Money flexibility and optimal consumption-leisure choice under price dispersion

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Abstract:

The synthesis of G.Sigler's rule of the optimal search with the classical individual labor supply model can incorporate the satisficing decision procedure in the neoclassical framework. Many psychological anomalies and puzzles like the paradox of little pre-purchase search for big-ticket items and the effect of sunk costs sensitivity get the purely economic rationale. This synthesis also enlarges the understanding of the phenomenon of money flexibility under price dispersion. The specific constraints of the search model establish the correspondence between elasticity of the marginal utility of labor income with regard to price dispersion, wage rates, and the propensity to search. The money flexibility under price dispersion discovers specific features of Veblen effect. When the smart shopping of luxuries results in price reductions, which are greater than the wage rate, the marginal utility of labor income becomes negative. The marginal utility of luxuries also becomes negative. And the total consumption-leisure utility is increasing only due to the increase in leisure time. The paper argues that the same mechanism underlies the phenomenon of money illusion due to the relatively excess money balances.

Keywords: money flexibility, price dispersion, consumption-leisure choice, Veblen effect, money illusion

JEL Classification: D11, D83.

Introduction

Although the issue of consumer search has become one of the most dynamic themes in modern economic thought during the past decades, the problem of its integration with the classical theory is still open. There is a possibility of a deeper methodological integration of

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[1] The author is grateful to readers of the earlier version, but he should apologize for its incompleteness. This version is complemented by the paragraph "Money flexibility and Money Illusion", which was not presented in the earlier version.

J.Stigler's original model of search (Stigler 1961) and the model of the individual labor supply. One of such approaches has been introduced earlier in the general form (Malakhov 2011). That approach enabled the synthesis of the search satisficing procedure and the optimal consumption-leisure choice (Malakhov 2012b), as well as the original explanation of anomalies of consumer behavior such as the paradox of little pre-purchase search for big-ticket items (Malakhov 2012a) and Veblen effect (Malakhov 2012c). Based on a wide review of the modern economic literature, those papers have also opened up the possibility of even deeper integration of the search model and the individual labor supply model, now in terms of money. This approach makes the turn to early fundamental methodological works of the neoclassical school. In this paper we pay attention to the Lagrangian multiplier and to the marginal utility of money in the consumption-leisure choice under price dispersion.

The Lagrangian multiplier in the search model

The optimal consumption-leisure choice under price dispersion can be presented by the utility function *(U(Q,H)* which is constrained by the equality of marginal costs of search (marginal loss of labor income) to its marginal benefit (marginal savings on price reduction), or

$$w\frac{\partial L}{\partial S} = Q\frac{\partial P}{\partial S} \quad (1)$$

where

w – the wage rate; Q – the quantity purchased; L – the labor time: S - the time of search;

$\partial L/\partial S$ – the propensity to search ($\partial L/\partial S<0$);

$\partial P/\partial S$ – the price reduction during the search ($\partial P/\partial S<0$).

We can formulate the following problem:

$$maxU(Q,H) \quad subject\ to \quad w = Q\frac{\partial P/\partial S}{\partial L/\partial S} \quad (2)$$

Here we consider the value of price reduction $\partial P/\partial S$ as a variable functionally independent from both the consumption Q and leisure H. This value simply states the fact that we have concluded the search at the certain satisficing price level, where we have spent time S=ΔS on the search and we have got the price reduction ΔP with respect to our willingness to pay. At this price level we determine how much we should buy for the period of time T until the next purchase. The chosen quantity Q determines the labor time L we need to restore our cash-in-the-pocket or money balances (Here, we can bypass the monetary theory by the simple appeal to the H.Leibenstein's understanding of the static analysis, who presented the static situation as one in which the order of events was of no significance (Leibenstein 1950, p.187), or by the come-back to old days when debts were recorded by shopkeepers in notebooks). And for the chosen time horizon of our consumption-leisure choice *T* (a day, couple of days, a week, a year) we finally determine the leisure time $H = T - S - L$ we will need to consume the chosen quantity *Q*.[2]

This search model of the optimal consumption-leisure choice can be presented with the help of the Cobb-Douglas utility function $U(Q,H) = Q^{-\partial L/\partial S} H^{\partial H/\partial S}$. This intuition is supported by the fact that the marginal rate of substitution of leisure for consumption in the search model (Eq.3.1) corresponds to the *MRS* value of the utility function with constant elasticity of substitution $\sigma = 1$ (Eq.3.2), because from the equation $L+S+H=T$ we get $(\partial L/\partial S + \partial H/\partial S) = -1$:

$$MRS(HforQ) = -\frac{dQ}{dH} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H \quad (3.1)$$

$$\frac{dQ}{dH} = \frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = \frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = \frac{QT}{(H-T)}\frac{1}{T} = \frac{Q}{H}\frac{H}{(H-T)} = \frac{Q}{H}\frac{(H-T+T)}{(H-T)};$$

$$\frac{dQ}{dH} = \frac{Q}{H}(1+\frac{T}{H-T}) = \frac{Q}{H}(1+\frac{1}{\partial L/\partial S}) = \frac{Q}{H}(\frac{\partial L/\partial S + 1}{\partial L/\partial S});$$

$$\partial L/\partial S = -a \Rightarrow \frac{dQ}{dH} = -\frac{Q}{H}(\frac{1-a}{a}) \quad (3.2)$$

Here the difference between the search model and the classical model of the individual labor supply is the analytical extraction of the time of search from the time of leisure and its addition, as we are going to see in the set of Eq. (4), to the time of labor.

Usually the search cuts not only the labor time but also the leisure time ($\partial H/\partial S<0$). The following pair of the inequalities ($-1<\partial L/\partial S<0;\ \partial H/\partial S<0$) describes the "common model" of consumer behavior. And this "common model" can be presented with the help of the Archimedes' principle. The search displaces both labor time and leisure time in the time horizon of the consumption-leisure choice, like ice displaces both whiskey and soda from the glass (Eq.4.2). The Archimedes' principle enables the understanding of the relationship between the labor and the search, or the function $L = L(S)$:

[2] We can present any local store by the given value of price reduction $\partial P/\partial S$. This means that we can analyze the functions $Q=Q(\partial P/\partial S)$ and $H=H(\partial P/\partial S)$. But inverse functions do not exist, because the location and the price policy of the local store don't depend on consumer's tastes and preferences. Here the consumer is the price-reduction-taker. If we presuppose that consumers can always adjust price reductions to their consumption patters ($|\partial P/\partial S|=\partial P/\partial S(Q)$; $\partial P/\partial S=\partial P/\partial S(H)$), consumption and leisure become perfect complements.

$$L(S) = T - H(S) - S \Rightarrow \partial L / \partial S = -\partial H / \partial S - 1 \quad (4.1)$$

$$dH(S) = dS\frac{\partial H}{\partial S} = -dS\frac{H}{T} \quad (4.2)$$

$$\frac{\partial L}{\partial S} = -\frac{\partial H}{\partial S} - 1 = \frac{H}{T} - 1 = \frac{H - T}{T} = -\frac{L+S}{T} \quad (4.3)$$

$$\frac{\partial L}{\partial S} = -\frac{L+S}{T} \Rightarrow \partial^2 L / \partial S^2 = -\frac{\partial L / \partial S + 1}{T} < 0 \quad (4.4)$$

$$\frac{\partial L}{\partial S} = \frac{H - T}{T} \Rightarrow \partial^2 L / \partial S \partial H = 1 / T \quad (4.5)$$

We can see that in the search model the value of the time horizon equals not to the chronological period, but to the period of the product lifecycle, i.e., to the moment of the next purchase of the same item (Eq.4.5). The greater time horizon increases not only labor time required for purchase, but also the leisure time required for consumption. So, the increase in the value of time horizon decreases the absolute value of *propensity to search* $|\partial L/\partial S|$.

The set of Eq. (4) simplifies the presentation of the values of marginal utilities and it explains the step-by-step derivation of the *MRS (H for Q)* in the Eq. (3):

$$MU_Q = \lambda\frac{\partial P / \partial S}{\partial L / \partial S} = \lambda\frac{w}{Q} \quad (5.1)$$

$$MU_H = -\lambda Q\frac{\partial P / \partial S}{(\partial L / \partial S)^2}\partial^2 L / \partial S \partial H = -\lambda\frac{w}{\partial L / \partial S}\partial^2 L / \partial S \partial H = \lambda\frac{w}{L+S}\frac{T}{T} = \lambda\frac{w}{L+S} \quad (5.2)$$

Now we can optimize the utility U^* with respect to the wage rate w. We also meet here the specific forms of elasticity of the search model, which we will need in the analysis of money flexibility:

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda\left[\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} + \frac{w}{L+S}\frac{\partial H}{\partial w}\right] \quad (6.1)$$

$$\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} = \frac{\partial P / \partial S}{\partial L / \partial S}\frac{1}{\partial P / \partial S}\left(\frac{\partial L}{\partial S} + w\frac{\partial(\partial L / \partial S)}{\partial w}\right) = 1 + \frac{w}{\partial L / \partial S}\frac{\partial(\partial L / \partial S)}{\partial w} = 1 + e_{\partial L/\partial S, w}$$

$$\frac{w}{L+S}\frac{\partial H}{\partial w} = -\frac{w}{L+S}\frac{\partial(L+S)}{\partial w} = -\frac{w}{(L+S)/T}\frac{\partial(L+S)/T}{\partial w} = -\frac{w}{\partial L / \partial S}\frac{\partial(\partial L / \partial S)}{\partial w} = -e_{\partial L/\partial S, w}$$

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial w} = \lambda\left[\frac{\partial P / \partial S}{\partial L / \partial S}\frac{\partial Q}{\partial w} + \frac{w}{L+S}\frac{\partial H}{\partial w}\right] = \lambda\left[1 + e_{\partial L/\partial S, w} - e_{\partial L/\partial S, w}\right] = \lambda \quad (6.2)$$

Then, we can re-arrange the Eq.(1) with the help of the Eq.(4.3):

$$-T\partial P / \partial S = w(L+S) = P_0 \quad (7)$$

The Eq.(7) gives us the value of *the price equivalent of the potential labor income w(L+S),* which the consumer would have earned if he spends all time of search on working.

The value of the price equivalent of the potential labor income simplifies the presentation of the value of the marginal utility of consumption. Now we can write for consumption and leisure

$$MU_Q = \lambda \frac{\partial P / \partial S}{\partial L / \partial S} = \lambda T \frac{\partial P / \partial S}{L + S} = \lambda \frac{P_0}{L + S} = \frac{\partial U^*}{\partial w} \frac{P_0}{T - H} \quad (8.1)$$

$$MU_H = \lambda \frac{w}{L + S} = \frac{\partial U^*}{\partial w} \frac{w}{T - H} \quad (8.2)$$

However, if we repeat this step-by-step utility optimization procedure with respect to the wage rate in the framework of the individual labor supply model, we get the same results for the classical consumption-leisure choice (Baxley and Moorhouse 1984):

$$\frac{\partial U^*}{\partial w} = \frac{\partial U}{\partial Q} \frac{\partial Q}{\partial w} + \frac{\partial U}{\partial H} \frac{\partial H}{\partial w} = \lambda P \frac{\partial Q}{\partial w} + \lambda w \frac{\partial H}{\partial w} = \lambda \left[P \frac{\partial Q}{\partial w} + w \frac{\partial H}{\partial w} \right]$$

$$wT = PQ + wH \Rightarrow T = P \frac{\partial Q}{\partial w} + w \frac{\partial H}{\partial w} + H \Rightarrow T - H = P \frac{\partial Q}{\partial w} + w \frac{\partial H}{\partial w}$$

$$\lambda = \frac{\partial U^*}{\partial w} \frac{1}{T - H} \quad (9)$$

and finally:

$$MU_Q = \lambda P = P \frac{\partial U^*}{\partial w} \frac{1}{T - H} \quad (10.1)$$

$$MU_H = \lambda w = w \frac{\partial U^*}{\partial w} \frac{1}{T - H} \quad (10.2)$$

So, the following graphical presentation, given with unusual attributes, simply represents the specific form of the graphical resolution of the optimal consumption-leisure choice for T = *24 hours* (Fig.1):

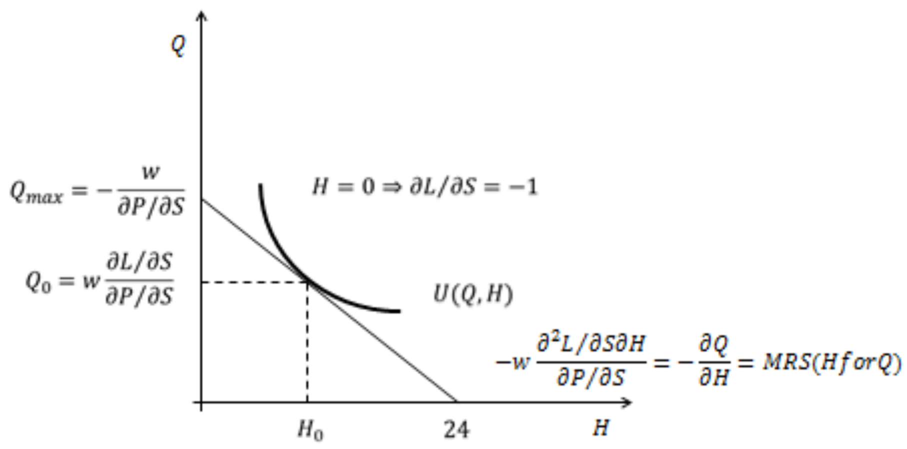

Fig.1."Common model" of search behavior

There are two important differences between two models. First, the value of the Lagrangian multiplier λ in the search model is exactly equal to the marginal utility of the wage rate, or $MU_w=\lambda$, while in the classical model we need to adjust it to the allocation of time between labor and leisure. Second, in the classical model the marginal utility of consumption is determined by the equilibrium purchase price P_e, while the marginal utility of consumption in the search model is determined by the price equivalent of the potential labor income $P_0=w(L+S)$.

If we re-arrange the Eq. (3.1) with the help of the Eq,(7), we can simplify the presentation of the marginal rate of substitution of leisure for consumption:

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{w}{\partial P/\partial S}\frac{1}{T} = \frac{w}{P_0} \quad (11)$$

The analysis of the satisficing decision procedure of the unit purchase (Malakhov 2012b) can clarify the concept of the price equivalent of the potential labor income (Fig.2):

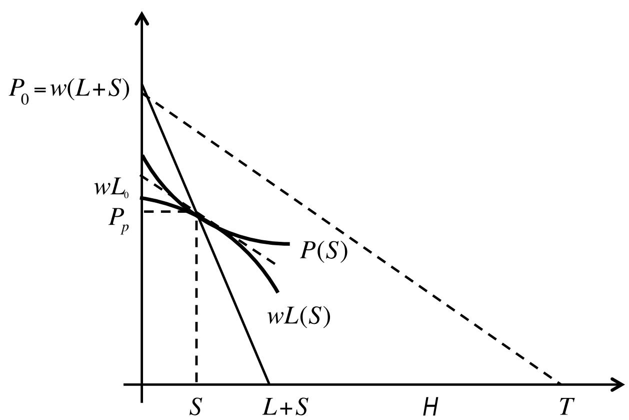

Fig.2.Satisficing decision procedure and optimal consumption-leisure choice

Here we get the shape of the *wL(S)* curve ($\partial^2L/\partial S^2 <0$) from the Eq.(4.4). The shape of the *P(S)* curve ($\partial^2P/\partial S^2 >0$) is based on the commonsense assumption of the diminishing efficiency of search. The both curves have the same tangent (dotted) line at the purchase price P_P. The purchase price P_P is equal to the labor income *wL,* where the wage rate *w* gives us the slope of the intersection with the time horizon axis of the consumption-leisure choice. So, the value of the labor income, reserved for the purchase before we start to search, $wL_0=wL+dwL(S)$, can represent our willingness to pay. And we can say that the value of price reduction $\partial P/\partial S$ simply represents the rate of decline in the willingness to pay during the search. If we link the points [P_0,T], we get another dotted line with the same $\partial P/\partial S$ slope in accordance with the Eq.(7).

This is more difficult to determine the price equivalent of the potential labor income. It looks like a high-order willingness to pay with regard to the reservation level. P.Diamond used that approach when he described the behavior of shoppers with high willingness to pay (Diamond 1987). On the other hand, the Eq.(11) looks corresponding to another P.Diamond's consideration, that positive search costs makes the equilibrium price equal to the monopoly price (Diamond 1971). However, the idea of the monopoly price seems not to be an appropriate explanation of the price equivalent of the potential labor income, because if a monopoly sets the price at the level of the potential labor income, consumers will increase their labor supply by the time of search, and this increase in labor supply will decrease the wage rate. The price equivalent of the potential labor income at lower wage rate will become unattainable.

We should keep in mind that this value is to some extent virtual, and it can be determined only *ex post*, when we conclude the search. This *ex ante – ex post* relationship between the reserved labor income and the potential labor income initiates the assumption that we have here the WTP-WTA relationship and the potential labor income represents the price that compensates not only monetary loss, which is equal to the purchase price $P_P=wL$, but also the loss of the time of search, calculated *ex post* on the base of the wage rate, or *wS*. However, this consideration needs the answer to the question whether, when search costs are positive, *the willingness to accept* can represent the equilibrium price.

Finally, it is easy to demonstrate that all above-presented equations are valid in the case when search costs are equal to zero, or $S=0$. Here, all properties of the search model become equivalent to the corresponding values of the classical individual labor supply model. The utility function takes the usual $U(Q,H)=Q^{L/T}H^{H/T}$ form, the *MRS (H for Q)* becomes equal to the *Q/L* ratio, the value of the marginal utility of consumption MU_Q and the value of the marginal utility of leisure MU_H become equal to the classical λP_P and the λw values, where the Lagrangian multiplier, according to Baxley and Moorhouse (1984), is equal to the $(\partial U^*/\partial w)/(T-H)$ ratio.

Indeed, when search costs are equal to zero, the purchase price is equal to the reserved labor income as well as to the potential labor income. The willingness to pay becomes equal to the willingness to accept. In addition, the classical individual labor supply model becomes a particular case of the search model, where the propensity to search $\partial L/\partial S$ is equal to the *(- L/T)* ratio.

Money flexibility and the Veblen effect

The Eq.(6.2), where $MU_w=\lambda$, tells us that the marginal utility of labor income per unit of time is equal to the marginal utility of an extra unit of money. However, the search process also produces the non-labor income through the reduction in prices of purchases. And in order to compare the efficiency of shopping in different stores we need to define the marginal utility of price reduction.

For better understanding and illustrative purposes, we will use the absolute value of price reduction $|\partial P/\partial S|$, keeping in mind that due to $\partial^2 P/\partial S^2 > 0$, the increase in the absolute value $|\partial P/\partial S|$ means the increase in price. (The replacement of the value $\partial P/\partial S$ to its absolute value $|\partial P/\partial S|$ as well as the replacement of the value $\partial L/\partial S$ to the absolute value $|\partial L/\partial S|$ does not matter.)

$$\frac{\partial U^*}{\partial|\partial P/\partial S|} = \frac{\partial U}{\partial Q}\frac{\partial Q}{\partial|\partial P/\partial S|} + \frac{\partial U}{\partial H}\frac{\partial H}{\partial|\partial P/\partial S|} = \lambda\left[\frac{w}{Q}\frac{\partial Q}{\partial|\partial P/\partial S|} + \frac{w}{L+S}\frac{\partial H}{\partial|\partial P/\partial S|}\right]$$
$$= \lambda\frac{w}{\partial P/\partial S}\left[\frac{|\partial P/\partial S|}{Q}\frac{\partial Q}{\partial|\partial P/\partial S|} + \frac{|\partial P/\partial S|}{L+S}\frac{\partial(T-(L+S))}{\partial|\partial P/\partial S|}\right]$$
$$= \lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|} - \frac{|\partial P/\partial S|}{(L+S)/T}\frac{\partial(L+S)/T)}{\partial|\partial P/\partial S|}\right]$$
$$= \lambda\frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|} - e_{|\partial L/\partial S|,|\partial P/\partial S|}\right] \quad (12)$$

The key equation (Eq.1) of the search model can convert this result. If we present it in the elasticity form with respect to the absolute value of price reduction $|\partial P/\partial S|$, we get for the constant wage rate ($\partial w/\partial|\partial P/\partial S|=0$):

$$Q\left|\frac{\partial P}{\partial S}\right| = w\left|\frac{\partial L}{\partial S}\right| \Rightarrow e_{Q,|\partial P/\partial S|} + 1 = e_{|\partial L/\partial S|,|\partial P/\partial S|}$$
$$e_{Q,|\partial P/\partial S|} - e_{|\partial L/\partial S|,|\partial P/\partial S|} = -1 \quad (13)$$

This elasticity form (Eq.13) gives us the value of the marginal utility of price reduction:

$$\frac{\partial U^*}{\partial |\partial P/\partial S|} = \lambda \frac{w}{|\partial P/\partial S|}\left[e_{Q,|\partial P/\partial S|} - e_{|\partial L/\partial S|,|\partial P/\partial S|}\right] = -\lambda \frac{w}{|\partial P/\partial S|} \quad (14)$$

The negative value of the marginal utility of price reduction $MU_{|\partial P/\partial S|}$ needs some methodological comments. The search produces the non-labor income and the greater absolute value of price reduction increases money balances. In this sense, the marginal utility of price reduction is positive. But the assumption of the decreasing efficiency of search, or $\partial^2 P/\partial S^2 > 0$, establishes the direct correspondence between the value of the purchase price and the value of price reduction. The greater absolute value of the price reduction corresponds to the greater purchase price. (The case, when consumers meet big discounts and find low purchase prices, is examined with regard to the reduction in the time horizon of consumption-leisure choice or shelf lives. (Malakhov 2012b). However, the increase in price decreases the utility and in order to avoid the confusion, we will use the notion of *the marginal disutility of price reduction* for the value $MU_{|\partial P/\partial S|} < 0$.

Its unusual form raises the question whether we have the same value of marginal utility of an extra unit of money when we compare labor income with non-labor income. However, it is easy to show that *pecunia non olet* and the value of the Lagrangian multiplier, or the utility of an extra unit of money, is the same for labor income as well as for non-labor income.

The Eq.(14) is very useful. It uncovers the anatomy of the search process. First of all, we can see that different levels of wage rate result in different values of the marginal disutility of price reduction. Moreover, their relationship is reciprocal. The greater wage rate results in the greater marginal disutility of price reduction. This consideration is very important. When the value of price reduction represents the rate of reduction of the purchase price with respect to the willingness to pay, the increase in the absolute level of price reduction $|\partial P/\partial S|$ first of all means the increase in the willingness to pay.

The increase in the willingness to pay happens due to the increase in the opportunity costs of search, i.e., in the wage rate. Trying to save time, individuals begin to buy in nearby stores at high prices.[3] However, they can keep their willingness to pay constant and they can continue to buy in a distant supermarket. If the value $|\partial P/\partial S|$ is constant, we expect that the marginal disutility of price reduction $MU_{|\partial P/\partial S|}$ is also constant.

In order to verify this assumption, let's analyze the total change in the marginal disutility of price reduction with regard to the change in the wage rate:

[3] Here we simply follow the Salop-Stiglitz's assumption, that only high-search-cost individuals make purchases in high-price stores (Stiglitz 1979, p. 141)

$$\frac{\partial MU_{|\partial P/\partial S|}}{\partial w} = -\left[\frac{\partial \lambda}{\partial w}\frac{w}{|\partial P/\partial S|} + \lambda\frac{\partial\left(\frac{w}{|\partial P/\partial S|}\right)}{\partial w}\right] = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (15)$$

We can see, that the marginal disutility $MU_{|\partial P/\partial S|}$ for consumers who continues to buy in the same place at the same price after the increase in their wage rates ($e_{|\partial P/\partial S|,w} = 0$) is constant only if the marginal utility of the wage rate is unit elastic, or $e_{\lambda,w} = -1$. If it is not, consumers will face either the increase or the decrease in the marginal disutility of price reduction, may be in the form of *the frustration from shopping* with respect to their income levels.

Here, we can address to the original R.Frisch's analysis of the phenomenon of money flexibility (Frisch 1959). It can give us some ideas about feelings of consumers with different income levels when they meet each other in the same supermarket. Things look like consumers with lower wage rate elasticity of the marginal utility of income $e_{\lambda,w}$, i.e., "the better-off part of the population", get greater disutility in supermarkets than "the median part of the population".

If we continue to follow the methodology of R.Frisch, we can simply re-write the Eq.(15), keeping in mind that the second cross derivatives of utility are equal, or $\partial MU_{|\partial P/\partial S|}/\partial w = \partial MU_w/\partial|\partial P/\partial S|$ and $MU_w = \lambda$:

$$\frac{\partial MU_{|\partial P/\partial S|}}{\partial w} = \frac{\partial \lambda}{\partial |\partial P/\partial S|} = -\left[\frac{\partial \lambda}{\partial w}\frac{w}{|\partial P/\partial S|} + \lambda\frac{\partial\left(\frac{w}{|\partial P/\partial S|}\right)}{\partial w}\right] = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \quad (16)$$

Then, we make one more step and we get the general relationship of the marginal utilities of wage rate and of consumption expenditures under the search process:

$$\frac{\partial \lambda}{\partial |\partial P/\partial S|} = -\frac{\lambda}{|\partial P/\partial S|}\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right] \Rightarrow \frac{\partial \lambda}{\partial |\partial P/\partial S|}\frac{|\partial P/\partial S|}{\lambda} = -\left[e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w}\right]$$

$$e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} + 1 - e_{|\partial P/\partial S|,w} = 0 \quad (17)$$

Really, the value $e_{\lambda,|\partial P/\partial S|}$ seems to be an appropriate representation of the concept of *the marginal utility of money expenditures* carefully derived by M.Blaug from the analysis of the Marshallian "Principles" (Blaug 1997, pp.322-323) and, in the same scrupulous manner, compared with the marginal utility of money by A.Abouchar (Abouchar 1982). However, even when the second cross derivatives are equal, they have different economic sense. From here we can leave the concept of the marginal disutility of price reduction and begin to use the concept of the price reduction elasticity of the marginal utility of money expenditures $e_{\lambda,|\partial P/\partial S|}$.

We can see that for the unit elastic marginal utility of income ($e_{\lambda,w}$ = *-1*) the choice of the high-price store after the increase in the wage rate makes the price reduction elasticity of the marginal utility of money expenditures $e_{\lambda,|\partial P/\partial S|}$ as expected *positive*. The purchase in the high-price store increases the marginal utility of money holdings with respect to the corresponding wage rate elasticity of price reduction, or:

$$e_{\lambda,|\partial P/\partial S|} = e_{|\partial P/\partial S|,w} \quad (18)$$

However, here we come to the real combinatorics' variety because every value of the Eq.(17) is a pair of other variables. It is very difficult to describe all possible changes in the Eq.(17). But it is worth looking at some of them.

First of all, we should take into consideration the proportional change in the marginal utility of an item to be bought and the change in the willingness to pay. This proportional change happens, for example, when we make the trade-off between the quality and the price. Again, the shift in the $|\partial P/\partial S|$ value means here the choice of high-price store with quality items. However, the assumption of the proportional change in the marginal utility of an item and in the willingness to pay for it results in the constant value of the marginal utility of consumption expenditures. And its price reduction elasticity $e_{\lambda,|\partial P/\partial S|}$ becomes equal to zero.

The Eq.(17) takes the following form:

$$e_{\lambda,w} = e_{|\partial P/\partial S|,w} - 1 \quad (19)$$

It means, that the inappropriate ambitious choice of a high-price store after the increase in the wage rate ($\partial|\partial P/\partial S|/\partial w>1$) can make the wage rate elasticity of price reduction greater than one, or $e_{|\partial P/\partial S|.w} >1$. In addition, the wage rate elasticity of the marginal utility of income becomes positive, or $e_{\lambda,w}>0$.

There are two possible explanations for the positive wage rate elasticity of the marginal utility of income $e_{\lambda,w}$. The first and most obvious explanation is that this is a symptom of the risk-seeking behavior, or $\partial\lambda/\partial w>0$. We can find many such examples in the real estate sector.

However, not all luxury items are risky. The Chateau Lafite Rothschild 1995 from Pauillac and the Opus XA from Arturo Fuente bought to celebrate new position can hardly be regarded as investments in risky assets. There is another consideration. We can ask ourselves whether a dollar spent on those items has really increased the utility, or, more definitely, whether the marginal utility of the increase in the wage rate $MU_w=\lambda$ has been positive.

Before we reject these seemingly absurd questions, let's analyze changes in marginal utilities of both consumption and leisure under this assumption.

If we come back to our preceding results, we can see that:

$$MU_Q = \lambda \frac{\partial P / \partial S}{\partial L / \partial S} \quad (20.1)$$

$$MU_H = -\lambda \frac{w}{\partial L / \partial S} \partial^2 L / \partial S \partial H \quad (20.2)$$

Even when the price reduction is unit elastic with respect to the wage rate ($e_{|\partial P/\partial S|.w} =1$), the ambitious choice of the high-price store ($\partial|\partial P/\partial S|/\partial w>1$), when consumers are attracted by deep discounts on big-ticket items, results in the following inequality: $w<|\partial P/\partial S|$. However, if we take the inequality $w<|\partial P/\partial S|$, we leave the "common model" of behavior and we come to the "leisure model" of behavior. The $w<|\partial P/\partial S|$ relationship changes the allocation of time. According to the key equation of the search model (Eq.1), the absolute value of the propensity to search becomes greater than one, or $|\partial L/\partial S|>1$ and $\partial L/\partial S<-1$. But now the relationship between leisure and search becomes positive, or $\partial H/\partial S>0$. The Archimedes' principle stops working. And this change means that the increase in the absolute value of the propensity to search $|\partial L/\partial S|$, i.e., its decrease in real terms, is followed now by the increase in leisure time, or $\partial^2 L/\partial S\partial H<0$.

This situation corresponds to the classical backward-bending effect. However, there is very important distinction between the classical model and the search model. Let's come back to the Eq.(20). The value $\partial^2 L/\partial S\partial H<0$ doesn't change the marginal utility of consumption, but it dramatically changes the marginal utility of leisure. The latter becomes *negative* and the leisure becomes "bad".

That conclusion had been done in the earlier analysis of Veblen effect (Malakhov 2012c). Things looked like we wasted time at parties after the purchase of a suit. However, that consideration was *dialectically incomplete*. We told about it, when it was discussed that either the purchase of a suit made parties "reasonable", or parties made "reasonable' the purchase of a suit.

If we presuppose that the marginal utility of the wage rate, i.e., value of the Lagrangian multiplier, becomes negative, we make another representation of the marginal utilities of both consumption and leisure in the Eq.(20). Now the consumption becomes "bad" whereas the marginal utility of leisure again becomes positive.

Of course, the classical utility optimization problem doesn't take place in the "leisure model" of behavior. However, consumers can continue to increase the total utility. But now they can increase the total utility only with the increase in leisure time. The party organized in order to celebrate the new position increases the total utility, where negative marginal utilities of the Chateau Lafite Rothschild 1995 from Pauillac and of the Opus XA from Arturo Fuente become invisible.

Moreover, the increase in leisure time, required to restore the positive utility of such a "luxury" consumption-leisure choice, automatically increases the absolute value of the price reduction $|\partial P/\partial S|$, i.e., the purchase price, and Veblen effect takes place (Fig.3):[4]

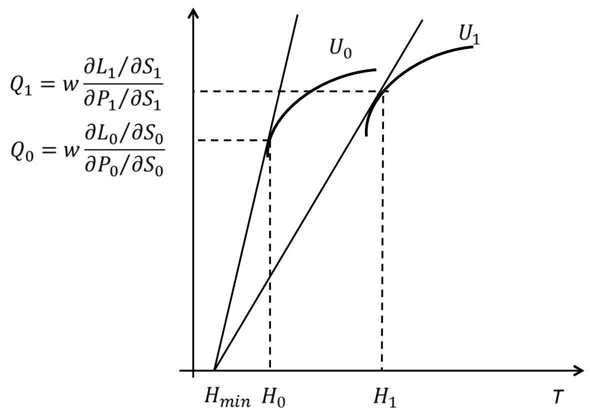

Fig.3."Leisure model" of search behavior and Veblen effect

It is very easy to follow all these considerations with the help of the value of the *MRS (H for Q),* derived earlier (Eq.3.1) and repeated here for illustrative purposes:

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (21)$$

Coming back to the key equation of the model (Eq.1) we can see that the "leisure model" of behavior ($\partial L/\partial S<-1$; $\partial H/\partial S>0$; $\partial^2 L/\partial S\partial H<0$) can be produced not only by luxury items with corresponding high absolute values of price reductions $|\partial P/\partial S|$. We can also see that the value of the lower wage rate can produce the same effect even on almost perfect markets with low absolute values of price reductions, but with very important quantities to be purchased, or $w<|Q\partial P/\partial S|$. This consideration brings the Veblen effect with the Giffen case.

In addition, the "leisure model" of behavior can enlarge our understanding of many economic phenomena. For example, the need to compensate the negative utility of a luxury item by the increase in leisure time, i.e., by the increase in time and/or in intensity of consumption, when a boat is "depreciated" by the number of trips or miles, illustrates the phenomenon of sunk costs sensitivity. But the most intriguing application of the "leisure model" of behavior is the explanation of the phenomenon of money illusion.

Money flexibility and money illusion

[4] Here we should keep in mind that usually there is a certain physical or psychological minimum level of leisure time where $\partial L/\partial S=-1$.

If we come back to the key equation of the search model and to its elasticity form with respect to the wage rate, we can see that the proportional increase in wages w and in price reductions $|\partial P/\partial S|$ doesn't exclude the possibility of the simultaneous increase in both consumption Q and propensity to search $|\partial L/\partial S|$:

$$Q\left|\frac{\partial P}{\partial S}\right| = w\left|\frac{\partial L}{\partial S}\right| \Rightarrow e_{Q,w} + e_{|\partial P/\partial S,w|} = 1 + e_{|\partial L/\partial S|,|w}$$

$$e_{Q,w} + 1 = 1 + e_{|\partial L/\partial S|,|w} \qquad (22)$$

If we re-arrange both the Eq.(13) and the Eq.(17), we have:

$$e_{\lambda,|\partial P/\partial S|} + e_{\lambda,w} = e_{|\partial P/\partial S|,w} - 1 = e_{Q,w} - e_{|\partial L/\partial S|,w} = 0 \quad (23)$$

If we take the "common model" of behavior *($-1<\partial L/\partial S<0$)*, presented graphically by the Cobb-Douglas utility function (Fig.1), where both consumption and leisure are normal goods, we can see that the Eq.(23) takes place only when $\partial Q/\partial w=0$ and $\partial|\partial L/\partial S|/\partial w=0$. Indeed, the derivative $\partial|\partial L/\partial S|/\partial w$ is negative in the "common model of behavior. The wage growth reduces *the propensity to search* and the search itself, because the latter becomes less attractive. However, the decrease in the time of search is greater than the increase in the labor time, because for the "common model" of behavior we have $dL(S)+dS+dH(S)=0$ where $\partial L/\partial S<0$ and $\partial H/\partial S<0$. And with the help of the Eq.(4.4) we can confirm the negative of the derivative $\partial|\partial L/\partial S|/\partial w$.

This consideration tells us that the phenomenon of money illusion doesn't take place in the "common model" of behavior. The proportional increase in wages w and in price reductions ($e_{|\partial P/\partial S|.w}=1$) changes neither the consumption nor the time allocation ($\partial Q/\partial w=0$; $\partial|\partial L/\partial S|/\partial w=0$), because the negative value $\partial|\partial L/\partial S|/\partial w<0$ makes its elasticity with respect to the wage rate $e_{|\partial L/\partial S|.w}$ also negative. And the Eq.(23) takes place only when $\partial Q/\partial w=0$ and $\partial|\partial L/\partial S|/\partial w=0$, of course, if consumption and leisure are not inferior.

But this is not true for the "leisure model" of behavior. There, the values $\partial|\partial L/\partial S|/\partial w$ and $e_{|\partial L/\partial S|.w}$ become positive, because now both the time of search and the time of leisure are increased by the important decrease in the labor time. So, the increase in the search time is less than the decrease in the labor time. But this means that the Eq.(23) takes place in the "leisure model" of behavior for any proportional increase in consumption Q and in propensity to search

$|\partial L/\partial S|$. However, the increase in propensity to search in the "leisure model" means the *increase in leisure time*. So, the unit price reduction elasticity ($e_{|\partial P/\partial S|.w}=1$) actually can be produced by an ambitious choice of an item ($\partial|\partial P/\partial S|/\partial w>1$) in the framework of the "leisure model" of behavior ($w<|\partial P/\partial S|$; $\partial L/\partial S<-1$; $\partial H/\partial S>0$; $\partial^2 L/\partial S\partial H<0$). And the phenomenon of money illusion takes place.

It can take place either when a suit makes parties "reasonable", i.e., when parties are "bad", and the price of the suit becomes the price for the "disposal of bad leisure", or when parties make "reasonable" the purchase of the suit, i.e., when the suit is "bad". In the case of the "bad" suit the "luxury model" of behavior changes not only the sigh of the Lagrangian multiplier λ. It also changes signs of both $e_{\lambda,P}$ and $e_{\lambda,w}$ values in the Eq.(23), however, without prejudice to the Eq.(23) itself.

It seems that the "leisure model" of behavior is the result of large money balances, when the marginal usefulness of money becomes negative (Friedman 1969, pp.17-18, Walsh 2003, p.45). However, the Eq.(23) pays attention not to the absolute value of wealth but to its relative value. Poor people can also suffer from money illusion, because they can also follow the "leisure model" of behavior as well as the Veblen effect (Moav and Neeman 2012). When "public is unwilling to hold securities at interest rates lower than those corresponding to a floor level" (Fellner 1956, p.949), people couldn't wait the restoration of the aggregative equilibrium with a fall of the general price level. On the contrary people could start to buy luxury items.

Conclusion

When market environment progressively relaxes budget constraints, the development of Stigler's rule of the equality of marginal costs of search and its marginal benefit becomes more and more important in the understanding of consumer behavior. The general approach to the elasticity of the marginal utility of money (Eq.17) can be developed in two ways. First, we can use the key equation of the search model (Eq.1) in order to show the relationship between the marginal utility of money and income elasticity of both consumption and leisure for the "common model" of behavior in the following form:

$$e_{\lambda,|\partial P/\partial S|}+e_{\lambda,w}+1-e_{|\partial P/\partial S|,w}=0$$

$$1-e_{|\partial P/\partial S|,w}=e_{Q,w}-e_{|\partial L/\partial S|,w}=e_{Q,w}+\frac{H}{L+S}e_{H,w}$$

$$e_{\lambda,|\partial P/\partial S|}+e_{\lambda,w}+e_{Q,w}+\frac{H}{L+S}e_{H,w}=0 \quad (24)$$

This equation can be useful in field studies of the elasticity of the marginal utility of income in public economics as well as for studies of optimal taxation and for measurements of inequality.

Second, if we presuppose that the price reduction itself is unit elastic, or $e_{|\partial P/\partial S|,P} = 1$, we can re-write the Eq.(17) in the following form:

$$e_{\lambda,P} + e_{\lambda,w} = e_{P,w} - 1 \quad (25)$$

This equation can give us the better understanding of many macroeconomic phenomena, money illusion, for example.

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