A General Coalition Structure: Some Equivalence Results

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Abstract

It is well known that in a differential information economy the free coalition formation may imply some theoretical difficulties. It does not suffice to say that a coalition can be formed by several agents. We define a set of all possible coalitions as the set of those coalitions that can be formed and joint by any agent. There exists, in this way, a rule imposed over coalition formation. We assume that only a subset $\mathcal{S}$ of Σ is alowed to form. In such way, we fix over the set of agents an aggregation rule for which the coalitions can be formed only if they belong to this subset. We have restricted the set of coalitions that can be joined by traders. The main result is the equivalence between two private core concept: the classical one for a differential information economy and the private core *restricted*.

Keywords: Differential information economy, restriction on coalition formation, private core.
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1 Introduction

The restriction of coalition formation is inflated by incomplete information. In a finite economy with N as the set of agents, it may happen that an

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agent will only know the preferences and endowments of a subset $K \subseteq N$ of people and can decide to form coalitions joint with agents from this group. Consequently, there is an upper maximum to the size of possible coalitions in the economy. Moreover, the formation of coalition may imply some theoretical difficulties, such as costs arising from forming a coalition or sharing information among agents. In fact, incompatibilities among different agents may arise and a big amount of information and communication might be needed to form a coalition. We will assume that only a subset $\mathcal{S}$ of the set of all possible coalitions in an economy is the set of admissible coalitions. We define the $\mathcal{S}$-core concept, as in Beloso-Garcia. We will extend to a model with both uncertainty and asymmetric informations the results showed in Okuda and Shitovitz.

There are some consequences of placing an upper limit on the set of possible coalitions. Intuitively the core will be larger. We call a core with an upper maximum a *restricted core*. The first study on this direction were made by Schmeidler [12], Vind [14] and Grodal [9].

We have remarked that the free coalition formation may imply some theoretical difficulties. It does not suffice to say that a coalition can be formed by several agents. We must take into account all limits imposed by the society to the aggregation in coalition. It is very simple to thing that agents are not free to form any coalition, especially in our framework. In fact, it is usually argued that the costs, which arise from forming a coalition, are not all negligible. Moreover, traders will form a coalition only if they know one to each other. Incompatibilities among different agents may arise and a big amount of information and communication might be needed to form a coalition. Thus, it will be not enough to say merely that several agents form a coalition.

We define a set of all possible coalitions as the set of those coalitions that can be formed and joint by any agent. There exists, in this way, a rule imposed over coalition formation. We assume that only a subset S of Σ is allowed to be formed. In such way, we fix over the set of agents a rule of aggregation for which the coalitions can be formed only if they belong to this subset. We have restricted the set of coalitions that can be joined by traders.

A coalition S is a measurable subset of T, such that $\mu(S) > 0$ which represents the size of coalition S. In the case of atomless economy, the size of a coalition S can be interpreted, following [12], as the amount of information and communication, or costs, needed in order to form the coalition S. Then, it may be meaningfully to consider those coalitions whose size converges to zero or, symmetrically, to one; that is, the coalitions that do not involve high costs to be formed.

The starting question is: suppose that in differential information economies a private allocation can be blocked, then "can it also be blocked by a coalition that is of a given structure"? Let $\mathcal{P} = (R_1, ..., R_k)$be a partition of the grand coalition, with k large enough. We will prove that an optimal private allocation x belongs to the core if and only if it cannot be improved upon by any coalition that includes at least one of the element of the partition $\mathcal{P}$. Under differentiability the dimension of the cone of the efficiency price vector is one, then the condition k large enough becomes $k \geq 2$. Our statements becomes, for any coalition R, a private allocation x belongs to the private core of a market if and only if it cannot be blocked by any coalition that contains R. Then, we can classifying core allocations with respect to the family of all coalitions that include one of the members of partition.

2 The model

We consider a Radner-type exchange economy $\mathcal{E}$ with differential information, with a finite number of types. The exogenous uncertainty is modeled by a measurable space $(\Omega, \mathcal{F})$, where Ω denotes a finite set of states of nature and the field $\mathcal{F}$ represents the set of all events. The space of traders is a measure space (T, Σ, μ), where T is the set of all traders, Σ is a σ-field of all coalitions, and μ is the Lebesgue measure. There is a finite number of goods, l, in each state. The information of traders $t \in T$ is described by a measurable partition $\Pi_t \in \Omega$. We denote by $\mathcal{F}_t$ the field generated by Π_t. If ω_0 is the true state of nature, trader t observes the member of Π_t which contains ω_0. Every traders $t \in T$ has a probability measure q_t on $\mathcal{F}$ which represents his *prior beliefs*: i.e. probability conditioned by their information set. The preferences of a trader $t \in T$ are represented by a state dependent utility function, $u_t : \Omega \times \Re^l_+ \to \Re$ such that $u_t(., \omega)$ is continuous, concave and strictly monotone a.e. in T. Moreover, each trader $t \in T$ has a fixed initial endowment $e_t : T \times \Omega \to \Re^l_+$, such that, $e(., \omega)$ is assumed to be μ-integrable in each state $\omega \in \Omega$ while $e(t, .)$ is $\mathcal{F}_t$-measurable, i.e. constant on each element of $\mathcal{F}_t$. The interpretation of this condition is that traders do not acquire any new information from their initial endowment. Let, for each $t \in T$, $M_t = \left\{ x_t : \Omega \to \Re^l_+ | \ x_t \text{ is } \mathcal{F}_\sqcup - \text{measurable} \right\}$ be the set of all $\mathcal{F}\ _t$-measurable selections from the random consumption set of agent t. Throughout the paper, we shall assume that $e(t, \omega) \gg 0$, and, for any function $x_t : \Omega \to \Re^l_+$, we will denote by $h_t(x) = \sum\limits_{\omega \in \Omega} q_t(\omega) u_t(\omega, x(\omega))$ the *ex-ante expected utility* from x of trader t.

Definition 2.1 *Let R be a fixed coalition. An allocation $x(t, \omega)$ is said to belong to the R-inclusive core if it cannot be improved upon by any coalition S that includes R; i.e. if there is no coalition S and an assignment y $\mathcal{F}_t$-measurable, $y : S \times \Omega \to \Re^l_+$ such that $R \subseteq S$, $\mu(S) > 0$, $\int_S y(t, .) d\mu \leq \int_S e(t, .) d\mu$ and $h_t(y(t, .)) > h_t(x(t, .))$ for almost every t in S.*

Definition 2.2 *A non-zero vector $p \ : \ \Omega \to \Re^l_+$ is an efficient price vector for the allocation $x(t, \omega)$ if μ a.e. in T , $x(t, \omega)$ is the maximal element of h_t over the efficiency set*

$$B^*_t(p) = \left\{ z \in M_t \ | \ \sum_{\omega \in \Omega} p(\omega) \cdot z(\omega) \leq \sum_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \right\}.$$

We denote the cone of all efficiency price vectors for an allocation $x(t, \omega)$ by $P(x, \succ_t) = \left\{ p \in \Re^{l \times n}_+ : x \succ_t y \Rightarrow \sum\limits_{\omega \in \Omega} p(\omega) \cdot x(t, \omega) \geq \sum\limits_{\omega \in \Omega} p(\omega) \cdot y(t, \omega) \right\}$ and its linear dimension by $r = \dim P$.

Definition 2.3 *Let $\mathcal{S} \in \Sigma$ be the subset of all admissible coalitions, with $\mu(S) > 0$ for every $S \in \mathcal{S}$. A feasible allocation $x(t, \omega)$ belongs to the $\mathcal{S}$-private core of $\mathcal{E}$ if it is not privately blocked by any coalition $S \in \mathcal{S}$.*

We denote this core as $\mathcal{S}$-$\mathbf{C}_p(\mathcal{E})$.

In each coalition S belonging to the subset $\mathcal{S}$ agents do not share their information, accordingly with the private blocking mechanism. Traders joint a coalition which belongs to $\mathcal{S}$, and they choose a private allocation over $\mathcal{S}$ which improves upon the allocation x.

From the definition of $\mathcal{S}$-core given $\mathcal{S}_1$, $\mathcal{S}_2 \subseteq \Sigma$ we can easily infer the following properties:

i) if $\mathcal{S}_1 \subseteq \mathcal{S}_2$ then $\mathcal{S}_2$-$\mathbf{C}_p(\mathcal{E})$$\subseteq$$\mathcal{S}_1$-$\mathbf{C}_p(\mathcal{E})$;

ii) $\mathcal{S}_1$-$\mathbf{C}_p(\mathcal{E})$$\cap$$\mathcal{S}_2$-$\mathbf{C}_p(\mathcal{E}) = (\mathcal{S}_1 \cup \mathcal{S}_2)$-$\mathbf{C}_p(\mathcal{E})$

From the property i) it is deduced that if the private core is non-empty, then so is the $\mathcal{S}$-private core. The property ii) implies that if $\Sigma = \bigcup_i \mathcal{S}_i$, then $\bigcap_i(\mathcal{S}_i - \mathbf{C}_p(\mathcal{E})) = \mathbf{C}_p(\mathcal{E})$. That is, for any partition $\mathcal{P}$ of the whole coalition set Σ the allocations belonging to the private core are exactly those allocations that belong to every $\mathcal{S}$-private core, with $S \in \mathcal{P}$, and the intersection of the S-private cores of a partition P does not depend on $\mathcal{P}$.

2.1 Some Technical Results

Given a fixed coalition $R \in \Sigma$, let

$$\mathcal{Q}_{\mathcal{R}} = \{S \in \Sigma : R \subseteq S\}$$

be the set of all coalitions which contain R. This structure define the only coalitions that can be formed as those containing R.
Define with $T \backslash \mathcal{Q}_{\mathcal{R}}$=$\{S \in \Sigma : R \cap S = \phi\}$.

Given this information structure, we turn to define the private core concept in a R-inclusive way.

Definition 2.4 *Let R be a fixed coalition. An allocation $x(t,\omega)$ is said to belong to the R-inclusive private core if it cannot be privately improved upon by any coalition $S \in \mathcal{S}$, with $\mathcal{S} = \mathcal{Q}_{\mathcal{R}}$; i.e. if there is no coalition S, with $\mu(S) > 0$, and a feasible assignment $y : S \times \Omega \to I\!\!B_+$, $\mathcal{F}_t$-measurable, such that*

i) *$R \subseteq S$,*

ii) *$h_t(y(t,.)) > h_t(x(t,.))$ for almost every t in S.*

Definition 2.5 *A feasible allocation $x(t,\omega)$ is individually rational if $h_t(x) \geq h_t(e)$ for almost every t in T.*

Definition 2.6 *A non-zero vector* $p\ :\ \Omega \to I\!B'_+$ *is an efficient price vector for the allocation* $x(t,\omega)$ *if* μ *a.e. in* T , $x(t,\omega)$ *is the maximal element of* h_t *over the efficiency set*

$$B_t^*(p) = \left\{ z \in M_t \mid \sum_{\omega\in\Omega} p(\omega)\cdot z(\omega) \le \sum_{\omega\in\Omega} p(\omega)\cdot x(t,\omega) \right\}.$$

We denote the cone of all efficiency price vectors for an allocation $x(t,\omega)$ by $P(x)$ and its linear dimension by $r = \dim P$[1].

We consider a finite and measurable partition $\mathcal{P} = (R_1, ..., R_k)$ of the grand coalition, with k large enough[2]. We prove that an optimal allocation x belongs to the core if and only if it cannot be improved upon by any coalition belonging to $\mathcal{Q}_{\mathcal{R}_\rangle}$ for all $i = 1, ...k$.

Lemma 2.7 *Let* $x(t,\omega)$ *be an allocation and let* p *be a non negative price,* $p \in I\!B'^{\Omega}_+$. *Then* p *is an efficient price vector for* x *if and only if* $p \cdot G^*(t) \ge 0$ *for almost all traders* t.

proof: The first implication is trivial.

Conversely, suppose that there exists a price p supporting the set $G^*(t)$ for almost all t in T. We want to show that $x(t,\omega)$ is the maximal element of the efficiency budget set $B_t^*(p)$ for almost all $t \in T$.

Suppose that $z \in B_t^*(p)$ and $h_t(z) > h_t(x)$. Then $\sum_{\omega\in\Omega} p(\omega)\cdot z(\omega) \le \sum_{\omega\in\Omega} p(\omega)\cdot x(t,\omega)$. By continuity, there exists $\alpha < 1$ such that $h_t(\alpha z) > h_t(x)$. Therefore, $\sum_{\omega\in\Omega} p(\omega)\cdot \alpha z(\omega) \ge \sum_{\omega\in\Omega} p(\omega)\cdot x(t,\omega) \ge \sum_{\omega\in\Omega} p(\omega)\cdot z(\omega)$. If $\sum_{\omega\in\Omega} p(\omega)\cdot z(\omega) > 0$ the contradiction $\sum_{\omega\in\Omega} p(\omega)\cdot z(\omega) > \sum_{\omega\in\Omega} p(\omega)\cdot \alpha z(\omega)$ follows. If $\sum_{\omega\in\Omega} p(\omega)\cdot z(\omega) = 0$ then $\sum_{\omega\in\Omega} p(\omega) x(t,\omega) = 0$. Since $x(t,\omega) \gg 0$ for almost all agents, $p(\omega) = 0$ for all $\omega \in \Omega$. Then, x is the maximal element of the efficient budget set. $\square$

[1] As it is shown in Grodal [9], it is always true that the linear dimension of the cone P of the efficiency price vectors is less than or equal to the number of commodities in the market, $l \cdot |\Omega|$, and that under classical assumption of differentiability and interiority $r = 1$.

[2] We refer to Okuda and Shitovitz [11]

Lemma 2.8 *For a given allocation $x(t,\omega)$, let F be a set-valued function such that $G^*(t) \subseteq F(t)$ for almost all traders t. If p is a non negative price such that $p \cdot \int F \geq 0$, then*

i) *(p, x) is an efficiency equilibrium,*

ii) *$p \cdot f(t) \geq 0$ for all integrable selections f of F and almost all $t \in T$.*

proof: For each $z \in I\!\!B_+^{\emptyset}$, let $G^{*-1}(z) = \{t \in T : z \in G^*(t)\}$ be the set of all agents t for which the allocation z belongs to the preferred set $G^*(t) = \{z \in M_t : h_t(z) > h_t(x)\} - x(t,.)$.
Then from $G^{*-1}(z) = \{t : h_t(z(.) + x(t,.)) > h_t(x(t,.))\}$ we infer that this set is measurable for each z. Let N be the set of all rational points $r \in Q^{\Omega}$, where Q is a dense and denumerable set of $I\!\!B$, for which $G^{*-1}(r)$ is null. Obviously, N is denumerable. Define with $S = \bigcup\limits_{r \in N} G^{*-1}(r)$. Then S is a null coalition. Suppose that for some $t \notin S$, there is a bundle $z(.) \in G^*(t)$ with $\sum\limits_{\omega \in \Omega} p(\omega) \cdot [z(t,\omega) - x(t,\omega)] < 0$. By continuity, we may find a rational point $r \in G^*(t)$ sufficiently close to z, so that we still have $\sum\limits_{\omega \in \Omega} p(\omega) \cdot r < 0$. Hence, for $t \notin S$ if $A = G^{*-1}(r)$ then $\mu(A) > 0$.
By desirability, for each $\epsilon > 0$, we have an integrable selection $f = r\chi_A + \epsilon q(t,.)\chi_{T \setminus A}$ from $G^*(t)$, where $q \in G^*(t)$. Hence, $f \in F(t)$. Therefore $0 \leq \sum\limits_{\omega \in \Omega} p(\omega) \cdot \int f = \sum\limits_{\omega \in \Omega} p(\omega) \cdot r\mu(A) + \epsilon \sum\limits_{\omega \in \Omega} p(\omega) \cdot \int\limits_{T \setminus A} q(t,\omega) \longrightarrow_{\epsilon \to 0} \sum\limits_{\omega \in \Omega} p(\omega) \cdot r\mu(A) < 0$ a contradiction.
Therefore, $\sum\limits_{\omega \in \Omega} p(\omega) \cdot G^*(t) \geq 0$ for almost all traders t, and by Lemma 2.7, (p, x) is an efficiency equilibrium.
Let f be an integrable selection from $F(t)$.
Define with $A = \left\{ t : \sum\limits_{\omega \in \Omega} p(\omega) \cdot f(t,\omega) > 0 \right\}$, then, for each $\epsilon > 0$, the integrable function $f = r\chi_A + \epsilon q(t,.)\chi_{T \setminus A}$ belongs to $F(t)$. Therefore $0 \leq \sum\limits_{\omega \in \Omega} p(\omega) \cdot \int f = \sum\limits_{\omega \in \Omega} p(\omega) \cdot \int_A f + \epsilon \sum\limits_{\omega \in \Omega} p(\omega) \cdot \int\limits_{T \setminus A} q(t,\omega) \longrightarrow_{\epsilon \to 0} \sum\limits_{\omega \in \Omega} p(\omega) \cdot \int_A f$. Therefore, $\sum\limits_{\omega \in \Omega} p(\omega) \cdot \int_A f \geq 0$, which implies by the definition of A that $\mu(A) > 0$. This completes the proof of the Lemma.

$\square$

3 The equivalence $\mathcal{C}_p(\mathcal{E}) = \mathcal{S} - \mathcal{C}_p(\mathcal{E})$

The purpose of this section is to prove the equivalence between two private core concept: the classical one for a differential information economy, and the private core restricted defined in the previous section.

Proposition 3.1 *Let $x(t,\omega)$ be an allocation. Then x is Pareto optimal if and only if there exists an efficient price vector $p \in I\!B'^{\emptyset}$ $(p \neq 0)$ such that $\sum_{\omega\in\Omega} p(\omega) \cdot \int_T x(t,\omega) = \sum_{\omega\in\Omega} p(\omega) \cdot \int_T e(t,\omega)$.*

proof: By contrary, suppose that x is not a Pareto optimal allocation. Then there exists an allocation $y : T \times \Omega \to I\!B_+$, with $y(t,.) \in M_t$ such that $\int_T y(t,.) \leq \int_T e(t,.)$ and $h_t(y) > h_t(x)$ for almost all $t \in T$. By assumption, there exists a supporting price $p : \Omega \to I\!B'_+$ such that $\sum_{\omega\in\Omega} p(\omega) \cdot y(t,\omega) > \sum_{\omega\in\Omega} p(\omega) \cdot x(t,\omega)$. By integrating over T, we get $\int_T p(.) \cdot y(t,.) > \int_T p(.) \cdot x(t,.)$. Since y is feasible, a contradiction follows.

For the converse, let us consider the correspondence G defined by $G(t) = \{z \in M_t : h_t(z(\cdot)) > h_t(x(t,\cdot))\}$.
We denote by $Z^*(t)$ the correspondence defined by $Z^*(t) = G(t) - e(t,\cdot)$ $\forall t \in T$. By Pareto optimal assumption, we know that $0 \notin \int_T Z^*(t)$. Therefore, by Separation hyperplane Theorem, there exists a price $p \neq 0$ such that $p \cdot \int_T Z^* \geq 0$, i.e. (p,x) is an efficient equilibrium.
Since $\int_T x(t,.)$ belongs to the closure of $\int_T G(t)$ for almost all $t \in T$, then $\int_T x(t,.) - \int_T e(t,.) \in \overline{\int_T Z^*}$ and do to feasibility the conclusion follows. □

Theorem 3.2 *Let $x(t,\omega)$ be a Pareto optimal allocation satisfying the smoothness assumption. Let $\mathcal{P}= (R_1,...,R_k)$ be a measurable partition of T. If $k \geq 2$, then x belongs to the private core if and only if x belongs to the R_i-inclusive private core for all i, $i = 1,...,k$.*

The proof of our results needs the following result:

Theorem 3.3 *Let $x(t,\omega)$ be an allocation and let R be a fixed coalition. Then x belongs to the R-inclusive core if and only if there exists an efficiency price vector $p : \emptyset \to I\!B'_+$ such that $\sum_{\omega\in\Omega} p(\omega) \cdot x(t,\omega) \leq \sum_{\omega\in\Omega} p(\omega) \cdot e(t,\omega)$ for almost each t in $T \backslash R$.*

proof: First assume that there exists an efficient price vector such that $\sum\limits_{\omega\in\Omega} p(\omega)\cdot x(t,\omega) \leq \sum\limits_{\omega\in\Omega} p(\omega)\cdot e(t,\omega)$ for almost each t in $T\backslash R$. Suppose by contrary that x does not belong to the R-inclusive private core, than there exist a coalition $S \supseteq R$ and a private allocation $y : T\times\Omega \to I\!B_+$, with $y(t,\omega)\in M_t$ such that $\int_S y(t,.) \leq \int_S e(t,.)$ and $h_t(y) > h_t(x)$ for almost all $t\in S$. Let define with z a private measurable allocation in this way

$$z = y_{\chi_S} + e_{\chi_{T\backslash R}}$$

then for almost every $t\in S$

$$\sum_{\omega\in\Omega} p(\omega)\cdot z(t,\omega) = \sum_{\omega\in\Omega} p(\omega)\cdot y(t,\omega) > \sum_{\omega\in\Omega} p(\omega)\cdot x(t,\omega)$$

and for almost every $t\in T\backslash S$

$$\sum_{\omega\in\Omega} p(\omega)\cdot z(t,\omega) = \sum_{\omega\in\Omega} p(\omega)\cdot e(t,\omega) \geq \sum_{\omega\in\Omega} p(\omega)\cdot x(t,\omega).$$

Then for almost all $t\in T$

$$\sum_{\omega\in\Omega} p(\omega)\cdot \int\limits_T z(t,\omega) > \sum_{\omega\in\Omega} p(\omega)\cdot \int\limits_T x(t,\omega)$$

and
$\sum\limits_{\omega\in\Omega} p(\omega)\cdot \int\limits_T z(\omega) = \sum\limits_{\omega\in\Omega} p(\omega)\cdot \int\limits_S y(t,\omega) + \sum\limits_{\omega\in\Omega} p(\omega)\cdot \int\limits_{T\backslash S} e(t,\omega) \leq \sum\limits_{\omega\in\Omega} p(\omega)\cdot \int\limits_T e(t,\omega)$, and the contradiction.

Let us look at the "only if" part. Assume that x belongs to the R-inclusive private core. Then x s Pareto optimal.
Define with $F(t)$ the correspondence:

$$F(t) = \begin{cases} G^*(t) & \text{for } t\in R \\ G^*(t)\cup[e(t,\omega)-x(t,\omega)] & \text{otherwise} \end{cases}$$

where $G^*(t) = \{z(.)-x(t,.)|z(.)\in M_t \text{ and } h_t(z(.)) > h_t(x(t,\omega))\}, \forall\, t\in T$.
By Pareto optimality $0\notin \int_T F(t)$.
From supporting hyperplane Theorem there exists a price $p:\Omega\to I\!B'_+$ such that $\sum\limits_{\omega\in\Omega} p(\omega)\cdot\int F(t) \geq 0$. By Lemma 2.8 p is an efficient price vector

for x. By monotonicity, there exists a measurable and integrable selection $f(t,.) = (e(t,.) - x(t,.))_{\chi_{T\setminus R}} + z(.)_{\chi_R}$, with $f(t,.) \in F(t)$ for almost all $t \in T$. Therefore, by lemma 2.8 $0 \leq p \cdot f(t,.) = p \cdot e(t,.) - p \cdot x(t,.)$ for almost all $t \in T \setminus R$ $\square$

Let us try to give an interpretation. If we consider a partition of T into two sets, namely R and its complement we will say that a strictly positive allocation belongs to the R-inclusive core if and only if it is possible for individuals belonging to $T \setminus R$ to choose the efficiency price vector $p(\omega)$, in each state of nature, so that the value of their bundle is less than or equal to the value of initial bundle. So that, despite of the measure of the fixed coalition R, agents in R are not willing to leave this coalition to join its complement and to gain.

Now we can show the demonstration of the main theorem:

proof: (Theorem 3.2)

Suppose that x belongs to each R_i-inclusive core. By theorem 3.3 there are efficient price vectors $p_i \geq 0$ for x, one for each R_i such that:

$$\sum_{\omega \in \Omega} p_i(\omega) \cdot x(t,\omega) \leq \sum_{\omega \in \Omega} p_i(\omega) \cdot e(t,\omega)$$

$\forall \quad i = 1, ...k$ and for almost all $t \in T \setminus R_i$. Such $p_i(\omega)$ are linearly dependent for all $\omega \in \Omega$, i.e., there exist $\alpha_1(\omega), ...\alpha_k(\omega)$ not all vanishing, with $\sum_{i=1}^{k} \alpha_i(\omega) p_i(\omega) = 0$ for all $\omega \in \Omega$. Let $I^+ = \{j : \alpha_j(\omega) > 0\}$ and $I^- = \{j : \alpha_j(\omega) < 0\}$. Since $p_i \geq 0$ for all $i = 1, ..., k$, I^+ and $I^-$are both nonempty. Let us define P by

$$P(.) = \sum_{i \in I^+} \alpha_i(.) p_i(.) = \sum_{i \in I^-} (-\alpha_i)(.) p_i(.)$$

P is the competitive price vector for x. Indeed,

i) P is an efficient price vector for x since by definition P is a convex cone.

ii) $\sum_{\omega \in \Omega} P(\omega) \cdot x(t,\omega) \leq \sum_{\omega \in \Omega} P(\omega) \cdot e(t,\omega)$ for almost each $t \in T$. In fact, let t be in T. Since $(R_1, ..., R_k)$ is a partition of T, there exists i_0 such that $t \in R_{i_0}$. Assume, w.l.o.g., that $i_0 \notin I^+$. Therefore, for every $j \in I^+$, we have $j \neq i_0$, in particular $t \notin R_j$ and therefore, by definition of the

$p_j(.)$, we have $\sum_{\omega\in\Omega} p_j(\omega)\cdot x(t,\omega) \le \sum_{\omega\in\Omega} p_j(\omega)\cdot e(t,\omega)$. Since $\alpha_j(\omega) > 0$ for $j \in I^+$, we have $\sum_{\omega\in\Omega} \alpha_j(\omega)p_j(\omega)\cdot x(t,\omega) \le \sum_{\omega\in\Omega} \alpha_j(\omega)p_j(\omega)\cdot e(t,\omega)$. Summing over I^+, we obtain the inequality

$$\sum_{\omega\in\Omega} P(\omega)\cdot x(t,\omega) = \sum_{\omega\in\Omega}\sum_{j\in I^+} \alpha_j(\omega)p_j(\omega)\cdot x(t,\omega) \le \\ \sum_{\omega\in\Omega}\sum_{j\in I^+} \alpha_j(\omega)p_j(\omega)\cdot e(t,\omega) = \sum_{\omega\in\Omega} P(\omega)\cdot e(t,\omega).$$

for almost each $t \in T$.

Now, by Theorem 3.3, x is a core allocation. $\square$

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