The pure logic of value, profit, interest

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11 May 2011

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Abstract

Standard economic models are based on axioms that epitomize the fundamental behavioral assumptions. This approach is trapped in a blind alley. The suggested change of perspective is guided by the question: what is the minimum set of nonbehavioral propositions for the consistent reconstruction of the evolving monetary economy? We start with three structural axioms and determine their real world implications. The differentiation of the axiom set leads to the structural value theorem. For the limiting case of the competitive structure a formal link to the classical and neoclassical value theories can be established.

JEL B12, B41, D00, E43

Keywords new framework of concepts, structure-centric, axiom set, structural value theorem, labour theory of value, marginalistic theory of value, profit, distributed profit, rate of interest, reproducibility

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J. S. Mill clearly enunciated the question than stands at the beginning of any and every scientific inquiry:

> What are the propositions which may reasonably be received without proof? That there must be some such propositions all are agreed, since there cannot be an infinite series of proof, a chain suspended from nothing. But to determine what these propositions are, is the *opus magnum* of the more recondite mental philosophy. (Mill, 2006b, p. 746)

Where and when the axiomatic method originated is uncertain. We know about it particularly from Euclid. It is certain, however, that in economics axiomatization commenced with Senior:

> It [the axiomatic method] was introduced to economics in A.D. 1836 by Nassau William Senior in his *Outline of the Science of Political Economy* and is today more or less consciously adopted by most economic theorists as the way of theorizing in economics. (Stigum, 1991, p. 4)

Euclid's path runs through the classical school (Halévy, 1960, p. 494), the neoclassical school (Jevons, 1911, p. 21) and reached an unprecedented level of Walrasian abstraction in the 1960s (Debreu, 1959, p. x). Whether the project of Wald, von Neumann, Debreu, Arrow, Hahn, McKenzie, and others (Ingrao and Israel, 1990), (Leonhard, 1995), (Weintraub, 1998) has met with success is no longer an open question. It is plain that 'anything based on this mock-up is unlikely to fly' (Hahn, 1981, p. 1036), see also (Ackerman, 2004), (Kirman, 1989, p. 126). Keynes, as so often, perspicuously articulated the challenge:

> The classical theorists resemble Euclidean geometers in a non-Euclidean world .... Yet, in truth, there is no remedy except to throw over the axiom of parallels and to work out a non-Euclidean geometry. Something similar is required to-day in economics. (Keynes, 1973, p. 16)

The crucial point is not axiomatization *per se* but the choice of axioms. One cannot *not* axiomatize but one can practice it imaginatively or mechanically, rigorously or sloppily, systematically or haphazardly, precisely to the point or beside it. As Clower put it:

> My opinion continues to be that axiomatics, like every other tool of science, is no better than its user, and not all users are skilled. (Clower, 1995, p. 308)

The thesis to start with suggests itself: neither neoclassical nor Keynesian economics possess a qualified axiomatic basis. Heterodox economists as complementary group mostly rebut the 'deductivist Euclidean methodology', recommend a pluralistic approach, and propose to give up the 'Euclidean hope' (Pålsson Syll, 2010, p. 52). But from the argument that the neoclassical behavioral axioms are unqualified does not logically follow that the axiomatic method is defective. An outstanding proponent of axiomatization like Poincaré simply did not accept Walras's hypotheses of selfishness and farsightedness as axioms (Kirman, 2009, p. 82). It only follows that some economists have not applied the method correctly, for whatever reasons (Hudson, 2010, p. 15-16). The point at issue is the real world content of axioms. The fact that the economy is an open system (Dow, 2010, pp. 272-274) does not on principle prevent axiomatization.

The second thesis says: human behavior does not yield to the axiomatic method. A behavioral axiom is a methodological oxymoron (cf. Cairnes 1875, p. v; Rosenberg, 1980; Hudík, 2011). Yet the axiomatization of the economy's basic structure is feasible. Accordingly, the first objective is to establish a formalism of maximum structural simplicity and generality. We start with an axiom set that is free of any behavioral specifications and subsequently approach the complexity of the real world by a process of consistent differentiation. The differentiation of the business sector in turn immediately leads to the question of what determines relative prices or, as the classics put it, exchange values. The question is in the following answered by the structural value theorem. The claim of generality entails that it should be possible to demonstrate that selected parts of the classical and neoclassical theory of value can be formally connected to the structural value theorem.

The consistent restatement in structural axiomatic terms of what is broadly referred to as theory of value is the purpose of the present paper. In the first part, the logical interdependencies of the key variables that formally embody the business and the household sector and the markets in between are expounded. In the second part, the real world implications with regard to the theory of value are made explicit. The analytical starting point is given with the structural axiom set in Section 1. In Sections 2 to 8 the structural value theorem, the profit ratio, and the rate of interest are derived for the competitive structure. Section 9 concludes.

1 Axioms and definitions

The first three axioms relate to income, production, and expenditures in a period of arbitrary length. For the remainder of this inquiry the period length is conveniently assumed to be the calendar year. It can be shown that the applicability of the axiom set does not depend on the chosen period length (for details see 2011b). Simplicity demands that we have at first one world economy, one firm, and one product. Quantitative and qualitative differentiation is obviously the next logical step after having worked out the implications of the following three axioms.

Total income of the household sector Y in period t is the sum of wage income, i.e. the product of wage rate W and total working hours L, and distributed profit, i.e. the product of dividend D and the number of shares N.

$$Y = WL + DN \quad |t \tag{1}$$

Output of the business sector O is the product of productivity R and working hours.

$$O = RL \quad |t \tag{2}$$

The productivity R depends on the underlying production process. The 2nd axiom should therefore not be misinterpreted as a linear production function.

Consumption expenditures C of the household sector is the product of price P and quantity bought X.

$$C = PX \quad |t \tag{3}$$

A set of axioms is a *tentative* formal starting point. The assessment comes on the next stage with the interpretation of the logical implications of the formal world and the comparison with selected data and phenomena of the real world. Axioms should have an intuitive economic interpretation (von Neumann and Morgenstern, 2007, p. 25). The economic meaning is rather obvious for the set of structural axioms. What deserves mention is that total income in (1) is the sum of wage income and *distributed profit* and not of wage income and profit. Profit and distributed profit have to be thoroughly kept apart.

By choosing *objective* structural relationships as axioms behavioral hypotheses are not ruled out. The structural axiom set is open to *any* behavioral assumption and not restricted to the standard optimization calculus (for details see 2011d).

Definitions are supplemented by connecting variables on the right-hand side of the identity sign that have already been introduced by the axioms (Boylan and O'Gorman, 2007, p. 431). They add no content to the set of axioms but merely facilitate the use of symbols. New variables are introduced with new axioms. With (4) wage income Y_W and distributed profit income Y_D is defined:

$$Y_W \equiv WL \qquad Y_D \equiv DN \quad |t. \tag{4}$$

With (5) the expenditure ratio ρ_E, the sales ratio ρ_X, the distributed profit ratio ρ_D, and the factor cost ratio ρ_F is defined:

$$\rho_E \equiv \frac{C}{Y} \qquad \rho_X \equiv \frac{X}{O} \qquad \rho_D \equiv \frac{Y_D}{Y_W} \qquad \rho_F \equiv \frac{W}{PR} \quad |t. \tag{5}$$

The axioms and definitions are consolidated to one single equation:

$$\rho_F \frac{\rho_E}{\rho_X}(1+\rho_D) = 1 \quad |t. \tag{6}$$

The period core (6) as absolute formal minimum determines the interdependencies of the measurable key ratios for each period. It asserts that the product of the constituents which characterize the firm, the market outcome, and the income distribution is always equal to unity. The period core is purely structural, i.e. free of any behavioral assumptions, unit-free[1] because all real and nominal dimensions cancel out, and contingent. Contingency means that it is open until explicitly stated which of the variables are independent and which is dependent. The form of (6) precludes any notion of causality; it states that the interdependence of the key ratios is subject to a 'conservation law' (cf. Mirowski, 1988, p. 3).

The factor cost ratio ρ_F summarizes the internal conditions of the firm. A value of $\rho_F < 1$ signifies that the real wage $\frac{W}{P}$ is lower than the productivity R or, in other words, that unit wage costs $\frac{W}{R}$ are lower than the price P or, in still other words, that the value of output per hour PR exceeds the value of input W. In this case the profit per unit is positive. Then we have the conditions in the product market. An expenditure ratio $\rho_E = 1$ indicates that consumption expenditures C are equal to income Y, in other words, that the household sector's budget is balanced. A value of $\rho_X = 1$ of the sales ratio means that the quantities produced O and sold X are equal in period t or, in other words, that the product market is cleared. In the special case $\rho_E = 1$ *and* $\rho_X = 1$ with budget balancing and market clearing the factor cost ratio ρ_F and with it the profit per unit is determined solely by the distributed profit ratio ρ_D. The period core (6) covers the key ratios about the firm, the market, and the income distribution and determines their interdependencies. The period core represents the pure consumption economy, that is, no investment expenditures, no foreign trade, and no taxes or any other state activity.

2 The structural value theorem (I)

The axioms and definitions have first to be differentiated for two firms. This formal exercise is referred to the Appendix. For the relative prices of two products then follows directly from (52) in combination with (44):

$$\frac{P_A}{P_B} = \frac{R_B}{R_A}\frac{L_B}{L_A}\frac{C_A}{C_B} \quad \text{if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{7}$$

If the markets for both products are cleared the price ratio is inversely proportional to the ratio of productivities and the ratio of labor inputs and directly proportional to

[1] "This procedure is in accordance with the principle of objectivity requiring that the whole theory and its interpretations have to be independent of the choice of the units of measurement. And this requirement is met, if the theory is unit-free, the necessary condition stated in Buckingham's Π-theorem." (Schmiechen, 2009, p. 176)

the ratio of the consumption expenditures for the two products. This implies a soft budget constraint, that is, the sum of consumption expenditures $C_A + C_B$ needs in the general case not be equal to income Y. For the special case of budget balancing $C = Y$, i.e. $\rho_{EA} + \rho_{EB} = 1$, it follows from (7):

$$\frac{P_A}{P_B} = \frac{O_B}{O_A} \frac{1}{\frac{1}{\rho_{EA}} - 1} \quad \text{if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{8}$$

Relative prices are equal to the inverse relation of the outputs of both firms multiplied with a factor that depends on the expenditure ratio for product A which is now complementary to the expenditure ratio for product B. For the special case with total expenditures divided equally between the two products, i.e. $\rho_{EA} = 0.5$, relative prices are simply equal to the inverse relation of the outputs. Or, as commonplace economics always had it (Niehans, 1994, p. 15), the relatively abundant product is cheap and the relatively scarce product is dear. If, on the other hand, outputs are numerically equal, then the relative price of product A varies directly with the expenditure ratio ρ_{EA}.

Relative prices depend according to (8) on the *objective* ratio of outputs, i.e. on supply, and on the *subjective* partitioning of consumption expenditures, i.e. on demand.

Since we have from the standard theory of consumer demand the marginalistic behavioral condition that the marginal rate of substitution MRS be equal to the price ratio we are in the position to synthesize the structural formalism and the marginalistic behavioral assumption. From the definition of the expenditure ratio (5) follows:

$$\frac{\rho_{EA}}{\rho_{EB}} = \frac{\frac{C_A}{Y}}{\frac{C_B}{Y}} = \frac{P_A X_A}{P_B X_B} \quad |t. \tag{9}$$

When, by applying the rule $MRS = \frac{P_A}{P_B}$, the optimal quantities X_A, X_B are determined in the usual way as coordinates of the tangential point of budget constraint and indifference map then the optimal partitioning of consumption expenditures ρ_{EA}, ρ_{EB} is also determined. This implies that *any* configuration of expenditure ratios can be formalized as a consumer optimum.

A straightforward result materializes as significant limiting case if the labor inputs of the two firms stand in the same proportion as the expenditures for both products:

$$\begin{gathered} \frac{P_A}{P_B} = \frac{R_B}{R_A} \\ \text{if} \quad \frac{L_A}{L_B} = \frac{\rho_{EA}}{\rho_{EB}} \quad \text{and if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \end{gathered} \tag{10}$$

If labor input is allocated according to the consumers' preferences, which are revealed by their expenditure ratios (cf. Marshall, 2009, p. 13), then relative prices are inversely proportional to the productivities in the two lines of production. Budget balancing is not required. Neither are speculations about the shape of a hypothetical production function required. The productivities in period t are measurable. We refer to this configuration as the *competitive structure*.

Now we define the exchange ratio as quotient of market clearing prices and the transformation ratio as quotient of productivities:

$$\rho_P \equiv \frac{P_A}{P_B} \quad \text{and} \quad \rho_R \equiv \frac{R_A}{R_B}$$

$$\frac{\text{units of } B}{\text{unit of } A} \leftarrow \frac{\frac{€}{\text{unit of } A}}{\frac{€}{\text{unit of } B}} \quad \text{and} \quad \frac{\text{units of } A}{\text{unit of } B} \leftarrow \frac{\frac{\text{units of } A}{h}}{\frac{\text{units of } B}{h}} \tag{11}$$

The exchange ratio is different from the price relation with regard to the dimension but has the same numerical value. The exchange ratio is the real counterpart of relative prices. In a strictly real analysis only exchange ratios are admissible. Likewise for the transformation ratio. In real terms (10) boils down to:

$$\rho_P = \frac{1}{\rho_R} \quad |t. \tag{12}$$

In the competitive structure the exchange ratio is inverse to the transformation ratio. This is the most elementary form of the objective relation between exchange and production. This real structural value theorem is entirely free of subjective connotations.

Let us summarize before we take profit into the picture. As a general structural result, which follows directly from the axioms, one has for relative prices:

$$\frac{P_A}{P_B} = \frac{X_B}{X_A}\frac{C_A}{C_B} \quad |t. \tag{13}$$

Expressed in ratios the structural value theorem takes these forms:

$$\rho_P = \frac{\rho_{XB}\,\rho_{EA}}{\rho_{XA}\,\rho_{EB}\,\rho_L\,\rho_R}(\text{i}) = \frac{\rho_{EA}}{\rho_{EB}\,\rho_L\,\rho_R}(\text{ii}) = \frac{1}{\rho_R}(\text{iii}) \quad |t. \tag{14}$$

Real economies will most probably be found between market clearing (ii) and the competitive structure (iii). In the competitive structure solely organization/technology stands in relation to relative prices respectively the exchange rate. Neither pleasure and pain, nor utility maximization and profit maximization play a

role in this state. Whether the budget is balanced or not is a matter of indifference. Obviously, it does not follow from the axiom set that this state is 'natural' and that the economy 'gravitates' towards it. This idea was an article of faith among the classics. Open to debate were only the nature of profit and the relation between natural price and the natural rates of wages, profit, and rent (Dobb, 1973, p. 44).

3 Formal touch points

The purely deductive result of (10) coincides with 'a doctrine of the utmost importance in political economy' (Ricardo, 1981, p. 13), according to which 'it is natural that what is usually the produce of two days or two hours labour, should be worth double of what is usually the produce of one day's or one hour's labour' (Smith, 2008, p 45). There is an alternative formulation of (10) that in combination with (2) formally embodies Adam Smith's proposition:

$$\frac{P_A}{P_B} = \frac{L_A^\star}{L_B^\star} \quad \text{with} \quad L_A^\star = \frac{L_A}{O_A} = \frac{1}{R_A} \quad |t. \tag{15}$$

The structural axiom set implies as a *limiting case* that relative prices are equal to the ratio of labor inputs per unit of the respective output (cf. Niehans, 1994, p. 89; Dmitriev, 1986, p. 81; Blaug, 1998, p. 110). In addition it implies the refutation of Jevons's sweeping critique of the classics. Jevons announced a new era of economic thinking by arguing that authority, i.e. Smith, Ricardo and J. S. Mill, had been on the wrong side and explained the nature of value by 'infinitely small amounts of pleasure and pain' (Jevons, 1911, p. vii). But since the subjective partitioning of consumption expenditures can always be expressed in marginalistic terms there is no contradiction between the 'Jevonian Law of Diminishing Utility' (Dobb, 1973, p. 109) and the classical labour theory of value in its most *elementary* form. In the competitive structure of the pure consumption economy the three value theories (classical, neoclassical, structural axiomatic) fall formally into one.

There is, of course, much more to classical and neoclassical value theory. The former is a substance theory the latter a field theory. The implications of these different conceptions have been elaborated at length by Mirowski (1995). Establishing the formal touch point with the labour theory of value does not entail that the more philosophical idea that value is created by labor is adopted (for Marx's theory of surplus value see 2011a).

4 Profit ratio equalization

In the structural axiomatic context the business sector's profit in period t is given with (16) as the difference between the sales revenues – for the economy as a whole

identical with consumption expenditures C – and costs – here identical with wage income Y_W:[2]

$$Q_{fi} \equiv C - Y_W \quad |t. \tag{16}$$

In explicit form, after the substitution of (3) and (4), this definition is identical with that of the theory of the firm:

$$Q_{fi} \equiv PX - WL \quad |t. \tag{17}$$

By applying the 1st axiom and the definitions (4) and (5) one arrives at:

$$Q_{fi} \equiv C - Y + Y_D \quad \text{or} \quad Q_{fi} \equiv \left(\rho_E - \frac{1}{1+\rho_D}\right) Y \quad |t. \tag{18}$$

To get rid of all absolute magnitudes the profit ratio ρ_Q is defined with (19) and this gives a succinct summary of the structural interrelation of the profit ratio, the expenditure ratio, and the distributed profit ratio for the business sector as a whole:

$$\rho_Q \equiv \frac{Q_{fi}}{Y_W} \quad \Rightarrow \quad \rho_Q \equiv \rho_E\,(1+\rho_D) - 1 \quad |t. \tag{19}$$

The overall profit ratio is positive if the expenditure ratio ρ_E is > 1 or the distributed profit ratio ρ_D is > 0, or both. The determinants of profit look essentially different depending on the perspective. For the firm price P, quantity X, wage rate W, and employment L in (17) seem to be all important; under the broader perspective of (18), which is formally equivalent, these variables play no role at all. The profit definition provokes a cognitive dissonance between the micro and the macro view that shall be dealt with in Section 6.

The first question is how profits are distributed between the two firms. The financial profit for each firm is given by:

$$\begin{array}{l} Q_{fiA} \equiv P_A X_A - W_A L_A \\ Q_{fiB} \equiv P_B X_B - W_B L_B \end{array} \quad |t. \tag{20}$$

Applying (3) and (2) one gets for relative profits in the competitive structure:

$$\frac{Q_{fiA}}{Q_{fiB}} = \left(\frac{1 - \dfrac{W_A}{P_A R_A}}{1 - \dfrac{W_B}{P_B R_B}}\right) \frac{C_A}{C_B} \quad \text{if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{21}$$

[2] Profits from changes in the value of financial and nonfinancial assets are neglected here. One member of the latter class is the stock of products which may change with regard to quantity and valuation price if the product market is not cleared in successive periods. This case is here excluded by the condition $\rho_X = 1$.

If the wage rates in the different lines of production are equal the numerical value in the brackets is one under the condition of (10) and the ratio of profits is equal to the ratio of consumption expenditures for the two products.

Equation (21) presupposes that the wage rate for all employees, i.e. inclusive management and executives, is equal. This is normally not so; hence W has to be taken as average wage rate that is given by:

$$W_A \equiv W_{A1}\frac{L_{A1}}{L_A} + \ldots + W_{Ai}\frac{L_{Ai}}{L_A} + \ldots + W_{An}\frac{L_{An}}{L_A} \quad |t. \tag{22}$$

From the purely formal standpoint it suffices that the *average* wage rates W_A and W_B are equal. It is obvious, however, that the differentiation of wage rates within a firm affects the partitioning of consumption expenditures if the individual expenditure ratios of different employees are different. To keep things simple, this interdependency between the distribution of wage rates within each firm and the partitioning of consumption expenditures between the two firms is ruled out with the assumption that the average expenditure ratios for both firms are, for the time being, independent from the distribution of wages within the firms.

For the comparison of firms with different size and different absolute profits the respective profit ratios are required. The profit ratio for the business sector as a whole (19) has been directly derived from the profit definition (16) and is adapted for a single firm as follows:

$$\rho_{QA} \equiv \frac{Q_{fiA}}{W_A L_A} \quad |t. \tag{23}$$

Combining (20) and (10) one gets for the relative profit ratios:

$$\frac{\rho_{QA}}{\rho_{QB}} = \frac{\dfrac{P_A R_A}{W_A} - 1}{\dfrac{P_B R_B}{W_B} - 1} = \frac{\dfrac{P_A R_A}{W_A} - 1}{\dfrac{P_A R_A}{W_B} - 1} \quad \text{if} \quad \rho_{\mathrm{XA}} = 1, \rho_{\mathrm{XB}} = 1 \quad |t. \tag{24}$$

If the (average) wage rates are equal the profitability of firms is equal in the competitive structure as defined by (10). There is, though, no such thing as a "law" of uniform profitability because there is nothing in the formalism that equalizes the average wage rates between firms. For the classics (Mill, 2006a, p. 472) and even more so for Walras (Morishima, 1977, pp. 82-83) profit equalization was self-evident. Of course, a formal proof of overall dynamic profit equalization could not be delivered. The classics simply added the principle of perfect competition to the principle of self-interest (Dmitriev, 1986, pp. 134-135) wherefrom in due time the general equilibrium emerged. If an equalizing mechanism exists it has to be separately identified and consistently combined with the axiom set.

5 The structural value theorem (II)

We now turn the question around and ask for the implications of equal profit ratios in the general case of different wage rates. Eq. (23) can be rewritten for both firms:

$$\begin{aligned} \rho_{QA} &\equiv \frac{\rho_{EA} Y}{W_A L_A} - 1 \\ & \qquad\qquad |t. \\ \rho_{QB} &\equiv \frac{\rho_{EB} Y}{W_B L_B} - 1 \end{aligned} \tag{25}$$

Under the condition of equal profit ratios this yields:

$$\frac{W_A L_A}{W_B L_B} = \frac{\rho_{EA}}{\rho_{EB}} \quad |t. \tag{26}$$

Equalization demands that the weighted labor inputs must be in proportion to the expenditure ratios. If wages rates are equal we are back at the initial condition for the competitive structure (10). Eq. (26) is taken as the the general condition for the competitive structure. Inserted in (7) the price relation in the competitive structure by consequence is:

$$\begin{gathered} \frac{P_A}{P_B} = \frac{\frac{W_A}{R_A}}{\frac{W_B}{R_B}} \\ \text{if} \quad \frac{W_A L_A}{W_B L_B} = \frac{\rho_{EA}}{\rho_{EB}} \quad \text{and if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \end{gathered} \tag{27}$$

The structural value theorem now states: relative prices are equal to the relation of unit wage wage costs in the competitive structure. This is the benchmark case. If wage rates are equal, then relative prices are equal to the inverse productivities as in (10). In both cases the profit ratios are equal.

The relation (27) cannot be expressed in a real economy. It is, however, possible to reformulate the weights in (26) as follows: $W_B = w_B W_A$; this eliminates the different wage rates in the condition of the competitive structure and gives the weighted labor input $L_B^{\bullet} = w_B L_B$. If $W_B > W_A$ then L_B simply gets a greater weight and we are back at the real value theorem. The general condition for the competitive structure is that weighted labor input, i.e. $L_A, L_B^{\bullet}$ is allocated in the same proportion as the expenditure ratios.

We therefore have two essentially different configurations: the arbitrary allocation of labor input and the competitive structure. In this configuration labor is allocated according to the household sector's preferences and the profit ratios are equal

between the firms. By translating the different wage rates into a weighting factor w_B the relation of unit wage costs of (27) turns into a weighted relation of productivities:

$$\frac{P_A}{P_B} = \frac{R_B^\bullet}{R_A}$$
$$\text{if} \quad \frac{L_A}{L_B^\bullet} = \frac{\rho_{EA}}{\rho_{EB}} \quad \text{and if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad \text{with} \quad W_B = w_B W_A \quad |t. \tag{28}$$

The point to emphasize is that in the perfect economic configuration all subjective elements are eliminated. In the structural axiomatic context value depends alone on weighted productivities. A change of preferences has no effect on relative prices only on the allocation of labor input. That means, we have to rethink the accustomed idea of the price mechanism.

Different profit ratios in different lines of production do not jeopardize the functioning of the system as a whole but must be taken as empirical normality. The market economy can exist for an indefinite time without equal profit ratios but not with losses. *This* is the *sine qua non*. The economy needs an adequate margin for erroneous decisions, counterproductive behavior, and all kinds of deviations from the benchmark structure. The safety buffer that ensures a reasonable stability is provided by profit. The structural precondition of a positive profit ratio for the economy as a whole is given with (19). If the expenditure ratio ρ_E is unity and the distributed profit ratio ρ_D is zero then the profit ratio for the business sector is zero. If profit ratios are not equal in this zero-profit economy the profit of one firm is equal to the loss of the other and this is not a comfortable situation over a longer time span. The axiomatic theory of profit is obviously fundamentally different from both Ricardo's 'Corn Theory of Profit' (Dobb, 1973, p. 70), (Blaug, 1998, p. 86) and Mill's abstinence theory (Mill, 2006a, p. 481).

When market clearing, budget balancing, and the equalization of profit ratios is assumed then the only subjectively chosen variable is the expenditure ratio for *one* product. The rest of the system is in this case determined by objective conditions. This economy deserves the predicate optimal because the partitioning of consumption expenditures can always be interpreted as optimal. In the case of budget balancing total profit is equal to distributed profit.

The optimal competitive structure can obviously be generalized for an arbitrary number of firms and products. In marked contrast to the classical approach the structural axiomatic approach asserts that a perfect competitive structure with all desirable properties is possible but *not* that the economy will attain this state sooner or later. This, though, is not a matter of primary concern. With regard to the proper functioning of the market economy the critical condition is that the expenditure ratio has to be greater than unity and/or the distributed profit ratio has to be greater than zero because a zero-profit economy – Walras's 'Ni bénéfice ni perte' – is not reproducible with more than one firm (for details see 2011f, pp. 10-14).

6 On profits

It is of utmost importance that profit Q_{fi} and distributed profit Y_D is clearly distinguished. The latter is a flow of income from the business to the household sector analogous to wage income. By contrast, profit is the difference of flows within the business sector. Profit is not connected to a factor input. So far we have labor input as the sole factor of production and wage income as the corresponding factor remuneration. Since for now the factor capital is nonexistent profit cannot be assigned to it in functional terms. And since profit cannot be counted as factor income (cf. Knight, 2006, p. 308-309, Schumpeter, 2008, p. 153) there is no place for it in the theory of income distribution. To mix up income and profit is a category mistake (for details see 2012).

The individual firm is blind to the structural relationship given by (18). On the firm's level profit is therefore subjectively interpreted as a reward for innovation or superior management skills or higher efficiency or toughness on wages or for risk taking or capitalizing on market imperfections or as the result of monopolistic practices. These factors play a role when it comes to the *distribution* of profits between firms and these phenomena become visible when similar firms of an industry are compared. Business does not 'make' profit; it redistributes profit. Because of this it is not wise to take the individual firm as analytical starting-point and then to generalize. The microeconomic approach is prone to the fallacy of composition.

The profit definition entails a cognitive dissonance between micro and macro, but no logical contradiction. In the first place, that is, irrespective of the distribution between individual firms total profit as a factor-independent residual (Ellerman, 1986, pp. 61-65) has nothing to do with price, costs, competition or any other factor that is crucial to the individual firm's management. The existence and magnitude of total profit is *not* explicable by the marginal principle.

Under the condition $\rho_E = 1$ profit Q_{fi} must, as a corollary of (18), be equal to distributed profit Y_D. The fundamental difference between the two variables is not an issue in this limiting case. The equality of profit and distributed profit is an implicit feature of equilibrium models and of general equilibrium theory in particular (Patinkin, 1989, p. 329), (Buiter, 1980, pp. 3, 7). These approaches cannot find a counterpart in reality because profit and distributed profit are never equal. Models that are based on the collapsed definition total income $\equiv$ wages + profits (cf. Keynes, 1973, p. 23; Kaldor, 1956, p. 95) are *a priori* unacceptable because profit and distributed profit is not the same thing.

The barter-economic notion of surplus stands in no relation to profit as determined by (16). Neither is the neoclassical equilibrium condition, profit rate = marginal productivity of capital, applicable in the pure consumption economy because we have profit but no capital. And, since profit and capital must not be treated like Siamese Twins, as they have by the classics, the tendency of the profit rate to fall is

also in need of a thorough revision. Neither the classicals nor the neoclassicals ever came to grips with profit (Desai, 2008, p. 10).

The simplest formula to express the relation between prices and cost of production is:

$$\text{price} \geq \text{cost of production per unit.} \tag{29}$$

The classical school rightly insisted on the view that production cannot continue for long if the price does not cover the costs and based their value theory on this incontestable fact (Dmitriev, 1986, p. 63). Neither vital necessity nor wishful thinking, though, makes profit to appear and therefore this view cannot explain *why* profits are in fact positive over a longer time span. When condition (29) is reformulated in structural axiomatic terms and somewhat tightened to exclude zero profits we get condition (i):

$$\text{(i)} \quad \rho_F \equiv \frac{W}{PR} < 1 \quad \Rightarrow \quad \text{(ii)} \quad \rho_E\,(1+\rho_D) > 1 \quad \text{if} \quad \rho_X = 1. \tag{30}$$

From the period core (6) then follows condition (ii) which states that the expenditure ratio has to be greater than unity and/or that the distributed profit ratio has to be greater than zero and this condition may be referred to as the life-formula of the pure consumption economy. This formula implies credit growth, at least over some initial periods. For the business sector as a whole to make a profit consumption expenditures C have in the simplest case, i.e. $\rho_D = 0$, to be greater than wage income Y_W. So that profit comes into existence in the pure consumption economy the household sector must run at first a deficit. This in turn makes the inclusion of the financial sector mandatory.[3]

It needs hardly emphasis that in the investment economy the process of profit generation appears more complex. This does not affect the nature of profit. It simply removes the *formal* necessity that the *households* have to run a deficit to get the economy going. This is then done by the investing firms.

7 The rate of interest

Profit, interest, and rent posed some intricate problems for the labor theory of value (Vianello, 2010). We have dealt with profit above and now include interest. This also gives a clue of how to deal axiomatically with rent which is not considered further here (for details see 2011h).

[3] The purchase of long lived consumption goods, e.g. houses, has to be subsumed under consumption expenditures. With regard to collateral there arises no problem for the banking industry and a sound credit expansion may – in principle – proceed for an indefinite time in the pure consumption economy.

To simplify matters it is supposed that all financial transactions are carried out without costs by the central bank. Money then takes the form of current deposits or current overdrafts (for details see 2011e).

The central bank consists of two units, the transaction unit and the banking unit that finances the households which in turn is the precondition for consumption expenditures to be greater than income, i.e. for an expenditure ratio $\rho_E > 1$. The output of the banking unit consists of the administrative services related to a certain number of one-period loans as given by (44), which is reproduced here:

$$O_B = R_B L_B \quad |t. \tag{31}$$

The output's dimension is loans processed per period. With regard to the peculiarities of the banking business it is formally necessary to relate the services to the value of loans. With $\hat{\mathsf{A}}$, as specified by (32), the average value per loan of the existing average stock of loans $\bar{\mathsf{A}}$ (which in turn follows from the cumulated overdrafts) is denoted:

$$\hat{\mathsf{A}}_B \equiv \frac{\text{average value of stock of loans}}{\text{number of loans in stock}} \equiv \frac{\bar{\mathsf{A}}_B}{n_S} \left[\frac{EUR}{loist}\right]. \tag{32}$$

Equation (31) is rewritten as:

$$O_B \hat{\mathsf{A}}_B = \left(R_B \hat{\mathsf{A}}_B\right) L_B \quad |t. \tag{33}$$

The relation between the number of processed loans and the number of loans in the banking unit's books defines the stock-flow ratio ρ_n:

$$\rho_n \equiv \frac{O_B}{n_S} \left[\frac{\frac{lopro}{period}}{loist}\right] \quad \Rightarrow \quad \rho_n \equiv 1 \left[\frac{1}{period}\right] \quad \text{if} \quad O_B = n_S. \tag{34}$$

Here we make the simplifying assumption that the whole stock is processed in each period which means that we make no distinction between new lending and the administration of the already existing stock. In this case the ratio ρ_n reduces to one and carries only a dimension.

From (33) in combination with (32) and (34) follows:

$$\underbrace{\bar{\mathsf{A}}_B \rho_n}_{O_B^\star} = \underbrace{R_B \hat{\mathsf{A}}_B}_{R_B^\star} L_B \quad |t. \tag{35}$$

The households' expenditures (52) consist now also of interest payments to the banking unit. The flow of services bought X is taken to be equal to the output $O_B^\star$ of

the banking unit. Under the condition of market clearing the expenditures are equal to the interest payments:

$$C_B \equiv P_B X_B \quad \Rightarrow \quad P_B \rho_n \bar{\mathsf{A}}_B \equiv \mathsf{I}_B \bar{\mathsf{A}}_B \quad \text{if} \quad \rho_{XB} = 1 \quad |t. \tag{36}$$

For the formal relation between the price P_B and the rate of interest follows:

$$P_B \rho_n \equiv \mathsf{I}_B \quad \left[\frac{1}{period}\right]. \tag{37}$$

The rate of interest thereby inherits the role of the price.

If the production structure is competitive then for relative prices holds:

$$\frac{P_A}{P_B} = \frac{R_B^\star}{R_A} \quad \text{if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{38}$$

After the substitution of (29) relative prices, here the relation of product price P to the rate of interest I, are finally given by:

$$\begin{aligned} &\frac{P_A}{\mathsf{I}_B} = \frac{R_B}{R_A}\frac{\hat{\mathsf{A}}_B}{\rho_n} \quad \Rightarrow \quad \mathsf{I}_B = \frac{R_A}{R_B}\left(\frac{\rho_n}{\hat{\mathsf{A}}_B^\star}\right) \\ &\text{if} \quad \rho_{XA} = 1,\ \rho_{XB} = 1,\ \hat{\mathsf{A}}_B^\star \equiv \frac{\hat{\mathsf{A}}_B}{P_A} \quad |t. \end{aligned} \tag{39}$$

The loan interest rate I_B is equal to the ratio of the productivity in the consumption goods producing firm A and of the productivity in the banking unit B. The latter is weighted with the deflated average loan value. In the competitive structure the interest rate is therefore equal to a relation of *real* variables. When the average loan value $\hat{\mathsf{A}}_B$ and the price P_A always move in step (as they do with perfect indexing of the *nominal value* of the loans), there is no effect on the rate of interest other than productivity changes. Inflation or deflation are of no consequence. Without perfect lockstep, however, the rate of interest and the price are positively related:

$$\mathsf{I}_B = \left(\frac{R_A}{R_B}\frac{\rho_n}{\hat{\mathsf{A}}_B}\right) P_A \quad \text{if} \quad \rho_{XA} = 1, \rho_{XB} = 1 \quad |t. \tag{40}$$

The loans are produced like any other good and the interest rate therefore truly reflects the production conditions in the respective industries.

The two functions of the central bank, transaction on the one hand (which has been assumed here to be costless) and the processing of one-period loans on the other, are neatly separated. The banking unit is the conceptual nucleus of the commercial banking industry and has eventually to be spun off from the central bank.

The financial profit of the business sector as a whole in period t is derived from the profit definition (17) and (36) as:

$$Q_{fi} \equiv P_A X_A - W_A L_A + \mathsf{I}_B \bar{\mathsf{A}}_B - W_B L_B \quad |t. \tag{41}$$

In the competitive structure the banking unit's profit ratio is equal to the production firm's profit ratio if the wage rates are equal. The banking unit earns its living like any other firm.

The salient point is that the (loan) rate of interest is not some kind of reward for abstinence or waiting but the price for the services of the banking unit. It is basically not different from any other price that the households pay for any other goods and services.

The classical school did not properly discriminate between the rate of interest and the profit rate (Walras, 2010, p. 423) and maintained that profits are 'an element in Cost of Production, in so far as they are spread over unequal lengths of time' (Mill, 2006a, p. 482). Profits are no costs of production as we have seen in Section 2 but interests can be when the business sector finances its operations with loans from the central bank (for details see 2011i). These interests for the working capital cancel out for the business sector as a whole and determine only the *distribution* of profits between the production firms and the central bank. The classical notion of profit, although erroneous from the vantage point of the structural axiom set, is realistic given the circumstances at that time when the roles of entrepreneur, owner, and financier fell into one (Blaug, 1998, p. 91). Nevertheless, the classics applied only parochial realism which is the defining characteristic of partial analysis. The classical capitalist's roles have to be separated analytically and reconfigured in the general axiomatic context. This has been done; what remains to be done is, of course, the inclusion of 'commodities made by machinery'. Without a full understanding of the elementary consumption economy, though, there is no chance of grasping the complexities of capital accumulation and their implications for the theory of value (for details see 2011g).

8 Can it be true?

> One of the most famous stories about Gauss depicts him measuring the angles of the great triangle formed by the mountain peaks of Hohenhagen, Inselberg, and Brocken for evidence that the geometry of space is non-Euclidean. (Brown, 2011, p. 565)

From the differentiated axiom set follows for the competitive structure that the exchange ratio for two products is equal to the (weighted) inverse productivity ratio (28). Since this statement has been deductively derived it is a theorem. A theorem

is different from a law in that it does not make any deterministic claim about real world phenomena. Thus the structural value theorem does not suggest that there exist market 'forces' that make it inevitably true 'in the long run' or 'on the average'. Nevertheless, it may turn out to be applicable to the real world like the Pythagorean Theorem.

First of all it is important to recall that we deal at the moment with a pure consumption economy. That is: no taxes, no foreign trade, no investment goods industry, and no secondary market for durable consumer goods (Ricardo's rare statues and peculiar wines; for details see 2011c). Second, the structural value theorem holds but for the product market. In fact the theorem says nothing at all about the labor market, therefore it is a matter of indifference whether the economy is in full employment or not. It does not matter either whether the economy experiences inflation or deflation. In the limiting case of the competitive structure the price relations reflect solely the state of organization/technology in different lines of production.

The conditions for the application of the structural value theorem constitute an ideal consumption economy and are plausible in this restricted domain. They allow for the following conditional prediction. If the allocation of the labor input between firms reflects the preferences of the consumers, as it ideally should with efficient markets, then the price ratios of the products are equal to the (weighted) inverse productivity ratios. If the preconditions are approximately realized then we should observe a relatively stable price structure that reflects the productivity development in different industries.[4]

Independently of this we should find that the general structural value theorem (14), i.e.

$$\rho_P = \frac{\rho_{XB}\,\rho_{EA}}{\rho_{XA}\,\rho_{EB}\,\rho_L\,\rho_R} \tag{42}$$

always holds *exactly*, that is, within the boundaries of measurement errors. In other words, it holds by virtue of pure deduction. What does that mean?

For a general answer let us briefly return to the Pythagorean Theorem. With a known baseline and two known angles one can calculate the unknown and not directly measurable distance to the moon. In physics a theorem is used as a calculating device (Georgescu-Roegen, 1971, p. 332). The usefulness of theorems is beyond question.

The application of a theorem, however, implicitly introduces a *new* claim. The first claim is that the Pythagorean Theorem is true_1, i.e. formally correct. By applying it to calculate the distance to the moon it is tacitly assumed that earth and moon are located in Euclidean space which is quite another claim that may or may not

[4] "Summarizing [the interrelations for Germany], we may conclude that relative productivity growth is the most important determinant of relative price growth, where causality may run the other way as well ..., but there is no proportional relation between the variables." (Rahmeyer, 1988, p. 229)

be true_2. While true_1 refers to the axioms, true_2 refers to reality (Rosenberg, 1994, pp. 225-229). Only when the properties of the space that is formally given with the axioms happen to be those of real space the calculation of the distance will yield the correct result. By innocently applying the Pythagorean Theorem we therefore implicitly make the really strong claim that the Euclidean axioms capture reality. If this happens to be the case, and as far as we know it does (Penrose, 2007, pp. 21, 29), then true_1 and true_2 amalgamate to true_0– 'beyond all reasonable expectations' (Wigner, 1979, p. 231); but see also (Velupillai, 2005).

Under the condition that the general structural value theorem (42) is true_0 it can readily be applied as a calculating device that enables valid inferences from known facts to unknown facts. This process is cumulative.

9 Conclusion

Behavioral assumptions, rational or otherwise, are not solid enough to be eligible as first principles of theoretical economics. Hence all endeavors to lay the formal foundation on a new site and at a deeper level actually need no further vindication. The present paper suggests three objective= nonbehavioral axioms that comprise ten measurable variables as groundwork for the formal reconstruction of the evolving monetary economy.

The analytical priority claim of the structural axiomatic approach rests on the simple fact that, since the structure that is given by the axiom set does not adapt to behavior, behavior has to adapt to structure. If behavioral and structural logic are at odds behavioral logic is conductive to frustrated plans and expectations. This is the normal state of economic affairs.

The present paper has two parts. In the first, the *logical interdependencies* of the key variables that formally embody the firm, the market, and the income distribution are developed. In the second, the *real world implications* for the theory of value are made explicit. The main results of the axiomatic inquiry are:

- From the differentiated structural axiom set follows the general structural value theorem which is *independent* of behavioral hypotheses.
- The competitive production structure as a limiting case of the general structural value theorem is characterized by an allocation of total working hours in exact proportion to the assignment of consumption expenditures to different products and epitomizes the consumers' optimum.
- Given a competitive structure, relative prices are in the simplest case equal to the inverse ratio of productivities. This configuration is formally consistent with both the marginalist theory of the consumer optimum and the classical labour theory of value.

- Given a competitive structure with different wage rates, relative prices are equal to the weighted inverse ratio of productivities.
- The characteristic of the competitive structure is that the profit ratios are equal. The profit ratio is different from the profit rate which is not defined in the pure consumption economy.
- Profit is not attributable to capital in the pure consumption economy. Profit is a factor independent residual that is determined by the expenditure ratio and the distributed profit ratio.
- In the structural axiomatic context the (loan) rate of interest is not some kind of reward for abstinence or waiting but the price for the loan processing services of the banking system.
- Profit is no part of the costs of production as Mill maintained. The profit ratio is qualitatively different from the rate of interest.

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Appendix: The differentiated axiom set

Differentiation of the axiom set (1) to (3) for period t:

$$Y = W_A L_A + D_A N_A + W_B L_B + D_B N_B \tag{43}$$

$$O_A = R_A L_A \qquad O_B = R_B L_B \tag{44}$$

$$C = P_A X_A + P_B X_B \tag{45}$$

$$Y_{WA} \equiv W_A L_A \qquad Y_{WB} \equiv W_B L_B \tag{46}$$

$$Y_{DA} \equiv D_A N_A \qquad Y_{DB} \equiv D_B N_B \tag{47}$$

$$Y_A \equiv Y_{WA} + Y_{DA} \qquad Y_B \equiv Y_{WB} + Y_{DB} \tag{48}$$

$$Y_W \equiv Y_{WA} + Y_{WB} \tag{49}$$

$$Y_D \equiv Y_{DA} + Y_{DB} \tag{50}$$

$$L \equiv L_A + L_B \tag{51}$$

$$C_A \equiv P_A X_A \qquad C_B \equiv P_B X_B \tag{52}$$

$$\rho_{EA} \equiv \frac{C_A}{Y} \quad \rho_{EB} \equiv \frac{C_B}{Y} \quad \rho_E \equiv \frac{C}{Y} \quad \Rightarrow \quad \rho_E \equiv \rho_{EA} + \rho_{EB} \tag{53}$$

$$\rho_{XA} \equiv \frac{X_A}{O_A} \qquad \rho_{XB} \equiv \frac{X_B}{O_B} \tag{54}$$

$$\rho_{DA} \equiv \frac{Y_{DA}}{Y_{WA}} \qquad \rho_{DB} \equiv \frac{Y_{DB}}{Y_{WB}} \qquad \rho_D \equiv \frac{Y_D}{Y_W} \tag{55}$$

$$\rho_{FA} \equiv \frac{W_A}{P_A R_A} \qquad \rho_{FB} \equiv \frac{W_B}{P_B R_B} \tag{56}$$

Axioms and definitions are consolidated to the period core for two firms:

$$\rho_{FA} \frac{\rho_{EA}}{\rho_{XA}} (1 + \rho_{DA}) + \rho_{FB} \frac{\rho_{EB}}{\rho_{XB}} (1 + \rho_{DB}) = 1 \tag{57}$$