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Bazhanov, Andrei

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Andrei V. Bazhanov*†‡

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*Institute of Mathematics and Computer Science, Far Eastern National University, Vladivostok, Russia, e-mail: bazhanov@imcs.dvgu.ru or Department of Economics, Queen's University, Kingston, ON K7L 3N6, Canada, phone: 1-(613)-533-2270, fax: 1-(613)-533-6668, e-mail: bazhanov@econ.queensu.ca
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Abstract A normative analysis of the problem of optimal extraction of a non-renewable resource is considered. The economy depends on the essential non-renewable resource and the rate of the resource extraction increases over time. At some instant the government gradually switches to a sustainable (in sense of non-decreasing consumption over time) pattern of the resource extraction. Different approaches are offered for the construction some curves of switching to decreasing paths of the resource depletion. Consumption paths have diverse behavior patterns along these curves, including a path of unlimited growth. A new approach to the Rawlsian maximin criterion which allows for growth of consumption is offered.

Keywords Non-renewable resource · Intergenerational justice · Hartwick rule · Optimal path of extraction · Generalized Rawlsian criterion

JEL Classification Numbers Q32 · Q38

1 Introduction

Theories are developing, governments are changing, and the criteria for the optimality of growth can change from time to time. Wassily Leontief [27] described the dynamic inconsistency of economic growth as follows: "...while each step, being determined by a conscious act of choice, satisfies certain maximizing conditions, ... sequence as a whole does not. Its path can be compared to the course of a dog running across a field toward his master, while the master walks along the road. The dog's path will usually describe a gentle arc, while the fastest way of joining his master would be to run along a straight, properly aimed intercepting line". In a number of applications we present examples, for which we will observe "regime shifts" where we must switch to a different path which is optimal according to new and different criteria or to different constraints. The problem of finding the optimal transition paths is standard in highway or railroad construction and in other engineering applications (see, e.g., [22]). The problem also is associated with so-called transition economics, as in [5], "...excess speed of closure" the public enterprises for developing private sector "...may slow down transition because of output contraction effects". And of course the construction of a transition path toward a sustainable path is a concern of resource and environmental economics. For example, C. Fisher, C. Withagen, and M. Toman [11] analyze a simple model which captures the main effects of the properties of a transition path from "dirty" to "clean" (but more costly) technology. W.D. Nordhaus in his works (see, e.g., [34]) considers a problem of optimal (in sense of utilitarian criterion) transition to economy with less emission of greenhouse gases for the case of a Cobb-Douglas aggregate technology. K. Farmer and R.Wendner [10] consider responses in a general two-sector model to parameter changes (policy shock). Of interest is that transition to the steady state can exhibit qualitatively different behavior (damped oscillation besides overshooting) for the model with heterogeneous capital in comparison with the case of homogenous capital.

We are going to consider the problem of transition path construction for a case of changing the growth path for world oil extraction for the Solow [43] model. Assume that we are not sure if it is possible to find perfect substitutes for non-renewable resource use and come up with alternative technologies during the period of the resource abundance. Then we must consider the problem of construction the optimal resource extraction path from the point of view of intergenerational justice. We show the existence of such a path in an example drawn from the class of rational functions. In the future we are going to examine when it might be appropriate to use such kind of curves as a way of switching smoothly to a sustainable extraction path (for example, Hartwick's curve). The aim of this paper is to examine consumption behavior along the transition path itself. The main result (Proposition 2) shows that depending on parameters of the path, consumption can decline to zero in finite time, asymptotically approach a non-negative constant, or grow to infinity. The second case we argue is "asymptotically optimal" in sense of Rawlsian maximin principle. There have been numerous attempts to reconcile Rawls's idea of supporting the least advantaged with the possibility of economic growth. Also there are other criteria which imply different patterns of consumption growth as the optimal behavior[1]. We present a new modification of Rawlsian criterion (Generalized Rawlsian Criterion, GRC) which implies a different interpretation of the "relevant social positions" for the comparison of persons or generations, and as a result implies growth of consumption. The main idea does not contradict various versions of the maximin criterion for intergenerational problems and we

[1]Different approaches to reconcile Rawls' principle with economic growth can be found, e.g., in [1], [6], [15], [35], [25], [26], [21]. One of the latest reviews on this subject can be found in a paper of N.V.Long [29]. He introduces the weighed Utilitarian-Rawlsian Criterion

can deal with overlapping generations or maximin in combination with the utilitarian criterion and possibly other desiderata. The basic assumption of our approach postulates that utility must depend not only on a current vector of static indicators of consumption but also on the prehistory (derivative) of individual consumption. History is involved in the personal estimation of an individual's consumption level. This postulate can be applied either to problems of intra- or intergenerational justice and so apparently it can help to solve the anomalous situation, namely when very attractive and plausible principles can not be extended (as Rawls confirms by himself [38], p.291) to intertemporal situations. We show with simple examples that GRC-optimal consumption can exhibit limited growth (or limited decline) and unlimited growth depending on the properties of the utility function. Thus, in general, transition paths of essential resource extraction can be adjusted to different optimality criteria but the constraints represented by difficulties in changing from oil-based technologies and from existing patterns of saving do not allow the system to switch rapidly. Since we do not know the estimations of parameters of such constraints, we do not consider them in our examples.

The social planner solves the Gray-Hotelling problem [13],[19] of maximization of the total utility from the resource use during the finite period T of the resource existence:

$$\int_0^T U(r(t))dt \to \max_{r(t)}.$$

For the case of the resource essential to production Solow [43] considered Rawlsian justice principle for the resource allocation between generations as a limiting special case of utilitarian criterion:

$$\max_{r(t)} \min_{t\in(0,\infty)} U(r(t)),$$

which leads to requirement of maximum constant consumption over time.

John Hartwick [17] showed the savings for the Solow [43] model must involve investing current exhaustible resource returns in reproducible capital in order to maintain constant per capita consumption over time. We review this for the case of a Cobb-Douglas technology. For the case with zero population growth, no capital depreciation, no technological progress, and zero extraction cost, we have output $q = f(k, r) = k^\alpha r^\beta$ where k is produced capital, r - current resource use, $r = -\dot{S}$, S - resource stock, α, $\beta \in (0, 1)$ are constants. Prices of capital and the resource are $f_k = \alpha \frac{q}{k}$, $f_r = \beta \frac{q}{r}$. Per capita consumption is $c = q - \dot{k}$. The Hartwick savings rule implies $c = q - r f_r$ or, substituting for f_r, $c = q(1 - \beta)$, which means that instead of $\dot{c} = 0$ we can check $\dot{q} = 0$.

From Hotelling rule $\frac{\dot{f}_r}{f_r} = f_k$ we have $\frac{\alpha\beta q}{k} + \frac{\dot{r}}{r}(\beta - 1) = f_k = \alpha \frac{q}{k}$ which in turn yields

$$\frac{\dot{r}}{r} = -\frac{\alpha q}{k}. \tag{1}$$

Then

$$\frac{\dot{q}}{q} = \alpha \frac{\dot{k}}{k} + \beta \frac{\dot{r}}{r} = \beta \left(\frac{\alpha q}{k} + \frac{\dot{r}}{r}\right) = 0, \tag{2}$$

which means that we really have $\dot{q} = \dot{c} = 0$.

Since from (2) $q = const$ and then $r f_r = \beta q = const$, we have $\dot{k} = \beta q = const$ for deriving $k(t)$ and we have (1) for $r(t)$. We can find two constants of integration k_0 for $k(t) = k_0 + \beta q t$ and the constant of equation

$$\frac{\dot{r}}{r} = -\frac{1}{\frac{k_0}{\alpha q} + \frac{\beta}{\alpha} t}$$

using initial conditions $r(0) = r_0$ and $s(0) = s_0$, where s_0 is the given resource stock which must be used for production over infinite time: $s_0 = \int_0^\infty r(t) dt$. Then we have

$$r(t) = r_0 \left[1 + \frac{r_0 \beta}{s_0(\alpha - \beta)} t\right]^{-\frac{\alpha}{\beta}}, \tag{3}$$

where $\alpha > \beta$ (Solow condition) and

$$\dot{r}(t) = -\ddot{s}(t) = -\frac{\alpha r_0^2}{s_0(\alpha-\beta)}\left[1+\frac{r_0\beta}{s_0(\alpha-\beta)}t\right]^{-\frac{(\alpha+\beta)}{\beta}}. \tag{4}$$

Since we assume that our economy depends on the resource essentially, we obtain path $r(t)$, asymptotically approaching zero and the path of extraction changes $\dot{r}(t)$ (or negative acceleration of stock $s(t)$ diminishing) also approaching zero, but starting from the negative value $\dot{r}_0 = -\frac{\alpha r_0^2}{s_0(\alpha-\beta)}$.[2]

Assuming that our economy has some "additional" savings, besides resource rent, it is possible to relax the assumption of zero population growth (as in J. Stiglitz [44] and G. Asheim, W. Buchholz, J. Hartwick, T. Mitra, and C. Withagen [3] papers), or zero capital depreciation. But in any case, if we assume, that

1) economy at every instant of time depends on resource (even if we gradually introduce substituting technologies and this dependence asymptotically approaches zero), and

2) we really want to maintain nondecreasing per capita consumption,

then rate of extraction $r(t)$ must tend to zero.

Capital - resource substitution is a fundamental topic in energy economics and there are some empirical evidences (see, e.g., [33], [36]) which can support the assumption that the elasticity of substitution between natural resources and capital exceeds unity. This implies that resource can be inessential. Though other investigations (e.g., [12], [30], and partly in [16]) show that energy and capital are rather strong complements than substitutes (elasticity is less than unity) and some researches find that this value is rather close to unity (e.g., [14],

[2]Path (3), asymptotically approaching zero, is necessary, but not sufficient condition of following Hartwick rule for Cobb-Douglas economy under the Hotelling rule assumption. By definition of $f(k,r)$ it can be seen, that if economy is extracting resource in accord with (3), and resource rent is consuming (total investments are less than resource rent), then $q(t)$ and $c(t)$ are asymptotically approaching zero, but from a greater starting value $c(0)$.

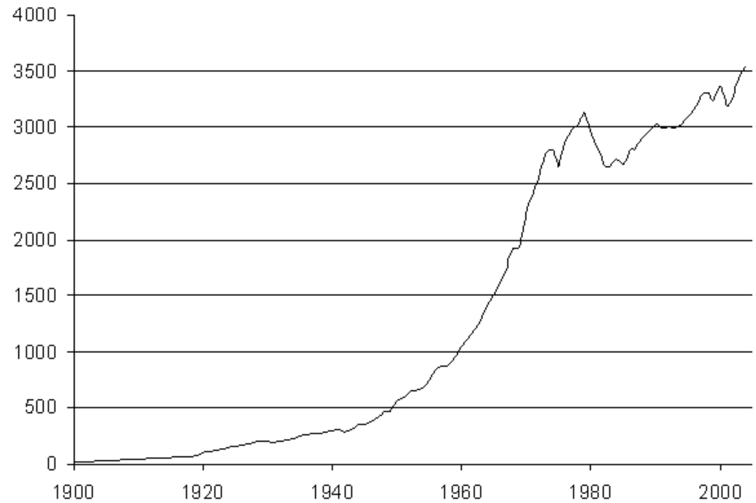

Figure 1: World oil extraction (mln.t./year)

[36]). In any case empirical evidences are not a proof and as P. Dasgupta and G. Heal noted "Past evidence may not be a good guide for judging substitution possibilities for large values of k/r"([7], p. 207). And so, we can assume that for the world economy oil is essential, especially taking into account that no adequate immediate substitutes are available for transportation fuels, a main area of oil use (see, e.g., [18], [31]). However, as we can see, for example, from oil extraction data in December issues of Oil and Gas Journal, rates of extraction are in fact both growing on the world level (see Fig. 1) and for the leading oil producers, not declining.

Assume that the government of our economy after period of oil-rent consumption and growing rate of extraction decided to conform to the intergenerational justice principle and switch at t_0 to some sustainable path of saving, e.g., to the Hartwick rule. An example with $\alpha = 0.3$ and $\beta = 0.05$ gives us behavior of $r(t)$ and $\dot{r}(t)$ for world oil extraction in Fig. 2 and Fig. 3.

An abrupt switch to the Hartwick rule means that people in oil-producing countries must instantly forget about this "additional" source of income and in a moment substantially re-

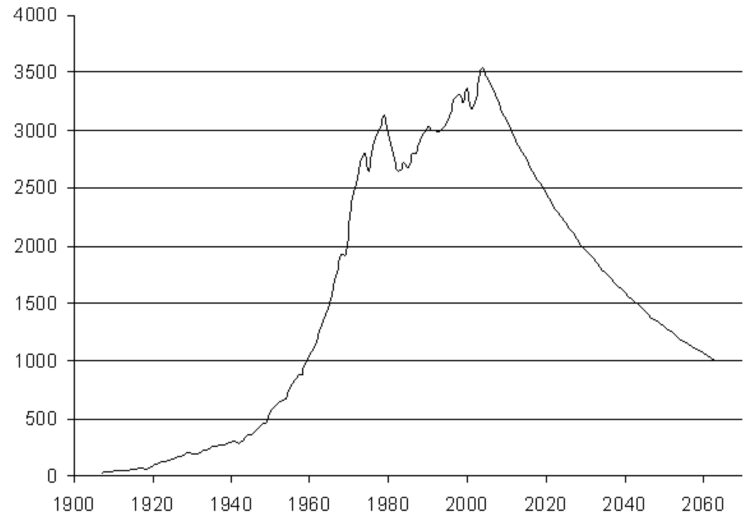

Figure 2: Historical data and Hartwick curve

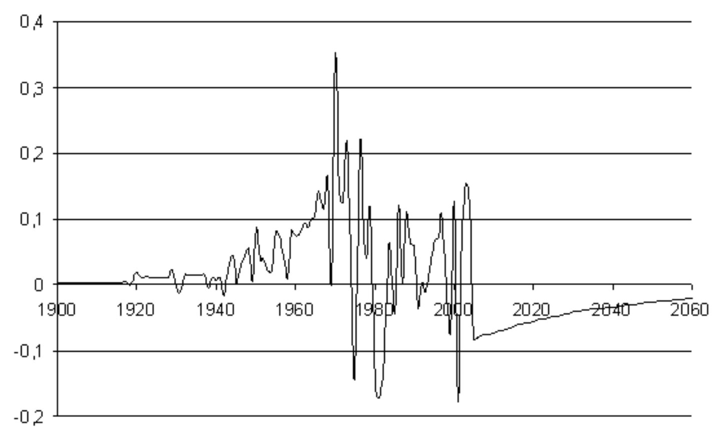

Figure 3: Accelerations for historical data and for the Hartwick curve

structure their living style. Moreover, countries must instantly reorganize their economies, because of the sharp decrease of consumption, which in turn leads to a decrease in production, a possible increase in unemployment, and a further decrease of demand and so on. Thus, for an economy not following the Hartwick rule, the sudden invocation of intergenerational justice creates the dilemma of choosing between two awkward futures: diminishing consumption to zero in the future because of the inevitable and increasing shortage of essential exhaustible resources or diminishing consumption to a sustainable level right from the moment of switching to the Hartwick rule.

Solow's model implies that oil-rent is invested from the very beginning and that there is no time gap between the moment of oil extraction and correspondent increase of reproducible capital according to the Hartwick rule. We can consider it as an adequate model if we assume that reproducible capital is a fund of some high-return securities and oil profit can be instantly invested in some shares or bonds. But suppose that money bills are not able to substitute gasoline in engines of our cars when we have shortage of oil. And the shortage will be the inevitable result of growing demand because of economic growth and decreasing, according to (3), supply of oil. It means, that in order to sustain non-decreasing output with the same structure, we must invest at least part of oil profit into development of oil-substituting technologies. In other words, we must create an "anti-oil market" with the oil rent. And under this assumption the model of instant investment can not be really adequate because of the difficulties of a rapid re-structuring. Historical examples show that the development and the introduction of coal-based technologies took decades despite the obvious benefit of the new technologies for economy. The same can be said about the switch from a coal to an oil economy. Now we must consider the problem of switching to technologies, based on renewable resources not because they are economically more preferable but just because of anticipated shortage of profitable but exhaustible raw materials. And this process will occur over decades, not months.

The second dimension of the impossibility of an instant switch to the Hartwick rule is the awkward requirement of an abrupt and very substantial change of saving patterns for oil producing countries. As an illustration we can compare non-renewable resource profit only from oil with the total amount of investments for a selection of countries. For example, oil gives Kuwait about 50% of GDP but gross fixed investments are only 6.6% of GDP. For Saudi Arabia these numbers are 45% and 16.3%, United Arab Emirates - 30% and 20.7%, Venezuela - 33% and 23.8%.[3] From leaders of oil producers only Norway can boast almost coinciding numbers (about 18.6%[4]), because of investing oil rent to Petroleum Fund.[5]

However, the well-known empirical research of Simon Kuznets [24] tells us that consumer behavior is very persistent over time despite changes of governments and government policy. Subsequent analyses, for example, the work of Duesenberry [9], tried to explain this phenomenon, and later papers examined why consumers do not react on "natural experiments" such as the Reagan cuts in taxes [37]. In any case, there is evidence that at least in the short run saving rate is very stable, and it is much more difficult to change it instantly, than to change a government policy toward maximin.

Hence, the problem of switching to sustainable path of essential resource extraction must take into account the next factors:

1) the curve must have a transition period, or period of a gradual slow-down in the rate of extraction;

[3] Source of information:
http://www.cia.gov/cia/publications/factbook/docs/profileguide.html (March 2006)

[4] Source of information: http://www.ssb.no/en/indicators/ (March 2006)

[5] Though there is no direct connection between this Fund and development of oil-substituting technologies.

2) there is a time lag between the moment of resource rent investment and correspondent increase in capital;

3) there is a non-zero period length for changing saving patterns from resource rent consumption to resource rent investment.

In this paper we suppose, for simplicity, that the third problem is already solved (as in Norway), and also we will temporarily neglect the influence of the second factor. So, we will concentrate on the question of the construction the trajectories for the transition period using various optimality criteria and examine consumption behavior along the paths.

2 Formulation of the problem

We have assumed that the technical restrictions do not allow us to change the rates of oil extraction instantly. The government's policy (a criterion of optimal extraction) can be changed much faster than the rates of extraction. The changes in the patterns of savings can also have their own rate since the reasons of these changes do not coincide with the reasons and the mechanisms of the oil-extraction rates changes. These differences in the speeds and in the patterns of changes can cause a deviation from an efficient path of extraction as a result of a policy shock. For example, if we follow the Hartwick savings rule and start to pursue the constant consumption criterion, and at the same time the rates of oil extraction grow, we surely follow an inefficient path of extraction. This is because an efficient path must satisfy the Hotelling rule ([7], pp. 213-219). In turn, Hotelling rule and Hartwick savings rule yield the unique path of extraction (3) which is decreasing for all $t \geq 0$ (Fig. 2).

Hence, if we are going to change policy and we do not want to urge people to change the savings patterns quickly, we inevitably will enter an inefficient path of extraction. In this paper we are going to analyze the case ("the worst case") when some reasons cause the deviation from an efficient path of extraction and we must find the optimal path across inefficient curves. We set down these assumptions below in the definitions 1 - 4, and the Proposition 1.

Definition 1 An *intertemporal program* $\langle f(t), c(t), k(t), r(t) \rangle_{t=0}^{\infty}$ is a set of paths $f(t)$, $c(t)$, $k(t)$, $r(t)$, $t \geq 0$ such that $f(t) = f[k(t), r(t)]$ and $c(t) = f(t) - \dot{k}(t)$.

Definition 2 For positive initial stock of capital and resource $(k_0, s_0) \gg 0$ the set of the programs $F = \{< f(t), c(t), k(t), r(t) >_{t=0}^{\infty}\}$ is *a feasible sheaf* at $t = 0$ and each of the paths $f(t)$, $c(t)$, $k(t)$, $r(t)$ is *a feasible path* if any program $< f(t), c(t), k(t), r(t) >_{t=0}^{\infty}$ from F for all $t \geq 0$ satisfies the conditions:

1) $(f(t), c(t), k(t), r(t)) \gg 0$;

2) $r(t), k(t), c(t)$ are continuously differentiable and $\sup_t |\dot{r}(t)| \leq \dot{r}_{\max} < \infty$;

3) $f(t)$ is twice continuously differentiable;

4) $\int_t^{\infty} r(t)dt \leq s(t)$;

5) $k(0) = k_0, c(0) = c_0, r(0) = r_0, \dot{r}(0) = A_0 \leq \dot{r}_{\max}$.

Definition 1 is based on the definition of the interior feasible path in [3]. The differences reflect our assumptions: a) population is constant; b) the speed of change of the extraction rate $\dot{r}$ is limited and continuous for all t including $t = 0$. Henceforth, a "program" and a "path" will refer to a feasible program and a feasible path.

Definition 3 ([7], p.214) A feasible program $< f(t), c(t), k(t), r(t) >_{t=0}^{\infty}$ from F is *intertemporally inefficient* if there exists a program $< \overline{f}(t), \overline{c}(t), \overline{k}(t), \overline{r}(t) >_{t=0}^{\infty}$ from F such that $\overline{c}(t) \geq c(t)$ for all $t \geq 0$ and $\overline{c}(t) > c(t)$ for some t.

Definition 4 ([7], p.214) A set of feasible programs $E = \{< f(t), c(t), k(t), r(t) >_{t=0}^{\infty}\}$ is a *set of efficient programs* if all the programs $< f(t), c(t), k(t), r(t) >_{t=0}^{\infty}$ from E are not inefficient.

Proposition 1 *If* $\dot{f}_r(0)/f_r(0) \neq f_k(0)$ *then* $F \cap E = \emptyset$.

Proof. Since $f(t)$ is twice continuously differentiable at $t = 0$, then there exists $\varepsilon > 0$ such that for any $t \in [0, \varepsilon)$ and for any feasible program $< f(t), c(t), k(t), r(t) >_{t=0}^{\infty} \in F$ the Hotelling rule is not satisfied: $\dot{f}_r(t)/f_r(t) \neq f_k(t)$. Necessity of the Hotelling rule for the efficiency of a program (see, e.g., [3], [7]) follows the assertion of the Proposition.

Since there is a mutual dependence of GDP percent change and oil extraction and supply, we can try to construct a path of extraction which asymptotically approaches zero and minimizes the maximum negative shock represented by GDP percent change. Generally, the technical aspects of a transition path must be considered as restrictions on the optimization problem. But for simplicity we can suppose that these restrictions are satisfied along the optimal path or, in other words, they are inactive. For the case when the optimal path violates some of the restrictions, we can project it on the set of feasible solutions (method of constraints relaxation).

According to (2) GDP percent change for our economy is

$$\frac{\dot{q}}{q} = \alpha \frac{\dot{k}}{k} + \frac{\beta}{r} \dot{r}.$$

If in the first period all resource rent was being consumed, then $\dot{k} = 0$ and

$$\frac{\dot{q}}{q} = \frac{\beta}{r} \dot{r}. \tag{5}$$

If we assume that in the second period oil rent is invested in oil-substituting technologies, then there is a time lag between the moment of oil extraction and the moment of capital increase. So, there must be a non-zero time period when despite the investment of oil rent in reproducible capital, output q satisfies (5).

Assume that the government "does its best" and manages to extract all oil profit from consumption and completely invests it in long-return technologies. Then, according to (4), in the beginning ($t_0 = 0$) of period 2 GDP percent change is

$$\left(\frac{\dot{q}}{q}\right)_0 = -\alpha\beta\frac{r_0}{s_0(\alpha-\beta)}. \tag{6}$$

For $\alpha = 0.3$, $\beta = 0.05$, and world oil reserves and extraction on January 1, 2005:[6] $r_0 = 70,899$ [1,000 bbl/day] $\times 365 = 25,878,135$ [1,000 bbl/year] (or 3.54495 bln t/year); $s_0 = 1,277,701,992$ [1,000 bbl] (or 175.0277 bln t[7]) we obtain the aggregate decline $\left(\frac{\dot{q}}{q}\right)_0 \approx -0.0012$ or -0.12% (annual).[8]

In this situation, when the growth of output in period 1 was based on non-renewable resource consumption, we can not already speak about intergenerational equity, because future generations will be worse off in any case - due to approaching shortage of the depleting resource, or because of the switch to the sustainable pattern of consumption. But we can consider the question of mitigating the negative consequences of the switch to a sustainable path. The amount of output decline in the second period, according to (5), is defined by the value of negative acceleration in the process of switching to the sustainable approach to extraction. Then we can try to find a path, which incorporates the gradual switch or "smooth breaking" of growing extraction, and along which the peak of negative acceleration $A(t) = \dot{r}(t)$ is minimal

[6] Worldwide Crude Oil and Gas Production // Oil&Gas Journal, Dec. 12, 2005, p.72.

[7] We use coefficient 1 ton of crude oil = 7.3 barrel.

[8] For growing rate of extraction $r(t)$ and diminishing $s(t)$ in the first period, the "cost" of switch to Hartwick rule, namely, r_0/s_0, which defines the value of negative shock for $(\dot{q}/q)_0$ in (6), is increasing over time.

in absolute value. So we can consider the problem of finding such a function $r^*(t)$, for which

$$\begin{aligned}
\min_t A^*(t) &= \max_{r(t)} \min_t A(t), \\
s.t. \quad r(0) &= r_0, \\
\int_0^\infty r(t)dt &= s_0,
\end{aligned} \tag{7}$$

where the last condition means that resource is essential.

3 Solving the problem

A solution of (7) can be found in the same class of rational functions as the Hartwick curve (3). The difference is in the numerator, which must depend on t with a different (negative) coefficient to control "smooth breaking" in the neighborhood of $t = 0$. Namely, we must find $A(t)$ in the form of

$$A(t, b, c, d) = \frac{A_0 + bt}{(1 + ct)^d}, \tag{8}$$

where $b < 0, c > 0, d > 1$ (for convergence $A(t) \to -0$ with $t \to \infty$), and then problem (7) resembles a problem of finding such b^*, c^*, d^*, that

$$\begin{aligned}
\min_t A(t, b^*, c^*, d^*) &= \max_{b,c,d} \min_t A(t, b, c, d), \\
s.t. \quad r(0) &= r_0, \\
\int_0^\infty r(t)dt &= s_0.
\end{aligned} \tag{9}$$

The first order condition on t gives us (taking into account $(1 + ct) > 0$)

$$t^* = \frac{A_0 cd - b}{bc(1 - d)}$$

and the minimum value of A (maximum negative acceleration) is

$$A(t^*, b, c, d) = \frac{b}{cd} \cdot \left[\frac{b(1-d)}{d(A_0 c - b)} \right]^{d-1}. \tag{10}$$

Corresponding to (8) $r(t)$ has a dependence on $b, c,$ and d in[9]

$$r(t) = \left\{ -\frac{1}{c(d-1)} \left[A_0 + \frac{b}{c(d-2)} \right] + \frac{b}{c(2-d)} t \right\} / (1+ct)^{d-1},$$

then $r_0 = -\frac{1}{c(d-1)} \left[A_0 + \frac{b}{c(d-2)} \right]$, which can be used to express b :

$$b = -c(d-2) \left[r_0 c(d-1) + A_0 \right], \tag{11}$$

and then the least acceleration (LA) curve has a dependence on c and d in

$$r(t) = r_0 \left\{ 1 + \left[c(d-1) + \frac{A_0}{r_0} \right] t \right\} / (1+ct)^{d-1}. \tag{12}$$

Coefficient c can be expressed from the condition that resource is finite and essential $s_0 = \int_0^\infty r(t) dt$:

$$\begin{aligned} \frac{s_0}{r_0} &= \int_0^\infty (1+ct)^{1-d} dt + \left[c(d-1) + \frac{A_0}{r_0} \right] \int_0^\infty \frac{t}{(1+ct)^{d-1}} dt \\ &= \frac{1}{c(d-2)} + \frac{r_0 c(d-1) + A_0}{r_0 c^2 (d-3)(d-2)}, \end{aligned}$$

which means that c is a solution of quadratic equation

$$\frac{s_0}{r_0} c^2 - \frac{2}{d-3} c - \frac{A_0}{r_0 (d-3)(d-2)} = 0.$$

The only relevant root (because we are looking for $c > 0$) is

$$c(d) = \frac{1}{s_0} \left[\frac{r_0}{d-3} + \sqrt{\frac{r_0^2}{(d-3)^2} + \frac{s_0 A_0}{(d-3)(d-2)}} \right]. \tag{13}$$

[9] Constant of integration for $\dot{r}(t) = A(t)$ must be zero for the convergence of $\int_0^\infty r(t) dt$, and also for the convergence note, that d actually must be greater than 3.

Substituting (13) into (11) and then from $b(d)$ and $c(d)$ in (10) we have a dependence of the minimum value of $A(t^*)$ on d :

$$A(t^*) = -\left[r_0 c(d)(d-2) + A_0\right] \cdot \left\{\frac{(d-2)[r_0 c(d)(d-1) + A_0]}{d[r_0 c(d)(d-2) + A_0]}\right\}^d$$

Denote $f(d) = \{\cdot\}$. Then the first order condition on d is:

$$A'_d = -r_0[c'_d(d)(d-2) + c(d)]f(d)^d - [r_0 c(d)(d-2) + A_0]f(d)^d\left[\ln f(d) + d\frac{f'_d(d)}{f(d)}\right] = 0.$$

Note, that $f(d) > 0$, because $d > 2, r_0 > 0, c > 0, A_0 > 0$. Then by dividing this equation through by the $-r_0 f(d)^d$ we have the equation for d:

$$[c'_d(d)(d-2) + c(d)] + \left[c(d)(d-2) + \frac{A_0}{r_0}\right]\left[\ln f(d) + d\frac{f'_d(d)}{f(d)}\right] = 0. \tag{14}$$

A numerical example based on data for recent world oil extraction gives a single positive root of equation (14) d =12.845. Then c =0.00527385, b =-0.019793 and plots of $r(t)$ and $A(t)$ are on Fig. 4 and Fig. 5.

Maximum negative acceleration along the LA curve is A^*_{LA} =-0.07604549 at t^* =22.81939, which is less in absolute value than maximum negative acceleration of the Hartwick curve A^*_H =-0.08466 right from the very start at $t^* = 0$.

4 Consumption Along Transition Curves

We are going to examine, for simplicity, the case when all resource rent is always invested in capital and there are no time lags between the moments of investment and the corresponding capital increase. The only reason for change the pattern of extraction is that sustainable (in sense of constant consumption) path of the essential resource extraction must be decreasing and asymptotically approaching zero.

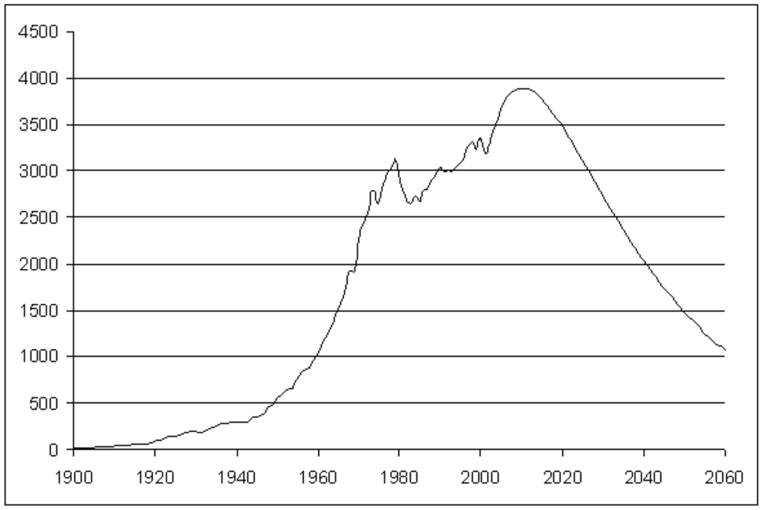

Figure 4: The least acceleration curve of the world oil extraction (from 2005)

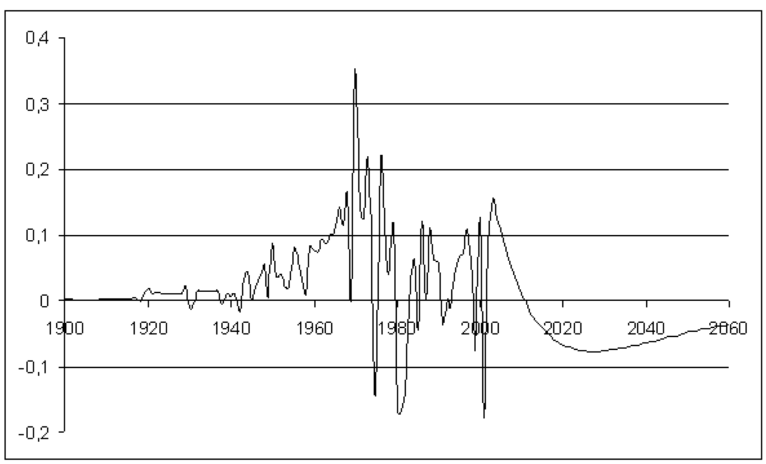

Figure 5: Accelerations of oil extraction along the LA curve

Note, that constant per capita consumption is the result of

1) total investment of oil rent in capital (with no time lag) and

2) fulfillment of the Hotelling rule.

The LA curve (12) satisfies only the first condition unlike the Hartwick curve (3) which is derived from the Hotelling rule and so satisfies it identically. Hence, to examine consumption behavior along some path we should check the fulfillment of the Hotelling rule along this curve. In common case $\dot{q} = f_k \dot{k} + f_r \dot{r}$. Then $\dot{f}_r = \beta d\left(\frac{q}{r}\right)/dt = \beta\left[f_k \frac{\dot{k}}{r} + f_r \frac{\dot{r}}{r}\right] - \beta \frac{\dot{r}q}{r^2}$. Dividing on $f_r = \beta \frac{q}{r}$ we have $\frac{\dot{f}_r}{f_r} = \frac{r\beta}{\beta q}\left[\frac{\alpha \dot{k}}{kr} + \frac{\beta q \dot{r}}{r^2}\right] - \frac{\dot{r}}{r} = \alpha \frac{\dot{k}}{k} - (1-\beta)\frac{\dot{r}}{r}$ or

$$\frac{\dot{f}_r}{f_r} = f_k \left[\frac{\dot{k}}{q} - \frac{(1-\beta)k\dot{r}}{\alpha q r}\right]. \tag{15}$$

Just to check, we can see, that for the Hartwick curve $[\cdot] \equiv 1$, because $\frac{\dot{r}}{r} = -\frac{\alpha q}{k}$ and $\dot{k} = \beta q$.

Hence, if $[\cdot] < 1$, then $\dot{q} > 0$, because $\frac{\dot{f}_r}{f_r} < f_k$, which follows $-\frac{\dot{r}}{r} < \frac{\alpha q}{k}$ or $\frac{\alpha q}{k} + \frac{\dot{r}}{r} > 0$. And the latter, using expression in the left hand side of (2), means $\dot{q} > 0$. In the same way, $[\cdot] > 1$ follows $\dot{q} < 0$ and, in general, $\operatorname{sgn} \dot{q} = \operatorname{sgn}\{1 - [\cdot]\}$. So, to examine long-run consumption $c = (1-\beta)q$ along the LA curve, we can check asymptotic behavior of $[\cdot]$.

Proposition 2 *If an economy with technology $q = k^\alpha r^\beta$ is such that*

1) resource rent is completely invested in capital;

2) there is no time lag between the moment of investment and correspondent increase in capital;

3) rate of extraction $r(t)$ is such that

$$\dot{r}(t) = \frac{A_0 + bt}{(1+ct)^d}, \; b < 0, \; c > 0, \; d > 3,$$

then the output q asymptotic behavior for different β is:

$$\lim_{t\to\infty} \operatorname{sgn} \dot{q}(t) = \begin{cases} -1, & \beta(d-2) \geq 1, \\ \operatorname{sgn}\left(1 - \frac{|b|(1-\alpha)\beta}{r_0\alpha\rho c[1-\beta(d-2)]}\right), & \beta(d-2) < 1, \end{cases} \tag{16}$$

where $\rho = c(d-1) + A_0/r_0, A_0 = \dot{r}(0), r_0 = r(0)$.

Proof of the proposition is in the appendix.

5 Numerical Examples

For the example with given $\alpha = 0.3, \beta = 0.05, r_0, A_0$ for the world oil extraction, and optimal (in sense of minimal negative output shock) $d^*, b(d^*)$, and $c(d^*)$

$$L(d, \alpha, \beta) = \frac{|b(d)|\,(1-\alpha)\beta}{r_0\alpha\rho c(d)[1-\beta(d-2)]} = 2.764 > 1,$$

which means, that consumption and output decrease in the long run along the LA curve (Fig. 6 and Fig. 7).

For $\alpha = 0.2, \beta = 0.05$ (estimations from [33]) we have $L(d, \alpha, \beta) = 1.543$ which also means decreasing to zero consumption in finite time. We can see from (16) that there are sets of α and β for which, given optimal values of $d^*, b(d^*)$, and $c(d^*)$, we have $L(d^*, \alpha, \beta) = 1$ or consumption tends to a constant along the LA curve. For example, $L(d^*, 0.325, 0.03) = L(d^*, 0.4337, 0.04) = 1$.

Selection of different values for (α, β) for which $\lim_{t\to\infty} \dot{c} = 0$ makes some sense if we wish to get a feeling of how far can some real extraction path be from the stable one, given that we don't know true values of α and β. In fact it is unrealistic to speak about short-term regulation of these magnitudes by the government's decisions. Our examples make the path of resource extraction look more controllable. We can try to fit the single free parameter d and

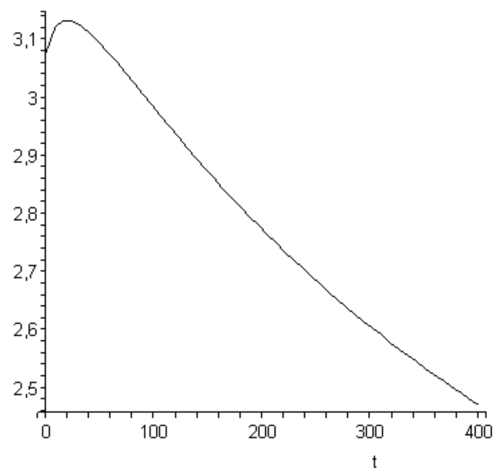

Figure 6: Consumption decrease along the LA curve $t \in (0, 400)$

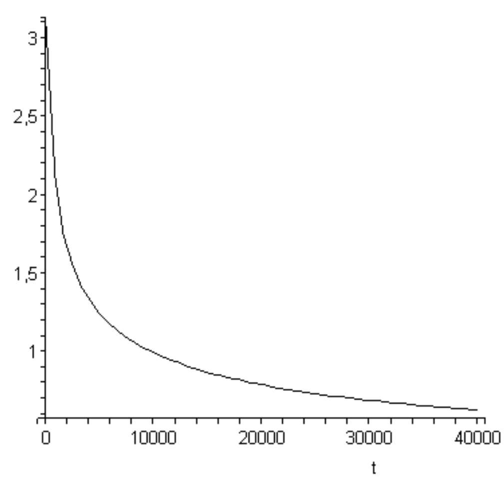

Figure 7: Consumption decrease along the LA curve $t \in (0, 40000)$

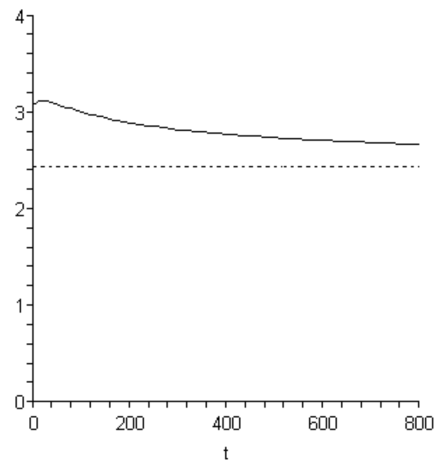

Figure 8: Consumption along the TCC or LD curve

recalculate $c(d)$ and $b(d)$ using our main criterion - constant consumption over time in the long run (asymptotically constant consumption) instead of the least negative output shock during transition period. In other words we should solve a system of equations: $L(d, \alpha, \beta) = 1$ plus equations for $b(d)$ and $c(d)$ ((11) and (13)). An example with $\alpha = 0.3$ and $\beta = 0.05$ gives us $d = 8.0, c = 0.01022, b = -0.023196$. In this case the maximum negative output shock takes place a little bit earlier ($t_{\max} = 20.1$) in comparison with $t_{\max} = 22.82$ for the LA curve; the value of the shock is larger ($A_{\max} = -0.0767$) in comparison with $A_{\max LA} = -0.07605$, but the shock is weaker than for the curve (3), for which $A_{\max} = -0.08466$.

To check that the level of consumption along this curve, which we can call the "Transition Constant Consumption" (TCC) curve, is far enough from zero, we can solve numerically differential equation for $k(t)$[10] and then plot $c(t)$ (Fig. 8). The value of constant consumption for the t, big enough, is around $c_{const} = 2.42801$.

[10]Numerical solution was obtained in Maple by the procedure $rkf45$.

The maximum value along the TCC curve is $c_{\max} = 3.126$ at $t = 16.6$. As we can see, $c_{const} < c_0 = 3.078$,[11] but rather far from zero. Since we have bounded decrease of consumption along this curve, we can call it also the "Limited Decline" (LD) curve.

6 Consumption Growth, Transition Curves, and Generalized Rawlsian Criterion

Another interesting case is the behavior of infinitely growing consumption when $L(d, \alpha, \beta) < 1$. What is the "cost" of this growth? And can it be "optimal" in some sense or is it just a result of overinvestment? An example with $L(7.5671, 0.3, 0.05) = 0.9$ gives us the long-run growing consumption (Fig. 9) with the same c_0, but $c_{\max} = 3.1248$ at $t = 16.2$ and $c_{\min} = 2.6817$ at $t_{\min} = 3035$. Consumption exceeds c_0 after $t = 1.144 \cdot 10^6$. The dash line is the asymptote for the LD curve (Fig. 8).

Negative effects or the "cost" of the long-run consumption growth along this curve, compared with the TCC or LD curve are:

1) $c_{\max}$ is a little bit less, than for the LD curve (3.1248 vs. 3.126);

2) peak of negative shock on output is a little bit stronger (-0.0769 vs. -0.0767) and takes place on 6 months earlier ($t = 19.65$ vs. $t = 20.1$).

The example is an illustration of the answer to the question "what is worse": a small decrease of consumption in the present or the depriving of oneself and (or) one's descendants of any prospects for improving their lives in the future. According to Rawls's maximin principle, it is obviously a pattern of overinvestment. But actually Rawls objected to applying his maximin principle (or as he call it "difference principle") to the questions of justice among generations

[11] G.B. Asheim [2] considers a theoretical example where the consumption decreases to a sustainable level $\overline{c} < c_0$ after a period of "over-consuming" with $c(t) > c_0$.

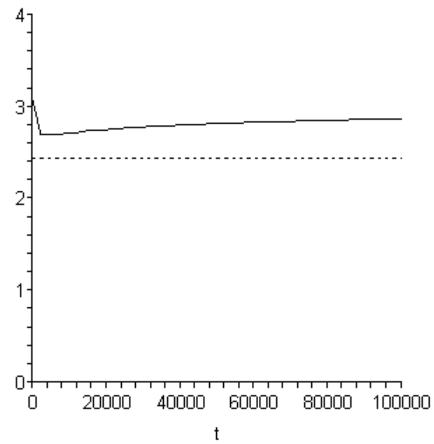

Figure 9: Long-run consumption growth for $L(d, \alpha, \beta) = 0.9$

because of unacceptable consequences: "The principle is inapplicable and it would seem to imply,... that there be no saving at all"([38], p.291). As a solution of the problem Rawls suggests that the difference principle must be restricted by the additional, "just savings principle"([38], p.285). But Rawls challenges the possibility of its construction in a precise form: "I believe that it is not possible... to define precise limits on what the rate of savings should be. How the burden of capital accumulation and of raising the standard of civilization and culture is to be shared between generations seems to admit of no definite answer" ([38], p. 286). And there is a question: why such a plausible and attractive principle for intragenerational questions can not be extended to problems of intergenerational justice? And why we can not deduce the just savings principle from the main principle? Where is the essential source of this contradiction?

According to Rawls it is very important to define precisely relevant positions of persons (or generations) for which we will test different theories of justice: "...selection of relevant positions is necessary for a coherent theory of social justice..."([38], p. 100). And he assumes that "...each person holds two relevant positions: that of equal citizenship and that defined by his place in the distribution of income and wealth"([38], p. 96). Income and wealth (exchangeable goods) are used by Rawls as indicators of relevant position in his successive works also (e.g., [39], p. 58, [40], p.76, p. 181). A.K. Sen [42] has suggested adding some not exchangeable goods such as measures of development of personal capacities, and, as T. Scanlon has offered, "the avoidance of chronic physical pain" ([41], p. 41). But there are a number of contributions supporting the idea that for estimating utility and, consequently, relevant position, it is not enough to calculate some vector of measurable static indicators. "We can ask,... how well a person's life is going and whether that person is...better off than he or she was a year ago" ([41], p. 18). And there is evidence that has "...documented the claim that people are relatively insensitive to steady states, but highly sensitive to changes..." and "...the main carriers of value are gains and losses rather than overall wealth" ([20], p. 148). We just can consider, e.g., two persons with identically the same level of consumption and other not exchangeable goods but the first one was a billionaire, who lost her fortune just yesterday and the second was a poor peasant, who improved her live due to a good luck or (and) diligence and a good education. Each will surely evaluate the quality of her life currently differently. And having no information about her position in society ("behind a veil of ignorance" [38], pp. 136-142), it looks quite plausible that any person will choose a theory according to which she will count on a maximum support from society in a most desperate situation and this "reflex" does not necessarily imply that she would base her "claim" on the worst static indicators in the current period.

An important element in Rawls's approach to defining the least advantageous person is that actually she is not a single person but rather a representative of the "least fortunate group"([38], p. 98). Applying the theory absolutely in the same way to different generations (time-component of the theory) we should consider the least advantageous generation as represented by a person who stands for the group in the "least fortunate period". This means that the person must live during some finite period of time and she doesn't know how long this period is. Alternatively if we consider this person as a model of all generations then we can assume that she lives infinitely. In either case it must be *the same* person; in the same way as when we compare utilities of contemporaries, we pick up them from *the same* time period. When we consider the dimension "people" we don't consider the dimension "time" and vise versa. And since estimation of a person's utility depends on her "progress", and only her "prehistory part" of this progress is available to affect this estimation, the question of savings can be solved within this period without considering representatives of other generations. We want to stress that the question of just savings can (not must) be solved within one generation and this means that we can introduce overlapping generations as an added complication in order to examine additional effects. Indeed there are persons who have no children but they do savings to buy a car or a house, there are families who have children but they frankly think that it will be much better for their children to be self-supporting, and nevertheless they also do savings to improve their own progress or prevent decline. And we can not say that these examples exhibit irrational behavior.

Hence, evaluation of one's life quality should include not only calculations of some static indicators, but rather such indicators combined with time-changes of variables (growth or recession) or differences in consumption from previous years. Assume, for simplicity, that utility from consumption has the form $u(c) = c$. For discrete time and taking into account the evaluation of some prehistory, the generalized Rawlsian criterion maximizes the minimal over all time t combinations of utilities from different periods, e.g., in the form of $w_0 c_t + \sum_{i=0}^{n} w_i (c_{t-i} - c_{t-i-1})$, where $n+1$ is a length of a "memory interval" (each person lives at least $n+1$ periods), the sum $\sum_{i=0}^{n} w_i(c_{t-i} - c_{t-i-1})$ is thought of as the person's memory of earlier utility levels or the "historical benchmark level" ([4] , p. 1257) and distribution of w_i over time ($\sum w_i = 1$, $w_i \in [0,1]$) depends on individual adjustment to changes in consumption. This form of individual welfare is close to K.J. Arrow [1], P. Dasgupta [6], J. Lane and T. Mitra [25], and N.V. Long [28]. The two differences are: 1) we take into account not the consumption of descendants (c_{t+1}) but the past consumption of the same person; 2) we evaluate past consumption not in absolute value but as a component in the process of comparing the past with our present condition (gains and losses). We want now to show that even for such an "egoistic" model, the generalized Rawlsian principle can imply intertemporal consumption growth. Taking into account "past experience" in its simplest form and making use of continuous time and following Rawls strictly[12], the corollary of the Generalized Rawlsian Criterion for intertemporal distribution ($c(t)$ is continuously differentiable) can be written, e.g.[13], as:

$$wc(t) + (1-w)\dot{c}(t) = \gamma = const \quad \text{for any } t,\ w \in [0,1], \tag{17}$$

It means (for $\dot{c} > 0$) that a person in the future with higher consumption and less growth must "feel the same emotional evaluation" or have the same utility level of her "position" as she feels in the present with less consumption and higher value of $\dot{c}(t)$. Note that for $\gamma < c_0 w$ we have a case of generalized "fair Rawlsian decline";[14] a person in the present has not only higher

[12] We assume, according to Rawls, the fulfillment of his first principle ("Each person is to have an equal right to the most extensive total system of equal basic liberties compatible with a similar system of liberty for all") and we will apply the second principle in absolutely the same way ("Social and economic inequalities are to be arranged so that they are...to the greatest benefit of the least advantaged..." [38], p. 302), but maximize the minimal value of some combination $c(t)$ with $\dot{c}(t)$.

[13] The author introduced an example of utility in form (17) independently of N.V.Long ([28], p.17) and $\dot{c}$ here has a different meaning. It is an estimation of person's "prehistory" which influences her evaluation of current consumption, rather then expected future consumption as it is in N.V. Long's interpretation.

[14] We have no proof that present economic growth is not a pattern of overshooting and so we must define not only a just path for growth but also a just path for decline.

level of consumption but also higher rate of decline, than she has in the future, so that the weighting according to (17) yields the same utility for each period. In the long run it reminds the consumption behavior along the LD curve after the point of maximum (Fig. 8).

Expressing $c(t)$ from (17) with $c_0 = c(0)$ we have

$$c(t) = \frac{1}{w}\left[\gamma - e^{-\frac{w}{1-w}t}(\gamma - c_0 w)\right] \tag{18}$$

with

$$\dot{c}(t) = \frac{1}{1-w}e^{-\frac{w}{1-w}t}(\gamma - c_0 w),$$

and then the path of net investment $N(t)$ is a corollary of the Rawlsian "difference principle". For utility in form (17) $N(t)$ was obtained by N.V. Long [28]:

$$N(t) = \dot{c}(t)g(t),$$

where

$$g(t) = \int_t^\infty e^{-\int_t^\tau \left[\frac{w}{1-w} + f_K(s)ds\right]} d\tau > 0.$$

Observe that $\lim_{t\to\infty} \dot{c}(t) = 0$ or we have a case of limited growth (Fig. 10) for $\gamma > c_0 w$ (even without overlapping generations as in [35] and without discounting of maximin as in [21]) and this curve is *desirable* in a sense "...that an extra bit of consumption at t is more valuable than the same *extra* bit at $t+1$, since individuals will, in any case, have more consumption at $t+1$." ([7] p. 284) But observe also that we have limited decline for $\gamma < c_0 w$, and identically constant consumption (as in the Hartwick rule) for $\gamma = c_0 w$.

So, for the LD curve we have the consumption behavior, which in the long run is "close" to generalized "fair Rawlsian decline" in sense that it also represents limited decline. And the

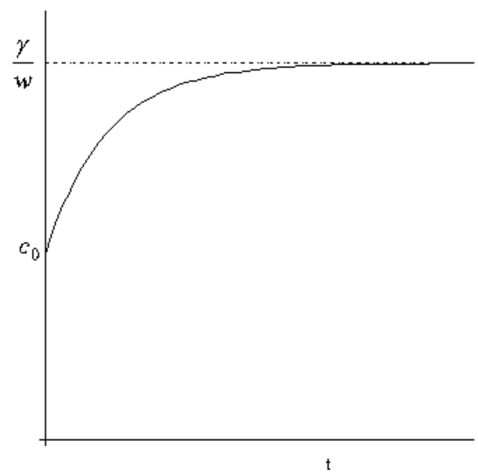

Figure 10: Generalized "fair Rawlsian growth" (18), $w = 0.5$

"cost" of the impossibility of an instantaneous switch to the Hartwick curve is an infinite but limited decline of consumption to the value which is less than c_0. In the sense of (17) this is the path which is "close" to optimal, except for the rate of its decline[15]. The value of w is supposed to be defined by the government. We do not claim that everybody favors this type of just path, particularly when it is apparent that rather small sacrifices in present can bring slow but *unlimited* growth in the long run (Fig. 9).

For those, who prefer this form of intertemporal distribution, the more appropriate consumption utility function would be the function with essential factors and the constant elasticities of marginal utility, e.g., the Cobb-Douglas case. Then the rule of intertemporal distribution is

$$c^w \dot{c}^{1-w} = \gamma = const, \tag{19}$$

which for $w = 1$ is also the corollary of the regular Rawlsian principle. Integration of (19) gives

[15] An exponent approaches an asymptote faster or, in other words, the tail of a rational function is "heavier".

us

$$c(t) = c_0 \left(1 + \mu t\right)^{\varphi} \tag{20}$$

where

$$c_0 = c(0), \mu = \frac{1}{1-w}\left(\frac{\gamma}{c_o}\right)^{\frac{1}{1-w}}, \varphi = 1 - w$$

or a pattern of unlimited (quasi-arithmetic, [3], p. 5) growth which (for w close to 1) looks like the part of the curve on Fig. 9 after the point of minimum.

Utility can be written more generally as a CES function, or as a function with a variable elasticity where the elasticity of factor substitution and w are to be chosen by the government.[16] Then the specific just savings principle can be deduced for the specific utility function and the extraction path (transition curve) can be adjusted to approach as close as possible (depending on constraints) the asymptotically optimal (in the long run) pattern of intertemporal distribution of consumption.

Rawls holds the asymmetry of intergenerational relations to be the reason for being unable to apply his maximin principle: "It is now clear why the difference principle does not apply to the savings problem. There is no way for later generations to improve the situation of the least fortunate first generation. The principle is inapplicable and it would seem to imply, if anything, that there be no saving at all. Thus, the problem of saving must be treated in another fashion" ([38], p. 291). Indeed intergenerational asymmetry influences the process of just intergenerational allocation. That is why Rawlsian just distribution among contemporaries allows inequality in utility unlike intergenerational allocation which implies the same utility (for

[16] And in turn, our moral evaluation of government activity in maintaining some rate of economic growth depends on the proximity of "our own values" to these parameters of the government's choice.

the combination of consumption and "progress") for the same person but generally not the same level of consumption and not a zero net saving rate. So using this approach for a definition of a person's relevant position we "reduce" the problem of just savings to the problem of the choice of a proper form of the utility function.

The choice of utility function involves another important problem, namely the problem of dynamic inconsistency. This means that for an infinitely lived person we do have a rather "good" function like (18) or (20). But if we reformulate the problem as a set of problems each of which is solved by a member of the overlapping or subsequent generations, we (depending on concrete form of utility function) can obtain a path which is continuous, but has points of discontinuous derivatives at the moments of time where another generation chooses its own optimal path using the same approach. And the resulting combination of these paths will generally not coincide with the path of an infinitely lived person. Or if we assume that an infinitely lived person will check and recalculate her path at each moment of time then we will observe that she always changes her preferences and this seems to contradict the usual assumptions of rational behavior. Therefore, for models with always rational agents we must pick a form of the utility function which 1) induces dynamically consistent paths (constraint); 2) in the best way reflects real agents' preferences (criterion). But then our agenda becomes extremely complicated. From our perspective dynamic inconsistency needs a separate consideration.[17] Moreover since preferences can often change over time even for rational agents [8] and in addition we have uncertainties with initial conditions (e.g., estimation of initial stock S_0), the optimal path of extraction will generally need a correction and an adjustment of the

[17] One of the first works on dynamic inconsistency was a paper of R. Strotz [46], for non-renewable resources this phenomena was shown by P. Dasgupta [6] and it is being discussed in a number of papers dealing with normative analysis of some activities (see, e.g., [8], [26], [32]).

parameters of the transition curve from time to time. And then the resulting path apparently will not be a "globally optimal" curve but rather will "...be compared to the course of a dog running across a field toward his master, while the master walks along the road"[27].

7 Concluding Remarks

It deems natural for the government, committed to switching to sustainable path, to consider the problem of minimizing the short-run negative consequences of moving to such a path. We observe that the so-called optimal short-run transition path involves an interval of zero consumption and this runs counter to the government's primary goal of pursuing intergenerational justice principle in the long run.

Consideration of the long-run consumption behavior along possible transition curves shows that even for inefficient curves there is a path of extraction with asymptotically constant (separated from zero) consumption over time. The short-run negative shock along this transition path is small for our numerical example based on observed world oil extraction data. Moreover, a "worsening " of the short-run situation (shortening the period of transition and introducing a stronger negative shock on output) yields the possibility of slow, but unlimited growth of consumption in the long run. In other words the transition curve (to be exact, the single free parameter - d) can be fitted to satisfy desirable qualitative behavior of consumption in accord with the various optimality criteria in the long run. And it again raises the long-standing question about the fairest ethical theory for the distribution of consumption across generations. If decreasing the rate of oil extraction is really necessary, which criterion we must follow? [18]

Aside from equivocation on our main welfare criterion there are some other questions and

[18] A very detailed analysis of different ethical theories is in [23]

limitations of the model we have presented.

(1) We examined transition curve as interior solution neglecting restrictions imposed by technical possibilities and difficulties connected with the saving rate changing. Constraints on the speed of changing savings behavior can restrict us from implementing even the path of extraction with asymptotically constant consumption not speaking of paths with unlimited growth in the long run (questions of optimal path existence and uniqueness).

(2) There is an interesting question of the optimal program stability with respect to initial conditions.

(3) There is one more interesting question of the comparative estimation of the consumption behavior, considered in this paper with the consumption behavior along the efficient transition paths.

(4) Transition curve can be constructed in different class of functions, e.g., as a solution of calculus of variation problem.

We also assumed that:

(5) Cost of extraction is zero and population is constant though it would be interesting to consider the problem of transition when these values are variables.

(6) There is no time lag between the moment of oil extraction and correspondent increment of man-made capital which is not true if we invest oil rent in the development of alternative technologies.

(7) All oil rent is invested into reproducible capital. In general, this is not observed and we should consider some period of increasing investments along some smooth (maybe hysteresis-like) curves and examine the influence of this curve on long-run consumption behavior.

(8) Losses and gains are symmetric in the utility function for definition of person's relevant

position though there is evidence that “Losses loom larger than gains...” ([20], p. 148).

We think that all these questions need special careful consideration in separate papers.

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8 Appendix (proof of the Proposition 2)

For the economy with a Cobb-Douglas production function $q = k^{\alpha} r^{\beta}$ and investment covered by resource rent in $\dot{k} = \beta q$, expression $[\cdot]$ in the right hand side of (15) is

$$\left[\frac{\dot{k}}{q} - \frac{(1-\beta)}{\alpha}\frac{k}{q}\frac{\dot{r}}{r}\right] = \beta - (1-\beta)\frac{k}{\alpha q}\frac{\dot{r}}{r}.$$

In the long run $\dot{r} < 0$ and $(1-\beta), k, \alpha, q, r > 0$. Then we can rewrite the last equation as

$$[\cdot] = \beta + (1-\beta)\frac{k}{\alpha q}\frac{|\dot{r}|}{r},$$

which means that $[\cdot] = 1$ if and only if $\frac{k}{\alpha q}\frac{|\dot{r}|}{r} = 1$, and generally

$$\operatorname{sgn}\{1 - [\cdot]\} = \operatorname{sgn}\left\{1 - \frac{k}{\alpha q}\frac{|\dot{r}|}{r}\right\}.$$

So, in order to examine the behavior of $q(t)$ and $c(t)$, we can compare $\frac{k}{\alpha q}\frac{|\dot{r}|}{r}$ or $\frac{1}{\alpha}k^{1-\alpha}r^{-(1+\beta)}|\dot{r}|$ to unity where $k(t)$ is an unknown function. An attempt to find $k(t)$ from the differential equation $\dot{k} = \beta k^{\alpha} r^{\beta}$ gives us $k^{1-\alpha} = \beta(1-\alpha)I(t)$, where

$$I(t) = \int r(t)^{\beta} dt = r_0^{\beta} \int \frac{(1+\rho t)^{\beta}}{(1+ct)^{\beta(d-1)}} dt = r_0^{\beta} I_1(t),$$

and $\rho = c(d-1) + A_0/r_0$. The integral $I_1(t)$ can be expressed in elementary functions using Chebyshev substitutions if we set $d = 12$ and $\beta = 0.1, 0.2, 0.3$, and so on. Thus, for $\beta = 0.2$

$$k_{0.2}^{1-\alpha}(t) = \frac{\beta(1-\alpha)r_0^{\beta}a_c^{1-10\beta}}{\rho(p+1)}\left[(a_c+b_c)^{(p+1)} - \left(\frac{a_c}{1+\rho t}+b_c\right)^{(p+1)}\right] + k_0^{1-\alpha}, \tag{21}$$

where $a_c = 1 - \frac{c}{\rho}, b_c = \frac{c}{\rho}, p = \beta(1-d)$. And for $\beta = 0.3$

$$k_{0.3}^{1-\alpha}(t) = \frac{\beta(1-\alpha)r_0^{\beta}a_c^{1-10\beta}}{\rho(p+2)(p+1)} \tag{22}$$

$$\times \left\{ (a_c+b_c)^{(p+1)}[a_c(p+1)-b_c] - \left(\frac{a_c}{1+\rho t}+b_c\right)^{(p+1)} \left[\frac{a_c(p+1)}{1+\rho t}-b_c\right]\right\} + k_0^{1-\alpha}.$$

k_0 must be the same for any curve, and so it can be evaluated for our economy using equation (1) for the Hartwick curve in terms of output $q_0 = q(0)$:

$$k_0 = -\frac{\alpha q_0 r_0}{A_{0H}},$$

where q_0 can be set equal to unity and $A_{0H} = \dot{r}_{Hart} < 0$ for the Hartwick curve. We can use (21) and (22) for a detailed analysis of capital (e.g., asymptote for $t \to \infty$), output, and consumption along some transition curve. We must restrict attention to the cases $\beta = 0.2$ or $\beta = 0.3$. The expression for $k(t)$, given $\beta = 0.1$ is lengthier than (21) or (22) and we will not consider it here. We are interested in cases with $\beta < 0.1$ in which according to Chebyshev theorem $k(t)$ can be expressed in elementary functions for relatively large values of d.[19]

However we can consider the asymptotic behavior of expression $k^{1-\alpha}r^{-(1+\beta)}|\dot{r}|$. Note, that $\lim_{t\to\infty} k^{1-\alpha}(t) = \infty$, since $\dot{k} = \beta q > 0$ for any $t \geq 0$. And the asymptotic behavior of $r^{-(1+\beta)}|\dot{r}|$ is

$$\begin{aligned}
\lim_{t\to\infty} r^{-(1+\beta)}|\dot{r}| &= \lim_{t\to\infty} \left\{\frac{(1+ct)^{d-1}}{r_0(1+\rho t)}\right\}^{1+\beta} \left|\frac{A_0+bt}{(1+ct)^d}\right| \\
&= \left(\frac{1}{r_0}\right)^{1+\beta} \lim_{t\to\infty} \frac{(1+ct)^{(d-1)(1+\beta)}}{(1+ct)^d} \cdot \frac{|A_0+bt|}{(1+\rho t)^{1+\beta}} \\
&= \left(\frac{1}{r_0}\right)^{1+\beta} \lim_{t\to\infty} \left(\frac{1+ct}{1+\rho t}\right)^{\beta} \cdot \frac{|A_0+bt|}{1+\rho t} \cdot (1+ct)^{\beta(d-2)-1} \\
&= \left(\frac{1}{r_0}\right)^{1+\beta} \cdot \left(\frac{c}{\rho}\right)^{\beta} \cdot \frac{|b|}{\rho} \cdot \lim_{t\to\infty}(1+ct)^{\beta(d-2)-1}
\end{aligned} \tag{23}$$

[19] An integral $\int \tau^{\beta}(a_c+b_c\tau^n)^p d\tau$, where $\tau = 1+\rho t$, can be expressed in elementary functions only in cases 1) p - integer; 2) $\frac{\beta+1}{n}$ - integer; 3) $\frac{\beta+1}{n}+p$ - integer. In our case $p = \beta(1-d)$ and for $\beta \in (0,1)$ and $n = 1$ we can use the third case. Then we have a condition: $\beta(2-d)+1 = N$ must be an integer. From the finiteness of the resource we have constraint $d > 3$ or $N < 1-\beta$ and so, the minimum feasible ($N = 0$) value of d for $\beta = 0.05$ is $d = 2-(N-1)/\beta = 22$. This is much larger than the optimum values for d either for the problem of minimizing the output negative shock (for the LA curve $d = 12.845$) or for the problem of asymptotically constant consumption (for the LD curve $d = 8.0$).

or

$$\lim_{t\to\infty} r^{-(1+\beta)}\left|\dot{r}\right| = \begin{cases} 0, & \beta(d-2)<1, \\ \left(\frac{1}{r_0}\right)^{1+\beta}\left(\frac{c}{\rho}\right)^{\beta}\frac{|b|}{\rho}, & \beta(d-2)=1, \\ \infty, & \beta(d-2)>1. \end{cases}$$

This means that for $\beta(d-2)\geq 1$ we have our output and consumption asymptotically decreasing:

$$\lim_{t\to\infty} \operatorname{sgn}\{1-\frac{1}{\alpha}k^{1-\alpha}r^{-(1+\beta)}\left|\dot{r}\right|\} = \lim_{t\to\infty} \operatorname{sgn}\dot{q} = -1$$

or the first expression in statement (16) of the proposition. The most interesting case is when $\beta(d-2)<1$. Then using (23) we have

$$\lim_{t\to\infty} k^{1-\alpha}r^{-(1+\beta)}\left|\dot{r}\right| = \infty\cdot 0 = \lim_{t\to\infty}\frac{k^{1-\alpha}}{1/\left(r^{-(1+\beta)}\left|\dot{r}\right|\right)} = \frac{\infty}{\infty}$$

$$= \lim_{t\to\infty}\frac{d\left[k^{1-\alpha}\right]/dt}{d\left[1/\left(r^{-(1+\beta)}\left|\dot{r}\right|\right)\right]/dt}$$

$$= \left(\frac{1}{r_0}\right)^{1+\beta}\left(\frac{c}{\rho}\right)^{\beta}\frac{|b|}{\rho}\lim_{t\to\infty}\left\{(1-\alpha)k^{-\alpha}\dot{k}/\frac{d\left[(1+ct)^{-\beta(d-2)+1}\right]}{dt}\right\}$$

$$\begin{aligned} &= \left(\frac{1}{r_0}\right)^{1+\beta}\left(\frac{c}{\rho}\right)^{\beta}\frac{|b|}{\rho}\frac{1-\alpha}{c[1-\beta(d-2)]}\lim_{t\to\infty}\frac{\beta r^{\beta}}{(1+ct)^{-\beta(d-2)}} \\ &= \left(\frac{1}{r_0}\right)^{1+\beta}\left(\frac{c}{\rho}\right)^{\beta}\frac{|b|}{\rho}\frac{(1-\alpha)\beta r_0^{\beta}}{c[1-\beta(d-2)]}\lim_{t\to\infty}\frac{(1+\rho t)^{\beta}}{(1+ct)^{\beta(d-1)}}\cdot(1+ct)^{\beta(d-2)} \\ &= \left(\frac{1}{r_0}\right)^{1+\beta}\left(\frac{c}{\rho}\right)^{\beta}\frac{|b|\,(1-\alpha)\beta r_0^{\beta}}{\rho c[1-\beta(d-2)]}\lim_{t\to\infty}\left(\frac{1+\rho t}{1+ct}\right)^{\beta}. \end{aligned}$$

And finally we have

$$\lim_{t\to\infty} k^{1-\alpha}r^{-(1+\beta)}\left|\dot{r}\right| = \frac{|b|\,(1-\alpha)\beta}{r_0\rho c[1-\beta(d-2)]},$$

where $b=b(c(d),d)$ (see (11)) and $c=c(d)$ (see (13)). Thus for $\beta(d-2)<1$

$$\lim_{t\to\infty}\operatorname{sgn}\dot{q} = \lim_{t\to\infty}\operatorname{sgn}\left\{1-\frac{1}{\alpha}k^{1-\alpha}r^{-(1+\beta)}\left|\dot{r}\right|\right\} = 1-\frac{|b|\,(1-\alpha)\beta}{r_0\alpha\rho c[1-\beta(d-2)]},$$

which is the second expression of (16).