Forecasting Stock Market Volatility: A Forecast Combination Approach

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Abstract

Recently, with the development of financial markets and due to the importance of these markets and their close relationship with other macroeconomic variables, using advanced mathematical models with complicated structures for forecasting these markets has become very popular. Besides, neural network models have gained a special position compared to other advanced models due to their high accuracy in forecasting different variables. Therefore, the main purpose of this study was to forecast the volatilities of TSE index by regressive models with long memory feature, feed forward neural network and hybrid models (based on forecast combination approach) using daily data. The results were indicative of the fact that based on the criteria for assessing forecasting error, i.e., MSE and RMSE, although forecasting errors of the feed forward neural network model were less than ARFIMA-FIGARCH model, the accuracy of the hybrid model of neural network and best GARCH was higher than each one of these models.

Key Words: Stock Return, Long Memory, Neural Network, Hybrid Models.
JEL: C14, C22, C45, C53.
1. Introduction

With the development of financial markets, increase in the number of investors in these markets and the close relationship between these markets and macroeconomic variables during the last two decades, have promoted forecasting of the price behavior of financial assets in the dynamic field of economy and the capital market into one of the most controversial issues in financial sciences because forecasting guides policy makers, planners, researchers and investors in correct and efficient evaluation and pricing of the assets, optimal allocation of the resources, and evaluation of the risk management performance. Basically, researchers and policy makers use different methods for forecasting economic variables. These methods are overall divided into two categories: classic and neural network. Classic methods like regressive and structural models, despite their relative success in forecasting different variables, the results have not been satisfactory to researchers in the field because these models rely on information obtained from historical events. As financial and economic issues result in the formation of nonlinear and complicated relationships in the stock market, using nonlinear and flexible models (such as the neural network models) can help to obtain impressive results in modeling and forecasting. On the other hand, the use of nonlinear and flexible models such as the neural network models and hybrid models of classic and neural network models has been a reaction to lack of consensus on the rejection or acceptance of the efficient markets hypothesis as one of the basic challenges to financial analysts since these methods are able to forecast the future trend of prices with an acceptable number of errors despite their high complexity.

On this basis, during the last recent years, applying hybrid models has become very popular in forecasting economic variables especially financial markets because, first of all, these models chiefly have a nonlinear structure and are capable of identifying, modeling and exact forecasting of the linear and nonlinear process. Second, financial markets have a complicated behavior and much volatility due to the clarity of the information, liquidity, and also the existence of speculators and investors with different decisions. This leads to a decrease in accuracy of linear models in modeling and forecasting these markets preparing the ground for the use of nonlinear models especially hybrid models.

In line with this, Delavari et al (2013); Sahin et al (2012); Gursen et al (2011); Soni (2011); Georgescu and Dinucă (2011); Mehrara et al (2010); Tong-Seng (2007); Ghiassi et al (2006); Sheta and Jong (2001) have tried to forecast the stock market in different parts of the world using artificial neural network model. Furthermore, Abounoori et al (2013); Ullah Khan and Gour (2013); Wang et al, (2012); Kumar and Kailas (2012); Wei et al (2011); Merh
et al (2010); Güreşen and Kayakutlu (2008); Sui et al (2007); Pai and Lin (2005) used various hybrid models for forecasting stock prices. The interesting point about these studies is that in all of them different neural network models have led to a higher forecasting accuracy compared to classic models.

Despite this, the present study attempts to combine these two types of models (classic and neural network models) and introduce an optimal model for forecasting volatilities of the Tehran Stock Exchange index. For this purpose, the classic model used in this study, i.e., the Conditional Heteroscedasticity model is based on long memory and also the neural network model is multi-layered feedforward neural network model. To this end, daily time series data from 5/1/1388 to 30/7/1390 were used from which 556 observations (approximately 90% of the observations) were utilized for estimations and 60 observations for out-of-sample forecasting.

2. Methodology
2.1. Long Memory
After many important studies were conducted on the existence of Unite Root and Cointegration in time series starting in 1980, econometrics experts examined other types and subtypes of non-stationary and approximate persistence which explain the processes existing in many of the financial and economic time series. Today, different studies have been and are being conducted on these processes including "Fractional Brownian Motion" and "Fractional Integrated Process" and the "processes with long memory" (Lento, 2009). Hurst (1951) for the first time found out about the existence of processes with long memory in the field of hydrology. After that, in early 1980s econometricians such as Granger and Joyex (1980) and Hosking (1981) developed econometric models dealing with long memory and specified the statistical properties of these models. During the last three decades, numerous theoretical and empirical studies have been done in this area. For example, (Mandelbrot, 1999; Lee and et al. 2006; Onali and Goddard 2009)’s studies can be mentioned as among the most influential in this regard.

The concept of long memory includes a strong dependency between outlier observations in time series which, in fact, means that if a shock hits the market, the effect of this shock remains in the memory of the market and influences market activists’ decisions; however, its effect will disappear after several periods of time (in the long term). Thus, considering the nature and the structure of financial markets such as the stock market, which are easily and quickly influenced by different shocks (economic, financial and political), it is possible to analyze the effects of these shocks and in a way determine the time of their
disappearance by observing the behavior of these markets (Los and Yalamova, 2004). Meanwhile, the long memory will be used as a means of showing the memory of the market. By examining the long memory, the ground will also be prepared for improvement of financial data modeling. Modeling volatilities of the market is done using GARCH-type models. These models will be briefly discussed after describing the tests for identifying the long memory feature.

### 2.2. Tests Used for Identifying the Long Memory Features

The most important step in estimating a model with long memory feature is examination of the existence of this feature in the return and volatility of the mentioned series. Identifying the existence of long memory feature via techniques such as ACF test, GPH test, etc. is possible; in the following section.

#### 2.2.1. ACF Test

This method is one of the most popular tests identifying the long memory feature first introduced by Ding and Granger (1996). In this test, autocorrelation graph decreases from a certain value very slowly or hyperbolically (not exponentially). Therefore, such time series have long memory feature. It means that these processes cannot be produced by determined and specific AR and MA lags because in these series, AR and MA have infinite order (Xio and Jin, 2007).

#### 2.2.2. The GPH Test (Spectral Density Method)

This method is based on Frequency Domain Analysis. In the framework of spectral and frequency domain analysis, the observed time series is weighted summation of the underlying time series which have different periodical patterns. Periodogram technique is used for differentiating between short and long memories. This technique was proposed by Gewek and Porter-Hudak (1983) and is often known as the GPH estimator. Overall, GPH statistics estimates the long memory parameter ($d$) which is based on the following periodogram regression:

$$
\ln[I(w_j)] = \beta_0 + \beta_1 \ln \left[ 4 \sin(w_j / 2) \right] + e_j
$$

where $w_j = 2\pi j / T$, $j = 1, 2, ..., n$ and $e_j$ represent residuals of the model and $w_j$ refers to Fourier Frequency Transformation ($n = \sqrt{T}$). Finally, $I(w_j)$ is a simple periodogram which is defined as follows:

$$
I(w_j) = \frac{1}{2\pi T} \sum_{t=1}^{T} e_t e^{-w_j t}
$$

Thus, the GPH statistic equals $-\hat{\beta}_1$. 

2.3. Volatility Modeling In Financial Markets
Correct modeling of the volatilities in the financial markets is an important issue in the discussions related to econometrics especially during the recent years. According to Poon and Granger (2003), although volatilities are not exactly the same as risk, when they are regarded as uncertainty, it turns into one of basic variables in many of the applied studies on financial markets since correct modeling of the volatilities is of considerable significance in investment, determining portfolios, optional trading, future markets, and risk management and forecasting future volatilities.

Engel (1982) by introducing Auto Regressive Conditional Heteroscedasticity (ARCH) model, and then Bollerslev (1986) by extending this model and presenting Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) model took a big step in correct modeling of the volatilities. Following that, many studies examined major characteristics of financial markets such as Conditional Heteroscedasticity, volatility clustering, excessive kurtosis and the existence of fat tail returns in financial markets using GARCH model. However, GARCH model was not able to explain the asymmetric features of the effect of shocks on volatilities. This problem was resolved with the introduction of asymmetric GARCH models.

Another feature of the financial markets such as the stock market is the existence of long memory feature in these markets. The logic behind this type of Conditional Heteroscedasticity models is that the long memory is a specific form of linear dynamicity and its modeling is not possible using linear methods. Therefore, there is a need for nonlinear models. Furthermore, considering the existence of the long memory feature, pricing the derivatives using traditional methods is not acceptable and statistical inferences derived from pricing models based on standard statistical tests such as Capital Asset Pricing lose their justification (Bollerslev et al, 2010).

2.3.1. Different Types of ARCH Models
Auto Regressive Conditional Heteroscedasticity (ARCH) models first proposed by Engel (1982) later on expanded by Bollerslev (1986) include the kind of models that are used for explaining the volatilities of a time series. Following that different types of ARCH models were introduced. They are divided into two groups: Linear (IGARCH and GARCH) and nonlinear models (EGARCH, TGARCH, PGARCH, FIGARCH, etc.).
I. Linear GARCH Models

Borlerslev (1986) started introducing the generalized model of ARCH, i.e., GARCH model based on Engel’s ARCH model. The distinguishing factor between these two models is the existence of variance lags in the conditional variance equation. In fact, GARCH model has a similar structure to ARMA. Stipulated forms of this model include:

\[ M_t = \mu_t + \varepsilon_t \]
\[ \varepsilon_t = z_t \sqrt{h_t}, \quad z_t \sim N(0,1) \]
\[ h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \]
\[ h_t = \sigma_t^2 \]

Equation (3) above is a mean equation which includes two sections; one of them is \( \mu_t \), which should be an appropriate structure for explaining mean equation, and the other is \( \varepsilon_t \), which is indicative of residuals in the model above which has heteroscedasticity variance and is consisted of two normal elements (\( z_t \) and conditional standard deviation (\( \sqrt{h_t} \))). As a matter of fact, \( h_t \) is a conditional variance equation that is estimated along with the mean equation to eliminate the problems related to the heteroscedasticity variance \( \varepsilon_t \). In the equation (4), \( \omega \) is the average of \( \sigma_t^2 \), the \( \varepsilon_{t-1}^2 \) coefficient indicates the effects of ARCH and \( h_{t-1} \) coefficient represents the effects of GARCH (Chuang et al., 2012). One of the most important features of this model is the existence of temporary shocks imposed on the time series under investigation (Kittiakarasakun and Tse, 2011).

Furthermore, the results of Engel and Borlerslev’s (1986) studies show that in some of the cases the GARCH equation mentioned above has a unit root. It means that, for example, in GARCH(1,1), the \( \alpha_1 + \beta_1 \) value is very close to one. In this case, the GARCH model is cointegrated and is called IGARCH. In these models, if there is a shock to the time series under investigation, it will have lasting effects and become noticeable in the long term (Poon and Granger, 2003).

II. Nonlinear GARCH Models or the FIGARCH Model

FIGARCH model was first proposed by Baillie (1996). In this model, a variable has been defined as fraction differencing, which ranges from zero and one. A General form of the FIGARCH (p,d,q) is as follows:

\[ (1 - L)^d \Phi(L) \varepsilon_t^2 = \omega + B(L) \vartheta_t \]
In equation (5), $\phi(L)$ is the function of appropriate lag (q), $B(L)$ is the function of appropriate lag (p), L is the lag operator, and d represents fraction differencing parameter. If $d=0$, the FIGARCH model will turn into GARCH, and if $d=1$, it will turn into IGARCH. It should be noted that in these models, the effects of the shocks are neither lasting as in IGARCH models nor temporary as in GARCH models; the effects are between these two extremes meaning that the effects of the shocks will decrease at a hyperbolic rate.

2.5. Neural Network Models
Despite its novelty, Artificial Intelligence (AI) has obtained the attention of scholars and researchers. Different types of artificial neural networks attempt to emulate the human mind or the learning process using computational methods, automate the process of knowledge acquisition from data and solve great and complex problems. Artificial neural networks have many applications such as data classification, function approximation, forecasting, clustering, and optimization (Ripley, 1996; Krose, 1996). Using artificial neural networks has many considerable advantages; first, neural networks have a high similarity with the human nervous system, and unlike the traditional methods, they are data-driven self-adaptive methods, which have only few assumptions for the problems. In other words, they are model-free; second, in addition to their high-speed information processing due to parallel processing, neural networks have a very high generalizations; finally, because neural networks have more comprehensive and more flexible functional forms compared to the traditional statistical methods, they are Universal functional approximates. Neural network models are distributed parallel processes with natural essence, and their main feature is the ability to model a complex non-linear relation without any presuppositions about the essence of the relationships between the data. There are two types of neural networks: dynamic and static networks (Deng, 2013; Chiang et al. 2004; Tsoi and Back, 1995).

2.5.1. Feed-Forward Neural Network Models
The simplest form of a neural network has only two layers, output layer and input layer. The networks act as an input-output system. In these systems, to calculate the value of the output neurons, the value of the input neurons is checked by a transfer function. Farley and Clark (1954) first used computational machines, then called calculators, to simulate a Hebbian network at MIT. Other neural network computational machines were created by Rochester, Holland, Habit, and Duda (Rochester et al, 1956). The studies on multi-layered neural networks date back to the initial works of Frank Rosenblatt (1958) on two-layer neural network or the perceptron, an algorithm for pattern recognition based on a two-layer learning
computer network using simple addition and subtraction (Werbos, 1975). Besides the input and output layers, the multi-layer neural networks use the hidden layer because it will improve the performance of the networks. First Rumelhart et al. in 1986 and since then many authors, such as Nielson (1987), Cybenko (1989), Funahashi (1989), Hornik et al. (1990), and White (1992), have demonstrated that Feed-forward neural network with one logistic activation function in the hidden layer and one linear activation function in the output neuron can approximate any function with the desired accuracy. Generally, although change in the transfer function is one of the distinguishing factors between different multi-layered feedforward neural network models, basically these models are a function of three major parts including:
1) Number of layers and neurons in each layer
2) The transfer function used
3) Weights of the artificial neural network

Based on the presented concepts, in the forecast combination approach used in this study, first of all volatilities of the stock market are modeled and predicted using the GARCH-type model and then using the best possible feedforward neural network model, more accurate forecasts of the time series will be presented. Therefore, considering the accuracy of the models is compared using forecasting errors criteria, they will be focused on in the following section.

2.6. Criteria for Comparing Forecasting Performance
On the whole, MSE and RMSE criteria are among the most frequently used criteria for comparing forecasting accuracy of the models among other criteria for fitting the accuracy of prediction. In this study, we used the MSE criterion for comparing forecasting accuracy of the models because this criterion has important features among which is taking account of the outlying data in comparing forecasting accuracy of the models. Besides, this criterion has a higher accuracy as against RMSE which shows the error differences as lower (Swanson et al., 2011).

\[
MSE = \frac{\sum (\hat{y}_i - y_i)^2}{n} = \frac{SSR}{n}
\]

3. Empirical Results
For the purpose of this study, we used daily data of the Tehran Stock Exchange (TSE) Index from 2009/25/03 to 2011/22/10. It should also be mentioned that the acronyms of the
variables used in this study include: TEDPIX (Tehran Exchange Dividend Price Index) and DLTED, showing the difference of the logarithm (return) of the Dividend Price Index.

3.1. Descriptive Analysis of the Data
Considering the importance of the utilized data in this study, before modeling the mentioned index, a descriptive statistics related to the data will be analyzed first (see Table 1 for details):

<table>
<thead>
<tr>
<th>Criterions</th>
<th>Accounting Value</th>
<th>Criterions</th>
<th>Accounting Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>observations</td>
<td>616</td>
<td>Jarque- Bra</td>
<td>9953.99 (0.000)</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00193</td>
<td>Box- Ljung Q(10)</td>
<td>108.81 (0.000)</td>
</tr>
<tr>
<td>S.D</td>
<td>0.00797</td>
<td>McLeod-Li Q(10)</td>
<td>241.25(0.000)</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.2684</td>
<td>ARCH (10)</td>
<td>8.9832 (0.000)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>22.1799</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Findings of Study

With a brief look at the above table, it can be found that the mean of time series return in Tehran Exchange Return in the period under investigation is 0.00193 and its standard deviation is 0.00797. By comparing these two, it can be realized that this time series has experience a high level of volatility during this period. The Jarque-Bera test indicates non-normal distribution of this time series. Besides, the kurtosis statistics also indicate that the distribution of the mentioned time series is fat tail. Observing the Liang-Box statistics (with ten lags), can find, the null hypothesis about the lack of a serial correlation between the terms of the time series be rejected. The McLeod-Lee statistics also reject the null hypothesis about the lack of Serial correlation between square of the time series return) which is, in fact, expressive of the existence of nonlinear effects in this time series. It should be mentioned that the results of Engel’s test were consistent with McLeod-Lee’s test and confirmed the hypothesis about conditional variance of the time series return.

3.2. Stationary Test
As the next step, stationary of the DLTED series (done to prevent creation of a spurious regression) will be assessed using different tests (see Table 3 for the results).
Table 2. The Results Related to Stationary of the Stock Return Series

<table>
<thead>
<tr>
<th>Test</th>
<th>Critical Stat.</th>
<th>Accounting Value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF$^1$</td>
<td>-1.9413</td>
<td>-16.586</td>
<td>Stationary</td>
</tr>
<tr>
<td>ERS$^2$</td>
<td>3.2600</td>
<td>0.9403</td>
<td>Non-Stationary</td>
</tr>
<tr>
<td>PP$^3$</td>
<td>-1.9413</td>
<td>-17.543</td>
<td>Stationary</td>
</tr>
<tr>
<td>KPSS$^4$</td>
<td>0.4630</td>
<td>0.590</td>
<td>Non-Stationary</td>
</tr>
</tbody>
</table>

Source: Findings of Study

If the long memory feature does not exist, it is expected that the series becomes stationary by first differencing, but the results of first differencing show that stock return series is stationary in ADF and PP tests while in the KPSS and also ERS test the results are indicative of non-stationary of the series (see Table 2 for the results). Such conditions might have been caused by the long memory feature in this series. For this reason, the long memory feature in the stock return series (by fractional differencing series) was further analyzed by the researchers. Besides, interpreting the Autocorrelation plot can also help to find if there is long memory in the stock return series; as shown in Fig. 1 below, the autocorrelation between different lags in the time series has not disappeared even after about 30 periods and, in fact, these autocorrelations in the series are decreasing at a very slow rate. This is anomalous to the behavior of autocorrelation of the stationary series in which the autocorrelations between different lags in the series decrease exponentially.

![Fig. 1. ACF Graph for Stock Return Series](image)

Source: Findings of Study

3.3. Examining the Fractal Market Hypothesis

Generally, dependence of the behavior of a market on the Efficient Markets Hypothesis depends on the significance of long memory parameter in its time series. In general, models that are based on long memory are highly dependent on the value of long memory parameter and also attenuation of the autocorrelation functions. On this basis, in the following

$^1$ Augmented Dickey–Fuller  
$^2$ Elliott, Rothenberg and Stock  
$^3$ Philips-Prone  
$^4$ Kwiatkowski–Phillips–Schmidt–Shin
subsections, the values of long memory parameter are estimated using the GPH. On the whole, this test is conforms to the frequency domain analysis and uses the Log-Period gram technique; this technique is a means for differentiating short and the long memory processes. It should also mentioned that slope of the regression line resulting from applying the Log-Period gram technique gives us the long memory parameter and if significant, the significance of the related feature in the stock return series can be inferred and the fractal markets hypothesis is confirmed. The results of this test have been provided in Table 3 below.

<table>
<thead>
<tr>
<th>Series</th>
<th>d-Parameter</th>
<th>t-stat.</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock series</td>
<td>0.14088</td>
<td>3.13</td>
<td>0.002</td>
</tr>
<tr>
<td>Stock return series</td>
<td>1.04695</td>
<td>12.3</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Source: Findings of Study

As shown in Table 3 above, the value for long memory parameter is non-zero (and also lower than 0.5) which is a confirmation of the existence of long memory in the stock return series. Therefore, two conclusions can be drawn from the above test: first, the fractal markets hypothesis is supported. The second conclusion is that this series should be fraction differenced once again so that modeling can be done in conformity with it. Therefore, although the existence of the long memory feature was confirmed in the return index, in the following sections, we will also focus on the existence of this feature in the stock volatility series. On this basis, in the following sections, different volatility models will be focused on using the long memory property.

3.4. Estimating the GARCH and FIGARCH Models

According to the results provided in Table 1, based on the ARCH test, the conditional heteroscedasticity effects are confirmed to exist in the return of TSE index and consequently, in order to eliminate the problems associated, the ARCH family models can be used. Therefore, in the next part, not only the long memory feature will be tested in the stock volatility index, there will also be a focus on modeling variance equation of the series using GARCH models including those with long memory (fractal) and the non-fractal ones, in both mean and variance equations. The results related to different forms have been presented in Table 4.

<table>
<thead>
<tr>
<th>Models</th>
<th>ARFIMA(1,1)</th>
<th>ARFIMA(1,2)</th>
<th>ARFIMA(2,1)</th>
<th>ARFIMA(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AIC</td>
<td>SBC</td>
<td>AIC</td>
<td>SBC</td>
</tr>
</tbody>
</table>
All the proposed models shown in Table 4 have been based on different mean equations with long memory and as shown in this table, different combinations include three general parts: the first part of that (at the top of the table) includes different non-fractal models of conditional variance heteroscedasticity, the second part includes the combination of a conditional variance heteroscedasticity with unit root model (IGARCH), and finally, the third part (down the table) includes the different types of fractal conditional variance heteroscedasticity models (FIGARCH).

By comparing information criteria related to different types of GARCH models, it can be easily found that the ARFIMA(1,2)-FIGARCH(BBM) model has the lowest Akaike and Schwarz information criteria, so, it is the best model for explaining the behavioral pattern of volatility in the stock series (see Table 5 for the coefficients for variables of this model and the statistics related to significance of these coefficients). Another conclusion to be drawn from the results shown in the table is the existence of the long memory feature in the stock volatility series. Furthermore, statistics related to examining the existence of variance heteroscedasticity in residuals of this model (statistics related to Liang-Box, McLeod-Lee and ARCH) have also been presented below the table with the estimation of this model.

**Table 5. Estimating ARFIMA-FIGARCH Model Results**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.002</td>
<td>0.0008</td>
<td>2.56</td>
<td>0.010</td>
</tr>
<tr>
<td>d-ARFIMA</td>
<td>0.18</td>
<td>0.014</td>
<td>12.85</td>
<td>0.000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.28</td>
<td>0.073</td>
<td>3.93</td>
<td>0.000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.09</td>
<td>0.008</td>
<td>-12.09</td>
<td>0.000</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.11</td>
<td>0.016</td>
<td>-6.47</td>
<td>0.000</td>
</tr>
<tr>
<td>Dummy(1)</td>
<td>0.06</td>
<td>0.009</td>
<td>6.16</td>
<td>0.000</td>
</tr>
<tr>
<td>Dummy(2)</td>
<td>0.04</td>
<td>0.005</td>
<td>7.84</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variance Equation</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.94</td>
<td>0.776</td>
<td>2.51</td>
<td>0.006</td>
</tr>
<tr>
<td>d-FIGARCH</td>
<td>0.31</td>
<td>0.031</td>
<td>10.06</td>
<td>0.000</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.56</td>
<td>0.259</td>
<td>2.19</td>
<td>0.028</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.75</td>
<td>0.154</td>
<td>4.85</td>
<td>0.000</td>
</tr>
</tbody>
</table>

| Log likelihood    | 1891.932    | Box-Ljung Q(10)| 12.06 (0.098) |
| Akaike            | -7.334374   | McLeod-Li Q(10)| 4.87 (0.771)  |
| Schwarz           | -7.258863   | ARCH(10)        | 0.0031 (0.955) |

Source: Findings of Study
According to the Table 5, there are some points worth mentioning. First of all, the dummy variables introduced in the mean equation of the above model indicate the existence of unusual shocks to the time series under investigation. Furthermore, in the model under investigation all the coefficients (except the constant) are significant at .95 level of confidence. The results of Liang-Box test show no sign of serial correlation in the residuals of this model. The existence of variance heteroscedasticity in the residuals was also negated based on the results from McLeod-Lee and ARCH test.

3.5. Comparing the Performance of Models in Accuracy of Forecasts

Considering the fact that, the main purpose of this study was to develop a new hybrid model that could yield more accurate forecasts of the return series, in this part of the study, there will be an attempt to combine the models mentioned above and present a model that could not only takes into consideration the theories related to the financial markets (such as the fractal markets hypothesis) but also benefits from the strength and flexibility of the models based on artificial intelligence. Thus, in order to achieve more accurate forecasts, the first lag of the forecasting results from FIGARCH model, were used as the input for the multi-layer feed forward neural network (MFNN) model and as a result, the improvement in forecasting stock volatility series will be focused upon. On the basis of MSE and RMSE criteria\(^1\), a comparison will be made between different models in their accuracy of out-of-sample forecasting (60 out-of-sample observations). The results produced by this model have been presented in the following table.

<table>
<thead>
<tr>
<th>Models</th>
<th>MSE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARFIMA-FIGARCH</td>
<td>4.31*10^(-5)</td>
<td>6.56*10^(-3)</td>
</tr>
<tr>
<td>MFNN(FIGARCH(-1))</td>
<td>3.47*10^(-5)</td>
<td>5.89*10^(-3)</td>
</tr>
</tbody>
</table>

Source: Findings of Study

The results shown in Table 6, are indicative of the fact that the hybrid model has the best performance in comparison with other model based on the forecasting error criterion.

4. Conclusions

Basically, one of the most important economic hypotheses in the field of financial markets in the unpredictability of changes in stock market price indices, known as random walk hypothesis in statistics. The forecasting models designed for stock prices are in fact a

\(^1\) MSE and RMSE are the most frequently used criteria for comparing models in accuracy of predictions among other criteria for assessing accuracy of prediction (Swanson & et al, 2011).
challenge to this hypothesis and try to show that despite the complexity of the trend of prices, their future trend can be forecasted with an acceptable number of errors. Among these models, hybrid models using economic theories and neural network models proved successful in forecasting series that have a complicated trend.

In the present study, ARFIMA-FIGARCH model and multi-layer feed forward neural network model (MFNN) were used to forecast Tehran Stock Exchange Price and Dividend Index (TEDPIX). Additionally, considering the importance of obtaining more accurate forecasts, an attempt was made to get more reliable results in forecasting stock Index in Tehran Stock Exchange using a novel hybrid intelligent framework based on feed forward neural network model and using the results from ARFIMA-FIGARCH model. Our finding suggested that, the combination of ARFIMA-FIGARCH and MFNN models yielded more acceptable forecasting results compared to the ARFIMA-FIGARCH model as this hybrid model takes into consideration not only the nonlinear and complicated structure of the stock series but also the memory-based feature of financial markets based on the fractal markets hypothesis. This results was, in fact, expected considering the high flexibility of neural network models and in contrast, the inflexible and in a way, imposed structure of regressive models such as ARFIMA in which any change in their coefficients happens only after a change in the time series under investigation (as a results of adaptation).

Finally, the policy makers, macroeconomic decision makers, and investors can be recommended to use a combination of neural network and ARFIMA-FIGARCH models as an appropriate method. Based on the findings of this study, some suggestions can also be made. First of all, considering the confirmation of the existence of long memory feature in Tehran Stock Exchange indexes series, paying attention to the fact that it has involved fraction-differencing (and led to the loss of a smaller portion of the available information in the series compared to when one-order fractioning is performed) can help improve the results of modeling and consequently economic forecasts because taking this feature into consideration means that although current shocks may exert a small part of their influence and effect at the very time or at maximum after some lags, the major part of the effects of these shocks can influence the behavior of the time series with this feature in future periods; therefore, as it was confirmed in this study and other studies, taking this feature into consideration leads to an improvement in the performance of models and can be offered to investors and financial markets and macroeconomic decision makers as an appropriate suggestion. Second, as using hybrid models has become popular during the recent years, the fact that applying combining complicated (nonlinear) methods and the long memory feature bring better results can be
further investigated in future studies. Finally, researchers are suggested to use this model in other volatile markets.

5- References


