simple return rate

$$\rho(r,\omega) \ = \mu\left((1+r)^{-1}\cdot \tilde{V}_t(\omega)\right) \tag{7}$$

– for the logarithmic return rate

$$\rho(r,\omega) \ = \mu\left(e^{-r}\cdot \tilde{V}_t(\omega)\right) \tag{8}$$

For any fuzzy probabilistic return we determine the parameters of its distribution. We have here:

– distribution of the expected return rate

$$\varrho(r) = \int_{-\infty}^{+\infty} \mu\left(r_0^{-1}\left(r,\tilde{V}_t(\omega)\right)\right) dF_V(y)\ ; \tag{9}$$

– the expected return rate

$$\bar{r} = \frac{\int_{-\infty}^{+\infty} r\cdot\varrho(r)dr}{\int_{-\infty}^{+\infty} \varrho(r)dr}\ . \tag{10}$$

The distribution of the expected return rate $\varrho\epsilon[0,1]^{\mathbb{R}}$ is a membership function of the fuzzy subset $\tilde{R}$ on the real line. This subset $\tilde{R}$ is called the fuzzy expected return rate. This rate represents both rational and behavioural aspects in the approach to estimating the expected benefits. We will use the following variance of the return rate to assess the risk uncertainty

$$\sigma^2 = \left(\int_0^{+\infty}\int_{-\infty}^{+\infty} \nu(r,y)dF_V(y)dr\right)^{-1}\cdot \int_0^{+\infty}\int_{-\infty}^{+\infty} r\cdot\nu(r,y)dF_V(y)dx \tag{11}$$

where

$$\nu\left(x,\tilde{V}_t(\omega)\right) = \begin{cases} max\{\rho(\bar{r}+\sqrt{x},\omega),\rho(\bar{r}-\sqrt{x},\omega)\} & x\geq 0\ . \\ 0 & x<0 \end{cases} \tag{12}$$

A detailed analysis of these relationships shows that this variance describes both rational and behavioural aspects of an assessment of the safety of the capital employed.

As in the case of a precisely defined return rate, there are such probability distributions of future value for which the variance of the return rate does not exist. We then replace this distribution with a distribution truncated on both sides, for which the variance always exists. This procedure finds its justification in the theory of perspective [15]. Among other things, this theory describes the behavioural phenomenon of the rejection of extremes.

2. The three-dimensional image of the risk

In the classical portfolio theory given by Markowitz [23] a normative investment strategy involves the maximization of the expected return rate $\bar{r}$ while its variance ς is minimizing. In this situation, each security is represented by pair $(\bar{r}, \varsigma^2)$. This pair represents rational premises for the evaluation of a security. It is implicitly assumed that the returns have a Gaussian distribution.

In this section the expected return rate r is replaced by the fuzzy return rate $\tilde{R}$, which also takes into account behavioural aspects of decision making in finance. In this way, we describe the imprecision risk. Imprecision is composed of the ambiguity and indistinctness.

The ambiguity is the lack of a clear recommendation one alternative between the various given alternatives. In accordance with suggestion given in [1], we will evaluate the ambiguity risk using the energy measure $d(\tilde{R})$ of the fuzzy expected return rate $\tilde{R}$. This measure is determined by the identity

$$\delta = d(\tilde{R}) = \frac{\int_{-\infty}^{+\infty} \varrho(x)dx}{1+\int_{-\infty}^{+\infty} \varrho(x)dx} \cdot . \tag{13}$$

Indistinctness is the lack of an explicit distinguishing amongst the given information and its negation. According to suggestion given in [3], we will evaluate the indistinctness risk using the entropy measure $e(\tilde{R})$ of the fuzzy expected return rate $\tilde{R}$. This measure is described as follows

$$\varepsilon = e(\tilde{R}) = \frac{\int_{-\infty}^{+\infty} min\{\varrho(x), 1-\varrho(x)\}dx}{1+\int_{-\infty}^{+\infty} min\{\varrho(x), 1-\varrho(x)\}dx}. \tag{14}$$

This uncertainty follows from an investor's lack of knowledge about future states of the financial market. This lack of this knowledge implies that no investor is sure of future profits or losses. The properties of such risk are discussed in a rich body of literature. In this paper, we will evaluate the uncertainty risk using the variance σ^2given by identity (11).

In this situation, we assign a three-dimensional vector $(\sigma^2, \delta, \varepsilon)$ to each fuzzy expected return rate $\tilde{R}$. This vector describes the risks, which are understood to be composed of the risks of uncertainty, ambiguity and indistinctness.

An increase in the ambiguity risk means that the number of recommended investment alternatives increases too. This increases the chance of selecting a recommended alternative, which involves opportunity cost.

An increase in indistinctness risk means that the distinctions between recommended and unrecommended alternatives are more blurred. It implies a higher probability of choosing unrecommended alternatives.

These observations show that an increase in the imprecision risk makes investment conditions noticeably worse. Using the three-dimensional description of risk $(\sigma^2, \delta, \varepsilon)$ facilitates the management of imprecision risk. It is desirable here to minimize each of the three risk assessments.

Using this three-dimensional description of risk enables the investigation of relationships between different types of risk. Here we can observe the empirical interaction between risks. Moreover, there is a formal correlation between the uncertainty risk and ambiguity risk. The number of recommended alternatives increases with ambiguity risk. In this way, there is more certainty that the recommended alternatives include the best investment decision. This means that the uncertainty risk decreases. In summary, the uncertainty risk and the ambiguity risk are negatively correlated.

3. Financial effectiveness

A security is called effective (ES) if it attains, for a given variance, the maximum expected return rate. In his classical portfolio theory, Markowitz assumed that the distribution of return rates is Gaussian. The set of ESs is given by the upper branch of the Markowitz curve, which is called the ES curve.

The set of ESs can also be specified by means of multicriteria comparison theory. Using this approach we can dispense with the assumption that the probability distribution of return rates is Gaussian. We define two preorders on the set of all securities. These preorders are the maximization of expected return rates and the minimization of variance. The set of ESs is then described as the Pareto optimal set for multicriteria comparison defined by the above preorders. If we additionally assume here that the distribution of the return rate is Gaussian, the set of ESs will coincide with the upper branch of the Markowitz curve. This means that the set of ESs is a generalization of the concept of the ES curve defined on the basis of classical Markowitz theory.

Any investment using an ES is an investment in security guaranteeing maximum returns with the minimal risk of capital loss. This is a standard goal of investors in normative theories of the financial markets. This poses some difficulties in application, since investors typically invest in securities which are outside of the ES set. Accordingly, from the viewpoint of these theories, they invest in ineffective securities. At the same time, these investors declare investing in ESs to be their normative goal. In this way, we find a paradox inherent the real financial markets.

This paradox is very common. It cannot be explained by a lack of sufficient knowledge of the real processes occurring in the financial markets and economic environment. The increasing professionalization of investor activity and fast development of information technology imply that full access to market information and the capacity to process data is available to all professional investors, who manage the vast majority of the volume of exchange trading.

This paradox may be explained in the following way. The normative aim of investing in ESs is declared by investors who invest only in securities similar to effective ones. The degree of effectiveness of a given security is equal to the degree of its similarity to an ES. In practice, this means that almost every commercially available security is effective to some extent. On the other hand, an ES is no longer traded on the stock exchange. All this explains the paradox of the divergence between a normative investor's purpose and the real goal of an investment strategy. Investors always act in a manner similar to an effective course of action.

Let us consider a normative model of investor activity. The set of all securities is denoted by the symbol $\mathbb{Y}$. The security $\check{Y}\epsilon\mathbb{Y}$ is represented by the pair $\left(\tilde{R}_Y, (\sigma_Y^2, \delta_Y, \varepsilon_Y)\right)$, where the individual symbols have the following meanings:

- $\tilde{R}_Y$ is fuzzy expected rate of return on security $\check{Y}$,
- σ_Y^2 is the variance of the rate of return on security $\check{Y}$,
- δ_Y is the energy measure of the fuzzy expected return rate $\tilde{R}_Y$,
- ε_Y is the energy measure of the fuzzy expected return rate $\tilde{R}_Y$.

The fuzzy expected return rate $\tilde{R}_Y$ is defined by the distribution of the expected return rate $\varrho_Y\epsilon[0,1]^{\mathbb{R}}$. On the set of fuzzy real numbers $\mathcal{F}(\mathbb{R})$ define the relation $\widetilde{K} \succcurlyeq \tilde{L}$, which reads:

Fuzzy real number $\widetilde{K}$ is greater or equal to fuzzy real number $\widetilde{L}$.

This relation is a fuzzy preorder defined by a membership function $\nu_Q\colon \mathcal{F}(\mathbb{R}) \times \mathcal{F}(\mathbb{R}) \to [0,1]$ which fulfils the condition

$$\nu_Q\left(\tilde{R}_Y, \tilde{R}_Z\right) = sup\{min\{\varrho_Y(u), \varrho_Z(v)\}: u \geq v\} \tag{15}$$

for any pair $\left(\tilde{R}_Y, \tilde{R}_Z\right)$ of fuzzy expected return rates.

In the next step we determine the multicriteria comparison $\mathcal{W}\epsilon\mathbb{Y}\times\mathbb{Y}$ based on the maximization of the fuzzy expected return rate and minimization of variance. We describe the relation formed in this way as the predicate $\breve{Y} \supseteq \breve{Z}$ which reads

$$\textit{Security } \breve{Y} \textit{ is no more effective than security } \breve{Z}. \tag{16}$$

In a formal way this multicriteria comparison is defined by the equivalence

$$\breve{Y} \supseteq \breve{Z} \Leftrightarrow \tilde{R}_Y \succcurlyeq \tilde{R}_Z \wedge \sigma_Y \leq \sigma_Z. \tag{17}$$

In this situation the relation $\mathcal{W}$ is a fuzzy preorder defined by its membership function $\nu_W: \mathbb{Y}\times\mathbb{Y} \to [0,1]$. For any pair of securities $\breve{Y},\breve{Z}\epsilon\mathbb{Y}$ this membership function is represented by the identity

$$\nu_W(\breve{Y},\breve{Z}) = \begin{cases} \nu_Q(\tilde{R}_Y,\tilde{R}_Z) & \sigma_Y \leq \sigma_Z \\ 0 & \sigma_Y > \sigma_Z \end{cases}. \tag{18}$$

The set $\tilde{\Phi}$ of ESs is equal to the Pareto optimal set defined by multicriteria comparison (17). The set $\tilde{\Phi}$ is represented by its membership function $\varphi: \mathbb{Y} \to [0,1]$ determined by the identity

$$\varphi(\breve{Y}) = inf\{\max\{\nu_W(\breve{Y},\tilde{Z}), 1-\nu_W(\tilde{Z},\breve{Y})\}: \tilde{Z}\epsilon\mathbb{Y}\}. \tag{19}$$

The value $\varphi(\breve{Y})$ can be interpreted as a truth value of the sentence:

$$\textit{The security } \breve{Y} \textit{is effective.} \tag{20}$$

We described above the case, when the investor determines the effective securities, taking into account only the risk of uncertainty. Now we focus our attention on the case, when the investor simultaneously takes into account the uncertainty risk and imprecision risk. Let us now consider the multicriteria comparison $\mathcal{L}\epsilon\mathbb{Y}\times\mathbb{Y}$ determined by the maximization fuzzy expected return rate and three criteria for the minimization of the risk measures described above. Formed in this way relation we describe as the predicate $\breve{Y} \sqsupseteq \breve{Z}$ which reads

$$\textit{Security } \breve{Y} \textit{ is no more strictly effective than security } \breve{Z}. \tag{21}$$

In a formal way this multicriteria comparison is defined by the equivalence

$$\breve{Y} \sqsupseteq \breve{Z} \Leftrightarrow \tilde{R}_Y \succcurlyeq \tilde{R}_Z \wedge \sigma_Y \leq \sigma_Z \wedge \delta_Y \leq \delta_Z \wedge \varepsilon_Y \leq \varepsilon_Z. \tag{22}$$

In this situation the relation $\mathcal{L}$ is fuzzy preorder defined by its membership function $\nu_L: \mathbb{Y}\times\mathbb{Y} \to [0,1]$. For any pair of financial instruments $\breve{Y},\breve{Z}\epsilon\mathbb{Y}$ this membership function is represented by the identity

$$\nu_L(\breve{Y},\breve{Z}) = \begin{cases} \nu_Q(\tilde{R}_Y,\tilde{R}_Z) & \sigma_Y \le \sigma_Z \wedge \delta_Y \le \delta_Z \wedge \varepsilon_Y \le \varepsilon_Z \\ 0 & \sim(\sigma_Y \le \sigma_Z \wedge \delta_Y \le \delta_Z \wedge \varepsilon_Y \le \varepsilon_Z) \end{cases}. \tag{23}$$

The set $\breve{\Psi}$ of strictly effective securities is determined as the Pareto optimal set defined by the multicriteria comparison (22). The set $\breve{\Psi}$ is represented by its membership function $\psi: \mathbb{Y} \to [0,1]$ determined by the identity

$$\psi(\tilde{Y}) = inf\{\max\{\nu_L(\tilde{Y},\tilde{Z}), 1 - \nu_L(\tilde{Z},\tilde{Y})\}: \tilde{Z}\epsilon\mathbb{Y}\}. \tag{24}$$

The value $\psi(\tilde{Y})$ can be interpreted as a truth value of the sentence:

The security $\breve{Y}$ is strictly effective. (25)

If investors considers purchase or sale the security $\breve{Y}$, then they can take into account the values $\varphi(\tilde{Y})$ and $\psi(\tilde{Y})$. Investors should limit the area of their investments to securities characterized by relatively high value of these indicators. Also investors should limit the sale of their securities to those for which these indicators have low values. The considerations presented in [11] suggest that individual investors use different values of these indicators at the same time. Such variation follows from the variation in subjective behavioural reasons for investment decisions.

4. Conclusions

It is shown above that an increase in imprecision risk makes investment conditions noticeably worse. Accordingly, imprecision should be considered as a risk which is relevant to the investment process.

This paper applies behavioural reasons for investment decision making to describe the similarity of individual securities to effective ones (ES). Such a result is obtained without the assumption that the probability distribution of the return rates is Gaussian. The normative theory presented here explains that the divergence between the normative investor's purpose and the real goal of an investment strategy is implied by behavioural aspects of the perception of the financial markets. Each of the paradoxes explained is apparent. This formal theory allows us to control the choice of securities similar to ES. This follows from the fact that using this theory we can determine the truth value of sentence (20) or (25).

Firstly, the results so obtained may be applied in behavioural finance theory as a normative model. Investing only in strictly efficient securities can be recognized as a normative investor's goal. This strategy results in the rejection of those investment alternatives which are admittedly attractive from the viewpoint of classical Markowitz theory, but the information gathered about them is unfortunately imprecise.

Secondly, the results presented above may provide theoretical foundations for constructing an investment decision support system.

Applications of the normative model presented above involve several difficulties. The main difficulty is the high formal and computational complexity of the tasks involved in determining the membership function for the set of effective securities. The computational complexity of the normative model is the price we pay for the lack of detailed assumptions about the return rate. On the other hand, the low logical complexity is an important attribute of the formal model presented in this paper.

The problem of finding a membership function for the set of effective securities can also be solved using econometric analysis of the financial markets. Examples of such solutions are presented in [26] and [28].

The author's main contribution in this paper is to propose two models of effective security with fuzzy probabilistic return. Moreover, the paper also offers an original three-dimensional image of the risk affecting fuzzy probabilistic return.

Acknowledgment

The author would like to thank to anonymous reviewer for his/her valuable suggestions

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