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Abstract

Patent licensing agreements among competing firms usually involve royalties which are often considered to be anticompetitive as they raise market prices. In this paper we propose simple tax policies than can alleviate the effect of royalties. Considering a Cournot duopoly where firms produce under decreasing returns and trade a patented technology, we show that the interaction of royalties with decreasing returns may generate the counter-intuitive result that market prices decrease in the magnitude of diseconomies of scale. In such cases there exist progressive quantity taxes on firms that weaken the effect of royalties and lower the market prices. These taxes collect sufficient revenue to compensate firms for their losses. As a result, it is possible to design deficit neutral tax-transfer schemes that strictly Pareto improve the welfare of consumers as well as firms.

Keywords: Decreasing returns; patent licensing; royalty; progressive quantity tax; deficit neutrality

JEL Classification: D43, D45, H21, L24

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1 Introduction

A patent grants an innovator monopoly rights over its innovation for a given period of time. It seeks to provide incentives to innovate as well as to diffuse innovations. Since Arrow (1962), it has been recognized that under a patent system, there is often a conflict between private and social incentives of innovation. An innovation that results in an efficient production technology may not yield its full benefits to consumers, as market prices may not fall significantly. This is due to the distortion created by specific licensing policies used by the patent holder to disseminate its innovation.[1] Specifically, it has been argued by Shapiro (1985) and Wang (1998) that if a patent holder is one of the incumbent firms in an oligopoly, it has incentives to use royalties to license its innovation to its rivals. Royalties give competitive edge to the innovator, but distort the innovation and lead to higher prices. One standard policy intervention to remedy this could be to provide either direct or indirect subsidies to firms, but fiscal constraints may prevent governments to follow such policies.

In this paper we identify situations in oligopolies where, with a patent system in place, the government can lower market prices through quantity taxation on firms. In addition to this surprising effect of taxation on market prices, the tax collects sufficient revenue to compensate firms for their potential losses. Consequently it is possible to design deficit neutral tax-transfer schemes that strictly Pareto improve the welfare of consumers as well as the firms in the market.

We carry out our analysis in a Cournot duopoly where both firms have increasing and linear marginal costs of production, i.e., the production technology exhibits decreasing returns to scale. One of the firms has a patent on a technological innovation that results in a downward shift of the marginal cost line. The patent holder can either use the new technology exclusively or license it to its rival. The available licensing policies consist of all combinations of upfront fees and royalties (two-part tariffs).

The starting point of the analysis is the characterization of optimal licensing policies. We show that regardless of the magnitude of the innovation, the patent holder has incentive to license the new technology to its rival. Moreover, the optimal licensing policy always involves positive royalties, which is also the case with constant returns (e.g., Shapiro 1985; Wang 1998). However, royalties interact with decreasing returns to generate certain counter-intuitive effects which are absent under constant returns. The rate of optimal royalty falls with the strength of decreasing returns. As a result, decreasing returns affects market price through two channels. Stronger decreasing returns have the direct effect of raising the price, but they have the indirect effect of lowering royalties which reduces the price. We identify robust regions where the indirect effect dominates the direct effect and consequently, stronger decreasing returns result in lower market price.

Consider the regions where stronger decreasing returns lead in lower price. Suppose the government introduces a *quantity tax* on each firm which is increasing and convex in the quantity it sells. As the marginal tax is increasing in quantity, this tax is "progressive" in nature. The introduction of this tax effectively strengthens the magnitude

[1]Kamien and Tauman (1984, 1986) and Katz and Shapiro (1985, 1986) first studied the problem of patent licensing in oligopolies. See Kamien (1992) for a survey.

of decreasing returns for firms. Within the parametric configuration of the model, we show that it is possible for the government to design this tax in a way that weakens the effect of royalties and reduces the market price. However, doing so necessarily makes one or both firms worse off. We show that the tax generates sufficient revenue to compensate firms for their losses. Consequently, it is then possible to design tax and transfer schemes (i.e. taxation together with lump-sum transfers to firms) that make consumers *as well as* the firms better off. The scheme finances itself and thus achieves deficit neutrality.

The perverse effects of taxation in imperfectly competitive markets have been also shown in de Meza et al (1995) who find that imposing an ad-valorem tax in Cournot markets can reduce prices. Their result is driven by the presence of increasing returns to scale. The introduction of the tax induces some firms to exit, allowing the remaining ones to exploit increasing returns to scale so efficiently that the post-tax price falls below the pre-tax level. Interestingly, our result is driven by the completely opposite effect of scale economies: decreasing returns (and their interaction with licensing policies).

Our paper relates to the literature that analyzes Pareto-improving tax policies in economies with imperfectly competitive markets. Konishi et al. (1990) consider tax-subsidy policies in general equilibrium models with one competitive and one non-competitive production sectors. Assuming that the competitive (non-competitive) sector operates under constant (increasing) returns to scale the authors provide specific tax-subsidy schemes that raise the welfare of the representative consumer. Dillen (1995) analyzes efficiency-restoring policies in general equilibrium frameworks under the assumption that firms compete in prices. Ushio (2000) studies the impact of small commodity taxes in a Cournot market with asymmetric firms (under partial equilibrium). Although taxation reduces consumer surplus, social welfare might increase as taxes shift production from inefficient to efficient firms (and hence substantial cost savings are created). Wang (2011) shows that in economies where all commodities are taxable, the optimal tax rates should equalize the Lerner indexes of commodities. The tax system raises welfare provided labor supply is sufficiently inelastic.

This paper is also indirectly related to a large literature on Pigouvian taxation (e.g., Pigou, 1932; Baumol, 1972; Greenwald and Stiglitz, 1986) which studies the role of taxes in correcting externalities generated by market activities. In a market where production results in negative externalities, Carlton and Loury (1980) [CL] argue that imposition of a Pigouvian tax (i.e., a per unit tax) on firms leads to socially efficient outcome in the short run, but in the long-run, a lump-sum transfer together with the unit tax is required to achieve efficiency. However, such tax-transfer schemes may raise deficits. Geanakoplos and Polemarchakis (2008) [GP] show that in (almost all) pure exchange economies with separable externalities, a tax-transfer scheme can be used to Pareto improve upon competitive equilibria. Although we study the role of taxes in the context of technology transfer rather than externalities, our paper shares the broad theme that taxes can be used to improve market outcomes. One key difference with both CL and GP is that we look at imperfectly competitive markets. Consequently, the choices of firms (e.g., the nature of licensing contracts in the technology transfer stage) affect market prices. We show that there exist progressive quantity taxes that "correct" the licensing contracts in a way that is beneficial for consumers.

The paper is organized as follows. Section 2 presents the model and the optimal licensing policy. Section 3 discusses the inverse relation between market price and decreasing returns. Section 4 analyzes tax-transfer schemes and their welfare impact. Concluding remarks are presented in the last section. All proofs are placed in the Appendix.

2 The model

• *Demand*: Consider a Cournot duopoly with firms 1 and 2. For $i = 1, 2$, let q_i be the quantity produced by firm i and let Q be the industry output, i.e., $Q = q_1 + q_2$. Denoting by p the market price, the inverse demand function of the industry is $p = \max\{a - Q, 0\}$ where $a > 0$.

• *Cost*: There is an existing technology under which both firms have the identical linear marginal cost function

$$\mu(q) = bq + c \text{ where } 0 < c < a \text{ and } b > 0 \tag{1}$$

As $\mu(q)$ is increasing in q, the technology exhibits decreasing returns to scale.

• *New technology*: One of the firms, say firm 1, is granted a patent for a new technology that leads to a reduction in the production cost. Specifically the new technology results in the marginal cost function

$$\mu^{\varepsilon}(q) = bq + c - \varepsilon \text{ where } 0 < \varepsilon < c \tag{2}$$

So ε is the magnitude of innovation.[2]

• *Licensing policies*: The patent holder (firm 1) carries out its production with the new technology. It may also license the new technology to its rival firm 2. The set of licensing policies available to firm 1 is the set of all combinations of a unit royalty $r \geq 0$ and an upfront fee $\alpha \geq 0$, so a typical policy is given by (r, α). If firm 2 does not have a license, its marginal cost is $\mu(q)$. If it has a license under a policy (r, α), its *effective* marginal cost becomes $\mu^{\varepsilon}(q) + r$. Notice that firm 2 will not accept a policy with $r > \varepsilon$. So it is sufficient to consider royalties $r \in [0, \varepsilon]$.

• *The game G_b*: The strategic interaction between the two firms is modeled as an extensive-form game G_b that has three stages. In the first stage, firm 1 decides whether to license the new technology to firm 2 or not. If firm 1 licenses, it offers a policy (r, α) where $r \in [0, \varepsilon]$ and $\alpha \geq 0$. In the second stage, firm 2 decides whether to accept or reject the licensing policy (if any is offered). Finally, in the third stage, the two firms compete in quantities. If firm 2 operates under a policy (r, α) and produces q units, it pays $rq + \alpha$ to firm 1.

In what follows we analyze the above game by confining to its Subgame Perfect Nash Equilibrium (SPNE) outcomes.

[2]We consider the innovation to be a downward shift of the marginal cost line which lowers c without affecting the slope b. This makes our analysis comparable to the existing literature as by taking $b = 0$, we have the standard case of constant returns.

2.1 Cournot equilibrium

We begin with the last stage of the game where firms compete as Cournot duopolists. If firm 2 does not have a license, the payoff of firm i is simply its profit in the Cournot duopoly. Denote this profit by $\widehat{\pi}_i(q_1, q_2)$. Then

$$\widehat{\pi}_i(q_1, q_2) = (a - q_1 - q_2)q_i - c_i q_i - bq_i^2/2, \quad i = 1, 2 \tag{3}$$

where $c_1 = c - \varepsilon$ and $c_2 = c$. Denoting by $\pi_i(r, q_1, q_2)$ the duopoly profit of firm i when firm 2 has a license with rate of royalty r, we have

$$\pi_1(r, q_1, q_2) = (a - q_1 - q_2)q_1 - (c - \varepsilon)q_1 - bq_1^2/2 \tag{4}$$

$$\pi_2(r, q_1, q_2) = (a - q_1 - q_2)q_2 - (c - \varepsilon + r)q_2 - bq_2^2/2 \tag{5}$$

When there is licensing, the total payoff Π_1 of firm 1 is the sum of its duopoly profit and licensing revenue. For firm 2, note from (5) that royalty r is part of its marginal cost and royalty payments are already included in its duopoly profit. So firm 2's total payoff Π_2 is its duopoly profit net of fixed fees. Consequently under a licensing policy (r, α), we have

$$\Pi_1(r, \alpha) = \pi_1(r, q_1, q_2) + rq_2 + \alpha, \;\; \Pi_2(r, \alpha) = \pi_2(r, q_1, q_2) - \alpha \tag{6}$$

To determine the SPNE of G_b, we need to consider the Nash equilibrium (NE) outcome of the Cournot duopoly where firms choose q_1, q_2 simultaneously and obtain profits given by (3) or (5). The equilibrium for this market is computed and presented in Lemma A2 in the Appendix.

Our analysis considers drastic as well as non-drastic innovations. A cost-reducing innovation is *drastic* (Arrow 1962) if it is significant enough to create a monopoly whenever one firm only uses the new technology; otherwise it is non-drastic. Lemma A2 in the Appendix shows that an innovation of magnitude ε is drastic if $\varepsilon \geq (b+1)(a-c)$ and it is non-drastic if $\varepsilon < (b+1)(a-c)$.

2.2 Technology transfer

Given the previous analysis, in this section we determine whether technology transfer occurs or not. Let $\widehat{q}_i, \widehat{\pi}_i$ denote the NE output and profit of firm i in the no-licensing scenario; let $q_i(r)$,$\pi_i(r)$ denote the NE output and profit of firm i if licensing occurs under royalty r; finally let π_M denote the monopoly profit under the new technology.

Consider a licensing policy (r, α). Using (6), the payoffs under this policy are given by

$$\Pi_1(r, \alpha) = \pi_1(r) + rq_2(r) + \alpha, \; \Pi_2(r, \alpha) = \pi_2(r) - \alpha$$

If firm 2 rejects the policy, then it obtains $\widehat{\pi}_2$. Hence for any $r \in [0, \varepsilon]$, it is optimal for firm 1 to set the fixed fee equal to $\alpha = \pi_2(r) - \widehat{\pi}_2$, making firm 2 just indifferent between accepting and rejecting the licensing offer. Therefore, if firm 1 decides to offer a license, its problem reduces to choosing $r \in [0, \varepsilon]$ to maximize

$$\Pi_1(r) = \pi_1(r) + rq_2(r) + \pi_2(r) - \widehat{\pi}_2 \tag{7}$$

The following proposition characterizes the outcome of the licensing interaction and the optimal licensing policy.

Proposition 1 *For any $b > 0$, the game G_b has a unique SPNE outcome. It has the following properties.*

(I) *Regardless of whether the innovation is drastic or non-drastic, licensing always occurs.*

(II) *Firm 2 always obtains the net payoff $\widehat{\pi}_2$, and is just indifferent between accepting and rejecting the licensing offer. This payoff is zero for drastic innovations and positive for non-drastic innovations.*

(III) $\exists\ \ell(b) < b + 1$, *which is decreasing with $lim_{b\to\infty}\ell(b) = 0$ such that*

(a) *If $\varepsilon \leq \ell(b)(a - c)$, the licensing policy has royalty ε and no fixed fee.*

(b) *If $\varepsilon > \ell(b)(a - c)$, the licensing policy has royalty $r_b(\varepsilon) < \varepsilon$ and fixed fee $\alpha = \pi_2(r_b(\varepsilon)) - \widehat{\pi}_2 > 0$. The royalty rate $r_b(\varepsilon)$ is decreasing in b.*

Proof See the Appendix. ■

For non-drastic innovations, firm 1 cannot drive its rival out of the market by not offering a license. By choosing a royalty policy $r = \varepsilon$ firm 1 obtains positive royalty revenue and at the same time ensures that 2 effectively operates under the old technology. Since offering no license is dominated by this specific licensing policy, licensing is generally optimal for non-drastic innovations. This result does not depend on the nature of scale economies.

For drastic innovations, firm 1 can become the monopolist by using the innovation exclusively. Still it prefers to license to its rival. To see the intuition, observe that under decreasing returns to scale, production of a large output by a single firm creates cost inefficiencies. Hence firm 1 has incentive to keep firm 2 in the market: the presence of two active firms increases efficiency and results in higher surplus than the monopoly profit, which firm 1 then extracts via a fixed fee.

The optimal mixture of fees and royalties is determined by two factors. First, firm 1 intends to create a relatively inefficient rival and second, it has to consider how the rival's efficiency affects its own marginal cost. Under decreasing returns to scale, these two factors work in opposite direction. A less efficient rival implies higher output and higher marginal cost for firm 1. The relation between the magnitude of innovation and the size of decreasing returns plays an important role to resolve the conflict, as we explain below.

Recall by Prop 1(III) that the threshold $\ell(b)$ is decreasing in b (the intensity of decreasing returns). Hence we can restate the two parts of Prop 1(III) in terms of b being low or high: If b is low then the impact of decreasing returns on firm 1's marginal cost is relatively low. Hence, the incentive of creating an inefficient rival is more important. Accordingly, a pure royalty policy with maximum royalty equal to ε is used. If b is high, the impact of decreasing returns is high too. Thus, firm 1 prefers to provide a certain degree of cost efficiency to firm 2 so that it (i.e., firm 1) avoids operating in relatively inefficient production zones. For this reason, the rate of royalty is set below ε.

Note that the above explain also why the royalty rate $r_b(\varepsilon)$ is decreasing in b. Higher values of b means the innovator faces higher marginal cost. By designing a royalty that decreases in b the innovator "commits" to a relatively smaller quantity (as the reduced royalty raises the quantity of the rival firm), thus staying in a relatively efficient production zone.

3 Post-licensing price

Let us now study how post-innovation market equilibrium (market price and industry output) behave vis-a-vis the intensity of diseconomies of scale. For our purposes, we shall focus on changes of b, i.e., the slope of marginal cost line. Note first that if the marginal cost of firm i is $\mu(q_i) = bq_i + c_i$, a higher value of b raises the intensity of diseconomies of scale[3] (or raises marginal cost) provided that $b + \partial q_i/\partial b > 0$. We can show that the latter inequality holds in our model.

Below we identify a paradox: given the licensing policy described in the previous section, we find ranges of parameter values under which an increase in b raises industry output and hence lowers market price.

Proposition 2 *Let $p^0(b)$ be the post-licensing Cournot price in the SPNE of G_b. There exist constants $\widehat{\varepsilon} \in (0, a-c)$ and $\widehat{b} > 0$ such that*

(I) *If $0 < \varepsilon \leq \widehat{\varepsilon}$, $p^0(b)$ is increasing for all $b > 0$.*

(II) *If $\widehat{\varepsilon} < \varepsilon < a - c$, $\exists\, \widetilde{b}(\varepsilon) \in (0, \widehat{b})$ such that $p^0(b)$ is increasing for $b \in (0, \widetilde{b}(\varepsilon))$, decreasing for $b \in (\widetilde{b}(\varepsilon), \widehat{b})$ and increasing for $b > \widehat{b}$.*

(III) *If $\varepsilon \geq a - c$, $p^0(b)$ is decreasing for $b \in (0, \widehat{b})$ and increasing for $b > \widehat{b}$.*

Proof See the Appendix. ■

Denote the optimal royalty by r^*. By Proposition 1, $r^* \in \{\varepsilon, r_b(\varepsilon)\}$. Let us write the industry output as $Q^0 = Q(b, r^*)$. An increase in b creates a direct and an indirect effect on Q^0: the first effect captures the standard impact of an increase in marginal cost and has a negative sign. The second effect operates via the royalty rate. Letting $\dfrac{dQ^0}{db}$ denote the overall marginal effect of b on Q^0, we have

$$\frac{dQ^0}{db} = \underbrace{\frac{\partial Q^0}{\partial b}}_{<0} + \underbrace{\frac{\partial Q^0}{\partial r}}_{<0} \cdot \underbrace{\frac{\partial r^*}{\partial b}}_{\leqq 0} \tag{8}$$

[3] As a measure of intensity of decreasing returns we take the cost elasticity, i.e., the ratio of marginal cost over the average cost.

If $r^* = \varepsilon$, the indirect effect of b is zero and industry output decreases (price increases) in b. If $r^* = r_b(\varepsilon)$, (8) becomes

$$\frac{dQ^0}{db} = \underbrace{\frac{\partial Q^0}{\partial b}}_{<0} + \underbrace{\frac{\partial Q^0}{\partial r}}_{<0} \cdot \underbrace{\frac{\partial r_b(\varepsilon)}{\partial b}}_{<0} \tag{9}$$

In this case, the indirect effect is positive as an increase in b reduces the level of royalty. As a result of the two opposite effects, output has an inverse U-shape in b. In other words, under royalty $r_b(\varepsilon)$, market price is U-shaped in b.

The specific ranges identified in Proposition 3 are due to the optimal licensing policy. Recall by Proposition 1 that if the magnitude of innovation is low, the optimal royalty is ε irrespective of b. So for small innovations, industry output always decreases in b; hence price increases in b. If the magnitude of innovation is large, the royalty charged is $r_b(\varepsilon)$ irrespective of b; market price is then U-shaped in b. Finally, if the magnitude is intermediate, the optimal royalty is ε for low b and $r_b(\varepsilon)$ for high b. So price is increasing for low b and it is U-shaped for high b.

Remark Our comparative statics exercise has focused on changes of b. Alternatively, we could examine the constant term of the marginal cost line (net of the magnitude of innovation). The impact on industry output would again be captured by an expression similar to (8). However, in this case the direct effect would always dominate the indirect effect.

4 Pareto-improving taxes

For this section we restrict to cases where the post-licensing price $p^0(b)$ is decreasing in b. Let $\varepsilon > \widehat{\varepsilon}$ and define

$$\underline{b}(\varepsilon) := \begin{cases} \widetilde{b}(\varepsilon) & \text{if } \widehat{\varepsilon} < \varepsilon < a - c \\ 0 & \text{if } \varepsilon \geq a - c \end{cases} \tag{10}$$

By Proposition 2, if $\varepsilon > \widehat{\varepsilon}$, then $p^0(b)$ is decreasing for $b \in (\underline{b}(\varepsilon), \widehat{b})$. We will show that in this case it is possible to design a *quantity tax* that lowers the market price and benefits consumers. Moreover, the tax will generate sufficient revenue to offset the potential losses of firms from this taxation.

Let $t(q)$ be the tax policy that applies to any firm, i.e., any firm that sells q units has to pay tax $t(q)$. Take

$$t(q) = \tau q^2/2 \text{ where } \tau > 0 \tag{11}$$

As $t'(q) = \tau q > 0$ for $q > 0$, the marginal tax under the policy above is increasing[4] in q, i.e., this tax policy is *progressive*. It has the effect of raising each firm's marginal cost by τq, so the strategic interaction between firms under the tax policy (11) is the same as the interaction when there is no tax but the marginal costs under existing and new technologies are $(b+\tau)q+c$ and $(b+\tau)q+c-\varepsilon$. Accordingly, under the tax policy

[4]See Atkinson and Stiglitz (1976) for a discussion on non-linear taxation.

(11) firms play the extensive-form game[5] $G_{b+\tau}$. Denoting by $p^\tau(b)$ the post-licensing price under tax (11), we conclude that

$$p^\tau(b) = p^0(b+\tau) \tag{12}$$

For $i = 1, 2$, let q_i^τ be the quantity and Π_i^τ the payoff of firm i at the SPNE of $G_{b+\tau}$. Let S^τ be the sum of payoffs of firms $1, 2$; T^τ the tax revenue and U^τ the sum of tax revenue and payoffs of the firms at the SPNE, i.e.,

$$S^\tau := \Pi_1^\tau + \Pi_2^\tau, T^\tau := \tau[(q_1^\tau)^2 + (q_2^\tau)^2]/2 \text{ and } U^\tau := S^\tau + T^\tau \tag{13}$$

Note that S^0, T^0, U^0 correspond to the values when there is no tax (i.e. $\tau = 0$) and in particular $T^0 = 0$.

Definition A *tax and transfer scheme*, denoted by $(t(.), f_1, f_2)$, is a scheme under which any firm $i = 1, 2$ (i) pays quantity tax according to function $t(q)$ and (ii) receives a lump-sum transfer f_i.

Proposition 3 *Let* $\varepsilon > \widehat{\varepsilon}$, $\tau \in (0, \widehat{b} - \underline{b}(\varepsilon))$ *and* $b \in I^\tau \equiv (\underline{b}(\varepsilon), \widehat{b} - \tau)$. *The introduction of tax* (11) *has the following effects.*

(I) $p^\tau(b) < p^0(b)$, *i.e., the tax lowers the post-licensing price.*

(II) $\exists\ \underline{\tau} \in (0, \widehat{b} - \underline{b}(\varepsilon))$ *such that for all* $\tau \in (0, \underline{\tau})$

- (i) $S^\tau < S^0$ *i.e., the tax reduces the sum of payoffs of two firms. Consequently it makes one or both firms worse off.*
- (ii) $T^\tau > S^0 - S^\tau$ *i.e., the tax generates sufficient revenue to offset the sum of losses of firms from the tax.*
- (iii) $\exists$ *a tax and transfer scheme* $(t(.), f_1, f_2)$ *where firm* $i = 1, 2$ *pays tax according to* (11) *and receives a lump-sum transfer* f_i *such that* (a) *the scheme makes consumers better off compared to no taxes,* (b) $\Pi_i^\tau + f_i > \Pi_i^0$ *for* $i = 1, 2$ *i.e., the scheme makes both firms better off compared to no taxes and* (c) $f_1 + f_2 = T^\tau$, *i.e., the scheme is deficit-neutral.*

Proof See the Appendix. ■

Remark Observe by Prop 3(II)(i) that the tax (11) makes at least one firm worse off. If it makes both firms worse off, then any tax and transfer scheme satisfying Prop 3(II)(iii) must have positive lump-sum transfer for both firms, i.e., $f_i > 0$ for $i = 1, 2$. However, if tax (11) makes firm i better off and firm j worse off, then f_i could be negative (i.e. i might be required to make a lump-sum transfer, which together with the tax revenue will compensate for the losses of j), but i would still be better off compared to no taxes.

The quantity tax we propose essentially strengthens the intensity of diseconomies of scale. As a result the rate of optimal royalty falls. Therefore, the tax affects the

[5] Recall that when there are no taxes, the game played between two firms under existing technology (1) and new technology (2) is given by G_b, whose SPNE has been characterized in Proposition 1.

market price directly (via the increase of diseconomies of scale) and indirectly (via the reduction of royalty). Given the parametrization of Proposition 3, the indirect impact dominates and price falls. The fall in price, and the subsequent increase in quantities sold, allows the government to collect sufficient revenue to compensate firms. Hence our tax-transfer scheme strictly Pareto improves the welfare of all agents in the market.

5 Conclusions

In this paper we show that in the presence of patent licensing in a duopoly, stronger diseconomies of scale can benefit consumers in terms of lower prices. Using this result we propose simple tax-transfer policies that reduce the post-licensing price and compensate firms for their possible losses from taxation. Furthermore, these schemes are deficit-neutral and hence do not impose any fiscal constraints.

We have kept our framework simple for clarity of presentation. The main driving force of our result is the interaction of royalties with decreasing returns. Stronger decreasing returns have the direct effect of raising the price, but they have the indirect effect of lowering royalties which in turn reduces the price. In our model there are regions where the indirect effect dominates the direct effect and consequently, stronger decreasing returns result in lower market price. It is of interest to see if these regions expand or contract in a general oligopoly model where the patent holder has more than one rival. Another line of inquiry would be to the see the extent to which our proposed tax-transfer schemes work when there is asymmetry of information between the government and firms regarding certain features (e.g., strength of decreasing returns or magnitude of the innovation) of the market. These questions are left for future research.

Appendix

Let $i, j = 1, 2$ and $i \neq j$. By (3) and (5), the profit function firm i is

$$\pi_i = (a - q_i - q_j)q_i - (c - \delta)q_i - bq_i^2/2 \tag{14}$$

for some $\delta \in [0, \varepsilon]$, where (a) for $i = 1$, $\delta = \varepsilon$ and (b) for $i = 2$, $\delta = 0$ if firm 2 does not have a license and $\delta = \varepsilon - r \in [0, \varepsilon]$ if it has a license with royalty r.

Lemma A1 *Suppose firm i has profit function* (14) *for some $\delta \in [0, \varepsilon]$. For any $b > 0$, firm i has a unique best response to any quantity q_j chosen by its rival firm j, given as*

$$B^\delta(q_j) = \begin{cases} (a - c + \delta - q_j)/(b + 2), \text{ if } q_j \in [0, a - c + \delta) \\ 0, \text{ if } q_j \geq a - c + \delta \end{cases} \tag{15}$$

Proof Follows by standard computations. ■

Lemma A2 below describes the Nash equilibrium (NE) of the duopoly game of section 2.1. In addition to the notation used in the main text, let $p(r)$ denote the NE price when licensing occurs under r; let $\hat{p}$ denote the NE under no licensing; and let p_M, q_m denote the monopoly price and quantity.

Lemma A2

(i) *The monopoly equilibrium where the monopolist has the new technology is given by* $p_M = c+[(b+1)(a-c)-\varepsilon]/(b+2)$, $q_M = (a-c+\varepsilon)/(b+2)$, $\pi_M = (b+2)[q_M]^2/2$. *Furthermore,* $p_M \gtreqless c \Leftrightarrow \varepsilon \lesseqgtr (b+1)(a-c)$.

(ii) *Suppose firm 2 has a license under rate of royalty r. The resulting Cournot duopoly game has a unique NE.*

(a) *If* $\varepsilon < (b+1)(a-c)$, *then* $p(r) = c+[(b+1)(a-c)-2\varepsilon+r]/(b+3)$; $q_1(r) = [(b+1)(a-c+\varepsilon)+r]/(b+1)(b+3)$, $q_2(r) = [(b+1)(a-c+\varepsilon)-(b+2)r]/(b+1)(b+3)$ *and* $\pi_i(r) = (b+2)[q_i(r)]^2/2$ *for* $i = 1, 2$.

(b) *If* $\varepsilon \geq (b+1)(a-c)$, *then* $\exists\ \overline{r}_b(\varepsilon) \equiv (b+1)(a-c+\varepsilon)/(b+2) \in (0, \varepsilon]$ *such that if* $r \in [0, \overline{r}_b(\varepsilon))$ *NE price, outputs and profits are same as in* (a). *If* $r \in [\overline{r}_b(\varepsilon), \varepsilon]$, *then firm* 2 *does not produce and* $p(r) = p_M$, $q_1(r) = q_M$, *i.e, a monopoly is created with firm* 1.

(c) *For* $\varepsilon < (b+1)(a-c)$, $q_1(r)q_2(r)$ *is decreasing for* $r \in [0, \varepsilon]$; *for* $\varepsilon \geq (b+1)(a-c)$, $q_1(r)q_2(r)$ *is decreasing for* $r \in [0, \overline{r}_b(\varepsilon)]$ *and* $q_1(r)q_2(r) = 0$ *for* $r \in [\overline{r}_b(\varepsilon), \varepsilon]$.

(iii) *Suppose firm 2 does not have a license. The resulting Cournot duopoly game has a unique NE.*

(a) *If* $\varepsilon < (b+1)(a-c)$, *then* $\widehat{p} = c+[(b+1)(a-c)-\varepsilon]/(b+3) > c$; $\widehat{q}_1 = [(b+1)(a-c)+(b+2)\varepsilon]/(b+1)(b+3)$, $\widehat{q}_2 = [(b+1)(a-c)-\varepsilon]/(b+1)(b+3)$ *and* $\widehat{\pi}_i = (b+2)[\widehat{q}_i]^2/2$ *for* $i = 1, 2$.

(b) *If* $\varepsilon \geq (b+1)(a-c)$, *then* $\widehat{p} = p_M \leq c$. *Firm* 2 *does not produce and a monopoly is created with firm* 1.

(c) $\pi_i(\varepsilon) = \widehat{\pi}_i$ *for* $i = 1, 2$ *and* $p(\varepsilon) = \widehat{p}$.

Proof (i) Follows by standard computations.

(ii) By Lemma A1, the best response function of firm 1 to q_2 is $B^{\varepsilon}(q_2)$. If firm 2 has a license with rate of royalty r, its best response function to q_1 is $B^{\varepsilon-r}(q_1)$. The results of (a)-(b) follow by solving the system of two best response equations. To prove (c), let $\rho(r) := q_1(r)q_2(r)$. Using the expressions of $q_1(r), q_2(r)$ from (a) and (b), we observe that whenever $\rho(r) > 0$, it is a inverse u-shaped quadratic function. Noting that $\rho'(0) = -(a-c+\varepsilon)/(b+3)^2 < 0$, it follows that $\rho'(r) < 0$ for $r \geq 0$, which proves the result.

(iii) As before, the best response function of firm 1 is $B^{\varepsilon}(q_2)$. If firm 2 does not have a license, its best response function is $B^0(q_1)$. The results of (a)-(b) follow by solving the system of two best response equations. The result of (c) follows by noting that $B^{\varepsilon-r}(q_1)$ equals $B^0(q_1)$ when $r = \varepsilon$. ■

Let $F_b(p)$ be the profit of the monopolist under the new technology (2) evaluated at price p, i.e.,

$$F_b(p) := (p - c + \varepsilon)Q - bQ^2/2 \text{ where } Q = \max\{a - p, 0\} \tag{16}$$

As $\pi_1(r) = p(r)q_1(r) - (c-\varepsilon)q_1(r) - b[q_1(r)]^2/2$ and $\pi_2(r) = p(r)q_2(r) - (c-\varepsilon+r)q_2(r) - b[q_2(r)]^2/2$, denoting $Q(r) := q_1(r) + q_2(r)$ to be the industry output, from (7) we have

$$\Pi_1(r) = (p(r) - c + \varepsilon)Q(r) - b[(q_1(r))^2 + (q_2(r))^2]/2 - \widehat{\pi}_2$$

Using (16) in the expression above, it follows that

$$\Pi_1(r) = F_b(p(r)) + bq_1(r)q_2(r) - \widehat{\pi}_2 \tag{17}$$

Since firm 2 obtains $\widehat{\pi}_2$ in any SPNE of G_b, it follows from (17) that if the royalty rate is r in an SPNE of G_b, then the sum of payoffs of two firms is

$$\Pi_1(r) + \Pi_2(r) = F_b(p(r)) + bq_1(r)q_2(r) \tag{18}$$

Proof of Proposition 1

(I) **Case 1** $\varepsilon \geq (b+1)(a-c)$ (drastic innovations): For this case without a license firm 2 obtains zero profit and firm 1 becomes a monopolist, i.e., $\widehat{\pi}_2 = 0$ and $\widehat{\pi}_1 = \pi_M$. By Lemma A2, $\exists\, \overline{r}_b(\varepsilon) \in [0, \varepsilon]$ such that if 1 chooses $r \in [\overline{r}_b(\varepsilon), \varepsilon]$, then $\Pi_1(r) = \widehat{\pi}_1 = \pi_M$. Therefore it is sufficient to consider policies with $r \in [0, \overline{r}_b(\varepsilon)]$.

Using the results of Lemma A2 in (17), it follows that

$$\Pi_1'(r) = [(b+1)^2(a-c+\varepsilon) - h(b)r]/(b+1)^2(b+3)^2 \text{ where } h(b) := (b+2)(b^2+4b+1) \tag{19}$$

Since $h(b) > 0$,

$$\Pi_1'(r) \gtreqqless 0 \Leftrightarrow r \lesseqqgtr r_b(\varepsilon) \equiv (b+1)^2(a-c+\varepsilon)/h(b) > 0 \tag{20}$$

As $\overline{r}_b(\varepsilon) - r_b(\varepsilon) = b(b+1)(b+3)(a-c+\varepsilon)/h(b) > 0$, we conclude that over $r \in [0, \overline{r}_b(\varepsilon)]$, the unique maximum of $\Pi_1(r)$ is attained at $r = r_b(\varepsilon)$. Thus, in particular, $\Pi_1(r_b(\varepsilon)) > \Pi_1(\overline{r}_b(\varepsilon)) = \pi_M$. This proves that for any $b > 0$, it is optimal for firm 1 to license to firm 2 with rate of royalty $r = r_b(\varepsilon)$ and fee $\pi_2(r_b(\varepsilon)) - \widehat{\pi}_2 = \pi_2(r_b(\varepsilon))$, so firm 2 obtains zero net payoff. This proves the result for $\varepsilon \geq (b+1)(a-c)$.

Case 2 $\varepsilon < (b+1)(a-c)$ (non-drastic innovations): By Lemma A2(iii)(c) in the Appendix, $\widehat{\pi}_i = \pi_i(\varepsilon)$ for $i = 1, 2$. In particular, if firm 1 does not license, it obtains $\widehat{\pi}_1 = \pi_1(\varepsilon)$. If it offers a license with royalty $r = \varepsilon$, then by (7), firm 1 obtains $\Pi_1(\varepsilon) = \pi_1(\varepsilon) + \varepsilon q_2(\varepsilon) + \pi_2(\varepsilon) - \widehat{\pi}_2 = \pi_1(\varepsilon) + \varepsilon q_2(\varepsilon) > \pi_1(\varepsilon)$ (since $q_2(\varepsilon) > 0$ for a non-drastic innovation). This proves that it is optimal for firm 1 to offer a license to firm 2.

(II) Follows by the discussion of page 5: Recall that $\alpha = \pi_2(r) - \widehat{\pi}_2$ and that $\widehat{\pi}_2 = 0$ if the innovation is drastic and $\widehat{\pi}_2 > 0$ if the innovation is non-drastic.

(III) For any $r \in [0, \varepsilon]$, using the results of Lemma A2 in (17), it follows that for this case too $\Pi'(r)$ is also given by (19) and (20). Denote $w(b) := b^3 + 5b^2 + 7b + 1$,

$\ell(b) := (b+1)^2/w(b)$ and note that both are positive, $\ell(b) < b+1$ and $\ell(b)$ is decreasing with $\ell(0) = 1$ and $\lim_{b\to\infty} \ell(b) = 0$. Observe that $\varepsilon - r_b(\varepsilon) = w(b)[\varepsilon - \ell(b)(a-c)]/h(b)$. Hence $\varepsilon \gtreqless r_b(\varepsilon) \Leftrightarrow \varepsilon \gtreqless \ell(b)(a-c)$.

If $\varepsilon > \ell(b)(a-c)$, then $\varepsilon > r_b(\varepsilon)$ and by (20), the unique maximum of $\Pi_1(r)$ is attained at $r = r_b(\varepsilon)$; the resulting fixed fee is thus positive. This proves the result for $\ell(b)(a-c) < \varepsilon < (b+1)(a-c)$.

If $\varepsilon \leq \ell(b)(a-c)$, then $r_b(\varepsilon) \geq \varepsilon$ and by (20), the unique maximum of $\Pi_1(r)$ is attained at $r = \varepsilon$ where the fixed fee is $\alpha = \pi_2(\varepsilon) - \widehat{\pi}_2 = \pi_2(\varepsilon) - \pi_2(\varepsilon) = 0$. This proves the result for $\varepsilon \leq \ell(b)(a-c)$.

The properties of $r_b(\varepsilon)$ with respect to b follows by standard computations. ■

Proof of Proposition 2 Recall that $p(r) = c + [(b+1)(a-c) - 2\varepsilon + r]/(b+3)$ is the Cournot price with royalty r when both firms are active in the market. Then,

$$p^0 = \begin{cases} p(r_b(\varepsilon)) \text{ if } \varepsilon > \ell(b)(a-c) \\ p(\varepsilon) \text{ if } \varepsilon \leq \ell(b)(a-c) \end{cases} \tag{21}$$

Noting that $r_b(\varepsilon) = (b+1)^2(a-c+\varepsilon)/(b+2)(b^2+4b+1)$, it follows that

$$\partial p(r_b(\varepsilon))/\partial b = y(b)(a-c+\varepsilon)/[h(b)]^2 \text{ where } y(b) := 2b^4 + 10b^3 + 15b^2 + 4b - 1$$

and $\exists\, \widehat{b} \in (0.15, 0.16)$ such that $\partial p(r_b(\varepsilon))/\partial b \gtreqless 0 \Leftrightarrow b \gtreqless \widehat{b}$. We consider the following cases.

Case 1 $\varepsilon \geq a-c$: Since $\ell(0) = 1$ and $\ell(b)$ is decreasing, for this case $\varepsilon > \ell(b)(a-c)$ for all $b > 0$ and by (21), $p^0 = p(r_b(\varepsilon))$. So for this case, p^0 is decreasing for $b \in (0, \widehat{b})$ and increasing for $b > \widehat{b}$ which proves (III).

Case 2 $0 < \varepsilon < a-c$: Since $\ell(0) = 1$, $\ell(b)$ is decreasing and $\lim_{b\to\infty} \ell(b) = 0$, $\exists\, \widetilde{b}(\varepsilon) > 0$ such that $\varepsilon \gtreqless \ell(b)(a-c) \Leftrightarrow b \gtreqless \widetilde{b}(\varepsilon)$. From the properties of $\ell(b)$, it follows that $\widetilde{b}(\varepsilon)$ is decreasing with $\lim_{\varepsilon\uparrow a-c} \widetilde{b}(\varepsilon) = 0$ and $\lim_{\varepsilon\downarrow 0} \widetilde{b}(\varepsilon) = \infty$. Therefore, $\exists\, \widehat{\varepsilon} \in (0, a-c)$ such that $\widehat{b} \lesseqgtr \widetilde{b}(\varepsilon) \Leftrightarrow \varepsilon \lesseqgtr \widehat{\varepsilon}$.

Case 2(a) $0 < \varepsilon \leq \widehat{\varepsilon}$: For this case, $\widehat{b} \leq \widetilde{b}(\varepsilon)$. If $b \in (0, \widetilde{b}(\varepsilon)]$, then $\varepsilon \leq \ell(b)(a-c)$ and by (21), $p^0 = p(\varepsilon)$, which is increasing in b. If $b > \widetilde{b}(\varepsilon)$, then $\varepsilon > \ell(b)(a-c)$ and $p^0 = p(r_b(\varepsilon))$ which is increasing in b (since $b > \widetilde{b}(\varepsilon) \geq \widehat{b}$). By continuity of p^0 it follows that p^0 is increasing for all $b > 0$. This proves (I).

Case 2(b) $\widehat{\varepsilon} < \varepsilon < a-c$: For this case, $\widehat{b} > \widetilde{b}(\varepsilon)$. If $b \in (0, \widetilde{b}(\varepsilon)]$, then $\varepsilon \leq \ell(b)(a-c)$ and $p^0 = p(\varepsilon)$, which is increasing in b. If $b > \widetilde{b}(\varepsilon)$, then $\varepsilon > \ell(b)(a-c)$ and $p^0 = p(r_b(\varepsilon))$, which is decreasing for $b \in (\widetilde{b}(\varepsilon), \widehat{b})$ and increasing otherwise. Therefore for this case p^0 is increasing for $b \in (0, \widetilde{b}(\varepsilon))$, decreasing for $b \in (\widetilde{b}(\varepsilon), \widehat{b})$ and increasing for $b > \widehat{b}$. This proves (II). ■

Lemma A3 *Let $\tau \in (0, \widehat{b} - \underline{b}(\varepsilon))$ and $b \in I^\tau \equiv (\underline{b}(\varepsilon), \widehat{b} - \tau)$. Consider the functions given in (13). As functions of τ, S^τ is decreasing and T^τ, U^τ are both increasing at $\tau = 0$.*

Proof As the SPNE price of $G_{b+\tau}$ is $p^0(b+\tau)$, from (18) it follows that the sum of payoffs of firms at the SPNE is

$$S^\tau = F_{b+\tau}(p^0(b+\tau)) - bq_1^\tau q_2^\tau \tag{22}$$

where $F_b(p)$ is given in (16). Since $b+\tau \in (\underline{b}(\varepsilon), \widehat{b})$, from the proof of Prop 2 we know that at the SPNE of $G_{b+\tau}$, firm 1 sets royalty $r_{b+\tau}(\varepsilon)$ where $r_b(\varepsilon)$ is given in (20). The quantities q_1^τ, q_2^τ and the price $p^0(b+\tau)$ is obtained from Lemma A2(ii) by replacing b by $b+\tau$ and taking $r = r_{b+\tau}(\varepsilon)$. Denoting $\kappa(b) := (a-c+\varepsilon)^2/2[h(b)]^2 > 0$, from (13) and (22),

$$\frac{\mathrm{d}S^\tau}{\mathrm{d}\tau}[\tau = 0] = -\kappa(b)(b+1)(2b^3+6b^2+3b+1) < 0,$$

$$\frac{\mathrm{d}T^\tau}{\mathrm{d}\tau}[\tau = 0] = \kappa(b)(2b^4+10b^3+15b^2+6b+1) > 0, \quad \frac{\mathrm{d}U^\tau}{\mathrm{d}\tau}[\tau = 0] = \kappa(b)2b(b^2+2b+1) > 0$$

This proves the result. ■

Proof of Proposition 3 (I) For any $b \in I^\tau$, both $b, b+\tau \in (\underline{b}(\varepsilon), \widehat{b})$ and by Prop 2, $p^0(b+\tau) < p^0(b)$. Using (12), it follows that $p^\tau(b) = p^0(b+\tau) < p^0(b)$.

(II) Since as function of τ, S^τ is decreasing and U^τ is increasing at $\tau = 0$ (see Lemma A3 of the Appendix), $\exists$ a sufficiently small $\underline{\tau} \in (0, \widehat{b} - \underline{b}(\varepsilon))$ such that for all $\tau \in (0, \underline{\tau})$:

$$S^\tau < S^0 \text{ and } U^\tau > U^0 \tag{23}$$

The first inequality of (23) proves (i). As $U^\tau = S^\tau + T^\tau$ and $T^0 = 0$, the last inequality of (23) implies

$$T^\tau > S^0 - S^\tau = (\Pi_1^0 - \Pi_1^\tau) + (\Pi_2^0 - \Pi_2^\tau) \tag{24}$$

which proves (ii).

Since lump-sum transfer do not affect market prices, (iii)(a) is immediate from (I). Denote $T^\tau - (S^0 - S^\tau) \equiv d > 0$ and take $f_i = \Pi_i^0 - \Pi_i^\tau + d/2$ for $i = 1, 2$. Then $\Pi_i^\tau + f_i = \Pi_i^0 + d/2 > \Pi_i^0$ for $i = 1, 2$ and by (24), $f_1 + f_2 = S^0 - S^\tau + d = T^\tau$. This proves (b) and (c).

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