Okun's Law as a Pi-to-1 ratio: A harmonic / trigonometric theory as to why Okun's Law works

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Okun's Law as a π-to-1 Ratio:
A Harmonic / Trigonometric Theory as to Why Okun's Law Works

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Abstract: *"Okun's Law" states a 3:1 proportion between percent growth in U. S. real GNP and percent decrease in the rate of unemployment. This paper argues that this ratio is actually a π:1 proportion, heretofore unrecognized because it is displayed through a form of mathematic / harmonic inverse.*

In Part One the Cartesian coordinate system is merged with the legal doctrines of actus reus (x-axis, actions) and mens rea (y-axis, thoughts). A unit circle of personal choice – including economic choice (trading vs. keeping) – may thereby be devised. This unit circle is then aggregated into a torus, half the circumference of which represents U.S. real GNP (π), the antipodal half-circumference its monetary value (π) and the radius the rate of employment necessary to its production (R = 1). Mainstream econometric analysis appears to support this theory of inverses with proximities of within 1.3%, 1.0%, 0.35%, 0.00105% and less than half a degree.

In Part Two this model of Okun's Law is connected closely to an analysis of the well-known Kondratiev Wave, a 56-year "Long Wave" of evolving social and economic relationships. This approach to macroeconomics is thereby aligned with a geometric, harmonic and trigonometric analysis of empirical data, rather than purely statistical methods. [1],[2]

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[1] Acknowledgements. This article is a sequel, a "Part Two," following upon a previous peer-reviewed article entitled "On the mathematic prediction of economic and social crises: towards a harmonic interpretation of the Kondratiev Wave," *Entelequia: Revista Interdisciplinar*, University of Malaga, Malaga, Spain, Issue 15, April, 2013, pp. 37-124, http://www.eumed.net/entelequia/en.art.php?a=15a02. Special thanks go to Dr. Arno Tausch, editor of this issue "Globalization of World System Research," http://www.eumed.net/entelequia/en.ant.php?a=15, for including this article in the monograph.

This second and subsequent paper is a reply to Dr. Edward Knotek's rhetorical question "How Useful is Okun's Law?" (*Economic Review* 2007) http://www.kc.frb.org/publicat/econrev/PDF/4q07Knotek.pdf made possible only because Dr. Knotek has been so generous with his time, information, insights and explanations vis-à-vis that article.

[2] This article contains 12,693 words, with an abstract of 200 words, paginated as a two page pdf view, odd numbered pages to the left, even numbered pages to the right. A majority of the diagrams used in this essay are taken from five working papers which develop "the Political Economy Wave" entitled "Predicting crises: Five essays on the mathematic prediction of economic and social crises," http://mpra.ub.uni-muenchen.de/43484/ . These diagrams are left in their original format and numbering scheme for easy reference to that set of papers, i.e. "5-3" or "2-4" etc. New diagrams introduced in this essay post-Introduction in Parts One and Two are without dashes, i.e. "Diagram 1." Early drafts of this paper are posted at the Munich Personal Repository, Papers No. 44843 and No. 44594. See also http://www.scribd.com/scott_albers_1 for additional related works adapting the ideas herein to a variety of fields.

JEL Classification: A12, A13, B00, B4, B59, C1, C19, C50, E1, E10, E19, E3, P16, Z0, Z00

Key words: Okun's Law, Kondratiev wave, unemployment, GNP growth, Long wave, trigonometric analysis, unit circle, Okun's coefficient, steady-state rate of growth, pi, phi, the Golden Mean, harmonic analysis

Introduction: Okun's Law

"Okun's Law" is an economic empirical regularity[3] which notes that – in the United States[4] – for every three percentage points of increase in real GNP the rate of employment increases by one percentage point, and that decreases of both take place at the same rate. This 3 : 1 proportion is generally referred to using a double negative, i.e. an increase of three percent in real GNP will lead to a one percent *decrease* in the rate of *un*-employment. Although first stated by Arthur Okun, at the time senior economist of President Kennedy's Council of Economic Advisors, "Okun's Law" has taken on a legend of its own, as "one of the most reliable empirical regularities in macroeconomics." (Tobin, 1983)

This 3:1 ratio was first presented in a 1962 paper by Arthur M. Okun entitled "Potential GNP: Its Measurement and Significance." The paper opens with the question: "How much output can the economy produce under conditions of full employment?" Dr. Okun writes:

> The basic technique I am reporting consists of a leap from the unemployment rate to potential output rather than a series of steps involving the several underlying factors (which might impact on potential output). Strictly speaking, the leap requires the assumption that, *whatever the influence of slack economic activity on average hours, labor force participation, and manhour productivity, the magnitudes of all these effects are related to the unemployment rate*. With this assumption, the unemployment rate can be viewed as a proxy variable for all the ways in which output is affected by idle resources. The measurement of potential output then is simplified into an estimate of how much output is depressed by unemployment in excess of four percent.

[3] The notion of an "empirical regularity" in economics is undefined in the literature. The notion of a *statistical* regularity is described by the observation that, although the throw of dice is indeterminable on a single throw, over many repeated throws various statistical regularities are observed. The notion that a π:1 ratio is to be found in the econometric data of the United States is at odds with the notion of a statistical regularity. As William Feller noted: "There is no place in our system for speculations concerning the probability that the sun will rise tomorrow. Before speaking of it we should have to agree on an (idealized) model which would presumably run along the lines 'out of infinitely many worlds one is selected at random...' Little imagination is required to construct such a model, but it appears both uninteresting and meaningless."

The phrase "stylized fact" is also used, defined as: "Something that has been observed to be true, or close to true, sufficiently often and in enough different contexts that an economic theory should be consistent with it. Those who present a set of stylized facts typically do not attempt to support them with data, but simply list them so as to motivate their theoretical analysis." Dictionary Central.com. See e.g. Heine, et al. (2005), "Stylized Facts and the Contribution of Simulation to the Economic Analysis of Budgeting."

[4] I undertake an economic analysis of the United States alone on the expectation that if a harmonic / trigonometric theory for Okun's Law can be derived for this largest and most dynamic economy a similar analysis of other countries may more readily be considered. Regional and cross-cultural evaluations of Okun's coefficient are available, see e.g. Moosa (1997); Lee (2000); Freeman (2000); Sogner and Stiassny (2002); Kennedy (2009); Oberst and Oelgemoller, (2013).

Dr. Okun then states his law.

> The answer I have to offer is simple and direct. In the postwar period, on the average, each extra percentage point in the unemployment rate above four percent has been associated with about a three percent decrement in real GNP.
>
> ... My own subjectively weighted average of the relevant coefficients is 3.2, yielding the following estimate of potential:

$$P = A(1 + .032 (U-4))$$

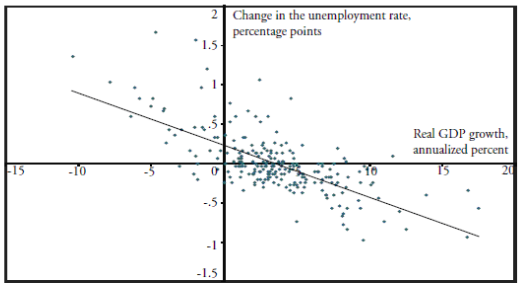
Recent Scholarship

Dr. Edward Knotek's article "How Useful Is Okun's Law?" (2007) proposes that Okun's Law is, at best, a helpful rule of thumb. As the title of the article suggests directly, Dr. Knotek describes in detail our present understanding of Okun's Law as both a mathematic equation and as a policy tool.

To make the point of his article Dr. Knotek organizes data sets which follow mainstream econometric methods as applied to well-known and easily available federal data bases covering a 60 year period of American economic history, i.e. the second quarter of 1947 through the third quarter of 2007. Charts One and Two graph the quarterly and annual data sets supporting the regularity of the relationship between changes in the size of real GNP (x-axis) and the corresponding effect this has on the rate of employment (y-axis).

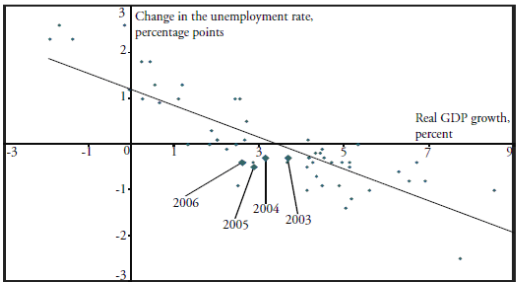
INTRODUCTORY DIAGRAM 1.
CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"

Chart 1
THE DIFFERENCE VERSION OF OKUN'S LAW, QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2
THE DIFFERENCE VERSION OF OKUN'S LAW, ANNUAL DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

The significance of these trend lines is developed throughout this essay. [5]

[5] The trend line for annualized quarterly data in Chart One as originally provided by Dr. Knotek (June 24, 2011) is y = .23094 + -0.066036x, giving a steady-state rate of growth (x-intercept) of 3.4971853. This rate was virtually identical to the 3.4969781 rate calculated at the time under this program of research. These rates will be used throughout this essay but see the Appendix for alternative trend lines and measurements.

Dr. Knotek then takes issue with the straight-forward association presented above between the rate of growth and the rate of unemployment. He makes the point that the historic associations underlying these trend lines merit much closer scrutiny. We will turn to this insight, illustrated by Chart Three below, near the end of this paper.

INTRODUCTORY DIAGRAM 2.
CHART THREE OF "HOW USEFUL IS OKUN'S LAW?"

Notes: Dates along the horizontal axis denote the last quarter in the sample period for each rolling regression. Each sample period is 13 years long.

Despite the issues brought forward by Dr. Knotek,[6] Okun's Law remains one of the most well-known and central findings of macro-economics.[7,8,9] Surprisingly, there is at present no theoretic structure,[10] or even agreed upon *ansatz*, [11],[12],[13],[14] to explain the apparently long-standing and vital 3:1 macroeconomic / mathematic relationship given by Okun's Law.

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[6] See, in accord, e.g. Meyer and Tasci (2012), "An Unstable Okun's Law, Not the Best Rule of Thumb."

[7] On the significance of Okun's law see also Freeman, 2000: "(M)easured correctly Okun's relationship continues to be perhaps the closest thing to a law that macroeconomics has." See also Blinder, 1997: "Is There a Core of Practical Macroeconomics that We Should All Believe? With emphasis on the adjective "practical" and the normative "should," my answer to the question of this session is a resounding yes. ... (Here) I will describe briefly the main practical elements that I think we should agree on, without worrying too much about their theoretical underpinnings ... Okun's Law. The other truly sturdy empirical regularity, Okun's Law, is even more atheoretial, if not indeed antitheoretical. This simple linear relationship between the percentage change in output and the absolute change in the unemployment rate presumably embodies productivity, labor-force participation and production-function considerations. On the surface, it seems to contradict the concavity of the latter. Nonetheless, it closes the loop between real output growth and changes in unemployment with stunning reliability."

[8] As to the current perception of the reliability of Okun's Law see a recent working paper published by the International Monetary Fund, Ball, et al., 2012: "Our principal conclusion is that Okun's Law is a strong and stable relationship in most countries. Deviations from Okun's Law occur, but they are usually modest in size and short-lived. Overall, the data are consistent with traditional models in which fluctuations in unemployment are caused by shifts in aggregate demand. There is one important qualification to the universality of Okun's Law. While a stable Law fits the data for most countries, the coefficient in the relationship - percent change in output on the unemployment rate - varies across countries. We estimate, for example, that the coefficient is –0.15 in Japan, –0.45 in the United States, and –0.85 in Spain. These differences reflect special features of national labor markets, such as Japan's tradition of lifetime employment and the prevalence of temporary employment contracts in Spain."

[9] The significance of Okun's Law may be considered in light of the central role "potential output" plays in many macroeconomic models. See Moosa 1997: 335-336. "Economists are interested in (Okun's law) not only because it appears to be a robust empirical regularity, but also due to its theoretical importance because the aggregate supply curve is derived by combining it with the Phillips curve. Moreover, the relationship has important implications for macroeconomic policy, particularly in determining the optimal or desirable growth rate."

This essay proposes that a steady-state rate of growth of 3.4969% per year applies to the United States, both as a matter of our analysis as well as the annualized quarterly data for Okun's Law. (Knotek 2007)

[10] See e.g. Kennedy, 2009:3. "Given the weak theoretical understanding of why the coefficient is greater than one, and has been so stable, it is somewhat surprising that a larger literature has not been produced specifically on this variable. For every one article on Okun's coefficient, there are five on the Non-Accelerating Inflation Rate of Unemployment (NAIRU), or twenty on the Phillips curve. Yet Okun's coefficient is as firmly embedded in practical macroeconomics as any other empirical relationship." A good review of the current literature is provided in this dissertation, pages 4-28.

[11] It is important to note at the outset that 'potential output' is connected quite closely to the pricing scheme, at least in concept. As pointed out in Congdon (2008): "The analysis (by Okun) proposed a concept of 'the GNP gap', which was obtained by distinguishing between potential and actual GNP. In his words, 'Potential GNP is a supply concept, a measure of productive capacity.' Nevertheless, '... it is not a measure of how much output could be generated by unlimited amounts of aggregate demand... The full employment goal must be understood as striving for maximum production without inflationary pressure.' Potential output differs from actual output because aggregate demand may not be sufficient to deliver full employment. It follows that 'If, in fact, aggregate demand is lower, part of potential GNP is not produced; there is unrealized potential or a 'gap' between actual and potential.'"

[12] As to challenges to the notion of "potential GDP" see the report of the Congressional Budget Office (2004), "A Summary of Alternative Methods for Estimating Potential GDP." "A spectrum of opinion exists among economists about the usefulness of measures of potential GDP for monetary and fiscal policy and for economic projections. Some economists do not think that the idea of potential output is useful, arguing that: (1) The concept is based on a flawed view of the causes of inflation, even in the short run. According to this argument, inflation is determined by growth in the money supply, not by where the economy is in the business cycle. (2) Potential GDP is so unstable and varies so much that it is impossible to estimate accurately, especially for recent years, and thus is not a helpful guide for policymaking or forecasting. (3) Policies to manage demand generally do more harm than good

On this point see Owyang and Sekhposyan (2012):

> Many macroeconomic textbooks contain a rule of thumb relating real output growth to changes in the unemployment rate. This relationship, called Okun's law after Okun (1962) typically assigns a 2- to 3- percentage point decrease in real gross domestic product (GDP) growth to a 1-percentage point increase in the unemployment rate. Unlike laws in the physical sciences (e.g. Newton's laws of motion) Okun's law is an (arguably loose) empirical correlation and is, in general, neither theoretically motivated[15] nor strictly adhered to in the data. As many of the reduced-form relationships build strictly on associations and not causation, Okun's law appears to vary depending on the sample period studied.[16]

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because of lags, uncertainties and political pressures. Hence, the size of the gap between actual and potential output ought to be irrelevant to policymakers. ... In CBO's view, the value of potential GDP is not restricted to short term fiscal and monetary policy. Potential output calculated with a growth model is a useful concept for gauging the economy's productive capacity and offers the best basis for projecting GDP over the 10-year horizon required by the budget process. Carefully estimated, potential GDP can provide the user with a reasonable sense of the economy's potential for growth."

[13] On the validity of the notion of "potential GNP" see also Penson and Webb (1981), "Gross National Product at Full Employment." "Some economists have questioned the meaning and usefulness of the concept of potential GNP because it explicitly ignores demand. Plosser and Schwert, for example, argue that potential GNP has little operational significance because: 'It is not an equilibrium concept, since there is no relationship with aggregate demand. Consequently 'potential GNP' cannot be viewed as representing the level of output which would prevail in the absence of any unexpected random shocks to aggregate supply or demand.' Gordon also faults estimates of potential GNP – a term he says has been discredited and is obsolete – because they do not explicitly relate to the behavior of wages and prices. This raises a question as to whether policy makers can realistically expect to see the output levels suggested by estimates of potential GNP if they adopt policies to fully employ available resources, since these estimates explicitly ignore the economic factors influencing producers' and consumers' decisions."

[14] A good example of the relationship which Okun's Law has been perceived to play vis-à-vis prices is found in Holloway (1989). "The Okun's Law equation and estimates of potential GNP derived from it have some significant implications for public policy. Currently, the most important one concerns potential inflationary pressures. The level of potential GNP addresses the noninflationary capacity constraints of the economy. As the economy tests those constraints, accelerating inflation is the likely consequence. The growth rate of potential GNP relative to the actual growth rate determines whether the economy is moving toward its productive capacity or is not growing as rapidly as the economy is capable of adding to capacity. In recent years, very rapid growth in actual GNP has greatly exceeded the rate of growth in potential GNP of 2.25 percent estimated in this note. As a consequence the unemployment rate has fallen sharply and the economy has moved ever closer to its productive capacity. Partially in response to this strength, the Federal Reserve initiated credit tightening moves several times in 1987 and 1988 out of fears of the likelihood of accelerating inflation. ..."

[15] Okun's Law is studied from a number of perspectives including (1) the difference version, (2) the gap version, (3) the dynamic version, and (4) production-function versions. (Knotek 2007) See Condon (2008), "Two Concepts of the Output Gap," for a thorough review of the philosophic and theoretical differences surrounding the "gap" approach to Okun's Law. "(O)ne concept of the gap was first advanced by Arthur Okun in 1962 and may be termed 'Keynesian', whereas the alternative concept stems from Milton Friedman's presidential address to the American Economic Association in 1967 and may be regarded as 'monetarist'. The argument here will be that over time the monetarist concept of the gap has ousted the Keynesian and that the consequent refurbishment of economists' understanding of the 'gap' notion has made a vital contribution to the so-called 'Great Moderation'.

[16] I propose herein that eight 7-year periods, in a recurring circuit of 56-years, are an essential part of calculating Okun's Law, and that the slopes of these periods are responsible collectively for the maintenance of the proportions of Okun's Law over the long term. (see Part Two)

The Need for a Theory

Three approaches attempt to provide a theoretic explanation for Okun's Law, but with limited success in deciphering the stability or ratio of "Okun's coefficient" over time. Briefly, (1) Prachowny's approach takes Okun's Law from the view of production, (2) Adachi's approach considers Okun's Law as the outcome of economic growth and (3) Lang and de Perretti's approach considers Okun's Law from the point of view of historic "hysteretic" development.

(1) In his 1993 paper "Okun's Law: Theoretical Foundations and Revised Estimates" Dr. Martin Prachowny uses production functions to derive Okun's Law.

> The relationship between unemployment changes and output must be derived from a production function for the economy as well as from ancillary relationship in the labor market. In natural logs, the production function is written as
>
> $$y = \alpha(k+c) + \beta(\gamma n + \delta h) + \tau$$
>
> where y is output, k is the capital input and c is its utilization rate, n represents the number of workers, h is the number of hours that they work; α and β are output elasticites and γ and δ are the contributions of workers and weekly hours to the total labor input; finally τ is a disembodied technology factor. Various constraints can be put on the elasticites...
>
> As to the actual 3:1 ratio he states:
>
> Okun's coefficient of three is derived from a complicated weighted sum of all other changes.
>
> He concludes:
>
> Arthur Okun's insights into the relationship between the demand for labor and the supply of output are every bit as important as the Phillips curve in understanding the Aggregate Supply curve for any macro-economy. However empirical work on Okuns Law seems not to have progressed very far beyond Okun's original estimates. This paper has attempt to remedy this unfortunate neglect, not by generating one more parameter value for Okun's original coefficient, but by focusing on the underlying production function that connects labor input as well as other facts of production to output of goods and services. In the process, it has been determined that changes in weekly hours and movements in capacity utilization, in addition to adjustments in the unemployment, are significant influences on the changes in the output gap. The next step in the analysis is to examine whether changes in hours or capacity utilization have the have inflationary effects as changes in unemployment.[17]

[17] See Attfield and Silverstone (1997), "Okun's Coefficient: A Comment," for a complete re-evaluation of Prachowny's approach. "(T)here is no evidence that any of the relations (suggested by Prachowny) are co-

(2) In his 2007 paper "Economic Growth and Unemployment – Theoretical Foundations of Okun's Law" Dr. Hideyuki Adachi suggests that Okun's Law may be derived from the Solow growth model. (Adachi, 2007). Dr. Adachi distinguishes his effort to tie Okun's Law to Solow's growth equations from Prachowny's production functions as follows.

> Prachowny (1993) attempts to provide theoretical foundations of Okun's Law by deriving the relationship between unemployment changes and output from a production function for the economy and ancillary relationship in the labor market. Quite differently from his approach, this paper attempts to provide theoretical foundation to Okun's Law by integrating it with growth theory.

He states:

> So far, Okun's Law remains to be an empirical observation rather than a result derived from theory. Moreover, this quantitative relationship varies depending on the countries and time periods under consideration. To identify what factors cause these differences, the theory that explains this empirical law is required. ... As far as I know... there is no literature that gives theoretical explanations of this law. [18] ...

He comments upon the need to deal with growth as a fundamental basis for the reliability of Okun's Law.

> Solow (2000) ... mentions the need to develop the medium-run macroeconomic theory that explains medium-run departure from the steady growth. For this purpose, he suggests the idea of using Okun's Law in growth theory, saying "what is wanted is an integration of Okun's Law and growth models, so that the events of the business cycle are directly linked to the evolutions of the growth path. This is not only useful for growth theory, but also for Okun's Law, because Okun's Law might be improved by this marriage, too."

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integrated, and therefore we can draw no conclusion from the results. ...What we can conclude from this analysis is that the value of the Okun's coefficient for the United States is around -2.25 using Gordon's output-unemployment data from 1967 to 1986. This finding, rather than Prachowney's estimated value for around -0.67, supports previous research."

[18] In light of the theoretic work by Adachi, it would appear that Freeman's (2000) reference to a theoretical basis for Okun's Law is premature. He states: "Since (1962) a number of papers have established theoretical foundations for Okun's Law (Clark, 1983; Gordon, 1984; Prachowny, 1993) and tested the stability of the 3:1 trade-off (Clark, 1983; Gordon, 1984; Adams and Coe, 1989; Holloway, 1989; Prachowny, 1993; Attfield and Silverstone, 1998). In general, Okun's Law has withstood most challenges, although current estimates of the trade-off fall into a range closer to 2:1 than 3:1 (Gordon, 1984; Attfield and Silverstone, 1998; Moosa, 1997) and vary according to the methods and specifications used. Variations notwithstanding, the stability of Okun's Law contrasts favorably with the Phillips curve, its counterpart in the unemployment-inflation space."

Freeman notes the uncertain basis for Okun's Law even as Okun himself interpreted it. "Writing almost two decades later, however, and shortly before his death, Okun (1981, p. 228) himself doubted his law's stability: 'During the late seventies, the three-to-one ratio (on real GNP to the unemployment rate) no longer approximated reality. If employers encounter an unusually deep recession and expect the subsequent period of slack to be especially long lasting, they are likely to cut back employment more nearly in proportion to the decline in output.' "

Dr. Adachi derives the following equation as the basis for Okun's Law,

$$\frac{\dot{Y}}{Y} = (\alpha + \lambda) - \frac{1}{1-u}(1+\varepsilon\sigma(n))\dot{u}$$

where Y is output; α is labor augmenting the technological progress, assumed to be proceeding at a constant rate; λ is the constant rate of growth in the labor population; u is the rate of unemployment; ε represents the sensitivity of the real wage rate to the tightness of the labor market; $\sigma(n)$ is the elasticity of substitution between labor and capital and n is the efficiency per unit of capital. He notes:

> As this equation shows, the rate of growth of output in the case of (u "dot") = 0 is equal to $\alpha+\lambda$, which is the steady growth rate of the model. The coefficient for (u "dot"), which is called Okun's coefficient, is equal to $(1+ \varepsilon(n))/(1-u)$. Since this value depends on u, Okun's coefficient in theory is not constant. However, this value will presumably change little within the relevant range of u.

Ultimately Dr. Adachi's finds his effort to be of limited success. He states:

> It is shown that the substantial difference of Okun's coefficient between the two countries (Japan = 3.38 and the United States = 0.39) may be attributed at least partly to the difference in the elasticity of the real wage rate the unemployment rate, i.e. the real wage flexibility.[19]
>
> However these two parameters are not enough to explain fully the size of Okun's coefficient. I consider it important to introduce the utilization of labor and capital into the model to achieve more perfect marriage of Okun's Law with growth theory. I plan to discuss about this attempt in another paper.[20]

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[19] The conclusion reached in this paper is that growth in the United States (x-axis intercept) runs on a 56-year cycle, and that unemployment (y-axis intercept) runs on a 14-year cycle. In consequence the slope of Okun's Law varies over time, but continually dances around a steady π:1 ratio as exhibited in the data. Regarding the effort to quantify Okun's coefficient itself see Weber (1995), "Cyclical Output, Cyclical Unemployment, and Okun's Coefficient: A New Approach." See also Penson and Webb (1981), "Gross National Product at Full Employment."

[20] In connection with this approach see Aghion and Howitt (1994), "Growth and Unemployment." In this study the authors: "ask() the question of how the rate of economic growth affects unemployment in the long run. The main consideration that leads us to think that this is an interesting question has to do with the re-allocation aspect of growth. In the long run, faster economic growth must come from a faster increase in knowledge. To the extent that the advancement of knowledge is embodied in industrial innovations it is likely to raise the job-destruction rate, through automation, skill-obsolescence, and the bankruptcies associated with the process of creative destruction. In short, increased growth is likely to produce an increased rate of job-turnover, and the search theories of Lucas and Prescott (1974) and Pissarides (1990) imply that an increased rate of job-turnover will result in a higher natural rate of unemployment. This conclusion is also consistent with the empirical results of Davis and Haltiwanger (1990) which show that periods of high unemployment tend to be periods of high job-turnover at the establishment level. It suggests the possibility of a positive long-run tradeoff between growth and employment, at least over some range."

Aghion and Howitt do not discuss Okun's Law. However in as much as we herein consider Okun's Law as playing a central role in maintaining the political, social and cultural development of the United States over recurring 56-year circuits, the article may be relevant to Okun's Law itself and its final understanding.

(3) A third approach is presented by Drs. Dany Lang and Christian de Peretti in their paper entitled "A strong hysteretic model of Okun's Law: theory and a preliminary investigation." (2009) Their approach takes issue with the straight-forward linear relationship posed by Okun's Law.[21]

> The underlying assumption that unemployment responds to growth shocks in a *linear* fashion can be regarded as open to question. Arguably, the response of unemployment to variations in growth need not be the same during booms as during recessions, and should depend on the *intensity* of economic fluctuations, and possible on the *past history* of the economic system. (at 446)... This implies that reactions of the unemployment to growth shocks can be *asymmetric*. (emphasis in the original)

This approach does not take up the 3:1 ratio presented by Okun's Law. Rather a mathematic approach is devised whereby the study of points in the graph may be understood.

> (T)he link between growth and unemployment may be hysteretic. According to the rigorous mathematical definition of hysteresis, due to Krasnosel'skii and Pokrovskii (1989) and Mayeregozy (1991), a process that has a memory of past shocks must possess two key properties to be characterized as hysteretic: remanence and a selective, erasable memory. Remanence occurs when the application of two successive shocks of the same magnitude, but of opposite signs, does not bring the system back to its initial position. The selective, erasable memory property means that only the non-dominated extremum values of the past shocks that have hit the system remain in its memory bank. In economics this definition of hysteresis can be called 'strong' or genuine hysticeses. It has been applied mainly, but not only, to the study of exchange rates dynamics (Amable et al. 991; Gocke 2003;) and to unemployment (Cross et al. 1998). As argued by Amable et al (1993, 1994) and Cross (1993, 1995), the multiple other uses of the term 'hysteresis' are inappropriate and these inappropriate uses can be found in economics only. The definition of hysteresis used in this paper is the only one that is used in physics (from where the term originates) and biology, and which respects the mathematical properties of the concept.

Thus one may locate three theoretic approaches to Okun's Law each of which may be the outgrowth of the structure of the law itself.

The y-axis of Okun's Law (employment) might be seen in Prachowny's paper which emphasizes the productive use of labor, capital, technology, etc. and the y-axis of Okun's Law, i.e. the "rate of employment," not simply of labor but of all productive aspects of the economy.

The x-axis of Okun's Law (growth) might be seen in Adachi's paper which emphasizes the Solow growth model: "increase in the size of real GNP."

Finally Lang and de Peretti emphasize the creation of various moments in time driving the data underlying the linear relationship of Okun's Law itself.

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[21] See also Cross (1993); Cross, et al (1998); Piscitelli, et al (2000); Gocke (2002); Hallett (2002).

It is difficult to see how these equations and approaches can account for the linear stability of Okun's Law over time or the 3:1 ratio suggested by U. S. data. Simply considering the variables collected we have:

Prachowny:

y is output,
k is the capital input and
c is its utilization rate,
n represents the number of workers,
h is the number of hours that they work;
α and β are output elasticites and
γ and δ are the contributions of workers and weekly hours to the total labor input;
τ is a disembodied technology factor.

Adachi:

Y is output;
α is labor augmenting the technological progress, assumed to be proceeding at a constant rate;
λ is the constant rate of growth in the labor population;
u is the rate of unemployment;
ε represents the sensitivity of the real wage rate to the tightness of the labor market,
n is the efficiency per unit of capital, and
$\sigma(n)$ is the elasticity of substitution between labor and capital.

In both theories many of the variables mentioned are difficult to assess and may be quite volatile over time. Intuitively it does not seem possible that the linear stability of Okun's Law would emerge easily from sets of such variables Moreover the hysteresis approach does not attempt to define any larger, linear dynamic into which the method of hysteresis fits.

Our approach examines more closely the econometric data presented by Dr. Knotek. I propose that a valid and workable theory of Okun's Law and its 3:1 ratio may be derived, not from first principles suggested by way of a mathematic or theoretical emphasis, but by first principles which are imposed upon us by the data itself.

The implications of deriving a π:1 ratio from the existing data are quite significant, in as much as so much of mathematic inquiry begins at the unit circle of trigonometric analysis. The result is a view of the economy as a living organism, one which over time collects and puts into place the various and subsidiary ratios mentioned above as necessary to maintain an overarching unity of development over time. By considering the unity of the picture first and the ratios which pertain thereto, we hope to provide a context to which all other aspects of the economy must find themselves subject.

Okun's Leap

Okun begins with the following clearly stated and central assumption.

> Strictly speaking, the leap (from employment rate to potential output) requires the assumption that, *whatever the influence of slack economic activity on average hours, labor force participation, and manhour productivity, the magnitudes of all these effects are related to the unemployment rate*.

As will be developed at greater length in this paper, Okun's leap from an individual consideration of various subsidiary "effects" to the proxy variable of a single, all-encompassing "national rate of unemployment" significantly simplifies the approach to the question posed. It is important to note however that the subsidiary "effects" leapt over ("average hours, labor force participation, and manhour productivity") continue to have a vital and independent standing as they are "related to the unemployment rate."[22]

An inverse and reciprocal relationship is implied by this leap. Just as the "rate of national unemployment" will always be tied to the personal and individual "effects" disregarded in Okun's Law, so will these personal and individual "effects" always be tied to the "rate of national unemployment" by a mathematic reciprocity. Put simply, as the national rate of unemployment goes up, the number of individual people unemployed goes up. As the number of people unemployed goes up thereby igniting a myriad of personal concerns, the more that national rate of unemployment becomes a matter of national concern and direct political consequence. It is hardly too much to say that public concern regarding job creation, social mobility and economic fairness implicated in high rates of unemployment are among the chief concerns of the government of the United States.

One might imagine the inverse relationship implied between the "personal" and the "national" if we let "Government" = 1. In this case the fraction "1/individual" might represent the individual as he/she relates to the nation. Inversely, the fraction "individual/1" would represent the nation as it relates to the individual.

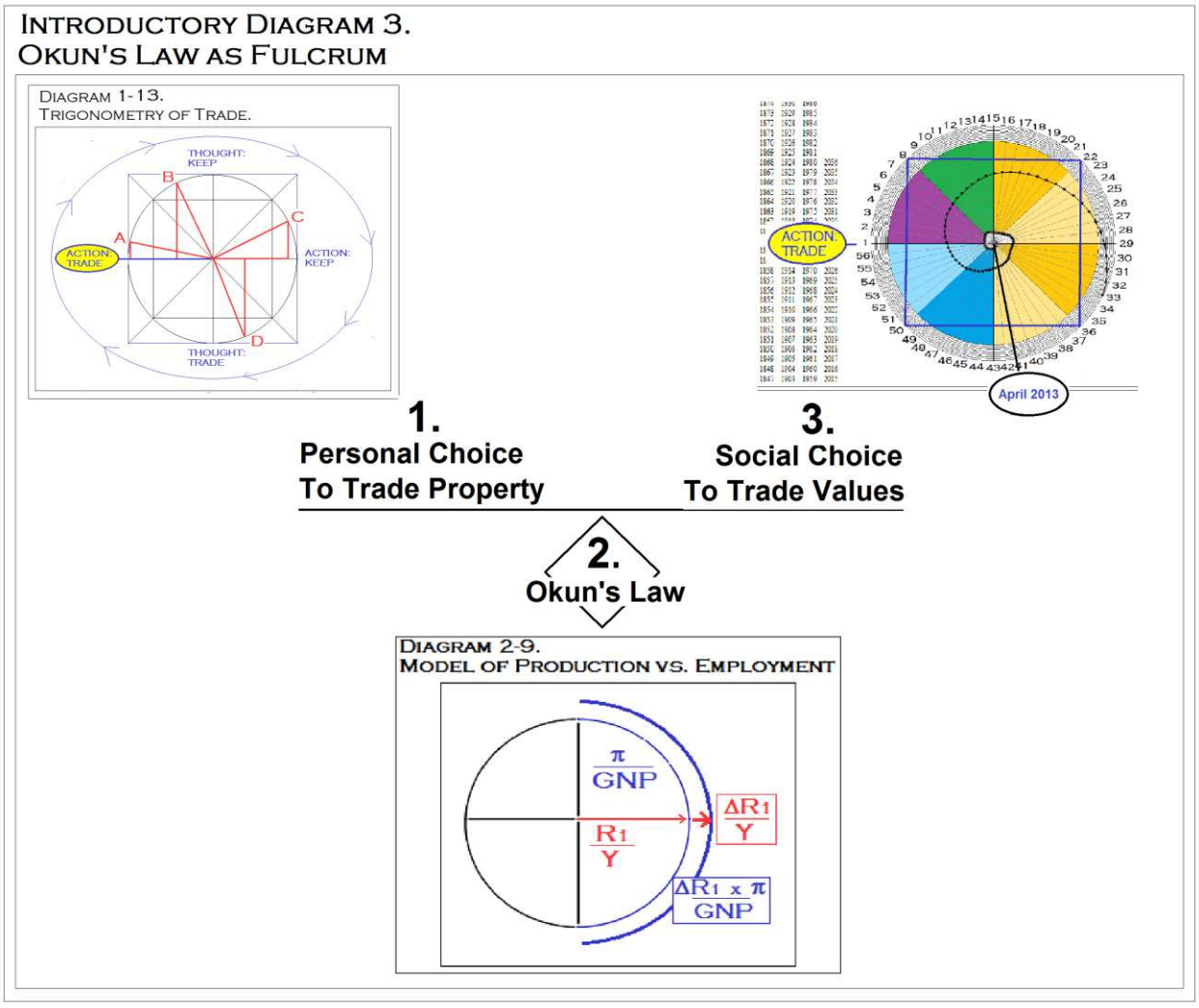
The risk inherent in Okun's approach is that the inverse relationship between the personal and the national might become obscured, taking on by unacknowledged acquiescence the nature of a "1/1," an impenetrable and un-investigable union. In this paper we will deal extensively with the inverse relationship between the "personal made national," and the "national made personal" as the mathematic foundation of our approach.

[22] See Adams and Coe (1989) and Gordon (1984) for a careful evaluation of the underlying factors underlying the national unemployment rate.

To this basic insight must be added the empirical fact that (1) the steady-state rate of growth given by the GNP Spiral (3.4969% per year, Albers & Albers, 2013) is virtually identical with the steady state rate of growth given for annualized quarterly growth given by Knotek. (3.4971% per year, Knotek, 2007, *supra*, footnote 5). In turn, (2) the GNP Spiral predicts this steady state rate of growth as a function of the bio-complexity of the United States.

A Description of Our Approach

Using these two central insights we propose that Okun's Law may be the result of a formal, highly mathematic balance between the individual's right to trade over the short term as influencing and influenced by social values chosen by the people of the United States over the long term. The larger society is thereby the fractal of the smaller, although both merge continually with each other, thereby providing a "proof" of this approach.



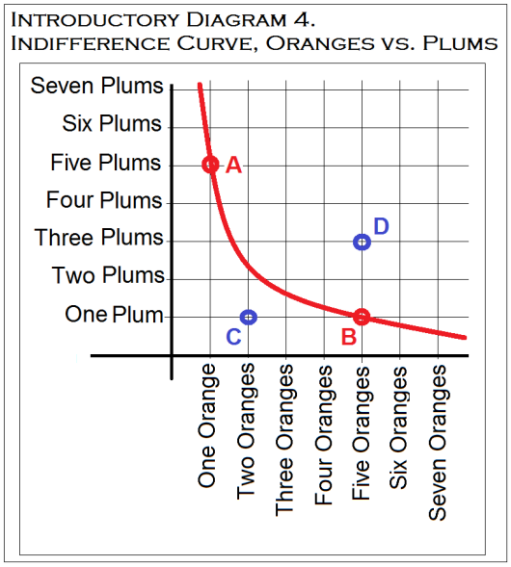
Conclusion to the Introduction: To Buy or Not To Buy

The balance between the personal and the social in the United States is provided by the individual choice of the citizen either to buy, or to refrain from buying, goods and services made available in the productive stream of commerce. This entails the notion of price and the relative demands people place upon economic production in view of their personal financial circumstances. It also connects directly to the type and location of jobs created and the employment rate which follows upon these decisions.

By way of introduction it is worth noting that in his course on the development of political morality Dr. Ian Shapiro (http://oyc.yale.edu/political-science/plsc-118/lecture-6) makes clear that the whole of neo-classical economics can be condensed into a study of the indifference curve. (Shapiro, 2003:38, 44-45)

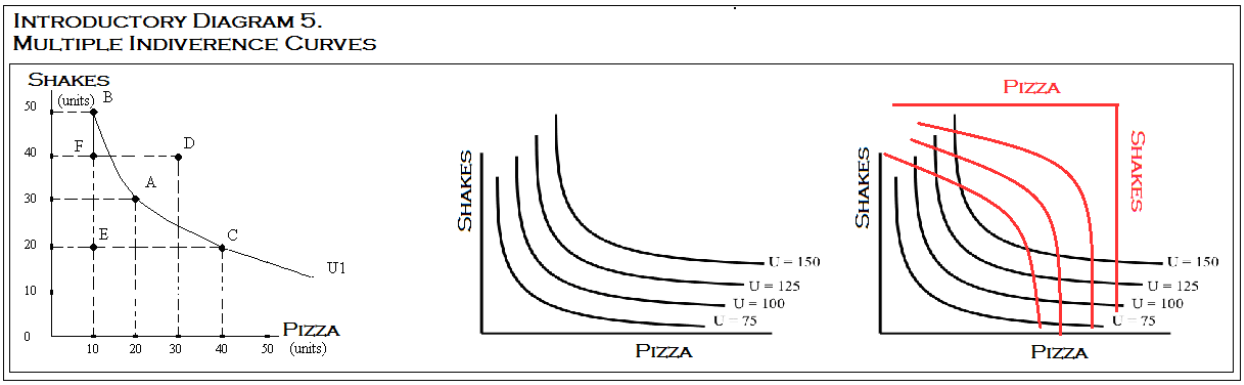
> The architects of neoclassical price theory, William Jevons (1835-1882), Leon Walras (1834-1910), Alfred Marshall (1848-1924), Francis Edgeworth (1845-1926), Knut Wicksell (1851-1926) and Vilfredo Pareto (1848) were principally interested in understanding the behavior of prices in market economics. ...The core notion here is that of an indifference curve. The intuition behind it is a syntheses of three ideas already discussed: that people want to maximize utility in Pareto's stripped-down sense, that their choices generally reflect the principle of diminishing marginal utility, and that they are minimally rational in that their ordering of their desires do not violate transitivity. ... Indifference means exactly what it says: someone is indifferent between two goods if exchanging one for the other would neither increase nor decrease his or her utility.

In the graph below points A and B represent John's indifference to possessing either five plums and one orange, or five oranges and one plum. Under these circumstances someone offering such a trade can be considered by John because he is indifferent to the choice between them.



Point C represents a collection of two oranges and one plum. If this point represents the bundle which John possesses, he will save this quantity because he is not indifferent to the choices available to him. Point D represents a collection of five oranges and three plums, a collection which John does not possess and therefore cannot trade. [23],[24]

The curve drawn below left represents the "indifference" for any consumer as to a choice between pizza and shakes. (Introductory Diagram 5, below) The "indifference curves" generated from this pair of dichotomies represents the willingness to trade one set of goods for different goods.



As increasing levels of affluence at provided, a map of multiple curves becomes possible. (center, Introductory Diagram 5, above)

The indifference curves of two competing trading partners may be explored by inverting the curve of one of the partners.[25] (right, Introductory Diagram 5, above)

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[23] The presentation of the indifference curve between plums and oranges is meant simply to give an intuitive notion of the indifference curve as originally envisioned by Pareto. See Lenfant, 2012:114-116: "The concept of the indifference curve was the touchstone of the escape from cardinalism and the psychological foundations of demand and choice. ... More precisely, what is meant here is the escape from a certain kind of psychology that was widespread in the late nineteenth century and the beginning of the twentieth century, that is, psychological assumptions taken from psychophysiology and experimental psychology and whose main figures were (or had been) Helmholtz, Weber, Fechner, and Wundt. ... The "ordinalist revolution" ... is grounded in a methodological transformation of economics that put the facts of objective experience as a foundation of economics and provided a research program for the ensuing years. Mathematically ordinalism is entirely based upon the idea that one can dispense with the use of a specific utility function and that no meaning shall be attached to utility measurement, except as an ordinal principle." (pp. 114-116)

[24] An excellent critique of the mathematization of indifference curves is found at Barnett (2003): "The purpose of this article is to demonstrate that neoclassical utility functions are an invalid means of analyzing consumer behavior for three reasons: first, and most important, because such functions, and their attendant rankings, are cardinal, not ordinal in nature; second, because, with respect to the set of bundles relevant to actual human beings, such functions are not continuous and therefore, not differentiable; and, third, such functions do not correctly, consistently, and properly include dimensions/units."

My use of indifference curves as an introductory point is simply to assert that, taken in the intuitive context wherein they are derived, they are useful in an evaluation of choices which people make on a day-to-day basis.

[25] Barnett 2003:42, footnote six, provides the following analysis of neoclassical economics and its use of indifference curves. "An anonymous referee comments that, 'It was the use of indifference curves by the victorious neoclassicists that permitted them to have ordinal utility and mathematical functions too. Indifference curves, invented by Edgeworth in the 1880s, made no advance among economists until it was noticed that they made it appear that one could advocate ordinal utility while doing mathematics.' "

"Pareto optimality" represents a qualitative evaluation of these relationships. Given an initial allocation of goods among a set of individuals, a change to a different allocation that makes at least one individual better off without making any other individual worse off is called a Pareto improvement. An allocation is defined as "Pareto efficient" or "Pareto optimal" when no further Pareto improvements can be made.[26]

The essential point to notice about each of these curves is that they assume that the "space" lying outside the curve is synonymous with a "refusal to trade" or "saving," and the points within the curve are synonymous with a "willingness to trade" or simply "trade."

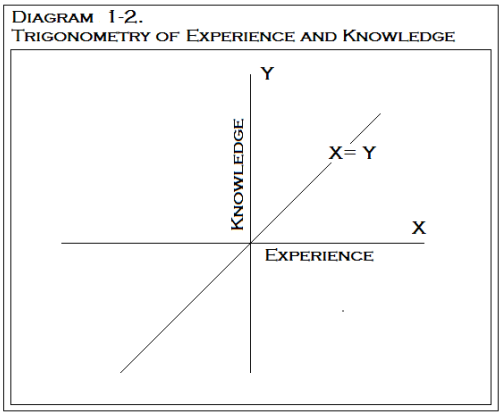
[26] Lenfant, 2012:118: " As is well known, Pareto's and Fisher's main idea was that knowledge of observed behavior was enough to derive the equilibrium of markets and the laws of a market economy. This idea was based upon the intuition that indifference curves were in principle obtainable from observed behavior and that indifference maps could be represented by indexing utility functions. Consequently, they expected to ignore the psychological foundations of choice and of price theory. ... I am not pointing out any contradiction per se between psychology and indifference curves. I am only stressing that indifference curves would be exploited in order to promote an ordinalist representation of utility and a behaviorist foundation for the theory of choice and demand."

The elaborate nature of indifference curves can be significantly simplified and expanded into other areas of social research through an investigation of the manner in which human beings associate a given thing with reality, a thing's actual existence.

For the purposes of these essays we will take as an axiomatic truth that all human life is based upon the presumed equivalence between that which we experience through the senses and that which we know to be real.[27] If "that which we experience" is given the variable "X" and "that which we know to be real" is given the variable "Y", we may state this equivalence as:

$$X = Y.$$

If we place this equation in a Cartesian coordinate system, we have the following 45 degree angle line, beginning at x = 0, y = 0 and extending on toward and infinite number of associations.



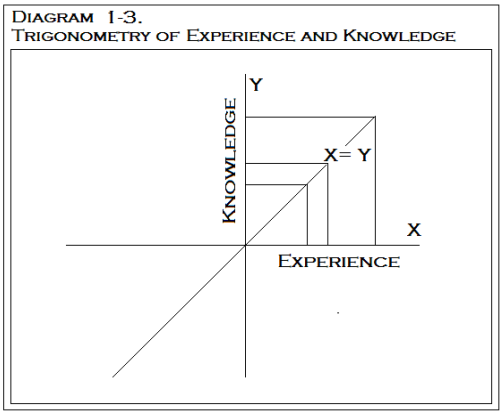


Diagram 1-2 is, in reality, the outcome of an infinite number of squares, wherein each corner point has a specific meaning. "X" represents our experience of something, "Y" represents our knowledge of the thing experienced, the point "(X, Y)" represents the interaction between our experience of the thing itself and our knowledge of the thing itself, and the origin of the graph "(0, 0)" represents the beginning association we make between experience and knowledge as fundamental assumptions of all inquiry.[28]

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[27] For a famous example of the meaning of this sentence, see Boswell, J. (1820). "After we came out of the church, we stood talking for some time together of Bishop Berkeley's ingenious sophistry to prove the nonexistence of matter, and that every thing in the universe is merely ideal. I observed, that though we are satisfied his doctrine is not true, it is impossible to refute it. I never shall forget the alacrity with which (Samuel) Johnson answered, striking his foot with mighty force against a large stone, till he rebounded from it -- "I refute it *thus*."

One might assert that the experience of reading a book and enjoying the imaginary world conveyed is not the same as "experiencing" or "knowing" anything about the world imagined.

Our point here is far more modest and direct. The "experience" referred in this essay is simply that of "reading the book" and the knowledge considered is simply that the person reading knows that he or she is reading a book. The equivalence understood between the *experience* of reading the book, and the *knowledge* that one *is* reading a book, is the equivalence with which we begin this analysis.

[28] See Ornstein, at 63: "In 1268, Roger Bacon, one of the founders of modern science, wrote (in his *Opus Maius....*), 'There are two modes of knowing, through argument and experience. Argument brings conclusion and compels us to concede them, but does not cause certainty nor remove doubts in order that the mind may remain at

Extension to the Jury Trial of a Criminal Case

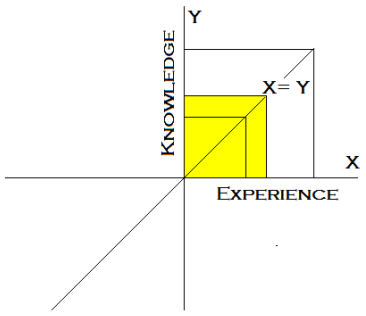
In the United States the jury trial of a case is premised on this same equation "X = Y," "experience" and "knowledge," taken to the next higher social level of the jury. The jury's reception and consideration of the evidence presented[29] indicates that this small group is the expansion of the smaller individual and included minds. In the jury's deliberation the jury demonstrates itself as being the larger, expanded, copied and congruent larger "fractal" of the individual mind.

Specifically, the jury's personal *experience* of the evidence as presented in trial represents the "X" of a trial proceeding.

The jury's evaluation of this evidence as understood through the prism of their own life experiences is the "Y" of the trial proceeding, their collective *knowledge* of the facts presented.

The final verdict given by the jury states its evaluation of the association between the "X" of the trial (the evidence presented) with the "Y" of the trial (the jury's evaluation of this evidence).

DIAGRAM 1-4.
TRIGONOMETRY OF EXPERIENCE AND KNOWLEDGE



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rest in truth, unless this is provided by experience.' These two modes are complementary (both are "right"), and together form the basis for the complete human consciousness."

[29] The law of evidence is an important branch of law within the United States. See Thayer 1898. "One who would state the law of evidence truly must allow himself to grow intimately acquainted with the working of the jury system and its long history." As taken from page 267, footnote 1 he states:

"At once, when a man raises his eyes from the common-law system of evidence, and looks at foreign methods, he is struck with the fact that our system is radically peculiar. Here, a great mass of evidential matter, logically important and probative, is shut out from the view of the judicial tribunals by an imperative rule, while the same matter is not thus excluded anywhere else. English-speaking countries have what we call a "Law of Evidence;" but no other country has it; we alone have generated and evolved this large, elaborate, and difficult doctrine. We have done it, not by direct legislation, but, almost wholly, by the slowly accumulated rulings of judges, made in the trying of causes during the last two or three centuries, - rulings which at first were not preserved in print but in the practice and tradition of the trial courts; and only during the last half or two-thirds of this period have they been revised, reasoned upon, and generalized by the courts in banc.

When one has come to perceive these striking facts, he is not long in finding the reason for them. … It is this institution of the jury which accounts for the common-law system of evidence, - an institution which English-speaking people have had and used, in one or another department of their public affairs, ever since the Conquest. Other peoples have had it only in quite recent times, unless, indeed they may belong to those who began with it centuries ago, and then allowed it to become obsolete and forgotten. England alone kept it, and, in a strange fashion, has developed it. "

This simple model may be expanded upon.

The criminal law of the United States is based upon a dichotomy between the criminal act alleged to have been committed – (the *actus reus* of the offense[30]) – and the mental intent – (the *mens rea* of the offense[31]) – associated with the crime. For example, the act of killing someone is a homicide if done with the intent to kill the individual. If the killing was the result of recklessly driving in a crowded street, the crime is less because the evil of the intent to harm was less. Differences in the consequence to the Defendant can be quite significant, depending upon the nature of the criminal act and mental intent found by the jury.

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[30] The significance of an actual *act* in violtion of the law was highlighted in the case of *Robinson v. California*, 370 U.S. 660 (1962). In this case the U.S. Supreme Court ruled that a California law making it illegal to be a drug addict was unconstitutional because the mere status of being a drug addict was not an *act* and thus not criminal. The Court held:

"It is unlikely that any State at this moment in history would attempt to make it a criminal offense for a person to be mentally ill, or a leper, or to be afflicted with a venereal disease. A State might determine that the general health and welfare require that the victims of these and other human afflictions be dealt with by compulsory treatment, involving quarantine, confinement, or sequestration. But, in the light of contemporary human knowledge, a law which made a criminal offense of such a disease would doubtless be universally thought to be an infliction of cruel and unusual punishment in violation of the Eight and Fourteenth Amendments. ...

"We cannot but consider the statute before us as of the same category. In this Court counsel for the State recognized that narcotic addiction is an illness. Indeed, it is apparently an illness which may be contracted innocently or involuntarily. We hold that a state law which imprisons a person thus afflicted as a criminal, even though he has never touched any narcotic drug within the State or been guilty of any irregular behavior there, inflicts a cruel and unusual punishment in violation of the Fourteenth Amendment."

[31] The Model Penal Code has provided a general scheme for mens rea in criminal cases since its promulgation in 1957. These levels of intent are:

Strict liability: the actor engaged in conduct and his mental state is irrelevant. Under Model Penal Code Section 2.05, this mens rea may only be applied where the forbidden conduct is a mere violation, i.e. a civil infraction.

Negligently: a "reasonable person" would be aware of a "substantial and unjustifiable risk" that his conduct is of a prohibited nature, will lead to a prohibited result, and/or is under prohibited attendant circumstances, and the actor was not so aware but should have been.

Recklessly: the actor consciously disregards a "substantial and unjustifiable risk" that his conduct is of a prohibited nature, will lead to a prohibited result, and/or is of a prohibited nature.

Knowingly: the actor is practically certain that his conduct will lead to the result, or is aware to a high probability that his conduct is of a prohibited nature, or is aware to a high probability that the attendant circumstances exist.

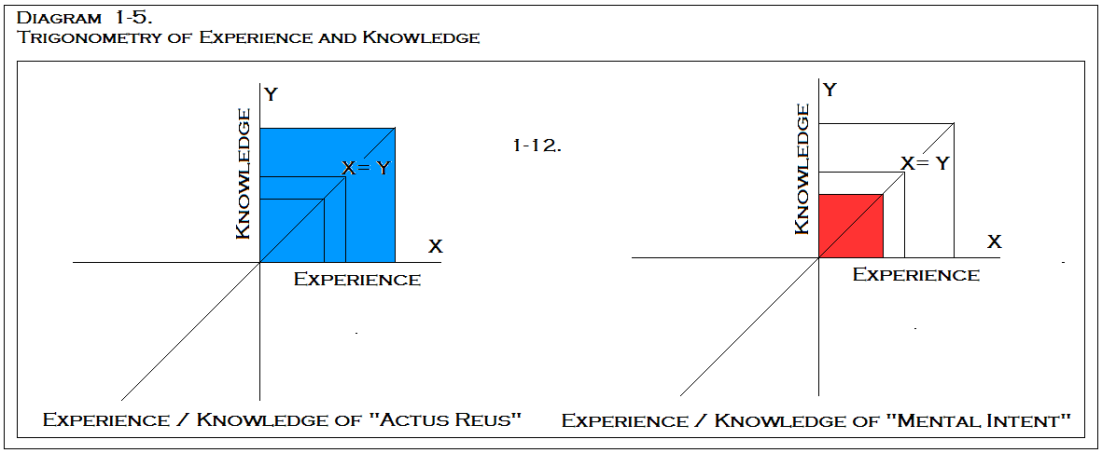
Purposefully: the actor has the "conscious object" of engaging in conduct and believes or hopes that the attendant circumstances exist.

Except for strict liability, these classes of mens rea are defined in Section 2.02(2) of the MPC.

The significance of these levels of mental intent and the actions to which they apply is well illustrated in the case of State of *Montana vs. Rothacher*, 901 P.2d 82, 86-87 (1995). In this case the court's prior decisions had left open the possibility that a homicide might be charged based upon a mens rea going simply to the act which created the crime, rather than the intent to commit the crime itself. The Montana Supreme Court reversed itself, as follows: "It is time to clear up this misperception of the state of mind which must be proven to establish deliberate or mitigated deliberate homicide before a significant injustice results. Our prior construction is clearly contrary to the plain language in the homicide statute and may, in the future, lead to serious and unjust perversion of its purpose. For these reasons, we conclude that the District Court erred when it instructed the jury that the State merely needed to prove that Rothacher acted purposely, without regard to the result that he intended. To the extent that our prior decisions in Sigler, McKimmie, and Byers are inconsistent with this opinion, they are overruled. District courts should not give a similar instruction in the future."

If we let the "actus reus" of any given offense equal a particular number – for example, 5 – then the jury's experience with the evidence presented as to the criminal act (X = 5) and the jury's understanding of that evidence (Y = 5) may be given as a square, in blue below.

Similarly, if we let the "mens rea" of the same offense equal a different number – for example, 3 – then the jury's experience with the evidence presented as to mental intent (X = 3) and the jury's understanding of that evidence (Y = 3) may be given as the red square below.[32]



The idea of giving physical "size" to the jury's experience in court with the evidence may be explained by comparing these experiences. One may readily imagine that prosecutor Jones, an obsessive-compulsive sort, might spend three days developing the actus reus of the case, replete with victim and expert testimony, etc. This is considerably different than might be the case put on by Prosecutor Smith who casually places before the evidence of the same charge a much lesser quantum of evidence, spending the bare minimum of time necessary to establish that a criminal act has occurred. As the jury experiences these differences in court, the outcome of the verdict will shift.

Likewise should Prosecutor Smith neglect to prove that a criminal mental state existed at the time of the alleged offense, it is possible that the proof of the crime as to mens rea may fail entirely. On the other hand, should the prosecutor Jones present proof of mens rea which includes confessions, eye-witness testimony, the testimony of co-conspirators, etc. the experience of the jury with this enlarged quantum of evidence will be fuller than with Smith.

The comparison of these different experiences with the evidence may be depicted by ever larger lengths along the x and y axis as to both the actus reus and mens rea of the charge. The point here is not to propose an absolute scale of proof but rather to suggest that there are very different quanta of proof going into these two essential elements of every criminal case. These

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32 The basic architecture underlying personal choice may be accessed through reference to the common law, an ongoing system of social, political and economic thought all of which is directed toward the maintenance of social order and progress. The central place of the American jury in the legal system of the United States provides a constant connection between the circumstances faced by the people and the laws governing the people. The central ideas of the common law in criminal cases – actus reus, mens rea – are profoundly important to economics because they state the fundamental social basis of common American understandings of human motivation and social judgment, much of which directly applies to very important matters of business, finance, morality and economics, as evolved over tens of thousands of jury trials. This wealth of information as to social and personal behavior is included in this model. It has proven to be both illustratively useful as well as mathematically helpful.

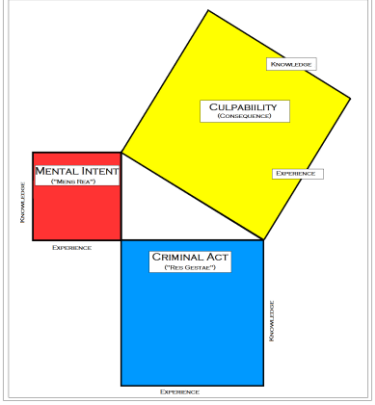
quanta are separate as to actus reus and mens rea but they are joined together in the jury's evaluation of the weight of the case against the Defendant.

The culpability, if any, of the Defendant for a crime is given in accordance with the sum of these two elements of proof. The full experience and knowledge summarized by the case will equal the sum of these two squares. Stating the jury's experience with the evidence of a criminal act as a positive distance "A" and the jury's experience with the evidence of mental intent as a positive distance "B", then the experience / knowledge represented by Culpability (C) associated with the verdict should equal the sum of these two things, or :

$$A^2 + B^2 = C^2$$

Geometrically, this equation may be portrayed with the proportions of the Pythagorean Theorem as follows.

DIAGRAM 1-6.
PYTHAGOREAN THEOREM OF SOCIAL CONDUCT



From the economic point of view, there is no difference between stating that "John purchased x" and "John is guilty of purchasing x." The relationship between the act and the thought which motivates the act, speaking economically, is the same as that of the court considering such an act criminally.

DIAGRAM 1-8.
PYTHAGOREAN THEOREM - SOCIAL AND ECONOMIC ACTIVITY

Micro-economics: The "Chooser – Available Choice" Model

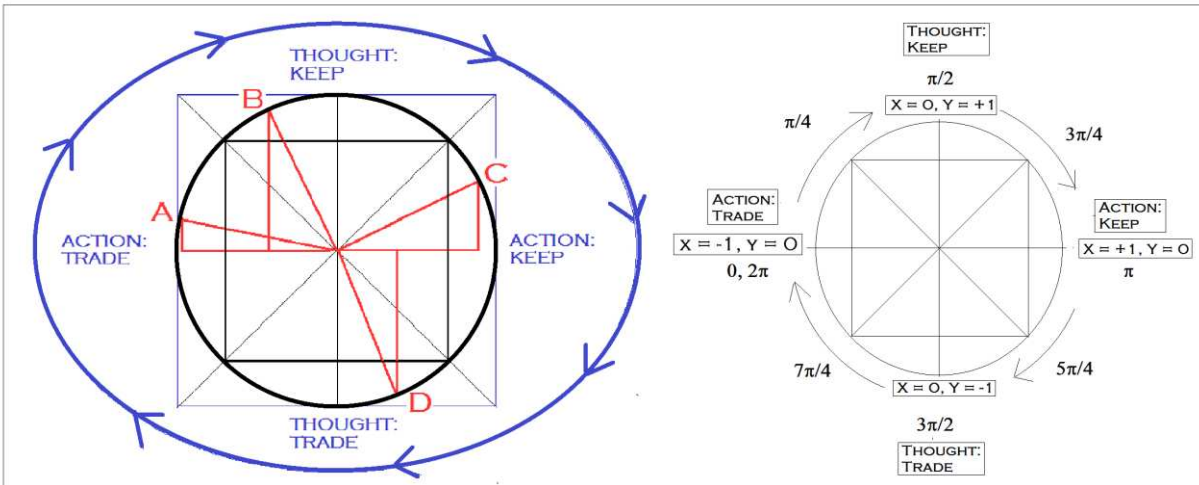
Each of the points within the plane of an indifference curve – both those on the curves and those outside the curve – represents a given decision to trade or to keep various properties. If we contrast the actions of *trading* a good versus *keeping* that same good, a set of dichotomies may be constructed which may be used to structure our understanding of economic development.

The first dichotomy – action, as comparable to the "actus reus" of criminal law – represents a tension between "Keeping" a particular good vs. "Trading" the good for something else. This is indicated in the circle below by the opposition of "Keep" at 3 o'clock and "Trade" at 9 o'clock. All economic life stems from the core principle that one may *act* freely in choosing either to keep a given property or to trade it for some other piece of property and that these transactions clearly affect the status of the property so owned or traded.

This is contrasted with a secondary dichotomy – thoughts, as comparable to the "mens rea" of criminal law – which represents a tension between one's mental "thoughts in favor of keeping" and "thoughts in favor of trading" a particular property, located at 12 o'clock and 6 o'clock respectively in the circle below. These are the mental pre-dispositions of every owner towards keeping or trading a given piece of property for something else.

Using the Pythagorean Theorem to structure the sum total of possible permutations between the "Action" aspect of a purchase, and the "Thought" aspect of a decision to Purchase, we may structure every possible balancing of these two with the "Purchase" itself.[33]

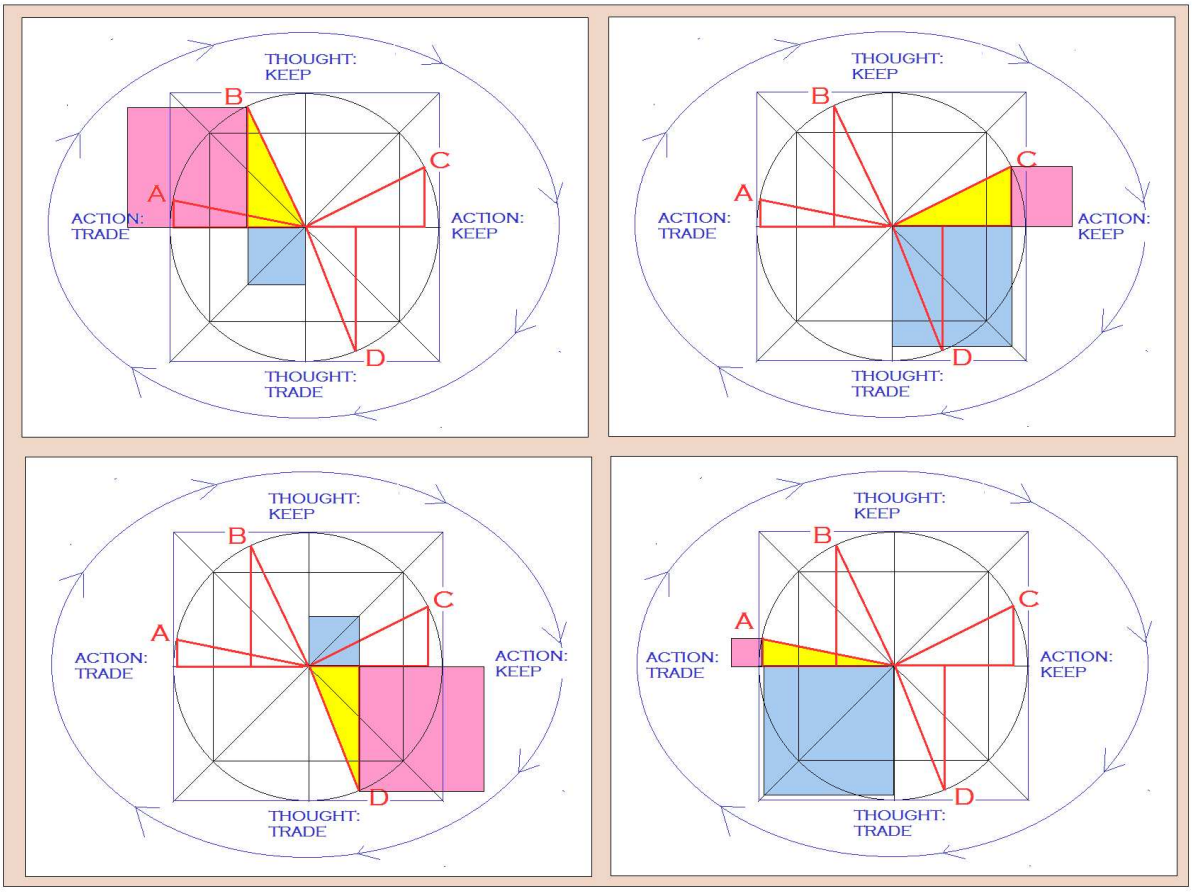
DIAGRAM 1-10.
TRIGONOMETRY OF TRADING VS. KEEPING PROPERTY



[33] The "clock-wise" direction of movement around the unit circle and the "9:00 o'clock" place of beginning the analysis as used in these essays are opposite that taken in most trigonometry textbooks. This approach does not alter the trigonometric identities considered in the slightest and provides an approach to the measurement of time which is consistent with the sense of the hands of a clock.

The Pythagorean relationships inherent in the association of Action and Thought as expressed previously create around the unit circle an infinite set of mathematic relationships wherein the actual possibility of a Purchase is set as the sum of some combination of Action and Thought.

DIAGRAM 1-11.
TRIGONOMETRIC RATIOS.



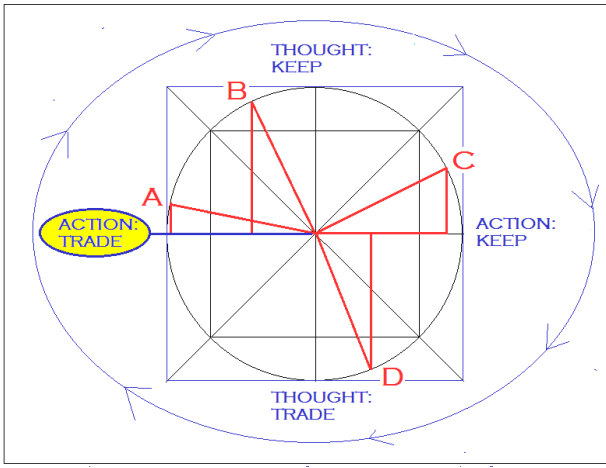
The unity of the underlying ego which selects these various points may be associated with the radius of this circle. If we give this radius the number "1" it represents the "unity" of the ego as a balancing radius between these two dichotomies of Action ("Trading" vs. Keeping") and Thoughts ("Thoughts related to Trading the property," "Thoughts related to Keeping the property"). An internal angle is thus constructed at the origin of the coordinate system.

The Significance of Trading

There is only one point along the Unit Circle where Action is wholly aligned with Trading, i.e. the point at 9:00. All other points along the unit circle are similar to one another in that there is some "Y" component connected to some mental aspect of trading and/ or keeping the object in question. This mental aspect must include some possibility of cancelling the action contemplated. Consequently only at 9:00 o'clock is the possibility of a "Trade" wholly equivalent with Action; at this point "Thought" is Zero and the Action "Trading" occurs.

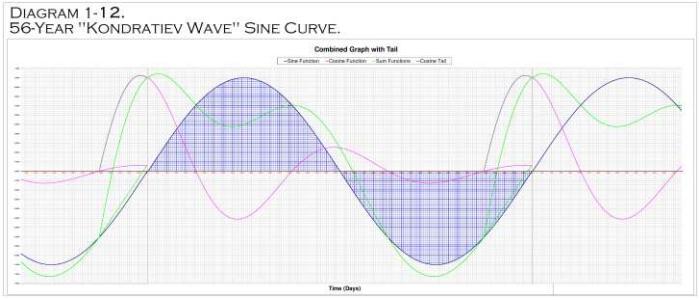
Conversely at 3:00 o'clock the Action undertaken is to "Keep" the property in question and the status quo is actively continued.[34]

DIAGRAM 1-13.
TRIGONOMETRY OF TRADE.



[34] If we consider the side opposite the internal angle as divided by the hypotenuse of "1" we set up a set of fractions which may be charted against an x-axis wherein the circumference of the circle is superimposed upon the x-axis in divisions associated with 2π. Beginning at 9 o'clock and moving clockwise, we have the following mathematic associations between various points along the unit circle, to wit, the sine curve.

DIAGRAM 1-12.
56-YEAR "KONDRATIEV WAVE" SINE CURVE.



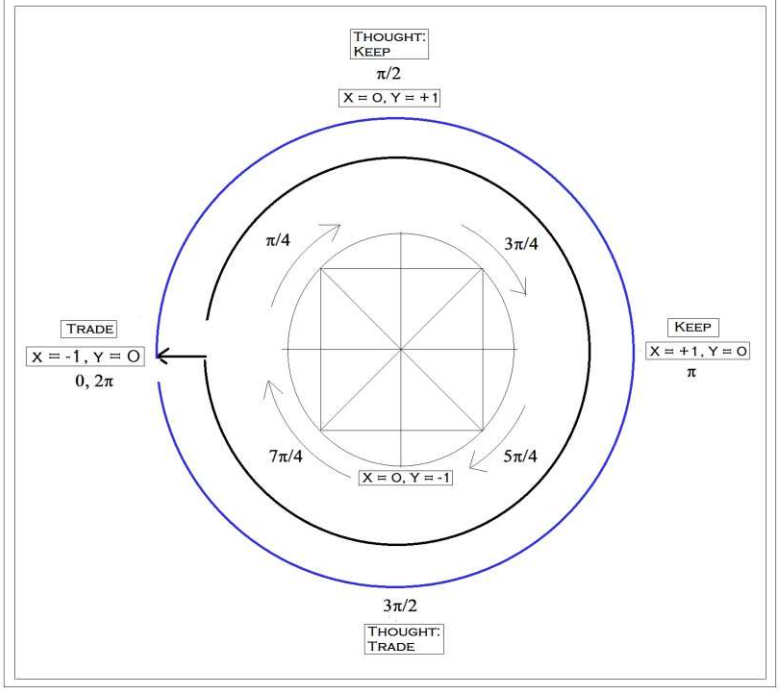
The equation for this wave is:

$$g(y) = \sin(y)$$

The unique aspect of this point at 9:00 o'clock creates an unavoidable change in the overall unit circle. The break which is presented at (x = -1, y = 0) creates a new and unknown element in the unit circle itself. Once the trade is made, the situation is no longer the way it was. Something new has taken place.[35]

In contrast, when the x-axis is directed toward "Keeping" a particular good, the point at which Thought = 0 will be that point most dedicated in favor of the status quo.[36]

DIAGRAM 1-14.
NEW CIRCLE ON TRADE.



[35] There is an analogy here to quantum mechanics in the "Schrodinger's Cat Thought Experiment." The second half of the third postulate of quantum mechanics states, roughly speaking, that observation changes the physical system. http://vergil.chemistry.gatech.edu/notes/quantrev/node20.html A physical system exists in as many state as possible until it is observed. Once the observation has been made, it changes into another state, one which can be unique or not.

Until one opens the box, the cat is both dead and alive. Opening the box (observing the state of the cat), indicates which state it is, and so changes the state of the physical system. In this essay, trading equates with the observation. By analogy, stating that with trade "something new has happened" one would indicate that the wave function describing the state of the cat has changed.

[36] As this relates to the use of indifference curves, at least in their original design by Pareto, see Lenfant 2012:119: "Pareto's own construction and discussion of indifference curves are developed in the Manual. ... Pareto (1900), 2008) already argued that indifference curves could be obtained through experiments or statistical studies. As long as statisticians have not established lines of indifference, 'for lack of more precise notion, the sciences possesses only some general data suggested by crude and everyday observations of facts.' ... So the final methodological position of Pareto is that the theoretical possibility of an empirical construction of indifference curves is at least enough for the foundation of the theory of choice. Eventually, when he comes to a precise description of indifference curves, Pareto appeals to "every day experience" and to introspection to discuss the shape of indifference curves."

The model will be referred to as the "chooser – available choice" model, as a way of presenting the unit circle and its radius of "1" – representing the "chooser" – and the number π – representing the "choices available" – in a simple and direct fashion. Our premise is that a radius originating at the center of the unit circle and moving toward any spot on the circle of possible choices divides the circle at a $1 : \pi$ ratio. Half of the circle constitutes "available choices" which will be associated with the point at which the radius and the circle intersect. This relationship will exclude an equivalent set of opposite choices on the opposing side of the circle.

In other words, one cannot simultaneously trade a good and keep the same good, or vice versa. The possible choices which *are* available toward any particular goal are those which are not directly undermining of whatever goal is chosen. The choices which are *not* available are those which are in some negative value, or opposite position, from this chosen goal. This same dynamic applies to any point of psychological consideration along the unit circle.

I conclude that it is possible to construct a simple and mathematically straight-forward model of micro-economic choices which is completely in accord with the available evidence of social behavior as evidenced by universal and legally required social understandings.

By drafting the experience and knowledge of a jury as the larger "fractal" of the individual mind, we have the ability to state a pattern of "mind" itself which is both useful and concrete in its form.

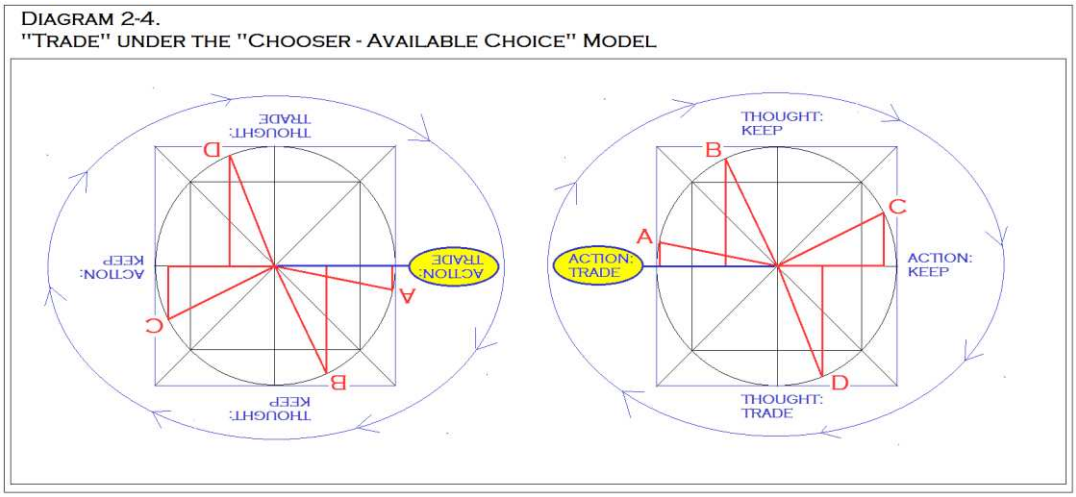
Macro-economics: The "Chooser - Available Choice" model in aggregate

The "chooser – available choice" model is the central point of departure for this model. If we invert this model such that the willingness to "trade" of one person meets the willingness to "trade" of a trading partner, we have a connection between two people indicating a mutual willingness to exchange goods or services with one another. (See discussion of Pareto efficiency *supra* and the inverted Edgeworth "box") The willingness and ability of persons to trade goods and their services with one another is the foundation for the entire economy.

Let us begin with a proposed willingness of Farmer Jones to part with two cows in return for three horses. This willingness is met by Farmer Smith who is willing to trade three specific horses which he owns in return for two specific cows belonging to Farmer Jones.

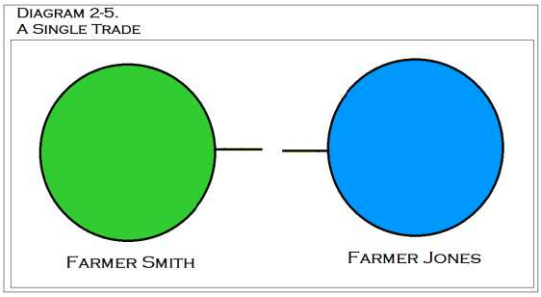
The fact that these two farmers have met with a match which in their minds is favorable to both is indicated by the fact that both have extended the 9:00 axis "Action : Trade" towards one another. As a result of this trade, Farmer Jones' two cows will be handed over to Farmer Smith, and Farmer Smith's three horses will be handed over to Farmer Jones.

DIAGRAM 2-4.
"TRADE" UNDER THE "CHOOSER - AVAILABLE CHOICE" MODEL



The following two circles simplify the basic ideas going into the above trade. Note that the early barter of horses for cows suggested by the circles below depicts trading at its most elementary level. Note that the trade itself must in some fashion state an improvement in the lives of the trading partners. Consequently the act of trading makes more efficient and useful the sum total of property within society because those who own the property are seeking ever more agreeable collections of that property by trading what they have for things which they desire but do not possess.

DIAGRAM 2-5.
A SINGLE TRADE

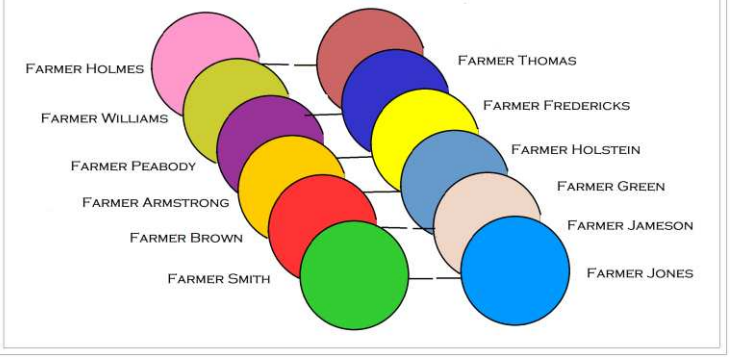


These trades represent a re-arrangement of property amongst those owning property. There is no "expansion" of the economy based upon this trade. However the usefulness of the property exchanged, in combination with the improved efficiency brought about by the trade, suggests that the natural rate of increase in any biologic organism – a farm, a household, a local market – will result from the full set of trades engaged in by all persons.

In short, the same property and the same traders exist after as well as before the trade. However the straight forward exchange of one set of property for another is conveyed by the model above.

There is no limit to the number of such trades which can be done over the course of any particular period of time. We may imagine two pipes running parallel, each suggesting the desire of one of two trading partners to enter into trade. Each trade may be listed in chronologic order and depicted as below.[37]

DIAGRAM 2-6.
THE STREAM OF TRADE



[37] The stream of trade considered in this paper is "Gross National Product" (GNP). This figure adds to Gross Domestic Product (GDP) the income receipts from the rest of the world minus payments to the rest of the world. The United States Bureau of Economic Analysis published the following table for these figures. (as taken from BEA 13-13, Table 9, http://www.bea.gov/newsreleases/national/gdp/2013/pdf/gdp4q12_3rd.pdf) Note that the difference between these is a multiple of (in billions of dollars) GNP = $16,130.8 / GDP = $15,864.1 = 1.016, or 1.6%, roughly $266 billion.

Table 9. Relation of Gross Domestic Product, Gross National Product, and National Income

[Billions of dollars]

Line		2010	2011	2012r	Seasonally adjusted at annual rates					Line
					2011	2012				
					IV	I	II	III	IVr	
1	**Gross domestic product**	**14,498.9**	**15,075.7**	**15,684.8**	**15,321.0**	**15,478.3**	**15,585.6**	**15,811.0**	**15,864.1**	1
2	Plus: Income receipts from the rest of the world	716.5	783.7	782.3	787.1	769.6	775.1	775.8	808.5	2
3	Less: Income payments to the rest of the world	507.2	531.8	539.3	523.1	554.7	527.8	532.7	541.8	3
4	**Equals: Gross national product**	**14,708.2**	**15,327.5**	**15,927.8**	**15,585.0**	**15,693.2**	**15,832.9**	**16,054.2**	**16,130.8**	4
5	Less: Consumption of fixed capital	1,873.4	1,936.8	2,011.7	1,966.6	1,984.9	2,004.8	2,019.8	2,037.4	5
6	Less: Statistical discrepancy	23.3	31.9	67.2	70.3	1.1	77.7	138.5	51.7	6
7	**Equals: National income**	**12,811.4**	**13,358.9**	**13,848.8**	**13,548.1**	**13,707.2**	**13,750.5**	**13,895.9**	**14,041.7**	7
8	Compensation of employees	7,970.0	8,295.2	8,565.8	8,340.1	8,495.7	8,527.7	8,577.6	8,662.1	8
9	Wage and salary accruals	6,404.6	6,661.3	6,880.7	6,692.4	6,825.9	6,849.2	6,888.5	6,959.3	9
10	Supplements to wages and salaries	1,565.4	1,633.9	1,685.1	1,647.7	1,669.8	1,678.5	1,689.1	1,702.8	10
11	Proprietors' income with inventory valuation and capital consumption adjustments	1,103.4	1,157.3	1,202.3	1,165.3	1,184.3	1,194.9	1,205.4	1,224.7	11
12	Rental income of persons with capital consumption adjustment	349.2	409.7	462.6	430.3	445.3	452.8	471.0	481.5	12
13	Corporate profits with inventory valuation and capital consumption adjustments	1,702.4	1,827.0	1,950.6	1,953.1	1,900.1	1,921.9	1,967.6	2,013.0	13
14	Net interest and miscellaneous payments	567.9	527.4	504.3	515.9	515.6	489.5	518.2	493.8	14
15	Taxes on production and imports less subsidies	998.0	1,036.2	1,069.2	1,047.1	1,067.7	1,069.8	1,067.8	1,071.3	15
16	Business current transfer payments (net)	140.0	132.6	128.0	127.4	130.5	127.9	123.8	129.7	16
17	Current surplus of government enterprises	–19.5	–26.5	–34.0	–31.1	–32.0	–34.1	–35.5	–34.5	17
	Addendum:									
18	Gross domestic income	14,475.6	15,043.8	15,617.5	15,250.7	15,477.1	15,507.9	15,672.6	15,812.5	18

r Revised

As reliable currency enters into circulation[38] persons engaged in trading have the further ability to makes trades of much greater complexity that a straight-forward barter. By saving the money obtained from prior trades people are able to amass a trading ability to trade which far exceeds the more clumsy and complicated trade of physical objects, herds of cattle or flocks of geese, etc.

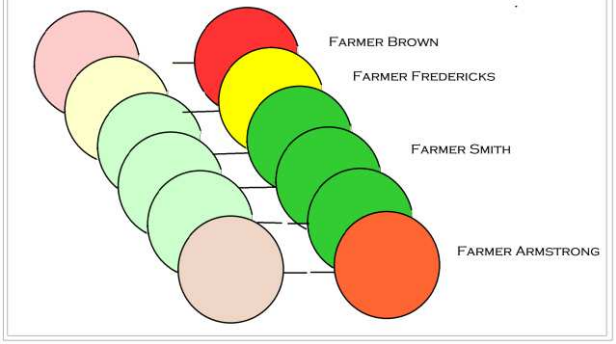
The ability to trade goods and services for currency permits the evaluation of the worth of the trade itself in relative terms visa vi all other trades, however subjective. A trade of $50 might represent an acre of land, a pair of mules, a suit of fine clothes or a suite of furniture. By "mirroring" the value of these various goods (or services), currency permits a much broader extent of trading and trading partners.

The pastel coloration below of the thing traded – money – is available to give a relative value to all the trades of an economy. These "trades" now become "sales," i.e. the surrender of something in return for currency.

The chronology of the trade is given be the difference in color, the red trade being first, the yellow being second, the green third, the orange fourth, etc. The pastel coloration indicates that in this case Farmer Jones did not trade goods for goods but rather money for goods (or services).

The size of the trade in question, its monetary value, is indicated by the number of circles used. For example Farmer Smith's trade of goods or services for money (three green circles) is three times as valuable in monetary terms as Farmer Brown's trade of goods and serves for money (one red circle), Farmer Frederick's trade of goods or services for money (one yellow circle) and Farmer Armstrong's trade of goods or services for money (one orange circle).

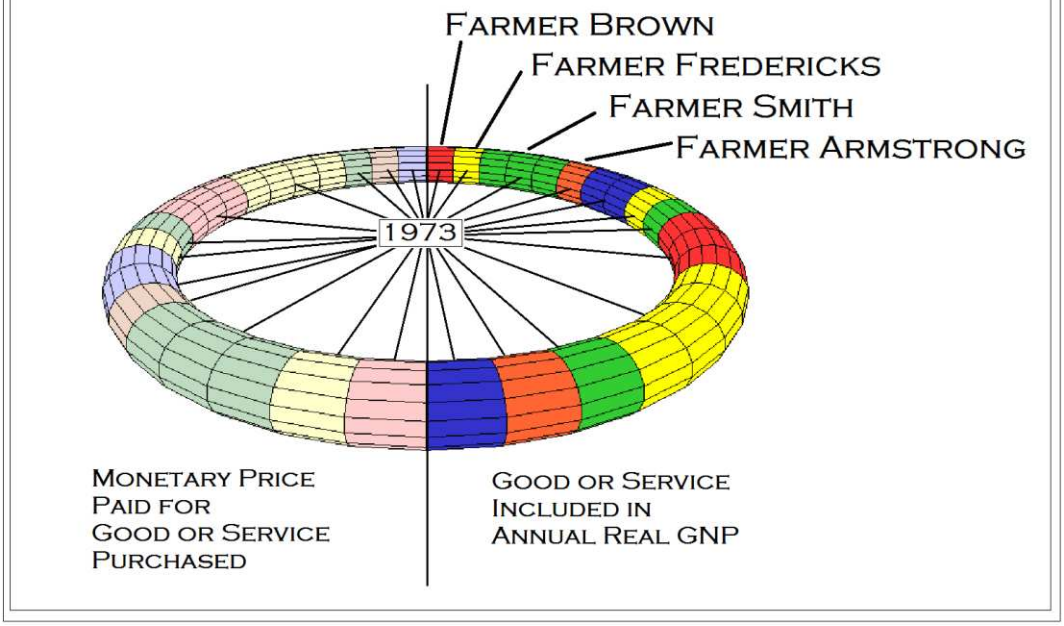
DIAGRAM 2-7.
CHRONOLOGIC SALES OF GOODS AND SERVICES



[38] See Penson and Webb (1981) on the importance of including capital into the determination of Okun's law. "The procedures used by the CEA (Council of Economic Advisors) assume that only the availability of labor and its productivity determine potential GNP. As Perry notes, however, 'it is hard to argue that capital should not be included in estimating potential output because everyone knows it belongs in the calculation.' Okun, in fact, also recognized that capital should be incorporated into the measurement of potential GNP when he stated 'I shall feel much more satisfied in the estimation of potential output when our data and our analysis have advanced to the point where ... the capital factor can be explicitly taken in into account.' ... All the procedures for estimating potential GNP, therefore either explicitly ignore the role of the current capital stock in the economy or implicitly assume the input shares for capital and labor are the same in each production sector of the economy. ... In measuring GNP at full employment, it is not enough to account only for the physical production process. One must also account for the changes in the relative prices of products and resources as the economy moves from current GNP to full employment GNP, and for the effects these price changes will have upon the economic decisions of producers and consumers'."

If we set an arbitrary division of the stream of trade at a single 365-day year, we can place the monetary and the "real" aspects of these sales of goods and services as oppositions antipodal from one another. The result is a circle of such sales. The length of half the circle indicates the monetary value of each of the sales of goods or services included in the year. If the size of these transactions is copied into the length of the circuit itself, we have the following. Because the connection of any particular sale of a good or service to the year "1973" is no greater than any other trade, we draw here a circle, i.e. that geometric construct in which all points in a plane lie equidistant from a single point. [39]

DIAGRAM 2-8.
UNITED STATES ANNUAL REAL GNP



The development of currency and its association with trade given above suggests that the "work" necessary for Farmer Smith or Farmer Jones to possess "trade-able items" has now become the "employment" of Farmer Smith and Farmer Jones as engaged "sales" of these items in a money-based, capitalistic society. In this fashion the use of currency which has turned "trades" into "sales" is in a direct relationship to the rate of employment, i.e. that employment necessary to sustain the full scope of sales given above.

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39 The 2010 real GNP for the United States was $2.27 trillion dollars in 1958 dollars with a population in the same year of 308,745,538 residents, for a GNP per capita of $7,355 per resident in 1958 prices. (See Essay Three, Data Set One, for figures as to real GNP. See 2010 Census for population figures.)

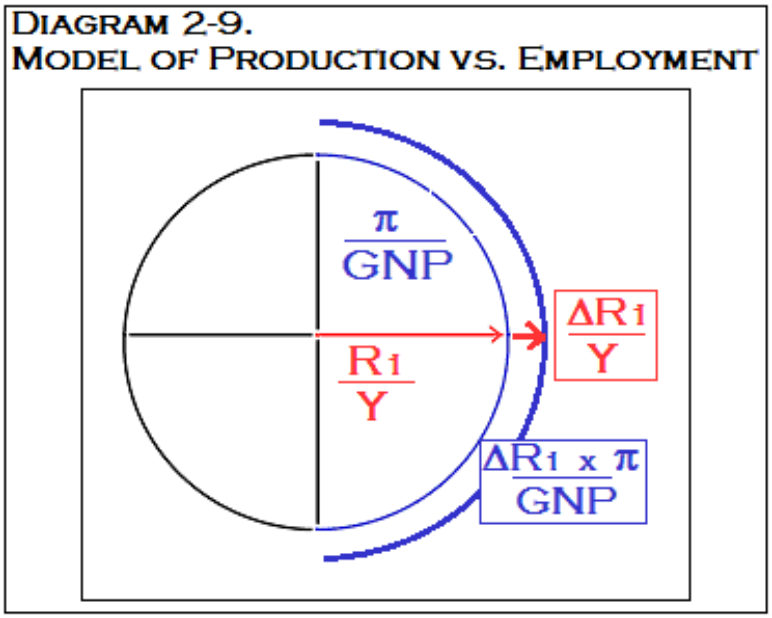
One might picture the relative size of these relationships by noting that if GNP per capita was set as the one inch radius of a pipe and the length of pipe set equal to U.S. real GNP, the pipe would run 406 miles (25,728,794 inches), roughly the distance from Chicago to Kansas City. To bend this pipe into the shape of half a circle would require a radius of 129 miles, roughly the distance from Washington D.C. to Philadelphia.

These proportions might be taken on a smaller scale. If a length of string representing 2010 real GNP was set equal to the length of a football field (3600 inches), the equivalent proportional thickness of the string would measure 0.00014 inches in a radial thickness. Spider silk measurements vary from 0.00012 to 0.00032 inches in diameter. The radius would run from the goal line to the 31.8 yard line.

In the above diagram 2-8 we have used the GNP per capita of the United States as a radius "r" of the generating circle and the rate of unemployment as the radius "R" generating the torus which swings the smaller circle in an arc around the center point "1973."

If this relationship is stated geometrically, it would appear necessary that an increase in the rate of employment from one year to the next (R = the radius of the circle = 1) will correlate geometrically to a necessary increase in the size of GNP (Y = half circumference = π) at the necessary ratio of 1 : π, as follows.

DIAGRAM 2-9.
MODEL OF PRODUCTION VS. EMPLOYMENT



The above diagram is therefore the basis for an understanding of <u>why</u> Okun's Law works. The π:1 ratio (3.14159:1 ratio) given above between "Percent Change in real GNP" and "Percent Change in the Rate of Employment" is a trigonometric outcome of necessary and straightforward social realities of longstanding duration within the economic history of the United States.

The relative wealth of Americans controlling trade may play a role in the maintenance of this ratio.[40]

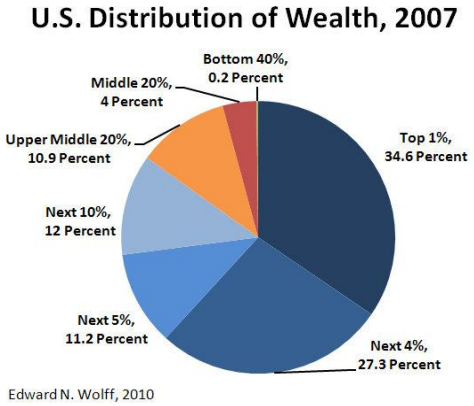
[40] Taking the analogy of a 100 yard length of spider silk stretched into a half circle on a football field, the present distribution of wealth in the United States in 20% quintiles is as follows: 1st = 84 yards, 2nd = 11 yards, 3rd = 4 yards, 4th = 7 inches, 5th = 3.6 inches. From the following chart one sees quite clearly that the top 1% of wealth owners control significantly more wealth (34.6%) than 90% of the rest of America combined (0.2% + 4% + 10.9% + 12% = 27.1%). (as taken from Wolff 2010)

Table 2. The Size Distribution of Wealth and Income, 1983–2007

		Percentage Share of Wealth or Income Held by:								
Year	Gini Coefficient	Top 1.0%	Next 4.0%	Next 5.0%	Next 10.0%	Top 20.0%	4th 20.0%	3rd 20.0%	Bottom 40.0%	All
A. Net Worth										
1983	0.799	33.8	22.3	12.1	13.1	81.3	12.6	5.2	0.9	100.0
1989	0.832	37.4	21.6	11.6	13.0	83.5	12.3	4.8	-0.7	100.0
1992	0.823	37.2	22.8	11.8	12.0	83.8	11.5	4.4	0.4	100.0
1995	0.828	38.5	21.8	11.5	12.1	83.9	11.4	4.5	0.2	100.0
1998	0.822	38.1	21.3	11.5	12.5	83.4	11.9	4.5	0.2	100.0
2001	0.826	33.4	25.8	12.3	12.9	84.4	11.3	3.9	0.3	100.0
2004	0.829	34.3	24.6	12.3	13.4	84.7	11.3	3.8	0.2	100.0
2007	0.834	34.6	27.3	11.2	12.0	85.0	10.9	4.0	0.2	100.0
B. Non-home Wealth										
1983	0.893	42.9	25.1	12.3	11.0	91.3	7.9	1.7	-0.9	100.0
1989	0.926	46.9	23.9	11.6	11.0	93.4	7.4	1.7	-2.5	100.0
1992	0.903	45.6	25.0	11.5	10.2	92.3	7.3	1.5	-1.1	100.0
1995	0.914	47.2	24.6	11.2	10.1	93.0	6.9	1.4	-1.3	100.0
1998	0.893	47.3	21.0	11.4	11.2	90.9	8.3	1.9	-1.1	100.0
2001	0.888	39.7	27.8	12.3	11.4	91.3	7.8	1.7	-0.7	100.0
2004	0.902	42.2	26.7	12.0	11.6	92.5	7.3	1.2	-1.1	100.0
2007	0.908	42.7	29.3	10.9	10.1	93.0	6.8	1.3	-1.0	100.0
C. Income (SCF)										
1982	0.480	12.8	13.3	10.3	15.5	51.9	21.6	14.2	12.3	100.0
1988	0.521	16.6	13.3	10.4	15.2	55.6	20.6	13.2	10.7	100.0
1991	0.528	15.7	14.8	10.6	15.3	56.4	20.4	12.8	10.5	100.0
1994	0.518	14.4	14.5	10.4	15.9	55.1	20.6	13.6	10.7	100.0
1997	0.531	16.6	14.4	10.2	15.0	56.2	20.5	12.8	10.5	100.0
2000	0.562	20.0	15.2	10.0	13.5	58.6	19.0	12.3	10.1	100.0
2003	0.540	17.0	15.0	10.9	14.9	57.9	19.9	12.1	10.2	100.0
2006	0.574	21.3	15.9	9.9	14.3	61.4	17.8	11.1	9.6	100.0

Source: Own computations from the 1983, 1989, 1992, 1995, 1998, 2001, 2004, and 2007 SCF.
For the computation of percentile shares of net worth, households are ranked according to their net worth; for percentile shares of non-home wealth, households are ranked according to their non-home wealth; and for percentile shares of income, households are ranked according to their income.

Graphically the net worth of Americans may be pictured as follows. (Wolff 2010, as placed on under a Creative Commons License 3.0 at: http://en.wikipedia.org/wiki/File:Global_Distribution_of_Wealth_v3.jpg)



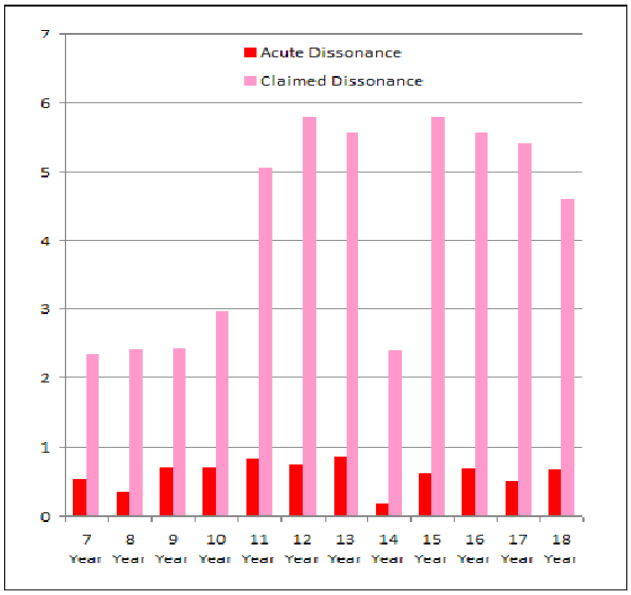
The amount of wealth controlled by the least wealthy 40% of Americans is a fraction of the difference between the GNP and the GDP of the United States (in linear measurements vis-à-vis the football field, 1.6 yards (= 57.6 inches) vs. 7.2 inches). Indeed for the lowest 20% this amount is actually negative in so far as debt outweighs assets. For popular YouTube videos on this point see: http://www.youtube.com/watch?v=QPKKQnijnsM, and: http://www.youtube.com/watch?v=EAOKIGJbg_c.

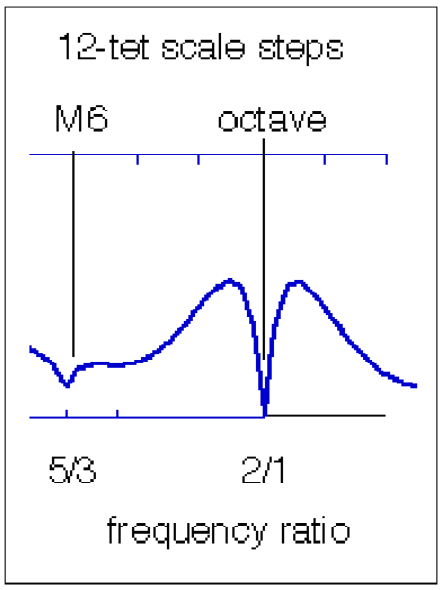
In essence this may mean that the financial identity of the United States – and its ability to function as an ongoing financial system – is far more dependent upon the influx of foreign capital (1.6% of GNP) than on the financial contribution of 40% of its citizens (0.2% of net worth). The long-standing difficulty of creating jobs in the United States may be exacerbated, and possibly connected directly, to considerations as to the use of GDP, rather than GNP, as the appropriate measurement of social progress.

The GNP Spiral

The macro-economic statement of annual GNP takes place in a context of years in sequence. Through the distinct similarity which ratios of U.S. real GNP using various "spreads of years" have with octaves of musical harmony, one may determine "octaves" of mathematic association within the economic data itself, falling at spreads of 14 years. (Albers & Albers, 2013) This is consonant with the onset of reproductive capabilities within the American citizenry; moreover it presents associations of both economics as well as politics.

DIAGRAM 3.11.
"ACUTE DISSONANCE" AND "CLAIMED DISSONANCE" WITH MUSICAL OCTAVE





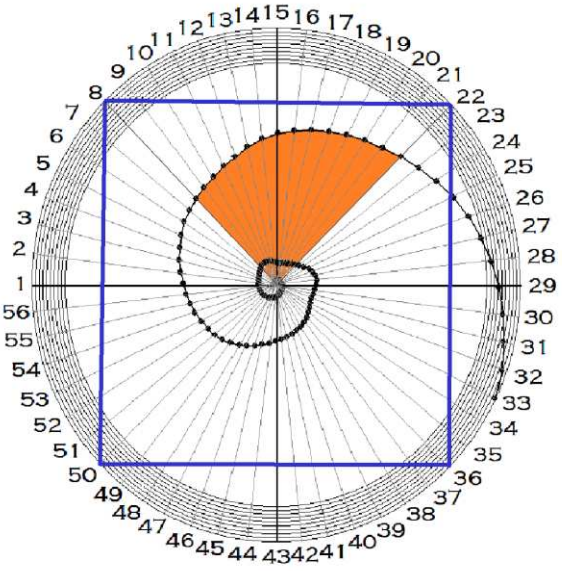
- RED	+ RED		Acute Dissonance	Claimed Dissonance
0	0.539222	7 Year	0.53922225	2.33166
0	0.328323	8 Year	0.32832275	2.39980
0	0.71348	9 Year	0.713480444	2.41405
0	0.702555	10 Year	0.702555	2.95466
-0.82882	0	11 Year	0.828815273	5.05443
-0.07077	0.680828	12 Year	0.751594667	5.77002
-0.07244	0.789312	13 Year	0.861753846	5.55129
-0.01764	0.134157	14 Year	0.151795	2.39229
-0.28313	0.338373	15 Year	0.6214992	5.77038
-0.16258	0.526958	16 Year	0.689539375	5.54327
-0.10785	0.394316	17 Year	0.502165176	5.40061
-0.04362	0.624485	18 Year	0.668103333	4.58002

Using 14 year periods arranged in a square it may be shown that the central quantitative fixture of the economy of the United States is the proportion 1:φ, as demonstrated cursorily in the following diagram. (Albers & Albers, 2011, 2013) The resulting "GNP Spiral" is an exactly 56-year pattern within the political, social and economic history of the United States which correlates generally with the well-known Kondratiev Wave or "Long Wave."

In other words, over the course of 14 years the real GNP of the United States increases on average in a 1 : 1.6180 ratio. This proportion is the famous "Golden Mean" of botanical arrangement, natural selection, pyramid construction, Greek art, Euclidean geometry, Renaissance painting, modular architecture, etc. The biologic, mystical, natural, mathematic, etc. associations, benign and otherwise, brought forward by this unexpected yet quite quantifiable fact are yet to be explored fully.

DIAGRAM 4-7. THE "GNP SPIRAL"

	Date	Date	Date	Date	Date	Date
14	1762	1818	1874	1930	1986	
13	1761	1817	1873	1929	1985	
12	1760	1816	1872	1928	1984	
11	1769	1815	1871	1927	1983	
10	1768	1814	1870	1926	1982	
9	1767	1813	1869	1925	1981	
8	1766	1812	1868	1924	1980	2036
7	1765	1811	1867	1923	1979	2035
6	1764	1810	1866	1922	1978	2034
5	1763	1809	1865	1921	1977	2033
4	1762	1808	1864	1920	1976	2032
3	1761	1807	1863	1919	1975	2031
2	1760	1806	1862	1918	1974	2030
1	1759	1805	1861	1917	1973	2029
56		1804	1860	1916	1972	2028
55		1803	1859	1915	1971	2027
54		1802	1858	1914	1970	2026
53		1801	1857	1913	1969	2025
52		1800	1856	1912	1968	2024
51		1799	1855	1911	1967	2023
50		1798	1854	1910	1966	2022
49		1797	1853	1909	1965	2021
48		1796	1852	1908	1964	2020
47		1795	1851	1907	1963	2019
46		1794	1850	1906	1962	2018
45		1793	1849	1905	1961	2017
44		1792	1848	1904	1960	2016
43		1791	1847	1903	1959	2015

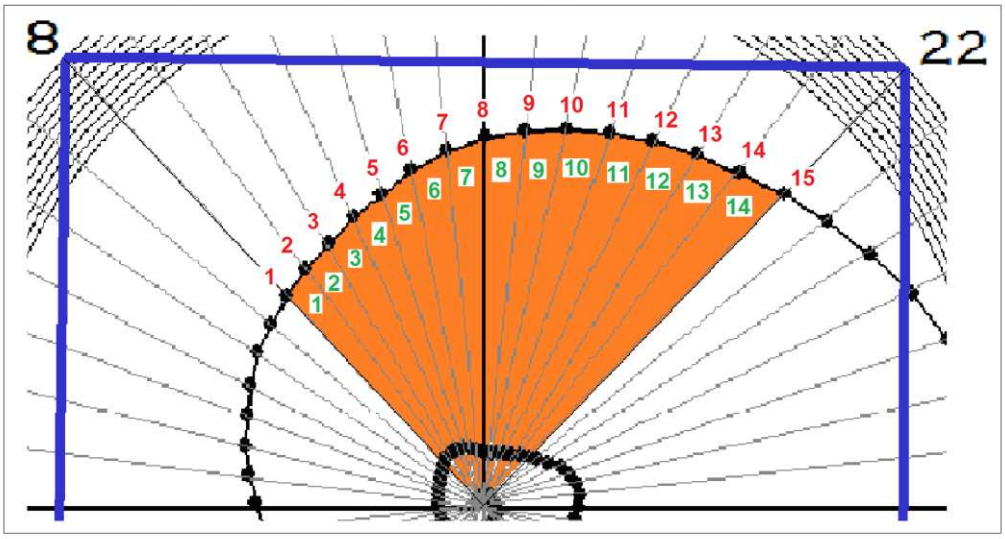


	Date	Date	Date	Date	Date
15	1763	1819	1875	1931	1987
16	1764	1820	1876	1932	1988
17	1765	1821	1877	1933	1989
18	1766	1822	1878	1934	1990
19	1767	1823	1879	1935	1991
20	1768	1824	1880	1936	1992
21	1769	1825	1881	1937	1993
22	1770	1826	1882	1938	1994
23	1771	1827	1883	1939	1995
24	1772	1828	1884	1940	1996
25	1773	1829	1885	1941	1997
26	1774	1830	1886	1942	1998
27	1775	1831	1887	1943	1999
28	1776	1832	1888	1944	2000
29	1777	1833	1889	1945	2001
30	1778	1834	1890	1946	2002
31	1779	1835	1891	1947	2003
32	1780	1856	1892	1948	2004
33	1781	1837	1893	1949	2005
34	1782	1838	1894	1950	2006
35	1783	1839	1895	1951	2007
36	1784	1840	1896	1952	2008
37	1785	1841	1897	1953	2009
38	1786	1842	1898	1954	2010
39	1787	1843	1899	1955	2011
40	1788	1844	1900	1956	2012
41	1789	1845	1901	1957	2013
42	1790	1846	1902	1958	2014

Using the above model – "the GNP Spiral" – repetitions of constitutional amendment in the lower left quadrant stand at a 18 liberal : 3 conservative ratio in relation to the upper right quadrant. Moreover the Golden Mean and its association with φ = 1.6180... is stated to within 3.4 parts of 10,000 – and under even more exacting analysis at 5.3 parts of 100,000 – with an explained steady-state rate of growth between 3.496 and 3.499 percent annually.

The above spiral, which mimics the spiral of galaxies and shellfish alike, brings forward numerous questions as to the nature of time in social systems. Here let us note that one of these aspects is that an additional inverse is implied. This inverse suggests that the running of a period of time, like the running of a race, can be looked at from two different but mathematically very complementary points of view.

DIAGRAM 4-7A. DETAIL.
ORIGIN OF THE 15/14 AND 14/15 RATIOS



In so far as the race begins with a starting line and ends with a finish line, the number of lines counted will be one more than the spaces held between the lines. In the above case highlighted in orange we count 15 lines creating 14 spaces. The fourteen spaces themselves contain a specific number days. To begin the count of days we start at the first day, indicating the starting line of the race. It is, however, the second line, not the first, which represents the end of the first year.

Consequently the period of time in orange might be measured as 14/15 (counting the time held within the boundaries). Conversely we may consider the same period as stated at its inverse, 15/14 (counting the number of boundaries holding the time period).

Okun suggests that the unemployment rate is to be taken as a proxy variable for a number of lesser and included features of employment (Okun, 1962: "average hours, labor force participation, and manhour productivity"). If these "included" factors are a form of inverse of the national rate of employment, then these "included" aspects of personal employment must be stated formally in the final calculation and understanding of Okun's Law.

It appears from the data that the inverses 14/15 and 15/14 above represent the personal "race through time" of the American citizenry as they are engaged as members of the national work force. This work force creates U. S real GNP over time through the personal element of the employment rate which was "leapt over" by Okun's approach. This personal aspect of employment is a necessary part of any understanding of *why* Okun's Law works. These inverses of 14/15 and 15/14 are a fundamental part of Okun's Law. In short,

If the GNP Spiral is governed by the "Golden Mean" as associated with the lifespan of American workers, then the π:1 relationship between employment and GNP must include as well the fractions 14/15 and 15/14 as representing these lifespans.

The Harmonic Inverse

As mentioned previously, Okun begins his analysis with the following assumption.

> Strictly speaking, the leap (from employment rate to potential output) requires the assumption that, *whatever the influence of slack economic activity on average hours, labor force participation, and manhour productivity, the magnitudes of all these effects are related to the unemployment rate*.

If this leap from the personal to the national is indeed an inverse of the leap from the national to the personal, then some effort must be made to identify the nature of an inverse relationship as it applies to the relationship between employment and growth.

For the purposes of the data analysis which follows it may first be considered that the positive numbers, $0<x$ may be divided arbitrarily into three groups, which we denominate for the purposes of this essay feminine ($0<F<1$), $1=1$, and masculine ($1<M$). Any $1/x = F$ must and always will have some number $x/1 = M$ by way of a multiplicative inverse, the product of which will be 1.

$$\frac{1}{x} \times \frac{x}{1} = 1$$

The word "progenic" may be introduced, as referring to the product of the above association of feminine and masculine numbers. By "progenic" ("P" as taken from the root word "progeny" signifying "child" or "children") we mean the number which is derived from a member of the feminine numbers and a member of the masculine numbers *as an intended result*, as contrasted with a number which appears in the data through statistical chance.

Two types of inverses may be noted. The first, a proper inverse, is given above. The second, a "Harmonic multiplicative inverse," may be created by taking a feminine number and calculating some M as the projenic product, rather than the number "1."

For example, should a "harmonic multiplicative inverse" be derived for the number ½ about the projenic number π, the algorithm $2/1 \times \pi = 6.28...$, will be the masculine number necessary, as follows:

$$\frac{1}{2} \longrightarrow \pi$$

$$\pi \times \frac{2}{1} = 2\pi$$

$$\frac{1}{2} \times 2\pi = \pi$$

To state clearly: a *proper* multiplicative inverse has as its progenic product the number "1," and a *Harmonic* multiplicative inverse has as its progenic product some number greater than 1, some "P," implying thereby the existence of some masculine father as determined to be always at some multiple greater than x/1.

By way of example, let us consider the simple process whereby a Harmonic multiplicative inverse may be procured for the number 1/46 about the projenic number phi = 1.6180... We would use the following straight-forward calculus:

$$\frac{1}{46} \longrightarrow \varphi$$

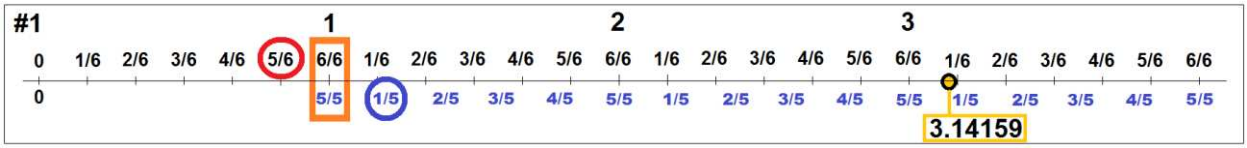
$$\varphi \times \frac{46}{1} = 46\,\varphi$$

$$\frac{1}{46} \times 46\,\varphi = \varphi$$

In the same fashion, taking the feminine number "5/6" a proper multiplicative inverse may be created by reversing the numerator and denominator and "6/5" is found to be the proper multiplicative inverse. (see #1, below):

$$\frac{5}{6} \times \frac{6}{5} = 1$$

As this might be placed on a number line, we have:



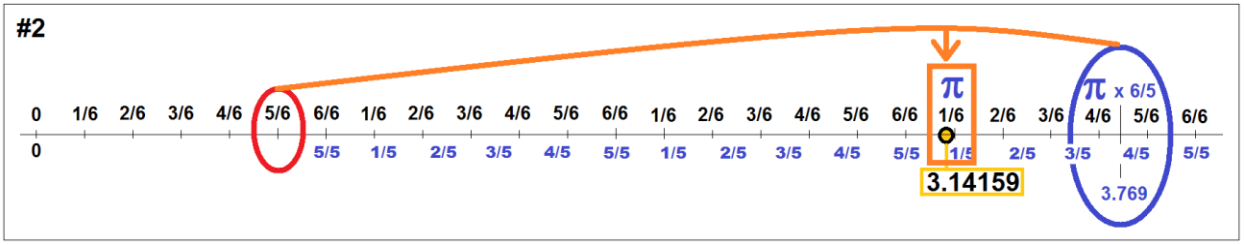
If a Harmonic multiplicative inverse about the progenic number π is intended, then multiplying π x 6/5 yields the following (see #2, below):

$$\frac{5}{6} \longrightarrow \pi$$

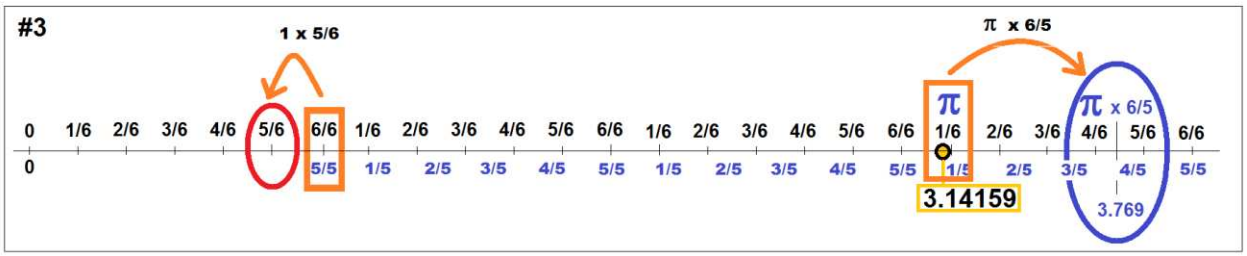
$$\pi \times \frac{6}{5} = \frac{6}{5}\pi$$

$$\frac{5}{6} \times \frac{6}{5}\pi = \pi$$

... or stated in the context of a number line:



Viewed in reverse, the progenic number P = π has been shown to be the progenic product of a Harmonic multiplicative inverse using 5/6 as the feminine number as follows: feminine number = "5/6 x 1," masculine number "6/5 x P." (see #3, below):



The fact that this relationship might be expressed in decimals rather than fractions does not alter the situation in the least. The following example, using decimals, is equivalent for the purposes of this commentary, to wit:

$$0.8333... \quad x \quad 1.2 \quad = 1$$

With this discussion of the concept of an inverse, let us consider the data which underlies Dr. Knotek's description of Okun's Law and the tables used.[41]

[41] For the original data sets used by Professor Knotek to create these graphs, see Appendix 1.

Data Analysis

This first table states the size of GDP as measured quarterly. These numbers form the basis for calculating GDP growth. Annual GDP growth is calculated as 100*((GDP in the fourth quarter of this year)/(GDP in the fourth quarter of last year)-1). Quarterly GDP figures are annualized according to the formula provided by the Bureau of Economic Analysis.

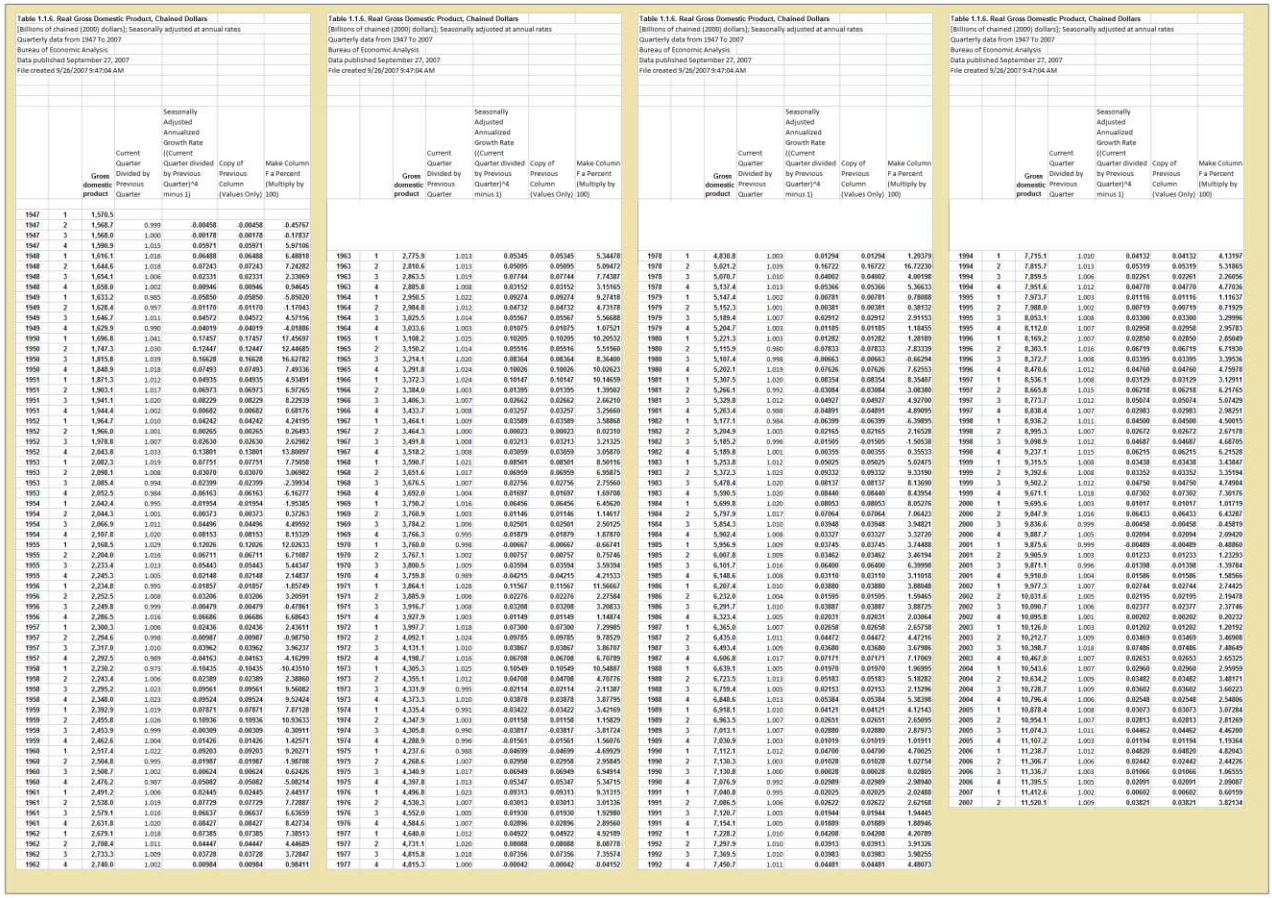


Table 1.1.6. Real Gross Domestic Product, Chained Dollars
[Billions of chained (2000) dollars]; Seasonally adjusted at annual rates
Quarterly data from 1947 To 2007
Bureau of Economic Analysis
Data published September 27, 2007
File created 9/26/2007 9:47:04 AM

		Gross domestic product	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate ((Current Quarter divided by Previous Quarter)^4 minus 1)	Copy of Previous Column (Values Only)	Make Column Fa Percent (Multiply by 100)
1947	1	1,570.5				
1947	2	1,568.7	0.999	-0.00458	-0.00458	-0.45767
1947	3	1,568.0	1.000	-0.00178	-0.00178	-0.17837
1947	4	1,598.9	1.015	0.05971	0.05971	5.97196
1948	1	1,616.1	1.016	0.06488	0.06488	6.48818
1948	2	1,644.6	1.018	0.07243	0.07243	7.24282
1948	3	1,654.1	1.006	0.02331	0.02331	2.33869
1948	4	1,658.0	1.002	0.00946	0.00946	0.94645
1949	1	1,633.2	0.985	-0.05850	-0.05850	-5.85020
1949	2	1,628.4	0.997	-0.01170	-0.01170	-1.17043
1949	3	1,646.7	1.011	0.04572	0.04572	4.57156
1949	4	1,629.9	0.990	-0.04019	-0.04019	-4.01886
1950	1	1,696.8	1.041	0.17457	0.17457	17.45697
1950	2	1,747.3	1.030	0.12447	0.12447	12.44685
1950	3	1,815.8	1.039	0.16628	0.16628	16.62782
1950	4	1,848.9	1.018	0.07493	0.07493	7.49336
1951	1	1,871.3	1.012	0.04935	0.04935	4.93491
1951	2	1,903.1	1.017	0.06973	0.06973	6.97265
1951	3	1,941.1	1.020	0.08229	0.08229	8.22939
1951	4	1,944.4	1.002	0.00682	0.00682	0.68176
1952	1	1,964.7	1.010	0.04242	0.04242	4.24195
1952	2	1,966.0	1.001	0.00265	0.00265	0.26493
1952	3	1,978.8	1.007	0.02630	0.02630	2.62982
1952	4	2,043.8	1.033	0.13801	0.13801	13.80097
1953	1	2,082.3	1.019	0.07751	0.07751	7.75058
1953	2	2,098.1	1.008	0.03070	0.03070	3.06982
1953	3	2,085.4	0.994	-0.02399	-0.02399	-2.39934
1953	4	2,052.5	0.984	-0.06163	-0.06163	-6.16277
1954	1	2,042.4	0.995	-0.01954	-0.01954	-1.95385
1954	2	2,044.3	1.001	0.00373	0.00373	0.37263
1954	3	2,066.9	1.011	0.04496	0.04496	4.49592
1954	4	2,107.8	1.020	0.08153	0.08153	8.15329
1955	1	2,168.5	1.029	0.12026	0.12026	12.02633
1955	2	2,204.0	1.016	0.06711	0.06711	6.71887
1955	3	2,233.4	1.013	0.05443	0.05443	5.44347
1955	4	2,245.3	1.005	0.02148	0.02148	2.14837
1956	1	2,234.8	0.995	-0.01857	-0.01857	-1.85749
1956	2	2,252.5	1.008	0.03206	0.03206	3.20591
1956	3	2,249.8	0.999	-0.00479	-0.00479	-0.47861
1956	4	2,286.5	1.016	0.06686	0.06686	6.68643
1957	1	2,300.3	1.006	0.02436	0.02436	2.43611
1957	2	2,294.6	0.998	-0.00987	-0.00987	-0.98750
1957	3	2,317.0	1.010	0.03962	0.03962	3.96237
1957	4	2,292.5	0.989	-0.04163	-0.04163	-4.16299
1958	1	2,238.2	0.973	-0.10435	-0.10435	-10.43593
1958	2	2,243.4	1.002	0.02389	0.02389	2.38860
1958	3	2,295.2	1.023	0.09561	0.09561	9.56082
1958	4	2,348.0	1.023	0.09524	0.09524	9.52424
1959	1	2,392.9	1.019	0.07871	0.07871	7.87178
1959	2	2,455.8	1.026	0.10936	0.10936	10.93633
1959	3	2,453.9	0.999	-0.00309	-0.00309	-0.30911
1959	4	2,462.6	1.004	0.01426	0.01426	1.42571
1960	1	2,557.4	1.022	0.09203	0.09203	9.20271
1960	2	2,504.8	0.995	-0.01987	-0.01987	-1.98708
1960	3	2,508.7	1.002	0.00624	0.00624	0.62426
1960	4	2,476.2	0.987	-0.05082	-0.05082	-5.08214
1961	1	2,491.2	1.006	0.02445	0.02445	2.44517
1961	2	2,538.0	1.019	0.07729	0.07729	7.72887
1961	3	2,579.1	1.016	0.06637	0.06637	6.63659
1961	4	2,631.8	1.020	0.08427	0.08427	8.42734
1962	1	2,679.1	1.018	0.07385	0.07385	7.38553
1962	2	2,708.4	1.011	0.04447	0.04447	4.44689
1962	3	2,733.3	1.009	0.03728	0.03728	3.72847
1962	4	2,740.0	1.002	0.00984	0.00984	0.98411

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Bureau of Economic Analysis
Data published September 27, 2007
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		Gross domestic product	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate ((Current Quarter divided by Previous Quarter)^4 minus 1)	Copy of Previous Column (Values Only)	Make Column Fa Percent (Multiply by 100)
1963	1	2,775.9	1.013	0.05345	0.05345	5.34478
1963	2	2,810.6	1.013	0.05095	0.05095	5.09472
1963	3	2,863.5	1.019	0.07744	0.07744	7.74387
1963	4	2,885.8	1.008	0.03152	0.03152	3.15165
1964	1	2,950.5	1.022	0.09274	0.09274	9.27418
1964	2	2,984.8	1.012	0.04732	0.04732	4.73178
1964	3	3,025.5	1.014	0.05567	0.05567	5.56688
1964	4	3,033.6	1.003	0.01075	0.01075	1.07521
1965	1	3,108.2	1.025	0.10205	0.10205	10.20532
1965	2	3,150.2	1.014	0.05516	0.05516	5.51560
1965	3	3,214.1	1.020	0.08364	0.08364	8.36480
1965	4	3,291.8	1.024	0.10026	0.10026	10.02623
1966	1	3,372.3	1.024	0.10147	0.10147	10.14659
1966	2	3,384.0	1.003	0.01395	0.01395	1.39502
1966	3	3,486.3	1.007	0.02662	0.02662	2.66250
1966	4	3,433.7	1.008	0.03257	0.03257	3.25660
1967	1	3,464.1	1.009	0.03589	0.03589	3.58868
1967	2	3,464.3	1.000	0.00023	0.00023	0.02310
1967	3	3,491.8	1.008	0.03213	0.03213	3.21325
1967	4	3,518.2	1.008	0.03059	0.03059	3.05870
1968	1	3,590.7	1.021	0.08501	0.08501	8.50116
1968	2	3,651.6	1.017	0.06959	0.06959	6.95875
1968	3	3,676.5	1.007	0.02756	0.02756	2.75560
1968	4	3,692.8	1.004	0.01697	0.01697	1.69708
1969	1	3,750.2	1.016	0.06456	0.06456	6.45620
1969	2	3,760.9	1.003	0.01146	0.01146	1.14617
1969	3	3,784.2	1.006	0.02501	0.02501	2.50125
1969	4	3,766.3	0.995	-0.01879	-0.01879	-1.87870
1970	1	3,760.0	0.998	-0.00667	-0.00667	-0.66741
1970	2	3,767.1	1.002	0.00757	0.00757	0.75746
1970	3	3,800.5	1.009	0.03594	0.03594	3.59394
1970	4	3,759.8	0.989	-0.04215	-0.04215	-4.21533
1971	1	3,864.1	1.028	0.11567	0.11567	11.56667
1971	2	3,885.9	1.006	0.02276	0.02276	2.27584
1971	3	3,916.7	1.008	0.03208	0.03208	3.20833
1971	4	3,927.9	1.003	0.01149	0.01149	1.14874
1972	1	3,997.7	1.018	0.07300	0.07300	7.29985
1972	2	4,042.1	1.011	0.09785	0.09785	9.78529
1972	3	4,131.1	1.010	0.03867	0.03867	3.86707
1972	4	4,198.7	1.016	0.06708	0.06708	6.70789
1973	1	4,305.3	1.025	0.10549	0.10549	10.54887
1973	2	4,355.1	1.012	0.04708	0.04708	4.70776
1973	3	4,331.9	0.995	-0.02114	-0.02114	-2.11387
1973	4	4,373.3	1.010	0.03878	0.03878	3.87795
1974	1	4,335.4	0.991	-0.03422	-0.03422	-3.42769
1974	2	4,347.9	1.003	0.01158	0.01158	1.15829
1974	3	4,305.8	0.990	-0.03817	-0.03817	-3.81724
1974	4	4,288.9	0.996	-0.01561	-0.01561	-1.56076
1975	1	4,237.6	0.988	-0.04699	-0.04699	-4.69829
1975	2	4,268.6	1.007	0.02958	0.02958	2.95845
1975	3	4,340.9	1.017	0.06949	0.06949	6.94914
1975	4	4,397.8	1.013	0.05347	0.05347	5.34715
1976	1	4,496.8	1.023	0.09313	0.09313	9.31335
1976	2	4,530.3	1.007	0.03013	0.03013	3.01336
1976	3	4,552.0	1.005	0.01930	0.01930	1.92980
1976	4	4,584.6	1.007	0.02896	0.02896	2.89560
1977	1	4,640.0	1.012	0.04922	0.04922	4.92189
1977	2	4,731.1	1.020	0.08088	0.08088	8.08778
1977	3	4,815.8	1.018	0.07356	0.07356	7.35574
1977	4	4,815.3	1.000	-0.00042	-0.00042	-0.04152

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		Gross domestic product	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate ((Current Quarter divided by Previous Quarter)^4 minus 1)	Copy of Previous Column (Values Only)	Make Column Fa Percent (Multiply by 100)
1978	1	4,830.8	1.003	0.01294	0.01294	1.29379
1978	2	5,021.2	1.039	0.16722	0.16722	16.72230
1978	3	5,070.7	1.010	0.04002	0.04002	4.00198
1978	4	5,137.4	1.013	0.05366	0.05366	5.36633
1979	1	5,147.4	1.002	0.00781	0.00781	0.78088
1979	2	5,152.3	1.001	0.00381	0.00381	0.38132
1979	3	5,189.4	1.007	0.02912	0.02912	2.91553
1979	4	5,204.7	1.003	0.01185	0.01185	1.18455
1980	1	5,221.3	1.003	0.01282	0.01282	1.28189
1980	2	5,115.9	0.980	-0.07833	-0.07833	-7.83339
1980	3	5,107.4	0.998	-0.00663	-0.00663	-0.66294
1980	4	5,202.1	1.019	0.07626	0.07626	7.62553
1981	1	5,307.5	1.020	0.08354	0.08354	8.35407
1981	2	5,266.1	0.992	-0.03084	-0.03084	-3.08380
1981	3	5,329.8	1.012	0.04927	0.04927	4.92700
1981	4	5,263.4	0.988	-0.04891	-0.04891	-4.89095
1982	1	5,177.1	0.984	-0.06399	-0.06399	-6.39895
1982	2	5,204.9	1.005	0.02165	0.02165	2.16528
1982	3	5,185.2	0.996	-0.01505	-0.01505	-1.50538
1982	4	5,189.8	1.001	0.00355	0.00355	0.35533
1983	1	5,253.8	1.012	0.05025	0.05025	5.02475
1983	2	5,372.3	1.023	0.09332	0.09332	9.33190
1983	3	5,478.4	1.020	0.08137	0.08137	8.13690
1983	4	5,590.5	1.020	0.08440	0.08440	8.43954
1984	1	5,699.8	1.020	0.08053	0.08053	8.05275
1984	2	5,797.9	1.017	0.07064	0.07064	7.06423
1984	3	5,854.3	1.010	0.03948	0.03948	3.94821
1984	4	5,902.4	1.008	0.03327	0.03327	3.32720
1985	1	5,956.9	1.009	0.03745	0.03745	3.74488
1985	2	6,007.8	1.009	0.03462	0.03462	3.46194
1985	3	6,101.7	1.016	0.06400	0.06400	6.39998
1985	4	6,148.6	1.008	0.03110	0.03110	3.11018
1986	1	6,207.4	1.010	0.03880	0.03880	3.88048
1986	2	6,232.0	1.004	0.01595	0.01595	1.59465
1986	3	6,291.7	1.010	0.03887	0.03887	3.88725
1986	4	6,323.4	1.005	0.02031	0.02031	2.03064
1987	1	6,365.0	1.007	0.02658	0.02658	2.65758
1987	2	6,435.0	1.011	0.04472	0.04472	4.47216
1987	3	6,493.4	1.009	0.03680	0.03680	3.67986
1987	4	6,606.8	1.017	0.07171	0.07171	7.17069
1988	1	6,639.1	1.005	0.01970	0.01970	1.96995
1988	2	6,723.5	1.013	0.05183	0.05183	5.18202
1988	3	6,759.4	1.005	0.02153	0.02153	2.15296
1988	4	6,848.6	1.013	0.05384	0.05384	5.38398
1989	1	6,918.1	1.010	0.04121	0.04121	4.12143
1989	2	6,963.5	1.007	0.02651	0.02651	2.65095
1989	3	7,013.1	1.007	0.02880	0.02880	2.87973
1989	4	7,030.9	1.003	0.01019	0.01019	1.01911
1990	1	7,017.1	1.002	0.00700	0.00700	0.70025
1990	2	7,130.3	1.003	0.01028	0.01028	1.02754
1990	3	7,130.8	1.000	0.00028	0.00028	0.02805
1990	4	7,076.9	0.992	-0.02989	-0.02989	-2.98940
1991	1	7,040.8	0.995	-0.02025	-0.02025	-2.02488
1991	2	7,086.5	1.006	0.02622	0.02622	2.62568
1991	3	7,120.7	1.005	0.01944	0.01944	1.94445
1991	4	7,154.1	1.005	0.01889	0.01889	1.88946
1992	1	7,228.2	1.010	0.04208	0.04208	4.20789
1992	2	7,297.9	1.010	0.03913	0.03913	3.91326
1992	3	7,369.5	1.010	0.03983	0.03983	3.98255
1992	4	7,450.7	1.011	0.04481	0.04481	4.48073

Table 1.1.6. Real Gross Domestic Product, Chained Dollars
[Billions of chained (2000) dollars]; Seasonally adjusted at annual rates
Quarterly data from 1947 To 2007
Bureau of Economic Analysis
Data published September 27, 2007
File created 9/26/2007 9:47:04 AM

		Gross domestic product	Current Quarter Divided by Previous Quarter	Seasonally Adjusted Annualized Growth Rate ((Current Quarter divided by Previous Quarter)^4 minus 1)	Copy of Previous Column (Values Only)	Make Column Fa Percent (Multiply by 100)
1994	1	7,715.1	1.010	0.04132	0.04132	4.13197
1994	2	7,815.7	1.013	0.05319	0.05319	5.31865
1994	3	7,859.5	1.006	0.02261	0.02261	2.26056
1994	4	7,951.6	1.012	0.04778	0.04778	4.77836
1995	1	7,973.7	1.003	0.01116	0.01116	1.11637
1995	2	7,988.8	1.002	0.00719	0.00719	0.71929
1995	3	8,053.1	1.008	0.03306	0.03306	3.29896
1995	4	8,112.8	1.007	0.02958	0.02958	2.95783
1996	1	8,169.2	1.007	0.02850	0.02850	2.85049
1996	2	8,303.1	1.016	0.06719	0.06719	6.71930
1996	3	8,372.7	1.008	0.03395	0.03395	3.39536
1996	4	8,470.6	1.012	0.04760	0.04760	4.75978
1997	1	8,536.1	1.008	0.03129	0.03129	3.12911
1997	2	8,665.8	1.015	0.06218	0.06218	6.21765
1997	3	8,773.7	1.012	0.05074	0.05074	5.07429
1997	4	8,838.4	1.007	0.02983	0.02983	2.98251
1998	1	8,936.2	1.011	0.04500	0.04500	4.50015
1998	2	8,995.3	1.007	0.02672	0.02672	2.67178
1998	3	9,098.9	1.012	0.04687	0.04687	4.68705
1998	4	9,237.1	1.015	0.06215	0.06215	6.21528
1999	1	9,315.5	1.008	0.03438	0.03438	3.43847
1999	2	9,392.6	1.008	0.03352	0.03352	3.35194
1999	3	9,502.2	1.012	0.04750	0.04750	4.74984
1999	4	9,671.1	1.018	0.07302	0.07302	7.30176
2000	1	9,695.6	1.003	0.01017	0.01017	1.01719
2000	2	9,847.9	1.016	0.06433	0.06433	6.43287
2000	3	9,836.6	0.999	-0.00458	-0.00458	-0.45819
2000	4	9,887.7	1.005	0.02094	0.02094	2.09420
2001	1	9,875.6	0.999	-0.00489	-0.00489	-0.48860
2001	2	9,905.9	1.003	0.01233	0.01233	1.23293
2001	3	9,871.1	0.996	-0.01398	-0.01398	-1.39784
2001	4	9,910.8	1.004	0.01586	0.01586	1.58566
2002	1	9,977.3	1.007	0.02744	0.02744	2.74425
2002	2	10,031.6	1.005	0.02195	0.02195	2.19478
2002	3	10,090.7	1.006	0.02377	0.02377	2.37746
2002	4	10,095.8	1.001	0.00202	0.00202	0.20232
2003	1	10,126.8	1.003	0.01202	0.01202	1.20192
2003	2	10,212.7	1.009	0.03469	0.03469	3.46908
2003	3	10,298.7	1.018	0.07486	0.07486	7.48649
2003	4	10,467.8	1.007	0.02653	0.02653	2.65325
2004	1	10,543.6	1.007	0.02960	0.02960	2.95959
2004	2	10,634.2	1.009	0.03482	0.03482	3.48171
2004	3	10,728.7	1.009	0.03602	0.03602	3.60223
2004	4	10,796.4	1.006	0.02548	0.02548	2.54806
2005	1	10,878.4	1.008	0.03073	0.03073	3.07284
2005	2	10,954.1	1.007	0.02813	0.02813	2.81269
2005	3	11,074.3	1.011	0.04462	0.04462	4.46200
2005	4	11,107.2	1.003	0.01194	0.01194	1.19364
2006	1	11,238.7	1.012	0.04820	0.04820	4.82043
2006	2	11,306.7	1.006	0.02442	0.02442	2.44226
2006	3	11,336.7	1.003	0.01066	0.01066	1.06555
2006	4	11,395.5	1.005	0.02091	0.02091	2.09087
2007	1	11,412.6	1.002	0.00602	0.00602	0.60159
2007	2	11,520.1	1.009	0.03821	0.03821	3.82134

It is important to mention at the outset that the character of GDP is quite different from the character of employment. Unlike employment, the measure of GDP begins with the fact that, like any object which grows, it has size. Sharing a commonality with the size of a dog, a flower or a tree, the measurement of GDP above is intended to give an estimate of the size of the economy as an objective entity.

This is important to mention because the *growth* of GDP is considered from two standpoints. The first is growth over the course of a year (Annual). The second, relating to quarterly GDP, is figured from a mathematic algorithm. In this algorithm (1) the current quarter is divided by the previous quarter, (2) this is then taken to the fourth power, (3) from this figure we subtract one and (4) make this number a percent by multiplying by 100.

The purpose of these procedures is to find the rate of growth of an object. Using estimates of the growth rate over quarters, which are four times as numerous as annual estimates, we might expect that these repeated quarterly annual-izations render a much more precise value

than is possible for annual data. And the measurement of this objective growth is quite unlike the nature of a measurement of quarterly and annual employment.

The next table below states the employment rate in months. For annual data, the change in the unemployment rate is the current December minus the previous December. For quarterly data, the change in the unemployment rate is the difference between subsequent quarterly averages.

Monthly Unemployment, Bureau of Labor Statistics
Labor Force Statistics from the Current Population Survey

Series Id: LNS14000000
Seasonally Adjusted
Series title: (Seas) Unemployment Rate
Labor force status: Unemployment rate
Type of data: Percent or rate
Age: 16 years and over

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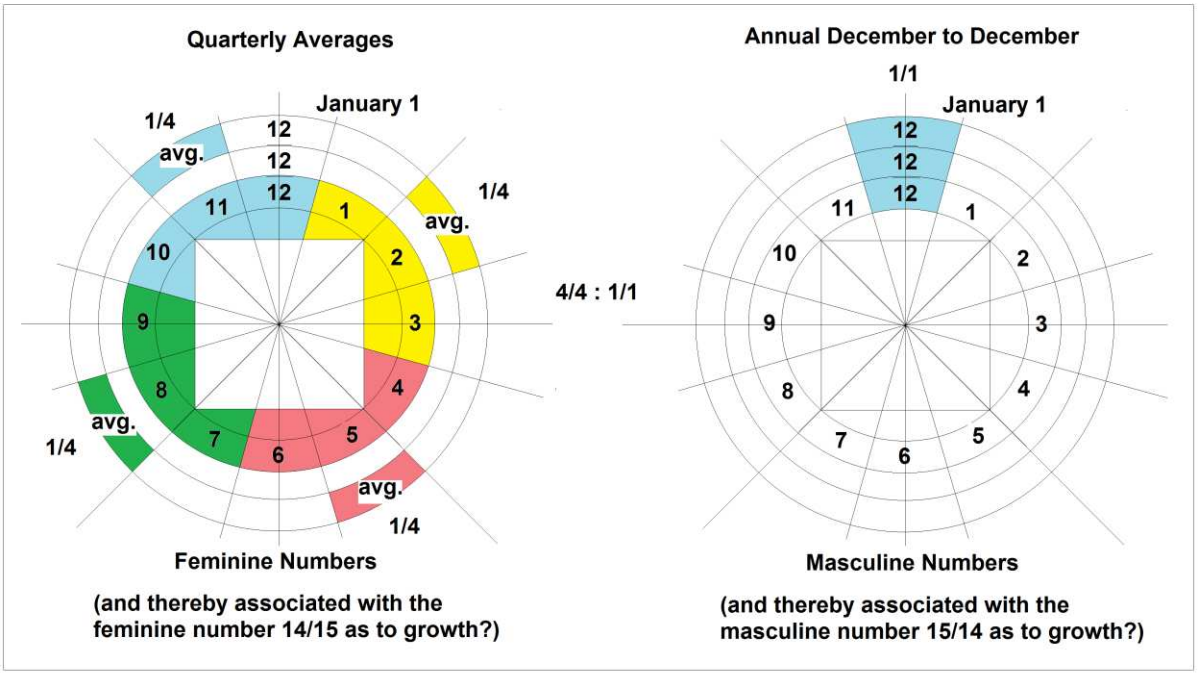
Year	Jan	Feb	Mar	1st Quarter Average (example: "1948.1")	Apr	May	Jun	2nd Quarter Average (example: "1948.2")	Jul	Aug	Sep	3rd Quarter Average (example: "1948.3")	Oct	Nov	Dec	4th Quarter Average (example: "1948.4")	Annual
1948	3.4	3.8	4	3.733333333	3.9	3.5	3.6	3.666666667	3.6	3.9	3.8	3.766666667	3.7	3.8	4	3.833333333	
1949	4.3	4.7	5	4.666666667	5.3	6.1	6.2	5.866666667	6.7	6.8	6.6	6.7	7.9	6.4	6.6	6.966666667	
1950	6.5	6.4	6.3	6.4	5.8	5.5	5.4	5.566666667	5	4.5	4.4	4.633333333	4.2	4.2	4.3	4.233333333	
1951	3.7	3.4	3.4	3.5	3.1	3	3.2	3.1	3.1	3.1	3.3	3.166666667	3.5	3.5	3.1	3.366666667	
1952	3.2	3.1	2.9	3.066666667	2.9	3	3	2.966666667	3.2	3.4	3.1	3.233333333	3	2.8	2.7	2.833333333	
1953	2.9	2.6	2.6	2.7	2.7	2.5	2.5	2.566666667	2.6	2.7	2.9	2.733333333	3.1	3.5	4.5	3.7	
1954	4.9	5.2	5.7	5.266666667	5.9	5.9	5.6	5.8	5.8	6	6.1	5.966666667	5.7	5.3	5	5.333333333	
1955	4.9	4.7	4.6	4.733333333	4.7	4.3	4.2	4.4	4	4.2	4.1	4.1	4.3	4.2	4.2	4.233333333	
1956	4	3.9	4.2	4.033333333	4	4.3	4.3	4.2	4.4	4.1	3.9	4.133333333	3.9	4.3	4.2	4.133333333	
1957	4.2	3.9	3.7	3.933333333	3.9	4.1	4.3	4.1	4.2	4.1	4.4	4.233333333	4.5	5.1	5.2	4.933333333	
1958	5.8	6.4	6.7	6.3	7.4	7.4	7.3	7.366666667	7.5	7.4	7.1	7.333333333	6.7	6.2	6.2	6.366666667	
1959	6	5.9	5.6	5.833333333	5.2	5.1	5	5.1	5.1	5.2	5.5	5.266666667	5.7	5.8	5.3	5.6	
1960	5.2	4.8	5.4	5.133333333	5.2	5.1	5.4	5.233333333	5.5	5.6	5.5	5.533333333	6.1	6.1	6.6	6.266666667	
1961	6.6	6.9	6.9	6.8	7	7.1	6.9	7	7	6.6	6.7	6.766666667	6.5	6.1	6	6.2	
1962	5.8	5.5	5.6	5.633333333	5.6	5.5	5.5	5.533333333	5.4	5.7	5.6	5.566666667	5.4	5.7	5.5	5.533333333	
1963	5.7	5.9	5.7	5.766666667	5.7	5.9	5.6	5.733333333	5.6	5.4	5.5	5.5	5.5	5.7	5.5	5.566666667	
1964	5.6	5.4	5.4	5.466666667	5.3	5.1	5.2	5.2	4.9	5	5.1	5	5.1	4.8	5	4.966666667	
1965	4.9	5.1	4.7	4.9	4.8	4.6	4.6	4.666666667	4.4	4.4	4.3	4.366666667	4.2	4.1	4	4.1	
1966	4	3.8	3.8	3.866666667	3.8	3.9	3.8	3.833333333	3.8	3.8	3.7	3.766666667	3.7	3.6	3.8	3.7	
1967	3.9	3.8	3.8	3.833333333	3.8	3.8	3.9	3.833333333	3.8	3.8	3.8	3.8	4	3.9	3.8	3.9	
1968	3.7	3.8	3.7	3.733333333	3.5	3.5	3.7	3.566666667	3.7	3.5	3.4	3.533333333	3.4	3.4	3.4	3.4	
1969	3.4	3.4	3.4	3.4	3.4	3.4	3.5	3.433333333	3.5	3.5	3.7	3.566666667	3.7	3.5	3.5	3.566666667	
1970	3.9	4.2	4.4	4.166666667	4.6	4.8	4.9	4.766666667	5	5.1	5.4	5.166666667	5.5	5.9	6.1	5.833333333	
1971	5.9	5.9	6	5.933333333	5.9	5.9	5.9	5.9	6	6.1	6	6.033333333	5.8	6	6	5.933333333	
1972	5.8	5.7	5.8	5.766666667	5.7	5.7	5.7	5.7	5.6	5.6	5.5	5.566666667	5.6	5.3	5.2	5.366666667	
1973	4.9	5	4.9	4.933333333	5	4.9	4.9	4.933333333	4.8	4.8	4.8	4.8	4.6	4.8	4.9	4.766666667	
1974	5.1	5.2	5.1	5.133333333	5.1	5.1	5.4	5.2	5.5	5.5	5.9	5.633333333	6	6.6	7.2	6.6	
1975	8.1	8.1	8.6	8.266666667	8.8	9	8.8	8.866666667	8.6	8.4	8.4	8.466666667	8.4	8.3	8.2	8.3	
1976	7.9	7.7	7.6	7.733333333	7.7	7.4	7.6	7.566666667	7.8	7.8	7.6	7.733333333	7.7	7.8	7.8	7.766666667	
1977	7.5	7.6	7.4	7.5	7.2	7	7.2	7.133333333	6.9	7	6.8	6.9	6.8	6.8	6.4	6.666666667	
1978	6.4	6.3	6.3	6.333333333	6.1	6	5.9	6	6.2	5.9	6	6.033333333	5.8	5.9	6	5.9	
1979	5.9	5.9	5.8	5.866666667	5.8	5.6	5.7	5.7	5.7	6	5.9	5.866666667	6	5.9	6	5.966666667	
1980	6.3	6.3	6.3	6.3	6.9	7.5	7.6	7.333333333	7.8	7.7	7.5	7.666666667	7.5	7.5	7.2	7.4	
1981	7.5	7.4	7.4	7.433333333	7.2	7.5	7.5	7.4	7.2	7.4	7.6	7.4	7.9	8.3	8.5	8.233333333	
1982	8.6	8.9	9	8.833333333	9.3	9.4	9.6	9.433333333	9.8	9.8	10.1	9.9	10.4	10.8	10.8	10.66666667	
1983	10.4	10.4	10.3	10.36666667	10.2	10.1	10.1	10.13333333	9.4	9.5	9.2	9.366666667	8.8	8.5	8.3	8.533333333	
1984	8	7.8	7.8	7.866666667	7.7	7.4	7.2	7.433333333	7.5	7.5	7.3	7.433333333	7.4	7.2	7.3	7.3	
1985	7.3	7.2	7.2	7.233333333	7.3	7.2	7.4	7.3	7.4	7.1	7.1	7.2	7.1	7	7	7.033333333	
1986	6.7	7.2	7.2	7.033333333	7.1	7.2	7.2	7.166666667	7	6.9	7	6.966666667	7	6.9	6.6	6.833333333	
1987	6.6	6.6	6.6	6.6	6.3	6.3	6.2	6.266666667	6.1	6	5.9	6	6	5.8	5.7	5.833333333	
1988	5.7	5.7	5.7	5.7	5.4	5.6	5.4	5.466666667	5.4	5.6	5.4	5.466666667	5.4	5.3	5.3	5.333333333	
1989	5.4	5.2	5	5.2	5.2	5.2	5.3	5.233333333	5.2	5.2	5.3	5.233333333	5.3	5.4	5.4	5.366666667	
1990	5.4	5.3	5.2	5.3	5.4	5.4	5.2	5.333333333	5.5	5.7	5.9	5.7	5.9	6.2	6.3	6.133333333	
1991	6.4	6.6	6.8	6.6	6.7	6.9	6.9	6.833333333	6.8	6.9	6.9	6.866666667	7	7	7.3	7.1	
1992	7.3	7.4	7.4	7.366666667	7.4	7.6	7.8	7.6	7.7	7.6	7.6	7.633333333	7.3	7.4	7.4	7.366666667	
1993	7.3	7.1	7	7.133333333	7.1	7.1	7	7.066666667	6.9	6.8	6.7	6.8	6.8	6.6	6.5	6.633333333	
1994	6.6	6.6	6.5	6.566666667	6.4	6.1	6.1	6.2	6.1	6	5.9	6	5.8	5.6	5.5	5.633333333	
1995	5.6	5.4	5.4	5.466666667	5.8	5.6	5.6	5.666666667	5.7	5.7	5.6	5.666666667	5.5	5.6	5.6	5.566666667	
1996	5.6	5.5	5.5	5.533333333	5.6	5.6	5.3	5.5	5.5	5.1	5.2	5.266666667	5.2	5.4	5.4	5.333333333	
1997	5.3	5.2	5.2	5.233333333	5.1	4.9	5	5	4.9	4.8	4.9	4.866666667	4.7	4.6	4.7	4.666666667	
1998	4.6	4.6	4.7	4.633333333	4.3	4.4	4.5	4.4	4.5	4.5	4.6	4.533333333	4.5	4.4	4.4	4.433333333	
1999	4.3	4.4	4.2	4.3	4.3	4.2	4.3	4.266666667	4.3	4.2	4.2	4.233333333	4.1	4.1	4	4.066666667	
2000	4	4.1	4	4.033333333	3.8	4	4	3.933333333	4	4.1	3.9	4	3.9	3.9	3.9	3.9	
2001	4.2	4.2	4.3	4.233333333	4.4	4.3	4.5	4.4	4.6	4.9	5	4.833333333	5.3	5.5	5.7	5.5	
2002	5.7	5.7	5.7	5.7	5.9	5.8	5.8	5.833333333	5.8	5.7	5.7	5.733333333	5.7	5.9	6	5.866666667	
2003	5.8	5.9	5.9	5.866666667	6	6.1	6.3	6.133333333	6.2	6.1	6.1	6.133333333	6	5.8	5.7	5.833333333	
2004	5.7	5.6	5.8	5.7	5.6	5.6	5.6	5.6	5.5	5.4	5.4	5.433333333	5.5	5.4	5.4	5.433333333	
2005	5.3	5.4	5.2	5.3	5.2	5.1	5	5.1	5	4.9	5	4.966666667	5	5	4.9	4.966666667	
2006	4.7	4.8	4.7	4.733333333	4.7	4.6	4.6	4.633333333	4.7	4.7	4.5	4.633333333	4.4	4.5	4.4	4.433333333	
2007	4.6	4.5	4.4	4.5	4.5	4.4	4.6	4.5	4.7	4.6	4.7	4.666666667	4.7	4.7	5	4.8	
2008	5	4.8	5.1	4.966666667	4.9	5.4	5.6	5.3	5.8	6.1	6.2	6.033333333	6.6	6.8	7.3	6.9	
2009	7.8	8.2	8.6	8.2	8.9	9.4	9.5	9.266666667	9.5	9.7	9.8	9.666666667	10.1	9.9	9.9	9.966666667	
2010	9.7	9.7	9.7	9.7	9.8	9.6	9.5	9.633333333	9.5	9.6	9.6	9.566666667	9.7	9.8	9.4	9.633333333	
2011	9	8.9	8.8	8.9	9	9.1	9.2	9.1	9.1	9.1	9.1	9.1					

The above chart is of specific interest as it relates to the calculation of quantities of monthly unemployment in both their "feminine" and "masculine" characteristics, or put another way, their "circumferential" and "radial" characteristics.

To make the distinction plain, let us imagine that the march of months within a year was made congruent to the 12 hours on the face of the clock. The manner in which the data for unemployment is collected and analyzed against itself partakes of the circular nature of a unit circle. In this way the average of each quarter is taken and compared with the average of each other quarter. This is portrayed in the left hand side of the following chart.

Contrast this with a single month, chosen from the twelve, and it alone being contrasted with the same month of the following year, and then the following year, and so on.

DIAGRAM 1.
FEMININE AND MASCULINE NUMBERS



On the left we have a circumferential relationship between quarterly data which itself relies upon a circular sense of time, a legitimate apportion-izing of something which itself is taken as a "1." On the right we have a distinctly different and radial view of time, one which does not accept any obvious limitation to its ongoing list of endless Decembers.[42]

Note that the estimation of a "quarterly" rate for unemployment takes as its beginning source of numeric encouragement the idea that it is 1/4th of something else, specifically a sub-part of a 12-month, four-quarter year. If we were to have a full year specified in quarters then numerically we would be interested in a year stated as 4/4 which, according to number theory, would equal a single year.

[42] It must be noted, however, that the GNP Spiral assumes a further circular aspect of time applying even to annual data. Consequently the 14/15 association of feminine numbers in this regard, and the 15/14 association with masculine numbers, remains connected to this approach. In short, if the annual data itself falls into a larger circumferential relationship, what relationship might this have to the quarterly data which are, at best, a sub-part of the GNP Spiral and its 1:φ ratio over a span of 56 years?

Conversely the statement of an "annual" rate of unemployment seeks not an association between the data and the year itself, but rather to an *on-going set of years in sequence*. Consequently the rate of one December is compared to the rate of the next December and measured. In contrast to the quarterly data – which by definition is part of some other wholeness – we might state annual data as a repeated sense of "1," each point repeating itself in endless time, a 1/1.

Here we enter into the intrigues and quiet thoughts of the numbers themselves. Placing both feminine and masculine numbers together we see above a hinted "radius : 2π" relationship between annual and quarterly approaches using a single data set describing unemployment and a second single data set describing GDP growth. Three questions arise.

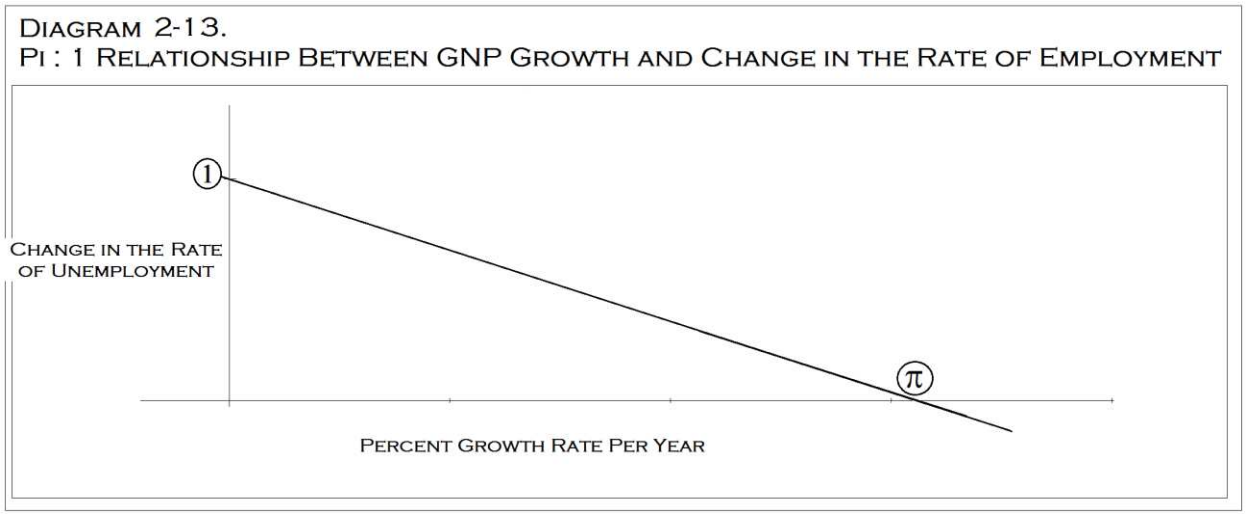
1. Do the feminine ($0<F<1$) numbers maintain a secret relationship with the quarterly employment figures, their circumferential sense of time and the fraction 14/15 as these relate to the GNP Spiral / Kondratiev Wave, perhaps "filling up" the space between moments of time?

2. Do the masculine ($1<M$) numbers share an equally hidden relationship with annual employment figures, their radial sense of time and the fraction 15/14 as these relate to the GNP Spiral / Kondratiev Wave, perhaps setting up "boundaries" separating moments of time?

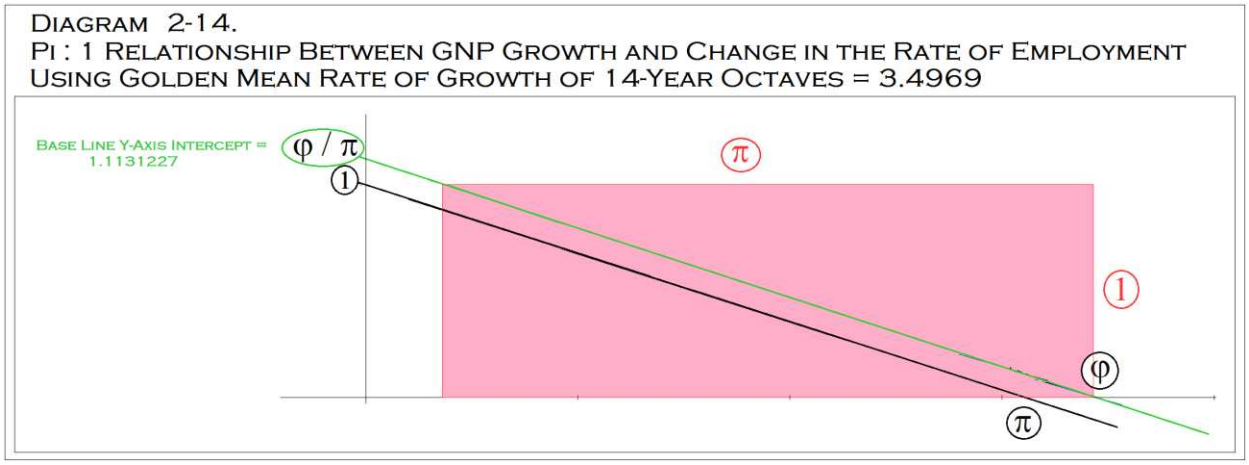
3. Under what circumstances might these secrets be revealed, secrets which although hidden, tentative and circumspect, might actually bear an inverse relationship of some sort to one another?

"The Harmonic Multiplicative Inverse Surprise"

The relationship which we are anticipating is that a 1: π relationship will exist between a percentage change in the rate of unemployment and the percentage growth of GNP. As the rate of growth increases on the x-axis, the rate of unemployment will go down on the y-axis. Setting this relationship as a straight-forward linear relationship, we have the following.



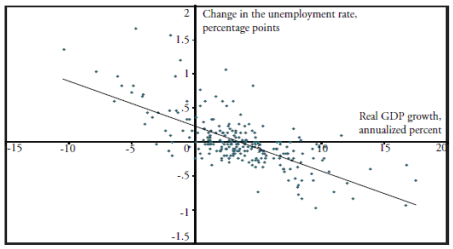
In order to establish a 1 : φ proportion over fourteen years the economy of the United States must possess a steady state rate of growth of approximately 3.4969% per year. As one calculates a 1 : π exchange between rates of unemployment and GDP growth under Okun's Law, one notices that the slope of the 1 : π relationship must remain the same, but that the y-intercept shifts slightly upwards, becoming not "1" but 3.4969 / π = 1.1131227.



Comparing this to the observed data calculated by Dr. Knotek, one notices that Chart One uses quarterly *growth data* which has been annualized. However quarterly *employment data* is *not* annualized.

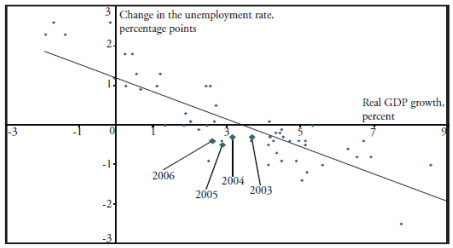
DIAGRAM 2-2.
CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"

Chart 1
THE DIFFERENCE VERSION OF OKUN'S LAW, QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2
THE DIFFERENCE VERSION OF OKUN'S LAW, ANNUAL DATA

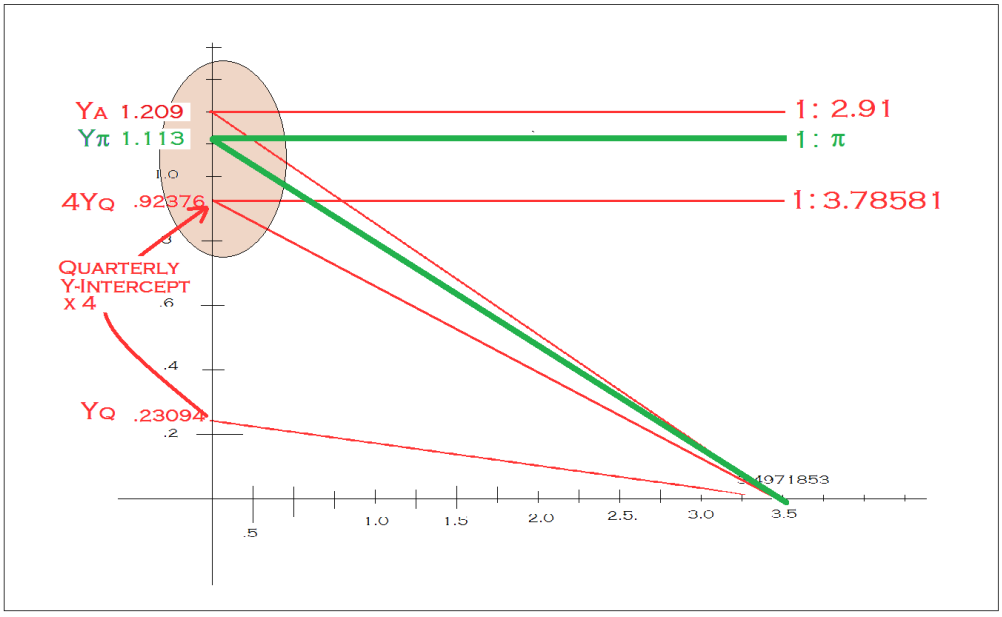


Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

We adjust the trend line for annualized quarterly data by multiplying quarterly employment data by four, thereby "annualizing" quarterly employment data. In this manner annualized quarterly data on growth is matched with "annualized" quarterly data on employment.

DIAGRAM 2-15.
MULTIPLYING QUARTERLY UNEMPLOYMENT RATE X 4

IMPLIED RATIOS BETWEEN
Y-INTERCEPT AND
X = 3.4551266 (ANNUAL X-INTERCEPT)
X = 3.4969781 (GOLDEN MEAN X-INTERCEPT)
X = 3.4972429 (QUARTERLY X-INTERCEPT)



If the steady state rate given for the Golden Mean proportion (3.4969 percent per year) is divided by π, the y-axis intercept is 1.1131227.

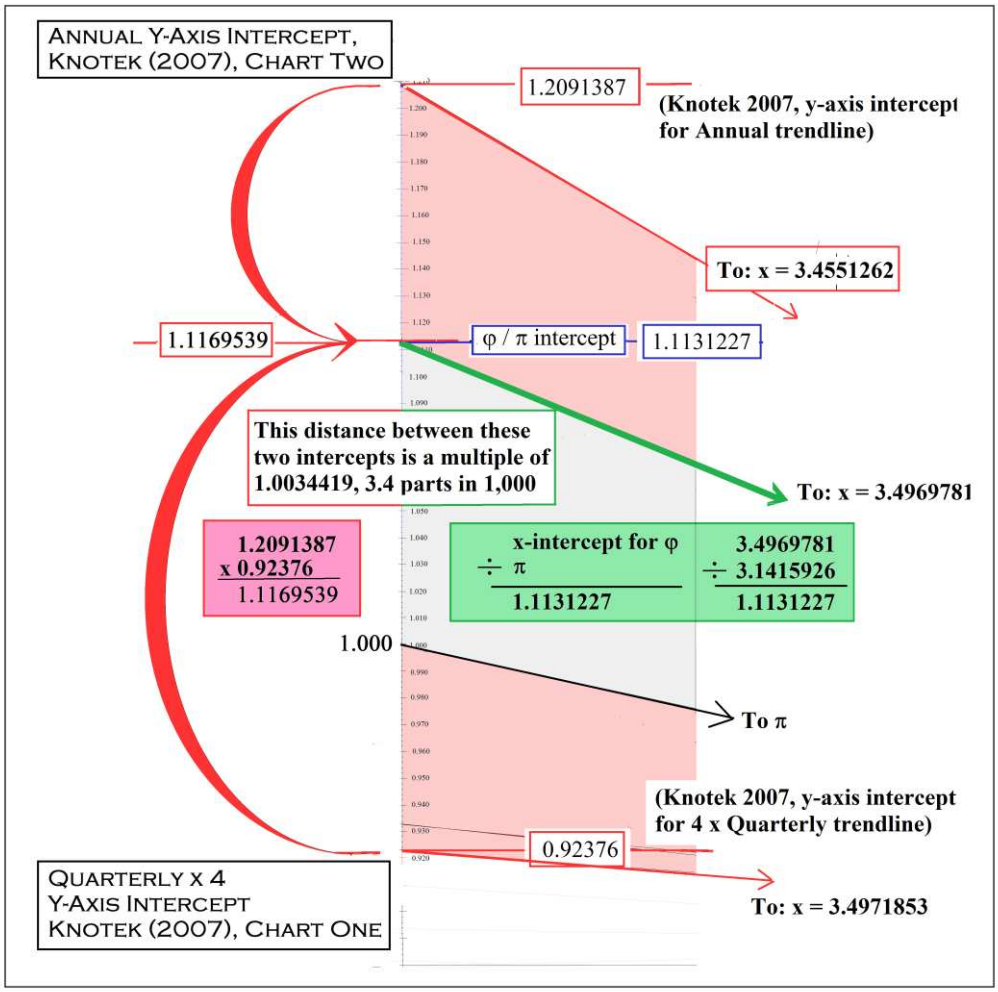
The "Annual" y-intercept given in Knotek 2007 is 1.2091387, and the "4 x Quarterly" y-intercept is 0.92376. We may multiply the two in order to test whether they are inverses around a common point. The multiple of these two intercepts is 1.1169539. The result is remarkable.

In short when the growth rate is zero (the y-axis), the y-axis intercepts for the Knotek:"Annual" and Knotek:"4 x Quarterly" trend lines create a "Harmonic multiplicative inverse" about a progenic y-axis intercept of 1.1169539.

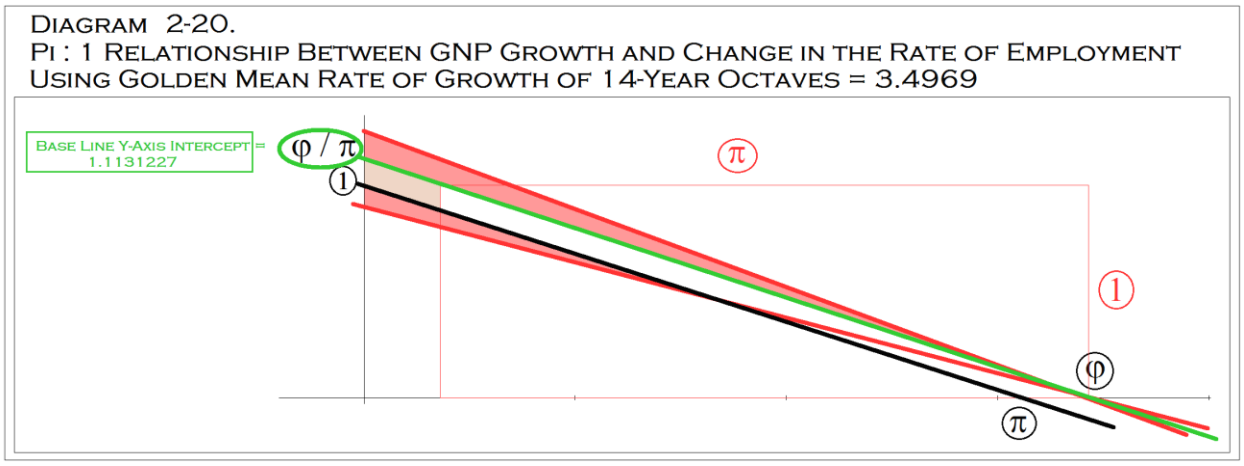
This is very proximate to the projected trend line connecting a 1:φ steady-state rate of growth with a π:1 slope for Okun's Law creating a y-axis intercept of 1.1131227.

These two y-axis intercepts are equal to one another to an accuracy of 0.34%, 3.4 parts in 1,000, or 99.65%. (See chart below)

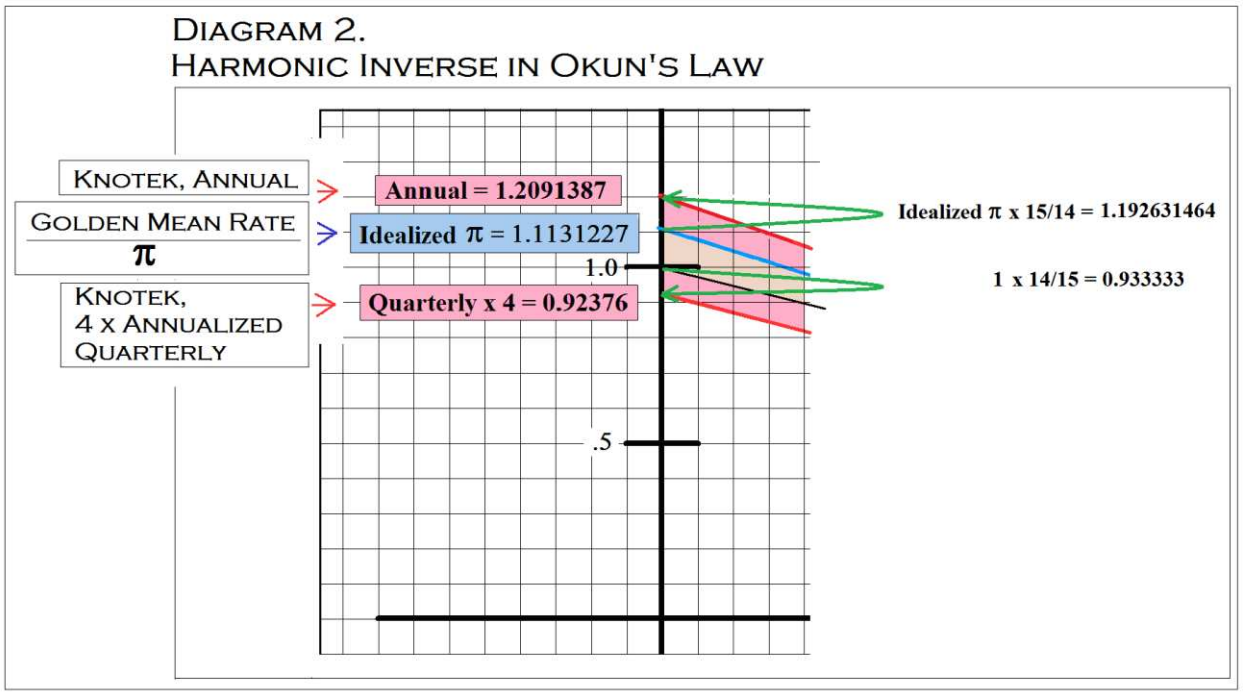
DIAGRAM 2.18. RELATIONSHIP BETWEEN φ AND π INTERCEPT (IN BLUE) AND KNOTEK'S ANNUAL AND "QUARTERLY X 4" Y-AXIS INTERCEPTS



In short, the possibility of two specific sets of numbers – feminine and masculine – as configured in the theory of a harmonic multiplicative inverse appears to generate a remarkable understanding of the econometric data underlying Okun's Law.[43]

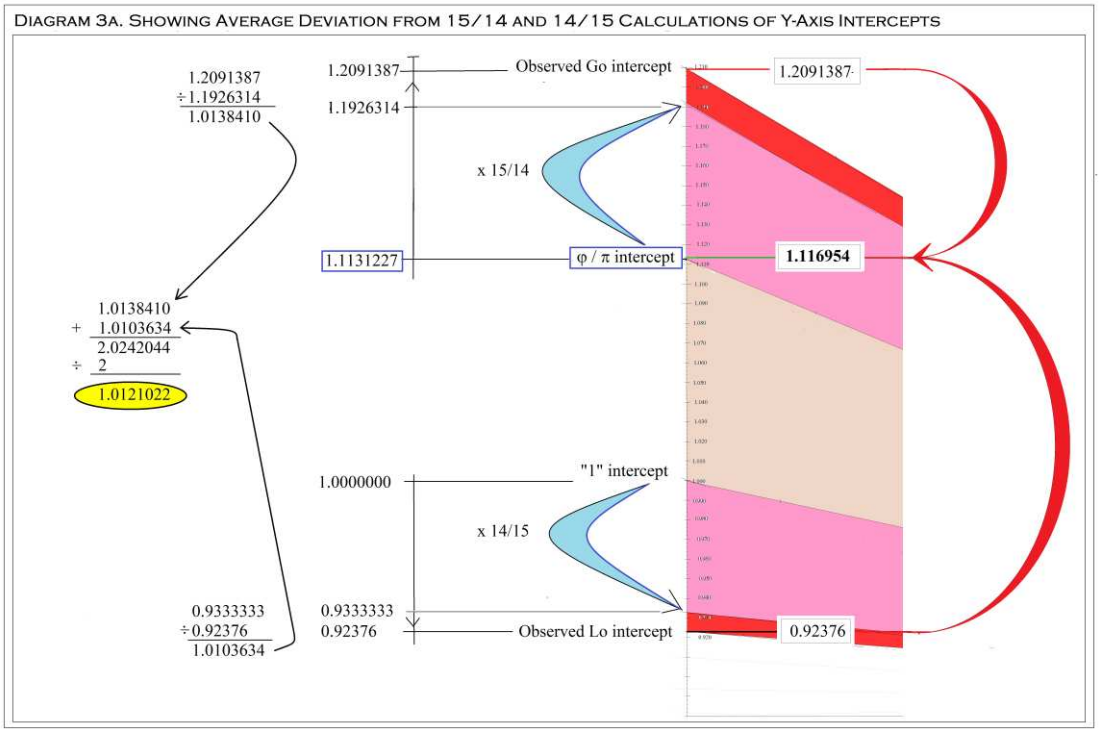


The advantages increase considerably if we connect the "1:π / 1:φ" trend line to an analysis of the Kondratiev Wave. The progenic π/φ intercept ("P") may be constructed from a feminine "14/15 x 1" as combined with a masculine "15/14 x P." (See "The Harmonic Inverse" *supra*) The resulting projections of Annual and Quarterly intercepts lie at variances from Knotek:4 x Quarterly at 1.0% and Knotek:Annual at 1.3%.

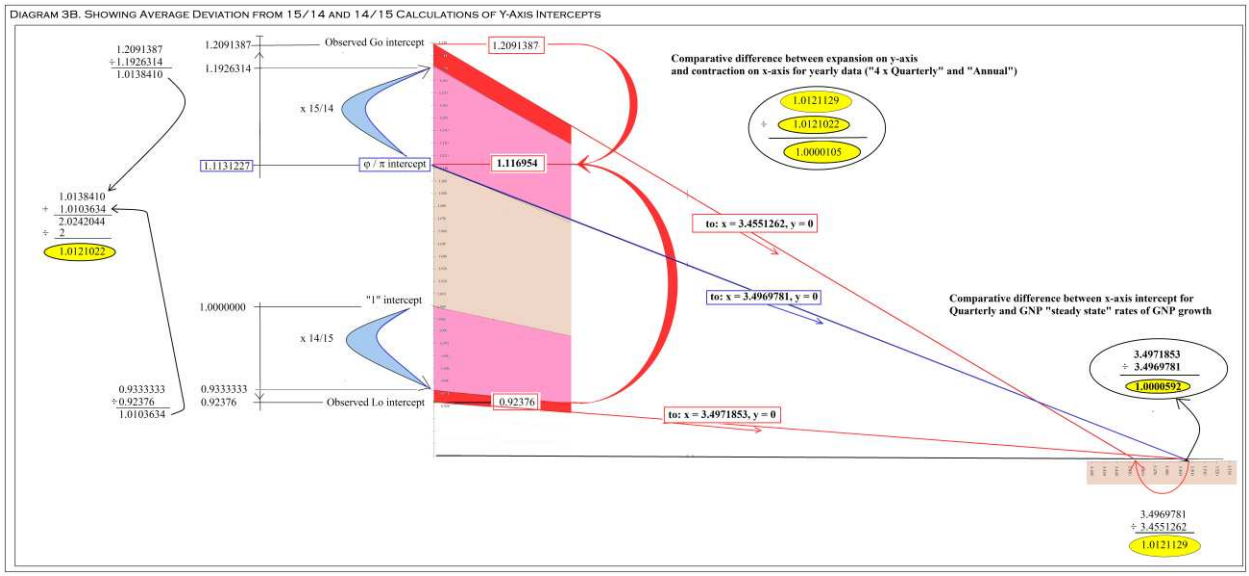


[43] A simple 3:1 ratio, with the same approach used, yields a y-intercept of 1.1656. This is contrasted with a π/φ intercept of 1.1131 / 1.1656 (at 95.49%) and an observed intercept of 1.1169 / 1.1656 (at 95.81%).

This yields an average expansion of 1.2% beyond the masculine and feminine inverses, or more specifically a multiple of 1.0121022, in yellow below.

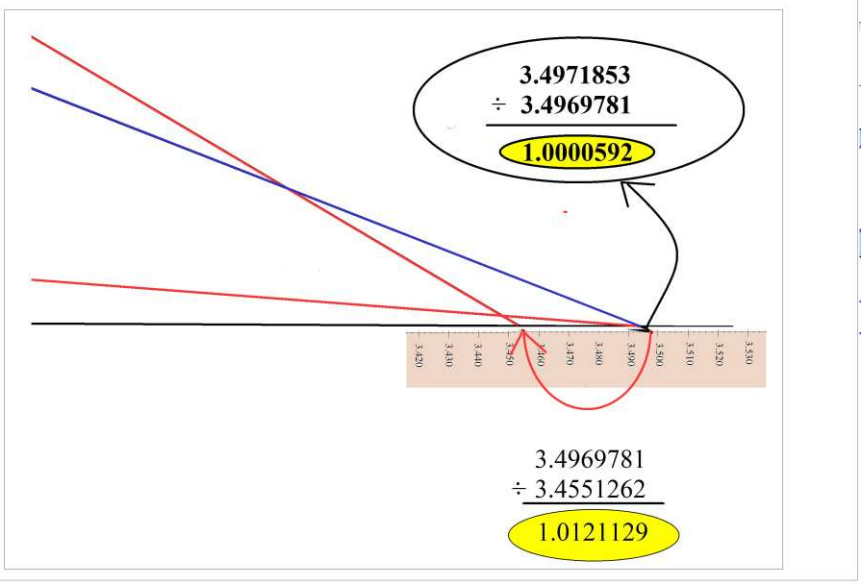


This y-axis deviation balances a similar deviation between growth rates along the x-axis. The steady state rate for Annual Data calculated by Dr. Knotek is 3.4551266. The steady state rate of growth calculated via the GNP Spiral (3.4969781) is greater than this number by a multiple of 1.0121129, virtually identical to the y-axis deviation stated above.



Let us consider more carefully the three rates we have for a steady state rate of growth, each of which constitutes an x-axis intercept. These are Knotek: Annual (3.4551262), Knotek: Annualized Quarterly (3.4971853) and the GNP Spiral (3.4969781).

DIAGRAM 3C.
COMPARATIVE DIFFERENCE BETWEEN X-AXIS INTERCEPT FOR STEADY-STATE RATES OF GROWTH:
KNOTEK ANNUAL, GNP SPIRAL, KNOTEK ANNUALIZED QUARTERLY



Dr. Knotek's data track slightly more than one complete circuit around the 56-year GNP Spiral, i.e. covering the second quarter of 1947 through the third quarter of 2007, a period of 60 years. This data misses the full range of GNP values available from the Department of Commerce (1869 through 1946), a period of 78 years. Moreover between 1869 and 1947 very large growth rates are found in GNP ratios. These larger ratios are included as a part of the calculation of the GNP Spiral.

Despite the incongruity of data sets Knotek:Annualized Quarterly (3.4971853) is virtually the same as that given for the GNP Spiral (3.4969781).

When the larger (Knotek: Annualized Quarterly = 3.4971853) is divided by the smaller (GNP Spiral = 3.4969781) a multiple of 1.0000592 is found, indicating a proximity between the two numbers of 5.9 parts in 100,000.[44]

[44] This result, as first pointed out by Dr. Knotek in an email of June 24, 2011, was the genesis of the correspondence resulting in this paper.

Given the absence of GNP data pre-dating 1947, one might expect that the Knotek:Annual must be smaller than that of the growth rate calculated by the GNP Spiral. Indeed the x-axis intercept for Knotek:Annual (3.4551262) retreats from the expected GNP Spiral x-axis intercept (3.4969781), the second being larger by a multiple of 1.0121129.

As noted previously, this compares to an expansion along the y-axis for unemployment averaging between feminine and masculine components of 1.0121022.

When the deviation along the x-axis 1.0121129 is divided by the deviation along the y-axis 1.0121022 a multiple of 1.0000105 results. This indicates that a balance between growth and employment along a $1{:}\varphi$ / $1{:}\pi$ trendline is accurate to within 1.05 parts in 100,000. It further suggests that while unemployment states a Harmonic multiplicative inverse, growth is not figured in such a way.

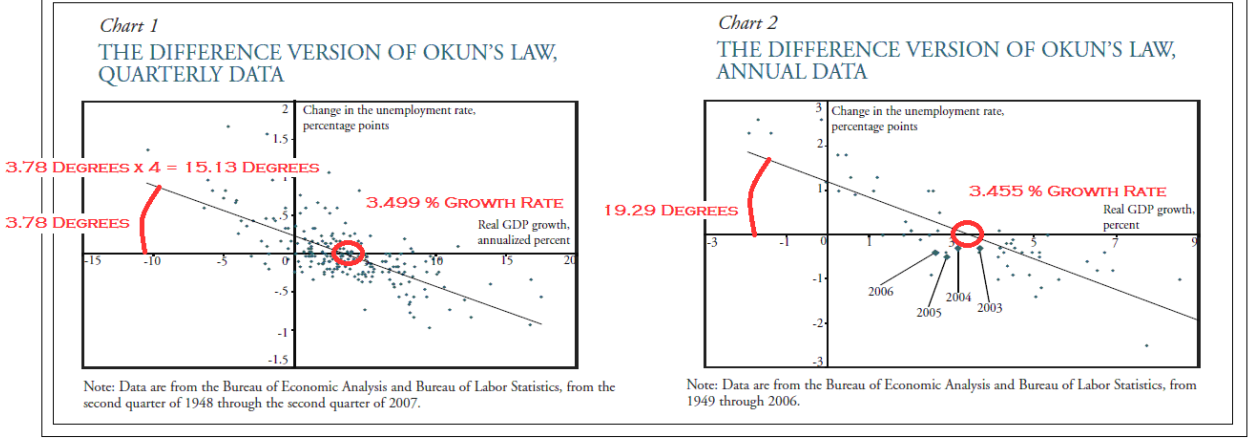
This leads to the following insight as to the operation of the harmonic multiplicative inverse and its impact upon the analysis of data surrounding Okun's Law.

When change in the rate of unemployment is zero, the rate of growth is seen clearly; there is no inverse at all to found in the growth data.

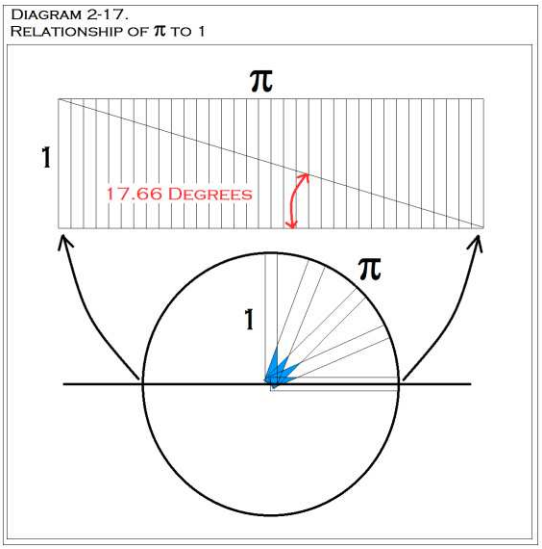
When the growth of GDP is zero, quarterly and annual rates of unemployment at in great flux and we see quite clearly the Harmonic multiplicative inverse in the unemployment data.

A second test of the harmonic multiplicative inverse may be found in the fact that the π:1 understanding of Okun's Law generates an angle bisecting that of Charts One and Two to within half a degree. These angles are 15.13 degrees for annualized quarterly data and 19.29 degrees for annual data.

DIAGRAM 2-16.
CHARTS ONE AND TWO OF "HOW USEFUL IS OKUN'S LAW?"



The angle created by the rectangle π : 1 bisects these two within one half of one degree, i.e. 17.66 degrees.



In other words, the slope of the angle bisecting the angles given in Charts One and Two is 17.213 degrees, less than half a degree from the slope of 17.66 degrees of a projected relationship between the constant π and 1 as projected by this approach.

Conclusion to Part One: Understanding of the Data

The same data, used in different guises, leads to two separate lines, each of which are the harmonic inverses necessary to bring together the π:1 relationship running Okun's law.

Let us consider how the mathematics of these lines might come about.

If John runs a race beginning at point "x", over space "y", and ending at point "z", the "rate" at which John has run will be:

$$\frac{z - x}{time}$$

If x=0 and z=5 miles from 0, and the race is over one hour, then John has run at the rate of (5 miles – 0)/one hour, or 5 miles per hour.

The "substance" or "effect" of running this distance is "y." In "y" we may count drops of sweat, hills and valleys, rocks, snakes dodged, etc. But whatever has happened to John between x and z (i.e. "y") is ignored by virtue of the (z-x)/time equation.

If we make "x" and "z" "markers" then the rate is composed of 2 markers / 1 substance. As this accounts for Okun's law, we have rate / effect. Nevertheless the number of markers which create the rate will always be one more than the number of effects or 2/1.

Okun disregards the notion of the two markers which create a single rate. By using the employment "rate" as a proxy variable for all subsidiary effects he inadvertently creates a "1/1" fraction between the "rate" and the lesser, included, subsidiary "effects." This poses a difficulty for econometrics because it leads to scores of evaluations for Okun's law without a consideration of what is being placed side-by-side for comparison.

We can just as easily consider the inverse of the above with the substance of the race (miles traversed, people applauding, mountains climbed, ankles broken, etc.) in a myriad of various "effects." But no matter how multitudinous the "effects" considered, the overall substance of these "effects" equates simply with the inverse of the first equation, i.e. substance / markers = effects / rate = ½.

It has proved salutary to see the chart of annual data as asking a "national employment rate" question (i.e. 15/14 x "y=φ/π" intercept) and the quarterly data as asking a personal "effects" question (14/15 x "y=1" intercept) both of which are engaged by Okun's approach. The questions posed are "harmonic inverses" of one another in a fashion not contemplated by Okun's law but implied nevertheless by the leap of logic which Okun makes at the beginning of his approach.

To clarify the "harmonic" approach to Okun's law advanced here let us imagine a musician sitting at a piano, holding down the damper pedal thereby "opening" all the strings to vibration, and playing a single "Middle C" on the keyboard. If the musician "sounds" the string and then stops its vibration while allowing all other strings to be vibrate clearly, one will hear the overtones of "middle C" quietly "humming" their various tones without any apparent effort by the musician. These overtones are the lengths of string which are mathematically close to the vibration of C itself. These strings resonate by "sympathetic vibration" or "sympathetic harmony" to Middle C, even after the Middle C string is silenced.

In a similar fashion arranging employment and growth rates in national data brings forth a "resonance" to the idea of growth vs. employment vis-à-vis *annual* data because it links multiple yearly "toruses" together into a common, overarching pattern, a 15/14.

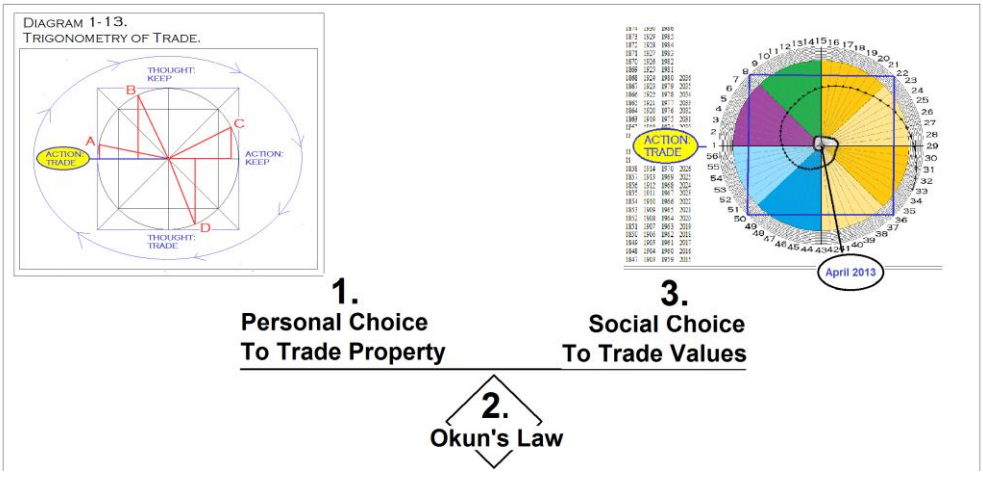
This is quite different than arranging growth and employment rates vis-à-vis *annualized quarterly data*, although technically they are supposed to be close to the same thing. This second chart sets up a sympathetic vibration amongst the data which is fundamentally different than that of the annual data because it seeks not a rate generated between end year dates but rather quarters of a single year which, in turn, are sub-portions of the single year wherein the torus was constructed.

It happens, however, that these questions are actually inverses surrounding a single topic raised by Okun's law, i.e. the proper understanding of growth vs. employment. This inverse relationship is further highlighted by the fact that the models given for immediate personal choice and long-term national choice are fractals of one another, mirror images of one another at differing scales. This may be considered more closely if we consider once again the "rates" vs. "effects" dichotomy set up by Okun at the beginning of his paper.

In other words, when growth is determined via the GNP Spiral to be a measurement of a rate of growth over 14 years (3.4969%) we may state this as emphasizing one of two aspects of this relation as presented by Okun,

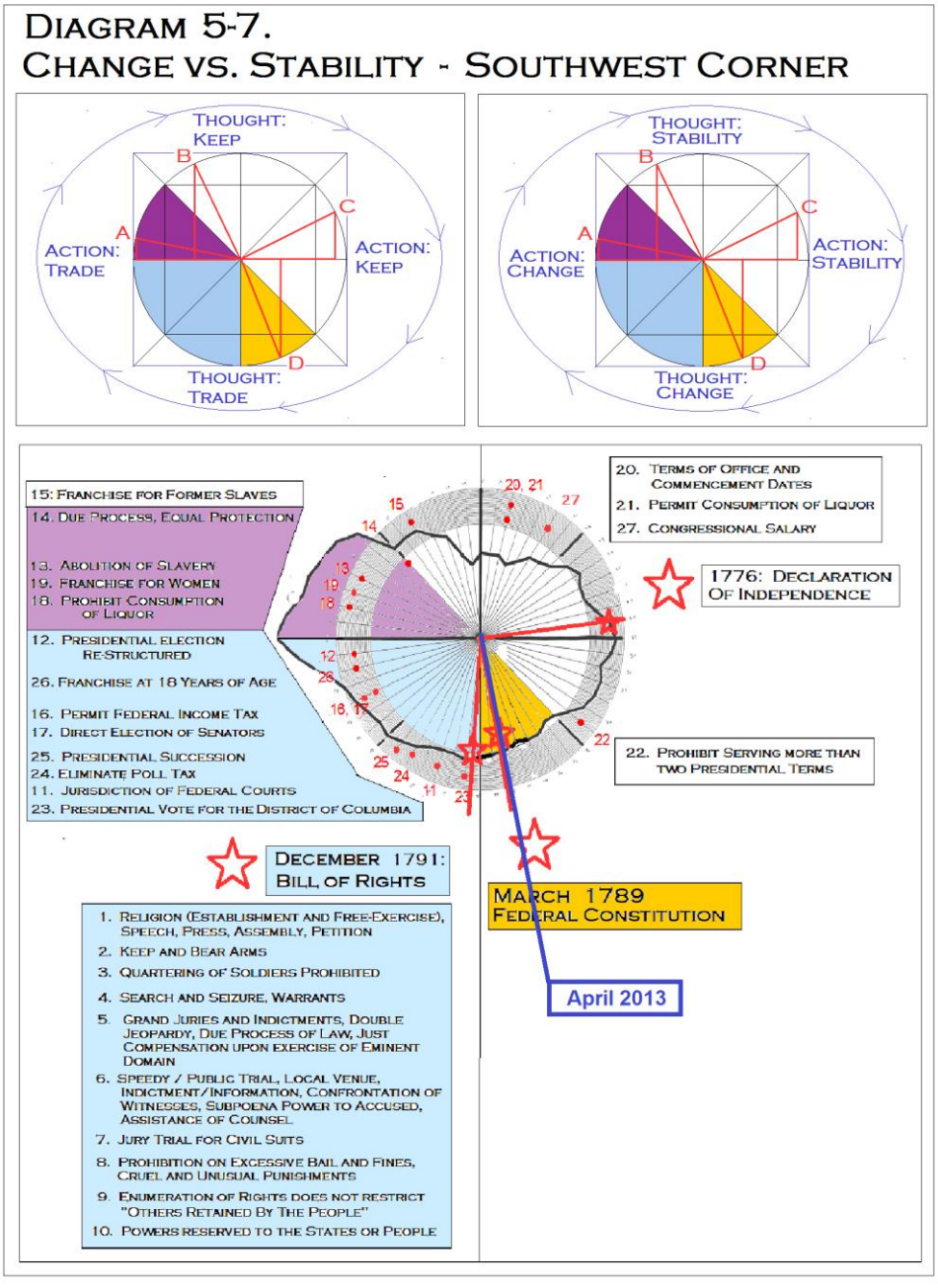
	National		*Personal*
i.e.	rate/effects	OR	effect/rate;
i.e.	15/14 x "$y=\varphi/\pi$" intercept	OR	14/15 x "$y = 1$" intercept.

The data used to evaluate these questions are the same data, but the harmonies which are raised from the data in response to these different questions or inverse points of view is the underlying basis for the strict and virtually identical correlations derived herein. Here the strict symmetry of the models given for personal choice in trading material goods and services and social choice in trading values over time may be key in maintaining the π:1 proportion which underlies both the ratio and stability of Okun's Law in the United States.

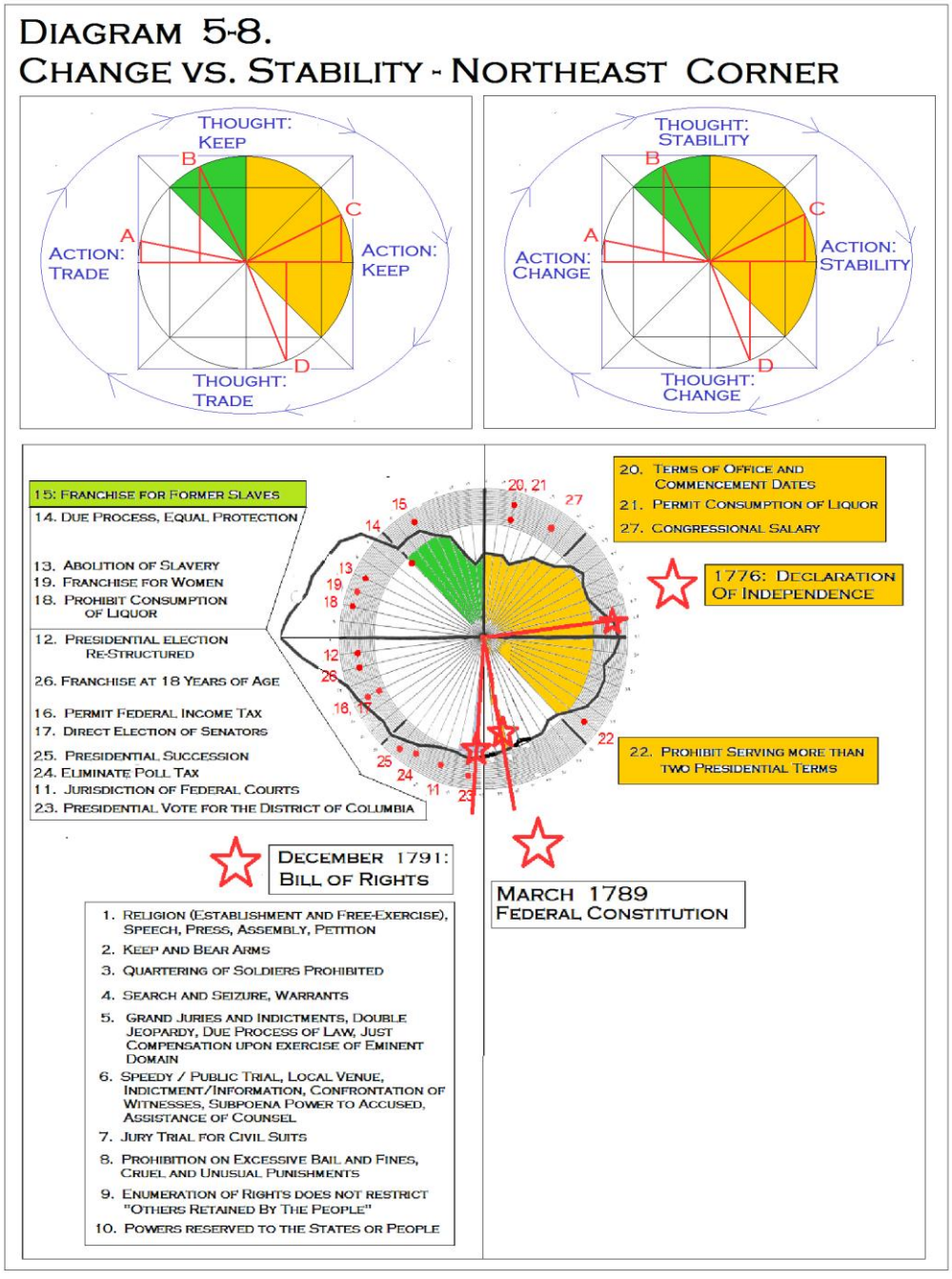


Part Two: The Kondratiev Wave

The GNP Spiral gives rise to an evaluation of the economic history of the United States. This may be stated in a circuit of 56 years, i.e. eight sections of 7 years each. (Albers & Albers 2013) The political and economic emphasis of these different periods has economic impact upon Okun's law. At the present time, we face a sea change in political attitudes. (April 2013) During comparable historic periods frustration with the political status quo has led to significant and enduring constitutional change. These changes alter the ground rules of economic engagement and permit the capitalistic enterprise to move forward. The following constitutional amendments are associated with historically comparable periods of time.



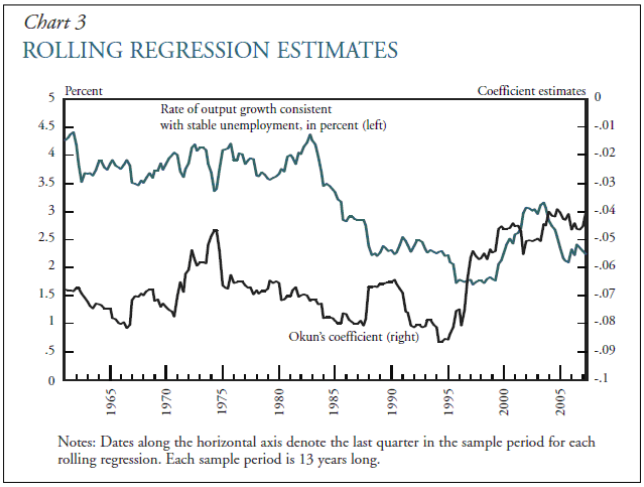
The period of time which we are leaving is one of tremendous conservatism, a period during which the rules previously laid down are made permanent to the satisfaction of a newly empowered political elite. Historically comparable periods of time are associated with the Articles of Confederation, the rise of slavery in the South and the westward expansion of the United States, the Gilded Age and power of the Robber Barons, and the international dominance of the United States post-World War II.



Consequently the straightforward presentation of a π:1 ratio within the data underlying Okun's Law also must take into account the tremendous underlying political change which this involves. These are part of Okun's Law as well. As Dr. Knotek argues, the stability of the trend lines provided in Charts One and Two mask the underlying dynamics of these relationships. He writes:

> One problem with a long time series – such as from 1948 to 2007 – is that history can hide changes in relationships. This is the case for Okun's law. The previous section (Charts 1 and 2) found considerable similarities between Okun's original estimate and an updated regression using a longer time series. This section shows that, when estimated over shorter time horizons, the relationship between changes in the unemployment rate and real output growth has varied considerably.
>
> To capture this variation, this article uses a technique called rolling regressions. A rolling regression estimates a particular relation over many different sample periods. Each regression produces a set of estimated coefficients. If the relationship is stable over time, then the estimated coefficients should be relatively similar from one regression to the next. Variations in the relation will appear as sizeable movements in the estimated coefficients.

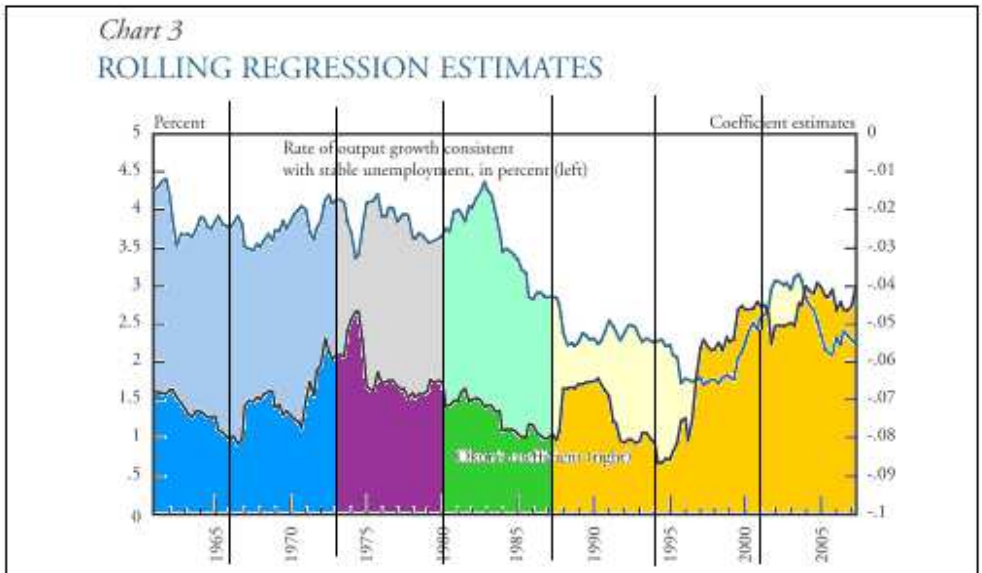
DIAGRAM 2-3.
CHART THREE OF "HOW USEFUL IS OKUN'S LAW?"



This apparent chaos may be understood by recalling that Okun's Law, being the consequence of the inverse between national and personal aspects of the economy, is itself intimately connected to the GNP Spiral and the passage of time within it. In consequence we are able to divide the time periods shown by Knotek's Chart Three above into quite specific, distinct *and predictable* periods of definite duration, each of which possesses its own unique political economy.

In short, using ideas proposed by Nicholai Kondratiev in 1925 we may develop a theory undergirding the seeming randomness of Knotek's Chart Three, one which brings light to the proper understanding of a short-term calculation of Okun's Law, without diminishing its long-term perspicuity.

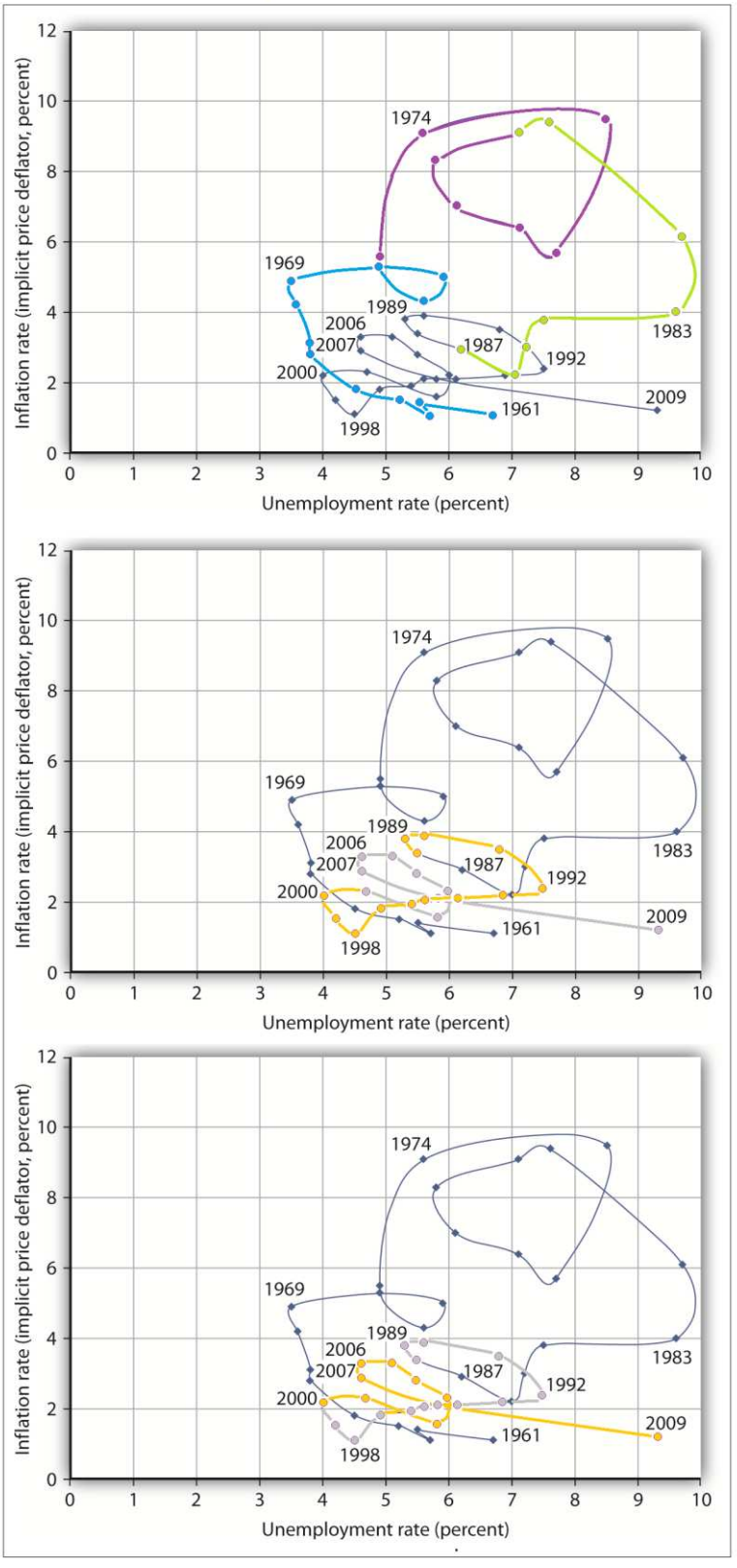
DIAGRAM 5-9.
ROLLING REGRESSIONS WITH HISTORIC CONTEXT



This coloration scheme further may be used to investigate the Phillips curve, i.e. the relation between unemployment and inflation. Comparing these two graphs simultaneously demonstrates important correlations between various periods of American economic history.[45] Quotes by Robert Gordon, with his permission, are provided next to the period described by these graphs as taken from his article "*The Demise of Okun's Law and of Procyclical Fluctuations in Conventional and Unconventional Measures of Productivity*," July 21, 2010.

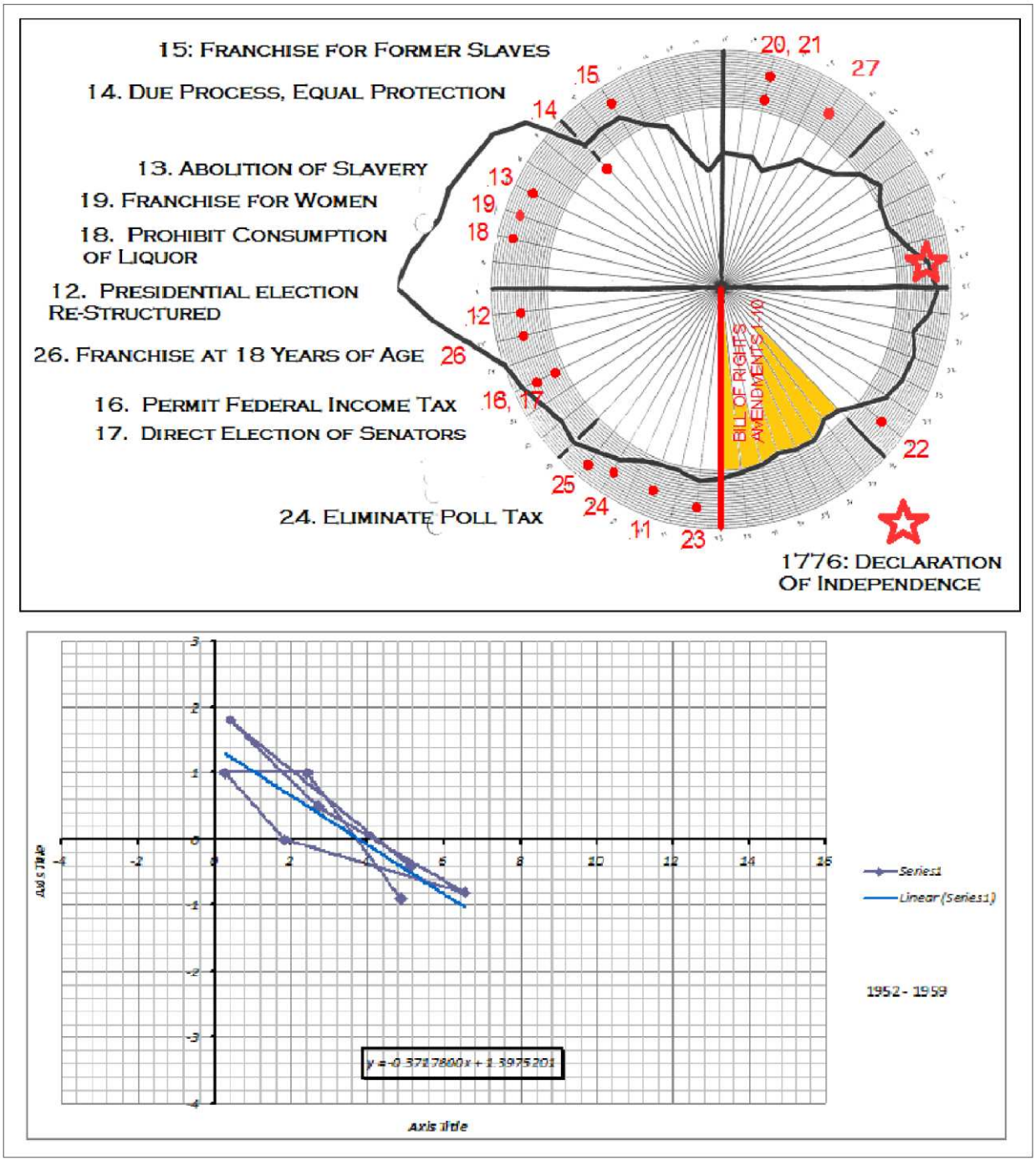
[45] See e.g Gordon, 2012: 35: "No paper can discuss or analyze cyclical gaps in output, hours, productivity, or employment until they have done their preliminary homework of determining the underlying growth trends from which the "gaps" are a deviation. ... Once the trends have been created, the ratios of actual to trend (or "gaps" can be examined. An important finding is that volatility in the cyclical gap for labour hours has gradually increased relative to the output gap."

DIAGRAM 4.
THE PHILLIPS CURVE ARRANGED BY 7-YEAR PERIODS

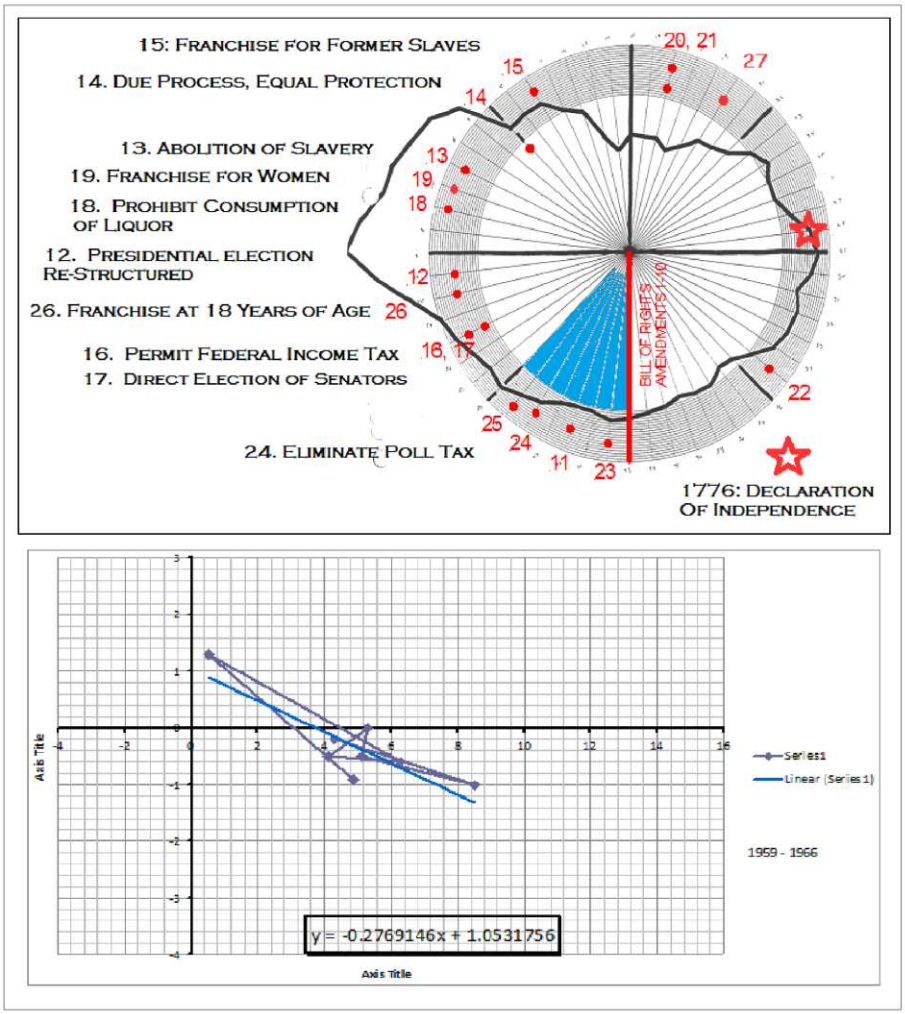


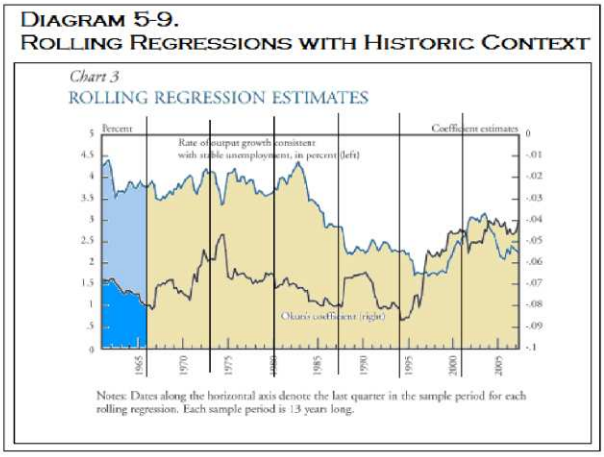
Using these separate portions of the circuit as guides, let us now re-evaluate Okun's Law as it moves around the 56-year circuit.

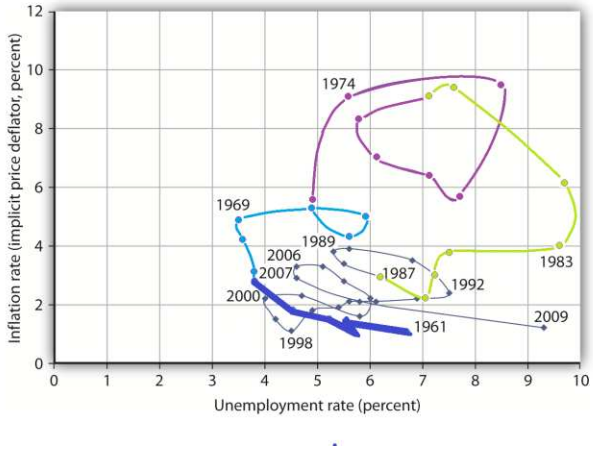
Early Evolving Revolution – 1952-1959
(Formerly 1784-1791, 1840-1847, 1896-1903)



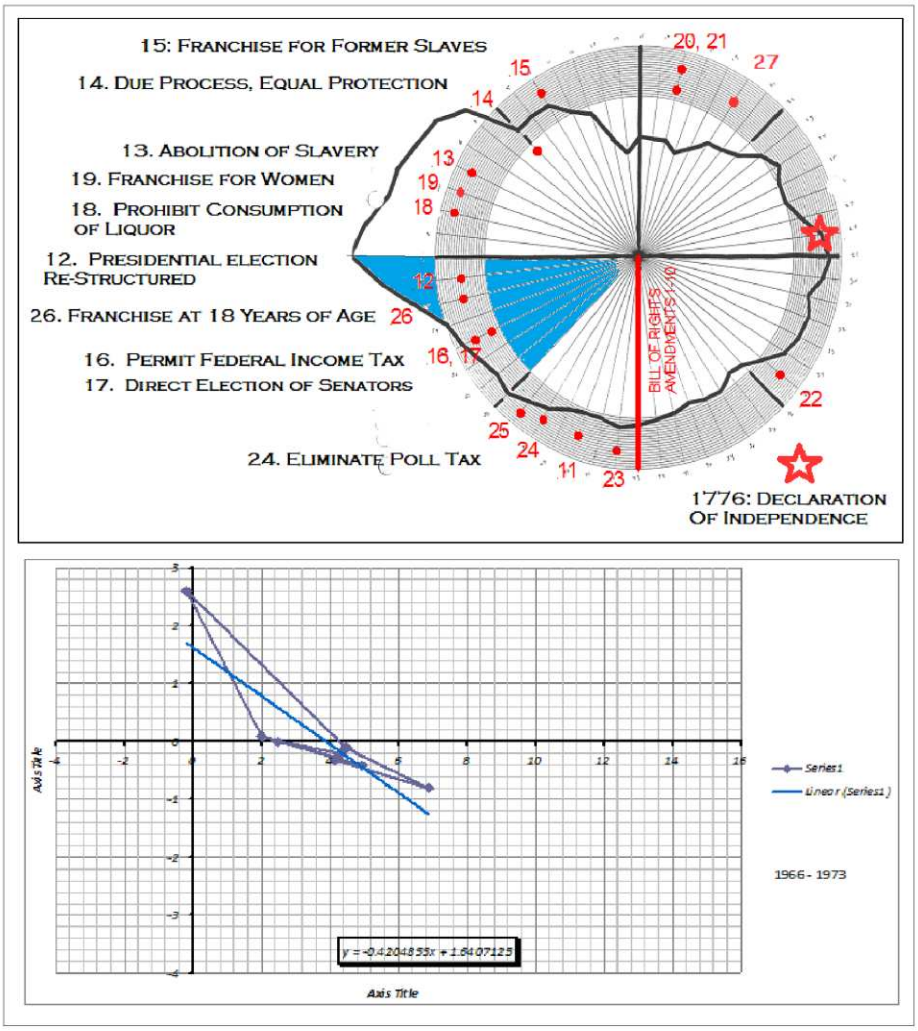
Late Evolving Revolution – 1959-1966
(Formerly 1791-1798, 1847-1854, 1903-1910)

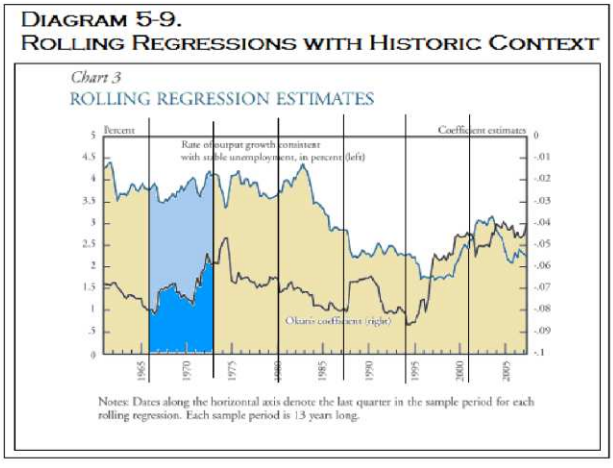


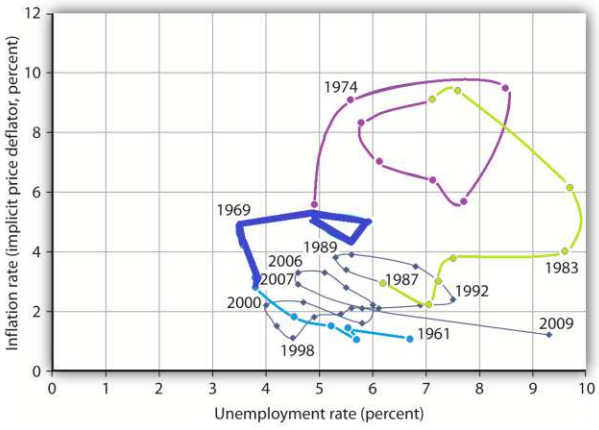




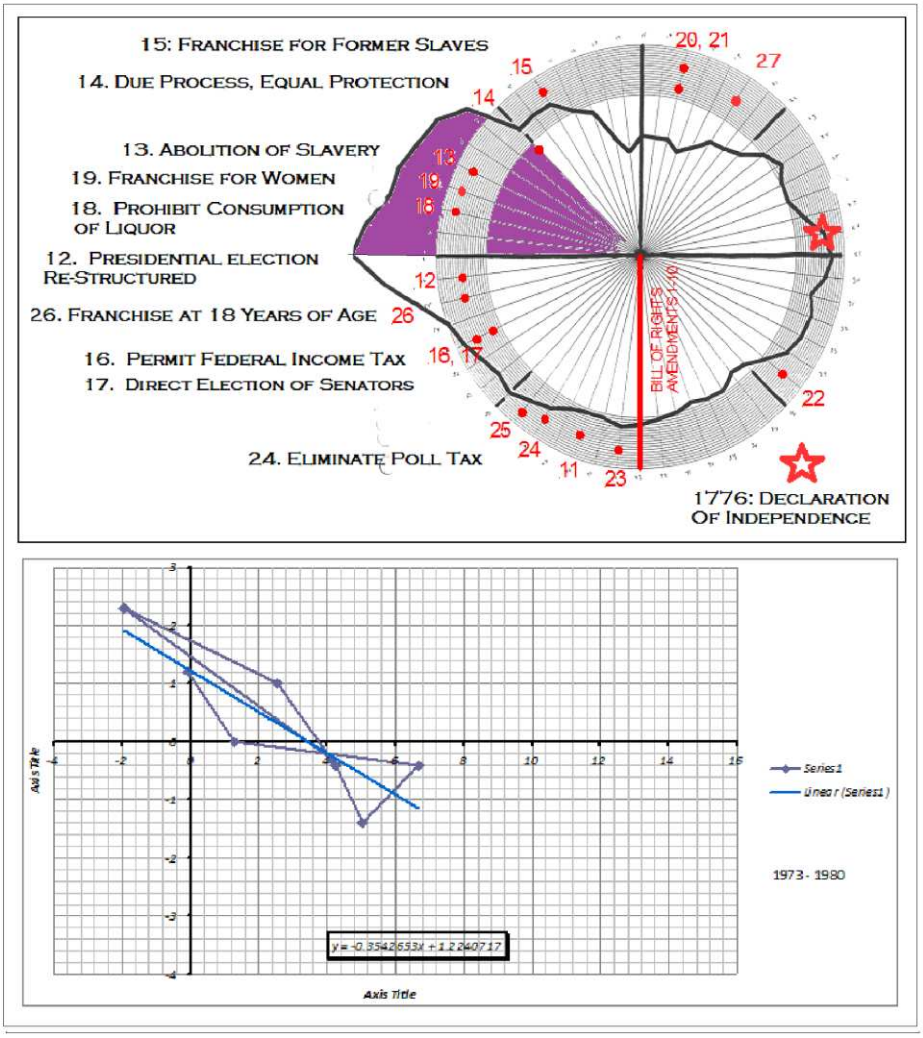
Early Revolution – 1966-1973
(Formerly 1798-1805, 1854-1861, 1910-1917)

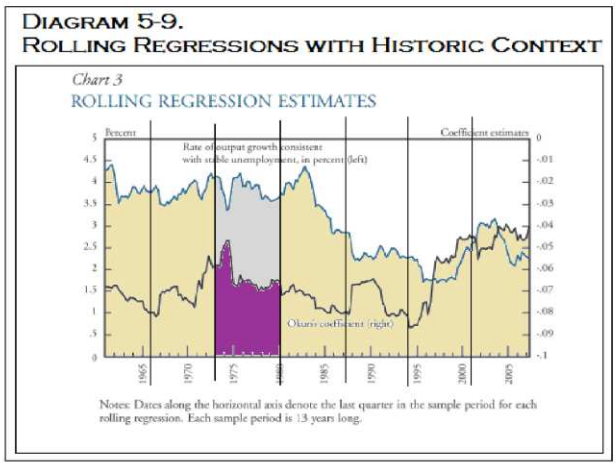


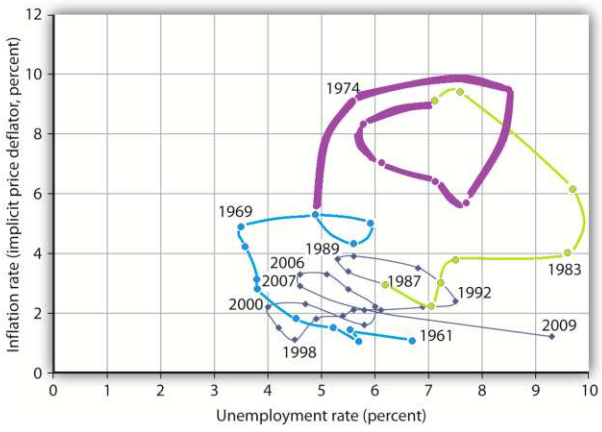




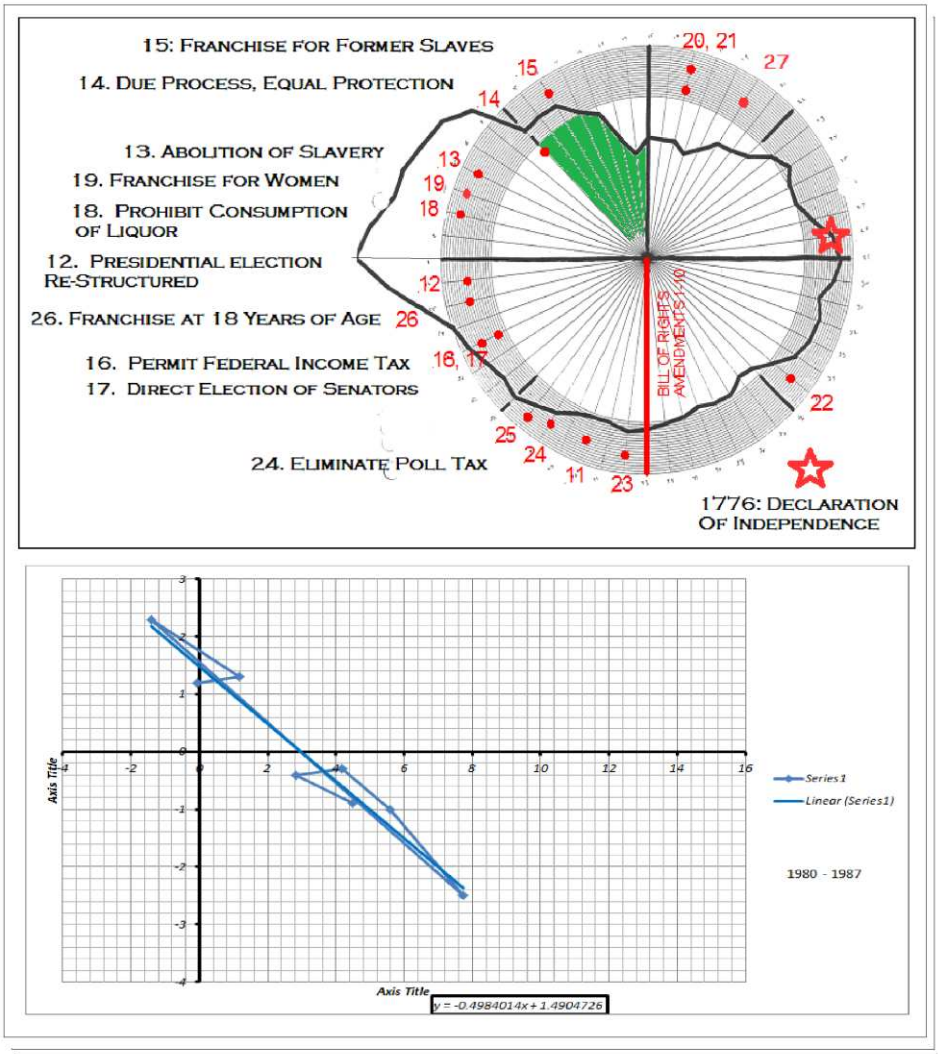
Late Revolution – 1973-1980
(Formerly 1805-1812, 1861-1868, 1917-1924)

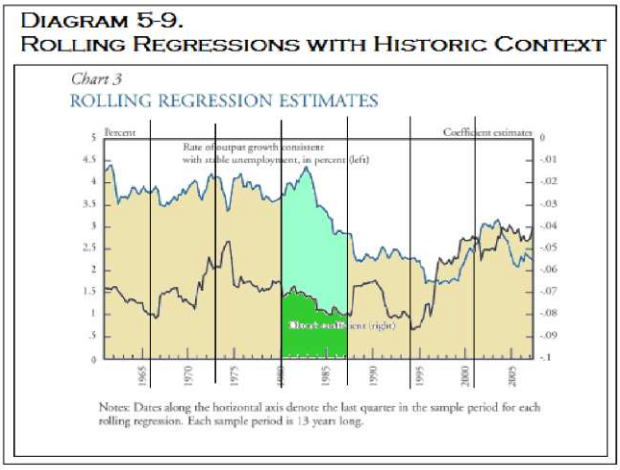


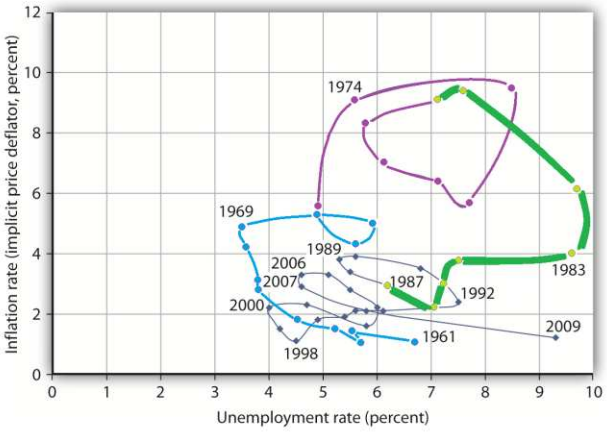




Early Evolving Consolidation – 1980-1987
(Formerly 1812-1819, 1868-1875, 1924-1931)



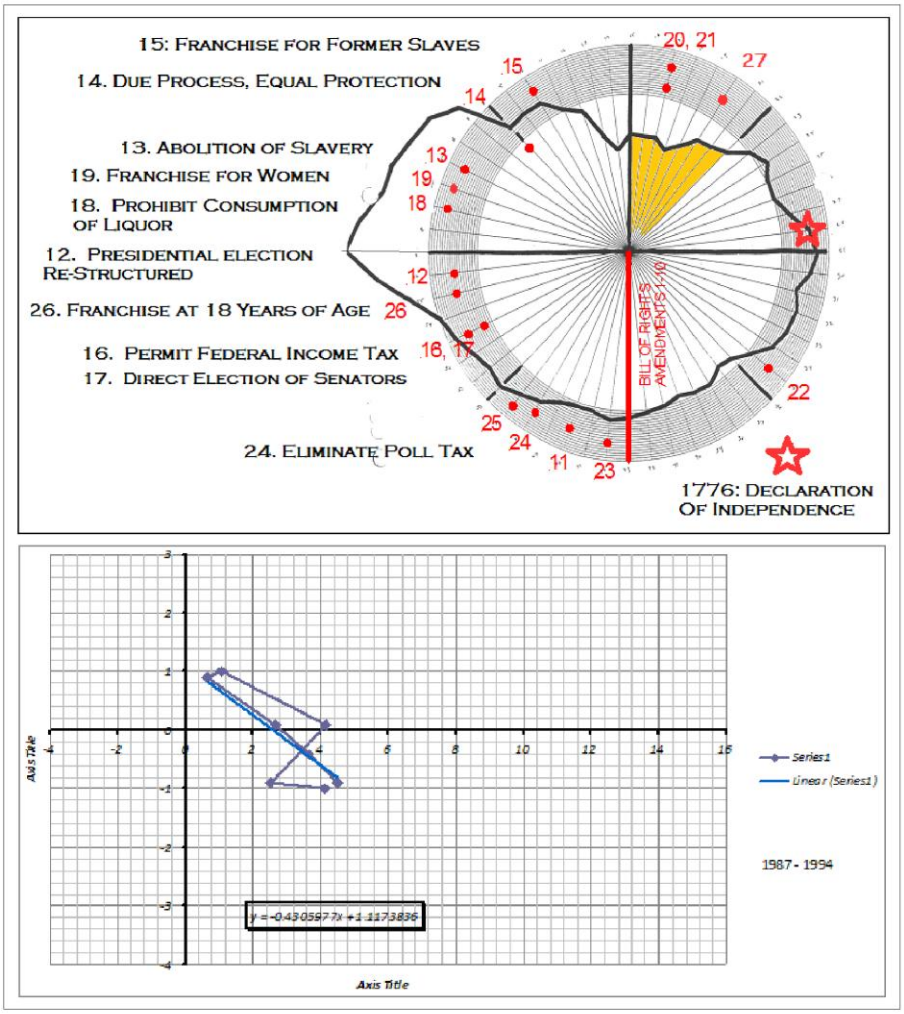


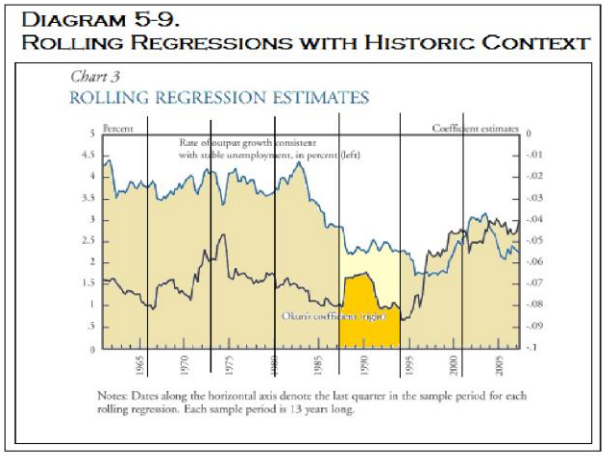


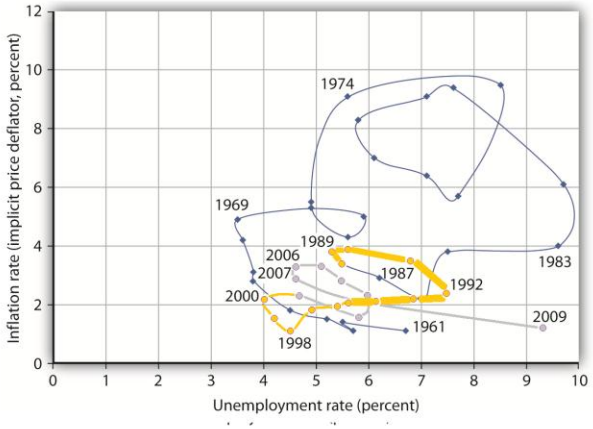
See e.g. Gordon 2010: 34.

> "Our basic conclusion is that Okun's Law was approximately correct for the cyclically volatile period between 1954 and 1986, but that since 1986 a marked structural shift has occurred in the responses of hours and productivity to cyclical fluctuations in real GDP."
>
> "This paper suggests a set of complementary hypotheses to explain these changes n behavior The most important of these is the "disposable worker" hypothesis. Starting in the 1990s business firms began to increase their emphasis on maximizing shareholder value, in part because of a shift in executive compensation toward stock options The overall shift in structural responses in the labor market after 1986 were caused by many of the same causes that have previously been proposed to explain the increase in American inequality. These include the role of the stock market in boosting compensation at the top, together with several forces that have increase income dispersion in the bottom 90 percent of the distribution. These include the declining minimum wage, the decline of unionization, the increase of imported goods, and the increased immigration of unskilled labor. Taken together these factors have boosted incomes at the top and have increased managerial power, while undermining the power of the increasingly disposable workers in the bottom 90 percent of the income distribution. As a result, employers can reduce labor hours with impunity and without restrain in response to a decrease in the output gap in contrast to the period before 1986 when their behavior was more constrained by the countervailing power of labor."

Late Evolving Consolidation – 1987-1994
(Formerly 1763-1770, 1819-1826, 1875-1882, 1931-1938)



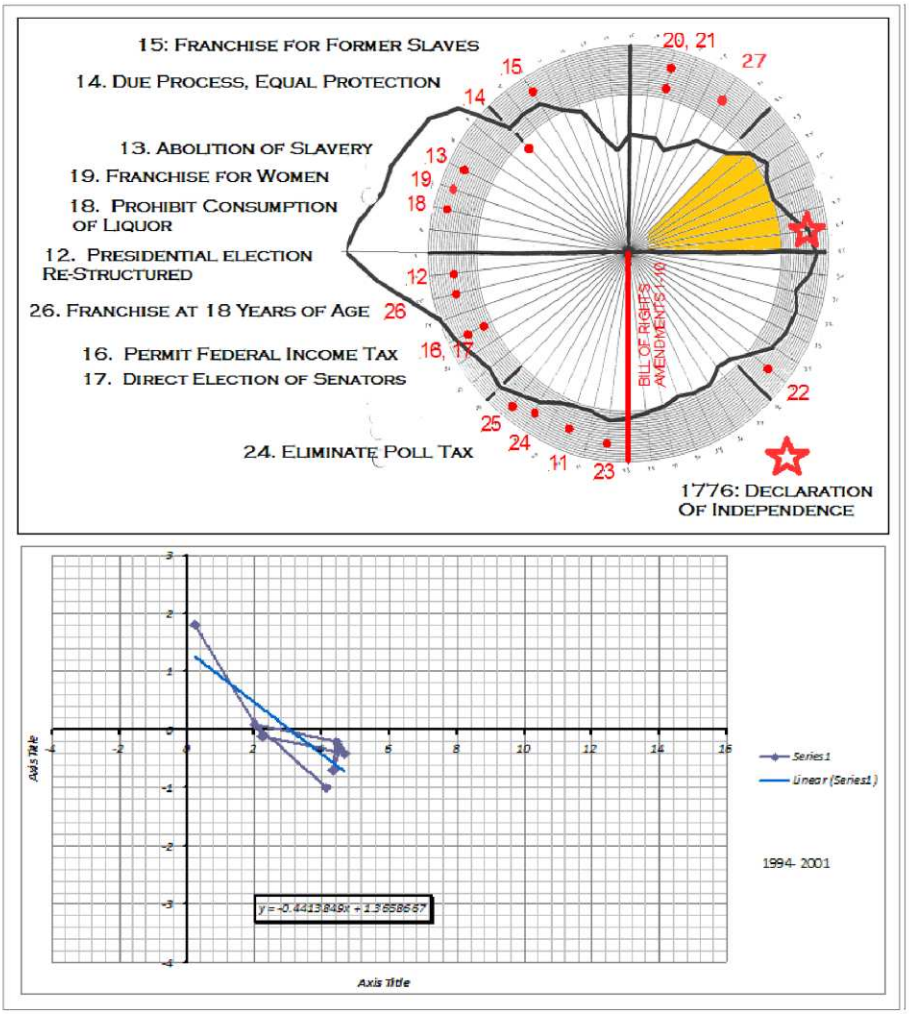


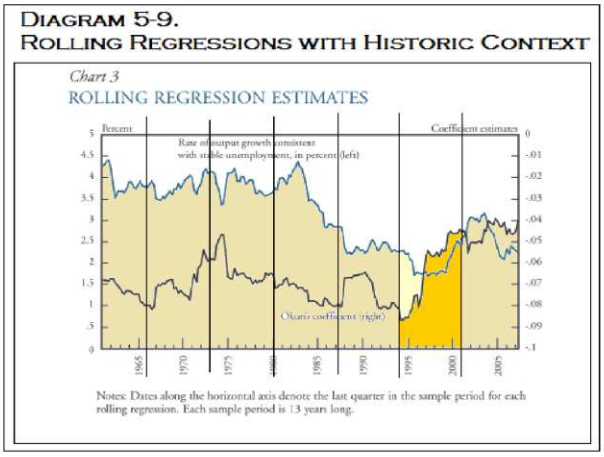


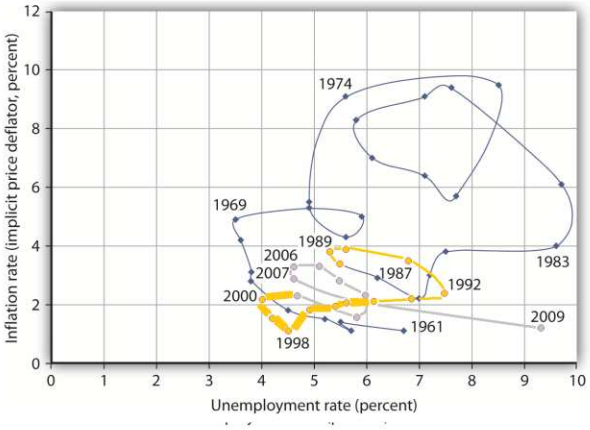
See e.g. Gordon 2010: 34.

> "The unique aspects of the recession/recover period of 2001-03 and the recession period of 2008-2009 require supplementary explanations. Our primary explanation for the large hours reductions in 2001 and the continuing reductions of 2002-03 combine two main hypotheses. As combination of increased reliance of executive pay on stock options, together with a collapse of profits and of the stock market, created a unique set of incentives to cut costs beyond anything that had been contemplated before Complacency and overhiring was replaced by desperation and job-shedding."

Early Consolidation – 1994-2001
(Formerly 1770-1777, 1826-1833, 1882-1899, 1938-1945)



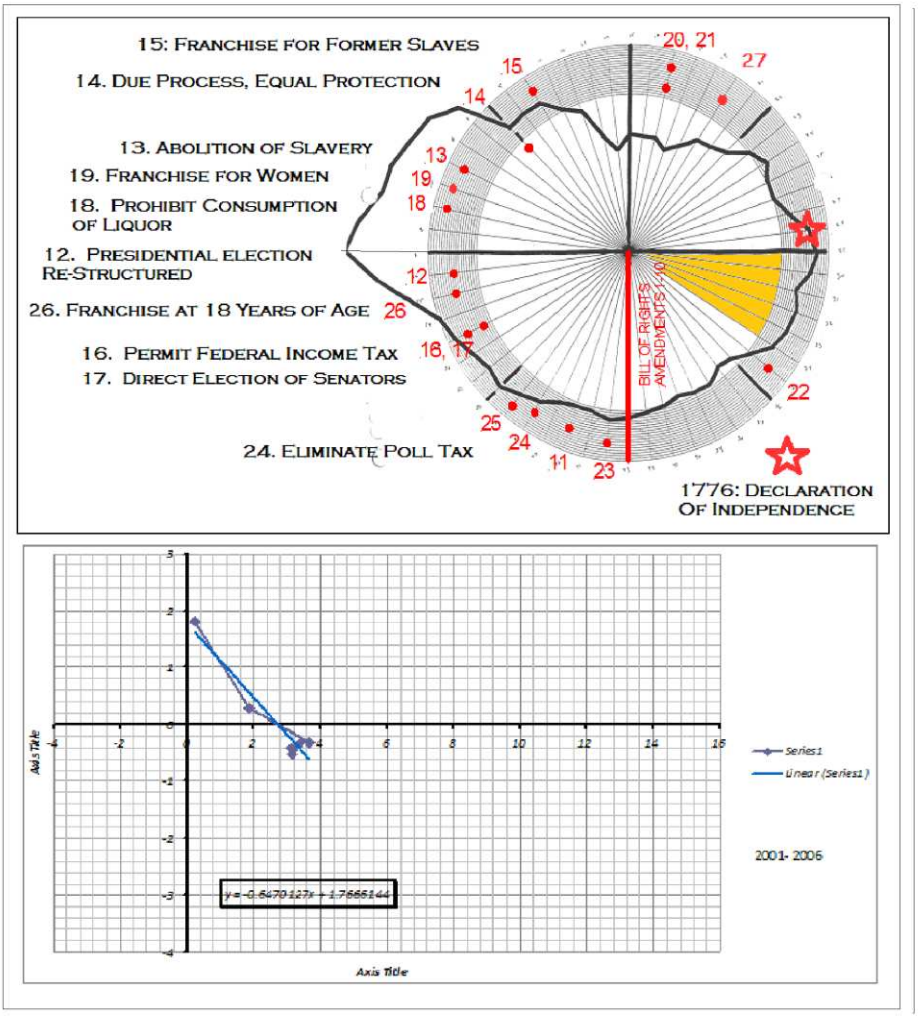


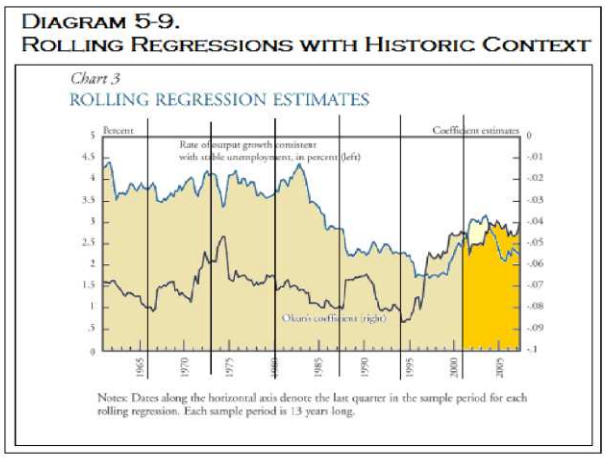


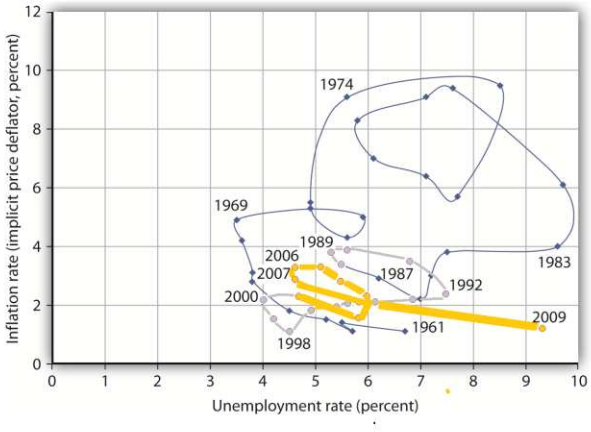
See e.g. Gordon 2010: 34.

> "The recent 2007-2009 recession involved the same mechanism, but with the added element of a much steeper decline in the output gap and a sense of sheer panic in the fall of 2008 and winter of 2009 that capitalism was on the verge of collapsing. For every deck chair that was thrown overboard in 2001-2003, perhaps three or four were tossed in 2008-2009. This comes out in the fact that the hours gap relative to trend in 2009-2010 was larger than the output gap, in contrast to 1982 when the hours gap was about two-thirds of the output gap."

Late Consolidation – 2001-2008
(Formerly 1833-1840, 1889-1896, 1945-1952)







Below are the equations for each of the slopes indicated by these trendlines.

DIAGRAM 5.
SLOPES AND INTERCEPTS IN 7-YEAR PERIODS

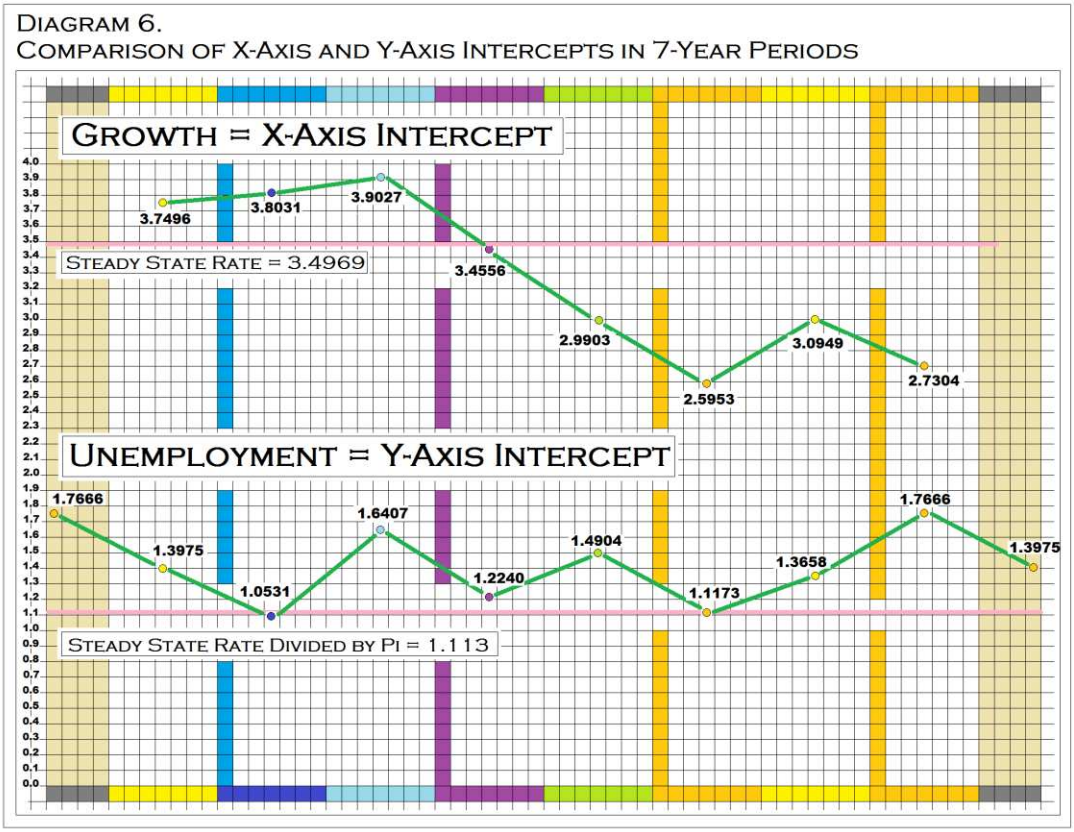
EARLY EVOLVING REVOLUTION, 1952-1959	Y = -0.3727x + 1.3975	X = 3.7496
LATE EVOLVING REVOLUTION, 1959-1966	Y = -0.2769x + 1.0531	X = 3.8031
EARLY REVOLUTION, 1966-1973	Y = -0.4204x + 1.6407	X = 3.9027
LATE REVOLUTION, 1973-1980	Y = -0.3542x + 1.2240	X = 3.4556
EARLY EVOLVING CONSOLIDATION, 1980-1987	Y = -0.4984x + 1.4904	X = 2.9903
LATE EVOLVING CONSOLIDATION, 1987-1994	Y = -0.4305x + 1.1173	X = 2.5953
EARLY CONSOLIDATION, 1994-2001	Y = -0.4413x + 1.3658	X = 3.0949
LATE CONSOLIDATION, 2001-2008	Y = -0.6470x + 1.7666	X = 2.7304
GOLDEN MEAN GROWTH / PI:1 EMPLOYMENT	Y = -0.3183x + 1.1131	X = 3.4970

Below are the x-axis and y-axis intercepts of Okun's Law as divided into eight seven-year periods.

The x-axis coordinates (growth) begin above the Golden Mean rate, but sink below it at the beginning of the evolving consolidation (in green) period. Throughout this and the subsequent consolidation period the rate of growth has been significantly less than is necessary to maintain the Golden Mean proportion over time.

Note as well that the y-axis coordinate (employment) appears to have a half-cycle of 7 years, and alternates quickly from high to low, as follows.

DIAGRAM 6.
COMPARISON OF X-AXIS AND Y-AXIS INTERCEPTS IN 7-YEAR PERIODS



It may be possible that we see above two separate cycles, the growth cycle extending over a period of 56-years and an employment cycle of 14 years. Note that the Evolving Revolution and Revolution periods have gentler slopes (pale blue, all are between -0.27 and -0.42) whereas Evolving Consolidation and Consolidation have much steeper slopes (light red, all are between -0.43 and -0.64). What do these different slopes, y-intercepts (unemployment) and x-intercepts (steady state rates of growth) mean?[46]

Growth rate, revolution

An x-intercept which is below 3.4969 will not be able to create the GNP spiral over time. This is the steady state rate given for annualized quarterly data (Chart One, x = 3.4971). The annual steady-state rate, however, is 3.4551. The data we are considering here comes from the annual data used in Knotek, Chart Two.

An x-intercept which is above 3.4969 (or, for annual data, 3.451) is significantly overheating the economy and placing great strain on the people in the United States. All of the x-intercepts involving evolving revolution or revolution are above this figure. (dark blue)

Slope, revolution

A negative slope of between -0.27 and -0.42 appears to represent flexibility in dealing with workers and the citizens of the United States. All of the slopes of involving evolving revolution or revolution fall into this range. (in pale blue)

Growth rate, consolidation

All x-intercepts involving consolidation are below this figure of 3.4551. None of them are capable of sustaining the growth rate of 3.4969 per year over the long term. (in dark red)

Slope, consolidation

A negative slope which falls outside the -0.42 range represents a willingness to be harsh and curtail many social and political rights, or alternatively, vigorously advance the financial prospects of a new and rising political elite. All the slopes involving evolving consolidation and consolidation are between -0.43 to -0.64. (in light red)

Unemployment, y-axis intercepts

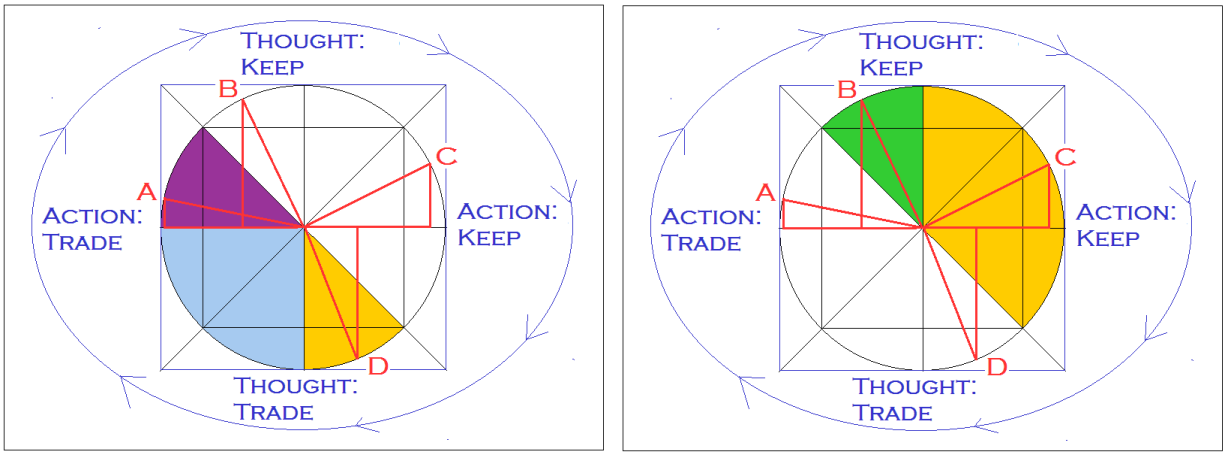
The the y-axis intercepts fluctuate considerably over time in 14-year stages during both revolution or consolidation periods. These rhythms may be combined with a different 56-year rhythm of x-axis intercepts and slopes so as to create trendline virtually identical to that given by the Golden Mean rate of growth over 14 years (3.4969 per year) as divided by π to obtain both the slope (-0.3183) and the y-axis intercept (y=1.1131).

[46] See e.g. Gordon, 2010: 35. "The two biggest recessions of the postwar period, 1981-1982 vs. 2008-2009, differ in the relative magnitude of the output and hours gap. In 1981-1982 the hours gap was only about 2/3 of the output gap, consistent with Okun's Law. In contrast in 2008-2009 the hours gap has been about 6/5 larger than the output gap, refuting Okun's Law."

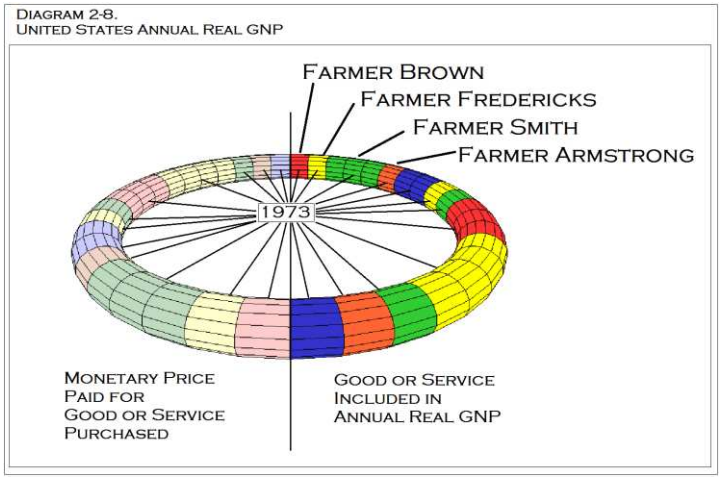
Conclusion

In conclusion, we began our description of Okun's Law with a formal structure combining action and thought in a geometric form,

Micro-economic Unit Circle

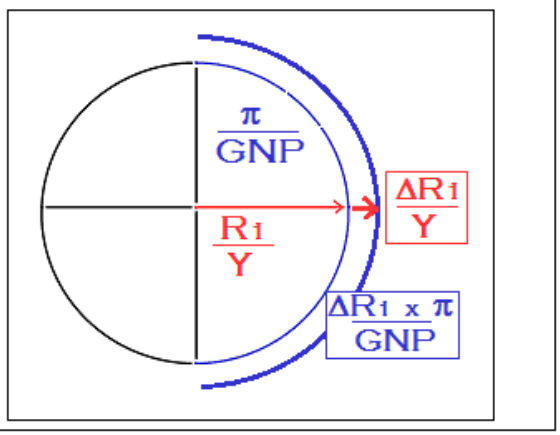


... and then aggregated this form over the period of a single year....



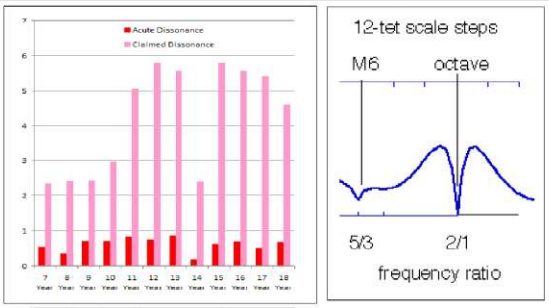
... to establish the 1:pi relationship which is at the heart of the 3:1 ratio pointed out by Okun in his analysis

DIAGRAM 2-9.
MODEL OF PRODUCTION VS. EMPLOYMENT



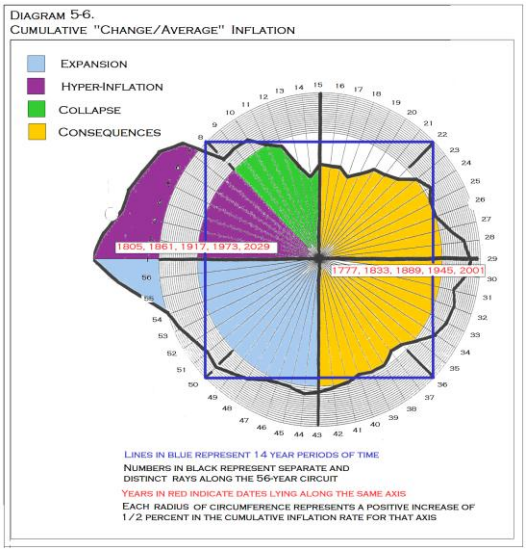
... thereby bringing into play the life-spans of the people responsible for the creation of the real GNP examined

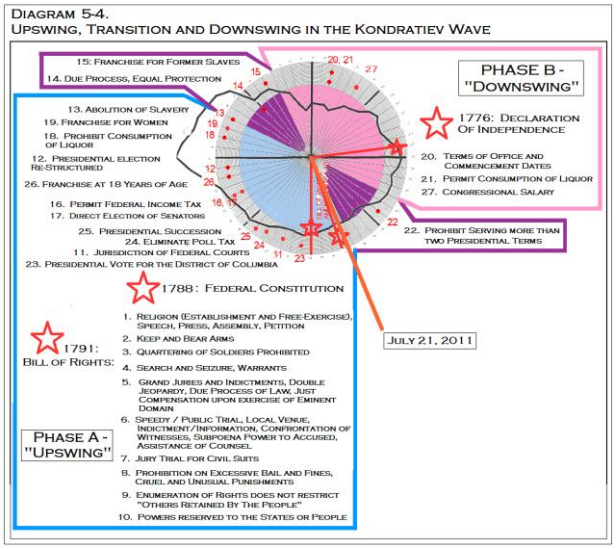
DIAGRAM 3.11.
"ACUTE DISSONANCE" AND "CLAIMED DISSONANCE" WITH MUSICAL OCTAVE



- RED	+ RED		Acute Dissonance	Claimed Dissonance
0	0.539222	7 Year	0.53922225	2.33166
0	0.328323	8 Year	0.32832275	2.39980
0	0.71348	9 Year	0.713480444	2.41405
0	0.702555	10 Year	0.702555	2.95466
-0.82882	0	11 Year	0.828815273	5.05443
-0.07077	0.680828	12 Year	0.751594667	5.77002
-0.07244	0.789312	13 Year	0.861753846	5.55129
-0.01764	0.134157	14 Year	0.151795	2.39229
-0.28313	0.336371	15 Year	0.6214992	5.77038
-0.16258	0.526958	16 Year	0.689539375	5.54327
-0.10785	0.394316	17 Year	0.502165176	5.40061
-0.04362	0.624485	18 Year	0.668103333	4.58002

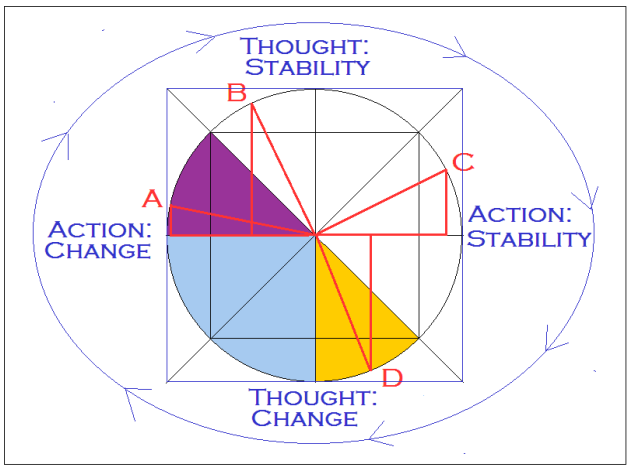
... and the political effect which these lives have on the environment of the United States

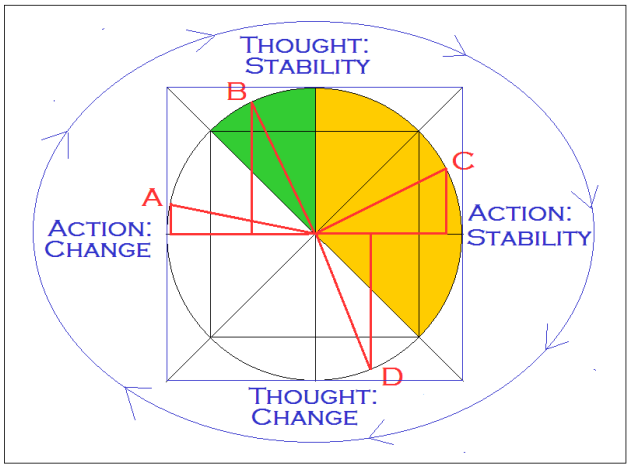




... leading in turn to the political re-statement of the original action vs. thought dichomoty with which we began the analysis. In short, making the larger the fractal of the smaller.

GNP Spiral





It would appear that Okun's Law is in fact a trigonometrically driven proportion. This accounts for its historic stability, heretofore completely un-described in either character or causation. This is demonstrated by a form of number theory engaging the set of feminine ($0<F<1$) and masculine ($1<M$) numbers using a trend line representing a $1:\varphi$ ratio for growth over time and a $1:\pi$ ratio for growth to employment as connected by a Harmonic multiplicative inverse. This view of the relationships is considerably enhanced the central tenets of the GNP Spiral generate masculine (15/14) and feminine (14/15) fractions which can be used to further interpret the interaction of time upon econometric data, however hidden these relationships might appear.

We now have a general theory of microeconomic trading goods and services which matches exactly a theory of macroeconomic trading of social values over a 56-year Kondratiev Wave using the intermediary of a π:1 value for Okun's Law.

And so it is that the two levels appear as fractals of one another, not unlike Russian dolls wherein the smaller builds into the greater and is contained and congruous to it.

DIAGRAM 1-13.
TRIGONOMETRY OF TRADE.

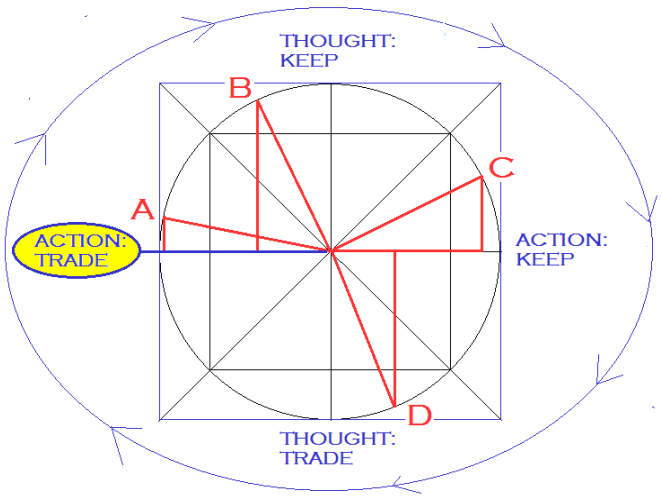
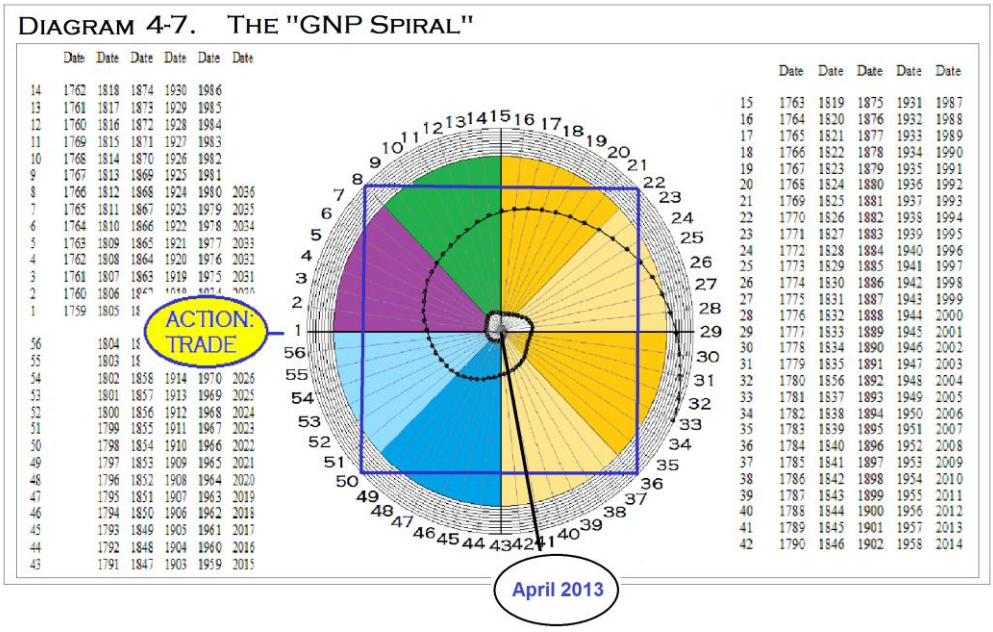


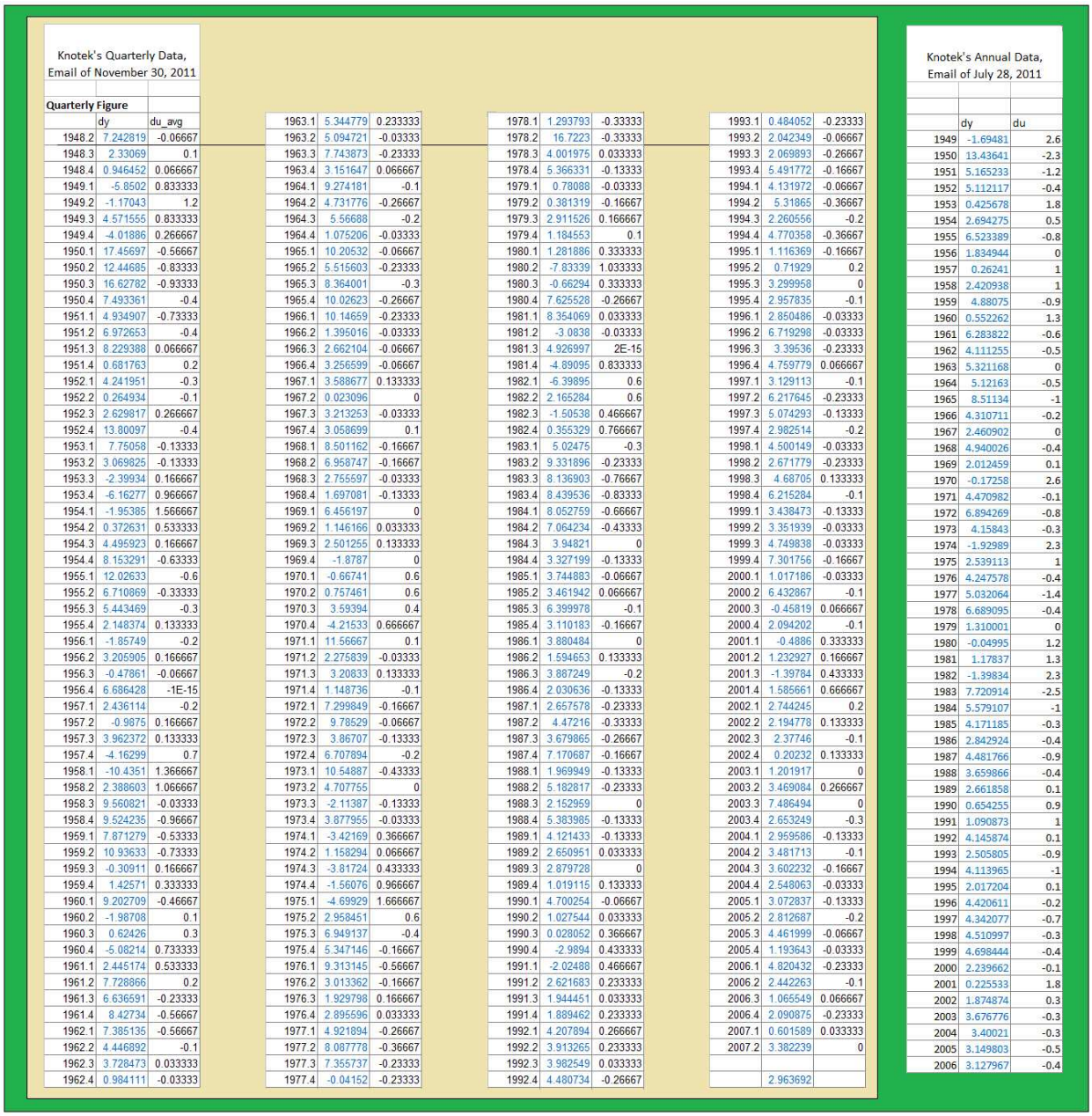
DIAGRAM 4-7. THE "GNP SPIRAL"



Scott A. Albers
Great Falls, Montana
April 30, 2013

APPENDIX

The following two emails provide Dr. Knotek's data sets for the annualized quarterly calculation (Chart One) and annual calculation (Chart Two) of Okun's Law.



Knotek's Quarterly Data, Email of November 30, 2011

Quarterly Figure

	dy	du_avg
1948.2	7.242819	-0.06667
1948.3	2.33069	0.1
1948.4	0.946452	0.066667
1949.1	-5.8502	0.833333
1949.2	-1.17043	1.2
1949.3	4.571555	0.833333
1949.4	-4.01886	0.266667
1950.1	17.45697	-0.56667
1950.2	12.44685	-0.83333
1950.3	16.62782	-0.93333
1950.4	7.493361	-0.4
1951.1	4.934907	-0.73333
1951.2	6.972653	-0.4
1951.3	8.229388	0.066667
1951.4	0.681763	0.2
1952.1	4.241951	-0.3
1952.2	0.264934	-0.1
1952.3	2.629817	0.266667
1952.4	13.80097	-0.4
1953.1	7.75058	-0.13333
1953.2	3.069825	-0.13333
1953.3	-2.39934	0.166667
1953.4	-6.16277	0.966667
1954.1	-1.95385	1.566667
1954.2	0.372631	0.533333
1954.3	4.495923	0.166667
1954.4	8.153291	-0.63333
1955.1	12.02633	-0.6
1955.2	6.710869	-0.33333
1955.3	5.443469	-0.3
1955.4	2.148374	0.133333
1956.1	-1.85749	-0.2
1956.2	3.205905	0.166667
1956.3	-0.47861	-0.06667
1956.4	6.686428	-1E-15
1957.1	2.436114	-0.2
1957.2	-0.9875	0.166667
1957.3	3.962372	0.133333
1957.4	-4.16299	0.7
1958.1	-10.4351	1.366667
1958.2	2.388603	1.066667
1958.3	9.560821	-0.03333
1958.4	9.524235	-0.96667
1959.1	7.871279	-0.53333
1959.2	10.93633	-0.73333
1959.3	-0.30911	0.166667
1959.4	1.42571	0.333333
1960.1	9.202709	-0.46667
1960.2	-1.98708	0.1
1960.3	0.62426	0.3
1960.4	-5.08214	0.733333
1961.1	2.445174	0.533333
1961.2	7.728866	0.2
1961.3	6.636591	-0.23333
1961.4	8.42734	-0.56667
1962.1	7.385135	-0.56667
1962.2	4.446892	-0.1
1962.3	3.728473	0.033333
1962.4	0.984111	-0.03333
1963.1	5.344779	0.233333
1963.2	5.094721	-0.03333
1963.3	7.743873	-0.23333
1963.4	3.151647	0.066667
1964.1	9.274181	-0.1
1964.2	4.731776	-0.26667
1964.3	5.56688	-0.2
1964.4	1.075206	-0.03333
1965.1	10.20532	-0.06667
1965.2	5.515603	-0.23333
1965.3	8.364001	-0.3
1965.4	10.02623	-0.26667
1966.1	10.14659	-0.23333
1966.2	1.395016	-0.03333
1966.3	2.662104	-0.06667
1966.4	3.256599	-0.06667
1967.1	3.588677	0.133333
1967.2	0.023096	0
1967.3	3.213253	-0.03333
1967.4	3.058699	0.1
1968.1	8.501162	-0.16667
1968.2	6.958747	-0.16667
1968.3	2.755597	-0.03333
1968.4	1.697081	-0.13333
1969.1	6.456197	0
1969.2	1.146166	0.033333
1969.3	2.501255	0.133333
1969.4	-1.8787	0
1970.1	-0.66741	0.6
1970.2	0.757461	0.6
1970.3	3.59394	0.4
1970.4	-4.21533	0.666667
1971.1	11.56667	0.1
1971.2	2.275839	-0.03333
1971.3	3.20833	0.133333
1971.4	1.148736	-0.1
1972.1	7.299849	-0.16667
1972.2	9.78529	-0.06667
1972.3	3.86707	-0.13333
1972.4	6.707894	-0.2
1973.1	10.54887	-0.43333
1973.2	4.707755	0
1973.3	-2.11387	-0.13333
1973.4	3.877955	-0.03333
1974.1	-3.42169	0.366667
1974.2	1.158294	0.066667
1974.3	-3.81724	0.433333
1974.4	-1.56076	0.966667
1975.1	-4.69929	1.666667
1975.2	2.958451	0.6
1975.3	6.949137	-0.4
1975.4	5.347146	-0.16667
1976.1	9.313145	-0.56667
1976.2	3.013362	-0.16667
1976.3	1.929798	0.166667
1976.4	2.895596	0.033333
1977.1	4.921894	-0.26667
1977.2	8.087778	-0.36667
1977.3	7.355737	-0.23333
1977.4	-0.04152	-0.23333
1978.1	1.293793	-0.33333
1978.2	16.7223	-0.33333
1978.3	4.001975	0.033333
1978.4	5.366331	-0.13333
1979.1	0.78088	-0.03333
1979.2	0.381319	-0.16667
1979.3	2.911526	0.166667
1979.4	1.184553	0.1
1980.1	1.281886	0.333333
1980.2	-7.83339	1.033333
1980.3	-0.66294	0.333333
1980.4	7.625528	-0.26667
1981.1	8.354069	0.033333
1981.2	-3.0838	-0.03333
1981.3	4.926997	2E-15
1981.4	-4.89095	0.833333
1982.1	-6.39895	0.6
1982.2	2.165284	0.6
1982.3	-1.50538	0.466667
1982.4	0.355329	0.766667
1983.1	5.02475	-0.3
1983.2	9.331896	-0.23333
1983.3	8.136903	-0.76667
1983.4	8.439536	-0.83333
1984.1	8.052759	-0.66667
1984.2	7.064234	-0.43333
1984.3	3.94821	0
1984.4	3.327199	-0.13333
1985.1	3.744883	-0.06667
1985.2	3.461942	0.066667
1985.3	6.399978	-0.1
1985.4	3.110183	-0.16667
1986.1	3.880484	0
1986.2	1.594653	0.133333
1986.3	3.887249	-0.2
1986.4	2.030636	-0.13333
1987.1	2.657578	-0.23333
1987.2	4.47216	-0.33333
1987.3	3.679865	-0.26667
1987.4	7.170687	-0.16667
1988.1	1.969949	-0.13333
1988.2	5.182817	-0.23333
1988.3	2.152959	0
1988.4	5.383985	-0.13333
1989.1	4.121433	-0.13333
1989.2	2.650951	0.033333
1989.3	2.879728	0
1989.4	1.019115	0.133333
1990.1	4.700254	-0.06667
1990.2	1.027544	0.033333
1990.3	0.028052	0.366667
1990.4	-2.9894	0.433333
1991.1	-2.02488	0.466667
1991.2	2.621683	0.233333
1991.3	1.944451	0.033333
1991.4	1.889462	0.233333
1992.1	4.207894	0.266667
1992.2	3.913265	0.233333
1992.3	3.982549	0.033333
1992.4	4.480734	-0.26667
1993.1	0.484052	-0.23333
1993.2	2.042349	-0.06667
1993.3	2.069893	-0.26667
1993.4	5.491772	-0.16667
1994.1	4.131972	-0.06667
1994.2	5.31865	-0.36667
1994.3	2.260556	-0.2
1994.4	4.770358	-0.36667
1995.1	1.116369	-0.16667
1995.2	0.71929	0.2
1995.3	3.299958	0
1995.4	2.957835	-0.1
1996.1	2.850486	-0.03333
1996.2	6.719298	-0.03333
1996.3	3.39536	-0.23333
1996.4	4.759779	0.066667
1997.1	3.129113	-0.1
1997.2	6.217645	-0.23333
1997.3	5.074293	-0.13333
1997.4	2.982514	-0.2
1998.1	4.500149	-0.03333
1998.2	2.671779	-0.23333
1998.3	4.68705	0.133333
1998.4	6.215284	-0.1
1999.1	3.438473	-0.13333
1999.2	3.351939	-0.03333
1999.3	4.749838	-0.03333
1999.4	7.301756	-0.16667
2000.1	1.017186	-0.03333
2000.2	6.432867	-0.1
2000.3	-0.45819	0.066667
2000.4	2.094202	-0.1
2001.1	-0.4886	0.333333
2001.2	1.232927	0.166667
2001.3	-1.39784	0.433333
2001.4	1.585661	0.666667
2002.1	2.744245	0.2
2002.2	2.194778	0.133333
2002.3	2.37746	-0.1
2002.4	0.20232	0.133333
2003.1	1.201917	0
2003.2	3.469084	0.266667
2003.3	7.486494	0
2003.4	2.653249	-0.3
2004.1	2.959586	-0.13333
2004.2	3.481713	-0.1
2004.3	3.602232	-0.16667
2004.4	2.548063	-0.03333
2005.1	3.072837	-0.13333
2005.2	2.812687	-0.2
2005.3	4.461999	-0.06667
2005.4	1.193643	-0.03333
2006.1	4.820432	-0.23333
2006.2	2.442263	-0.1
2006.3	1.065549	0.066667
2006.4	2.090875	-0.23333
2007.1	0.601589	0.033333
2007.2	3.382239	0
	2.963692	

Knotek's Annual Data, Email of July 28, 2011

	dy	du
1949	-1.69481	2.6
1950	13.43641	-2.3
1951	5.165233	-1.2
1952	5.112117	-0.4
1953	0.425678	1.8
1954	2.694275	0.5
1955	6.523389	-0.8
1956	1.834944	0
1957	0.26241	1
1958	2.420938	1
1959	4.88075	-0.9
1960	0.552262	1.3
1961	6.283822	-0.6
1962	4.111255	-0.5
1963	5.321168	0
1964	5.12163	-0.5
1965	8.51134	-1
1966	4.310711	-0.2
1967	2.460902	0
1968	4.940026	-0.4
1969	2.012459	0.1
1970	-0.17258	2.6
1971	4.470982	-0.1
1972	6.894269	-0.8
1973	4.15843	-0.3
1974	-1.92989	2.3
1975	2.539113	1
1976	4.247578	-0.4
1977	5.032064	-1.4
1978	6.689095	-0.4
1979	1.310001	0
1980	-0.04995	1.2
1981	1.17837	1.3
1982	-1.39834	2.3
1983	7.720914	-2.5
1984	5.579107	-1
1985	4.171185	-0.3
1986	2.842924	-0.4
1987	4.481766	-0.9
1988	3.659866	-0.4
1989	2.661858	0.1
1990	0.654255	0.9
1991	1.090873	1
1992	4.145874	0.1
1993	2.505805	-0.9
1994	4.113965	-1
1995	2.017204	0.1
1996	4.420611	-0.2
1997	4.342077	-0.7
1998	4.510997	-0.3
1999	4.698444	-0.4
2000	2.239662	-0.1
2001	0.225533	1.8
2002	1.874874	0.3
2003	3.676776	-0.3
2004	3.40021	-0.3
2005	3.149803	-0.5
2006	3.127967	-0.4

The following states the annual measures of GNP as compared with Dr. Knotek.

Change in Annual Unemployment vs. Change in Annual GDP

Quarterly GDP, Bureau of Economic Analysis

Table 1.1.6. Real Gross Domestic Product, Chained Dollars

[Billions of chained (2000) dollars]; Seasonally adjusted at annual rates

Quarterly data from 1947 To 2007

Bureau of Economic Analysis

Data published September 27, 2007

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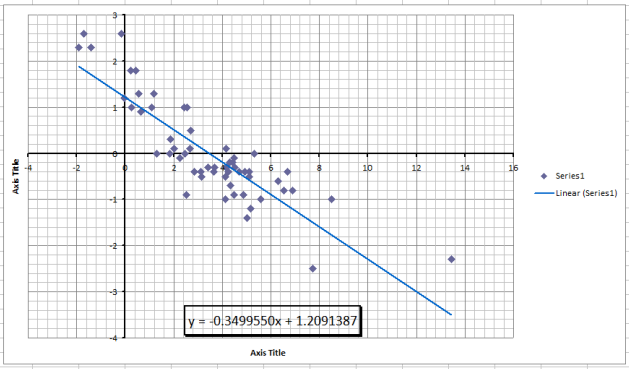
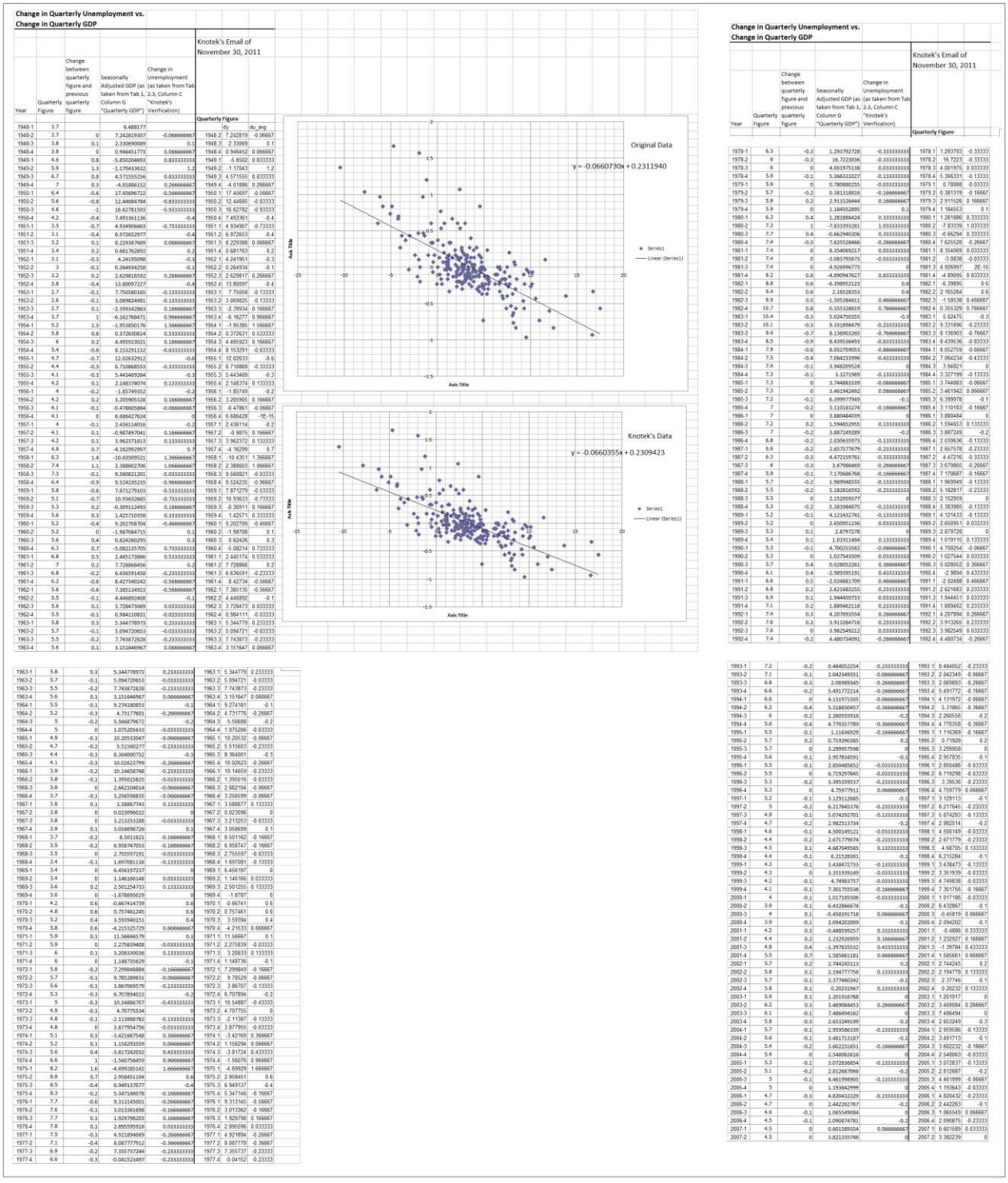
					Albers' GDP Calculations from Table 1.1.6, September 27, 2007			Knotek's Annual Data as provided through correspondence		
		Gross domestic product			Quarter 4 divided by Quarter 4	Minus 1	Multiply 100		dy	du
1947	4	1,590.9								
1948	4	1,658.0	1948	4	1.042177	n=m-1	n=mx100		dy	du
1949	4	1,629.9	1949	4	0.983052	-0.01695	-1.694813	1949	-1.69481	2.6
1950	4	1,848.9	1950	4	1.134364	0.134364	13.436407	1950	13.43641	-2.3
1951	4	1,944.4	1951	4	1.051652	0.051652	5.1652334	1951	5.165233	-1.2
1952	4	2,043.8	1952	4	1.051121	0.051121	5.1121168	1952	5.112117	-0.4
1953	4	2,052.5	1953	4	1.004257	0.004257	0.4256777	1953	0.425678	1.8
1954	4	2,107.8	1954	4	1.026943	0.026943	2.6942753	1954	2.694275	0.5
1955	4	2,245.3	1955	4	1.065234	0.065234	6.5233893	1955	6.523389	-0.8
1956	4	2,286.5	1956	4	1.018349	0.018349	1.8349441	1956	1.834944	0
1957	4	2,292.5	1957	4	1.002624	0.002624	0.2624098	1957	0.26241	1
1958	4	2,348.0	1958	4	1.024209	0.024209	2.4209378	1958	2.420938	1
1959	4	2,462.6	1959	4	1.048807	0.048807	4.8807496	1959	4.88075	-0.9
1960	4	2,476.2	1960	4	1.005523	0.005523	0.5522618	1960	0.552262	1.3
1961	4	2,631.8	1961	4	1.062838	0.062838	6.283822	1961	6.283822	-0.6
1962	4	2,740.0	1962	4	1.041113	0.041113	4.1112547	1962	4.111255	-0.5
1963	4	2,885.8	1963	4	1.053212	0.053212	5.3211679	1963	5.321168	0
1964	4	3,033.6	1964	4	1.051216	0.051216	5.1216301	1964	5.12163	-0.5
1965	4	3,291.8	1965	4	1.085113	0.085113	8.5113397	1965	8.51134	-1
1966	4	3,433.7	1966	4	1.043107	0.043107	4.3107115	1966	4.310711	-0.2
1967	4	3,518.2	1967	4	1.024609	0.024609	2.4609022	1967	2.460902	0
1968	4	3,692.0	1968	4	1.0494	0.0494	4.9400261	1968	4.940026	-0.4
1969	4	3,766.3	1969	4	1.020125	0.020125	2.0124594	1969	2.012459	0.1
1970	4	3,759.8	1970	4	0.998274	-0.00173	-0.1725832	1970	-0.17258	2.6
1971	4	3,927.9	1971	4	1.04471	0.04471	4.4709825	1971	4.470982	-0.1
1972	4	4,198.7	1972	4	1.068943	0.068943	6.8942692	1972	6.894269	-0.8
1973	4	4,373.3	1973	4	1.041584	0.041584	4.15843	1973	4.15843	-0.3
1974	4	4,288.9	1974	4	0.980701	-0.0193	-1.9298928	1974	-1.92989	2.3
1975	4	4,397.8	1975	4	1.025391	0.025391	2.5391126	1975	2.539113	1
1976	4	4,584.6	1976	4	1.042476	0.042476	4.2475783	1976	4.247578	-0.4
1977	4	4,815.3	1977	4	1.050321	0.050321	5.0320639	1977	5.032064	-1.4
1978	4	5,137.4	1978	4	1.066891	0.066891	6.6890952	1978	6.689095	-0.4
1979	4	5,204.7	1979	4	1.0131	0.0131	1.3100012	1979	1.310001	0
1980	4	5,202.1	1980	4	0.9995	-0.0005	-0.0499548	1980	-0.04995	1.2
1981	4	5,263.4	1981	4	1.011784	0.011784	1.1783703	1981	1.17837	1.3
1982	4	5,189.8	1982	4	0.986017	-0.01398	-1.3983357	1982	-1.39834	2.3
1983	4	5,590.5	1983	4	1.077209	0.077209	7.7209141	1983	7.720914	-2.5
1984	4	5,902.4	1984	4	1.055791	0.055791	5.5791074	1984	5.579107	-1
1985	4	6,148.6	1985	4	1.041712	0.041712	4.1711846	1985	4.171185	-0.3
1986	4	6,323.4	1986	4	1.028429	0.028429	2.8429236	1986	2.842924	-0.4
1987	4	6,606.8	1987	4	1.044818	0.044818	4.4817661	1987	4.481766	-0.9
1988	4	6,848.6	1988	4	1.036599	0.036599	3.6598656	1988	3.659866	-0.4
1989	4	7,030.9	1989	4	1.026619	0.026619	2.6618579	1989	2.661858	0.1
1990	4	7,076.9	1990	4	1.006543	0.006543	0.6542548	1990	0.654255	0.9
1991	4	7,154.1	1991	4	1.010909	0.010909	1.0908731	1991	1.090873	1
1992	4	7,450.7	1992	4	1.041459	0.041459	4.1458744	1992	4.145874	0.1
1993	4	7,637.4	1993	4	1.025058	0.025058	2.5058048	1993	2.505805	-0.9
1994	4	7,951.6	1994	4	1.04114	0.04114	4.1139655	1994	4.113965	-1
1995	4	8,112.0	1995	4	1.020172	0.020172	2.0172041	1995	2.017204	0.1
1996	4	8,470.6	1996	4	1.044206	0.044206	4.4206114	1996	4.420611	-0.2
1997	4	8,838.4	1997	4	1.043421	0.043421	4.3420773	1997	4.342077	-0.7
1998	4	9,237.1	1998	4	1.04511	0.04511	4.5109975	1998	4.510997	-0.3
1999	4	9,671.1	1999	4	1.046984	0.046984	4.6984443	1999	4.698444	-0.4
2000	4	9,887.7	2000	4	1.022397	0.022397	2.2396625	2000	2.239662	-0.1
2001	4	9,910.0	2001	4	1.002255	0.002255	0.2255327	2001	0.225533	1.8
2002	4	10,095.8	2002	4	1.018749	0.018749	1.8748739	2002	1.874874	0.3
2003	4	10,467.0	2003	4	1.036768	0.036768	3.6767765	2003	3.676776	-0.3
2004	4	10,796.4	2004	4	1.03147	0.03147	3.1470335	2004	3.40021	-0.3
2005	4	11,107.2	2005	4	1.028787	0.028787	2.8787374	2005	3.149803	-0.5
2006	4	11,395.5	2006	4	1.025956	0.025956	2.5956137	2006	3.127967	-0.4



The following states the quarterly data for annualized real GNP and quarterly employment, as contrasted with that of Dr. Knotek.



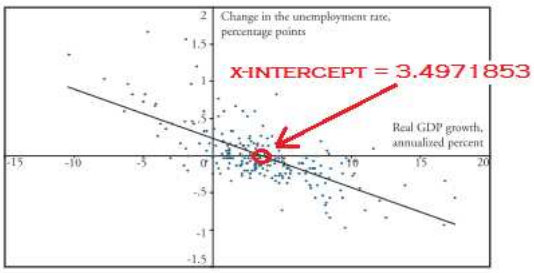
In Albers and Albers 2013:108-109 (pages 69-70 in the preprint edition) we stated the following:

> As noted previously, to figure the annual increase implied by the GNP Spiral, we may use the formula for simple interest compounded annually…
>
> $$FV = PV\ (1+r)^t$$
>
> … ; state a present value (PV) of $1,000,000; a time period (t) of 14 years; and the future value (FV) as given below in proportion to the varying numbers derived in the GNP Spiral. These assumptions give us the following interest rates (r).

	Future Value		Interest rate
x= Circle Analysis:	$1,618,590	interest rate is:	3.4995226
x= Square:	$1,618,120	interest rate is:	3.4973756
x= Golden Mean:	$1,618,033	interest rate is:	3.4969781

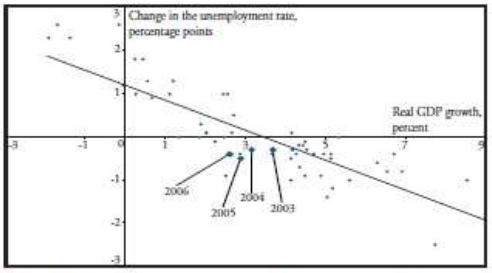
> The above "rates of growth" may be contrasted with one of the central empirical regularities of mainstream economics, i.e. Okun's Law. This rule proposes a roughly 3:1 ratio between increases in real GNP and decreases in the rate of unemployment in the economy of the United States. A trend line may be devised for quarterly data between the second quarter of 1948 and the second quarter of 2007 which gives the slope of this relationship as:
>
> $$y = .23094 + -0.066036x$$
>
> A "steady state" rate of economic growth may be figured for the x-intercept, i.e. that rate of growth which occurs when there is no change in the rate of employment. (y = 0). Using the above equation and trend line, this x-intercept is 3.4971853. (Knotek, 2007, with additional correspondence by the author)

Chart 1
THE DIFFERENCE VERSION OF OKUN'S LAW, QUARTERLY DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from the second quarter of 1948 through the second quarter of 2007.

Chart 2
THE DIFFERENCE VERSION OF OKUN'S LAW, ANNUAL DATA



Note: Data are from the Bureau of Economic Analysis and Bureau of Labor Statistics, from 1949 through 2006.

As these figures relate to the annual rate of growth necessary to sustain all values investigated above we have:

Analysis:	Future Value	Promixity to Phi	Rate:	Comparison to Okun's x-intercept at 3.4971853
Circle:	$1,618,590	1.00034424	3.4995226	1.000668337
Columns:	$1,618,200	1.00010321	3.4977411	1.000158927
Square:	$1,618,120	1.00005376	3.4973756	1.000054415
Okun's Law x-axis:	$1,618,078	1.00002781	3.4971853	1
Golden Mean:	$1,618,033	1	3.4969781	0.999940752

When this "steady state" rate of growth under Okun's Law is placed among the "rates of growth" calculated by the GNP Spiral, the x-intercept generates a future value in proximity to the Golden Mean of 2.7/100,000 parts, closer than all other values.

It is possible to calculate additional decimal places using the data sets and econometric techniques provided by Dr. Knotek. Taken to eight places the resulting trend lines for quarterly and annual data for the charts above have the equations.

Quarterly trendline:
y = 0.23094226 + -0.06603552x; x-intercept = 3.4972429

Annual trendline:
y = 1.20913875 + -0.34995497x; x-intercept = 3.4551266

The rounding of the Annualized Quarterly trend line results in a slightly decreased rate of steady state growth. The steady state growth rate in an "un-rounded" trend line increases very slightly, i.e. from 3.4971853 to 3.4972429.

Subsequent to the publication of "How Useful Is Okun's Law?" (Knotek 2007) the Department of Labor amended various rates of unemployment. In addition, the seasonally adjusted rate of growth for 2007-2 has been amended slightly.

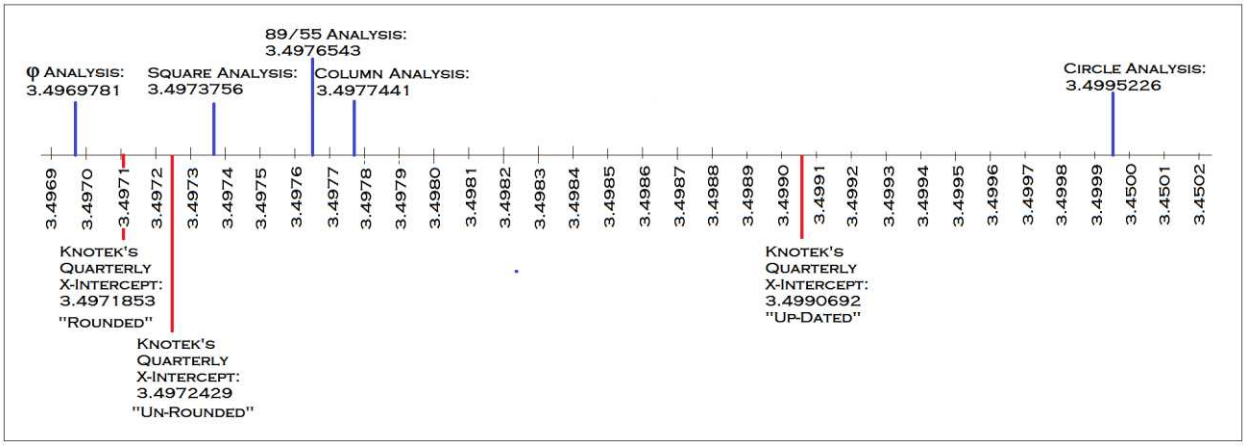
Change in Quarterly Unemployment vs. Change in Quarterly GDP

Year	Quarterly Figure	Change between quarterly figure and previous quarterly figure	Seasonally Adjusted GDP (as taken from Tab 1, Column G "Quarterly GDP")	Change in Unemployment (as taken from Tab 2.3, Column C "Knotek's Verification)	Knotek's Email of November 30, 2011 Quarterly Figure		
2004-1	5.7	-0.1	2.959586339	-0.133333333	2004.1	2.959586	-0.13333
2004-2	5.6	-0.1	3.481713187	-0.1	2004.2	3.481713	-0.1
2004-3	5.4	-0.2	3.602231651	-0.166666667	2004.3	3.602232	-0.16667
2004-4	5.4	0	2.548062616	0	2004.4	2.548063	-0.03333
2005-1	5.3	-0.1	3.072836854	-0.133333333	2005.1	3.072837	-0.13333
2005-2	5.1	-0.2	2.812687066	-0.2	2005.2	2.812687	-0.2
2005-3	5	-0.1	4.461998905	-0.133333333	2005.3	4.461999	-0.06667
2005-4	5	0	1.193642999	0	2005.4	1.193643	-0.03333
2006-1	4.7	-0.3	4.820432229	-0.233333333	2006.1	4.820432	-0.23333
2006-2	4.7	0	2.442262767	-0.1	2006.2	2.442263	-0.1
2006-3	4.6	-0.1	1.065549084	0	2006.3	1.065549	0.066667
2006-4	4.5	-0.1	2.090874781	-0.2	2006.4	2.090875	-0.23333
2007-1	4.5	0	0.601589354	0.066666667	2007.1	0.601589	0.033333
2007-2	4.5	0	3.821335748	0	2007.2	3.382239	0

Although Chart 1 (Knotek 2007) would appear only very slightly altered as a result of these changes, the trendline again changes, with an additional increase in the value of the x-intercept as follow

$$y = -0.0660730x + 0.2311940$$
$$\text{x intercept} = 3.499069211$$

It must be pointed out that the range between these prospective "rates of growth" is very narrow. The greatest figure (Circle Analysis: 3.4995226) exceeds the least figure (φ Analysis: 3.4969781) by a multiple of 3.4995226 / 3.4969781 = 1.0007276, or 0.072%, or 7.2 parts in 10,000. As these are plotted on an x-axis the following x-intercepts appear, all falling within a range of 7.2 parts in 10,000:



One can see immediately that the small deviations between data sets have had a striking effect on the order of quarterly intercepts and their proximity to the Golden Mean intercept. The claim that Knotek's Annualized Quarterly intercept as given in Chart One is closer to the Golden Mean analysis than the Square Analysis can no longer be made. Nevertheless the very tight range of values given for these different interpretations of the data supports the central theme of this essay, to wit: that Okun's Law is a trigonometrically derived function with geometric properties underlying it.

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