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The Treasury bill futures market is a subject that has been of great interest to financial economists. This paper analyzes prices and volumes in the T-bill futures market using a physical analogy called Brownian motion. The results are similar to those obtained in previous studies of stock markets. For prices, the T-bill futures market failed to exhibit the presence of resistance and support levels, indicating that chartists could not profit by looking for such levels. For low volumes, T-bill futures exhibited lognormal behavior patterns, meaning that new investors are attracted to markets in proportion to the volume already present. This means that for financial futures, the first exchange to establish a new type of futures contract is the one most likely to be successful, since its competitors will have a difficult time competing with an established high level of trading volume.

SINCE FUTURES TRADING in U.S. Treasury bills began on January 6, 1976, on the International Monetary Market (IMM) of the Chicago Mercantile Exchange, the subsequent explosion in trading volume has been paralleled by an explosion of research studies of the financial futures markets. For example, the futures markets have been used to examine the expectations theory of the term structure of interest rates by Branch [1], Burger, Lang, and Rasche [2], Chow and Brophy [4], Lang and Rasche [12], and Poole [15].

Other research studies have proceeded along lines that were similar to those used in previous analyses of agricultural commodity futures. Various tests of the efficiency of the market have been made by Capozza and Cornell [3], Dale [6], Puglisi [16], Rendleman and Carabini [17], and Vignola and Dale [19]. A test of the "cost of carry" hypothesis of Holbrook Working [21, 22], which is the theoretical basis for understanding pricing in most futures markets, was done by Vignola and Dale [20].

The hedging effectiveness of the market has been examined by Dale and Vignola [7], Ederington [9], and Ederington and Plumly [10]. The nature of the market participants was determined in a survey by Hobson [11]. Finally, a set of popular mechanical trading rules was tested on Treasury bill futures by Dale and Workman [8].

One remaining question has been the rather esoteric one of whether or not price movements in these markets exhibit patterns known as Brownian motion.[1] This paper analyzes the Treasury bill futures market in terms of Brownian motion, and discusses the behavioral implications that this type of motion has for price and volume movements.

PERIODICITY IN PRICES

The use of statistical mechanics for analyzing securities prices was first done by Osborne [13, 14]. One of the hypotheses that he tested was that, contrary to the hypothesis of random movements, prices might instead exhibit what is known as the phenomenon of resistance and support, or "Taussig pen-

See footnotes and references at end of text.

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umbras."[2] There are two aspects of this phenomenon, known as congestion and reflection.

The congestion aspect states that there are price ranges in which a security spends an inordinate amount of time, which means that there are more transactions in this region than would be expected by chance. The reflection aspect states that there are price levels at which a security is more likely to be reflected, or turned back in a direction opposite to its previous motion, up or down.

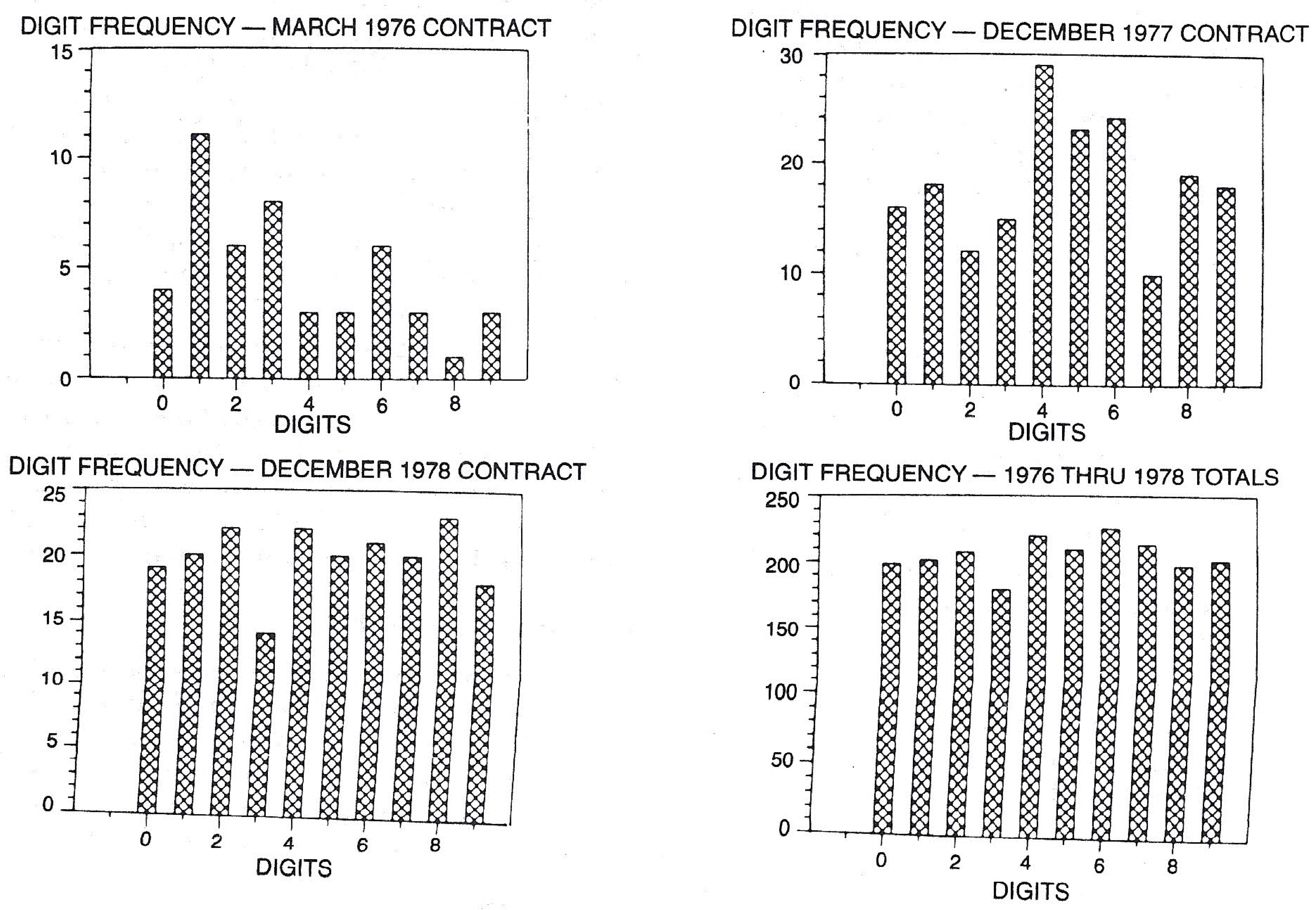
There are two behavioral assumptions behind the hypothesis that prices will have resistance and support levels. One is that many people tend to place buy and sell orders at the same prices, the other is that people don't like to take losses. For example, if there are numerous sell orders at the same price above the current market price, then it will be difficult for the price to go higher than the cluster of sell orders. If, however, the price does break through the resistance level, then those who did buy at that price will be reluctant to sell at a loss on the way back down, and the former resistance level will become a support level. This change from a resistance to a support level corresponds to the physical concept of Brownian motion in the presence of partially reflecting barriers.

Following Osborne, we will test separately for the existence of congestion and reflection. To search for congestion in Figure 1 we have plotted the distribution of closing prices, using only the last digit.[3] If traders had a pronounced tendency to place buy and sell orders on particular prices, such as, say, those ending in 0 or 5, than this would show up on Figure 1 and would mean that the market is subject to price congestion.

However, the three year grand totals show that there has been a very uniform occurrence of digits over the three year period. Only the very first contract, for March 1976 delivery, shows what might be considered a deviation from uniformity, but there were only 48 trading days on that contract, so this is not a surprising result. Only 2 of the other 11 contracts are shown, since they all exhibit the same high degree of uniformity. This result means that the T-bill futures market has exhibited less price congestion than have the markets for many stocks.

One technical point deserves mention here, concerning the way in which the closing price is determined. Since it would be nearly impossible to determine exactly what the last trade of the day is on a given futures contract, the settlement price is taken to be the midpoint of the closing range, where the closing range is determined by the last 60 seconds of trading (This is in contrast to many agricultural commodity futures, which have closing ranges of only 30 seconds). If the midpoint is not a permis-

Figure I



sible price, it is rounded to the price nearest the previous day's close. A very small and unscientific sample makes it appear to the author that in spite of the very high volume of trading, the closing range frequently consists of a single price, or at most a range of 2 or 3 points, so that the method of determining the settlement price does not seem to have biased the results.

Turning to the possibility of reflection, if there are partially reflecting barriers in the one-dimensional field in which prices move, then maxima and minima will tend to cluster on the barriers, more so than might be expected for random walks without such barriers. For our purposes, a high must be a local maximum, meaning that it is greater than both the nearest different preceding and following price. Clearly, if the market closes on the high price for the day, it may very well open at an even higher price on the following day. Thus, again following Osborne, we first "censor" the data, i.e., data was dropped for days in which the market closed on its high or low.

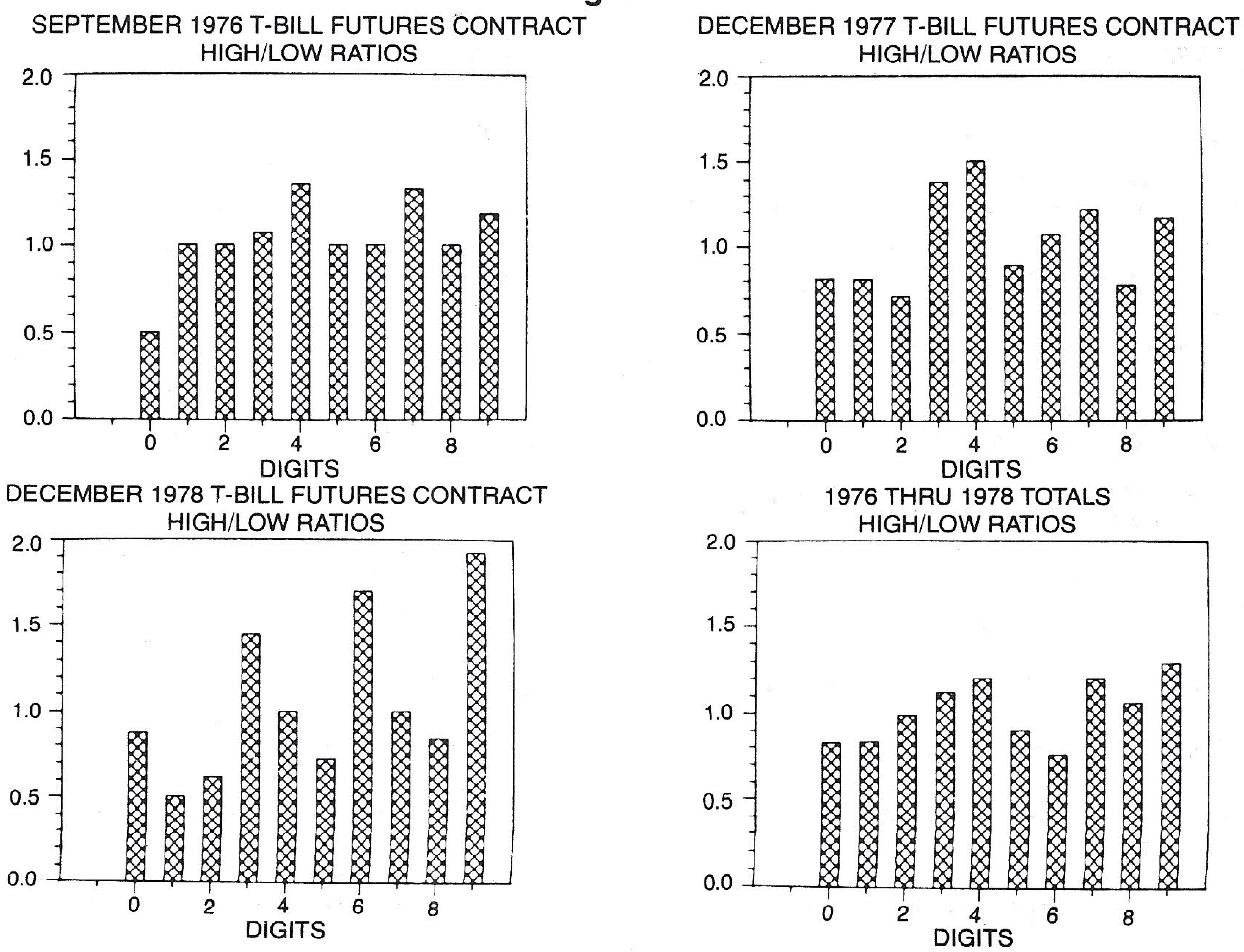
Figure 2 shows the ratios of the number of highs to the number of lows plotted against the last digit of the price. If there were a tendency for maxima or minima to cluster on particular digits, then this would show up on the charts as significant deviations from 1.0. The physical analogue would then be one in which prices move in the same way as particles would diffuse on a one-dimensional atomic lattice with scattering centers at locations corresponding to the preferred digits. Increasing the temperature of the particles (corresponding to a greater trading volume) would tend to diminish the importance of the scattering centers (preferred digits).

As good as the type of physical analogy just described might be for many stocks, Figure 2 clearly shows that clustering has not been present to any significant extent (September 1976 was the first contract with enough data to give meaningful results). Only 3 contracts and the grand totals are shown, because there is no appreciable difference in any of the other contracts. Thus, we have found no evidence for the existence of resistance and support levels in the Treasury bill futures market.[4]

PERIODICITY IN VOLUME

One question that has some theoretical interest is that of what type of distribution the volume of futures contracts has followed. Specifically, the question is one of whether or not the distribution of volume has more closely followed a normal, or a lognormal, distribution. A lognormal behavior pattern would imply that the increments of daily volume are proportional to the volume already present.

Figure II



This in turn would mean that new investors are attracted to markets that already have high volume.

Figures 3 and 4 show the cumulative probability distributions for the first three years of trading of the IMM 90-day T-bill futures contract. The diagrams are drawn so that if the cumulative probability distribution is a straight line, the underlying volume population is assumed to have that particular type of distribution.

In the figures, only for the first two contracts in 1976 do the lognormal plots appear to be straighter than the normal plots; from the September 1976 contract on, the normal probability distribution appears to be just as good as the lognormal. In addition, in most cases there are points at very high volumes where the curves deviate from being straight lines.

There are both mathematical and behavioral explanations for these results. Mathematically, as the volume increases, the ratio of the standard deviation to the mean becomes smaller, so the lognormal distribution approximates the normal distribution by the central limit theorem. On the other hand, in the early history of the market, when volume was very low, new traders may actually have become attracted by the gradually increasing investor interest in the market, which was accompanied by volatile interest rates in the spot market. Finally, at very high volumes, the deviation from normal distributions may be due to a violation of one of the standard assumptions, namely that transactions occur independently of each other, and that transactions will be approximately equally spaced in time.

On an intraday basis, the assumption of equally spaced transactions is clearly violated. The IMM publishes the correct volume a few days after the fact. However, they also estimate the volume for the same trading day, as follows. The Chicago Mercantile Exchange has its Treasury bill pit supervisor estimate a "volume factor" each morning. This means that he makes a Bayesian subjective probability assessment (i.e., a guess) of the average transaction size for the day. For example, assume he estimates the volume factor to be 17. This means that whenever a trade takes place, whether it actually is for a single contract or fifty contracts, the estimated daily volume will be increased by 17. If the subsequent estimated volume turns out to be significantly different from the actual volume, the pit supervisor may change the daily volume factor. In any case, on an intraday basis the trading has an unsurprising tendency to ebb and flow, and the transactions are certainly not equally spaced in time.

For day to day comparisons, however, as opposed to intraday analyses, the assumption of independence of the volume of transactions is a more defen-

Figure III. A

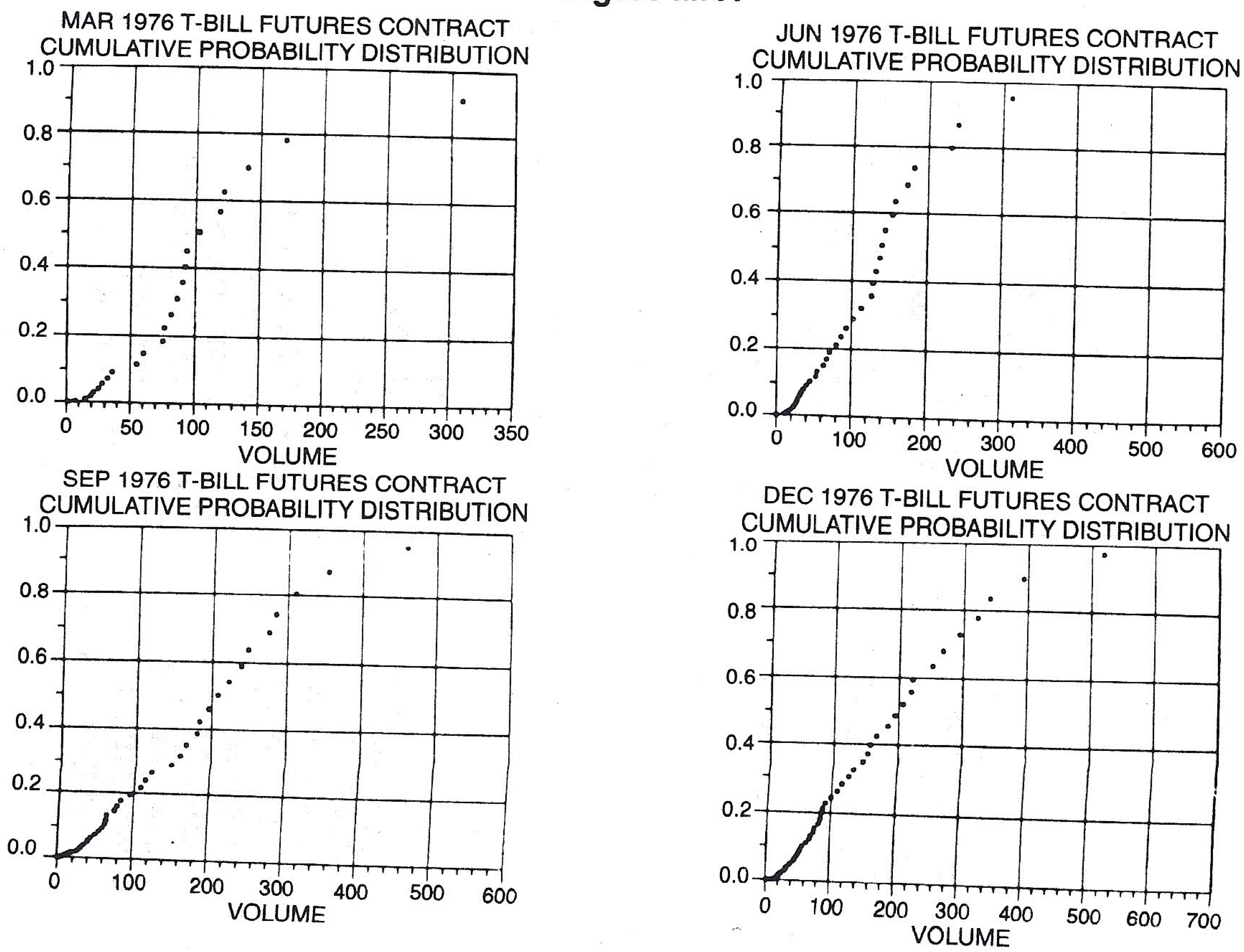


Figure III. B

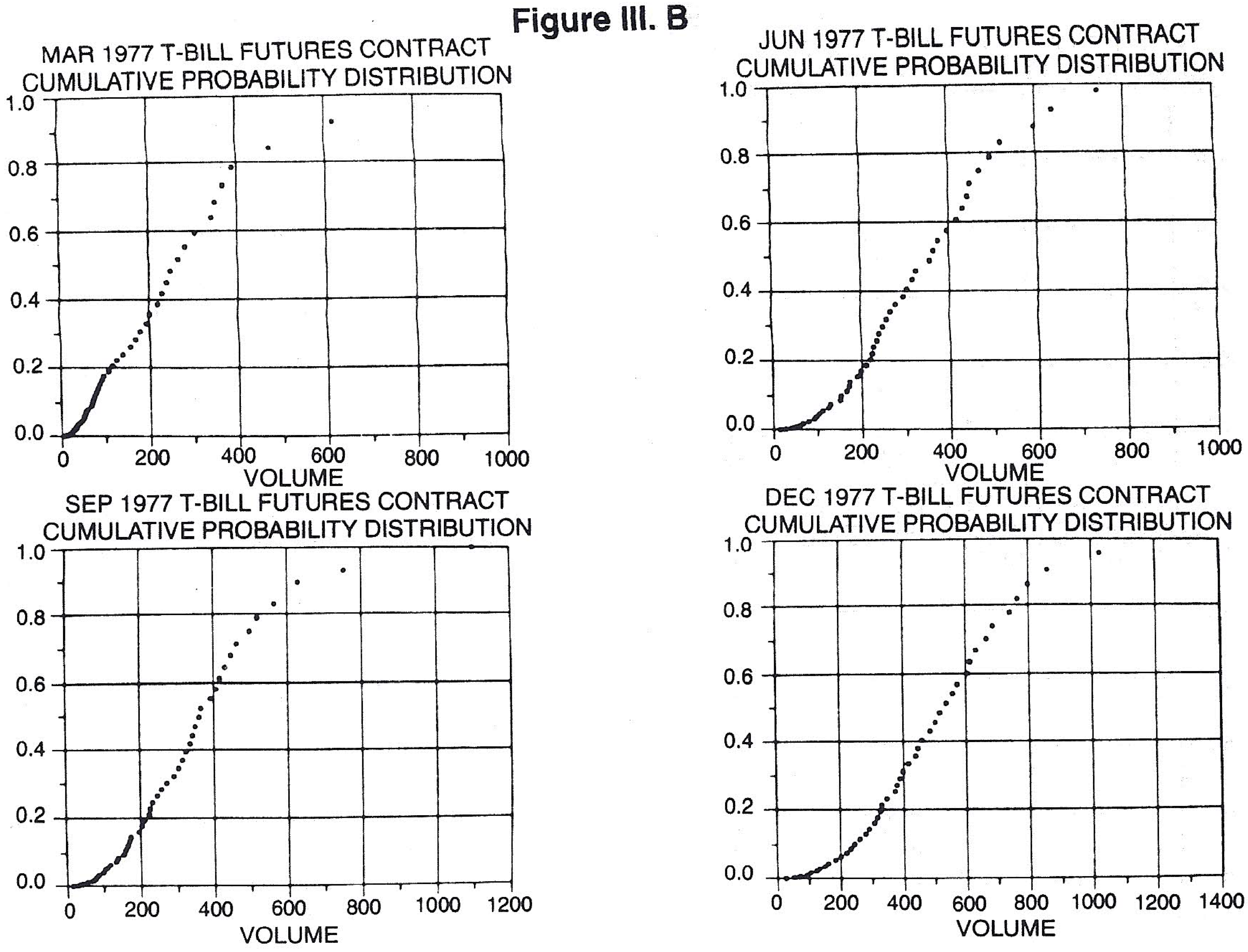


Figure III. C

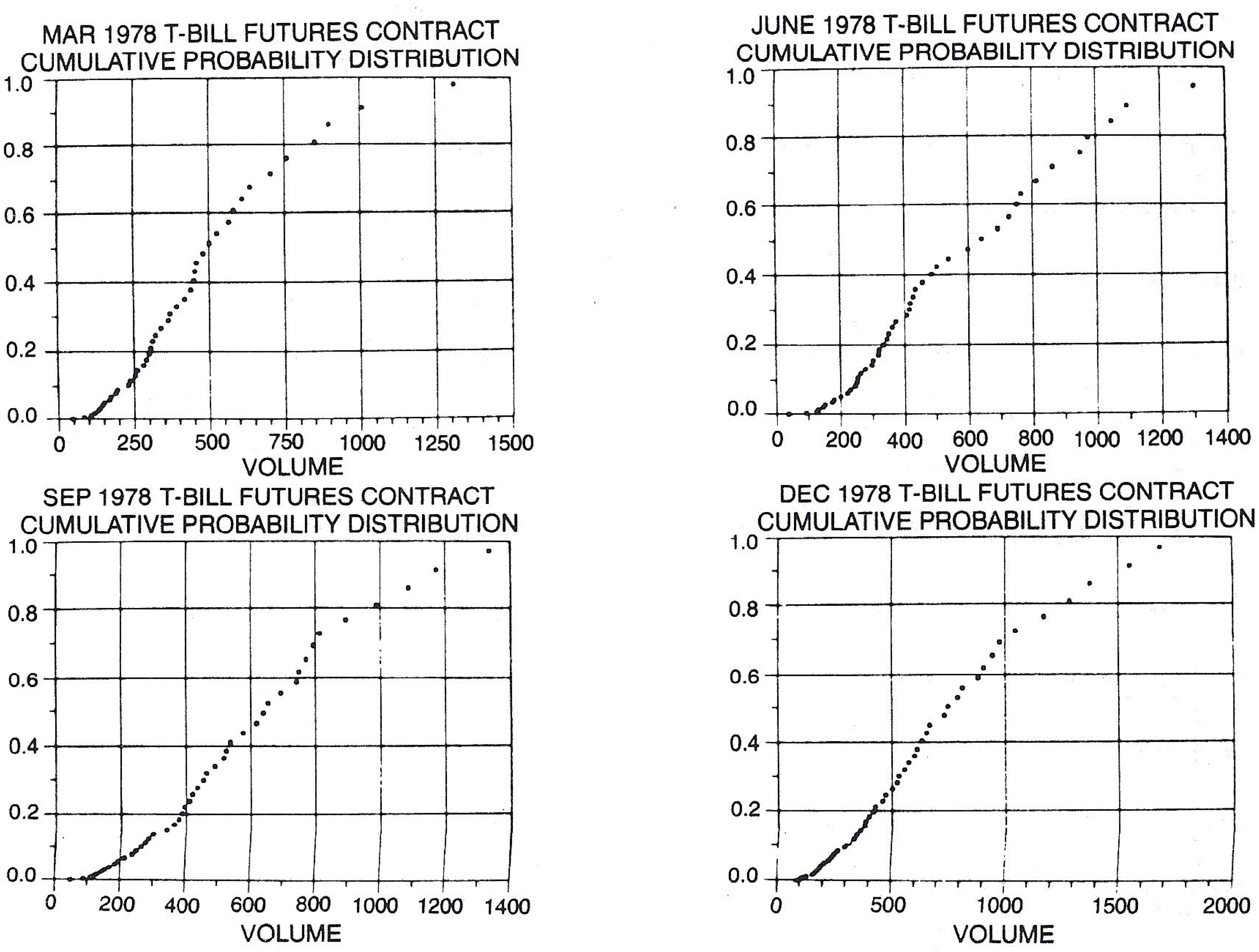


Figure IV. A

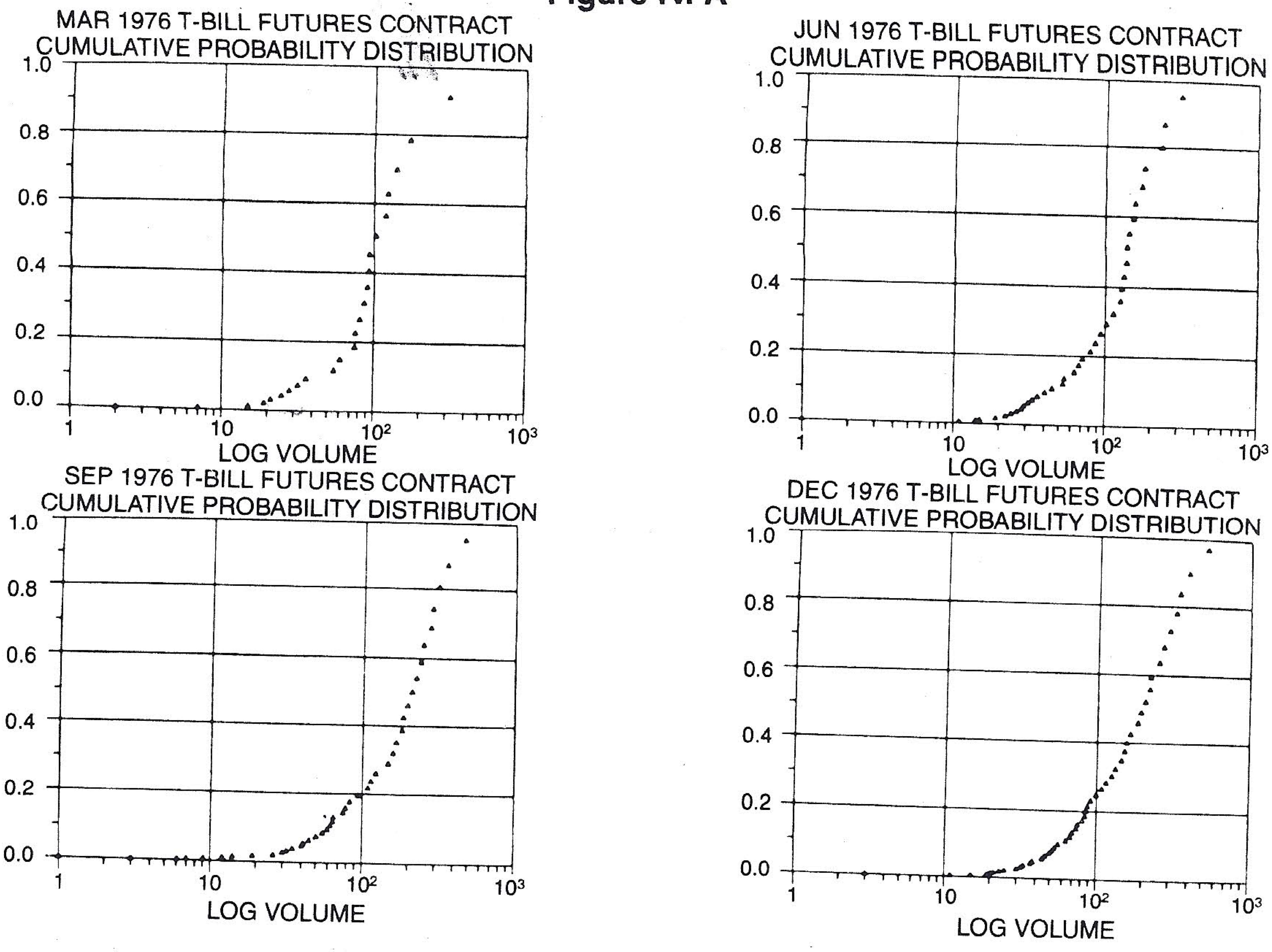


Figure IV. B

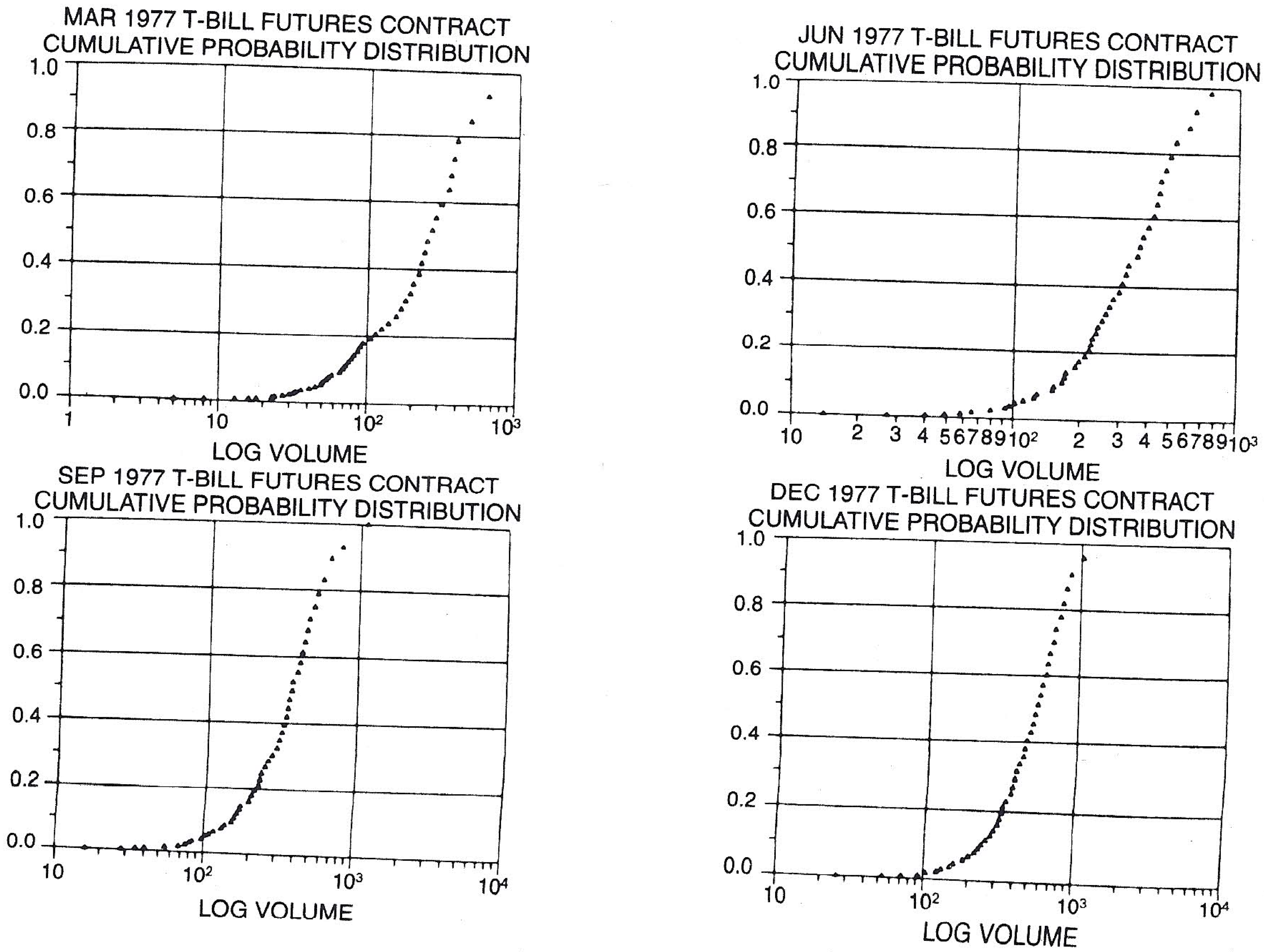
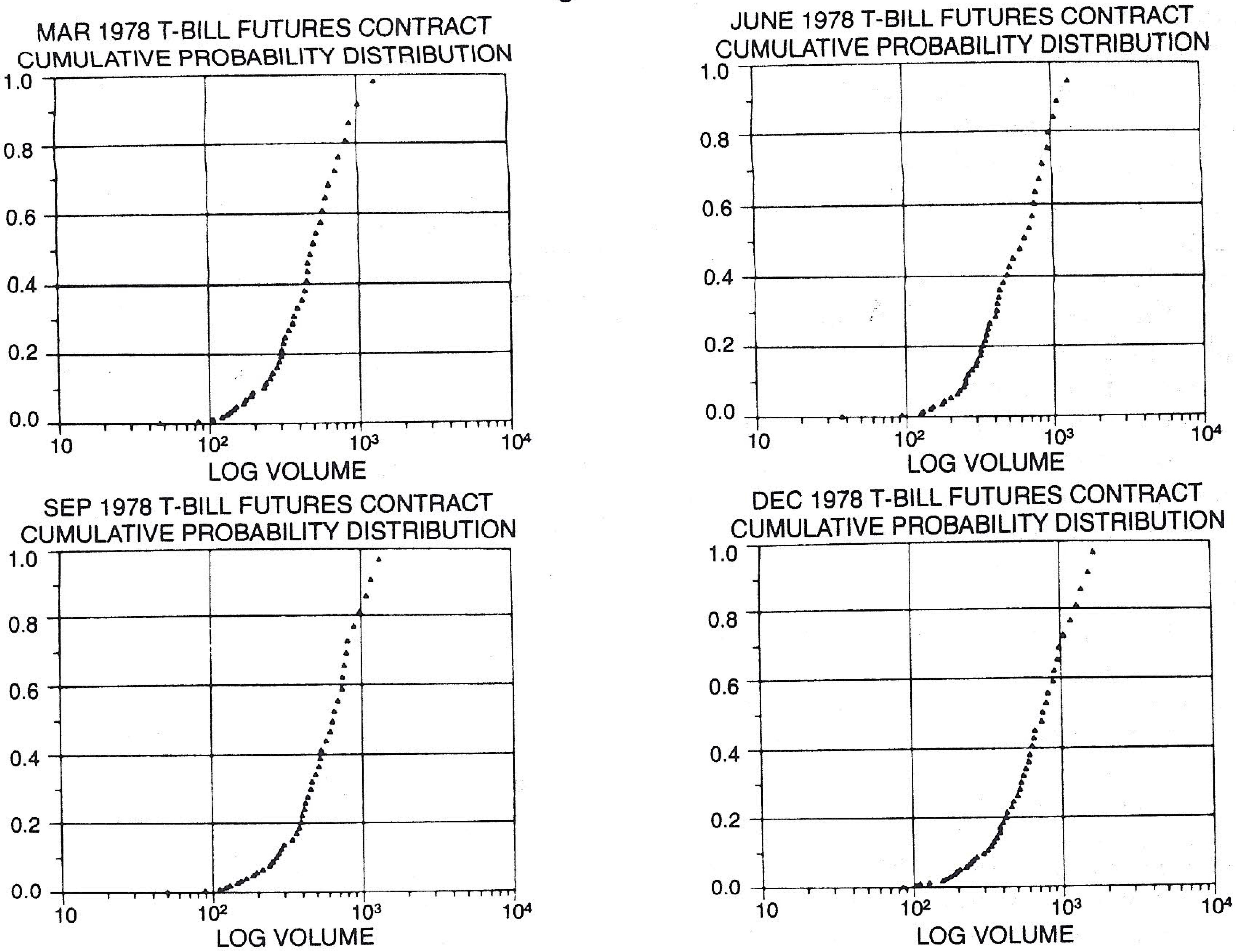


Figure IV. C



sible one, and helps explain the good fits the present work has obtained for the intermediate volume ranges. When very high volumes occur, it is sometimes due to some sudden economic shock, such as a change in Federal Reserve policy that may take several days to be fully assessed by the market. In this case the assumption of independence of daily volumes would be violated, accounting for the deviations from straightness of the curves. The important point is that this is the same type of result that has been found in analyses of stock markets.

CONCLUSIONS

This paper has subjected the IMM 90-day Treasury bill futures market to tests for the existence of price congestion and partially reflecting barriers. Neither of these market imperfections was found. Also, the trading volume was examined, to see if the underlying distribution was either normal or lognormal. The results showed a lognormal distribution at relatively low volumes, and a normal distribution at the higher volumes that have been present after the first few contracts. At very high volumes, the distribution was neither normal nor lognormal, which is the same type of result that has been obtained for stock prices.

These results suggest several interesting topics for future research. For example, it will be interesting to examine the progress of the two new 90-day T-bill futures contracts that are listed on the ACE and the Comex. Historically, new futures contracts have been able to compete successfully with similar established contracts only if they have some desirable distinguishing characteristic, such as different types of deliverable commodities. The two new T-bill futures contracts have different delivery months than the IMM, so it will be interesting to watch their development. When enough data becomes available, the new contracts may be subjected to the same tests that have been used in the present work. Assuming the volume in the new contracts increases over time, a comparison can be made to see how long it takes to achieve an underlying normal distribution. Whether or not the transition is faster or slower than it was for the IMM contract may have behavioral implications which will be useful in studying the ramifications of the proliferation of financial futures contracts.

Competing agricultural futures contracts frequently survive because transportation costs between regions have the effect of segmenting the marketplace. Transportation costs of Treasury bills, however, are negligible, so they would not be a factor if, say, an exchange started trading in a futures contract which exactly duplicated the terms and conditions of the IMM contract, except for having settlement take place in a different city.

One significant effect of zero transportation costs, however, has been to make the T-bill futures markets more of a delivery mechanism than is the case for agricultural commodity markets. If the upward trend in deliveries continues, there may eventually be severe problems due to an inadequate deliverable supply of Treasury bills. If the two new T-bill futures contracts result in an increasing interest in and use of all these contracts for making and taking delivery of T-bills, then the deliverable supply problems might ultimately be very serious. This is still just conjecture, however; to this point there is no evidence from the present work that the IMM T-bill futures market has been functioning any worse than have financial markets in many stocks.

FOOTNOTES

[1]The term "Brownian motion" refers to the random movement of microscopic particles which are suspended in liquids or gases. The same mathematics which describes this type of motion may also be used to describe the random price changes in many types of securities.

[2]Taussig's [18] theories of futures markets were not refuted at the level of theory until Working [23] expounded his theory of "anticipatory prices." All subsequent empirical work has supported Working's theory.

[3]The present work uses data solely for the 90 day T-bill futures contract traded on the IMM. In June 1979, trading in 90-day T-bill futures began on the New York Commodity Exchange (Comex) and the Amex Commodities Exchange (ACE), with both exchanges using different delivery months than the IMM. Data is much too limited at this time to allow any analytical work on these new contracts.

[4]Speculators frequently try to take advantage of resistance and support levels by devising various mechanical trading rules, which would automatically give buy or sell signals. The fact that the present work shows that such levels do not exist in the Treasury bill futures market is consistent with the results of Dale and Workman [8], who tested a variety of popular trading rules and showed that in the long run they all produced losses.

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