# Toilet Seat Rules: Why you Shouldn't Care

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*This paper analyzes the issue of choosing a socially efficient rule on how to leave the toilet seat. Leaving the seat as it is after usage is found to be the best rule over a wide parameters space. Using a loss function minimization approach, factors such as relative toilet usage, frequency of the down position, relative gender importance and cost elasticity to seat movements are considered. Leaving the seat as it is after usage proves to be dominating a large set of other rules that entail no strategic interaction.* (JEL D7, H4)

## I. INTRODUCTION

A common field of arguments between males and females is represented by how to leave the toilet seat. The issue is rarely discussed on the efficiency ground. An usual solution is to keep the toilet seat down (down rule, DO), which is a female friendly rule: they do not have to move the seat at all. This rule, though, is particularly costly to males since they must move the seat twice *almost* any time they use the toilet. However, an up rule (UP) is particularly costly to females, who must *always* move the seat twice and entails some cost to males as well, provided that they need to use the seat in the down position sometimes. We take these two simple rules as our extremes. Many other rules can be thought; for instance, any rule that is a combination of the previous two could be adopted (introducing also some stochastic elements into the analysis). A rule that seems particularly important in practice, for its simplicity, is the rule of leaving the toilet seat as it is after usage, which I shall call the "don't care" rule (DC). This rule is of special interest because it asks no commitment. Hence, it is unlikely that people are going to cheat or deviate from this rule. Even if simple, the DC rule introduces some probabilistic aspects, given that the expected number of toilet seat movements depends on the possibility of having two consecutive toilet operations of the same kind (down-down or up-up). This aspect can be simplified. In fact, the DC rule worst case is that of perfect alternation of toilet operations, which implies one seat movement for each toilet usage. This case is thus equivalent to the adoption of a fourth rule stating that one seat movement must be done at each toilet usage, the "one movement" rule (OM). The OM rule entails no uncertainty, while the DC worst case occurs with some probability. The OM rule can be shown to dominate the UP and the DO rules, and as a consequence the DC rule is dominating all the others in a wide range of situations. In what follows we focus only on the UP, DO and DC rules; the OM rule is discussed in the Appendix.

Related literature

Toilet seat literature is almost inexistent. There are many discussions on the Internet, but only some of them "rigorously" tackle the issue and deserve to be cited. Harter (2005) and Siddiqi (2007) analyze the toilet seat problem with a game theory approach, where the game can be cooperative or not. Indeed, the possibility of cheating, the possibility of using the toilet in the "wrong" way, the costs of yelling to each other etc. represent interesting empirical aspects. We claim that they should not be relevant for efficiency if we consider the cost of moving the toilet seat as the relevant one. Moreover, we are interested in finding the most efficient rule, not in discussing the rule that is going to be adopted. Hence, no strategic interaction is taken into account. As a consequence, we can treat males and females sharing a toilet as a single individual, i.e. a couple or a group, whose objective is to minimize its aggregate cost. Our approach is very similar to Venkataraman (1999) and Choi (2002, 2011). In their papers, they essentially use a cost minimization approach getting to results that are a particular case of our results under some specific parameters specification. In Venkataraman (1999) there is a link to a web-based computer simulation that allows you to set your parameters and find the optimal rule. In Choi (2002, 2011) there is a formal proof that his results are robust to consideration of any other rule and to heterogeneous costs within the same gender.

## II. THE LOSS FUNCTION APPROACH

The basic framework of the model assumes two genders, M and F, sharing a toilet. The toilet is assumed to be already shared. M uses the toilet $n_M$ times and F uses it $n_F$ times, during some time interval. The share of M using the toilet is then $s_M=n_M/n$ and the share of F is $(1-s_M)$, where $n=n_M+n_F$. These shares represent percentage usage. One can think to one M and one F using the toilet a different number of times or to the relative number of males and females using the toilet once; in both cases the shares can be interpreted as probabilities. Genders use the toilet in a different way. M uses it both with the seat in the up position and in the down position; F uses it in the down position only. The share of times M uses the seat in the down position is $\delta\in[0,1]$. We assume that people do never use the toilet in the "wrong" way, so that we rule out the possibility of F using it with the seat up or M using it down (up) while the "proper" position would have been up (down). This simplifies the analysis since using the toilet in the "wrong" way would introduce some costs on M and/or F, and would need some game theory analysis as well. People are assumed to know how to use the toilet and that they would use it properly even without sharing it with others. This assumption is due to the existence of a social etiquette on how genders are supposed to use toilet seat: this analysis takes it as given. Let us call operation 1 using the toilet with the seat up and operation 2 using it with the seat down. Operation 1 occurs

$s_1=s_M(1-\delta)$ times, while operation 2 occurs $(1-s_1)$ times. Again these shares can be thought as probabilities. Operations 1 and 2 entail a cost or a loss that depends on the number of toilet seat movements given the toilet seat rule adopted. Each seat movement is set to require an effort of 1 to both genders. The number of seat movements, or the effort (E), is then a function of the toilet seat rule adopted (R): $E_i=E_i(R)$ where i=1,2. The cost elasticity with respect to effort (or number of seat movements) is constant and equal among genders ($\beta \geq 1$). The loss functions of operations 1 and 2 and for M and F are defined as follows:

(1) $$L_1 = s_1 \left[E_1(R)\right]^{\beta}$$

(2) $$L_2 = \left(1-s_1\right)\left[E_2(R)\right]^{\beta}$$

(3) $$L_M = s_M \left\{(1-\delta)\left[E_1(R)\right]^{\beta} + \delta\left[E_2(R)\right]^{\beta}\right\}$$

(4) $$L_F = \left(1-s_M\right)\left[E_2(R)\right]^{\beta}$$

Note that the second factor of each of the above equations represents the respective average or unit loss function: $\underline{L}_1=[E_1(R)]^{\beta}$, $\underline{L}_2=\underline{L}_F=[E_2(R)]^{\beta}$, $\underline{L}_M=(1-\delta)\underline{L}_1+\delta\underline{L}_2$. We focus only on the three basic rules mentioned above, hence R={UP, DO, DC}. If the rule adopted is UP, operation 1 needs no seat movement, while operation 2 needs 2 movements; the opposite holds under the DO rule. The UP and DOWN rules entail no uncertainty about the number of movements. If instead the rule is DC, the number of movements is 0 if one finds the seat as needed, 1 otherwise; these events occur with some probability. Given that one has to take operation 1, the probability to find the seat in the desired position is $s_1$; whereas the probability to move the seat once is $(1-s_1)$; vice versa for operation 2. Therefore, under the DC rule, we can only evaluate the expected number of seat movements. However, this number is less or equal than 1, so we could also study the one movement (OM) rule as the certainty equivalent of the DC worst case of perfect alternation of toilet operations (see the Appendix). Finally, we can write effort (expected effort under the DC rule) as:

(5) $$E_1(R)=\begin{cases}0 & if \quad R=UP\\ (1-s_1) & if \quad R=DC\\ 2 & if \quad R=DO\end{cases} \qquad E_2(R)=\begin{cases}2 & if \quad R=UP\\ s_1 & if \quad R=DC\\ 0 & if \quad R=DO\end{cases}$$

Being interested in efficiency we want to minimize the social cost of using a shared toilet. Let us treat M and F jointly as an individual, i.e. a couple or a group (subscript c). Assume that M has a weight equal to α and F has a weight equal to (1−α), with α∈[0,1]. This parameter can be interpreted as the relative loss or the relative importance of genders or as their decision power. Alternatively, if we give genders the same weight, it can represent the relative cost of a unit effort between genders or, if we set β=1, the relative cost of a seat movement. These alternative views are useful in some particular cases and allow the comparison with Choi's (2002, 2011) results. The aggregate loss function is simply the weighted sum of the individual loss functions since they have been already defined in terms of total toilet usage. The minimization problem can be written as:

(6) $$\min_R L_c=\alpha L_M+(1-\alpha)L_F$$

The aggregate loss function depends on the particular rule adopted; in our context it can take three values since we focus on a set of three possible rules. These values are the following functions of the four parameters α, β, δ and $s_M$:

(7) $$L_c^{UP}=\left[\alpha s_M\delta+(1-\alpha)(1-s_M)\right]2^{\beta}$$

(8) $$L_c^{DO}=\alpha s_M(1-\delta)2^{\beta}$$

(9) $$L_c^{DC}=\alpha s_M\left[(1-\delta)(1-s_1)^{\beta}+\delta s_1^{\beta}\right]+(1-\alpha)(1-s_M)s_1^{\beta}$$

III. GENERAL RESULTS

The optimal rule depends on parameters and no rule is always dominating the others. The DO rule is likely to be dominating if F has a very high relative importance (small α), M toilet usage is relatively low (small $s_M$), M operation 2 frequency is very high (large δ) and elasticity of cost with respect to effort is very low (small β). On the contrary, the UP rule is likely to be dominating if M has a very high relative importance (large α), M toilet usage is relatively high (large $s_M$), M operation 2 frequency is very low (small δ) and cost elasticity with respect to effort is very low (small β). The DC rule can be shown to be dominating in many intermediate cases and if cost elasticity with respect to effort is relatively high (large β). The following conditions can be derived:

(10) $$L_c^{UP} < L_c^{DO} \Leftrightarrow -\alpha s_M \Delta \underline{L}_M^{UP-DO} > (1-\alpha)(1-s_M)\Delta \underline{L}_F^{UP-DO}$$

(11) $$L_c^{DC} < L_c^{DO} \Leftrightarrow -\alpha s_M \Delta \underline{L}_M^{DC-DO} > (1-\alpha)(1-s_M)\Delta \underline{L}_F^{DC-DO}$$

(12) $$L_c^{DC} < L_c^{UP} \Leftrightarrow \alpha s_M \Delta \underline{L}_M^{DC-UP} < -(1-\alpha)(1-s_M)\Delta \underline{L}_F^{DC-UP}$$

where $\Delta\underline{L}^{R'-R}=\underline{L}^{R'}-\underline{L}^{R}$ denotes the difference in the average loss functions under two different rules. Note that the above inequalities depend on the relative increment in the average loss function. This result is meaningful. It states that, all else equal, the optimal rule depends on the relative marginal loss due to a change in the rule. Considering rules in pairs; for UP to dominate DO, the female's relative loss increment must be relatively small, *ceteris paribus*. Put differently, UP is more efficient than DO only if the male's loss reduction (gain) is bigger than the female's loss increment. Similarly, for DC to dominate DO, the female's loss increment must be relatively small. Finally, for DC to dominate UP, the female's gain must be relatively large. The logic of this result is intuitive: the optimal rule is the most cost saving one. The following lemma synthesizes our result:

**Lemma**: the most efficient rule is the one that provides the lowest individual relative increment in average cost depending on relative importance of genders and relative toilet usage.

This result applies in general and is due to the interaction of three main factors. First, the individual cost, which depends on preferences. Second, the relative importance of genders, which depends on social norms and bargaining power. Third, the relative toilet usage, which depends on medical and/or demographic factors.

## III. PARTICULAR CASES

The conditions derived in the previous section are general, but of little practical use. In fact, since the best rule depends on parameters, we need to evaluate them in order to clearly identify the best rule in a given situation. In this section, we characterize our results in practice by providing a graphical analysis of two particular cases. First we show that the DC rule is dominating in the most basic symmetric case for intermediate values of parameters. Then, setting plausible values for all our parameters, we study a realistic case and show that the DC rule dominance area is even larger than in the basic case.

### Basic case

The most basic case is obtained by setting $\beta=1$ and $\delta=0$. This case is linear in seat movement effort and symmetric among genders, but still is probably a reasonable approximation of reality. This case reproduces the results of Choi (2002, 2011), being also in line with the work of Venkataraman (1999). Setting $\beta=1$ means that the cost of moving the seat is proportional to the number of movements. This simplification, in terms of our model, enhances an additional interpretation of parameter $\alpha$ as the cost of one movement to M with respect to total cost. If $c_M$ is the cost of one movement to M and $c_F$ is the cost to F, then $c_M/c_F=\alpha/(1-\alpha)$. Setting $\delta=0$ corresponds to ignoring that males use the seat in the down position sometimes. This also implies that $s_M=s_1$, that is operation 1 is taken only by M and operation 2 is taken only by F. Under this basic case, the loss functions for the three rules simplify and yield the following dominance conditions:

(13) $$L_c^{UP} < L_c^{DO} \Leftrightarrow \alpha > 1 - s_M$$

(14) $$L_c^{DC} < L_c^{DO} \Leftrightarrow 2\alpha > 1 - s_M$$

(15) $$L_c^{DC} < L_c^{UP} \Leftrightarrow 2\alpha < 2 - s_M$$

Interpreting α as the cost of moving the seat, the above conditions can be rewritten as:

(16) $$L_c^{UP} < L_c^{DO} \Leftrightarrow c_M s_M > c_F (1 - s_M)$$

(17) $$L_c^{DC} < L_c^{DO} \Leftrightarrow c_M (1 + s_M) > c_F (1 - s_M)$$

(18) $$L_c^{DC} < L_c^{UP} \Leftrightarrow c_M s_M < c_F (2 - s_M)$$

The loss functions are illustrated in figure 1, for $\alpha=0.5$ (genders have the same importance), and in figure 2, for $s_M=0.5$ (same toilet usage). Conditions are illustrated in figure 3 in the $(s_M, \alpha)$ space.

FIGURE 1
Basic case: Loss functions and male toilet usage.

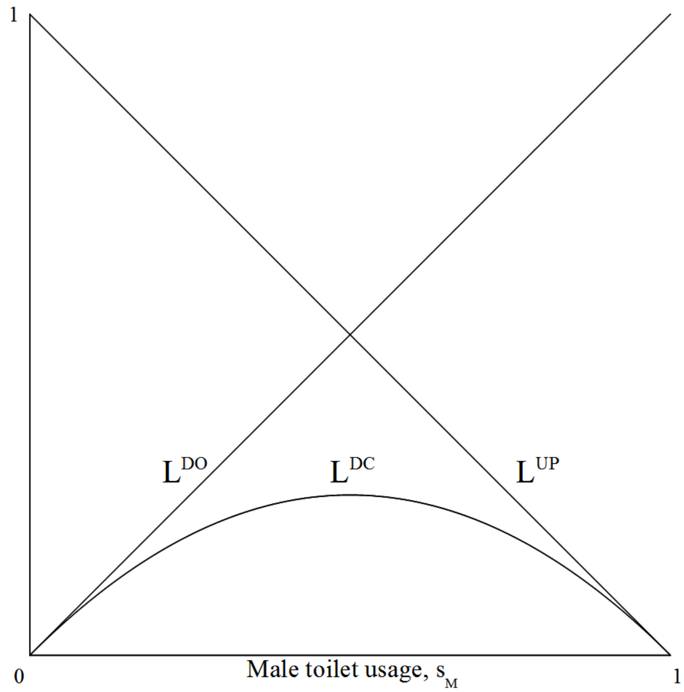



FIGURE 2
Basic case: Loss functions and male importance.

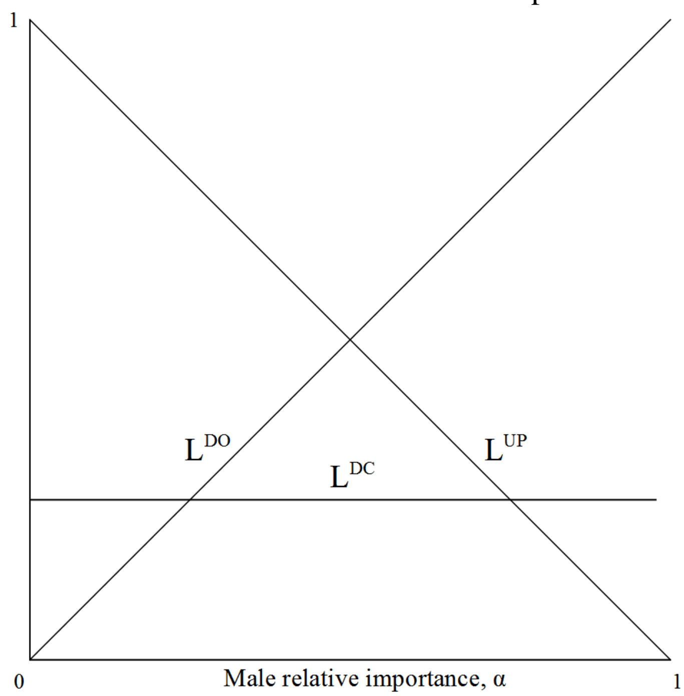
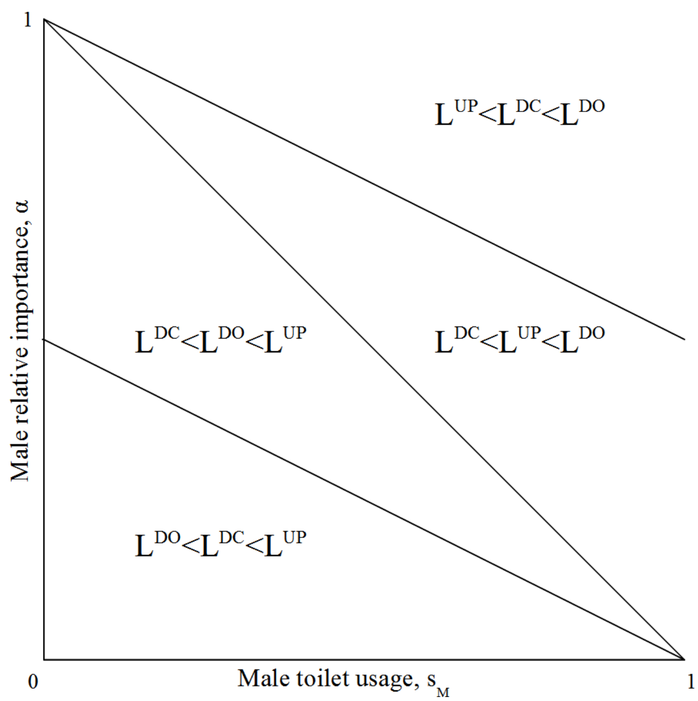



FIGURE 3
Basic case: Rules dominance areas in the ($s_M$, α) space.




Figures 1-3 show that the DC rule is dominating for many intermediate values of parameters α and $s_M$ or when a relatively large α is compensated by a relatively small $s_M$, and vice versa. Under the basic case with intermediate values of parameters, the conclusion of the model is that the DC rule is the most efficient one for the group as a whole. Although this result is likely to be a good approximation of reality in many circumstances, our framework allows to easily set up a more plausible case.

Plausible case

A realistic case is obtained by setting parameters to values that are empirically relevant. For instance, for a representative person (a male to our purpose) the normal (from a medical perspective) number of operations of type 1 goes from 5 to 8 per day, while the number of operations of type 2 goes from 3 per week to 3 per day. Therefore it is likely to have $\delta \in (0.08, 0.27)$. Let's take the mean value for simplicity and set $\delta = 0.175$. Empirical evidence on the parameter β is more difficult to gauge; we argue that it is reasonable to set it slightly greater than one. In fact, changing the seat position before using the toilet is probably done automatically if needed, assuming people use the toilet "properly"; while moving the seat after usage implies higher commitment. Hence, the second seat movement is probably costlier than the first, even though the physical effort is probably the same. This idea also explains the formulation used in the model, in which effort is the same as number of movements, while the cost is increasing more than proportionally with effort. We shall assume that the second toilet seat movement, that is the movement taken after usage, implies a cost that is 10% higher than the first. As a consequence we set $\beta$=1.07. This is a conservative parameterization, in fact any $\beta$ higher than one makes the DC rule relatively more efficient, hence using a "low" $\beta$ makes our results more robust. The loss functions are illustrated in figure 4, for $\alpha$=0.5 (genders have the same importance), and in figure 5, for $s_M$=0.5 (same toilet usage). Conditions are illustrated in figure 6 in the ($\alpha$, $s_M$) space.

FIGURE 4
Plausible case: Loss functions and male toilet usage.

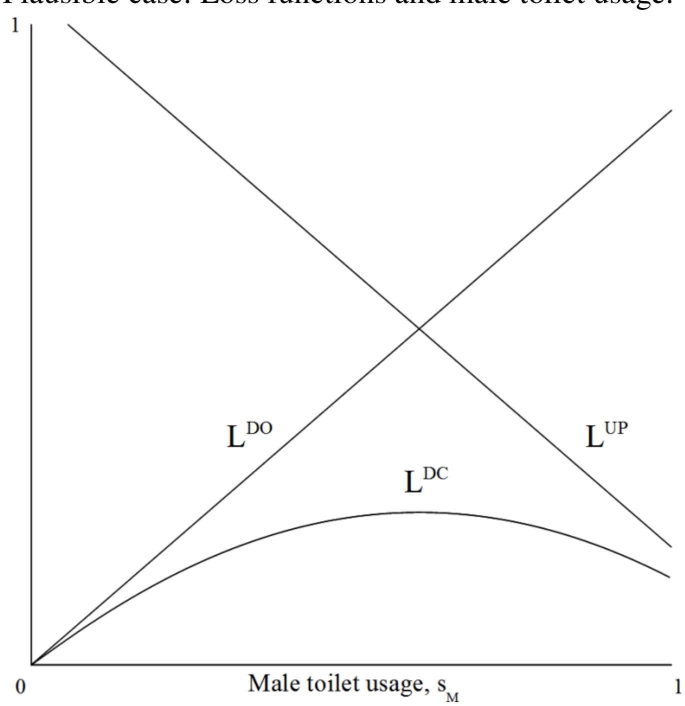



FIGURE 5
Plausible case: Loss functions and male importance.

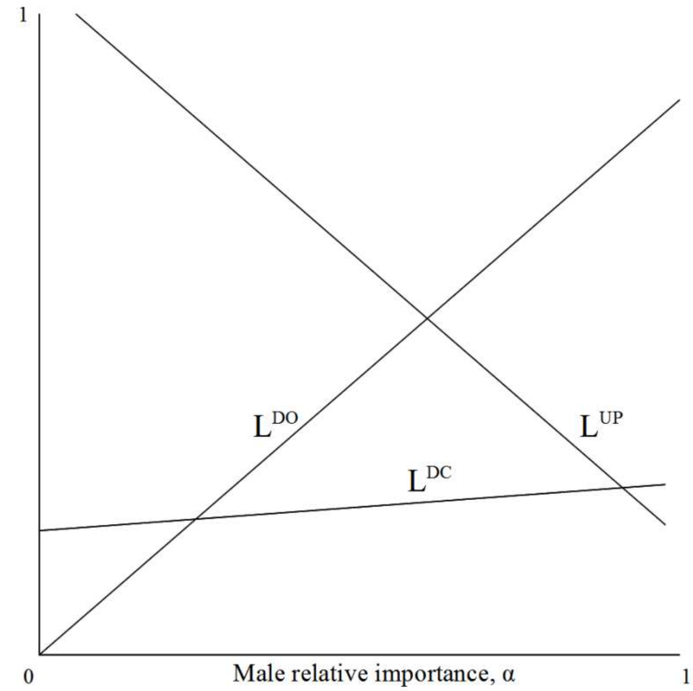
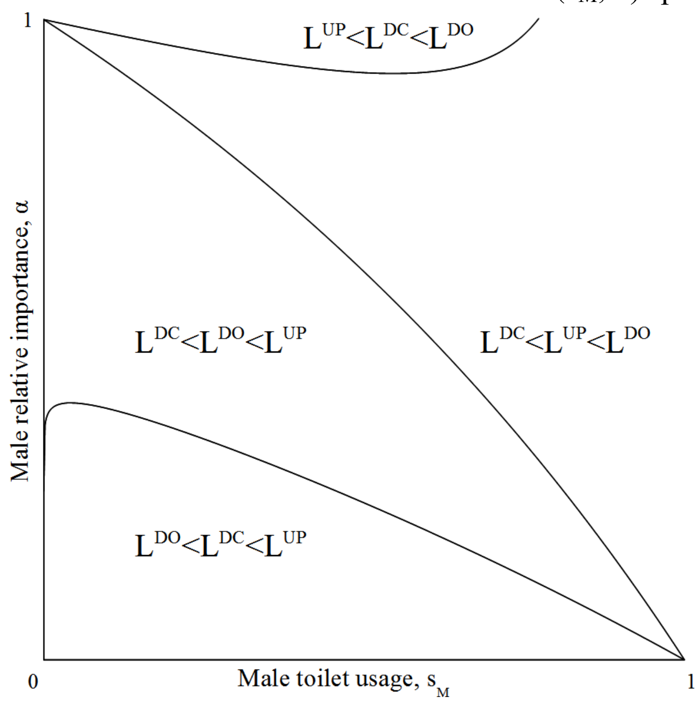



FIGURE 6
Plausible case: Rules dominance areas in the ($s_M$, α) space.




As in the basic case, figures 4-6 show that the DC rule is dominating for many intermediate values of parameters. The effect of having $\delta>0$ is to create an asymmetry between the UP and DO rules: it makes the UP rule dominance area smaller. Even in the case of a toilet that is used only by males, $s_M$=1, the DC rule results more efficient, provided that $\delta$ is large enough. The effect of $\beta$>1 is to enlarge the area of the DC dominance. It makes the expected loss under the DC rule smaller than the loss under the other rules, for a wide parameters space. Finally, as in the basic case, also under this plausible case the conclusion of the model is that the DC rule is the most efficient one if parameters take intermediate values. Moreover, the DC dominance area is even larger than in the basic case.

## IV. CONCLUSION

A loss function minimization approach is employed in order to model the cost of sharing a toilet and the socially optimal rule is proved to depend on parameters. Four parameters are taken into account: the frequency to which males use the seat down, the cost elasticity with respect to number of seat movements, the gender relative importance and the gender relative toilet usage. The optimal rule depends on the combination of these four parameters. Three basic rules are considered: keeping the seat up (UP rule), keeping it down (DO rule) and a “don't care” rule (DC). The focus on these three rule is due to the fact that one of them is likely to be adopted in practice for their simplicity. Moreover, it can be shown that one of the rules analyzed is always dominating any other possible rule. The general result of the model is that the socially optimal rule is the one providing the lowest relative increment in average cost. While this result is intuitive, the parameters space under which one rule is better than the others is not immediately apparent. Two particular cases are presented in order to show how parameters affect the choice of the optimal rule: a basic case and a plausible case. In the basic case, the frequency to which males use the seat down and the elasticity of cost with respect to the number of seat movements are set respectively equal to zero and equal to one. This simplification implies ignoring their effect. Thus, the optimal rule is just the one that minimizes the number of seat movements depending on the gender relative importance and the gender toilet usage. Graphical analysis shows that the DC rule is dominating for many intermediate values of parameters. This case is in line with Choi (2002, 2011) and with Venkataraman (1999). In the plausible case, two specific values for the frequency to which males use the seat down and the cost elasticity with respect to number of seat movements. The first parameter is set considering what is usually considered a normal habit in the medical science. Having no evidence on the second parameter, we argue that it is reasonable to set it slightly greater than one since the second toilet seat movement requires a higher commitment. The parameter is set such that the second seat movement entails a 10% higher cost than the first. As this is a conservative value, it does not affect much the result. Graphical analysis highlights the effect of the two parameters. Considering that males use the seat down with a certain frequency, the first parameter introduces a source of asymmetry between genders; while assuming that the cost increases more than proportionally with number of seat movements, the effect of the second parameter is to widen the space under which the DC rule is optimal. The conclusion is again that the DC rule is dominating for many parameters intermediate values, but the DC dominance area is wider than in the basic case. Finally, all the results presented in the paper show that the DC rule is the most efficient one, provided that genders importance is comparable and if the toilet usage is similar. Inasmuch as this is realistic, as we claim, then the DC rule is the best rule to choose in order to minimize the aggregate cost of two genders sharing a toilet. Furthermore, the DC rule is the only one that asks no commitment which implies that cheating is unlikely, thus its adoption represents a stable cooperative equilibrium.

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## APPENDIX

In this appendix we provide the analysis of a fourth possible rule, the one movement rule (OM). Under this rule, any toilet operation simply requires one seat movement, to be done either before or after usage; as a consequence the effort is always equal to one and $\delta$ can be ignored. This simplifies the analysis since the parameter $\beta$ can be ignored as well: the loss under OM rule is independent from $\beta$ and $\delta$. The loss function for the OM rule depends only on $\alpha$ and $s_M$ and is given by the following expression:

(A1) $$L_c^{OM} = \alpha s_M + (1-\alpha)(1-s_M)$$

Note that the OM rule can be thought as the certainty equivalent of the DC rule worst case of perfect alternation of toilet operations. Hence, the DC rule is expected to dominate the OM rule. With respect to the UP and DO rules, we expect the OM rule to be dominating if $\beta$>1, since it lowers the cost efficiency of the former rules. Comparing equation (A1) with the other loss functions yields the following conditions:

(A2) $$L_c^{OM} < L_c^{UP} \Leftrightarrow \alpha s_M \left(1-\delta 2^{\beta}\right) > (1-\alpha)(1-s_M)\left(2^{\beta}-1\right)$$

(A3) $$L_c^{OM} < L_c^{DO} \Leftrightarrow \alpha s_M \left[(1-\delta)2^{\beta}-1\right] > (1-\alpha)(1-s_M)$$

(A4) $$L_c^{OM} < L_c^{DC} \Leftrightarrow \alpha s_M \left[1-(1-\delta)(1-s_1)^{\beta}-\delta s_1^{\beta}\right] < (1-\alpha)(2-s_M)\left(s_1^{\beta}-1\right)$$

The first two conditions, (A2) and (A3), are shown in figure 7, in which $\beta$=1.5 and $\delta$=0.2 in order to have a clear picture. The third condition, (A4), is never verified, implying that DC rule is always dominating OM rule, as expected. The conclusion of this additional analysis is that the

OM rule is dominating the UP and the DO rules for some intermediate parameters values, if and only if β>1. The parametric space over which OM is dominating though is much smaller than that of DC rule. The final result is that, if parameters take intermediate values, the DC is dominating the OM and OM is dominating the UP and the DO rules.

FIGURE A1
One movement rule: Rules dominance areas in the ($s_M$, α) space.