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A Growth Perspective on Foreign Reserve Accumulation

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Abstract

Based on a dynamic open-economy macroeconomic model, this paper aims at understanding the contribution of domestic financial under-development to foreign reserve accumulation in some emerging market economies, especially in China. It is argued that foreign reserve accumulation is a consequence of a growth strategy induced by strong capital investment in a financially constrained economy. It is further proved using a Ramsey problem that purchasing international reserves is a welfare-improving policy in terms of production efficiency gains when the domestic economy faces two sources of frictions: credit constraint and capital controls. In fact, when domestic firms are occasionally credit-constrained and they do not have a direct access to international financial market, they need domestic saving instruments to increase their retained earnings so that they can sufficiently invest in capital. The central bank/government plays the role of financial intermediary and provides domestic firms with a liquid public bond, thus relaxing the domestic credit constraint. The proceeds of domestic public bonds are then invested abroad due to the limited scope of domestic financial market and a depressed domestic interest rate, leading to foreign reserve stockpiling. The speed of foreign reserve accumulation will slow down with the domestic financial deepening and the development of the domestic financial market.

JEL Classification: F41, F43
Keywords: Foreign reserves, capital controls, credit constraints, domestic savings, capital investment, economic growth, Chinese economy

1 Introduction

1.1 Research background

Over the last decade, we observe a surge in foreign reserve accumulation throughout the world and in particular in emerging market economies. The latter became the main reserve holders, taking over the place of advanced countries around 2005. As Figure 1 shows, the share of foreign reserves held by emerging and developing economies has doubled in eight years since 2000, rising from 33% to 67% of total foreign reserves in the world.
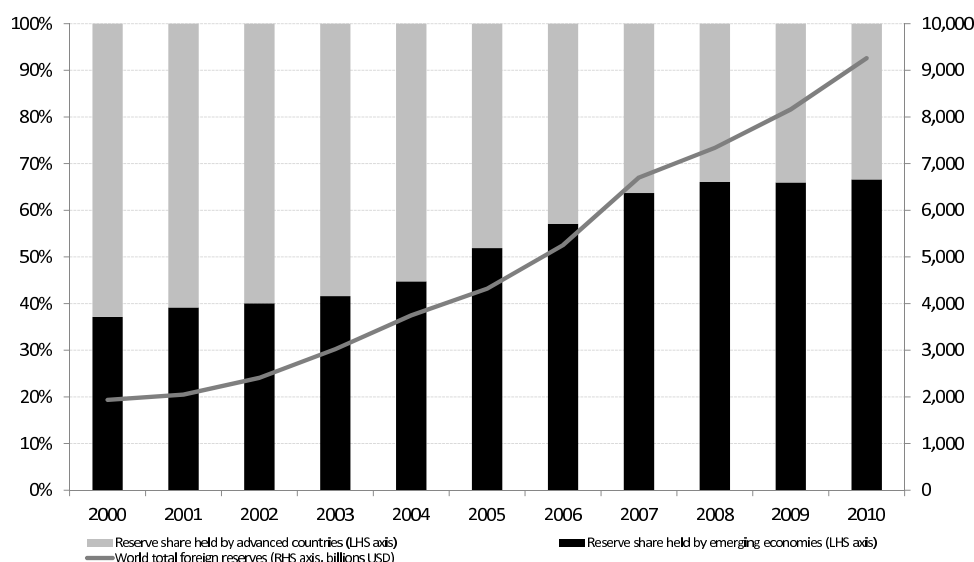
Figure 1: Evolution of the world reserves

Facing this trend of rapid accumulation of foreign reserves in the last decade, a renewed literature revived the debate regarding the determinants of the demand for foreign assets and the reserve adequacy ratio. Since very recently, there seems to be a consensus deeming that the motives of holding foreign reserves can be time-varying and specific to individual countries or country groups. An empirical paper by Ghosh et al. (2012) and a report of the Independent Evaluation Office (IEO) of the International Monetary Fund (IMF) confirmed this broad view.

Evidence has shown that countries accumulate reserves for many reasons that cannot be captured in single indicators or models, and sources of vulnerability

evolve over time. Reserve adequacy assessments and advice therefore need to rely on a broad range of indicators which should be adjusted when circumstances change (IEO (2012)).

In this paper, I propose a general equilibrium model to analyze a particular factor, *the under-development of the domestic financial market*, which may motivate a central bank to purchase foreign assets. Despite the extensive academic and policy debates with regard to international reserves, a theoretical model on the relationship between financial market under-development and reserve accumulation remains elusive. My analysis is based on three facts that we observe in the data.

Fact 1 *Emerging market economies which have accumulated a large stock of foreign assets are those which have experienced a fast economic growth.*

Figure 2 shows a clear positive relationship between the average growth rate of real GDP per capita and that of foreign reserve stocks between 1980 and 2010 for 24 emerging economies.
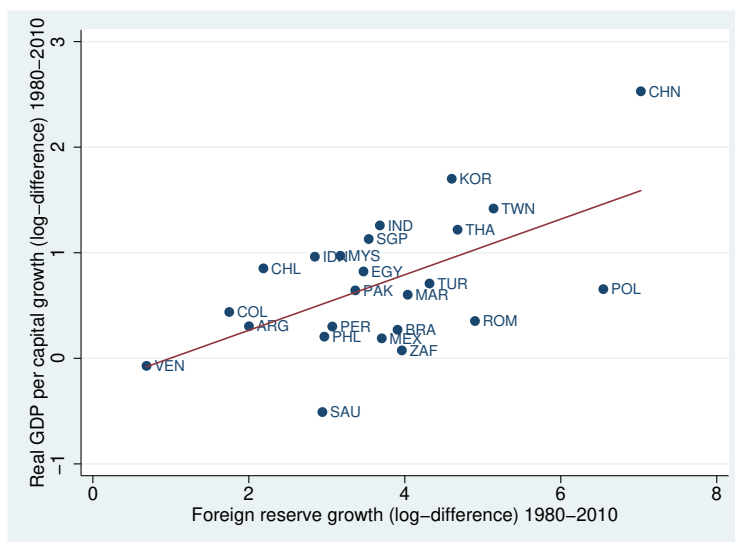
Figure 2: Foreign reserve accumulation vs. real GDP growth Data source: the World Bank WDI database for the real GDP series and the IMF IFS database for the foreign reserves series (Total Reserves Minus Gold, line 1 l.)

Indeed, most foreign reserve holders are middle income countries [1] (upper and lower middle income countries all combined according the the World Bank's classification). We

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[1] 14 out of the 20 biggest countries in terms of the foreign reserves to GDP ratio (average between 2000 to 2008) are middle income countries.

can think that they are in the phase of fast economic catch-up. The positive relationship between real economic growth rate and demand for foreign reserves suggests that holding reserves may be part of economic catch-up strategies.

Fact 2 *Among the middle income countries which have accumulated foreign reserves in the past decades, many have a relatively under-developed domestic private financial market.*

In some developing and emerging economies, bank loans constitute the major source of corporate financing, as the scope of the domestic equity and bond market is limited and domestic financial assets are thus scarce. Therefore, some domestic firms have to rely on retained earnings to finance their production activities. Figure 3 illustrates the public and private bond to GDP ratios compared with the foreign reserves to GDP ratio in selected countries. It seems like the lower the supply of domestic private bond and the higher the supply of domestic public bond, the more important is the level of foreign reserves. One natural conjecture about this is that the public bond constitutes an important source of domestic financing when the private market is very constrained. With a limited scope of the domestic financial market, public bond proceeds are highly likely invested abroad in the form of foreign reserves.

Fact 3 *In some reserve holding countries, we observe a persistent - and sometimes increasingly large - gap between gross domestic savings and gross domestic loans.*

The gap between gross domestic savings and gross domestic loans signals some degrees of credit frictions in these countries. The credit borrowing channel being constrained, the need to save cannot be satisfied without the intervention of the public sector. We illustrate this point using Chinese data (see Figure 4). As long as the wedge between domestic savings and loans exists, the domestic financial market is repressed and the domestic real interest rate is lower than the world interest rate. From here, we ponder whether there exist some domestic structural deficiencies which caused domestic excess savings, leading to a current account surplus and foreign reserve stockpiling.

Based on these three stylized facts, I shall make two points in this paper: 1) Financial

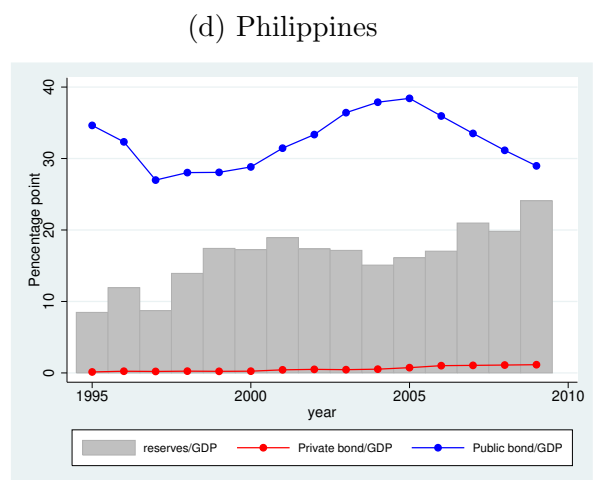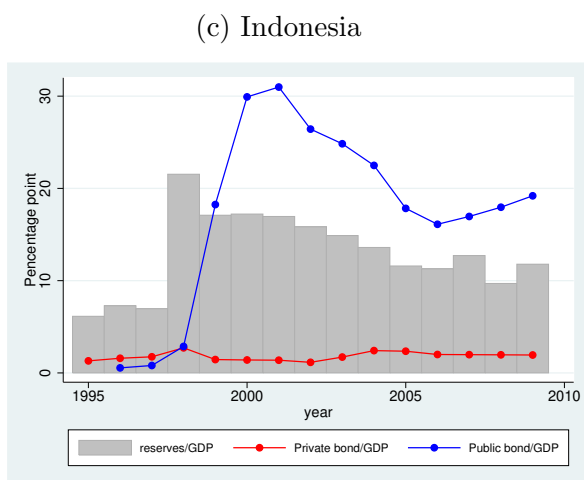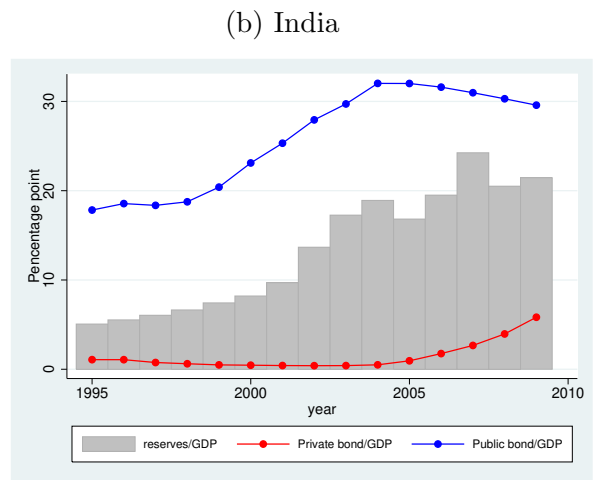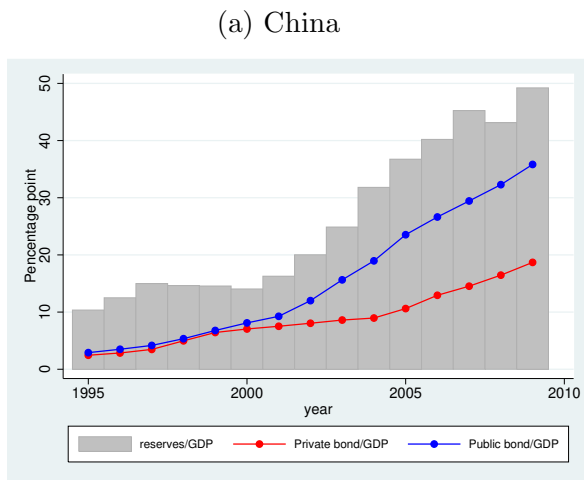(a) China

(b) India

(c) Indonesia

(d) Philippines

Figure 3: Domestic bond market vs. foreign reserves in selected countries

repression is Pareto sub-optimal for fast-growing emerging market economies with credit constraints. As some domestic firms heavily rely on retained earnings for financing their production, a repressed domestic interest rate would dampen firms' retained earnings. 2) Purchasing foreign reserve assets constitutes a second-best growth strategy in an economy where the domestic financial market is under-developed (both for the lack of domestic financial assets and for the existence of credit constraints) on the one hand and where there are controls on capital flows.

It is worthwhile emphasizing the importance of capital controls to trigger foreign reserve holding. In fact, capital controls constitute a way to discriminate public and private capital flows, so that public capital outflows (i.e. purchasing foreign assets) would not be entirely offset by private capital inflows (i.e. private external borrowing). This imperfect substitutability can also appear by setting an ad-hoc external borrowing constraint of domestic firms based on the asymmetric information facing international investors (such as in Benigno and Fornaro (2012)). With this regard, China constitutes an interesting illustration for my analysis, as in this country both financial market frictions and capital controls are present. Therefore, in the subsequent sections, I will mainly focus on the example of China.
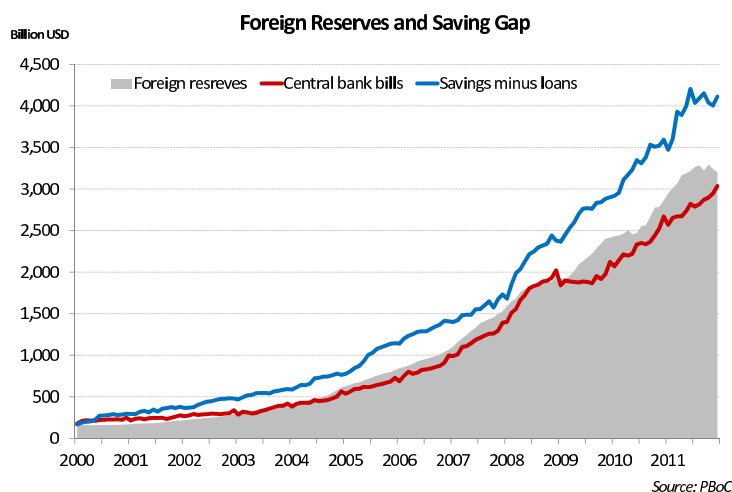

Figure 4: Foreign reserves and gap between savings and loans in China

In the scope of this paper, I rule out the aspect of international trade and exchange rate policies. This choice is explained by the need to focus on the relationship between domestic financial market under-development and foreign reserve holding. Moreover,

there is no consensus in academia on the (reverse) causality between foreign reserve holding and exchange rate stability. We do not know whether keeping the domestic currency undervalued is the objective of foreign reserve accumulation or both of them are side effects of some other macroeconomic consideration.

1.2   A growth story of foreign reserve accumulation

Through which channel do foreign reserves affect economic growth? I argue that the effect of foreign reserve accumulation goes through the fixed capital formation and production efficiency gains.

Many academic works show that domestic capital accumulation remains the main driver of economic catch-up in emerging economies - especially in East Asia - as the neo-classical growth theory predicts. Based on this viewpoint, even the Chinese spectacular growth rate is not a 'miracle'; the contribution of fixed capital formation looms large (Yu (1998), Nelson and Pack (1999), Song et al. (2011)).

Figure 5 shows a clear positive relationship between foreign reserves to GDP ratio and gross fixed capital formation to GDP ratio in China from 1980 to 2010. Figure 6 illustrates that the year-on-year growth rate of foreign reserves and that of gross fixed capital formation coincide very closely since the mid-1990s in China. If we further test the Granger causality of these two variables, we find that the growth rate of foreign reserves leads that of gross fixed capital formation.
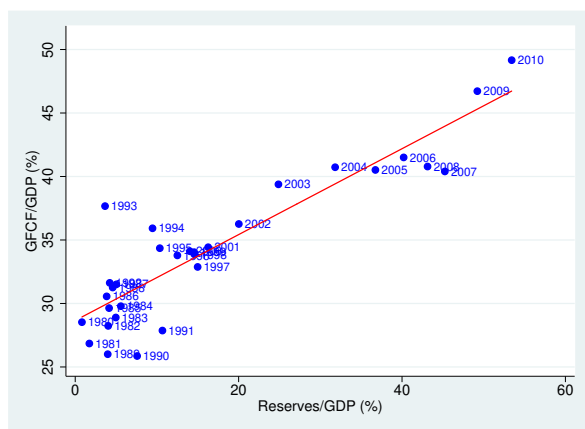

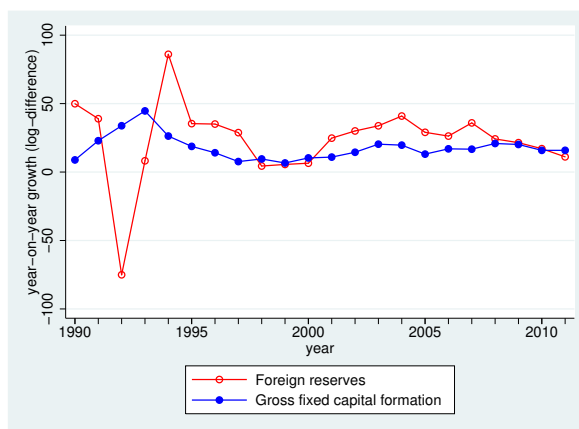Figure 5: Foreign reserves vs. GFCF (%GDP)

Figure 6: Annual growth rate of foreign reserves and of GFCF

As illustrated in Figure 7, the story I tell in this paper is as follows: in a financial-ly constrained economy, entrepreneurs need to make precautionary savings when they harvest an output in order to overcome the credit constraint when they face investment opportunities. In an fully open economy, this leads to a gap between domestic savings and investment, thus current account surplus. However, if in addition there were capital controls, entrepreneurs would not have direct access to international financial market in order to make precautionary savings. In this case, the central bank can play the role of financial intermediary and provide a domestic liquid bond with which domestic entrepreneurs can save. By issuing domestic liquid bonds, the central bank alleviates the credit constraint facing investing entrepreneurs because firms can save more with the public bond (volume increases) and domestic interest rate is less repressed (price increases). As the domestic financial market is limited in its scope, the central bank can only invest its bond proceeds abroad, in the form of foreign reserve assets. A further motivation for the central bank to invest the bond proceeds abroad is that the world interest rate is higher than the domestic interest rate as long as the domestic financial market is con-strained and the domestic interest rate depressed. With the provision of domestic liquid bonds, entrepreneurs will have higher retained earnings with which they can invest more in capital regardless to the institutional credit constraint. Interestingly, capital plays a dual role in this economy: it is not only the input for production, it is the collateral for credit borrowing as well. With higher capital accumulation, the credit constraint is further relaxed, all pushing the economy to achieve the first best level. There is indeed a 'financial accelerator' effect in the model.

As we can see in subsequent sections, foreign reserve accumulation constitutes a second-best strategy during economic transition in the presence of credit and capital account constraints. With this regard, the level of foreign reserves will increase until the capital deepening is strong enough and/or the domestic private financial market is sufficiently developed.
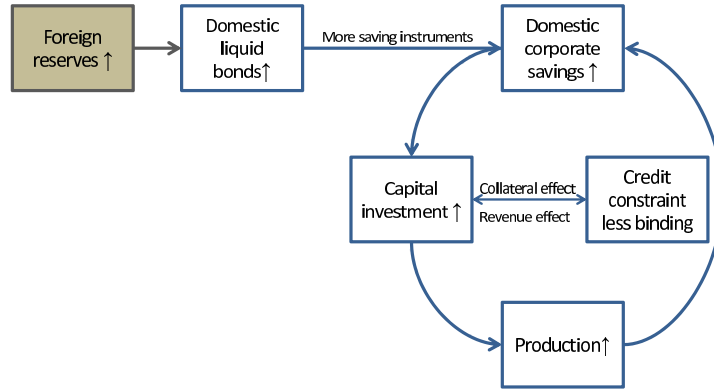
Figure 7: Storyline

1.3 Review of the literature

This paper is related to two broad strands of literature: the motives of foreign reserve accumulation, especially from a domestic structural viewpoint on the one hand and the economic growth in the presence of credit constraints on the other hand.

A country may accumulate foreign reserves for reasons of different nature: to constitute a buffer stock for the sake of crisis prevention or/and crisis management (*precautionary approach*[2],Bianchi et al. (2012)), to intervene in the foreign exchange market for the defense of a fixed exchange rate or for preventing the domestic currency from appreciation (*mercantilist approach*[3]), or to hold foreign reserves due to domestic structural deficiencies (*structural approach*). My work is related in particular to the last approach.

According to this strand of literature, countries accumulate foreign assets as a natural consequence of the wedge between domestic savings and investment. As Gourinchas and Jeanne (2007) put it, 'the allocation puzzle is a saving puzzle.' According to this paper, the distortions in the domestic financial sector may induce domestic precautionary savings. The distortions can be understood either in terms of the capacity of a country to supply safe assets (Caballero et al. (2008)) or in terms of the credit constraint facing domestic firms. In the presence domestic credit constraint, the economy in its whole needs to make precautionary savings, generating a positive gap between domestic savings and investment, source of capital outflows. Bacchetta and Benhima

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[2]See Aizenman and Lee (2007),Aizenman and Hutchison (2010), Jeanne and Rancière (2011), Benigno and Fornaro (2012)

[3]See Dooley et al. (2003), Korinek and Serven (2010), Benigno and Fornaro (2012)

(2010), Wen (2011) and Song et al. (2011) are the most relevant references with this regard. Based on an open economy with a credit constraint facing firms which invest, Bacchetta and Benhima (2010) show that the demand for foreign bonds is a complement to domestic investment rather than a substitute. The reason behind is that foreign bonds constitute supplementary corporate saving assets for credit-constrained firms. The retained earnings can be used to invest in capital even if firms' borrowing capacity can be constrained. Therefore, in a fast growing economy, the demand for foreign bonds increases with domestic capital formation. In the same vein, Song et al. (2011) shed light on how a constrained access to bank loans of private firms compared to state-owned firms can lead to a large level of corporate precautionary savings, thus to a persistent current account surplus. As there is a gradual transformation of state-owned firms into private ones, the overall precautionary savings increase, so do foreign reserves. Finally, Wen (2011) tries to 'make sense of China's excessive foreign reserves' by looking at household savings. He argues that a large uninsured risk (e.g. the lack of a sound social safety net), severe borrowing constraints, and rapid income growth can create an excessive level of high household saving rates and large current account surpluses for emerging economies. Therefore, foreign reserve accumulation is a natural consequence of a country's fast economic catch-up with an inefficient financial system. As all the three papers look at the demand for foreign assets in a small open economy setting with no capital controls, the government/central bank is absent. The public and private capital flows are thus perfect substitutes. What they examine is rather the change in a country's net foreign assets rather than the central bank's foreign reserves per se. My current work differs from theirs by targeting to the central bank's reserve assets. Moreover, foreign reserves are only a residual of private precautionary savings in these models while in my paper I argue that the social planner may have an optimal policy to boost domestic capital investment through purchasing foreign reserves.

In parallel, there is a strand of literature examining how policies of the government/central bank help alleviate domestic credit constraint. A seminal paper by Woodford (1990) demonstrates the welfare-improving effects of public debts in promoting domestic

capital investment in the presence of credit constraints. This paper is based on a financial autarky setting.

One contribution of my current work is to combine the above-mentioned two approaches and the features of both a small open economy and a closed economy setting. To put it in a different way, I examine the relationship between the demand for foreign assets and domestic capital formation in an economy with capital controls; only the public sector, e.g. the central bank, has a full access to international financial market. Private firms are forbidden to borrow/lend abroad directly. The imperfect substitutability thus requires the introduction of a central bank in the model. The model I present below will allow us to examine whether accumulating foreign reserves constitutes an optimal policy for the central bank. This paper is however not the first one which introduces a social planner (e.g. government or central bank) to discriminate public and private capital flows. Bacchetta et al. (2012) and Benigno and Fornaro (2012) also examine the foreign reserve accumulation in the presence of capital controls (Bacchetta et al. (2012)) or when private flows are not perfectly substitutable to public flows based on their respective credibility (Benigno and Fornaro (2012)).

My paper differs from Bacchetta et al. (2012) as I choose to use a production model so that I can focus on the contribution of reserves to domestic capital formation, driver of economic catch-up in emerging market economies. The introduction of capital is very important for two reasons. First, by introducing domestic capital formation, I add production efficiency gains when examining the optimal policy of foreign reserve accumulation in the presence of credit constraint and capital controls, in addition to redistributive effects and consumption smoothing gains. According to the literature of growth theory, the gains from production efficiency have a much more persistent and stronger impact on growth than that from pure consumption smoothing. This is thus a major contribution of my paper compared to Bacchetta et al. (2012) who only focus on consumption smoothing gains. Secondly, the introduction of capital formation makes my model setting more relevant for countries with large stock of foreign reserves, such as China, as in my model, households are allowed to finance directly firms through their savings.

My paper differs from Benigno and Fornaro (2012) as I model in a different manner the imperfect substitutability between public and private capital flows on the one hand and I consider a different motive of foreign reserve accumulation from their paper. While Benigno and Fornaro (2012) study the precautionary and mercantilist motives of foreign reserve accumulation, I focus on the imperfection of domestic financial market.

This paper is organized as follows. Section 2 describes the model setting. Section 3 analyzes the reserve accumulation and capital formation in a decentralized economy. Section 4 introduces the optimal policy of the central bank. Section 5 presents a numerical analysis. Section 6 presents some policy implications and concludes.

2   Model setting

Using a general equilibrium model with heterogeneous agents, the baseline model economy is a small economy with a single consumption good. There are three types of agents in the economy. Two symmetric entrepreneurs, a 'hand-to-month' labor and an active central bank. We describe their behaviors as follows.

2.1   Entrepreneurs and their optimal behavior

Two symmetric infinitely lived entrepreneurs produce and consume in this economy and they are occasionally constrained for credit borrowing every two periods when they face productive investment opportunities. In fact, each entrepreneur changes status every two periods, alternating between a 'saving period (denoted S)' and an 'investing period (denoted I)', according to the following Markov chain.

$$\begin{pmatrix} I_{t+1} \\ S_{t+1} \end{pmatrix} = \boldsymbol{M} \begin{pmatrix} S_t \\ I_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} S_t \\ I_t \end{pmatrix}$$

Entrepreneurs' budget constraints

A typical entrepreneur in his saving period receives the output from production (capital is chosen a period ahead) but does not have immediate productive investment plan available and thus chooses to save part of the revenue for future investment. On the other hand, in the investing period, an entrepreneur has immediate productive plans but does not have sufficient funding to invest, therefore it needs to borrow to invest in physical capital stock.

A typical entrepreneur who starts at time t with a saving period will have the following budget constraints every two periods.

$$\text{At } t : \Pi_t - r_t L_t = c_t^S + S_{t+1} \tag{1}$$

$$\text{At } t+1 : r_{t+1} S_{t+1} + L_{t+2} = c_{t+1}^I + K_{t+2} \tag{2}$$

That is, the saving entrepreneur uses his disposable income Π_t to reimburse loans subscribed a period ahead $r_t L_t$. It consumes c_t^S and makes savings S_{t+1} (in the form of lending to the investing entrepreneur or of central bank bonds). In the next period, there is a productive plan available, a saving entrepreneur becomes an investing entrepreneur and uses his retained earnings to consume c_t^I and to invest in capital stock K_{t+2}. r_t is the domestic gross interest rate ($r_t > 1$, for all t). As the revenue during investing stage is limited (no output available), the investing entrepreneur needs to make a loan L_{t+2} from the other entrepreneur who is in his saving period (due to symmetry).

The other symmetric entrepreneur who starts at time t with an investing period will have two reversed budget constraints as compared with above.

$$\text{At } t: r_t S_t + L_{t+1} = c_t^I + K_{t+1}$$

$$\text{At } t\text{+1}: \Pi_{t+1} - r_{t+1} L_{t+1} = c_{t+1}^S + S_{t+2}$$

Π_t denotes the wealth of the entrepreneur at the beginning of his saving period t. It comprises the production that the entrepreneur harvests using the capital stock invested a period ahead and the labor hired at the current period, net of labor cost.

$$\Pi_t = F(A_t, K_t, N_t) - w_t N_t \tag{3}$$

A_t, K_t, N_t and w_t denote respectively the labor-enhanced productivity, capital stock, labor force and per capita wage.

The production function $F(K_t, A_t, N_t)$ is a standard neoclassical production function: increasing in all arguments, concave and homogeneous of degree one. We use $F_{K,t}$ and $F_{N,t}$ to denote the marginal product of capital and of labor respectively.

Moreover, we introduce a passive fiscal policy (out of the hands of the central bank) in the model by allowing distortionary taxes on capital and labor. Denote $\tau_{K,t}$ and $\tau_{N,t}$ marginal rate of tax on capital and labor. Then we can express the after-tax marginal product of capital and labor, $\tilde{F}_{K,t}$ and $\tilde{F}_{N,t}$ as follows:

$$\tilde{F}_{K,t} = F_{K,t} - \tau_{K,t} \tag{4}$$

$$\tilde{F}_{N,t} = F_{N,t} - \tau_{N,t} \tag{5}$$

Therefore, based on the chapter 15 of Ljungqvist and Sargent (2004), we can rewrite the production function before and after tax payment in terms of the inputs and their (after-tax) marginal products:

$$F(A_t, K_t, N_t) = F_{K,t}K_t + F_{N,t}N_t \tag{6}$$

$$\tilde{F}(A_t, K_t, N_t) = \tilde{F}_{K,t}K_t + \tilde{F}_{N,t}N_t \tag{7}$$

The advantage of doing so, as underlined by Ljungqvist and Sargent (2004), is that we can treat $\tilde{F}_{K,t}$ as a new variable independent from $F_{K,t}$, similarly for $\tilde{F}_{N,t}$ and $F_{N,t}$.

Entrepreneurs' program

Entrepreneurs maximize the intertemporal utility function:

$$\sum_{t=0}^{\infty} \beta^t \left(U(c_t^S) + \beta U(c_{t+1}^I) \right) \tag{8}$$

As the two entrepreneurs are perfectly symmetric, we only need to look at the program of the entrepreneur who starts with a saving period at time t.

Notice that one of the market failure of the model economy is that there is a credit constraint facing the entrepreneur in his investing stage. This constraint is motivated by the classic argument of the asymmetric information. Namely, as the lender (the entrepreneur in his saving period) cannot guarantee the repayment of his debtor, he requires a collateral when making loans.

This credit constraint in some emerging market economies are institutional in nature. For example, in China, domestic private companies have a very restrictive access to bank

loans because of their status. The biggest Chinese banks are state-owned and prefer to lend money to state-owned firms, as Song et al. (2011) details.

Following this argument, we model the credit constraint as below:

$$L_{t+2} \leq \frac{\psi \Pi_{t+2}}{r_{t+2}} \tag{9}$$

The maximum loan that an investing entrepreneur can get is conditional on the discounted value of the disposable income that the entrepreneur will obtain next period when he harvest an output. ψ denotes the tightness of credit constraint, $\psi \in [0, 1]$. The smaller is ψ, the tighter the constraint is.

To obtain the optimal behavior of an entrepreneur in a decentralized economy, we maximize the equation (8) subject to (1), (2) and (9). And we obtain the following first order conditions. Note that $\lambda_{t+2} u'(c_{t+2}^S)$ denotes the Lagrange multiplier associated with the credit constraint at time $t + 2$.

$$(1 + \psi \lambda_t)\tilde{F}_{K,t} = r_t(1 + \lambda_t) \tag{10}$$

$$\tilde{F}_{N,t} = w_t \tag{11}$$

$$u'(c_t^S) = \beta r_{t+1} u'(c_{t+1}^I) \tag{12}$$

$$u'(c_{t+1}^I) = \beta r_{t+2} u'(c_{t+2}^S)(1 + \lambda_{t+2}) \tag{13}$$

Equation (10) tells us that the after-tax marginal product of capital is equal to the cost of capital r_t augmented by a coefficient $\frac{1+\lambda_t}{1+\psi \lambda_t}$, which is related to the credit constraint. Equation (11) equalizes the after-tax marginal product of labor with unit wage. Equations (12) and (13) are Euler equations. The Euler equation (13) which relates the marginal utility of the consumption of an entrepreneur in investing state to that of the same entrepreneur in saving stage depends clearly on the tightness of the credit constraint.

According to the Kuhn-Tucker conditions, the Lagrangian multiplier with respect to the credit constraint verifies:

$$\lambda_{t+2}u'(c_{t+2}^S)(r_{t+2}L_{t+2} - \psi\Pi_{t+2}) = 0$$

Therefore, the credit constraint may be binding $(r_{t+2}L_{t+2} = \psi\Pi_{t+2})$ or non binding $(\lambda_{t+2} = 0)$.

Notice that, when the credit constraint is binding, the investing entrepreneur at each period cannot borrow as much as they would like to do, therefore they cannot absorb all the funding the saving entrepreneur is willing to provide. This leads to a gap between saving and loans in each period. The central bank can thus play a role in order to close this gap. The strategy of the central bank is described later on.

2.2   Labor market

For simplicity, we assume that the labor supply is perfectly inelastic, namely $N_t^{supply} = 1$.

On the demand side, from the first order condition (11), we know that the marginal product of labor, $\tilde{F}_{N,t}$, which is equal to wage w, is decreasing in labor demand N_t^{demand}. Therefore, we obtain a conventional decreasing labor demand function.

For a given level of technology A_t and capital stock K_t, the labor demand and supply functions uniquely pin down the wage and the equilibrium labor on the labor market.

Therefore, we are allowed to use this result from the labor market equilibrium $N_t^{supply} = N_t^{demand} = 1$ henceforth.

Using this result and equation (11), we can rewrite the after-tax operational income of family companies:

$$\Pi_t = \tilde{F}_{K,t}K_t + \tilde{F}_{L,t}N_t - w_tN_t \tag{14}$$

$$= \tilde{F}_{K,t}K_t \tag{15}$$

Note that as the labor is inelastic in the model, the marginal tax on labor does not have distortionary effect. Any change in the marginal tax rate of labor is equivalent to a

lump-sum tax (transfer) to the labor force which is out of our scrutiny.

2.3 The central bank's Behavior

The hallmark of this paper is that we explicitly model the behavior of the central bank in the model economy. We argue that the central bank can provide a liquid central bank bond, which is used as a supplementary saving instrument for the saving entrepreneur. As we will show later on, this may be a second-best strategy of the central bank when the domestic financial market is under-developed.

General setting

Several characteristics about the central bank in our model needs to be underlined. First of all, the central bank is a consortium of the monetary authority and the commercial banks. This setting is plausible as in China the banking system is mainly state-owned. In fact, according to Walter and Howie (2012), more than 90 percent of total commercial banking assets is state-controlled in China.

Secondly, the central bank is the only player which has access to international financial market in our model economy. This situation is dubbed by Bacchetta et al. (2012) as a 'semi-open' economy. This way of modeling corresponds to what we observe about China's capital account management. In fact, China not only accumulates foreign reserves but also closely control capital inflows and outflows. The private sector does not have a full access to the international financial market. Within the scope of the paper, we assume that the capital controls which make domestic private capital flows and public capital flows imperfectly substitutable are institutional in nature and cannot be removed in the short-run. To our understanding, the need of setting capital controls are closely related to the under-development of the domestic financial market which is the focus of our analysis.

Thirdly, the central bank does not control fiscal policy which is in the hands of the government. As the State is the ultimate shareholder of the central bank, we argue that distortionary taxes on capital and labor are used to balance the central bank's

intertemporal budget constraint.

Finally, the central bank is forbidden to finance the government debt by law. Namely, the central bank cannot purchase any government bonds.

The central bank's flow budget constraint is described as follows:

$$\underbrace{r_t^* B_t^* + B_{t+1} + T_t}_{\text{resources}} = \underbrace{r_t B_t + B_{t+1}^* + \frac{\Phi}{2} \left(B_{t+1}^* - B_t^* \right)^2}_{\text{expenses}} \tag{16}$$

, where $T_t = \tau_{K,t} K_t + \tau_{N,t}$

The central bank receives tax revenue T_t from entrepreneurs as well as returns on foreign reserve holdings $r_t^* B_t^*$ from abroad and domestic liquid bond proceeds B_{t+1}. On the side of expenses, the central bank needs to pay back the liquid bond issued a period ahead $r_t B_t$ and to decide the amount of foreign reserves to purchase B_{t+1}^*. Notice that accumulating foreign reserves is not costless. A quadratic adjustment cost $\left(B_{t+1}^* - B_t^* \right)^2$ is introduced to take into account the costs of foreign reserve holding and to prevent explosive solution in small open economy model (following Schmitt-Grohe and Uribe (2003)). We further assume that the central bank does not consume.

Interaction between the private sector and the central bank

What is the role played by the central bank in our model economy. We present below the balance sheets of the central bank and of the private sector for illustration.

In fact, by providing a liquid central bank bond, the saving entrepreneur can save up to first-best level with this supplementary saving instrument. Therefore the retained earnings of this entrepreneur in the investing period will increase, allowing the entrepreneur to invest more in capital stock.

The central bank's choice of purchasing foreign reserve assets

The use of public debt, e.g. government or central bank bonds, has proved to be a welfare-improving strategy to raise domestic capital formation, as Woodford (1990) demonstrates. In this paper, I further argue that in order to provide this liquid bond,

Saving entrepreneur

Assets	Liabilities
central bank bond (B)	*Own capital* (*Savings S*)
Loans (L)	

central bank

Assets	Liabilities
Foreign reserves (B*)	*central bank bond* (B)

Investing entrepreneur

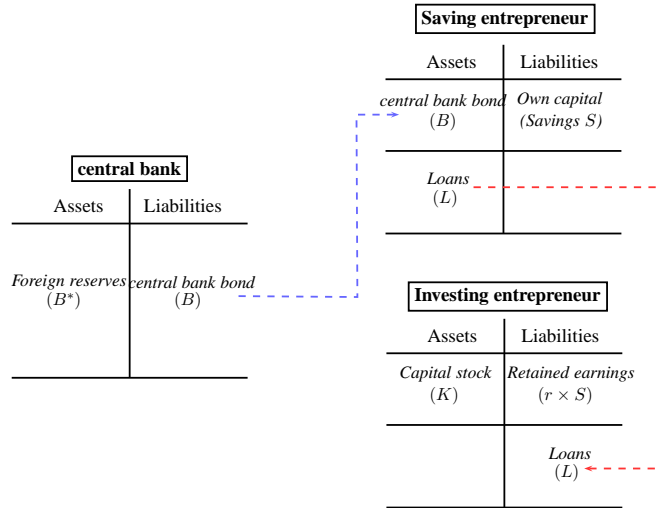| Assets	Liabilities
Capital stock (K)	*Retained earnings* ($r \times S$)
	Loans (L)

Figure 8: Balance sheet of the central bank and the private sector

the central bank needs to invest the bond proceeds abroad. There are not available alternatives. As a result, massive foreign reserve accumulation is a natural consequence of a growth strategy aiming at promoting domestic capital formation.

Based on the experience of China, several arguments can be advanced to explain why the central bank has to invest bond proceeds in foreign assets. First, the domestic financial market is very shallow in scope; as a result the domestic investment opportunities are limited. According to Walter and Howie (2012), in China 'the banks are the financial system' and 'it is bank lending and [public] bond issuance that keep the engine of Chinas state-owned economy revving at high speed.' Besides the bank deposits and loans (which are integrated into the private sector in our model), the private bond market and equity markets are very small. For example, in 2007 which was a record year for Chinese equity financing, more than 123 billion of U.S. dollar was raised on the equity market, but in the same year, banks extended new loans totaling 530 billion of dollars and total debt issues in the bond market accounted for another 355 billion. Now, if we look at more carefully at the Chinese bond market, we observe that this market is dominated by public obligations, i.e. government debt, central bank bond and policy bank bond issuance. Corporate and commercial papers are very recent financial products and their weight on domestic bond market is negligible (around 5%) compared to the public issuance (Figure 9).

Moreover, the central bank cannot directly buy government debt. This is prohibited

20
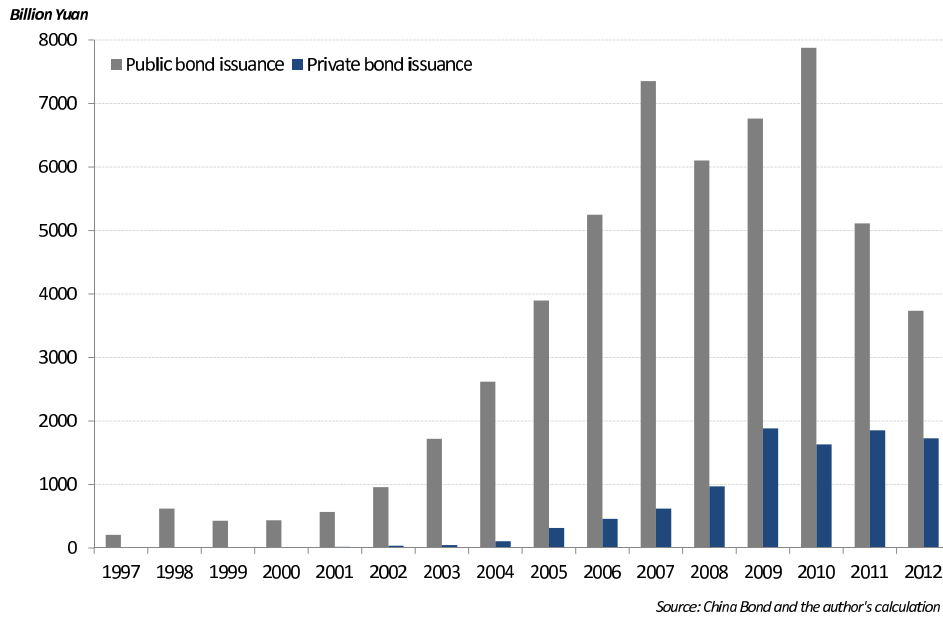
Figure 9: Public bonds vs. private bonds

by the Law of the People's Bank of China of 1995 and its amendment in 2003. The proportion of the central bank bonds has steadily increased from 2002 to 2010, a period when Chinese foreign reserves have rapidly increased and the government's capacity of issuing new debts was limited.

Figure 10: Decomposition of the Chinese bond market

Finally, the domestic financial system is mainly state-owned. If the central bank invested bond proceeds domestically, the money would just be going around in circles.

The central bank's policy

In order to promote domestic economic growth, the central bank has mainly three instruments: domestic interest rate, liquid bond issuance and foreign reserve purchasing, as indicated in the table below.

Table 1: Different variables

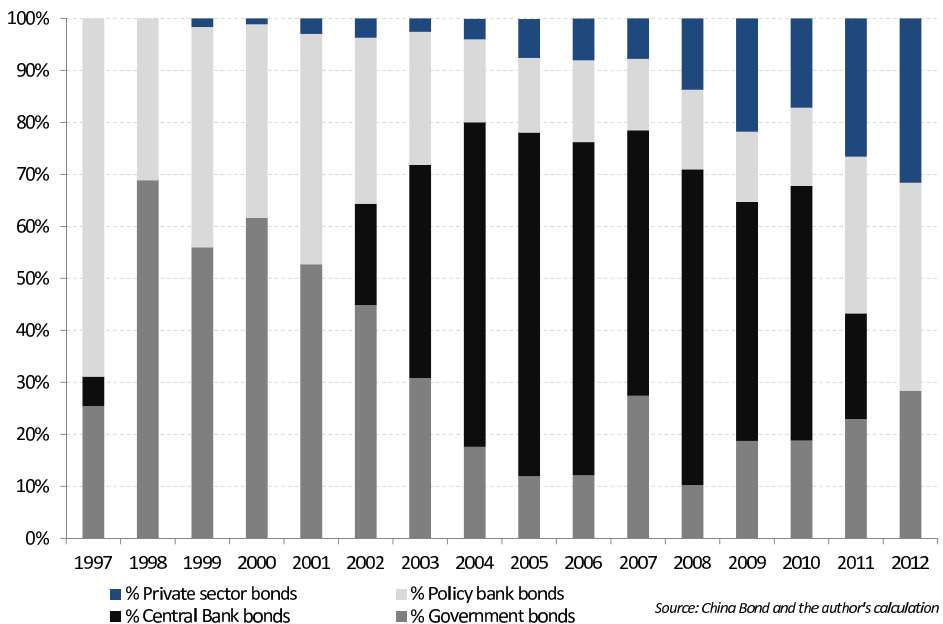| Central Bank policy	Market variables	Exogenous fiscal policy
r_{t+1}	c_t^S, c_t^I	$\tau_{K,t}, \tau_{N,t}$
B_t	K_{t+1}	
B_t^*	S_{t+1}, L_{t+1}	

The usefulness of the instruments depends on the policy regime that a country has. We will below compare several policy regimes in order to see whether accumulating foreign reserves consists of an optimal policy for the central bank. Different policy regimes are defined as follows, according to Bacchetta et al. (2012).

Table 2: Policy regimes

		Policy Instruments	
Policy regime	Characteristics	Foreign Reserves	Interest Rate
Financial autarky	Only fiscal policy & public debt	$B^* = 0$	$r \in \Re^+$
Open Economy	Saving opportunity abroad	$B^* \in \Re^+$	$r = r^*$
Semi-open economy	Controls on private flows	$B^* \in \Re^+$	$r \in \Re^+$

Notice that in this model, the financial intermediary is absent in this model. However, the results that I derive in the subsequent sections will not change if a perfectly competitive banking sector is introduced. Therefore, to keep the model tractable, I decide to put the financial sector aside in this paper.

2.4 Market clearing

On the financial market

The amount of the liquid bonds that the central bank provides is equal to the difference between the savings of the saving entrepreneur and the loans asked by the investing entrepreneur at each period, namely,

$$\underbrace{B_t}_{\text{supply of liquid bonds}} = \underbrace{S_t - L_t}_{\text{demand for liquid bonds}} \quad (17)$$

A positive B_t is motivated by the fact that the investing entrepreneur is financially constrained. Otherwise, the borrowing need L_t oughts to absorb all the savings S_t on the financial market, leading to $B_t = 0$.

On the good market

Output produced by the saving entrepreneur is consumed by both entrepreneurs and the labor force, to invest in capital investment or to buy foreign reserves. We thus have the following aggregate clearing condition:

$$F(A_t, K_t) = c_t^S + c_t^I + w_t + K_{t+1} - (r_t^* - 1)B_t^* + \underbrace{B_{t+1}^* - B_t^*}_{\text{current account balance}} + \frac{\Phi}{2}\left(B_{t+1}^* - B_t^*\right)^2 \quad (18)$$

3   Market equilibrium without public intervention

In this section, I present some fundamental characteristics of the model which can be analytically derived in a decentralized market equilibrium. I can show the saving entrepreneur tends to make precautionary savings, knowing that he will be constrained for borrowing next period in his investment stage. Moreover, I demonstrate that when the financial constraint is strong enough ($\psi < \frac{1}{2}$), in steady state, the constraint is binding and the constrained steady state is uniquely determined. Finally, I will show how the central bank policy can ameliorate the capital formation in the model economy. In order to derive analytical results, I use log-utility and Cobb-Douglas production function henceforth, namely $U(c_t^i) = log(c_t^i), i \in (S, I)$ and $F(K_t, A_t) = K_t^\alpha A_t^{1-\alpha}$.

Definition 1 *A perfect foresight market equilibrium in a decentralized economy is a sequence of prices $\{w_t, r_t\}$ and an allocation $\{c_t^S, c_t^I, K_{t+1}, S_{t+1}, L_{t+1}\}$, for a given policy set $\{B_t^*, B_t, \tau_{K,t}, \tau_{N,t}\}$ (t = 0, 1, 2, ...), and for given initial conditions $\{K_0, L_0, S_0, B_0, B_0^*\}$, such that for all $t \geq 0$:*

1. *Given prices and the government policy, $\{c_t^S, c_t^I, K_{t+1}, S_{t+1}, L_{t+1}\}$ satisfy the first order conditions (10) to (13)*

2. *Government budget constraint is verified (16)*

3. *Financial and good markets are cleared: (17) and (18)*

Notice that depending on the economic policy regime, the central bank can use all or part of its policy instruments $\{r_t, B_t^*, B_t\}$.

With the log-utility and equations (1), (2),(12), (13), we can show that the optimal consumption derived from each entrepreneur's utility maximization program represents a constant share of the entrepreneur's initial wealth at the beginning of the period (See Appendix A).

$$c_t^S = (1 - \beta)(\Pi_t - r_t L_t) \tag{19}$$

$$c_t^I = (1 - \beta)r_t S_t \tag{20}$$

Then, we obtain:

$$S_{t+1} = \beta(\Pi_t - r_t L_t) \tag{21}$$

$$K_{t+2} - L_{t+2} = \beta r_{t+1} S_{t+1} \tag{22}$$

3.1 Entrepreneurs' precautionary savings

We can first show a common result as the literature of domestic financial under-development is used to present: due to the credit constraint entrepreneurs save more than in a perfect market without credit frictions. This extra savings correspond to the 'precautionary savings' of the saving entrepreneur who anticipates the credit constraint that he will face a period later when there will be a productive investment plan.

Indeed, when the credit constraint is binding, the savings made by the saving entrepreneur is determined using (1), (2) and the optimal intertemporal allocation from the saving stage to the investing stage 12:

$$S_{t+1} = \frac{1}{1 + \beta} \left[\underbrace{\beta(1 - \psi)\tilde{F}_{K,t} K_t}_{\text{predetermined}} + \frac{K_{t+2}}{r_{t+1}} - \frac{L_{t+2}}{r_{t+1}} \right] \tag{23}$$

As the credit constraint is binding, $L_{t+2} \leq L_{t+2}^*$, L_{t+2}^* being the level of borrowing when the credit constraint is absent. Therefore, $S_{t+1} \geq S_{t+1}^*$, S_{t+1}^* being the private savings in the absence of any credit constraint. The saving entrepreneur saves thus more compared to the first-best situation where the financial market is frictionless.

3.2 Capital formation

How does the capital stock evolve over time? From (22), the capital accumulation consistent with optimal consumptions satisfies:

$$K_{t+2} = \underbrace{\beta r_{t+1} S_{t+1}}_{\text{retained earnings}} + \underbrace{L_{t+2}}_{\text{loans}} \tag{24}$$

$$= \beta^2 r_{t+1}(1 - \psi)\tilde{F}_{K,t}K_t + \frac{\psi \tilde{F}_{K,t+2}K_{t+2}}{r_{t+2}} \tag{25}$$

Notice that the retained earnings and the loans are reversely affected by interest rates. An increase in domestic interest rate raises the retained earnings of the saving entrepreneur (revenue effect) but reduces the loans that the investing entrepreneur may make (price effect). Therefore, for any entrepreneur, the stock of capital is affected by two-period interest rates. Whether the capital stock increases or decreases depends on which effect dominates.

We also observe a financial accelerator effect in equation (25). An initial positive shock on the retained earnings (either an increase in r_{t+1} or an increase in initial capital stock $\tilde{F}_{K,t}K_t$) will lead to an increase in capital formation at t_{t+2}, which in turn drives up loans. As a result, contemporary capital formation further increases, generating a virtuous cycle.

From (25), one can easily show that ceteris paribus

$$\frac{\partial K_{t+2}}{\partial r_{t+1}} > 0$$
$$\frac{\partial K_{t+2}}{\partial r_{t+2}} < 0$$

3.3 Steady state

I now study the steady state of the decentralized economy. I normalize all policy variables by the disposable income $\Pi = \tilde{F}_K K$. Namely we define $b^* = \frac{B^*}{\Pi}$, $b = \frac{B}{\Pi}$ and $\tilde{\tau} = \frac{T}{\Pi}$.

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From the central bank's budget constraint (16), we know that

$$(r^* - 1)b^* + \tilde{\tau} = (r - 1)b \qquad (26)$$

Given international and domestic interest rates, we observe from the equation above that if the central bank wants to provide liquid assets in the domestic economy, it can achieve this objective either by raising distortionary taxes or by purchasing foreign reserves. The foreign reserve purchasing allows the central bank to provide liquid assets without distorting the economy.

Uniqueness of the binding steady state

Now, we show that the steady state is uniquely determined.

From (12) and (13), we know that in steady state

- the credit constraint is binding when $r < \frac{1}{\beta} \left((\beta r)^2 = \frac{1}{1+\lambda} < 1 \right)$

- the credit constraint is unbinding when $r = \frac{1}{\beta}$

Proposition 1 *The financially constrained steady state is uniquely determined if $0 \leq b < \beta(1 - 2\psi)$.*

Proof. See Appendix B ∎

From Proposition 1, we can conclude that the credit constraint is not binding when $b \geq \beta(1 - 2\psi)$; the central bank can achieve this goal by raising b^* or $\tilde{\tau}$. The credit constraint is binding when $0 < b < \beta(1 - 2\psi)$. We also observe that with the credit constraint, the domestic interest rate is repressed and lower than the world interest rate $\frac{1}{\beta}$. When the central bank provides liquid bonds which were scarce in the economy, it lowers the price of the liquid bonds and pushes the domestic interest rate up to the world interest rate level. r is indeed increasing in b: $r = \frac{\psi}{\beta(1-\psi)-b}$

We can further prove that the coefficient of the tightness of the credit constraint is important for whether the credit constraint is binding in the steady state. When $\psi \geq \frac{1}{2}$, the credit constraint is unbinding in the steady state. When $\psi < \frac{1}{2}$, the credit constraint is strictly binding in the steady state (See Appendix C for detail).

We can further look at the different steady states in different policy regimes described in Table 2.

Financial autarky   In a closed economy where $B^* = 0$, if $\psi \geq 1/2$, the credit constraint does not bind in the steady state. In this case, $\beta r = 1$, namely, the domestic interest rate is equal to the world interest rate (assuming the same time preference). $K = A(\alpha\beta)^{\frac{1}{1-\alpha}}$, capital stock achieves its first-best level, assuming that there is no distortionary tax $\tau_K = 0$. And the consumptions are perfectly smoothed, namely the consumptions in the saving stage and the investing stage are equalized $c^I = c^S$.

If $\psi < 1/2$, the credit constraint binds in the steady state. The Lagrange multiplier associated with the credit constraint is thus positive, $\lambda = \frac{1}{\beta^2 r^2} - 1 > 0$. We have the domestic interest rate less than the world interest rate, namely $\beta r = \frac{\psi}{1-\psi} < 1$. Capital stock cannot achieve the first-best level, $K = A\left[\alpha\left((1-\psi)\beta^2 r + \frac{\psi}{r}\right)\right]^{\frac{1}{1-\alpha}}$. As for the consumptions, that of the investing stage is always smaller than that of the saving stage, $\frac{c^I}{c^S} = \frac{\psi}{1-\psi} < 1$.

In fact, in a financial autarky, with the productivity growth, the more capital the better. There is thus a strong demand for liquid assets. However, investors are constrained and the central bank does not have sufficient resources to provide domestic assets without distorting the domestic economy. Therefore, the economy may be constrained if the credit constraint is very tight.

Open economy   One big difference compared to the financial autarky is that the domestic interest rate is always equal to the world interest rate in the steady state, $r = r^* = \frac{1}{\beta}$. In this case the credit constraint in never binding, leading to $b = b^* = \beta(1 - 2\psi)$, in the absence of distortionary taxes. In an open economy, both the central bank and th saving entrepreneur have access to international financial market, therefore, the saving entrepreneur can make deposits abroad. In aggregate, there are sufficient foreign assets to increase saving revenues of domestic entrepreneurs. Therefore, in steady state, the economy is unconstrained.

Table 3: Decentralized economy

Economic Structure	Steady State	ψ
Financial autarky	constrained	$\psi < \frac{1}{2}$
	unconstrained	$\psi \geq \frac{1}{2}$
Open Economy	unconstrained	$\forall \psi$

The capital formation in the steady state

In the steady state, how the capital stock evolves over time with policy variables? Previously we demonstrate that the capital stock is composed of retained earnings and borrowing and that retained earnings are increasing in the interest rate while borrowing is decreasing in the interest rate. The capital stock thus depends on both the revenue effect and of the price effect of interest rates.

Proposition 2 *The capital stock represents a U-shape curve as a function of βr. Purchasing foreign reserves, which leads to a higher supply of domestic liquid assets and thus a higher interest rate, helps increase the capital stock and raise the production in the economy.*

Proof. See Appendix D ∎

For simplicity, we ignore distortionary taxes which will only shift the U-shape curve in Figure 11 vertically but do not change the shape of the curve.

From Figure 11, we observe that the capital stock achieves its first-best level if $\beta r = 1$ (i.e. when the credit constraint is not binding) or if $\beta r = \frac{\psi}{1-\psi}$. In the first case where $\beta r = 1$, the economy is in the steady state with an unconstrained credit constraint (i.e. either in a full open economy or in a financial autarky with $\psi \geq \frac{1}{2}$). In the second case where $\beta r = \frac{\psi}{1-\psi}$, the economy is in a financially autarkic steady state where the central bank does not provide any liquidy saving instruments.

When $\frac{\psi}{1-\psi} \leq \beta r \leq \sqrt{\frac{\psi}{1-\psi}}$, the curve is decreasing. when the central bank starts to provide liquid bonds, this raises the interest rate, leading to a more stringent credit constraint and thus a lower level of borrowing. At the same time, as the financial market is under-developed, the saving entrepreneur cannot save as much as he wants short of
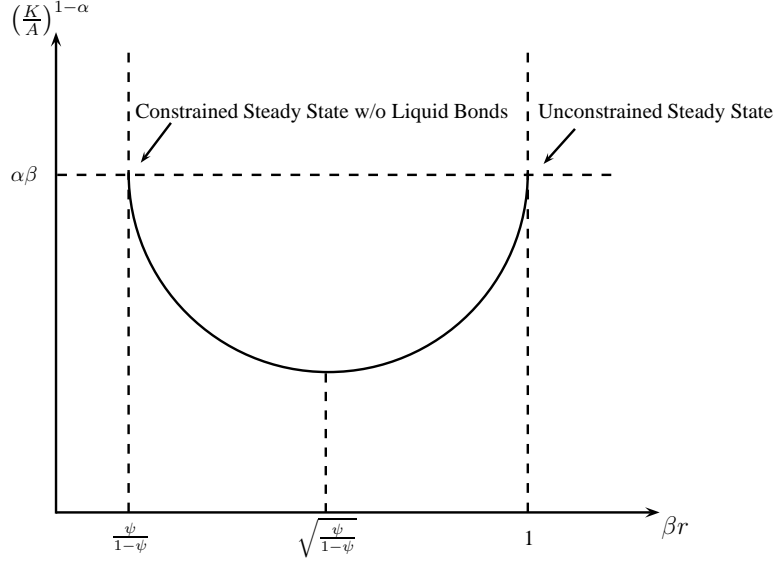
Figure 11: U-shape capital stock

sufficient saving instruments on the market. Therefore, on this part of the curve, the price effect of the interest rate dominates the revenue effect, leading to a decrease in the capital stock.

When $\sqrt{\frac{\psi}{1-\psi}} \leq \beta r \leq 1$, the curve is increasing. This is because when the central bank provides more and more liquid assets, the price of these assets further decrease and raise even more the interest rate, any entrepreneur in his saving stage can save up to the first-best level and the retained earnings increase with the interest rate. Bear in mind that the retained earnings are used for capital investment a period later when the entrepreneur faces investment opportunities. The revenue effect dominates the price effect on this part of the curve.

We can gain a more precise insight of how capital formation evolves with the interest rate by examining the following equation (derived in Appendix D):

$$\tilde{F}_K = \frac{r}{(\beta r)^2 + \psi[1 - (\beta r)^2]} \tag{27}$$

We identify three effects which affect the capital stock in the economy:

- Opportunity cost of capital, r

- Revenue effect, $(\beta r)^2 = \frac{1}{1+\lambda}$

- Wealth effect, $\psi[1 - (\beta r)^2]$

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The first effect, opportunity cost of capital, is standard: the higher is r, the higher is the cost of investing in capital stock compare to buying bonds, the less capital is demanded. The other two effects are particular in our model.

The second term $(\beta r)^2$ is related to the credit constraint in the economy. As entrepreneurs are occasionally credit-constrained, they have incentives to partly rely on retained earnings (precautionary savings) in order to be able to invest in capital even they are constrained. As the retained earnings are increasing in interest rate, this effect has a positive impact on capital stock (i.e. revenue effect). Notice that the square appears as the capital is invested every two periods. In an extreme case, when $\psi = 0$ (the constraint extremely binding), the second effect clearly dominates the first one. As a result, a higher supply of the liquid bonds leads to a higher domestic interest rate, unambiguously increasing the capital stock.

The third effect stems from the fact that the loans that an investing entrepreneur can make is conditional on the discounted valued of the future production. An increase in r decreases the borrowing capacity of investing entrepreneurs, as it lowers the discounted value of the collateral. The interest rate has thus a negative price effect on capital stock. As described above, the price effect dominates the revenue effect when the supply of liquidity is very low, as in this case the collateral value of capital becomes so large that it dominates the other effects. However, the capital formation has a feedback loop effect on the credit constraint. Namely, more capital invested provides stronger collateral in the next period, allowing investing entrepreneurs to borrow more money during the investing stage and leading to a looser credit constraint. As the credit constraint becomes less stringent, this third effect becomes less important and is dominated by the revenue effect. The tightness of the credit constraint can also dampen the collateral effect on capital, making the revenue effect stronger than the price effect.

As long as the central bank provides liquid bonds B by purchasing foreign reserves B^*, it can push up domestic interest rate to the world interest rate and to push domestic capital formation to its first-best level.

4 The central bank's optimal policy

Until now we have analyzed the fundamental characteristics of our model and its steady states, both when the credit constraint is binding or unbinding. We have also examined two policy regimes - financial autarky and open economy. In the first case, the central bank, along with a latent government setting tax rates, uses public bond and interest rate to adjust the economy. In the second case, when the economy is fully open, the private sector has a direct access to international financial market to eliminate the domestic credit constraint. As a result, the central bank is somewhat bypassed by the private sector.

Now we examine the central bank's policy in a semi-open economy, namely when the central bank is the only one which can borrow or lend abroad. In such a policy setting, can the public intervention of the central bank improve the welfare of the economy? We use here the Ramsey problem presented in Chapter 15 of Ljungqvist and Sargent (2004).

4.1 General setting

We assume that the central bank optimizes the consumptions of contemporaneous entrepreneurs in their different stages as well as the labor income, according to the following objective function. Remember that in a semi-open economy, the central bank can decide both the amount of foreign assets to purchase, B_{t+1}^*, and the domestic interest rate r_{t+1}.

$$\max_{\{S_{t+1}, L_{t+1}, B_{t+1}^*, K_{t+1}, c_t^S, c_t^I, w_t, r_{t+1}, \lambda_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t^S) + u(c_t^I) + \omega u(w_t) \right\} \tag{28}$$

Definition 2 (Ramsey problem) *The central bank's optimal policy consists of choosing a sequence of policy variables $\{B_{t+1}^*, B_{t+1}, r_{t+1}\}_0^\infty$ such that:*

- *The corresponding competitive equilibrium allocation (10) to (13) maximizes the welfare function (28)*

- *Individual entrepreneurs' budget constraints, (1) and (2), are verified*

- *Government budget constraint (16) and resource constraint (18) are verified*

- *Bond market* (17) *is cleared*

The way to solve the Ramsey problem follows the "Primal Approach" described by Erosa and Gervais (2001).

The full Ramsey problem is presented below. Equation (11) is used to substitute $\tilde{F}_{N,t}$ by w_t.

$$
\begin{aligned}
\max_{\{S_{t+1}, L_{t+1}, B^*_{t+1}, K_{t+1}, c^S_t, c^I_t, w_t, r_{t+1}, \lambda_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \Bigg\{ & u(c^S_t) + u(c^I_t) + \omega u(w_t) \\
& + \eta^S_t \left[\tilde{F}_{K,t} K_t - r_t L_t - c^S_t - S_{t+1} \right] \\
& + \eta^I_t \left[r_t S_t + L_{t+1} - c^I_t - K_{t+1} \right] \\
+ \eta^G_t \left[F_{K,t} K_t + F_{N,t} - c^S_t - c^I_t - w_t - K_{t+1} - B^*_{t+1} + r^*_t B^*_t - \frac{\Phi}{2} \left(B^*_{t+1} - B^*_t \right)^2 \right] & \\
& + \theta^S_t \left[\frac{1}{c^S_t} - \beta r_{t+1} \frac{1}{c^I_{t+1}} \right] \\
& + \theta^I_t \left[\frac{1}{c^I_t} - \beta r_{t+1}(1 + \lambda_{t+1}) \frac{1}{c^S_{t+1}} \right] \\
& + \rho_t \left[(1 + \psi \lambda_t) \tilde{F}_{K,t} - r_t(1 + \lambda_t) \right] \\
& + \Lambda_t \left[\psi \tilde{F}_{K,t} K_t - r_t L_t \right] \Bigg\}
\end{aligned}
$$

We then need to derive first order conditions with respect to all endogenous variables.

$$FOC(S_{t+1}) : \eta_t^S = \beta r_{t+1} \eta_{t+1}^I \tag{29}$$

$$FOC(L_{t+1}) : \eta_t^I = \beta r_{t+1} \left(\eta_{t+1}^S + \Lambda_{t+1} \right) \tag{30}$$

$$FOC(B_{t+1}^*) : \eta_t^G \left[1 + \Phi(B_{t+1}^* - B_t^*) \right] = \beta r_{t+1}^* \eta_{t+1}^G \left[1 + \frac{\Phi(B_{t+2}^* - B_{t+1}^*)}{r_{t+1}^*} \right] \tag{31}$$

$$FOC(K_{t+1}) : \beta \eta_{t+1}^S \tilde{F}_{K,t+1} - \eta_t^I - \eta_t^G + \beta \eta_{t+1}^G F_{K,t+1} + \beta \psi \Lambda_{t+1} \tilde{F}_{K,t+1} = 0 \tag{32}$$

$$FOC(c_t^S) : \frac{1}{c_t^S} - \eta_t^S - \eta_t^G - \frac{\theta_t^S}{(c_t^S)^2} + \frac{\theta_{t-1}^I r_t (1 + \lambda_t)}{(c_t^S)^2} = 0 \tag{33}$$

$$FOC(c_t^I) : \frac{1}{c_t^I} - \eta_t^I - \eta_t^G - \frac{\theta_t^I}{(c_t^I)^2} + \frac{\theta_{t-1}^S r_t}{(c_t^I)^2} = 0 \tag{34}$$

$$FOC(r_{t+1}) : \beta \eta_{t+1}^I S_{t+1} - \beta \left(\eta_{t+1}^S + \Lambda_{t+1} \right) L_{t+1} - \beta \frac{\theta_t^S}{c_{t+1}^I} - \beta \theta_t^I \frac{(1 + \lambda_{t+1})}{c_{t+1}^S} - \beta \rho_{t+1} (1 + \lambda_{t+1}) = 0 \tag{35}$$

$$FOC(\lambda_{t+1}) : \frac{-\theta_t^I \beta r_{t+1}}{c_{t+1}^S} + \beta \rho_{t+1} \left(\psi \tilde{F}_{k,t+1} - r_{t+1} \right) = 0 \tag{36}$$

We can also derive the first order conditions with respect to $\tilde{F}_{K,t+1}$ or/and $\tilde{F}_{N,t+1}$ (w_t), if we allow the central bank to set tax rates as well.

$$FOC(\tilde{F}_{K,t}) : \eta_{t+1}^S K_{t+1} + \rho_{t+1} \psi \lambda_{t+1} + \Lambda_{t+1} \psi K_{t+1} \tag{37}$$

$$FOC(w_t) : \frac{\omega}{w_t} - \eta_t^G = 0 \tag{38}$$

4.2 Steady state

When the central bank actively uses its policy instruments $\{B_{t+1}, B_{t+1}^*, r_{r+1}\}$ to adjust the economy, what is the steady state it can achieve?

Financial autarky

In the financial autarky, the first order condition (31) is no longer relevant, $B^* = 0$. Whether the steady state is financially constrained depends thus on, as in the decentralized economy, the tightness of the credit constraint.

Proposition 3 *The central bank can achieve the steady state with an unconstrained credit constraint when the coefficient of the tightness of the credit constraint $\psi \geq \frac{1}{2}$. That is, $\lambda = \Lambda = 0$, $\beta r = 1$ and $c^S = c^I$. When $\psi < \frac{1}{2}$, the steady state with the Ramsey policy is constrained. That is, $\lambda > 0$, $\Lambda > 0$, $\beta r < 1$ and $c^I < c^S$.*

Open economy

Proposition 4 *The central bank can achieve the steady state with an unconstrained credit constraint in an open economy by using an optimal policy à la Ramsey. That is, the central bank can achieve the same steady state as the decentralized economy does in an open economy.*

Proof. See Appendix E ∎

4.3 Economic transition in a semi-open economy

We have seen so far that the steady state does not change in the Ramsey program compare to the decentralized economy, both in a financial autarkic setting or in an open economy. How about the economic transition in a semi-open economy towards a steady state where the economy will be fully open? In a semi-open economy, the central bank has two instruments in its hands: central bank bond and interest rate. Therefore, we argue that the Ramsey program in a semi-open economy is welfare improving compare to a financial autarky or an open economy.

Indeed, the foreign reserve accumulation is a second-best strategy of economic growth, taking into account both credit constraint and capital controls. A combined use of capital controls and foreign reserve holding allows the central bank to have sufficient funding to provide liquid assets within the country and at the same time to be able to adjust domestic interest rate whenever it is necessary.

To see this, we should pay attention to the first order condition (35).

$$FOC(r_{t+1}): \underbrace{\eta_{t+1}^{I} S_{t+1} - \left(\eta_{t+1}^{S} + \Lambda_{t+1}\right) L_{t+1}}_{\text{redistributive effects}} - \underbrace{\frac{\theta_t^S}{c_{t+1}^I} - \frac{\theta_t^I \left(1 + \lambda_{t+1}\right)}{c_{t+1}^S}}_{\text{intertemporal choices}}$$

$$- \underbrace{\rho_{t+1} \left(1 + \lambda_{t+1}\right)}_{\text{opportunity cost}} = 0$$

When the capital account is closed and the central bank accumulates foreign reserves, it can control domestic interest rates subject to different policy objectives. When the central bank increases domestic interest rate, this is beneficial for the investor of the next period as the revenue from savings will be higher (first term). However, this will be harmful for the saver of the next period as it has to pay higher interests of loans they made during the investment stage (second term). It will also change intertemporal behavior of entrepreneurs, leading to more savings and less consumption, thus decreasing social welfare (third and fourth terms). Finally, a higher domestic interest rate implies higher opportunity cost, therefore the capital investment will decrease (the last term). The first two terms translate a redistributive trade-off that the central bank has to face. This redistributive effects also imply welfare costs to the economy. Indeed, if the current investors are very constrained and the central bank thinks the long run steady state can be achieved soon, it has incentives to decrease the interest rate to alleviate the interest burden of investing firms. If on the contrary the central bank believes that the economy will still be constrained for a long period, it has incentives to rise domestic interest rate, so that the current savers may transfer a larger amount of interest revenue to the future when they will be constrained.

Depending on the tightness of the credit constraint and the speed of convergence to the steady state, the central bank can decide to favor savers or investors in order to enhance the social welfare.

Denote $\mathcal{H}_t$ the left hand side of the equation (35). We can show that

$$\mathcal{H}_t = \left(\sum_{i=1}^{\infty} \Lambda_{t+2i}\right) S_{t+1} - \left(\sum_{i=1}^{\infty} \Lambda_{t+2i+1}\right) L_{t+1} \tag{39}$$

In sum, when $\mathcal{H}_t$ is different from 0, the central bank has incentives to intervene and the open economy does not provide an optimal equilibrium. If $\mathcal{H}_t > 0$, the central bank has incentives to raise interest rate to favor savers. If $\mathcal{H}_t < 0$, the central bank has incentives to lower interest rate. In general, as Table 4 shows, with financial frictions an open economy is suboptimal during transition while an economy with closed capital account and sufficient foreign reserves can achieve higher social welfare during transition.

Table 4: Static vs. Dynamic approach

| | Central Bank Intervention | |
Economic Structure	Steady state	Dynamics
Financial autarky	depending on ψ	suboptimal
Open Economy	unconstrained	suboptimal
Semi-open	unconstrained	optimal

We illustrate this the optimality of the central bank's policy with a numerical study.

5 Numerical study

5.1 Calibration

We simulate our model in a semi-open economy using Dynare 4.3.2 (see Adjemian et al. (2011)). The shock comes from an unanticipated productivity growth which is exogenous to the model. From this exercise, we will see how during a fast growing economic transition, the foreign reserve accumulation, combined with a closed capital account, helps the model economy to achieve the first-best level of capital stock.

The shock on A_t follows the process below:

$$A_t = (1 + g_t)A_{t-1} \tag{40}$$

$$g_t = \sigma g_{t-1} + \epsilon_t \tag{41}$$

The accelerator of the growth rate, σ_t, is less than 1, ensuring the growth rate converges to 0. The growth rate g_t can be disturbed by an exogenous shock denoted ϵ_t. We assume that ϵ_t is a white noise and we denote g_0 the initial technology growth rate.

I calibrate the model based on the recent literature on economic growth and foreign reserve accumulation in an economy with financial constraints. Song et al. (2011) is an important reference as it is the most recent theoretical paper on the strategy of economic growth in China in the presence of financial constraint. However, I need to point out that I do not pretend to quantitatively explain the foreign reserve accumulation in China since the early 2000s due to the simplicity of the model. I aim at illustrating with my numerical results a qualitative analysis of the foreign reserve policy in an economy with both financial and capital account frictions.

Table 5: Calibration

Symbol	Meaning	Value	Remarks
α	share of capital in the production	0.33	
β	discount fact	0.95	
ψ	credit constraint tightness	0.1	keeping the constraint binding
g_0	initial growth rate	0.10	targeting an annual growth rate of 10%
σ	technology growth rate autocorrelation coefficient	0.6	generating a lasting growth path
ω	share of the labor income in the Ramsey objective function	1	
ϕ	adjustment cost of purchasing reserves	0.01	

5.2 The demand for foreign assets as a response to the credit constraint

First of all, my model can replicate the results that the existing literature has illustrated about the foreign reserve accumulation in the presence of domestic credit constraints (in a fully open economy): the more stringent is the credit constraint, the higher domestic savings are, leading to a current account surplus and foreign reserve stockpiling.

The blue line in Figure 12 and 13 represents the situation where $\psi = 0.05$ (tight credit constraint) while the black line represents the opposite situation where $\psi = 0.45$ (loose credit constraint). Notice that a tighter credit constraint encourages entrepreneurs to save more, for that the central bank has to provide a larger level of domestic liquid bond. Due to the constrained domestic financial market, the provision of central bank bonds requires more investment in foreign reserves. It is interesting to see that in case where the credit constraint hardly binds (the black line), the demand for domestic central bank bond and for foreign assets is very low.

In terms of welfare, we observe that entrepreneurs are better off regardless to their stage, as the supply of central bank bond increases entrepreneurs' retained earnings and lowers their debt burden. As for capital accumulation, there is a gap, during transition only, between the levels of capital stock when the credit constraint is very binding (blue line) and when the constraint does not bind strongly (black line). Namely, a less

constrained domestic financial market allows the capital accumulation to achieve the unbinding steady state slightly faster than when the constraint is more binding.
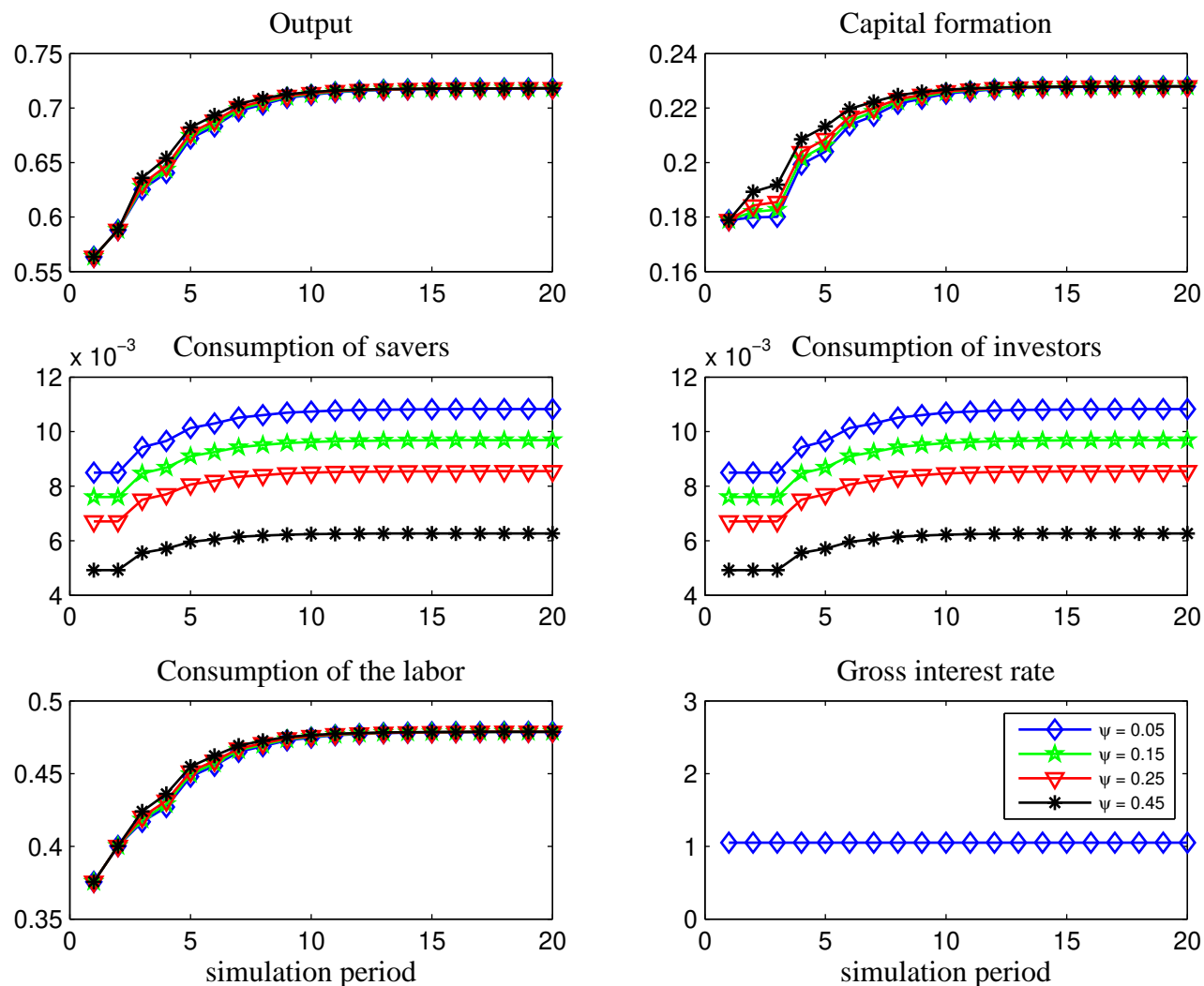

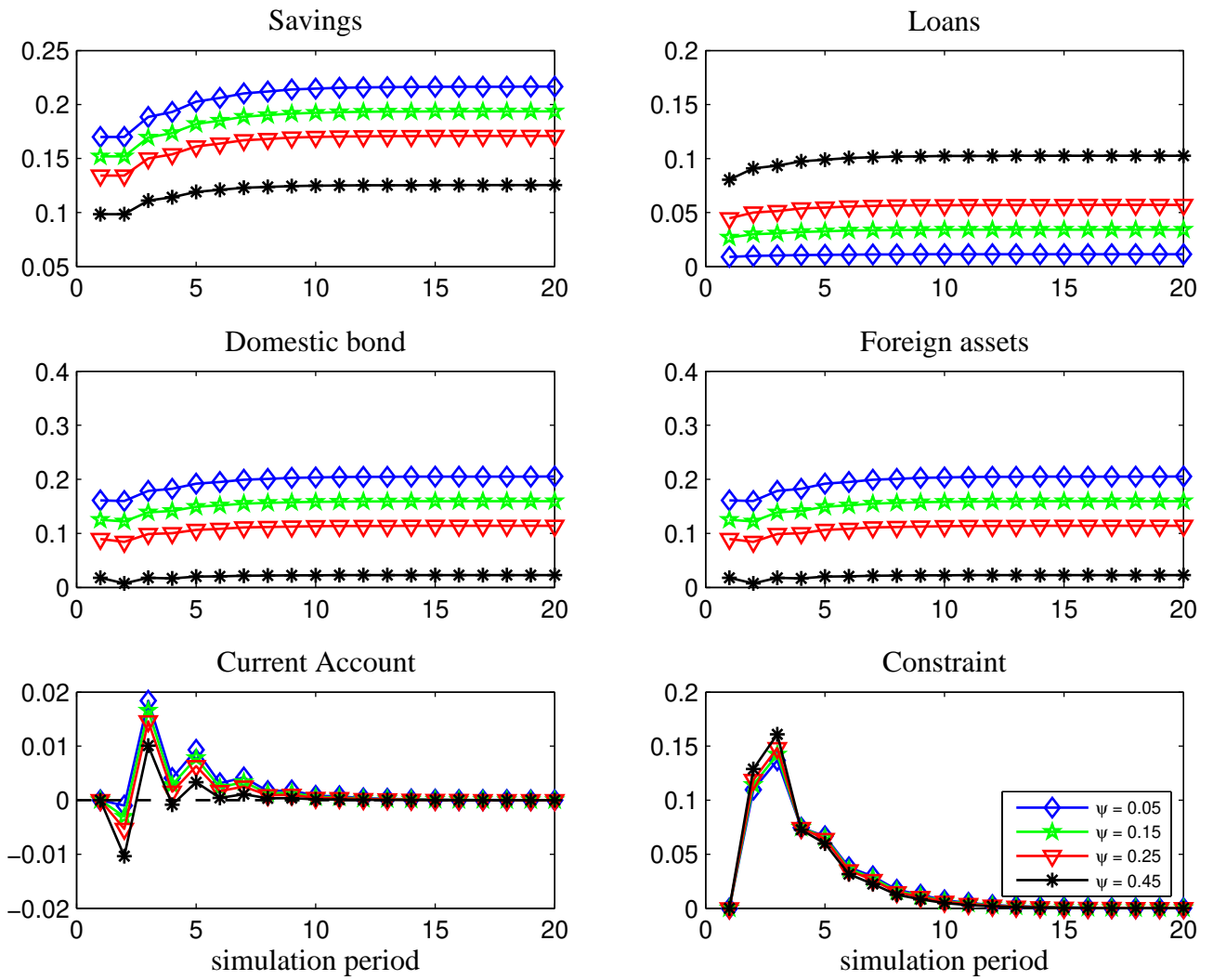Figure 12: Responses to the credit constraint tightness in open economy - 1

Figure 13: Responses to the credit constraint tightness in open economy - 2

5.3 Economic transition in a semi-open economy

We can see from Figure 14 that the central bank's optimal policy in an economy with capital controls allows a fast transition than in an open economy. By accumulating more foreign reserves during transition (B^*), the central bank is able to issue more domestic liquid assets (B) and with capital controls it raises domestic interest rate (r). This leads to higher domestic corporate savings (S) and thus higher capital investment (K). The revenue effect of the interest rate overrides the price effect as we observe that the loans asked by the investing entrepreneur decreases as the domestic interest rate increases. Domestic consumption levels are also higher for both types of entrepreneurs. Notice that the accumulation of foreign reserves will stabilize when the financial deepening (the capital-to-output ratio) is sufficiently large.

If we look at Chinese data, we can observe that along with massive foreign reserve accumulation in China the supply of the central bank's assets (e.g. central bank bonds or required reserves) has increased massively (see Figure 4). The surge in the central bank's assets has absorbed the liquidity in private sector and has forced entrepreneurs to save instead of consuming more. In this way, capital is channeled to productive plans, boosting an investment-driven growth.
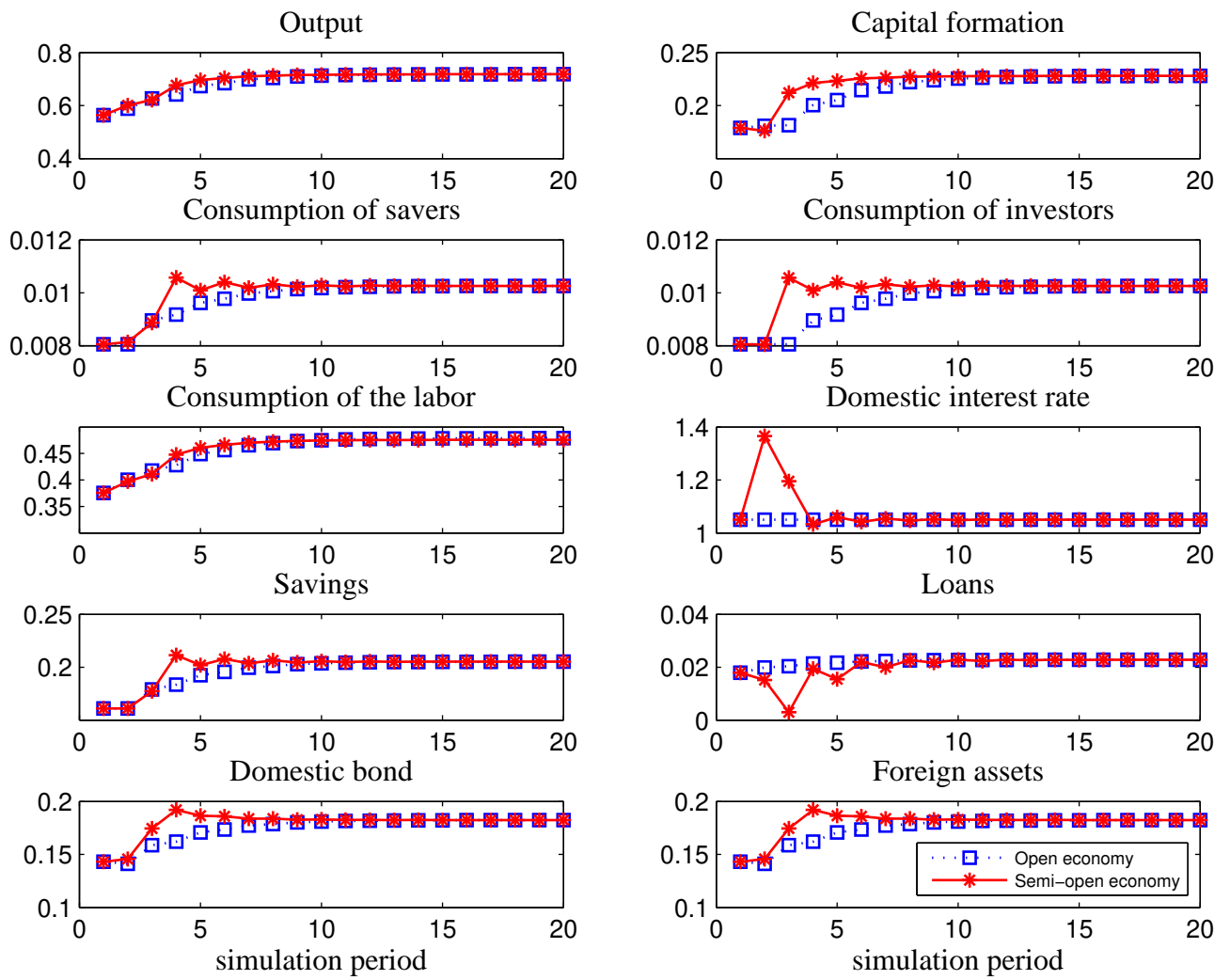
Figure 14: Simulation: semi-open economy vs. open economy

6 Conclusion

Based on the model presented in this paper, we can partly attribute the rapid accumulation of foreign exchange reserves in China and in some other emerging market economies to the under-development of the financial market there. During the economic transition, these countries may have a strong demand for liquid assets to support domestic capital accumulation. However, in the absence of a sound financial market, the central banks in emerging market economies have to serve as a financial intermediary to provide domestic liquid assets and thus alleviate the financial constraint facing some domestic firms. The bond proceeds of the public bond issuance are invested abroad in the form of foreign exchange reserves short of domestic investment opportunities.

This paper also advocates the (temporary) use of capital controls (especially in terms of controls on inflows). It is proved that foreign reserve accumulation is a second-best growth strategy only if public and private capital flows are imperfectly substitutable. A combined use of capital controls and foreign reserve accumulation allows the central bank to insure that firms save enough to invest in fixed capital, leading to a higher level of domestic production.

The demand for foreign assets will at last diminish when the economy approaches its steady state and the growth rate slows down. Moreover, a further development of domestic financial market, especially in terms of multiplication of domestic assets available for investment, will help mitigate the demand for foreign assets.

We need to also notice that this paper only considers the domestic welfare effects associated with foreign reserve accumulation. The spill-over effects of such a policy are left unexamined. Moreover, this model is based on the assumption of the neoclassical growth model: the growth rate stems from a high capital investment. In the aftermath of the global financial crises, many countries started to rethink of their growth model. In this context, if accumulating foreign reserves and controlling capital flows might be useful during economic transition, they should not hinder any structural reforms aiming at developing domestic financial market and strengthening domestic consumption. Indeed, as this paper shows, an open economy can always achieve the unconstrained steady state

without distorting the economy. A sound domestic financial market will contribute to a more sustainable economic growth in emerging market economies.

In the case of China, we observe some noticeable reforms to modernize domestic financial market since the mid-2000s. The call for the development of bond markets in China started only in 2005 with the support of the Chinese Communist party's 'Nine Articles (Walter and Howie (2012)).' On the same year, short-term corporate debt was allowed to be issued. There was an immediate issuing of RMB 142 billion (US$17 billion). This amount tripled in 2008. At the same time, a financial industry association - the National Association of Financial markets Institutional Investors (NAFMII) - was established to manage corporate bond issuance. We need to wait until April 2008 for the issuance of a thee- to five-year medium term note to be permitted. In spite of these recent reforms, the share of private financial instruments is still very small compared to the that of public debt. There is still a long way to go for China to fully develop its domestic financial market. We may expect that Chinese foreign reserves will still increase, maybe this time with a slower pace.

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A   Proof

Proof. It is easy to derive from the inter-temporal budget constraints of the saving entrepreneur and of the investing entrepreneur that:

$$c_t^S = (1-\beta) \left[\sum_{j=0}^{\infty} \tilde{F}_{K,t+2j} K_{t+2j} \prod_{i=1}^{2j} \left(\frac{1}{r_{t+i}} \right) - \sum_{j=0}^{\infty} K_{t+2j+2} \prod_{i=1}^{2j+1} \left(\frac{1}{r_{t+i}} \right) - r_t L_t \right] \quad (42)$$

$$c_t^I = (1-\beta) \left[\sum_{j=0}^{\infty} \tilde{F}_{K,t+2j+1} K_{t+2j+1} \prod_{i=1}^{2j+1} \left(\frac{1}{r_{t+i}} \right) - \sum_{j=0}^{\infty} K_{t+2j+1} \prod_{i=1}^{2j} \left(\frac{1}{r_{t+i}} \right) + r_t S_t \right] \quad (43)$$

In a two-period economy, only the saving entrepreneur harvest an output. Therefore,

$$c_t^S = (1-\beta)(\Pi_t - r_t L_t)$$
$$c_t^I = (1-\beta) r_t S_t$$

∎

B   Proof of Proposition 1

Proof. In a decentralized economy, the central bank bonds and tax rates are considered as exogenous. We know that the central bank has to have a balanced budget (16) and the financial market has to be cleared (17). The steady state of the model when the credit constraint is binding is determined by the following conditions:

- Central Bank's budget balance

- Financial market clearing

Using (17), (21) and the binding credit constraint:

$$B = S - L \tag{44}$$

$$\Rightarrow B = \beta(\Pi - rL) - L \tag{45}$$

$$\Rightarrow B = \beta(1 - \psi)\Pi - \frac{\psi\Pi}{r} \tag{46}$$

$$\Rightarrow b = \beta(1 - \psi) - \frac{\psi}{r} \tag{47}$$

The financial market clearing shows a negative linear relationship between b and $\frac{1}{r}$. Moreover, using (26),

$$(r - 1)b = (r^* - 1)b^* + \tilde{\tau} \tag{48}$$

$$\Rightarrow b = \frac{1}{r - 1}[(r^* - 1)b^* + \tilde{\tau}] \tag{49}$$

$$\Rightarrow b = \left(\frac{1}{1 - \frac{1}{r}} - 1\right)v(r^*, b, \tilde{\tau}) \tag{50}$$

The balanced central bank's budget shows a non-linear relationship between b and $\frac{1}{r}$.

Therefore, we can graphically demonstrate the uniqueness of the steady state when the credit constraint is binding.

Figure 15 shows that the there may be two solutions of $\frac{1}{r}$, corresponding respectively to $b \geq 0$ (the smaller root) and $b \leq 0$ (the bigger root). We focus here on the first case, a positive supply of central bank bonds and the gross interest rate bigger than 1. This equilibrium corresponds to $b \geq 0$. The interest rate is uniquely determined, $\beta < \frac{1}{r} < \min(1, \frac{\beta(1-\psi)}{\psi})$. For small value of ψ, for example $\psi \leq \frac{1}{2}$, $\frac{1}{\beta}\frac{\psi}{1-\psi} < r < \frac{1}{\beta}$.

Therefore, when $0 < b < \beta(1 - 2\psi)$, the economy has a unique binding steady state, provided that $\psi \leq 0.5$. ∎
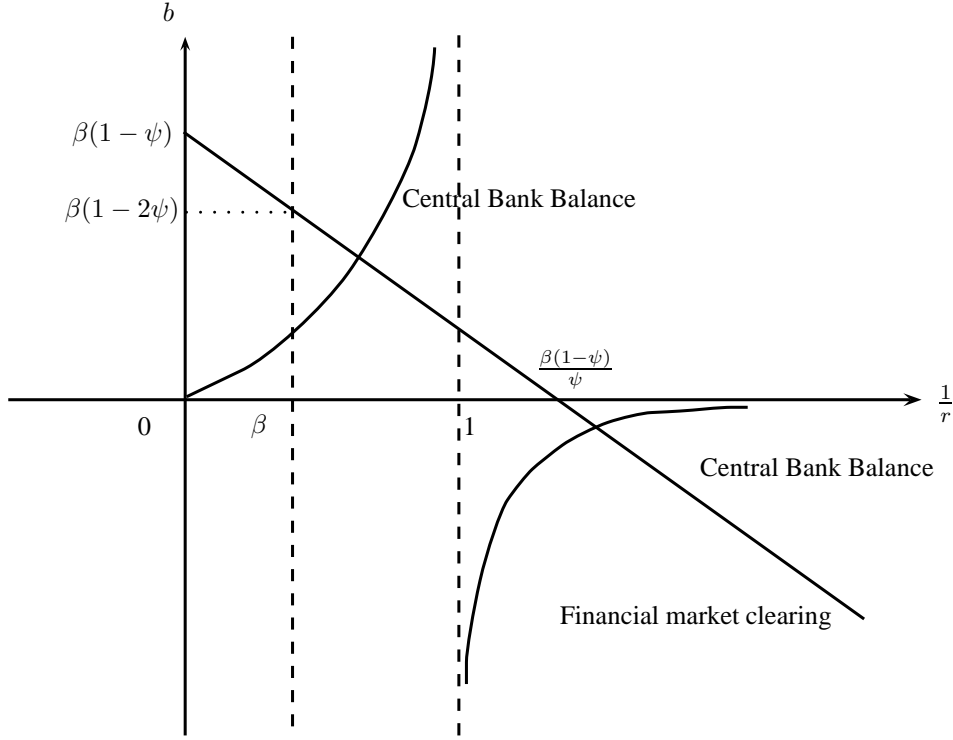
Figure 15: Uniqueness of binding steady state

C Proof

Proof. In a steady state when the credit constraint is not binding, we know that $S = L$ and $\beta r = 1$.

From (21) and (22), we have:

$$S = \beta(\tilde{F}_K - rL)$$

$$\Rightarrow S = L = \frac{1}{2}\beta\tilde{F}_K K$$

When the credit constraint is not binding, we know that $rL \leq \psi\tilde{F}_K K$, namely $\frac{1}{2}\tilde{F}_K K \leq \psi\tilde{F}_K K$.

Therefore, in a steady state with an unbinding credit constraint, we have:

$$\psi \geq \frac{1}{2}$$

∎

D   Proof of Proposition 2

Proof. Assume we are in a constrained state steady. Then from (12) and (13), we can derive:

$$1 + \lambda = \frac{1}{\beta^2 r^2} \tag{51}$$

Using this result, along with the first order condition of capital (10), we obtain equation (27).

$$\tilde{F}_K = \frac{r}{(\beta r)^2 + \psi[1 - (\beta r)^2]}$$

Namely,

$$\left(\frac{K}{A}\right)^{1-\alpha} = \alpha\beta \frac{1}{\beta\tau_K + \frac{1}{\beta r(1-\psi) + \frac{\psi}{\beta r}}} \tag{52}$$

In absence of tax (subsidy) on capital, the equation (52) becomes:

$$\left(\frac{K}{A}\right)^{1-\alpha} = \alpha\beta \left(\beta r(1-\psi) + \frac{\psi}{\beta r}\right) \tag{53}$$

Graphically, from its first-best level, K firstly decreases in βr then increases to go back to the first best. ∎

E   Proof of Proposition 4

Proof. In the open economy steady state, from (31), we have $r^* = \frac{1}{\beta}$, leading to $r = r^* = \frac{1}{\beta}$, namely $\beta r = \beta r^* = 1$.

Using Ramsey first order conditions (29) and (30), we have $\eta^S = \eta^I$ and $\Lambda = 0$.

From the definition of the Ramsey problem, we know that the private first order conditions (10) to (13) are fulfilled, we can derive thus $c^S = c^I$ and $\lambda = 0$.

Substitute these results into (34) and (33), we have $\theta^S = \theta^I = 0$. Using (35), we have $S = L$. There, the steady state is not credit-constrained. All the saving of the saving

entreprimer goes to the investing entrepreneur in the form of private loans.

Therefore, in the long run, the economy is not financially constrained in an open economy with the intervention of the central bank.

From (38), $\eta^g = \frac{\omega}{w} > 0$. Given that $\eta^S = \eta^L \geq 0$, we have

$$F_K = \tilde{F}_K = \frac{1}{\beta} \tag{54}$$

Namely, there is no distortionary tax on capital and $K = A(\alpha\beta)^{\frac{1}{1-\alpha}}$, achieving the first-best level as described in the decentralized economy with the U-shape curve. ∎