Monetary policy conduct: A hybrid framework

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This paper examines whether a small-open-economy, DSGE-based, New-Keynesian model can provide a natural framework for monetary policy conduct with the use of a hybrid monetary-policy regime. Allowing for some inflation inertia, a small-open-economy version of the Calvo sticky-price model to investigate hybrid inflation/price-level targeting was developed. This paper explores the properties of monetary policy in terms of Taylor interest-rate rules and conduct welfare analysis on various specifications. The study’s analyses show that hybrid targeting outperforms other specifications and produces quantitatively good results by lowering output and inflation variabilities when compared to regimes targeting the price levels or the inflation rate.

Key words: Small-open economy, monetary policy, hybrid targeting.

INTRODUCTION

Over the past two decades, many developed economies have been radically transformed. These countries, after shifting to a new monetary policy regime, now show a low rate of inflation, and the private sector is more concerned about costs as well as being more productive and efficient than during the 1990’s. As a result of the unsatisfactory monetary policy performance, countries such New Zealand, Canada, Australia, Sweden and U.K. introduced policies targeting inflation rate to achieve price stability. However, the definition of price stability and how to achieve it is still controversial and the meaning of price stability remains unclear. The debate has mainly focused on whether the inflation rate or the price-level path should be the policy target. More specifically, under inflation targeting (hereafter, IT), the central bank tries to bring the inflation rate to the target, while it aims to bring the price level to its initial level at the time the regime is established with price-level targeting (hereafter, PT). An alternative method considers a hybrid target (hereafter, HT) based on a weighted average of an inflation target and price-level target. This paper addresses this particular issue in a small open economy setting.

The difference between the three regimes can be captured in their effects on price level. The IT regime aims to maintain a stable path for future inflation even if this leads to a unit root (non stationarity) in the price level. The PT regime implies a stable path for price level leading at the same time to a stationary price level around the targeted value and a stationary inflation rate around zero. The hybrid regime combines the characteristics of IT and PT by incorporating an average of inflation and price level target. This policy (HT) targets an average inflation of several forthcoming periods rather than targeting one-period ahead (IT) or an infinite horizon (PT). The present work lays out a small-open-economy model with Calvo-type staggered price setting. Moreover, the benchmark model allows for some inflation inertia by including price indexation to past inflation. Introducing price indexation, results in a lagged inflation term in the price equation and therefore, a better fit for inflation persistence. The resulting specification enables the study to focus on the monetary-policy implications of different regimes namely inflation, price-level and hybrid targeting. Also, the welfare implications of these policy regimes were addressed. The small-open-economy representation takes into consideration the possibility that international trade and financial assets will affect the evolution of the domestic economy. From a theoretical point of view, the ‘small open economy’ assumption is particularly attractive, as many of the results that are obscured in a closed economy are more readily apparent here. Additionally, there are additional shocks (the exchange rate for

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example) that are not relevant to a closed economy. Thus, foreign shocks, such as changes in terms of trade, can alter domestic business-cycle fluctuations, giving rise to further dynamics within the model, which may lead the monetary authority to explicitly take these kinds of fluctuations into account (Lubik and Schorheide, 2003). The study's results suggest that allowing for some base drift in the price level (HT), small-open-economy welfare is improved. Indeed, these findings are consistent with the fact that hybrid inflation/price-level targeting performs well and provides an alternative method for conducting successful monetary policy in the case of a small-open economy without the shortcomings of the other monetary-policy regimes considered in this work (IT and PT). Following a hybrid targeting rule, a central bank generates lower variabilities in domestic inflation and output gap and reports significant benefits when adopting a HT regime.

**Inflation rate versus price-level target: The debate**

Throughout the last decade, inflation targeting was widely adopted as a framework for monetary policy. Several industrialized countries formally or informally adopted IT targeting and thus, far most enjoy low inflation, price stability and satisfactory real-growth records (before the current crisis). In contrast, 'conventional wisdom', as Svensson (1999) called it, has been sceptical of price-level targeting. The main argument against PT is that it induces both higher short-run inflation and output variability compared to IT (Fischer, 1994; Haldane and Salmon, 1995). However, Dittmar et al. (1999) and Svensson (1999) argued that PT has advantages over IT, since, with PT inflation, variability becomes lower, assuming output variability is at least moderately persistent. The controversy mainly concerns the definition of price stability and more particularly how price stability can be maintained in practice. For instance, monetary authorities should choose paths for either price level or the inflation rate allowing, in the latter case, for a base drift in the price level. More recently, Nessen (2002) and Nessen and Vestin (2000) suggest that the central bank should target average inflation over several periods. Batini and Yates (2003), Cecchetti and Kim (2003) and Kobayashi (2004) investigated another novel proposal that combines IT and PT in a mixed regime, called hybrid inflation/price-level targeting. In this proposal, inflation volatility becomes lower when compared to PT and IT regimes. Indeed, Batini and Yates (2003) introduce a new perspective on the analysis of price-level and inflation targets by considering a hybrid target, which is a weighted average of an inflation and a price-level target. They do not, however, use a utility-based, welfare-loss function as an evaluation criterion. In their analysis of price-level versus inflation targeting under different model specifications, policy rules and loss functions of the central bank, Batini and Yates (2003) found that the more forward-looking the model, the less noticeable the difference between the reaction functions of inflation and price-level targeting; thus, making the performance of such rules highly dependent on the degree of forward-looking behaviour. Using Fuhrer and Moore's (1995) model to explore the implications of these regimes for the United Kingdom, Batini and Yates (2003) examined both a set of simple rules feeding back from alternative combinations of price-level and inflation deviations from a given target and a set of optimal control rules obtained under the assumption that policy makers minimize a loss function which penalizes a mixed price-level/inflation target. Despite the contribution of these theoretical works, however, few studies have directly evaluated the HT regime using open-economy models. An analysis of HT in a small-open-economy environment is relevant, especially given the IT- and PT-regimes' shortcomings, as well as the implications of these weaknesses for central banks.

This work departs from the above-mentioned literature in at least two dimensions, extending the works of Batini and Yates (2003) and Kobayashi (2004). First, the study considered a New-Keynesian environment combined with inflation inertia where the model's dynamics are enriched by allowing for price indexation rather than considering them only in a pure Calvo-staggered fashion. Second, a utility-based and welfare-loss function as an evaluation criterion to evaluate the hybrid monetary-policy regime was used. By combining these new features, new insights on the price-level versus inflation targeting debate were obtained. The potential implications of these insights were discussed later in this paper. In line with previous research on monetary-policy analysis, the study adopts the New-Keynesian framework, a model that many macroeconomic studies have indeed frequently

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1. Domestic policy decisions do not indeed have any impacts on the rest of the world, allowing us to abstract from strategic interactions between the domestic economy and the rest of the world.
3. Canada, Australia, New Zealand, Sweden, the United Kingdom (UK) and other industrialized countries have adopted an IT regime. The current economic crisis cast, however, doubt about those performances and an old debate on the role that asset prices should play in the monetary policy strategy of price stability oriented monetary authorities is revived.
5. The first known example of an implicit target for price stability was in terms of price-level targeting, as adopted by Sweden in the 1930s (see Berg and Jonung, 1999).
6. Batini and Yates (2003) explored the implications of the HT regime using a reduced form of the Fuhrer and Moore (1995) model which is not built on microfoundations that are as compelling as the Calvo model. Like the original Taylor model, the Fuhrer-Moore model is based on some arbitrary but superficially plausible assumptions about the form of labour contracts (Mankiw, 2001).
employed. The most important feature of this model is the appearance of terms that reflect the forward-looking behaviour of representative agents. This leads, for example, to a stabilization-bias problem that occurs if monetary authorities apply discretionary monetary policy (Clarida et al., 2000). Most of the literature to date uses the new classical model to assess the properties of the HT regime and confirms its advantages (Kobayashi, 2004). However, the use of New-Keynesian models in analyzing the HT regime is only in its early stages.\(^8\) In this paper, an attempt was made to investigate this framework and provide evidence that will assist in discriminating between hybrid regimes and other kinds of monetary-policy targeting.

The recent development in new-open-economy macroeconomics, originating with Obstfeld and Rogoff (1995), has led to a wealth of literature in which micro-founded and optimization-based models are used for policy analysis in the open economy. These studies highlight the role of the terms of trade in the transmission of business cycles (Corsetti and Pesenti, 2001). As in Gali and Monacelli (2005), a small-open-economy version of the Calvo sticky-price model is considered to show how the equilibrium dynamics can be reduced to a simple representation in domestic inflation and output gap. The model used here further explores this avenue and extends Gali and Monacelli’s (2005) framework to account for HT targeting. The resulting setting was used to analyze the macroeconomic implications of three alternative rule-based policy regimes for the small-open economy: a CPI-inflation-based Taylor rule, pure price-level targeting and a hybrid- inflation/price-level-based rule.

In the study’s empirical work, the New Keynesian framework was used in a calibrated DSGE model, applying the hybrid monetary-policy rule. The study calibrated key parameters to match some broad characteristics of the Canadian data. Since analytical solutions are often not available for this regime and empirical literature has not reached a consensus about key parameters, the study must then rely on a calibrated model. Subsequently, a welfare analysis of the various monetary-policy regimes considered in this study was conducted and their impulse-response functions were compared. The paper proceeds in the following manner: Part 2 sketches the model’s derivation as suggested by the microfoundations presented by Gali and Monacelli (2005). Part 3 provides details on the quantitative methodology and discusses the results. Part 4 introduces welfare analysis and provides some results. Part 5 presents the concluding remarks.

**THE MODEL**

The study constructs a model that is a variant of a dynamic New-Keynesian model applied to a small-open economy, following Clarida et al. (2002) and Galìi and Monacelli (2005). In order to make this paper self-contained, key structural equations are presented in this part. The model has three sectors: (1) a continuum of profit-maximizing and monopolistically-competitive firms (owned by consumers who include their shares in their portfolios) operating a constant return-to-scale technology and making staggered price decisions in the spirit of Calvo (1983), (2) an infinitely-lived representative household which maximizes a utility function defined over a composite consumption-good and labour supply and (3) a central bank which sets the monetary policy through an interest rate that targets both the price level and the inflation rate in a hybrid formula.

**Firms’ problem**

**Production technology**

There is a continuum of identical monopolistically-competitive firms. The production function for firm \(i\) that produces a differentiated good \(Y_i\) is

\[
Y_i = A_i N_i, \tag{1}
\]

Where \(i \in [0, 1]\), \(Y_i\) and \(N_i\) are the firm’s \(i\)’s specific output and labour input, respectively. \(A_i = \log(A_i)\) is a total-factor productivity index driven by an \(AR(1)\) exogenous stochastic process, \(A_i = \rho_i A_{i-1} + \varepsilon_{a,i}\), where \(\varepsilon_{a,i}\) is a white noise with mean 0 and variance \(\sigma_{a}^2\). The cost minimization problem leads to the expression of real marginal cost in terms of home prices as

\[
MC_i = \frac{W_i (1 - \tau)}{A_i P_{H,i}}. \tag{2}
\]

Where \(W_i\) and \(A_i\) are a total-factor productivity index driven by an \(AR(1)\) exogenous stochastic process, \(A_i = \rho_i A_{i-1} + \varepsilon_{a,i}\), where \(\varepsilon_{a,i}\) is a white noise with mean 0 and variance \(\sigma_{a}^2\). The cost minimization problem leads to the expression of real marginal cost, which is common across domestic firms, is given by

\[
\hat{MC}_i = \hat{W}_i - \nu - \hat{P}_{H,i} - \hat{A}_i, \tag{2}
\]

Where \(\hat{P}_{H,i}\) and \(\hat{W}_i\) stand respectively for the deviations of domestic price and wage rate from their steady-state values. \(\nu = -\log(1 - \tau)\), where \(\tau\) is an employment subsidy created to exactly compensate for the monopolistic competition distortion. The employment subsidy exactly offsets the combined effects of the firm’s market power and the terms-of-trade distortions in the steady-state. In this case, there is only one effective distortion left in the small-open-economy model, namely sticky prices.

Let \(Y_i\) define the aggregate index for domestic output and \(N_i\), the aggregate employment. \(Y_i\) and \(N_i\) can be expressed in terms of
an individual firm's output as follows
\[ Y_t = \left[ \int_{0}^{\tilde{\xi}} Y(i) \frac{d\xi}{\tilde{\xi}^2} \right]^{\frac{1}{\tilde{\xi}^{-1}}} \text{ and } N_t = \int_{0}^{\tilde{\xi}} n_i(i) di = \int_{0}^{\tilde{\xi}} \frac{Y(i)}{A_i} di \]

Where \( \tilde{\xi} > 1 \) is the elasticity of substitution among goods within each category. Moreover, defining \( Z_t = \int_{0}^{\tilde{\xi}} Y(i) \frac{d\xi}{\tilde{\xi}^2} \) yields
\[ N_t = \frac{Y_t Z_t}{A_t}. \] In loglinear form (up to a first order approximation), aggregate output reduces to
\[ \hat{y}_t = \hat{a}_t + \hat{n}_t, \]

where the variables \( \hat{y}_t, \hat{a}_t, \) and \( \hat{n}_t \) represent the deviations of output, total-factor productivity and employment from a symmetric steady-state.

**Price setting**

Price-setting behaviour follows Calvo (1983) and Yun (1996) in that only a fraction \( 1 - \psi \) of firms adjust their prices each period. Indeed, firms are not allowed to change their prices unless they receive a signal allowing them to re-optimize prices. Following Christiano et al. (2005), prices set by firms that do not receive a random price-change signal are indexed to past inflation.\(^{11}\)

Furthermore, Christiano et al. (2005) assume that prices are fully indexed to past inflation, but empirical models that allow for partial indexation (following Smets and Wouters, 2003) often find that the best-fitting value for the degree of price indexation is positive but less than one. The partial indexation allows the study to have some inflation inertia, leeway, which can make the model more robust for policy and welfare analysis, especially if it is interested in welfare evaluation of inflation costs. Erceg et al. (2000) used indexation to the steady-state inflation rate, allowing them to compute a linearized equation for inflation combining with the expected future and lagged inflation. This equation differs from the forward-looking inflation process obtained under the standard Calvo model. Let \( P_{H.t}^b \) be the price set by firm \( i \) adjusting its price in period \( t \) facing a probability \( \psi^k \) of keeping its price unchanged for \( k \) periods (for \( k = 0, 1, 2, \ldots \)). \( P_{H.t}^b \) defines the price chosen by the remaining fraction \( \psi \) of firms not optimally adjusting their prices at time \( t \). The (log) price \( \hat{p}_{H.t}^b \) is set according to the simple and backward-looking rule
\[ \hat{p}_{H.t}^b = \hat{p}_{H,t-1} + \gamma_p \hat{p}_{H,t-1}, \]

while the new price must satisfy the following equation;
\[ P_{H.t}^s = \mu + (1 - \beta \psi) \sum_{k=0}^{\infty} (\beta \psi)^k E_t \{ mc_{t+k} + P_{H,t+k} \}. \]

Where \( \gamma_p \) is the coefficient of price indexation and \( \mu \) is the steady-state markup.\(^{12}\) The dynamics of the domestic price index are then given by
\[ P_{H.t}^n = \left[ \psi (P_{H,t-1}^b)^{1-\gamma} + (1-\psi)(P_{H,t-1}^n)^{1-\gamma} \right]^{1/\gamma}, \]

which can be loglinearized to obtain an expression for the domestic inflation as follows:
\[ \hat{\pi}_{H,t} = \psi \gamma_p \hat{\pi}_{H,t-1} + (1-\psi)(\hat{p}_{H,t-1}^n - \hat{p}_{H,t-1}). \]

Combining (6) with the differentiated version of (5) yields the aggregate supply equation
\[ \dot{\pi}_{H,t} = \frac{\beta}{1 + \beta \gamma_p} E_t \{ \dot{\pi}_{H,t+1} \} + \frac{\gamma_p}{1 + \beta \gamma_p} \hat{\pi}_{H,t-1} + \kappa mc_t, \]

where \( \kappa = (1-\beta \psi)(1-\psi)/\psi(1+\beta \gamma_p) \). \( mc_t \) represents the log-deviation of the real marginal cost. Equation (7) shows that the domestic inflation dynamic has both forward-looking and backward-looking components. The real marginal costs faced by the firms are also an important determinant of domestic inflation. Note that with \( \gamma_p = 0 \), this equation reverts to the standard open-economy supply equation.

Moreover, assuming that the degree of price stickiness \( \psi \) is identical across economies, the firms in the rest of the world (ROW) face simple Calvo-style price-setting behaviour. For simplicity and without loss of generality, it is assumed, throughout the study's analysis, that the degree of price indexation in the ROW \( \gamma_p^* \) is equal to zero.\(^{13}\)

**Households**

The small-open economy is inhabited by a continuum of infinitely-lived households where the representative household seeks to maximize the expected utility
\[ U_t = \sum_{i=0}^{\infty} \beta^i U(C_t, N_t), \]

Where \( N_t \) is hours worked and \( C_t \) is a composite consumption index defined by
\[ C_t = \left[ \left( 1 - \alpha \right) \beta \left( \bar{C}_{H.t} \right) \sigma^{\theta} + \alpha \beta \left( \bar{C}_{F.t} \right) \sigma \right]^{\sigma}. \]

The elasticity of substitution between the indices of home and

\(^{12}\)The forward-looking pricing decision is related to the fact that firms that adjust their price in any period do so for a random number of periods. The price is then set as a markup over the average of expected future marginal costs.

\(^{13}\)Setting \( \gamma_p^* \), so that it is equal to the domestic price-indexation coefficient (or \( \gamma_p^* = 0 \)) does not significantly change the policy-evaluation results.
foreign goods is given by \( \theta > 0 \). \( C_{H,j} \) is the consumption index of \( j \) domestic goods defined by the CES aggregator

\[
C_{H,j} = \left[ \int_0^1 (C_{H,j}(j))^{\frac{1}{\xi}} \, dj \right]^{\frac{\xi}{\xi-1}}.
\]

Likewise, the consumption index \( C_{F,j} \) is the index of \( j \) imported goods given by

\[
C_{F,j} = \left[ \int_0^1 (C_{F,j}(j))^{\frac{1}{\xi}} \, dj \right]^{\frac{\xi}{\xi-1}},
\]

where the elasticity of substitution among goods within the two indices \( \xi \) is greater than one.

Maximization of expected utility is subject to a sequence of budget constraints of the form

\[
\int_0^1 P_H(j)C_{H,j}(j)\,dj + \int_0^1 P_F(j)C_{F,j}(j)\,dj + E_t\{O_{t+1}D_{t+1}\} \cdot D_t = 0.
\]

(10)

Where \( P_H(j) \) is the price of domestic good \( j \) and \( P_F(j) \) is the price of imported good \( j \) expressed in home currency. \( D_{t+1} \) is the nominal payoff in period \( t+1 \) of the portfolio held at the end of period \( t \) (including firm shares), \( W_t \) is the nominal wage rate and \( T_t \) is lump-sum transfers/taxes. \( O_{t+1} \) is the stochastic discount factor for one-period-ahead nominal payoffs relevant to the domestic household.

Given the constant elasticity of the substitution aggregator for \( C_{H,j} \) and \( C_{F,j} \), the optimal allocation for good \( j \) is provided by the following demand functions:

\[
C_{H,j}(j) = \left( \frac{P_H(j)}{P_{H,j}} \right)^{\frac{1}{\xi}} C_{H,j}, \quad \text{and} \quad C_{F,j}(j) = \left( \frac{P_F(j)}{P_{F,j}} \right)^{\frac{1}{\xi}} C_{F,j}.
\]

(11)

Note that the above functions define the quantities consumed for each type of good, where \( P_{H,j} \) and \( P_{F,j} \) are the domestic and foreign price indices expressed in domestic currency. \( P_H(j) \) and \( P_{F,j} \) are then given by the following expressions:

\[
P_{H,j} = \left[ \int_0^1 (P_H(j))^{\frac{1}{\xi}} \, dj \right]^{\frac{\xi}{\xi-1}}, \quad \text{and} \quad P_{F,j} = \left[ \int_0^1 (P_F(j))^{\frac{1}{\xi}} \, dj \right]^{\frac{\xi}{\xi-1}}.
\]

(12)

Combining (11) and (12), the study obtained

\[
\int_0^1 P_H(j)C_{H,j}(j)\,dj = P_H C_{H}, \quad \text{and} \quad \int_0^1 P_F(j)C_{F,j}(j)\,dj = P_F C_{F}.
\]

Similarly, it can be shown that the optimal allocations between domestic and imported goods are provided by relations

\[
C_{H,j} = (1-\alpha)\left( P_{H,j} \right)^{1-\theta} C_t, \quad \text{and} \quad C_{F,j} = \alpha\left( P_{F,j} \right)^{1-\theta} C_t
\]

\[
\text{Where } P_t = \left( 1-\alpha \right) P_{H,j} + \alpha P_{F,j} \text{ is the overall consumer price index (CPI), or in loglinearized form, } \hat{P}_t = (1-\alpha)\hat{P}_{H,j} + \alpha \hat{P}_{F,j} \text{.}
\]

Accordingly, total consumption expenditures for the domestic household are given by

\[
P_{H,j}C_{H,j} + P_{F,j}C_{F,j} = P_t C_t.
\]

(13)

Substituting this relationship back into (10), the intertemporal budget constraint can be rewritten as

\[
P_t C_t + E_t\{O_{t+1}D_{t+1}\} \cdot D_t = W_t N_t + T_t.
\]

(14)

To solve the household’s optimization problem, the following functional form for the utility function was introduced

\[
U(C_t, N_t) = \left( \frac{C_t^\sigma}{1-\sigma} - N_t^{\sigma (1-\sigma)} \right) \frac{1}{1-\sigma} \phi. \quad This yields the following set of first order conditions. First, the intratemporal optimality condition
\]

\[
C_t^\sigma N_t^{\sigma (1-\sigma)} = \frac{W_t}{P_t}
\]

states that at any period of time \( t \), the marginal utility of consumption is equal to the marginal value of labour. On the other hand, intertemporal optimization (for all states and dates) implies the following Euler equation with regards to consumption:

\[
E_t O_{t+1} = \beta \left( \frac{C_{t+1}^\sigma}{C_t^\sigma} \right) \left( \frac{P_{t+1}}{P_t} \right) = 1
\]

(15)

which, in loglinearized form, yields,

\[
\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} (r_t - E_t \hat{r}_{t+1} - \rho) \quad \text{where } \rho \equiv -\log \beta \text{ is the time discount factor.}
\]

It is assumed that households in the rest of the world face the same optimization problem as the one outlined above where influence from the domestic economy is negligible; that is, relative to the ROW economy, the size of the small-open economy is negligible.

**Inflation, terms of trade and the real exchange rate**

This section sets out the relationships between inflation, terms of trade and the exchange rate. It is assumed that the law of one price holds for all goods (including imported goods), at all times implying that

\[
P_{F,j}(j) = e_j P_{F,j}(j) \quad \text{for all } j \in [0,1],
\]

(17)

**Footnotes:**

14Then the Lagrangian expression for this problem is given by

\[
\max \ E_t \left\{ \sum_{C_t, N_t, O_{t+1}} \beta \left[ \frac{C_t^\sigma}{1-\sigma} N_t^{\sigma (1-\sigma)} - \hat{c}_t (D_t + W_t N_t + T_t - P_{t+1} C_t - E_t \{O_{t+1}D_{t+1}\})] \right] \right\}
\]

15This assumption allows us to treat the ROW economy as a closed economy.
Where $\epsilon_t$ is the bilateral nominal exchange rate and $P_{F,t}(j)$ is the price of good $j$ produced in a foreign country, as expressed in terms of the foreign currency. Substituting (12) back into (17) yields the following expression for the foreign price index

$$P_{F,t} = \epsilon_t \left[ \int_0^1 (P_{F,t}(j))^{1-\eta} \, dj \right]^{1-\eta}. $$

Similarly, if the foreign price index is defined as $P^*_t = \left[ \int_0^1 (P_{F,t}(j))^{1-\eta} \, dj \right]^{1-\eta}$, the relation between the home price of imported goods and the foreign price index in loglinearized form around a steady-state can be written as

$$\hat{p}_{F,t} = \hat{e}_t + \hat{p}^*_t. $$

In addition, using the terms-of-trade definition $S_t = \epsilon_t P_t^*/P_{H,t}$, and loglinearizing around a symmetric steady-state yields

$$\hat{s}_t = \hat{e}_t + \hat{p}^*_t - \hat{p}_{H,t}. $$

Thus, terms of trade may be defined as the price of foreign goods per unit of home good. Since, by definition, the real exchange rate (in loglinearized form) is given by $\hat{q}_t = \hat{e}_t + \hat{p}^*_t - \hat{p}_t$, substituting into equation (19) yields the following relation between the real exchange rate and the terms of trade given by the price levels:

$$\hat{q}_t = \hat{s}_t + \hat{p}_{H,t} - \hat{p}_t. $$

On the other hand, using the definition of the price indices, it can be shown that

$$\frac{P_t}{P_{H,t}} \hat{p}_t - \frac{P_t}{P_{H,t}} \hat{p}_{H,t} = \left[ (1 - \alpha) + \alpha(S)^{1-\eta} \right]^{1/\eta} S^{1-\eta} \alpha \hat{s}_t. $$

Further, assuming that purchasing power parity (PPP) holds in the steady-state, that is, $S = \frac{P_t}{P_{H,t}} = 1$, and combining this equation with (19) and (21), the relation between the domestic price level and the CPI can be written as follows:

$$\hat{p}_t - \hat{p}_{H,t} = \alpha \hat{s}_t. $$

As a result, the study arrived at an identity-linking CPI inflation ($\hat{s}_t$), domestic inflation ($\hat{p}_{H,t}$) and the change in the terms of trade:

$$\hat{\pi}_t = \hat{p}_{H,t} + \alpha \Delta \hat{s}_t. $$

The difference between the CPI and domestic inflation is proportional to the change in the terms of trade and the coefficient of proportionality increases with the degree of openness, $\alpha$. Furthermore, substituting (22) back into (20) yields an expression for the real exchange rate as a function of terms of trade, that is,

$$\hat{q}_t = (1 - \alpha) \hat{s}_t, $$

which establishes a relation between real exchange rate and terms of trade, depending on the degree of openness of the SOE.

**International risk sharing**

In the study’s model, it is assumed that a complete security market actually exists in the world, where the expected nominal return from risk-free bonds, in domestic currency terms, must be the same as the expected domestic-currency return from foreign bonds, that is, $E_t O_{t+1} = E_t O_{t+1}^* \epsilon_{t+1}$. The Euler equation also holds for the foreign representative household and must equate the intertemporal optimality condition for the domestic household.\(^{16}\) This yields a relationship between the domestic and foreign level of consumption in terms of (log) real exchange rate ($Q_t$), that is,

$$C_t = \frac{1}{\epsilon_{t+1}} \frac{C_{t+1}}{C_{t+1}} C_t^{1/\eta}, \quad \epsilon^{-\eta} = \frac{1}{\epsilon_{t+1}} \frac{C_{t+1}}{C_{t+1}} C_t^{1/\eta}. $$

Finally, replacing $C_{t+1}$ and $C_{t+1}^*$ with their respective expression yields the following optimal allocation for the imported good:

$$C_t^* = \alpha Q_t^{-\eta} C_t. $$

Therefore, the relation in (25) can be rewritten as $C_t = \Phi_t C_t^{1/\eta}$, where $\Phi_t$ depends on the initial condition of the country’s asset position. If symmetric initial conditions between home and foreign country are assumed with zero foreign asset holdings for the small-open economy, without loss of generality, the study will obtain $\Phi = 1$ so that the loglinearized form leads to

$$\hat{e}_t = \hat{c}_t + \frac{1}{\sigma} \hat{q}_t. $$

Substituting (24) back into (27) then yields

$$\hat{c}_t = \hat{c}_t + \frac{1 - \alpha}{\sigma} \hat{s}_t, $$

which links both consumption variables to the terms of trade.

**Uncovered interest parity**

The assumption of complete securities markets points to another

\(^{16}\) Using the fact that $E_t O_{t+1} = \beta E_t \left( \frac{C_{t+1}}{C^*_t} \right)^{\sigma} \left( \frac{P_t^{\prime \prime}}{P_{t+1}^{\prime \prime}} \right) \left( \frac{E_{t+1}}{E_t} \right)$, combining this equation with its domestic counterpart, substituting in $Q_t = \epsilon_t P_t^*/P_t$, and rearranging terms to get the next equation.
important relationship and the uncovered interest parity (UIP) condition. Using the previous Euler equation, which also holds for foreign households, that is, \( \beta R^t E_1 (\frac{C^{t+1}_{i+1}}{C_i} - \sigma (\frac{P^t_{i+1}}{P^t_i})) = 1 \), or put in another way,

\[
R_t^{-1} = \beta E_1 (\frac{C^{t+1}_{i+1}}{C_i} - \sigma (\frac{P^t_{i+1}}{P^t_i}))
\]

(29)

and substituting (29) back into the Euler equation yields the price of a riskless bond dominated in foreign currency as

\[
R_t^{-1} = E_1 (O_{i,t+1}, e_{i,t+1})
\]

(30)

Since, by definition, \( R_t^{-1} = E_t O_{i,t+1} \), (30) implies that

\[
E_t [O_{i,t+1} [R_t - R^t (e_{i,t}/E_t)]] = 0, \loglinearizing around the perfect-foresight steady-state yields the asset pricing equation for nominal bonds, which implies that the interest rate differential is related to the expected exchange rate depreciation

\[
\hat{r}_t - \hat{r}_t^* = E_t \{ \Delta \hat{e}_{i,t+1} \},
\]

(31)

where \( \hat{e}_i \) is the deviation of the nominal exchange rate from its steady-state value. Thus, the UIP condition for the nominal exchange rate holds in equilibrium, meaning that the risk premium is assumed to be constant in the steady-state.

**Monetary policy**

To close the model, it is assumed that the central bank sets the nominal interest rate following a Taylor-type interest-rate rule. However, money does not appear in either the household utility function or in the budget constraint. Indeed, recent research on monetary policy adopts this modeling strategy (Gali and Monacelli, 2005). In this kind of model, money plays the role of a unit of account only. Moreover, the influential work by Taylor (1993) uses an interest rate feedback from output and inflation to approximate monetary policy. Recently, Woodford (2000) demonstrated that the interest rate rule is consistent with nominal demand determinacy for forward-looking models even when money demand is not present in the model. In addition, in an open-economy model, the exchange rate is affected by the difference between domestic and foreign nominal interest rates and the expected future exchange rates, via an interest rate parity condition (Svensson, 1998). The real exchange rate will then affect the relative price of domestic and foreign goods, which in turn affects both domestic and foreign demand for domestic goods and hence contributes to movements in CPI inflation. Likewise, the exchange rate affects the domestic currency prices of imported final goods included in the CPI price. In this way, monetary policy can affect both the CPI price and the CPI inflation rate. Consequently, when analyzing the study’s model under HT targeting, a monetary rule that incorporates both the price level and the inflation rate is considered, whereas the historical rule is only an inflation-based rule.

In the present paper, an analysis was done on the macroeconomic implications of three alternative monetary-policy regimes for the small-open economy: a policy that aims at fully stabilizing CPI inflation (IT), a policy that stabilizes CPI price level (PT) and a policy that combines price-level and inflation targeting (HT). As in Gali and Monacelli (2005), it is assumed that the world monetary authority succeeds in fully stabilizing world prices and the output gap; hence, \( \gamma^*_i = \pi^*_i = 0 \) is assumed for all \( t \) which is optimal for the closed economy under the study’s assumptions.

**Inflation targeting**

Inflation targeting is the policy which responds to deviations of the CPI inflation rate from the target. This regime acts in stabilizing the CPI inflation rate around the inflation-target path. Moreover, IT involves price-level drift and consequently price non-stationarity. A Taylor rule representation was adopted (Taylor 1993) where the interest rate (\( \hat{r}_t \)) reacts to inflation deviations from its target (\( \hat{\pi}_t \)) and output deviations from potential (output gap, \( \hat{X}_t \)), that is

\[
\hat{r}_t = \rho + \phi_\pi \hat{\pi}_t + \phi_X \hat{X}_t,
\]

where \( \rho \), \( \phi_\pi \) and \( \phi_X \) are policy parameters. The study analyzed the properties of the equilibrium of the small open-economy when this policy rule is used and its performance with PT and HT was compared.

**Price-level targeting**

Price-level targeting itself is a policy that systematically responds to deviations of the CPI price index from a predetermined long-run path. PT is then a policy stabilizing the price level (\( \hat{P}_t \)) around the target path, which implies stationarity for the price index and an inflation rate around zero inflation. In the study’s analysis, PT with a fixed price-level target was considered (steady-state price level, \( \bar{P} \)), that is, \( P_t = \bar{P} \) or in log deviation \( \hat{P}_t = 0 \). Thus, PT yields price-level stability around the steady-state price and zero inflation.

**Hybrid targeting**

A hybrid inflation/price-level targeting policy combines elements of both previous regimes. This regime embeds both an inflation and price-level target, allowing therefore for some base drift in the price path. As in Batini and Yates (2003), it is assumed that the monetary policy follows the generalized hybrid inflation/price-level target

\[
\hat{r}_t = E_t \{ \hat{\pi}_{t+1} \} + \phi_p (E_t \hat{P}_t - \chi \hat{P}_{t-1}) + \phi_X \hat{X}_t,
\]

(32)

Where \( \hat{r}_t \) denotes the short-term nominal interest rate, \( \hat{\pi}_t \), \( \hat{P}_t \) are defined in the same way as stated above and \( \hat{X}_t \) is the output gap. \( \chi \in (0,1] \) is the key parameter that defines the spectrum of targets between price-level and inflation targeting. When \( \chi = 0 \), the policy makers target the price level only and when \( \chi = 1 \), only the level of inflation rate is targeted. For \( 0 < \chi < 1 \), the target is a hybrid regime targeting both the price-level and the inflation-rate level.\(^{19}\) The degree of price-level drift, \( \chi \), in Batini and Yates’ [2003] model is treated as a choice variable for the government and no optimal value for this parameter is derived by the authors.

\(^{19}\)We only consider this last scheme in our analysis, that is, \( 0 < \chi < 1 \). We also study the welfare implications of varying \( \chi \) in the unit interval.
However, Røisland (2006) shows that, within a model with inflation inertia due to price indexation, an HT regime can be adopted to achieve optimal policy identical to the one under commitment if the monetary authority sets the degree of price-level drift equal to the degree of price indexation. Hence, the optimal degree of price-level drift in the HT rule is equal to the degree of price indexation.

Equilibrium determination

Aggregate demand

World output and consumption

Combining the market clearing condition for the ROW economy, \( \hat{y}_t = \hat{c}_t^* \), with the Euler equation for the foreign household's consumption, \( \hat{c}_t^* = E_t \hat{c}_{t+1}^* - \frac{1}{\sigma} (r_t^* - E_t \hat{r}_{t+1}^* - \rho) \), leads to a version of the new IS equation in the case of sticky-price models:

\[
\hat{y}_t = E_t \hat{y}_{t+1}^* - \frac{1}{\sigma} (r_t^* - E_t \hat{r}_{t+1}^* - \rho). \tag{33}
\]

This IS equation shows that the foreign output is related negatively to the world interest rate and positively to the expected foreign CPI inflation.

Small-open-economy output, consumption and trade balance

Market clearing for domestic goods requires that

\[
Y_t(i) = C_{Ht,i}(i) + C_{Ht,i}(i), \quad \text{where } Y_t(i), C_{Ht,i}(i) \text{ and } C_{Ht,i}(i) \text{ are respectively, the production, home and foreign demand for home produced good } i. \text{ Moreover, based on preference symmetry between the home and the foreign country, it can be shown that}
\]

\[
C_{Ht,i} = \alpha \left( \frac{P_{Ht,i}}{P_{t}} \right)^{-\xi} \left( \frac{P_{F,i}}{P_{t}} \right)^{-\omega} C_t^i. \tag{34}
\]

Substituting (34) back into the market clearing condition above, the study gets

\[
Y_t(i) = \left( \frac{P_{Ht,i}}{P_{t}} \right)^{-\xi} (1-\alpha) \left( \frac{P_{Ht,i}}{P_{t}} \right)^{-\omega} C_t^i + \alpha \left( \frac{P_{Ht,i}}{P_{t}} \right)^{-\xi} \left( \frac{P_{F,i}}{P_{t}} \right)^{-\omega} C_t^i
\]

for all \( i \in 0,1 \) and \( t \).

Using the fact that \( S_t \equiv P_{F,t} / P_{H,t} \), the aggregate output can be shown to reduce to

\[
Y_t = \left( \frac{P_{Ht,i}}{P_{t}} \right)^{-\xi} \left( \frac{P_{F,i}}{P_{t}} \right)^{-\omega} C_t^i \left[ (1-\alpha) + \alpha S_t \right]. \tag{35}
\]

Log-linearizing (35) while making use of \( \hat{p}_t = \hat{P}_{H,t} = \alpha \hat{S}_t \) yields

\[
\hat{y}_t = \hat{c}_t + \alpha \hat{S}_t + \alpha (\theta - 1) \hat{q}_t. \tag{36}
\]

Equation (36) states that the relation between output and consumption in terms of the exchange rate and terms-of-trade variables is governed by the degree of openness of the economy.

Furthermore, notice that by using \( \hat{q}_t = (1-\alpha) \hat{S}_t \), expression (36) can be rewritten as

\[
\hat{y}_t = \hat{c}_t + \frac{\alpha \omega}{\sigma} \hat{S}_t. \tag{37}
\]

Where \( \omega = \xi \sigma + (\sigma \theta - 1)(1-\alpha) \). Using the fact that \( \hat{c}_t = \hat{y}_t + (1-\alpha) \hat{S}_t \), equation (37) becomes

\[
\hat{y}_t = \hat{y}_t^* + \frac{1}{\sigma} \hat{c}_t. \tag{38}
\]

Where \( \sigma = \sigma |(1-\alpha) + \alpha \omega| / \sigma \) and the subscript in \( \sigma \) is meant to emphasize the dependence of this parameter on the degree of openness of the economy (\( \alpha \)). Finally, a version of the new IS equation for the SOE can be computed by combining Euler equation (16) with (23) and (37), which yields

\[
\hat{y}_t = E_t \{ \hat{y}_{t+1} \} - \frac{1}{\sigma} (\hat{c}_t - E_t \{ \hat{r}_{H,t+1} \} - \rho) - \frac{\alpha \omega}{\sigma} E_t \{ \hat{S}_{t+1} \}. \tag{39}
\]

This SOE equation is different from its closed-economy version because it depends on the small economy's degree of openness and on foreign output.\(^{20}\)

Moreover, net exports \( (NX) \) are related to domestic output in terms of steady-state output \( (Y) \) through the following equation

\[
nX_t = \left( \frac{1}{Y_t} \right) (Y_t - \hat{P}_{H,t} C_t). \tag{40}
\]

Combining the linearized version of (40) with (22), (28) and (37) yields

\[
nX_t = (1-\Lambda) \{ \hat{y}_t - \hat{y}_t^* \}
\]

Where \( \Lambda = \sigma |(1-\alpha) + \alpha \omega| / \sigma \). The relationship between net exports and output differential is ambiguous and depends on the value of \( \Lambda \). If \( -1 < \Lambda < 1 \), a positive output differential generates a trade surplus favourable to the small-open economy and with \( \Lambda > 1 \) or \( \Lambda < -1 \), the trade surplus is favourable to the foreign country. Following Galí and Monacelli (2005), \( -1 \leq \Lambda \leq 1 \)

\(^{20}\)It's easy to see that with \( \alpha = 0 \), we can obtain the closed-economy version.
is needed to satisfy the Marshall-Lerner conditions.21

**Deriving the new Keynesian Phillips curve (NKPC)**

Price stickiness is the only source of suboptimality in the equilibrium allocation. Indeed, as shown by Galí and Monacelli (2005), the employment subsidy neutralizes the market power distortion. By not assigning any explicit value to monetary holding balances, the monetary distortion that would pull monetary policy towards the Friedman rule is eliminated. Inflation inertia is also introduced in the model by the price behaviour. The resulting model is then consistent with what has been termed the NKPC. The determination of the real marginal cost as a function of domestic and foreign output is complex due to the wedge between some aggregate variables, namely output versus consumption and domestic price versus consumer price indexes. The study indeed has

\[
mc_t = -\nu + \hat{\phi} y_t - \hat{\phi}_t, 
\]

(41)

where the last equality makes use of (3) and (28). According to (41), real marginal cost is increasing as it concerns the terms of trade, domestic and world output and is decreasing with regards to technology. Hence, the wealth and employment effects on real wages combined with the changes in the product wage and then the impacts on real wages lead to changes in marginal cost through its direct effect on labour productivity. It follows from equation (38) that in this case, real marginal cost is given by

\[
mc_t = -\nu + (\sigma - \sigma_a) y_t^* - (1 + \phi) \hat{\phi}_t, 
\]

(42)

In what follows, focus is on equation (7) in deriving a NKPC representation for the small-open economy in terms of the output gap and domestic inflation given the study’s price schemes. Let the output gap22 \( \bar{x}_t \) be defined as the deviation of domestic output \( \hat{x}_t \) from its ‘natural’ level \( \bar{x}_t \). Formally,

\[
\bar{x}_t = \hat{x}_t - \bar{x}_t, 
\]

where natural output has been computed by imposing the restriction \( mc_t = -\mu \) for all \( t \) in equation (42) and solving for domestic output, that is,

\[
-\mu = -\nu + (\sigma - \sigma_a) y_t^* - (1 + \phi) \hat{\phi}_t, 
\]

which, after some algebraic manipulations, yields the natural output level as

\[
\bar{y}_t = \Omega + \Gamma y_t^* + \Psi \hat{\phi}_t, 
\]

(43)

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where \( \Omega = (\nu - \mu)/(\sigma_a + \phi) \), \( \Gamma = (\sigma_a - \sigma)/(\sigma_a + \phi) \) and \( \Psi = (1 + \phi)/(\sigma_a + \phi) \). Equation (43) states that the natural output for the small-open economy is determined by world output and productivity, as well as domestic markup. In addition, a relationship between real marginal cost and the output gap can be derived according to

\[
m_c = -\nu + (\sigma - \sigma_a) \hat{y}_t + (\sigma - \sigma_a)(\Omega + \Gamma y_t^* + \Psi \hat{\phi}_t) + (\sigma_a - \sigma) y_t^* - (1 + \phi) \hat{\phi}_t. 
\]

Where natural output has been substituted for its value in (43). By rearranging terms, the study gets

\[
mc_t = (\sigma - \sigma_a) \bar{y}_t, 
\]

(44)

which can be combined with equation (7) to derive a NKPC in terms of the output gap

\[
\bar{x}_t = \frac{\beta}{1 + \beta \gamma p} E_t (\bar{x}_{t+1}) + \frac{\gamma p}{1 + \beta \gamma p} \bar{x}_{t+1} + \bar{\delta}_t, 
\]

(45)

where \( \bar{\delta} = \kappa(\sigma_a + \phi) \). Notice that with the degree of openness \( \alpha \) and the coefficient of price indexation \( \gamma_p \) set to zero (that is, \( \alpha = 0 \) and \( \gamma_p = 0 \)), equation (45) reverts to the standard, purely forward-looking NKPC. The relation (45) also makes it clear that the standard formulation of NKPC based on the output gap assumes no price indexation to past inflation and hence, there is no inflation inertia in the model.

The equilibrium dynamics for the small-open economy in terms of output gap and domestic inflation can be completed by writing a version of the IS equation in terms of the output gap. Indeed, by combining (39) and (43), it can be shown that

\[
\bar{x}_t = E_t \{ \bar{x}_{t+1} \} - \frac{1}{\sigma_a}(\hat{r}_t - E_t (\hat{x}_{t+1}) - \rho) + \Gamma (\rho_a - 1) \hat{\phi}_t + \alpha(\Psi + \Theta)E_t (\hat{y}_t^*), 
\]

(46)

where \( \Theta = (\omega - 1) \). If the natural interest rate is defined as

\[
\hat{r}_t \equiv \rho - \sigma_a \Gamma (\rho_a - 1) \hat{\phi}_t + \alpha(\Psi + \Theta)E_t (\hat{y}_t^*), 
\]

where the degree of openness and the expected world output affect the natural rate of interest, then the new IS equation has the following form:

\[
\bar{x}_t = E_t \{ \bar{x}_{t+1} \} - \frac{1}{\sigma_a}(\hat{r}_t - E_t (\hat{x}_{t+1}) - \rho), 
\]

(47)

Equation (47) relates the output gap in the forward-looking equation to the interest rate, domestic inflation and the natural interest rate. To solve this model, a loglinear approximation of the equilibrium conditions was made around a balanced-trade and zero-inflation steady-state.23 The dynamic properties of the model crucially depend on the monetary policy used. Indeed, with the Taylor rule, where the \( \chi \) parameter takes the value 1, the persistent inflation

21The Marshall-Lerner conditions apply if and only if the sum of the import and export elasticities is greater than one.

22In our model, we have to handle three definitions of output: a measure of output, natural output (which we get in an economy with no imperfection or nominal rigidity) and finally the output gap, which is the difference between the output and the natural output.

23The markup is also assumed to be constant at steady state \( \mu = \frac{\bar{\epsilon}}{\bar{y} - 1} \) in order to derive the equilibrium conditions.
response to a technology shock implies that this shock will have a permanent effect on price level, which will then have a unit root, mirrored by a unit root in the nominal exchange rate. In this case, following Gali and Monacelli (2005), targeting inflation rate and the monetary authority seeks to stabilize CPI inflation. Such a policy only requires that \( r_t - E_t\{r_{t+1}\} = \phi_p (E_t \xi_t) + \phi_y \pi_t \), is set for all \( t \).

Moreover, following Woodford (1999) and Bullard and Mitra (2002), the study’s analysis focuses on the case where \( \phi_p \) and \( \phi_y \) have non-negative values. Thus, the necessary and sufficient condition for a stable allocation path is given by

\[
\delta(\phi_p - 1) + (1 - \beta)\phi_y \neq 0.
\]

Furthermore, it is assumed that the foreign country pursues an optimal policy, imposing a constant foreign-price level at equilibrium. The model’s dynamics can be stable in this case, even with non-stationary prices; otherwise, with \( 0 \leq \gamma < 1 \), the price level is \( I(0) \) and the stable allocation-path condition (48) holds at equilibrium.

In the following sections, the study will first set the model parameters as calibrated to the Canadian economy and before analyzing the welfare implications of each regime, the impulse response functions and second-moment statistics will be computed.

### QUANTITATIVE RESULTS

#### Model calibration

Baseline calibration of the model is based on recent literature and closely follows Gali and Monacelli (2005). The parameter values used by this study are intended to reflect Canadian data. A labour supply elasticity of about 1/3 was used which sets \( \phi = 3 \) and a steady-state markup \( \mu = 1.2 \), meaning that the elasticity of substitution between different domestic goods \( \xi \) is 6. The average-price adjustment period by firms is set to four quarters and then the study sets the sticky-price parameter \( \psi \) to 0.75, while the degree of openness of the economy \( \alpha \) is set to 0.4. The discount factor \( \beta \) is assumed to be equal to 0.99 and the elasticity of substitution between domestic and foreign goods \( \theta \) takes the value 1.5 according to Backus et al. (1995), and Gali and Monacelli (2005), who use the special case where \( \theta = \sigma = 1 \).

The remaining parameters are somewhat difficult to determine. Indeed, there is no consensus among open-economy researchers about the values attributed to the intertemporal rate of substitution \( \sigma \). Cochrane (1997) uses values between one and two, while Yun (1996) and Gali and Monacelli (2005) calibrated their models with model similar to the one studied in this paper (using \( \sigma = 1 \). The study follows Erceg et al. (2000) and sets this parameter to 1.5. The price indexation parameter characterizes the backward-looking component in the NKPC. A 0.5 value for this parameter is frequently found in empirical estimates of the NKPC (see forSmets, 2003), Christiano et al. (2005) and Gali et al. (2001) use indexation to the past period’s inflation rate but set the indexation parameter to one (full indexation). In contrast, estimates reported by Smets and Wouters (2003) and Sahuc (2004) using U.S. and Euro area data show that \( \gamma_p \) ranges from 0.40 to 0.64 with larger values for the U.S. case. The latter authors argue that partial indexation is data consistent and appears to capture inflation dynamics. Following Justiniano and Preston (2007), who estimated a small-open-economy Canadian data), this parameter is set at \( \gamma_p = 0.55 \), so that the backward-looking coefficient is 0.40 and the forward-looking term is 0.71 in the price equation. The calibration for the policy-rule parameters follows Batini and Yates (2003) and Taylor (1993). The study sets \( \varphi_p, \varphi_f, \) and \( \varphi_y \) to 0.5. As discussed in Batini and Yates, the parameter \( \chi \) calibration is quite difficult. Indeed, using a range of values within the interval [0,1], the authors show that the value of \( \chi \) depends on the size of the inflation tax, the cost of indexation and the length of nominal contracts, but they do not however derive an optimal value for this coefficient. As discussed in section 2.3 and following Røisland (2006), the study sets \( \chi \) equal to the degree of price indexation to past inflation. Moreover, Gali and Monacelli (2005), using the Canadian labour productivity for the period 1963Q01 - 2002Q04 as a proxy for domestic productivity, estimate the stochastic properties of technology shock to be: \( \rho_c = 0.66 \) with a standard deviation of about one percent. Finally, these estimates are used and the residual correlation is set to zero, that is, \( corr(e_t, e_t') = 0 \). Table 1 illustrates a summary of model-parameter calibration.

#### Impulse response functions and second moment analysis

Impulse response functions (IRFs) play an important role in describing the impact that shocks have on macroeconomic variables. To further understand this role, the IRFs of the main variables have been simulated using three rules: IT, PT and HT. The results are reported in Figures 1 to 3, displaying impulse responses

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24 As shown by Bullard and Mitra (2003), this condition rules out Eigen values on the unit circle.

Table 1. Model calibration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\mu$</th>
<th>$\varphi$</th>
<th>$\xi$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
<th>$\alpha$</th>
<th>$\psi$</th>
<th>$\lambda$</th>
<th>$\varphi_{\rho}$</th>
<th>$\varphi_{\pi}$</th>
<th>$\varphi_{y}$</th>
<th>$\rho_{A}$</th>
<th>$\sigma_{A}$</th>
<th>$\rho_{A*}$</th>
<th>$\sigma_{y*}$</th>
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<td>3</td>
<td>6</td>
<td>1.5</td>
<td>1.5</td>
<td>0.4</td>
<td>0.75</td>
<td>0.55</td>
<td>0.5</td>
<td>0.5</td>
<td>0.66</td>
<td>1%</td>
<td>0.76</td>
<td>1%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Impulse response functions to domestic productivity innovations.
Figure 2. Impulse responses to a foreign productivity shock under HT, IT and PT regimes.
up to 15 quarters. Figure 1 displays the impulse responses to a one percent positive technology shock under HT, IT and PT regimes. The output gap response function has the same patterns for all three regimes, with a hump-shaped pattern and initial negative responses ranging from -1.4 (IT) to -0.3% (PT). The peaks are reached after three to five quarters and then the IRFs revert slowly to steady-state. Inflation responses (domestic
and CPI-based inflation) display different patterns depending on the policy that is been targeted. While domestic inflation has approximately the same response as the output gap under HT and IT, the IRF under PT is quite different -0.3 Indeed, showing a small variation, domestic inflation under PT has an initial value of reaches a 0.2% peak in about 1.5 periods and then rapidly reverts to the steady-state. The response of CPI inflation under PT is flat at the steady-state level. The study can intuit that this is because the price targeting rule imposes a path for the price level at its steady-state value, which prevents both CPI and domestic inflation from displaying more variability when the model is hit by a transitory technology shock.

The main difference occurs with regard to the initial response to the shock, which is zero for CPI inflation under the HT regime and negative under IT with hump-shaped responses. Therefore, the monetary authority has the same response under all three rules, stabilizing inflation when the technology shock occurs. The same patterns are displayed by the domestic and CPI price-level responses with a hump-shaped domestic price response under HT and IT. The unit root in the price level is then mirrored by the unit root in the exchange rate. However, the responses of those three variables are quite different during HT and IT targeting, where after a while the path reverts to initial values. The initial fall in the domestic and CPI price responses under HT and IT are followed by hump-shaped patterns (more pronounced for IT targeting) with a slow increase toward steady-state values. Furthermore, the impact on foreign aggregates is negligible by construction, implying that the world interest rate remains unchanged. There is an anticipated domestic currency appreciation induced by the uncovered parity (UIP). Thus, the exchange rate depreciation explains the paths followed by the inflation rates that rise in the shock period and then revert back to initial levels.

The nominal interest rate shows a different response. With an initial response to the shock that is negative, it increases in a hump-shaped pattern to attain a peak in about five periods and then returns to steady-state values under all regimes. This can be intuited to mean that after the economy has been hit by a technology shock, the optimal monetary authority response will increase the nominal interest rate by a larger amount than the increase in inflation, resulting in an initial increase in the real interest rate level.

The terms of trade and net exports display similar paths, where initial positive responses and decreases reach steady-state values persistently. This leads to a stationary behaviour for those variables, which is defined as a property of the model. The nominal exchange rate moves in the wrong direction, especially under HT and PT. The dynamic effects of foreign technology shocks are displayed in Figure 2. In this case, the foreign monetary authority reacts to shocks by lowering the world interest rate to stabilize inflation. The domestic authorities react in the same way by reducing their own interest rate to counteract the real appreciation caused by the foreign policy, followed by a gradual depreciation until both interest rates converge to their steady-state levels.

Moreover, the output gap and domestic inflation responses display hump-shaped patterns under all targeting rules. CPI inflation responses are different given the rule followed by the monetary authority. While the terms-of-trade variable is more stable under HT and PT targeting, responses persistently remain above initial levels for this variable under IT targeting. The same patterns are displayed for net exports under the three regimes. The decline in domestic and CPI prices is more accentuated with this shock under IT. The nominal interest rate response takes the hump-shaped form and then reverts to the initial value. The main difference between home and foreign technology shock responses is registered for the exchange rate, while the response under all regimes persistently remains above the initial levels.

Finally, the response functions of the macro variables to unit innovations in the policy shocks reveal that all variables display approximately the same patterns under HT and PT targeting. However, the initial domestic and CPI price responses under IT targeting are persistently above the steady-state levels for more than 10 periods and then revert to the steady-state. Interestingly, the figure shows persistent exchange rate responses slightly below the initial values for all regimes. This can be explained by the negligible effect of the policy innovations on foreign variables. Indeed, a rise in the nominal interest rate is followed by an instant currency appreciation and an anticipated depreciation since the world interest rate remains unchanged. Asset and goods generate such movements in exchange rate and price levels. In order to conclude the quantitative analysis, the second moments for some macro variables under the three regimes are shown in Table 2. For each variable, standard deviation in percentage points was reported. The second moment analysis confirms the IRF visual analyses. Indeed, the IT regime requires more volatility in CPI and domestic price levels than that shown under the other regimes. Terms of trade are more stable under IT, where their volatility is about two times lower than that for the PTs. It can be intuitively predicted that under IT, the price level should follow the I(1) process. Hence, price adjustment after the occurrence of shocks is carried out very sluggishly, leading to sluggish inflation behaviour. In fact, lagged price levels have little direct influence on current price levels. In this case, the price adjustment made after shock occurrences inevitably entails sharp inflation.

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28One can believe that monetary contraction generates appreciation for the domestic currency. Thus, capital outflows cause demand for foreign exchange to increase and not to fall, as is the case here, especially under PT.

29With our earlier assumption about the foreign monetary policy that stabilizes price levels at equilibrium, a reduction in the world interest rate implies an appreciation of home currency.
fluctuation. Furthermore, the hybrid target can be set taking into account both inflation and its corresponding price level, such that past price levels affect current price levels, but their influence is not as strong when under IT. In this case, the price level path will lie between those under IT and PT. As pointed out by Kobayashi (2004), it can be said that implementing hybrid targeting can lead to relatively moderate inflation volatility by appropriately incorporating both the sluggish nature of inflation adjustment under IT and the rapid nature of inflation response under PT. More generally, it is found that across regimes, higher inflation and greater output gap volatilities result in higher welfare scores. These findings are in line with the results obtained by Galí and Monacelli (2005).

Welfare analysis of alternative regimes

The analysis of welfare implications for different monetary policy rules has become an important field of study (Taylor, 1999). The main concern is how important it is for policy makers to have access to a set of tools that allow them to predict the effects of switching from one policy rule to another. It would thus be worthwhile to investigate the welfare implications of the hybrid regime and compare them to other monetary-policy targeting schemes considered in this work. The application of the quadratic approximation of the objective function is complex and cannot be simply derived in an open-economy model with sticky prices. A popular measure thus uses inflation and output gap volatility, in addition to the utility function.

Furthermore, a welfare-maximizing central bank may target CPI inflation and price or a combination of specific price and inflation paths. In fact, the key difference in approaches to inflation/price-level targeting concerns a stable and long-run price level compared to maintaining a particular rate of inflation. These rule-based approaches have different welfare implications. Aoki (2001) and Devereux and Engel (2000) show that in a closed economy with sticky prices and backward-looking behaviour, optimal policy entails the perfect stabilization of the inflation rate. In fact, Svensson (1999) shows that if the monetary authority has a price-level targeting objective, then this may reduce inflation variability without affecting output variability. This 'free-lunch' result depends on substantial endogenous output persistence in the New-Classical Philips curve. Dittmar and Gavin (2000) extend this analysis to the case where expectations are forward-looking in a New-Keynesian Philips curve. They show that the free-lunch argument was applied without the need for persistence terms. Thus, the assigning of a price-level targeting objective by the central bank appears to improve welfare if expectations are forward-looking or if there is substantial endogenous persistence. Likewise, Vestin (2000) argues that in a purely forward-looking model, price-level targeting will provide more efficient outcomes than inflation targeting. Concerning a closed-economy model, Nessen and Vestin (2000) suggest that hybrid targeting will provide better outcomes than targeting inflation, only if the Phillips curve has forward- and backward-looking components.

The evaluation of household welfare in the small-open economy can be expressed as a fraction of steady-state consumption. Here, the study follows Galí and Monacelli (2005), who derive a second-order approximation to the domestic consumer utility function in a SOE model. 30 This second order approximation, 31 expressed as a fraction of steady-state consumption, reveals that the expected welfare losses of any policy in terms of domestic inflation and output gap variances are then given by:

\[ \Xi = \frac{(1-\alpha)}{\kappa} [\xi \text{var}(\hat{x}_{H}) + (1+\phi)\text{var}(\hat{x}_{r})]. \]

30 See Appendix 4 in Galí and Monacelli (2004) for details on the welfare-loss function derivations. However, the derivation is restricted to the special case of log utility and unit elasticity of substitution between different goods (i.e. \( \sigma = \eta = \theta = 1 \)) in deriving an exact expression; otherwise, its derivation is more complex. We use this approximation for the purpose of comparing different regimes without loss of generality. For more discussion about welfare analysis in the loglinearized model, refer to Kim and Kim (2003) and Schmitt-Grohe and Uribe (2004).

31 After dropping terms independent of policy and those of high order and computing the unconditional expectation of this approximation.
Using this expression, different monetary policies can be compared to assess their welfare implications and highlight welfare costs among regimes. Table 3 shows welfare losses associated with three different regimes: HT, IT and PT. It is assumed that the central bank wants to minimize variations in domestic inflation ($\pi^H$). Indeed, since most of the countries that use inflation targeting are likely to target CPI inflation rather than home inflation (namely producer-price inflation), HT has been essentially compared to the CPI inflation targeting regime (IT in the text). Entries for loss functions are percentage units of steady-state consumption. There are five panels in this table. In the first panel, welfare losses were reported under the study’s benchmark parameterization. The remaining panels display the effects of using different policy parameter values ($\chi$) and lowering respectively, the degree of economy openness ($\alpha$), the steady-state mark-up ($\mu$) and the elasticity of labour supply ($\phi$) (Gali and Monacelli, 2005).

The results show that under the study’s benchmark parameterization, the reduction in welfare loss results from a decrease in output and domestic inflation volatility varying from an IT to an HT regime. On the other hand, the CPI inflation targeting leads to a much higher level of losses in the welfare-loss function than those obtained by the two other regimes. In fact, as usually found in the literature, welfare losses are quantitatively small for all regimes. As compared to the benchmark case and using different policy parameters, the HT regime implies substantially larger welfare losses as one gets closer to extreme values corresponding to either IT (with $\chi=0.85$) or PT (with $\chi=0.25$). As one gets closer to the extremes, the PT targeting performs well, lowering both inflation and output gap variabilities. This finding is in line with recent studies of monetary policy showing PT targeting outperforming IT targeting (for example Svenson 1998; Vestin, 2000; Røisland, 2006); the effect of lowering the degree of economy openness is considered next. This has a general effect on decreasing both domestic inflation and output gap volatilities, leading to low welfare losses.

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Table 3. Welfare losses under alternative policy regimes.

<table>
<thead>
<tr>
<th>Variable</th>
<th>HT regime</th>
<th>IT regime</th>
<th>PT regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark $\mu=1.2$, $\varphi=3$, $\alpha=0.4$ and $\chi=0.55$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var (Domestic inflation)</td>
<td>0.043546</td>
<td>0.181292</td>
<td>0.061595</td>
</tr>
<tr>
<td>Var (Output gap)</td>
<td>0.013347</td>
<td>0.085585</td>
<td>0.048945</td>
</tr>
<tr>
<td>Welfare loss ($\Xi$)</td>
<td>-0.929222</td>
<td>-3.904543</td>
<td>-1.350430</td>
</tr>
<tr>
<td>$\mu=1.2$, $\varphi=3$, $\alpha=0.4$ and $\chi=0.25$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var (Domestic inflation)</td>
<td>0.067517</td>
<td>0.222545</td>
<td>0.057849</td>
</tr>
<tr>
<td>Var (Output gap)</td>
<td>0.001193</td>
<td>0.004340</td>
<td>0.001566</td>
</tr>
<tr>
<td>Welfare loss ($\Xi$)</td>
<td>-1.417321</td>
<td>-4.672170</td>
<td>-1.215020</td>
</tr>
<tr>
<td>$\mu=1.2$, $\varphi=3$, $\alpha=0.4$ and $\chi=0.85$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var (Domestic inflation)</td>
<td>0.001255</td>
<td>0.035528</td>
<td>0.019625</td>
</tr>
<tr>
<td>Var (Output gap)</td>
<td>0.048605</td>
<td>1.295282</td>
<td>1.238130</td>
</tr>
<tr>
<td>Welfare loss ($\Xi$)</td>
<td>-0.084637</td>
<td>-2.299402</td>
<td>-1.215020</td>
</tr>
<tr>
<td>Low degree of openness $\mu=1.2$, $\varphi=3$, $\alpha=0.25$ and $\chi=0.55$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var (Domestic inflation)</td>
<td>0.026702</td>
<td>0.141287</td>
<td>0.025688</td>
</tr>
<tr>
<td>Var (Output gap)</td>
<td>0.005782</td>
<td>0.047865</td>
<td>0.019376</td>
</tr>
<tr>
<td>Welfare loss ($\Xi$)</td>
<td>-0.708635</td>
<td>-2.299402</td>
<td>-0.702432</td>
</tr>
<tr>
<td>Low steady-state mark-up and Low elasticity of labor supply $\mu=1.1$, $\varphi=10$, $\alpha=0.4$ and $\chi=0.55$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Var (Domestic inflation)</td>
<td>0.177927</td>
<td>1.400579</td>
<td>0.118844</td>
</tr>
<tr>
<td>Var (Output gap)</td>
<td>0.009838</td>
<td>0.070990</td>
<td>0.011315</td>
</tr>
<tr>
<td>Welfare loss ($\Xi$)</td>
<td>-3.763743</td>
<td>-29.60564</td>
<td>-2.529602</td>
</tr>
</tbody>
</table>

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32Recently, Røisland (2006) has shown that under the HT regime the central-bank loss function should be modified to take the form $L = (\hat{\pi}_t - \chi \hat{\pi}_t) - 2 + \lambda \chi^2$, where $\chi$ is as in the text and $\lambda$ is a modified weight on the output gap. This assumption cannot be used here since we aim to compare various targeting regime. We relay this case to future work.

33Kollman (2002) and Smets and Wouters (2003) are recent examples of papers in which monetary-policy welfare implications are investigated.
under all regimes. This can be intuited to the fact that the decrease in volatilities and the resulting welfare values are essentially generated by movements in small-open-economy variables such as terms of trade and exchange rate which have low effects in a ‘quasi-open economy’ (with small α). In this case, HT and PT deliver lower welfare losses than the IT regime.

Finally, the study explored the effects of lowering both the mark-up to 1.1, which leads to a larger penalization of inflation variability in the loss function and the elasticity of labour supply to 0.1, which implies a larger penalization of output gap volatility. This leads to a similar output gap volatility compared to the other scenarios considered here, and in turn to an amplification of the volatility of domestic inflation, which implies higher welfare losses for all three regimes. Interestingly, the IT leads to a significantly larger loss compared to the two other regimes and gives a loss function value up to 10 times higher than the HT and PT. In comparison with PT, an HT regime leads to a larger welfare loss, meaning that the results may be sensitive to the model assumption, as pointed out by Galí and Monacelli (2005) and Schmitt-Grohé and Uribe (2001).34

CONCLUDING REMARKS

This paper investigates hybrid inflation/price-level targeting from a New-Keynesian perspective. To this end, generalizations of the models proposed by Galí and Monacelli (2005) and Monacelli (2003) are calibrated to the Canadian economy. Both papers develop a small-open-economy model incorporating many of the microfoundations appearing in a closed economy within the New-Keynesian framework (Clarida et al., 2000; Woodford, 2003) recently used for the analysis of monetary policy. The model's open-economy version allows for the possibility that international trade in goods and financial assets affects the evolution of the domestic economy, thus giving rise to richer dynamics within the model, given the study’s assumption of complete security markets. Furthermore, in light of the considerable attention paid in recent macroeconomic literature to monetary-policy formulations in terms of interest rate rules, the study adopts this formulation to construct three regimes. In addition, for the purpose of comparison of the hybrid regime, the IT and PT regimes were analyzed. The study’s results show that hybrid targeting can lead to a successful monetary policy strategy, yet, without any major loss in the welfare function.

In overall, an HT regime seems to be an appropriate method of conducting monetary policy if the monetary authorities want to achieve price stability. In fact, the long-run anchor for a central bank is clearly price stability. The problem is whether then it should target price level or variations in this price level (inflation rate). Recent literature on PT shows that anchoring the price level to a long-run price-level path is a good idea, given agents' expectations (forward and/or backward looking). However, since conservative central bankers seem to need more time to reach this point, it is believed that an intermediate way should be hybrid targeting and that more research is needed before PT can be implemented or even considered for implementation, as it is the case, for example, in Canada. Likewise, in this kind of model, including more nominal rigidities, particularly sticky wages or some type of wage indexation would be expected to change the results obtained in a significant manner. Further research is therefore necessary in order to establish the manner in which these frictions would likely alter this finding.

REFERENCES


34The authors argue that the welfare ranking among different monetary policies may be sensitive to distortions in the economy.