Edgeworth equilibria: separable and non-separable commodity spaces

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EDGEWORTH EQUILIBRIA: SEPARABLE AND NON-SEPARABLE COMMODITY SPACES

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Abstract. Consider a pure exchange differential information economy with an atomless measure space of agents and a Banach lattice as the commodity space. If the commodity space is separable, then it is shown that the private core coincides with the set of Walrasian expectations allocations. In the case of non-separable commodity space, a similar result is also established if the space of agents is decomposed into countably many different types.

1. Introduction

One of the classical result in economic theory is Aumann's equivalence theorem in a deterministic economy with a contimuun of agents and finitely many commodities, see [2]. Many extensions of this result have been obtained in the literature. Firstly, an extension of this result to an economy with an atomless measure space of agents and finitely many commodities can be found in [14]. In the context of infinite dimensional commodity space, the relation between the core and the set of Walrasian allocations are more interesting, since preferences and endowments are more diverse, and thus blocking become more difficult, refer to [11]. Rustichini and Yannelis [21] extended this result to an economy whose commodity space is a separable Banach lattice. In [2], Aumann also pointed out that many real markets are indeed far from being perfect; such a market is probably best represented by a mixed model, in which some agents are points in a continuum and others are individually significant. One of the key results on the equivalence between the core and the set of Walrasian allocations in a mixed economy was established by Shitovitz in [22]. To be precise, he showed that if there exist at least two large agents and all of them have the same initial endowment and preference, then the core coincides with the set of Walrasian allocations. Similar results in mixed economies also came out in [6, 9, 10]. In all of these results, the feasibility was defined without free disposal and in terms of Bochner integrable functions. In contrast to so far mentioned positive results, Podczeck [17] and Tourky and Yannelis [23] constructed counterexamples of atomless economies to show that the classical core-Walras equivalence theorem in [2] may fail under desirable assumptions when the commodity space is a non-separable ordered Banach space and the feasibility is defined by Bochner integrable functions. However, when feasibility is defined in terms of Pettis integral, Podczeck [69] obtained a positive result for a ceratin class of commodity spaces without requiring that those commodity spaces are separable.

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In [20], Radner extended the notion of Walrasian allocation to the case of differential information and it is known as *Walrasian expectations allocation*. Due to different information and communication opportunities among agents, several alternative core concepts have been introduced, refer to [25, 27]. In [27], Yannelis introduced the notion of *private core*, which was based on the fact that agents have no access to the communication system, that is, each member of the coalition uses only his own private information whenever a coalition blocks an allocation. It is also essential to mention that under standard assumptions, the private core is non-empty, Bayesian incentive compatible and rewards the information superiority of agents (see [15, 27]). Dealing with differential information, Einy et al. [7] first extended Aumann's equivalence theorem to the case of the private core and the set of Walrasian allocations, where the free disposal feasibility assumption was used. Later, this result was further generalized to a differential information economy with an atomless measure space of agents and an ordered separable Banach space having an interior point in its positive cone as the commodity space in [8]. In addition to these equivalence results with free disposal, Angeloni and Martins-da-Rocha [1] obtained an equivalence result between the private core and the set of Walrasian allocations in an atomless economy with finitely many commodities and without free disposal feasibility assumption.

The paper is organized as follows. In Section 2, a general description on the model of this paper is given. In Section 3, the main result of this paper is presented, where an extension of the main result in [21] to a differential information economy with a separable commodity space and the free disposal feasibility assumption is given. Section 4 deals with the equivalence theorem in an economy with a non-separable commodity space and an atomless measure space of agents with only countably many different types.

2. Differential information economies

An atomless model of pure exchange economy $\mathscr{E}$ with differential information is presented. The exogenous uncertainty is described by a measurable space $(\Omega, \mathscr{F})$, where Ω is a set of states of nature containing m elements and the σ-algebra $\mathscr{F}$ of Ω denotes the set of possible events. The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$, there is uncertainty over the states and agents make contracts that are contingent on the realized state at $\tau = 1$. Let the space of agents be a measure space (T, Σ, μ) with a complete, finite and positive measure μ, where T is the set of agents, Σ is a σ-algebra of measurable subsets of T whose economic weights on the market is given by μ. Throughout the paper, the commodity space Y of $\mathscr{E}$ is a Banach lattice having a quasi-interior point. The order on Y is denoted by a partial order $\leq$ and the positive cone $Y_+ = \{x \in Y : x \geq 0\}$ of Y denotes the *consumption set* of each agent t in each state ω.The symbol $x \gg 0$ (resp. $x > 0$) means that x is a quasi-interior (resp. a non-zero) point of Y_+. Let $Y_{++} = \{x \in Y_+ : x \gg 0\}$. Each agent has some private information, which is described by a partition Π_t of Ω. The interpretation is that if ω is the true state of nature then agent t cannot discriminate the states in the unique element $\Pi_t(\omega)$ of Π_t containing ω. Let $\mathscr{F}_t$ be the σ-algebra generated by Π_t. Each agent t is also associated with a *state-dependent* utility function $U_t : \Omega \times Y_+ \to \mathbb{R}$, a *random initial endowment* $a(t, \cdot) : \Omega \to Y_+$ and a *prior*, which is given by a probability measure $\mathbb{Q}_t$ on Ω. The quadruple $(\mathscr{F}_t, U_t, a(t, \cdot), \mathbb{Q}_t)$ is called the *characteristics* of the agent $t \in T$. Thus, $\mathscr{E}$ can be defined by

$$\mathscr{E} = \{(\Omega, \mathscr{F});\ (T, \Sigma, \mu);\ Y_+;\ (\mathscr{F}_t, U_t, a(t, \cdot), \mathbb{Q}_t)_{t \in T}\}.$$

A function $x : \Omega \to Y_+$ is interpreted as a random consumption bundle in $\mathscr{E}$. The *ex ante expected utility* of an agent t for a given random consumption bundle x is defined by $\mathbb{E}^{\mathbb{Q}_t}(U_t(\cdot, x(\cdot))) = \sum_{\omega \in \Omega} U_t(\omega, x(\omega))\mathbb{Q}_t(\omega)$. An *assignment* in $\mathscr{E}$ is a function $f : T \times \Omega \to Y_+$ such that $f(\cdot, \omega)$ is Bochner integrable for all $\omega \in \Omega$. It is assumed that a is an assignment. An assignment f in $\mathscr{E}$ is called an *allocation* if $f(t, \cdot) \in L_t$ μ-a.e., where $L_t = \{x \in (Y_+)^\Omega : x \text{ is } \mathscr{F}_t\text{-measurable}\}$. An assignment is *$S$-feasible* if $\int_S f(\cdot, \omega)d\mu \leq \int_S a(\cdot, \omega)d\mu$ for all $\omega \in \Omega$. This feasibility condition is also known as *S-feasibility with free disposal*. However, if the last inequality is replaced with an equality, then it is named as *S-feasibility without free disposal* or *S-exact feasibility*. For simplicity, if f is T-(exactly) feasible then it is termed as (exactly) feasible. Throughout the paper, the following assumption on the initial endowments is posed.

$(\mathbf{A}_1)$ $a(t, \cdot) \in L_t$ and $a(t, \omega) \gg 0$ for all $(t, \omega) \in T \times \Omega$.

This is a standard assumption and has been used in many references, see [3, 4, 8, 12]. Let $\mathfrak{P}$ denote the family of all partitions of Ω. For any $\mathscr{Q} \in \mathfrak{P}$, let $T_{\mathscr{Q}} = \{t \in T : \Pi_t = \mathscr{Q}\}$. It is assumed that $T_{\mathscr{Q}} \in \Sigma$ for all $\mathscr{Q} \in \mathfrak{P}$. For any coalition S, put $\mathfrak{P}_S = \{\mathscr{Q} \in \mathfrak{P} : S \cap T_{\mathscr{Q}} \neq \emptyset\}$ and $\mathfrak{P}(S) = \{\mathscr{Q} \in \mathfrak{P}_S : \mu(S \cap T_{\mathscr{Q}}) > 0\}$. Since $L_t = L_{t'}$ if $t, t' \in T_{\mathscr{Q}}$, the notation $L_{\mathscr{Q}}$ is employed to denote the common value of L_t for all $t \in T_{\mathscr{Q}}$. For any allocation f, define a function $P_f : T \to Y_+^\Omega$ by

$$P_f(t) = \left\{x \in L_t : \mathbb{E}^{\mathbb{Q}_t}(x) > \mathbb{E}^{\mathbb{Q}_t}(f(t, \cdot))\right\}.$$

For any $n \geq 1$, the $(n-1)$-simplex of $\mathbb{R}^n$ is defined as

$$\Delta^n = \left\{x = (x_1, ..., x_n) \in \mathbb{R}_+^n : \sum_{i=1}^n x_i = 1\right\}.$$

Consider a function $\varphi : (T, \Sigma, \mu) \to \Delta^m$ defined by $\varphi(t) = \mathbb{Q}_t$ for all $t \in T$. For each $\omega \in \Omega$, define a function $\psi_\omega : T \times Y_+ \to \mathbb{R}$ by $\psi_\omega(t, x) = U_t(\omega, x)$. The following assumptions on the priors and the state-dependent utilities of agents will be needed, the last two of which can be found in [3, 4, 8].

$(\mathbf{A}_2)$ For each $(t, \omega) \in T \times \Omega$, $U_t(\omega, \cdot) : Y_+ \to \mathbb{R}$ is monotone in the sense that $U_t(\omega, x + y) > U_t(\omega, x)$ if $x, y \in Y_+$ with $y > 0$.

$(\mathbf{A}_3)$ The function φ is measurable, where Δ^m is endowed with the Borel structure.

$(\mathbf{A}_4)$ For each $\omega \in \Omega$, the function ψ_ω is Carathéodory, that is, $\psi_\omega(\cdot, x)$ is measurable for all $x \in Y_+$, and $\psi_\omega(t, \cdot)$ is norm-continuous for all $t \in T$.

The graph of P_f is defined by $\mathrm{Gr}_{P_f} = \{(t, x) \in T \times Y_+^\Omega : x \in P_f(t)\}$, which is the intersection of $\bigcup\{T_{\mathscr{Q}} \times L_{\mathscr{Q}} : \mathscr{Q} \in \mathfrak{P}_T\}$ and $\{(t, x) \in T \times Y_+^\Omega : \mathbb{E}^{\mathbb{Q}_t}(x) > \mathbb{E}^{\mathbb{Q}_t}(f(t, \cdot))\}$. Under $(\mathbf{A}_3)$ and $(\mathbf{A}_4)$, the function $(t, x) \mapsto \mathbb{E}^{\mathbb{Q}_t}(x)$ is Carathéodory. Thus, the set $\{(t, x) \in T \times Y_+^\Omega : \mathbb{E}^{\mathbb{Q}_t}(x) > \mathbb{E}^{\mathbb{Q}_t}(f(t, \cdot))\}$ is $\Sigma \otimes \mathscr{B}(Y^\Omega)$-measurable, where $\mathscr{B}(Y^\Omega)$ is the Bore σ-algebra of Y^Ω. This further implies $\mathrm{Gr}_{P_f} \in \Sigma \otimes \mathscr{B}(Y^\Omega)$.

As usual, a *coalition* of $\mathscr{E}$ is a set $S \in \Sigma$ with $\mu(S) > 0$. If S and S' are two coalitions of $\mathscr{E}$ with $S' \subseteq S$, then S' is called a *sub-coalition* of S. Further, a coalition S *privately blocks an allocation* f in $\mathscr{E}$ if there is an S-feasible allocation g such that $\mathbb{E}^{\mathbb{Q}_t}(g(t, \cdot)) > \mathbb{E}^{\mathbb{Q}_t}(f(t, \cdot))$ μ-a.e. on S. Following [27], the *private core* of $\mathscr{E}$, denoted by $\mathscr{PC}(\mathscr{E})$, is the set of feasible allocations which are not privately blocked by any coalition. A *price system* is a non-zero function $\pi : \Omega \to Y_+^*$, where Y_+^* is the positive cone of the norm-dual Y^* of Y. The *budget set* of agent t with respect to a price system π is defined by

$$B_t(\pi) = \{x \in L_t : \mathbb{E}[\langle x, \pi\rangle] \leq \mathbb{E}[\langle a(t,\cdot), \pi\rangle]\}.$$

where for all $x \in Y_+^\Omega$,

$$\mathbb{E}[\langle x, \pi\rangle] = \sum_{\omega\in\Omega} \langle x(\omega), \pi(\omega)\rangle.$$

A *Walrasian expectations equilibrium* of $\mathscr{E}$ is a pair (f, π), where f is a feasible allocation and π is a price system such that $f(t,\cdot) \in \operatorname{argmax}\left\{\mathbb{E}^{\mathbb{Q}_t}(x) : x \in B_t(\pi)\right\}$ μ-a.e., and

$$\mathbb{E}\left[\left\langle \int_T f d\mu, \pi \right\rangle\right] = \mathbb{E}\left[\left\langle \int_T a d\mu, \pi \right\rangle\right].$$

In this case, f is called a *Walrasian expectations allocation* and the set of such allocations is denoted by $\mathscr{W}(\mathscr{E})$. It is well known that $\mathscr{W}(\mathscr{E}) \subseteq \mathscr{PC}(\mathscr{E})$. The opposite inclusion requires some assumptions on the characteristics of agents and it will be derived in the next two sections.

3. The Edgeworth Equilibria with Separable Commodity Spaces

In this section, it is assumed that Y is separable. When $\mathrm{int}Y_+ \neq \emptyset$, the equivalence between $\mathscr{W}(\mathscr{E})$ and $\mathscr{PC}(\mathscr{E})$ was established in [8] under $(\mathbf{A}_1)$-$(\mathbf{A}_4)$. The purpose of this section is to explore this result in an economy with Y_+ has an empty interior. However, such a result is not true under $(\mathbf{A}_1)$-$(\mathbf{A}_4)$, in general, as follows from the following example in [21].

Example 3.1. Consider the deterministic economy

$$\mathscr{E} = \left\{(T, \Sigma, \mu); \ell_2^+; (U_t, a(t))_{t\in T}\right\}$$

where (**i**) $T = [0, 1]$ and Σ is the σ-algebra of Lebesgue measurable subsets of T with the Lebesgue measure μ; (**ii**) ℓ_2^+ is the consumption set of each agent, where ℓ_2^+ is the positive cone of the space ℓ_2 (the space of real sequences $\{a_n : n \geq 1\}$ equipped with the norm $\|\{a_n : n \geq 1\}\|_2 = (\sum_{n\geq 1} |a_n|^2)^{\frac{1}{2}} < \infty$); and (**iii**) for all $t \in T$, $U_t(x) = \sum_{n\geq 1} n^{-2}(1 - \exp(-n^2 x_n))$ and $a(t) = \left\{\frac{1}{n^2} : n \geq 1\right\}$. Let $\mathscr{W}^e(\mathscr{E})$ and $\mathscr{PC}^e(\mathscr{E})$ be the set of Walrasian expectations allocations and the private core when the S-exact feasibility condition is used. Assume $(\mathbf{A}_1)$-$(\mathbf{A}_4)$. So, $\mathscr{W}^e(\mathscr{E}) = \emptyset$ and $\mathscr{PC}^e(\mathscr{E}) = \{a\}$, refer to [21]. It is now claimed that $\mathscr{PC}(\mathscr{E}) = \{a\}$. To see this, let $f \in \mathscr{PC}(\mathscr{E})$. By $(\mathbf{A}_2)$, one can show that $\int_T f d\mu = \int_T a d\mu$ and thus, $f \in \mathscr{PC}^e(\mathscr{E})$. This implies that $f = a$ and the claim is verified. If $a \in \mathscr{W}(\mathscr{E})$, the only candidate as a supporting price for the allocation a is the multiple of $(1, 1, ...)$ which are not in ℓ_+^2. So, $\mathscr{W}(\mathscr{E}) = \emptyset$.

To establish the equivalence theorem, similar assumptions and techniques used in [21] are employed in the rest of this section.

Definition 3.2. [21, 26] Let $\omega \in \Omega$, $v > 0$ and U be an open convex solid neighborhood of 0 in Y. Suppose that C is the open cone spanned by $v + U$. The bundle v is called an *extremely desirable bundle* with respect to U at state ω if $x \in Y_+$ and $y \in (C + x) \cap Y_+$ together imply $U_t(\omega, y) > U_t(\omega, x)$ μ-a.e.

($\mathbf{A}_5$) For each $\omega \in \Omega$, there is a $v(\omega) > 0$ such that $v(\omega)$ is an extremely desirable bundle with respect to some open convex solid neighborhood $U(\omega)$ of 0 in Y.

($\mathbf{A}_6$) Suppose that $\delta_1, ..., \delta_m$ are positive numbers with $\sum_{i=1}^m \delta_i = 1$. If $x_i \in Y_+$ and $x_i \notin \delta_i U(\omega)$ for all $1 \le i \le m$, then $\sum_{i=1}^m x_i \notin U(\omega)$.

Let $U = \left(\frac{1}{m} \bigcap_{\omega \in \Omega} U(\omega)\right)^m$, and C and $C(\omega)$ be the open convex cones spanned by $\sum_{\omega \in \Omega} v(\omega)\mathbf{1}_\Omega + U$ and $v(\omega) + U(\omega)$ respectively for all $\omega \in \Omega$.

Theorem 3.3. *Assume* ($\mathbf{A}_2$)-($\mathbf{A}_6$) *and that* $f \in \mathscr{PC}(\mathscr{E})$. *Let* $g : S \times \Omega \to Y_+$ *be defined by* $g(t, \omega) = y_i(\omega)$ *if* $(t, \omega) \in S_i \times \Omega$, *where for each* $1 \le i \le m$ *there is some* $\mathscr{Q} \in \mathfrak{P}(S)$ *such that* $S_i \subseteq S \cap T_{\mathscr{Q}}$ *and* $\mu(S_i) = \eta$. *Assume further that* $g(t, \cdot) \in L_t$ *and* $\mathbb{E}^{\mathbb{Q}_t}(g(t, \cdot)) > \mathbb{E}^{\mathbb{Q}_t}(f(t, \cdot))$ μ*-a.e. on* S. *Then* $\int_S (g - b) d\mu \notin -C$, *where*

$$b = \sum_{i=1}^m \left(\frac{1}{\eta} \int_{S_i} a d\mu\right) \chi_{S_i}.$$

Proof. Assume the contrary. Then

$$\sum_{i=1}^m (y_i - a_i)\eta \in -\alpha \left(\sum_{\omega \in \Omega} v(\omega)\mathbf{1}_\Omega + U\right),$$

where $a_i = \frac{1}{\eta} \int_{S_i} a d\mu$ and $\alpha > 0$. So there is an element $w \in \frac{\alpha}{\eta} U$ such that

$$\sum_{i=1}^m y_i + u + w = \sum_{i=1}^m a_i \ge 0,$$

where $u = \frac{\alpha}{\eta} \sum_{\omega \in \Omega} v(\omega)\mathbf{1}_\Omega$. Since $\sum_{i=1}^m y_i + u \ge 0$ and U is solid, one has $w^- \in \frac{\alpha}{\eta} U$ and $\sum_{i=1}^m y_i + u \ge w^-$. For any m-tuple $\sigma = (\sigma_1, \cdots, \sigma_m)$ of positive real numbers with $\sum_{i=1}^m \sigma_i = 1$,

$$w^- \le \sum_{i=1}^m (y_i + \sigma_i u).$$

By the Riesz decomposition property, one obtains a finite set $\{w_1^\sigma, \cdots, w_m^\sigma\}$ such that $w^- = \sum_{i=1}^m w_i^\sigma$ and $0 \le w_i^\sigma \le y_i + \sigma_i u$ for all $1 \le i \le m$. Let $I_{\mathscr{Q}} = \{i : S_i \subseteq S \cap T_{\mathscr{Q}}\}$ for all $\mathscr{Q} \in \mathfrak{P}(S)$. Pick an $i \in I_{\mathscr{Q}}$ and note that $y_i + \sigma_i u$ is $\mathscr{Q}$-measurable. Define $d_i^\sigma : \Omega \to Y_+$ by $d_i^\sigma(\omega) = \sup\{w_i^\sigma(\omega') : \omega' \in \mathscr{Q}(\omega)\}$. Obviously, d_i^σ is $\mathscr{Q}$-measurable and $d_i^\sigma \le y_i + \sigma_i u$. Let

$$z_i^\sigma = y_i + \sigma_i u - d_i^\sigma \text{ and } c^\sigma = \frac{\sigma_i \alpha}{\eta} \sum_{\omega \ne \omega' \in \Omega} v(\omega')\mathbf{1}_\Omega \text{ for all } \omega \in \Omega.$$

Fix an $\omega \in \Omega$. Put

$$\delta_i^\sigma = \operatorname{dist}\left(z_i^\sigma(\omega), (C(\omega) + c^\sigma(\omega) + y_i(\omega)) \cap Y_+\right),$$

and consider a continuous function $f : \Delta^m \to \Delta^m$ defined by

$$f(\sigma) = \left(\frac{\sigma_1 + \delta_1^\sigma}{1 + \sum_{j=1}^m \delta_j^\sigma}, \cdots, \frac{\sigma_m + \delta_m^\sigma}{1 + \sum_{j=1}^m \delta_j^\sigma}\right).$$

By Brouwer's fixed point theorem, one obtains a $\sigma^* = (\sigma_1^*, \cdots, \sigma_m^*) \in \Delta^m$ satisfying $\delta_i^{\sigma^*} = \sigma_i^* \sum_{j=1}^m \delta_j^{\sigma^*}$ for all $1 \le i \le m$. It is claimed that $\sum_{j=1}^m \delta_j^{\sigma^*} = 0$. Otherwise, $\delta_i^{\sigma^*} = 0$ if and only if $\sigma_i^* = 0$. Define the set $J = \{i : \delta_i^{\sigma^*} = 0\}$. Pick an $i \in J$, then

$z_i^{\sigma^*}(\omega) = y_i(\omega) - d_i^{\sigma^*}(\omega)$. If $d_i^{\sigma^*}(\omega) > 0$, then one has $U_t(\omega, y_i(\omega)) > U_t(\omega, z_i^{\sigma^*}(\omega))$ for $t \in S_i$ and hence

$$z_i^{\sigma^*}(\omega) \notin \text{cl}\left(\left(C(\omega) + c^{\sigma^*}(\omega) + y_i(\omega)\right) \cap Y_+\right).$$

By definition, $\delta_i^{\sigma^*} > 0$, which is a contradiction with the fact that $i \in J$. Thus $d_i^{\sigma^*}(\omega) = 0$ for all $i \in J$. Pick an $i \notin J$, then

$$z_i^{\sigma^*}(\omega) \notin \left(C(\omega) + c^{\sigma^*}(\omega) + y_i(\omega)\right) \cap Y_+.$$

Consequently,

$$y_i(\omega) + \frac{\sigma_i^* \alpha}{\eta} v(\omega) - d_i^{\sigma^*}(\omega) \notin C(\omega) + y_i(\omega),$$

which further implies that $d_i^{\sigma^*}(\omega) \notin \frac{\sigma_i^* \alpha}{\eta} U(\omega)$ for all $i \notin J$ and so $\sum_{i=1}^m d_i^{\sigma^*}(\omega) \notin \frac{\alpha}{\eta} U(\omega)$. Note that $d_i^{\sigma^*}(\omega) \leq \sum_{\omega \in \Omega} w_i^{\sigma^*}(\omega)$ and so

$$\sum_{i=1}^m d_i^{\sigma^*}(\omega) \leq \sum_{\omega \in \Omega} w^-(\omega).$$

Since $\sum_{\omega \in \Omega} w^-(\omega) \in \frac{\alpha}{\eta} U(\omega)$ and $\frac{\alpha}{\eta} U(\omega)$ is solid, $\sum_{i=1}^m d_i^{\sigma^*}(\omega) \in \frac{\alpha}{\eta} U(\omega)$, which is a contradiction. Thus the claim is verified, which means that $\delta_i^{\sigma^*} = 0$ for all $1 \leq i \leq m$. Hence,

$$z_i^{\sigma^*}(\omega) \in \text{cl}\left((C(\omega) + c(\omega) + y_i(\omega)) \cap Y_+\right)$$

for all $1 \leq i \leq m$. By $(\mathbf{A}_5)$, one has $U_t(\omega, z_i^{\sigma^*}(\omega)) \geq U_t(\omega, c(\omega) + y_i(\omega))$, which together with $(\mathbf{A}_2)$ gives $\mathbb{E}^{\mathbb{Q}_t}(z_i^{\sigma^*}) > \mathbb{E}^{\mathbb{Q}_t}(y_i)$ for all $t \in S_i$ and $1 \leq i \leq m$. Define $h : T \times \Omega \to Y_+$ by

$$h(\cdot, \omega) = \sum_{i=1}^m z_i^{\sigma^*}(\omega) \mathbf{1}_{S_i}.$$

Clearly, $h(t, \cdot) \in L_t$ and $\mathbb{E}^{\mathbb{Q}_t}(h(t, \cdot)) > \mathbb{E}^{\mathbb{Q}_t}(f(t, \cdot))$ μ-a.e. on S and

$$\int_S h d\mu \leq \eta \left(\sum_{i=1}^m y_i + u - w^-\right) \leq \eta \left(\sum_{i=1}^m y_i + u + w\right) = \eta \sum_{i=1}^m a_i = \int_S a d\mu.$$

This contradicts with $f \in \mathscr{PC}(\mathscr{E})$ and the proof has been completed. $\square$

Theorem 3.4. *Assume* $(\mathbf{A}_1)$-$(\mathbf{A}_6)$. *If the correspondence* $F : T \rightrightarrows Y_+^\Omega$ *defined by* $F(t) = \{x - a(t, \cdot) : x \in P_f(t)\} \cup \{0\}$ *for all* $t \in T$, *then* $\text{cl} \int_T F d\mu \cap -C = \emptyset$.

Proof. It is sufficient to prove that $\int_T F d\mu \cap -C = \emptyset$. Assume the contrary and let $\int_T h d\mu \in \int_T F d\mu \cap -C$. Put

$$S = \{t \in T : h(t, \cdot) \neq 0 \text{ and } h(t, \cdot) = g(t, \cdot) - a(t, \cdot) \text{ for some } g(t, \cdot) \in P_f(t)\}.$$

Then $S \in \Sigma$, $\mu(S) > 0$ and $\int_S (g - a) d\mu \in -C$. Without loss of generality, one can assume that $\mathfrak{P}(S) = \mathfrak{P}_S$. Pick an $\mathscr{Q} = \{A_1, ..., A_k\} \in \mathfrak{P}(S)$ and let $\omega_j \in A_j$ for all $1 \leq j \leq k$. Since $g(\cdot, \omega_j) \in L_1(\mu_{S \cap T_{\mathscr{Q}}}, Y_+)$, there is a monotonically increasing sequence $\left\{h_n^{(\mathscr{Q}, \omega_j)} : n \geq 1\right\}$ of simple functions converging pointwise to $g(\cdot, \omega_j)$ such that

$$\lim_{n \to \infty} \int_{S \cap T_{\mathscr{Q}}} \left\| g(\cdot, \omega_j) - h_n^{(\mathscr{Q}, \omega_j)}(\cdot) \right\| d\mu = 0.$$

Define $g_n : S \times \Omega \to Y_+$ by $g_n(t,\omega) = h_n^{(\mathscr{Q},\omega_j)}(t)$ if $(t,\omega) \in (S \cap T_{\mathscr{Q}}) \times A_j$. So, $\{g_n : n \geq 1\}$ is a monotonically increasing sequence of simple functions converging pointwise to g and $\lim_{n\to\infty} \int_S \|g(\cdot,\omega) - g_n(\cdot,\omega)\| d\mu = 0$ for all $\omega \in \Omega$. Let

$$\widetilde{S}_n = \{t \in S : g_n(t,\cdot) \in P_f(t)\} \text{ and } S_n = \bigcup \{\widetilde{S}_n \cap T_{\mathscr{Q}} : \mathscr{Q} \in \mathfrak{P}(S_n)\}.$$

By the continuity and the monotonicity of $\mathbb{E}^{\mathbb{Q}_t}$, one obtains $S_n \subseteq S_{n+1}$ for all $n \geq 1$ and $\mu(S \setminus \bigcup_{n\geq 1} S_n) = 0$. Pick an $n \geq 1$. Let $g_n|_{S_n} = \sum_{i=1}^m y_i \chi_{R_i}$, where for all $1 \leq i \leq m$ there are some $\mathscr{Q} \in \mathfrak{P}(S_n)$ and $\eta_n > 0$ such that $R_i \subseteq S_n \cap T_{\mathscr{Q}}$ and $\mu(R_i) = \eta_n$. Assume that

$$b_n = \sum_{i=1}^m \left(\frac{1}{\eta_n} \int_{R_i} a d\mu \right) \chi_{R_i}.$$

By Theorem 3.3, $\int_{S_n} (g_n - b_n) d\mu \notin -C$. Note that $\| \int_{S_n} (g_n - b_n) d\mu - \int_S (g-a) d\mu \| \to 0$ as $n \to \infty$. Since C is open, $\int_S (g-a) d\mu \notin -C$, which is a contradiction. □

Next, the main result of this section is presented. In its proof, the following result in [13] is employed: If $\varphi : T \times Y \to \mathbb{R}$ is Carathéodory, then

$$\int_T \inf_{x \in F(\cdot)} \varphi(\cdot, x) d\mu = \inf \left\{ \int_T \varphi(\cdot, f(\cdot)) d\mu : f \text{ is an integrable section of } F \right\}.$$

Theorem 3.5. *Assume* $(\mathbf{A}_1)$-$(\mathbf{A}_6)$. *Then* $\mathscr{W}(\mathscr{E}) = \mathscr{PC}(\mathscr{E})$.

Proof. By Theorem 3.4 and the separation theorem, there is a non-zero element $\pi \in (Y_+^*)^\Omega$ such that $\mathbb{E}[\langle y, \pi \rangle] \geq 0$ for all $y \in \int_T F d\mu$. Since $\mathrm{Gr}_F \in \Sigma \otimes \mathfrak{B}(Y^\Omega)$,

$$\int_T \inf \{\mathbb{E}[\langle z, \pi\rangle] : z \in F(\cdot)\} d\mu = \inf \left\{ \mathbb{E}[\langle y, \pi \rangle] : y \in \int_T F d\mu \right\} \geq 0.$$

Since $\inf \{\mathbb{E}[\langle z, \pi\rangle] : z \in F(t)\} \leq 0$ for all $t \in T$, one has $\inf \{\mathbb{E}[\langle z, \pi\rangle] : z \in F(t)\} = 0$ μ-a.e. Thus, $\mathbb{E}[\langle x, \pi\rangle] \geq \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ for all $x \in P_f(t)$ and μ-a.e. By $(\mathbf{A}_2)$, one obtains $\mathbb{E}[\langle f(t,\cdot), \pi\rangle] \geq \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ μ-a.e. This together with feasibility of f further yield $\mathbb{E}[\langle f(t,\cdot), \pi\rangle] = \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ μ-a.e. Thus,

$$\mathbb{E}\left[\left\langle \int_T f d\mu, \pi \right\rangle \right] = \mathbb{E}\left[\left\langle \int_T a d\mu, \pi \right\rangle \right].$$

To complete the proof, one needs to verify that $f(t,\cdot) \in \operatorname{argmax} \left\{ \mathbb{E}^{\mathbb{Q}_t}(x) : x \in B_t(\pi) \right\}$ μ-a.e. By $(\mathbf{A}_1)$, $\sum_{\omega\in\Omega} \langle a(t,\omega), \pi(\omega)\rangle > 0$ for all $t \in T$. Select some $t \in T$ satisfying $\mathbb{E}[\langle x, \pi\rangle] \geq \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ for all $x \in P_f(t)$. If $\mathbb{E}[\langle x, \pi\rangle] = \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ for some $x \in P_f(t)$, then $\lambda x \in P_f(t)$ and $\mathbb{E}[\langle \lambda x, \pi\rangle] < \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ for some $0 < \lambda < 1$, which is a contradiction. So, (f, π) is a Walrasian expectations equilibrium of $\mathscr{E}$. □

4. The Edgeworth Equilibria with non-separable commodity spaces

In this section, the equivalence between the private core and the set of Walrasian expectations allocations is provided in an asymmetric information economy with countably many characteristics. Since the commodity space in this section is not necessarily separable, the negative result obtained in [17, 23] is not valid in this economic setting. In [19], some necessary and sufficient conditions were given for the core-Walras equivalence theorem in a deterministic economy with an atomless measure space of agents and a Banach lattice as the commodity space. In fact, Podczeck [19] obtained the equivalence between the core and the set of

Walrasian allocations under some properties of the commodity space. In contrast, the equivalence theorem in this section does not require such properties. To see the equivalence theorem, let $\{(\mathscr{F}_i, U_i, a(t,\cdot), \mathbb{Q}_i) : i \geq 1\}$ be the set of different characteristics available in $\mathscr{E}$ and T_i be the set of agents in T having the same characteristics as $(\mathscr{F}_i, U_i, a(t,\cdot), \mathbb{Q}_i)$. Suppose that $T_i \in \Sigma$ for all $i \geq 1$. Note that the measurability conditions in $(\mathbf{A}_3)$ and $(\mathbf{A}_4)$ are trivially satisfied. For any allocation f in $\mathscr{E}$, let $\hat{f} = \Xi(f)$ be an allocation defined by

$$\hat{f}(t,\omega) = \begin{cases} f(t,\omega), & \text{if } (t,\omega) \in T_i \times \Omega, \mu(T_i) = 0; \\ \frac{1}{\mu(T_i)} \int_{T_i} f(\cdot,\omega) d\mu, & \text{if } (t,\omega) \in T_i \times \Omega, \mu(T_i) > 0. \end{cases}$$

Fix an i with $\mu(T_i) > 0$, and define $\hat{f}_{T_i}(\omega) = \hat{f}(\cdot,\omega)$ for all $(t,\omega) \in T_i \times \Omega$. The following lemma is similar to Theorem 3.5 in [6], and is essential for the equivalence theorem.

Lemma 4.1. *Suppose* $(\mathbf{A}_1)$-$(\mathbf{A}_2)$ *and that* U_t *is continuous and concave for each* $t \in T$. *If* $f \in \mathscr{P}\mathscr{C}(\mathscr{E})$, *then* $\mathbb{E}_t^{\mathbb{Q}}(f(t,\cdot)) = \mathbb{E}_t^{\mathbb{Q}}(\hat{f}(t,\cdot))$ μ*-a.e.*

Proof. By ignoring a μ-null subset of T, one can choose a separable closed linear subspace Z of Y^{Ω} such that $f(T,\cdot) \subseteq Z$. Assume that there exist an i_0, a coalition $D \subseteq T_{i_0}$ such that $\mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_{i_0}}) > \mathbb{E}^{\mathbb{Q}_t}(f(t,\cdot))$ for all $t \in D$. For any $r \in \mathbb{Q} \cap (0,1)$, let

$$D_r = \left\{ t \in D : \mathbb{E}^{\mathbb{Q}_t}\left(r\hat{f}_{T_{i_0}}\right) > \mathbb{E}^{\mathbb{Q}_t}(f(t,\cdot)) \right\}.$$

Note that D_r is the projection of

$$\left(D \times \left\{x \in Y_+^{\Omega} : \mathbb{E}^{\mathbb{Q}_t}\left(r\hat{f}_{T_{i_0}}\right) > \mathbb{E}^{\mathbb{Q}_t}(x)\right\}\right) \cap \{(t, f(t,\cdot)) : t \in D\}$$

on D. By the projection theorem, one has $D_r \in \Sigma$, and $D = \bigcup\{D_r : r \in \mathbb{Q} \cap (0,1)\}$. Thus, one can find a $r_1 \in \mathbb{Q} \cap (0,1)$ satisfying $\mu(D_{r_1}) > 0$. Let $r_2 = \frac{\mu(D_{r_1})}{\mu(T_{i_0})}$. Then $0 < r_2 \leq 1$. For each $\omega \in \Omega$, put

$$\upsilon(\omega) = r_1 r_2 \left(\int_T f(\cdot,\omega) d\mu - \int_T a(\cdot,\omega) d\mu \right) - r_2(1 - r_1) \int_{T_{i_0}} a_{i_0}(\omega) d\mu.$$

Clearly, $\upsilon(\omega) \in -Y_{++}$ for all $\omega \in \Omega$. So, there is an $\varepsilon > 0$ such that

$$\upsilon(\omega) + B(0, 2\varepsilon) \subseteq -Y_{++}$$

for all $\omega \in \Omega$. Applying Lemma 3.3 in [3], one has a coalition $E \subseteq T \setminus T_{i_0}$ of $\mathscr{E}$ such that $\mu(E) < \mu(T \setminus T_{i_0})$ and $\|d(\omega)\| < \varepsilon$ for all $\omega \in \Omega$, where

$$d(\omega) = \int_E (f(\cdot,\omega) - a(\cdot,\omega)) d\mu - r_1 r_2 \int_{T \setminus T_{i_0}} (f(\cdot,\omega) - a(\cdot,\omega)) d\mu.$$

Let $S = D_{r_1} \cup E$. Pick an $u \in B(0,\varepsilon) \cap Y_{++}$ and define $g : T \times \Omega \to Y_+$ by

$$g(t,\omega) = \begin{cases} f(t,\omega) + \frac{u}{\mu(E)}, & \text{if } (t,\omega) \in E \times \Omega; \\ r_1 \hat{f}_{T_{i_0}}, & \text{otherwise.} \end{cases}$$

Then, $g(t,\cdot) \in P_f(t)$ μ-a.e. on S, and

$$\int_S g(\cdot,\omega) d\mu = \int_E f(\cdot,\omega) d\mu + r_1 r_2 \int_{T_{i_0}} f(\cdot,\omega) d\mu + u$$

for all $\omega \in \Omega$. It can be easily verified that for all $\omega \in \Omega$,

$$-v(\omega) + \int_S (g(\cdot,\omega) - a(\cdot,\omega))d\mu = d(\omega) + u \in B(0, 2\varepsilon).$$

Hence,

$$\int_S a(\cdot,\omega)d\mu - \int_S g(\cdot,\omega)d\mu \gg 0$$

for all $\omega \in \Omega$, which contradicts with the fact that $f \in \mathscr{PC}(\mathscr{E})$. Thus, $\mathbb{E}^{\mathbb{Q}_t}(f(t,\cdot)) \geq \mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_i})$ μ-a.e. on T_i for all $i \geq 1$. Suppose that there is a coalition $R \subseteq T_{i'}$ for some $i' \geq 1$ such that $\mathbb{E}^{\mathbb{Q}_t}(f(t,\cdot)) > \mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_{i'}})$ for all $t \in R$. By Jensen's inequality, one has

$$\mathbb{E}^{\mathbb{Q}_t}\left(\frac{1}{\mu(R)}\int_R f(\cdot,\cdot)d\mu\right) > \mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_{i'}})$$

and

$$\mathbb{E}^{\mathbb{Q}_t}\left(\frac{1}{\mu(T_{i'} \setminus R)}\int_{T_{i'}\setminus R} f(\cdot,\cdot)d\mu\right) \geq \mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_{i'}}).$$

Let $\delta = \frac{\mu(R)}{\mu(T_{i'})}$. Since

$$\hat{f}_{T_{i'}} = \frac{\delta}{\mu(R)}\int_R f(\cdot,\cdot)d\mu + \frac{1-\delta}{\mu(T_{i'} \setminus R)}\int_{T_{i'}\setminus R} f(\cdot,\cdot)d\mu,$$

$\mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_{i'}}) > \mathbb{E}^{\mathbb{Q}_t}(\hat{f}_{T_{i'}})$, which is a contradiction. Thus, $\mathbb{E}^{\mathbb{Q}_t}(f(t,\cdot)) = \mathbb{E}^{\mathbb{Q}_t}(\hat{f}(t,\cdot))$ μ-a.e. □

Theorem 4.2. *Suppose* $(\mathbf{A}_1)$-$(\mathbf{A}_2)$ *and that* U_t *is continuous and concave for each* $t \in T$. *Then* $\mathscr{W}(\mathscr{E}) = \mathscr{PC}(\mathscr{E})$.

Proof. To complete the proof, one only needs to verify that $\mathscr{PC}(\mathscr{E}) \subseteq \mathscr{W}(\mathscr{E})$. Pick an element $f \in \mathscr{PC}(\mathscr{E})$ and note that

$$H = \operatorname{cl}\left(\bigcup\left\{\int_S P_f d\mu - \int_S a d\mu : S \in \Sigma,\ \mu(S) > 0\right\}\right)$$

is a non-empty convex subset of Y^Ω, refer to [5, Theorem 1]. Since $H \cap -Y^\Omega_{++} = \emptyset$, by the separation theorem, there is a non-zero element $\pi \in (Y^*_+)^\Omega$ such that for any coalition S,

$$\mathbb{E}[\langle y, \pi\rangle] \geq \mathbb{E}\left[\left\langle \int_S a d\mu, \pi \right\rangle\right]$$

for all $y \in \int_S P_f d\mu$. By Lemma 4.1, $\mathbb{E}^{\mathbb{Q}_t}(f(\cdot,\cdot)) : T_i \to \mathbb{R}$ is constant μ-a.e. for all $i \geq 1$. Pick an $i \geq 1$ and a $y_i \in P_f(t)$ μ-a.e. on T_i. If $y_i \in B_t(\pi)$ for $t \in T_i$, then one can construct some $z_i \in B_t(\pi)$ such that $z_i \in P_f(t)$ μ-a.e. on T_i and

$$\mathbb{E}\left[\left\langle \int_{T_i} z_i d\mu, \pi \right\rangle\right] < \mathbb{E}\left[\left\langle \int_{T_i} a_i d\mu, \pi \right\rangle\right],$$

which is a contradiction. Thus, one has $\mathbb{E}[\langle y_i, \pi\rangle] > \mathbb{E}[\langle a_i, \pi\rangle]$, which further implies $\mathbb{E}[\langle f(t,\cdot), \pi\rangle] \geq \mathbb{E}[\langle a_i, \pi\rangle]$ μ-a.e. Using the feasibility of f, one can show that $\mathbb{E}[\langle f(t,\cdot), \pi\rangle] = \mathbb{E}[\langle a(t,\cdot), \pi\rangle]$ μ-a.e. Thus, (f, π) is a Walrasian expectations equilibrium in $\mathscr{E}$. □

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