Non-Parametric methods: An application for the risk measurement

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An application for the risk measurement[♦]

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Resumen

Actualmente, las instituciones financieras están expuestas a diferentes tipos de riesgos lo que ha acrecentado la necesidad de contar con nuevas herramientas analíticas para la administración del riesgo, destacándose el Valor en Riesgo (VaR). Existen diferentes métodos de cálculo; sin embargo, al existir una necesidad de contar con herramientas que modelen el comportamiento de los mercados financieros de manera más precisa, el presente trabajo propone el cálculo del VaR utilizando métodos de estimación no paramétricos (kernel) para portafolios con una distribución caracterizada por colas gordas. La evidencia muestra que al ajustar la variación del valor de un portafolio con estos métodos se modela de manera más precisa el comportamiento del mismo ya que no se asume un comportamiento predeterminado.

Abstract

Currently, the financial institutions are exposed to different types of risks, which has increased the need for new analytical instruments for the risk management, being one of most developed the Value at Risk (VaR). There are different methods of calculation; however, as it was affirmed, there exists an increasing need to be provided with analytical tools that shape the behavior of the financial markets in a more accurate way, in this sense, the present work proposes the calculation of the VaR using non-parametric methods (kernel estimator) for portfolios characterized by heavy-tailed distributions. The evidence shows that the behavior of the changes of a portfolio's return can be estimated in a more precise way since there is not assumption about the distribution, as in case of a normal distribution.

JEL: G32, C14

Keywords: Value at Risk, Non-parametric methods

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1. Introduction

In the financial literature, risk is defined as the volatility of unexpected outcomes, generally the value of assets or liabilities of interest.[1] Financial Institutions are exposed to various types of risks, which can be broadly classified into financial and nonfinancial risks. The financial risks include: the market risk, the credit risk and operational risk.

In the last five decades, the theory and the practice of risk management have developed enormously, it has developed to the point where risk management is now regarded as a distinct sub-field of the theory of finance and is one of the more intensely discussed topic not just for the finance agents or regulatory entities but also for specialists in the academic field.

One factor behind the rapid development of risk management was the high level of instability in the economic environment within which firms operated. A volatile environment exposes firms to greater financial risk, and therefore provides an incentive for firms to find new and better ways of managing this risk.

Another factor contributing to the transformation of risk management is the huge increase in trading activity since the late 1960s. Furthermore, there have been massive increases in the range of instruments traded over the past three decades, standing out the rapidly growth of derivative instruments.

A third contributing factor to the development of risk management was the rapid advance in the state of information technology.

[1] Jorion (2001)

Consequently, all of these factors have created the need for new financial instruments and analytical tools for risk management, being one of the most developed the Value-at-Risk (VaR).

Best (1998) define VaR as the maximum loss that may be experienced on a portfolio with a given level of confidence. One of the most used methods to calculate the VaR is the delta-normal method, which assumes that the portfolio exposures are linear and that the risk factors are jointly normally distributed. However, researchers have long held reservations about the strong assumptions made in parametric models about the distributions. In this regard, the techniques of non-parametric estimation discard essentially all fixed assumptions about the functional form and distribution, where the centerpiece is the kernel density estimator.[2]

Therefore, this paper considers estimation of a probability density function of a given portfolio's returns using the non-parametric estimations and compares it with the traditional parametric method.

The document proceeds as follows: Section 2 introduces the VaR methods, the non-parametric techniques, the kernel estimation and the application to the risk management of a investment portfolio. The estimation of the VaR for a given portfolio using the kernel estimation is described in Section 3. The performance of this technique is compared with the traditional parametric estimation. Finally, Section 4 summarizes and concludes.

[2] Green (2003)

2. The risk measurement

2.1. Value at Risk (VaR)

Value at Risk (VaR) is a statistical measure of the risk that estimates the maximum loss that may be experienced on a portfolio with a given level of confidence (Best, 1998). VaR always comes with a probability that says how likely it is that losses will be smaller than the amount given. VaR is a monetary amount which may be lost over a certain specified period of time. It is typically calculated for one day time period and is often calculated with 95% confidence. 95% confidence means that, on average, only 1 day in 20 would you expect to lose more than the VaR calculated, due to market movements. Thus the typical definition becomes: *the maximum amount of money that may be lost on a portfolio in 1 day, with 95% confidence* (Figure N° 1).

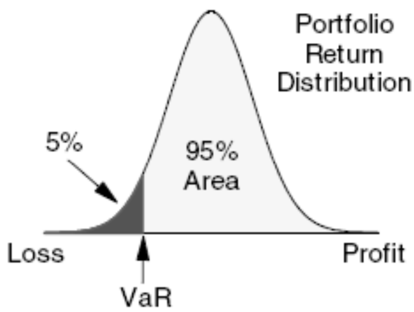


Figure N° 1

2.1.1. VaR methods

Approaches to VaR basically can be classified into two groups[3]:

i. Local-valuation methods, they measure risk by valuing the portfolio once, at the initial position and using local derivatives to infer possible movements. The delta-normal method uses linear derivatives and assumes normal distributions

ii. Full-valuation methods, they measure risk by fully repricing the portfolio over a range of scenarios

a) Delta-Normal Method

The delta-normal method is the simplest VaR approach. It assumes that the portfolio exposures are linear and that the risk factors are jointly normally distributed. As such, it is a local valuation method.

Because the portfolio return is a linear combination of normal variables, it is normally distributed. Using matrix notations, the portfolio variance is given by:

$$\sigma^2(R_p) = x'\Sigma \quad x$$

Where x: vector of portfolio exposures

Σ: variance-covariance matrix

If the portfolio volatility is measured in dollars, VaR is directly obtained from the standard normal deviate α that corresponds to the confidence level c:

[3] Jorion, P. (2001)

$$VaR = -\left[E(R_p) - Z_\alpha * \sigma(R_p)\right]$$

The delta-normal method simplifies the process by:

- Specifying a list of risk factors
- Mapping the linear exposure of all instruments in the portfolio onto these risk factors
- Aggregate these exposures across instruments
- Estimating the covariance matrix of the risk factors
- Computing the total portfolio risk

b) Historical simulation Method

Historical simulation takes a portfolio of assets at a particular point in time and then revalues the portfolio a number of times, using a history of prices for the assets in the portfolio. The portfolio revaluations produce a distribution of profit and losses which can be examined to determine the VaR of the portfolio with a chosen level of confidence.

Define the current time as t; we observe data from 1 to t. The current portfolio value is P_t, which is a function of the current risk factors:

$$P_t = P\left[f_{1,t}, f_{2,t}, \dots, f_{N,t}\right]$$

We sample the factor movements from the historical distribution, without replacement

$$\Delta f_i^k = \left\{\Delta f_{i,1}, \Delta f_{i,2}, \dots, \Delta f_{i,t},\right\}$$

From this we can construct hypothetical factor values, starting from the current one

$$f_i^k = f_{i,1} + \Delta f_{i,t}$$

which are used to construct a hypothetical value of the current portfolio under the new scenario:

$$P^k = P\left[f_1^k, f_2^k, \ldots, f_N^k,\right]$$

We can now compute changes in portfolio values from the current position $R^k=(P^k-P_t)/P_t$. We sort the returns and pick the one that corresponds to the cth quantile, $R_p(c)$. VAR is obtained from the difference between the average and the quantile:

$$VaR = E\left[R_p\right] - R_p(c)$$

c) Monte Carlo Simulation method

The Monte Carlo simulation method is basically similar to the historical simulation, except that the movements in risk factors are generated by drawings from some distribution. Instead of:

$$\Delta f_i^k = \left\{\Delta f_{i,1}, \Delta f_{i,2}, \ldots, \Delta f_{i,t},\right\}$$

now we have:

$$\Delta f^k \sim g(\theta)$$

Where g is the joint distribution (e.g. a normal or Student's) and θ the required parameters. The risk manager samples from this distribution and then generates pseudo-dollar returns as before. Finally, the returns are sorted to produce the desired VAR.

A comparison of VaR methods is provided in Appendix A.

2.1.2. VaR parameters

To measure VAR, we first need to define three parameters, the confidence level, the horizon and the base currency.

a) Confidence level

The higher the confidence level the greater the VAR measure. Varying the confidence level provides useful information about the return distribution and potential extreme losses. It is not clear, however, whether one should stop at 99%, 99.9%, 99.99% and so on. Each of these values will create an increasingly larger loss, but less likely.

According to RMG (1999): "there is nothing magical about confidence levels. In choosing confidence levels for market risk, companies should consider worst-case loss amounts that are large enough to be material, but that occur frequently enough to be observable".

Therefore, the usual recommendation is to pick a confidence level that is not too high, such as 95 to 99 percent.

b) Horizon

The longer the horizon the greater the VAR measure. This extrapolation depends on two factors, the behavior of the risk factors, and the portfolio positions.

To extrapolate from a one-day horizon to a longer horizon, we need to assume that returns are independently and identically distributed. This allows us to transform a daily volatility to a multiple-day volatility by multiplication by the square root of time. We also need to assume that the distribution of daily returns is unchanged for longer horizons, which restricts the class of distribution to the so-called "stable" family, of which the normal is a member. If so, we have:

$$VaR(T\ days) = VaR(1\ day) * \sqrt{T}$$

In practice, the horizon cannot be less than the frequency of reporting of profits and losses (P&L). Typically, banks measure P&L on a daily basis, and corporate on a longer interval (ranging from daily to monthly). This interval is the minimum horizon for VAR.

c) Base currency

The base currency for calculating VaR is typically the currency of equity capital and reporting currency of a company.

2.2 Methods of density estimation

Denote *f*=*f*(x) as the continuous density function of a random variable X at a point , and x_1, x_2,…, x_n, are the observations drawn from *f*. Two general methods have been advanced for the estimation of *f*.[4]

2.2.1 Parametric estimators

Parametric methods specify a form for *f*, say, the normal density,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

[4] Pagan (1999)

Where the mean μ and the variance σ^2 are the parameters of *f*. An estimation of *f* can be written as:

$$\hat{f}(x) = \frac{1}{\hat{\sigma}\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{x-\hat{\mu}}{\hat{\sigma}}\right)^2\right]$$

Where μ and σ^2 are estimated consistently from data as

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \text{and} \qquad \hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu})^2$$

respectively.

2.2.2 Nonparametric estimation

A disadvantage of the parametric method is the need to stipulate the true parametric density of *f*. In the nonparametric alternative *f(x)* is directly estimated without assuming its form. According to Davidson (2004), estimation by nonparametric methods refers to a variety of estimation techniques that do not explicitly involve estimating parameters.

2.2.2.1 Histogram

The histogram is one such estimator and it is one of the oldest methods of density estimation. But, although the histogram is a useful method of density estimation, it has the drawback of being discontinuous. Given a sample x_t, t=1, …, n, of independent realizations of a random variable X, let x_k be the midpoint of the kth bin and let h be the width of the bin (bandwidth). The distances to the left and right boundaries of the binds are h/2. The frequency count in each bin is the number of observations in the sample which fall in the range $x_k \pm h/2$. Collecting terms, we have our estimator:

$$\hat{f}(x) = \frac{1}{n}\sum_{t=1}^{n}\frac{1}{h}I\left(x - \frac{h}{2} \leq x_i < x + \frac{h}{2}\right) = \frac{1}{n}\frac{frequency\ in\ bin\ k^{th}}{width\ of\ bin\ k^{th}}$$

Where I(.) denotes an indicator function, which equals 1 if the statement is true and 0 if it is false, and the notation f(x) is motivated by the fact that the histogram is an estimate of a density function. Thus the value of the histogram at x is the proportion of the sample points contained in the same bin as x, divided by the length of the bin. It is thus quite precisely the density of sample points in that segment.

In the limit with just one bin, the histogram is completely smooth, being constant over the sample range. In the other limit of an infinite number of bins, the histogram is completely unsmooth, its values alternating between zero and infinity. Neither limit is useful, what we seek is some intermediate degree of smoothness.

2.2.2.2 Kernel

The idea of the histogram is:

$$\frac{1}{n * interval\ length}\#\{obs.that\ fall\ into\ a\ small\ interval\ \boldsymbol{containing}\ x\}$$

The idea of the kernel density estimator is:

$$\frac{1}{n * interval\ length}\#\{obs.that\ fall\ into\ a\ small\ interval\ \boldsymbol{around}\ x\}$$

Now consider the interval [x-h, x+h), the interval length is 2h, thus we have:

$$\widehat{f_h}(x) = \frac{1}{n * 2h} \#\{x_i \in [x-h, x+h)\} = \frac{1}{2hn} \sum_{i=1}^{n} I(|x - x_i| \leq h)$$

$$= \frac{1}{hn} \sum_{i=1}^{n} \frac{1}{2} I\left(\left|\frac{x - x_i}{h}\right| \leq 1\right)$$

$$\widehat{f_h}(x) = \frac{1}{hn} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

Where K(u) is the kernel function

In this case, $K(u) = \frac{1}{2} I(|u| \leq 1)$ is the uniform kernel function, which assigns weight ½ to each observation in the interval around x. Points outside the interval assigns the weight 0. Other alternatives for kernel functions are:

Table N° 1

Kernel	**K(u)**
Epanechnikov	$\left(\frac{3}{4}\right)(-u^2 + 1)\, I(|u| \leq 1)$
Quadratic	$\left(\frac{15}{16}\right)(1 - u^2)^2\, I(|u| \leq 1)$
Triangular	$(1 - |u|)\, I(|u| \leq 1)$
Gauss	$(2\pi)^{-1/2}\, exp\left(-u^2/_2\right)$

2.2.2.3 Choosing the bandwidth

The Kernel density estimator is very sensitive to the value of the bandwidth parameter h. Moreover, since k(u) is nonlinear, we should expect a bias in a finite sample. Therefore, the larger is the bandwidth, the greater is the bias, but at the same time, the smaller is the variance. For too large value of h, it gives rise to oversmoothing. This suggests that, to make bias small, h should be small. However, when h is too small, the estimator suffers from undersmoothing, which implies that the variance of the kernel estimator is large. Thus any choice of h inevitably involves a tradeoff between the bias and the variance. This might suggest a search for an optimal bandwidth. Two popular choices for h are:

$$h = 1.06 * \hat{\sigma} * n^{-1/5}$$

$$h = IQR = 0.79 * (\hat{q}_{0.75} - \hat{q}_{0.25}) * n^{-1/5}$$

Where IQR: Interquartile Range

The first one is also called the "Rule of thumb" bandwidth. According to Davidson and MacKinnon (2004), it makes sense to use σ to measure the spread of the data. In fact, the value of IQR is optimal for data that are normally distributed when using a Gaussian kernel. He suggests use a combination of both as a rule of thumb:

$$h = 10.9 * min(\hat{\sigma}, IQR/1.349) * n^{-1/5}$$

2.2.2.4 Application[5]

Denote the portfolio allocation as follows:

[5] Scaillet, O. (2008)

$$w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = [w_1 \quad \dots \quad w_n]'$$

Where w_i is the proportion of the total portfolio invested in security i

Therefore, the total portfolio return is:

$$R_p = w'R_t = \sum_{i=1}^{n} w_i * R_{i,t}$$

Define VaR as follows:

$$P[w'R_t + VaR(a, \alpha) < 0] = \alpha$$

$$P[-w'R_t > VaR(a, \alpha)] = \alpha$$

$$P[-w'R_t > VaR(a, \alpha)] = \int_{VaR(a,\alpha)}^{+\infty} f(z)\, dz$$

Where *f(z)* is the density of the loss portfolio return

Replacing the unknown *f(z)* by its kernel estimate we have:

$$\hat{f}(z) = \frac{1}{Th} \sum_{i=1}^{T} K\left(\frac{z_t - z}{h}\right)$$

The idea is to solve the following equation for the estimation of VaR(a, α):

$$\int_{\hat{V}aR(a,\alpha)}^{+\infty} \hat{f}(z)\, dz = \alpha$$

$$\int_{\hat{V}aR(a,\alpha)}^{+\infty} \hat{f}(z)\, dz = \frac{1}{T} \sum_{t=1}^{T} \varphi\left(\frac{z_t - \hat{V}aR(a,\alpha)}{h}\right)$$

Therefore,

$$\hat{V}aR(a,\alpha) = argmin\left(\frac{1}{T}\sum_{t=1}^{T}\varphi\left(\frac{z_t - \hat{V}aR(a,\alpha)}{h}\right) - \alpha\right)^2$$

2.2.2.5 Sensitivity of VaR

To calculate the sensitivity of VaR to changes in each market factor, we have to calculate the partial derivative:

$$\frac{\partial VaR(w,\alpha)}{\partial w} = \begin{bmatrix} \frac{\partial VaR(w,\alpha)}{\partial w_1} \\ \vdots \\ \frac{\partial VaR(w,\alpha)}{\partial w_n} \end{bmatrix}$$

$$\frac{\partial VaR(w,\alpha)}{\partial w} = E[-R_t| - w'R_t = VaR(w,\alpha)] = \begin{bmatrix} E\left[-R_{1,t}\right| - w'R_t = VaR(w,\alpha)\big] \\ \vdots \\ E\left[-R_{n,t}\right| - w'R_t = VaR(w,\alpha)\big] \end{bmatrix}$$

The estimation of a conditional mean by kernel estimation is:

$$E\left(Y_{i,t}\middle|X_{1,t} = x_1\right) = \frac{\frac{1}{Th}\sum_{t=1}^{T} y_{i,t}K\left(\frac{x_{1,t} - x_1}{h}\right)}{\frac{1}{Th}\sum_{t=1}^{T} K\left(\frac{x_{1,t} - x_1}{h}\right)}$$

Therefore, we get:

$$\frac{\partial VaR(w,\alpha)}{\partial w} = \frac{\frac{1}{Th}\sum_{t=1}^{T}(-R_t)K\left(\frac{-w'R_t - \hat{V}aR(w,\alpha)}{h}\right)}{\frac{1}{Th}\sum_{t=1}^{T} K\left(\frac{-w'R_t - \hat{V}aR(w,\alpha)}{h}\right)}$$

3. Empirical procedure

In this section, we proceed to compute the VaR of a portfolio composed by the stocks of five companies: Johnson & Johnson, Merck & Co. Inc., Bank of America, JP Morgan and

Apple Inc., all of them are members of the Standard & Poor's Index. First, we used the traditional delta-normal method, which assumes a normal distribution. Then, we compute the VaR using the proposed method (kernel estimator).

As we mentioned above, the experiments are based on the weekly returns of five companies' stocks for the period 1999 – 2010. The total return of the portfolio is the weighted sum of the individual returns, where the weights are the percentage participation of each company in the total portfolio.

3.1 Value at Risk (VaR)

3.1.1 Delta-Normal Method

As we mentioned in previous section, the delta-normal method assumes that the portfolio exposures are linear and that the risk factors are jointly normally distributed.

Therefore the VaR relies on the computation of the expected values and the variance-covariance matrix (i.e., the standard deviations and correlations) of the returns of the different risk factors.

The first step is to choose the market factors; in this case we are going to analyze the individual contribution of each stock in the total portfolio. It is necessary to mention that one can choose as a market factor other alternatives like the market sector (industry, sovereign and no sovereign, etc.). The second step consists in calculating the expected values and the standard deviations of, and correlations between, changes in the values of the market factors. In matrix form we have:

Expected Value: $E(R_p) = x'E(R)$

Variance-Covariance Matrix $\sigma^2(R_p) = x'\Sigma\, x$

Where $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ is the vector of participation of each of the n market factors in the total portfolio

Therefore, the portfolio composition is:

	Weight	*Media*
Johnson & Johnson	5%	0,000946
Merck & Co Inc	55%	-0,002896
Banc of America	5%	0,000371
JP Morgan	5%	0,002255
Apple Inc	30%	-0,002012
Portfolio	100%	-0,002018

The Variance – Covariance Matrix is:

	Johnson & Johnson	*Merck & Co Inc*	*Banc of America*	*JP Morgan*	*Apple Inc*
Johnson & Johnson	0,000173	0,000140	0,000175	0,000069	0,000001
Merck & Co Inc	0,000140	0,000943	0,000276	0,000130	-0,000041
Banc of America	0,000175	0,000276	0,001282	0,000581	0,000344
JP Morgan	0,000069	0,000130	0,000581	0,000625	0,000263
Apple Inc	0,000001	-0,000041	0,000344	0,000263	0,002759

Finally, we obtain the VaR with an α level of confidence for the period t:

$$VaR = -[E(R_p) - Z_\alpha * \sigma(R_p)]$$

Z(95%)	1,6449
Mean	-0,0020
Standard Deviation	0,0241
VaR (weekly)	0,0417
VaR (annual)	0,3007

Interpretation: Assuming 95% of confidence and a 1-week horizon, a VaR of 4.17% means that, only 1 week in 20 would you expect to lose more than 4 pp due to market movements. In order to express the weekly VaR in 1-year horizon we proceed in the following way:

$$VaR(1\ year) = VaR(1\ week) * \sqrt{52}$$

3.1.2 Non-parametric method (kernel estimator): Compute of VaR and sensitivity

In this section we propose the use of the non-parametric method: Kernel estimation, in order to calculate the VaR and the sensitivity of this measure to each risk factor (risk decomposition)[6]. The empirical procedure will consider the same market factors chosen for the Delta-Normal Method. The method proceeds as follows:

Step 1: Estimating the PDF and CDF of portfolio returns

Using a kernel estimator, we compute the probability density function (PDF) of the returns of the liquidity portfolio. Recall, our kernel estimator is given by:

$$\widehat{f_h}(x) = \frac{1}{hn}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

$$K(u) = (2\pi)^{-1/2}\, exp\left({}^{-u^2}/_{2}\right)$$

Where K(u) is the Gaussian kernel function

[6] All the computation was made using the program MATLAB version 7.4

In this case, we are estimating the PDF of the returns of a portfolio, which means that the dominion of the data is (-∞, +∞), therefore we propose the Gaussian kernel to approximate this type of data.

As we explained in previous section, the Kernel density estimator is very sensitive to the value of the bandwidth parameter h. Therefore, the rule selected is the "Rule of thumb":

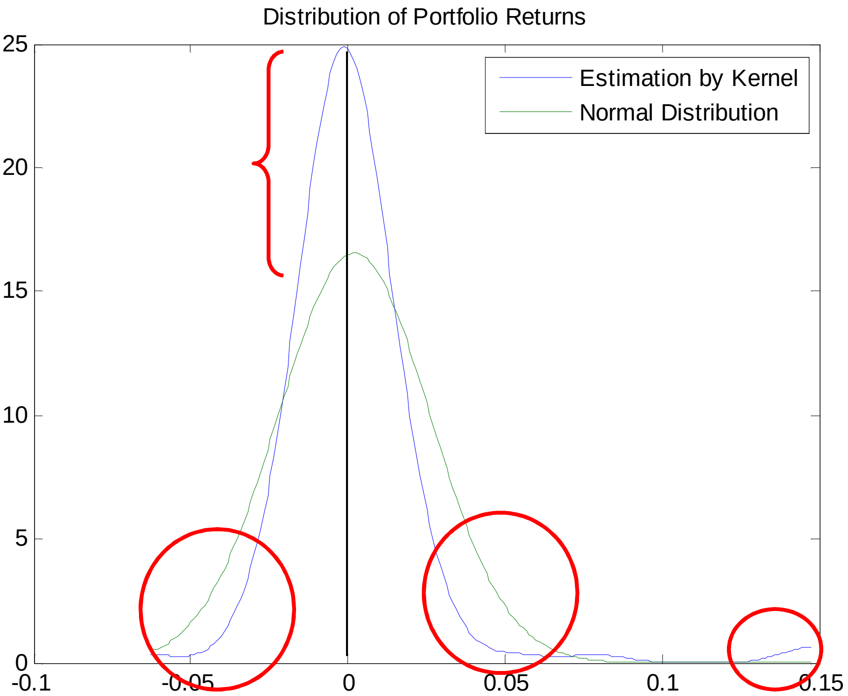
$$h = 1.06 * \hat{\sigma} * n^{-1/5}$$

Where $\hat{\sigma}$ is the empirical standard deviation of the data.

Figure N° 1 represents the PDF of the portfolio's return estimated by kernel and the familiar bell-shape normal distribution. As we can observe, the kernel estimation presents a positive skewed distribution, a leptokurtic shape and fat-tails, comparing to the normal distribution.[7] This means that the distribution of the returns of the portfolio does not follow a normal distribution, as it was assumed in the estimation of VaR using the Delta-Normal method. Therefore, the misspecification of the distribution of the portfolio's returns can lead us to biased estimations.

[7] For more detail, see Appendix B

Figure N° 2



The next table compares the values of the kurtosis and the skewness for a normal distribution and the values for our portfolio's returns:

Table N° 2

	Normal Distribution	*Portfolio*
Skewness	0,000	3,430
Kurtosis	3,000	22,173

Step 2: Estimation of VaR and sensitivities

Once it is computed the Kernel Density Function, we use it to calculate the VaR, solving the following equation for the value that minimize it:

$$\frac{1}{T}\sum_{t=1}^{T}\varphi\left(\frac{z_t-\hat{V}aR(a,\alpha)}{h}\right)=\alpha$$

$$\hat{V}aR(a,\alpha)=argmin\left(\frac{1}{T}\sum_{t=1}^{T}\varphi\left(\frac{z_t-\hat{V}aR(a,\alpha)}{h}\right)-\alpha\right)^2$$

Therefore,

Skewness	3,4297
Kurtosis	22,1731
VaR (weekly)	0,0271
VaR (annual)	0,1954

The next step is the compute of the sensitivities of VaR to each factor of risk, according to the following equation:

$$\frac{\partial VaR(w,\alpha)}{\partial w}=\frac{\frac{1}{Th}\sum_{t=1}^{T}(-R_t)K\left(\frac{-w'R_t-\hat{V}aR(w,\alpha)}{h}\right)}{\frac{1}{Th}\sum_{t=1}^{T}K\left(\frac{-w'R_t-\hat{V}aR(w,\alpha)}{h}\right)}$$

		VaR (weekl)	
	Weight	*Gauss*	*Non-parametric*
Johnson & Johnson	5%	0,006	0,009
Merck & Co Inc	55%	0,040	0,017
Banc of America	5%	0,024	0,013
JP Morgan	5%	0,012	0,014
Apple Inc	30%	0,059	0,024

Interpretation: Assuming 95% of confidence and a 1-week horizon, a VaR of 2.7% means that, only 1 week in 20 would you expect to lose more than 2 pp due to market movements. Regarding the risk decomposition, we observe that the largest contribution is from the

Apple Inc. securities, with an weekly contribution to risk of 2,4% and 5,9%, according to the Non-parametric and Delta-Normal methods, respectively.

4. Concluding remarks

The Valute at Risk (VaR), one of the most used measures of risk, is defined as: the maximum amount of money that may be lost on a portfolio in a predetermined period of time, with an α level of confidence. In order to estimate the VaR, two choices must be made: the method to be used and the parameters selected.

In this document we analyze two methods: the parametric and non-parametric methods. The former relies on the strong assumption that we know *a priori* what functional form is appropriated for describing the distribution associated with the random variable. Then the complete description of the random variable then merely requires the estimation of some parameters. Therefore, in this particular case we have assumed that the changes in the underlying market factors of a given portfolio are described by a multivariate normal distribution and we have based the VaR calculation on a linear approximation of the portfolio value. Moreover, the VaR relies on the computation of the expected values and the variance-covariance matrix of the returns of the different risk factors. However, if this assumption is not satisfied, it will yield biased estimates.

On the other hand, the non-parametric method makes no assumptions about the distribution of changes in the market factors, thus overcome with the potential problem of biased estimates of the Delta-Normal method. Comparing both methods, we can see that the parametric method treats the parametric model as exact, whereas the nonparametric estimation treats it as an approximation. Consequently, the apparent precision of parametric estimates is misleading unless the parametric model is known to be correct.

For that reason, we have proposed the use of nonparametric estimations, specifically the Gaussian Kernel estimator, in order to have a more accurate measure of the probability density function; and consequently of risk. Comparing with a normal distribution, the kernel estimation presents a positive skewed distribution, a leptokurtic shape and fat-tails, which means that the distribution of the returns does not follow a normal distribution. Comparing the VaR under the two methods, we found that the Delta-Normal method overestimates the Kernel VaR.

Likewise; we can use this method to decompose the total risk, measured by the VaR, in its components according to the choice of different market factors. It shows us that the largest contributors are the Apple Inc. securities, with an weekly contribution to risk of 2,4% and 5,9%, according to the Non-parametric and Delta-Normal methods, respectively.

Finally, a very useful application of this framework, for future investigation, is the Risk Budgeting. Once the estimation of the VaR and the risk decomposition is made, we can set limits, or risk budgets, on the quantity of risk assigned to each market factor and lastly establish asset allocations based on the risk budgets.

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6. Appendices

Appendix A

Comparison of VaR methods[8]

Table N° 1

Methodology	Advantage	Disadvantage
Parametric	• Fast and simple calculation • No need for extensive historical data (only volatility and correlation matrix are required)	• Less accurate for nonlinear portfolios, or for skewed distributions
Monte Carlo simulation	• Accurate for all instruments • Provides a full distribution of potential portfolio values (not just a specific percentile) • Permits use of various distributional assumptions (normal, T-distribution, normal mixture, etc.), and therefore has potential to address the issue of fat tails (formally known as "leptokurtosis") • No need for extensive historical data	• Computationally intensive and time-consuming (involves revaluing the portfolio under each scenario) • Quantifies fat-tailed risk only if market scenarios are generated from the appropriate distributions

8 RiskMetrics Group (1999)

Historical simulation	• Accurate for all instruments • Provides a full distribution of potential portfolio values (not just a specific percentile) • No need to make distributional assumptions (although parameter fitting may be performed on the resulting distribution) • Faster than Monte Carlo simulation because less scenarios are used	• Requires a significant amount of daily rate history • Difficult to scale far into the future (long horizons) • Coarse at high confidence levels (e.g., 99% and beyond) • Somewhat computationally intensive and time-consuming (involves re-valuing the portfolio under each scenario, although far less scenarios are required than for Monte Carlo) • Incorporates tail risk only if historical data set includes tail events

Appendix B

Skewness

In probability theory and statistics, skewness is a measure of the asymmetry of the probability distribution. The skewness value can be positive or negative. A negative value indicates that the tail on the left side of probability density function is longer that the right side and the bulk of the values lie to the right of the mean. A positive skew indicates that the tail on the right is longer than the left side and the bulk of the values lie to the left of the mean. A zero value indicates that the values are relatively evenly distributed on both sides of the mean.

Kurtosis

In probability theory and statistics, kurtosis is a measure of the "peakedness" of the probability distribution of a real-valued random variable. Higher kurtosis means more of the variance is the result of infrequent extreme deviations. A high kurtosis distribution has a sharper peak and longer, fatter tails, while a low kurtosis distribution has a more rounded peak and shorter thinner tails.

A distribution with positive excess kurtosis is called *leptokurtic*. In terms of shape, a leptokurtic distribution has a more acute peak around the mean (that is, a higher probability than a normally distributed variable of values near the mean) and fatter tails (that is, a higher probability than a normally distributed variable of extreme values). A distribution with negative excess kurtosis is called *platykurtic*. In terms of shape, a platykurtic distribution has a lower, wider peak around the mean (that is a lower probability than a normally distributed variable of values near the mean) and thinner tails (if viewed as the height of the probability density, that is a lower probability than a normally distributed variable of extreme values).