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A comment on: "Efficient propagation of shocks and the optimal return on money" ☆

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Abstract

Lotteries are introduced into Cavalcanti and Erosa (2008) [2], a version of Trejos and Wright (1995) [4] with aggregate shocks. Lotteries improve welfare and eliminate the two notable features of the optimum with deterministic trades: over-production and history-dependence. Moreover, the optimum can be supported by buyer take-it-or-leave-it offers.

JEL classification: C78; D61; D82; E30; E40; E50

Keywords: Random matching model of money; Aggregate shock; Optimal allocation; History-dependence; Lottery

1. Introduction

Cavalcanti and Erosa [2] (CE, hereafter) study optima in a version of Trejos and Wright [4]. They introduce into it *i.i.d.* aggregate shocks to preferences, shocks with a two-point support. They show that for an interval of intermediate magnitudes for the discount factor, the ex ante optimum over all individually rational (IR) and *deterministic* trades displays two properties: output

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☆ Cavalcanti and Erosa (2008) [2].
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is higher than the first-best when the shock is such that the first-best output is low and there is history-dependence—that is, promised utilities play a role. We show that if lotteries are allowed, then higher ex ante utility is achieved and neither property holds at an optimum.[2]

The role of lotteries in the CE setting is easily explained. Consider the situation in which the shock is such that the first-best level of output is high and in which the planner would like to weaken the seller IR constraint by making the current acquisition of money more valuable. Absent lotteries, CE achieve that by promising the current seller more output than the first-best in the future when he is a buyer and the shock is such that the first-best level of output is low.[3] With lotteries, the current acquisition of money can be made more valuable by having the buyer surrender money with some probability in that future situation.

2. Model

The model is [2] except that lotteries are allowed in trade. Time is discrete, dated as $t \geqslant 0$, and there is a unit nonatomic measure of agents. At the beginning of every period, the economy is hit by an aggregate shock s with support $\{l, h\}$, low or high, which, as described below, affects preferences. The shock s is i.i.d. over time and the probability of state s is $\pi_s (> 0)$.

Each agent maximizes the discounted sum of expected utility with discount factor $\beta \in (0, 1)$. At each date, if an agent produces $y \geqslant 0$ amount of good, the utility cost is y. If an agent consumes $y \geqslant 0$ amount of good when the current state is s, the period utility he gets is $u_s(y)$, where $u_s : \mathbb{R}_+ \to \mathbb{R}$ is differentiable, strictly increasing, strictly concave, and satisfies $u_s(0) = 0$, $u_s'(0) = \infty$ and $u_s'(\infty) = 0$. We also assume that u_s is bounded, above by $\bar{u}$, and that $u_l' < u_h'$.[4] Define the first-best output levels by $y_s^* \equiv \arg\max\{u_s(y) - y\}$ or $u_s'(y_s^*) = 1$. That is, the first-best output maximizes the sum of utilities of the consumer and the producer. It follows that $y_l^* < y_h^*$.

In each period, after the aggregate state is observed, agents are randomly matched in pairs. With probability $1/N$, an agent is a consumer, with probability $1/N$, the agent is a producer, and with probability $1 - 2/N$, the match is a no-coincidence meeting, where $N \geqslant 2$.

There exists a fixed stock of indivisible, perfectly durable money, the per capita amount of which is denoted $m \in (0, 1)$. Individual money holdings are restricted to $\{0, 1\}$. In meetings, agents' money holdings are observable, but any other information about an agent's trading history is private.

3. The planner's problem and the solution

We study the mechanism-design problem studied by CE; the planner chooses an allocation to maximize welfare subject to a notion of implementability.

The realization of the date-t aggregate shock is denoted s_t and a history up to date t is denoted $s^t \equiv (s_0, s_1, \ldots, s_t)$. Let $S^t \equiv \{s_0\} \times \{l, h\}^t$ denote the set of possible histories up to date t

2 Berentsen, Molico and Wright [1] are the first to introduce lotteries into matching models of money.

3 This over-production in turn leads to history-dependence. See Proposition 10 of their paper for details.

4 One way to get the linear cost function and the bounded utility function is as follows: suppose that the utility and the cost from consuming and producing z amount are given by a possibly unbounded function $\tilde{u}_s(z)$ and a convex function $\tilde{c}(z)$, respectively. Suppose further that there is a bound $\bar{z}$ on production in a sense that $\lim_{z \to \bar{z}} \tilde{c}(z) = \infty$. Then changing the unit of goods nonlinearly by $y \equiv \tilde{c}(z)$ leads to the bounded utility function $u_s(y) \equiv \tilde{u}_s(\tilde{c}^{-1}(y))$ and the linear cost function $c(y) \equiv \tilde{c}(\tilde{c}^{-1}(y)) = y$ with no bound on y.

conditional on the initial state s_0, and let $p(s^t) \equiv \pi_{s_1}\pi_{s_2}\cdots\pi_{s_t}$, the probability of event s^t. It is assumed that the initial state is given and that $p(s_0) = 1$.

An allocation is $\{y(s^t), q(s^t)\}_{s^t}$, where $y(s^t) \in \mathbb{R}_+$ is output (produced by the producer and consumed by the consumer) and $q(s^t) \in [0, 1]$ is the probability that the consumer transfers money to the producer.[5] The welfare criterion is

$$\sum_{t=0}^{\infty}\sum_{s^t\in S^t}\beta^t p(s^t)\left[u_{s_t}\left(y(s^t)\right) - y(s^t)\right], \tag{1}$$

where $u_s(y) - y$ is the social gain, or the sum of period utility of the consumer and the producer.

Because people can exit a meeting without trade and with no further punishment, the planner is subject to IR constraints for the producer and for the consumer. In order to state the IR constraints in a simple way, let $v_j(s^t)$ denote the expected discounted utility of an individual with money holdings $j \in \{0, 1\}$ after history s^t and before being matched. These satisfy

$$\begin{aligned} v_1(s^t) = {} & \frac{1-m}{N}\left[u_{s_t}\left(y(s^t)\right) + q(s^t)\beta\left(\pi_l v_0(s^t, l) + \pi_h v_0(s^t, h)\right)\right] \\ & + \left(1 - \frac{1-m}{N}q(s^t)\right)\beta\left(\pi_l v_1(s^t, l) + \pi_h v_1(s^t, h)\right), \end{aligned} \tag{2}$$

and

$$\begin{aligned} v_0(s^t) = {} & \frac{m}{N}\left[-y(s^t) + q(s^t)\beta\left(\pi_l v_1(s^t, l) + \pi_h v_1(s^t, h)\right)\right] \\ & + \left(1 - \frac{m}{N}q(s^t)\right)\beta\left(\pi_l v_0(s^t, l) + \pi_h v_0(s^t, h)\right). \end{aligned} \tag{3}$$

The IR constraints for the producer and the consumer are expressed as[6]

$$y(s^t) \leqslant q(s^t)\beta R(s^t) \leqslant u_{s_t}\left(y(s^t)\right), \tag{4}$$

where

$$R(s^t) \equiv \pi_l r(s^t, l) + \pi_h r(s^t, h), \tag{5}$$

and

$$r(s^t) \equiv v_1(s^t) - v_0(s^t).$$

The planner's problem is as follows.

Definition 1. An allocation $\{y(s^t), q(s^t)\}_{s^t}$ is *implementable* if there exists a sequence $\{v_0(s^t), v_1(s^t)\}$ that satisfies conditions (2)–(4) and $v_i(s^t) \in [0, \bar{u}/(1-\beta)]$. An allocation is *optimal* if it maximizes (1) among the set of implementable allocations. An allocation is *history-independent* if it depends only on the current state, in which case the allocation is characterized by four numbers: (y_l, q_l, y_h, q_h).

[5] Because goods are divisible and agents are risk-averse, lotteries over output do not improve welfare. The proof is somewhat analogous to Proposition 3 of [1].

[6] Expressing IR constraints by using v_j implicitly relies upon the principle of one-shot deviation. That principle applies because there is discounting and u_s is bounded.

Our result is

Proposition 1. *Let*

$$\frac{1}{\beta_l} \equiv 1 + \frac{1-m}{N} \cdot \frac{\pi_l[u_l(y_l^*) - y_l^*] + \pi_h[u_h(y_l^*) - y_l^*]}{y_l^*}, \tag{6}$$

$$\frac{1}{\beta_h} \equiv 1 + \frac{1-m}{N} \cdot \frac{\pi_l[u_l(y_l^*) - y_l^*] + \pi_h[u_h(y_h^*) - y_h^*]}{y_h^*}. \tag{7}$$

There is an optimal allocation and it is history-independent. Moreover, $0 < \beta_l < \beta_h < 1$ *and the optimal allocation is as follows*; *if* $\beta \leqslant \beta_l$, *then* $y_l = y_h \leqslant y_l^*$ *and* $q_l = q_h = 1$; *if* $\beta \in (\beta_l, \beta_h)$, *then* $y_l = y_l^*$, $y_h < y_h^*$, $q_l < 1$ *and* $q_h = 1$; *and finally if* $\beta \geqslant \beta_h$, *then* $y_l = y_l^*$ *and* $y_h = y_h^*$.

This differs from CE for intermediate magnitudes for β; state-l output is kept first-best and lotteries are necessary.[7] Moreover, one can see in the proof that the optimum can be supported by buyer take-it-or-leave-it offers. Hence, the trades are not only IR, but also coalition-proof for the pairs in meetings.

4. Proof of the proposition

The proof proceeds as follows. First, an upper bound on $R(s^t)$ (see (5)) is established. (The candidate for the upper bound, which depends on β, is provided in Lemma 1. Then, Lemma 2 shows that the candidate is, in fact, an upper bound.) Then, the proposition is proved by constructing the optimum in terms of that upper bound.

Lemma 1. *Let*

$$g(R;\beta) \equiv \beta R + \frac{1-m}{N}\Big[\pi_l \max_{0\leqslant q_l\leqslant 1} H_l(q_l\beta R) + \pi_h \max_{0\leqslant q_h\leqslant 1} H_h(q_h\beta R)\Big],$$

where $H_s(x) \equiv u_s(x) - x$. *The function* $g(\cdot;\beta)$ *has a unique positive fixed point, denoted* $\bar{R}(\beta)$. *Moreover,* $\beta\bar{R}(\beta)$ *is strictly increasing with* $\beta_l\bar{R}(\beta_l) = y_l^*$ *and* $\beta_h\bar{R}(\beta_h) = y_h^*$, *which implies* $\beta_l < \beta_h$.

Proof. Note that

$$\arg\max_{q_s\in[0,1]} H_s(q_s\beta R) = \begin{cases} y_s^*/\beta R & \text{if } \beta R \geqslant y_s^*, \\ 1 & \text{otherwise.} \end{cases}$$

It follows that

$$g(R;\beta) = \begin{cases} \beta R + \frac{1-m}{N}[\pi_l H_l(\beta R) + \pi_h H_h(\beta R)] & \text{if } \beta R \leqslant y_l^*, \\ \beta R + \frac{1-m}{N}[\pi_l H_l(y_l^*) + \pi_h H_h(\beta R)] & \text{if } \beta R \in [y_l^*, y_h^*], \\ \beta R + \frac{1-m}{N}[\pi_l H_l(y_l^*) + \pi_h H_h(y_h^*)] & \text{if } \beta R \geqslant y_h^*. \end{cases} \tag{8}$$

Therefore, $g(R;\beta)$ is continuous and strictly increasing in R. Moreover, it follows by direct computation that $\partial g(R;\beta)/\partial R$ exists, is infinite at $R = 0$, is weakly decreasing in R, and that

[7] It is not hard to show that in the model without aggregate shocks, optima can be attained without the use of lotteries. In this sense, the CE model is a simple monetary model in which lotteries are necessary.

$\partial g(R;\beta)/\partial R = \beta$ for $R \geqslant y_h^*/\beta$. Then, $g(0;\beta) = 0$ implies that there is a unique $R > 0$, denoted $\bar{R}(\beta)$, such that $R = g(R;\beta)$. Also, because $g(R;\beta)$ is strictly increasing in β for $R > 0$, it follows that $\bar{R}(\beta)$ is strictly increasing in β. Finally, continuity of $\bar{R}(\beta)$ follows from the implicit function theorem.

Now consider the equations, $\beta\bar{R}(\beta) = y_s^*$. It follows from the above characterization of g that $\beta\bar{R}(\beta) \to \infty$ as $\beta \to 1$. That, $\beta\bar{R}(\beta) = 0$ at $\beta = 0$, and continuity of $\beta\bar{R}(\beta)$ imply existence of a solution. Also, monotonicity of $\beta\bar{R}(\beta)$ implies that the solution is unique and increasing in y_s^*.

Finally, the closed-form expressions for the β_s are obtained by solving the equations $y_s^*/\beta = g(y_s^*/\beta;\beta)$ for β. In particular, by (8), for $s = l$, that equation is

$$y_l^*/\beta = y_l^* + \frac{1-m}{N}\bigl[\pi_l H_l\bigl(y_l^*\bigr) + \pi_h H_h\bigl(y_l^*\bigr)\bigr],$$

while for $s = h$, it is

$$y_h^*/\beta = y_h^* + \frac{1-m}{N}\bigl[\pi_l H_l\bigl(y_l^*\bigr) + \pi_h H_h\bigl(y_h^*\bigr)\bigr]. \qquad \square$$

Lemma 2. *If $\{y(s^t), q(s^t)\}_{s^t}$ is implementable, then $R(s^t) \leqslant \bar{R}(\beta)$ for all s^t.*

Proof. For any s^{t-1} and s_t,

$$\begin{aligned} r\bigl(s^{t-1}, s_t\bigr) &= \frac{(1-m)u_{s_t}(y(s^t)) + my(s^t)}{N} + \left(1 - \frac{q(s^t)}{N}\right)\beta R\bigl(s^t\bigr) \\ &\leqslant \frac{(1-m)u_{s_t}(q(s^t)\beta R(s^t)) + mq(s^t)\beta R(s^t)}{N} + \left(1 - \frac{q(s^t)}{N}\right)\beta R\bigl(s^t\bigr) \\ &= \beta R\bigl(s^t\bigr) + \frac{1-m}{N}\bigl[u_{s_t}\bigl(q\bigl(s^t\bigr)\beta R\bigl(s^t\bigr)\bigr) - q\bigl(s^t\bigr)\beta R\bigl(s^t\bigr)\bigr] \\ &\leqslant \beta R\bigl(s^t\bigr) + \frac{1-m}{N}\max_{0\leqslant q\leqslant 1}\bigl[u_{s_t}\bigl(q\beta R\bigl(s^t\bigr)\bigr) - q\beta R\bigl(s^t\bigr)\bigr] \\ &\equiv g_s\bigl(R\bigl(s^t\bigr)\bigr), \end{aligned} \tag{9}$$

where the first equality follows from the definition of $r(s^t)$ (see (3) and (2)), and the first inequality from the first inequality in (4), the producer IR constraint. Hence, we have

$$\begin{aligned} R\bigl(s^{t-1}\bigr) &= \pi_l r\bigl(s^{t-1}, l\bigr) + \pi_h r\bigl(s^{t-1}, h\bigr) \\ &\leqslant \pi_l g_l\bigl(R\bigl(s^{t-1}, l\bigr)\bigr) + \pi_h g_h\bigl(R\bigl(s^{t-1}, h\bigr)\bigr) \\ &\leqslant g\bigl(\max\bigl\{R\bigl(s^{t-1}, l\bigr), R\bigl(s^{t-1}, h\bigr)\bigr\}\bigr), \end{aligned}$$

where the first inequality follows from (9) and the second inequality because g_s is increasing. Therefore,

$$R\bigl(s^{t-1}\bigr) \leqslant g\bigl(R\bigl(s^{t-1}, s_t\bigr)\bigr) \quad \text{for either } s_t = l \text{ or } s_t = h. \tag{10}$$

Now, suppose, by way of contradiction, that $R(s^t) > \bar{R}(\beta)$ for some s^t. Then, by (10), there exists s^{t+1} such that $R(s^{t+1}) \geqslant f(R(s^t))$, where $f = g^{-1}$. Because f is increasing, by induction there exists a continuation of s^t such that $R(s^{t+n}) \geqslant f^{(n)}(R(s^t))$ for all n. Moreover, it follows from the properties of g that $f(R(s^t)) > R(s^t)$ and that f is convex. Therefore, the sequence $R(s^{t+n})$ is unbounded, which violates the definition of implementability. $\square$

Proof of Proposition 1. We consider, in turn, three exhaustive cases.

Case 1: $\beta \leqslant \beta_l$.

Consider the allocation $(y_s, q_s) = (\beta\bar{R}(\beta), 1)$, $s = l, h$. By construction, this satisfies the first inequality in (4). Also, $\beta\bar{R}(\beta) \leqslant y_l^* < y_h^*$ (see Lemma 1) implies $u_s(y_s) = u_s(\beta\bar{R}(\beta)) \geqslant \beta\bar{R}(\beta) = \beta\bar{R}(\beta)q_s$. Therefore, the second inequality in (4) is also satisfied. Hence, this allocation is implementable.

Now because $\beta\bar{R}(\beta) \leqslant y_l^*$ and $u_s(y) - y$ is increasing in y for $y \in [0, y_l^*]$, any better allocation must have higher production after some history. However, the bound on $R(s^t)$ and $q_s = 1$ implies that higher production violates the first inequality in (4).

Case 2: $\beta_l < \beta < \beta_h$.

Consider the allocation $(y_h, q_h) = (\beta\bar{R}(\beta), 1)$ and $(y_l, q_l) = (y_l^*, \frac{y_l^*}{\beta\bar{R}(\beta)})$, where $y_l^* < \beta\bar{R}(\beta) < y_h^*$ (see Lemma 1) guarantees $q_l < 1$. By construction, this satisfies the first inequality of (4). Also, $u_l(y_l) = u_l(y_l^*) \geqslant y_l^* = \beta\bar{R}(\beta)q_l$, and $\beta\bar{R}(\beta) \leqslant y_h^*$ implies $u_h(y_h) = u_h(\beta\bar{R}(\beta)) \geqslant \beta\bar{R}(\beta) = \beta\bar{R}(\beta)q_h$. Therefore, the second inequality of (4) is also satisfied. Hence, the allocation is implementable.

Now because $\beta\bar{R}(\beta) < y_h^*$ and $u_h(y) - y$ is increasing in y for $y \in [0, y_h^*]$, any better allocation must have higher production after some history s^t with $s_t = h$. (After histories with $s_t = l$, $y(s^t) = y_l^*$, so there is no room for improvement.) However, the bound on $R(s^t)$ and $q_h = 1$ implies that higher production violates the first inequality in (4).

Case 3: $\beta_h \leqslant \beta$.

Consider the allocation $(y_s, q_s) = (y_s^*, \frac{y_s^*}{\beta\bar{R}(\beta)})$, $s = l, h$, where $y_l^* < y_h^* \leqslant \beta\bar{R}(\beta)$ (see Lemma 1) guarantees $q_s \leqslant 1$. By construction, this satisfies the first inequality of (4). Also, $u_s(y_s) = u_s(y_s^*) \geqslant y_s^* = \beta\bar{R}(\beta)q_s$ implies that the second inequality of (4) is satisfied. Hence, the allocation is implementable. It is optimal because it is first-best.[8] □

5. The optimal choice of *m*

Given the result that the optimal allocation is history-independent, we now consider the optimal choice of m.[9] For that purpose, we now express β_s in (6)–(7) and $\bar{R}(\beta)$ in Lemma 1 as $\beta_s(m)$ and $\bar{R}(m, \beta)$, respectively, to make explicit their dependence on m. Suppose that the planner chooses m before the initial shock s_0 is realized. The planner maximizes the product $E(m) \cdot I(m, \beta)$, where $E(m) \equiv m(1 - m)/N$, is the frequency of *trade* meetings, and

$$I(m,\beta) \equiv \begin{cases} \frac{1}{1-\beta}\{\pi_l H_l(\beta\bar{R}(m,\beta)) + \pi_h H_h(\beta\bar{R}(m,\beta))\} & \text{if } \beta < \beta_l(m), \\ \frac{1}{1-\beta}\{\pi_l H_l(y_l^*) + \pi_h H_h(\beta\bar{R}(m,\beta))\} & \text{if } \beta_l(m) \leqslant \beta < \beta_h(m), \\ \frac{1}{1-\beta}\{\pi_l H_l(y_l^*) + \pi_h H_h(y_h^*)\} & \text{if } \beta_h(m) \leqslant \beta. \end{cases}$$

[8] In this range, the outputs are unique but q's are not. This is similar to what happens in Trejos and Wright for high discount factor. Here, (q_l, q_h) is chosen to maximize $R(s^t)$, which is equivalent to buyer take-it-or-leave-it offers.

[9] Similar discussions are found in previous models without aggregate shock and lotteries: in Trejos and Wright [4], where consumer and producer have a specific Nash bargaining with equal bargaining powers, and in Cavalcanti and Wallace [3], where the planner chooses the optimal allocation.

$E(m)$ is increasing for $m < 0.5$, a maximum at $m = 0.5$, and decreasing for $m > 0.5$, while $I(m, \beta)$ is decreasing in m, because the cutoff values $\beta_s(m)$ are increasing and the maximum return $\bar{R}(m, \beta)$ is strictly decreasing in m.

One immediate result is that if $\beta \geqslant \beta_h(0.5)$, then the unique optimal quantity is 0.5. Otherwise, the optimal quantity is less than 0.5, as can be seen from following first-order condition, a necessary condition for the optimal m:

$$\begin{aligned} 0 &= \frac{\partial E}{\partial m} \cdot I(m) + E(m) \cdot \frac{\partial I}{\partial m} \\ &= \frac{1-2m}{N} \cdot I(m, \beta) + \frac{m(1-m)}{N} \cdot \frac{\partial I}{\partial m}. \end{aligned} \tag{11}$$

The first term, the 'extensive margin effect,' is zero at $m = 0.5$, while the second term, the 'intensive margin effect,' is negative at $m = 0.5$, because $\partial I/\partial m|_{m=0.5} < 0$ due to $\beta < \beta_h(0.5)$ and $\partial \bar{R}/\partial m < 0$.

6. Extension to more than two states

The extension of our results to the case of more than two states is straightforward. Let the support of the preference shock s be $\{1, 2, \ldots, d\}$, where $y_1^* < \cdots < y_d^*$. Then, let

$$\frac{1}{\beta_s} \equiv 1 + \frac{1-m}{N} \cdot \frac{\sum_{i \leqslant s} \pi_i H_i(y_i^*) + \sum_{i \geqslant s+1} \pi_i H_i(y_s^*)}{y_s^*}$$

for $s = 1, \ldots, d$. The candidate for the optimal allocation is as follows.

$$\begin{aligned} &\text{If } \beta \in (0, \beta_1] \quad \text{then } (y_i, q_i) = (\beta\bar{R}, 1), \quad i = 1, \ldots, d; \\ &\text{if } \beta \in [\beta_s, \beta_{s+1}] \quad \text{then } (y_i, q_i) = \begin{cases} (y_i^*, \frac{y_i^*}{\beta\bar{R}}), & i = 1, \ldots, s, \\ (\beta\bar{R}, 1), & i = s+1, \ldots, d; \end{cases} \\ &\text{if } \beta \in [\beta_d, 1) \quad \text{then } (y_i, q_i) = \left(y_i^*, \frac{y_i^*}{\beta\bar{R}}\right), \; i = 1, \ldots, d, \end{aligned}$$

where $\bar{R} = \bar{R}(\beta)$ is the unique positive solution to $R = g(R; \beta)$ and

$$g(R; \beta) \equiv \beta R + \frac{1-m}{N} \sum_{s=1}^{d} \pi_s \max_{0 \leqslant q_s \leqslant 1} H_s(q_s \beta R).$$

The proof is essentially the same as that for two states.

References

[1] A. Berentsen, M. Molico, R. Wright, Indivisibilities, lotteries, and monetary exchange, J. Econ. Theory 107 (2002) 70–94.

[2] R. Cavalcanti, A. Erosa, Efficient propagation of shocks and the optimal return on money, J. Econ. Theory 142 (2008) 128–148.

[3] R. Cavalcanti, N. Wallace, A model of private bank-note issue, Rev. Econ. Dynam. 2 (1999) 104–136.

[4] A. Trejos, R. Wright, Search, bargaining, money and prices, J. Polit. Economy 103 (1995) 118–141.