An Econophysics Model for Investments using the Law of the Electric Field Flow (Gauss' Law)

Spanulescu, Ion and Popescu, Ion and Stoica, Victor and Gheorghiu, Anca

Hyperion University, Hyperion University, Hyperion University, Hyperion University

25 May 2012

AN ECONOPHYSICS MODEL FOR INVESTMENTS USING THE LAW OF THE ELECTRIC FIELD FLOW (GAUSS' LAW)

Ion SPÂNULESCU*, Ion POPESCU*,
Victor STOICA* and Anca GHEORGHIU*

Abstract. *In this paper a new econophysics model of investment processes is proposed and discussed. For this purpose an analogy between the electric field flow and the investment supplying flow with credits for the considered investment is used.*

Keywords: *econophysics, electrical field, Gauss' Law, investment process, investment field.*

1. Introduction

In order to model the economic phenomena and processes, besides the mathematical methods, which led to econometrics, the researchers resorted to models based on the similarity of the economic phenomena and processes with physics laws and processes, which led to the emergence of the new interdisciplinary science, namely econophysics.

The majority of studies and papers published or communicated to econophysics conferences or symposiums present the results of the economic modelling using mostly mathematical statistics methods and statistical physics methods (see for instance [1-14]). As a result, most researchers and analysts refer only to this part of econophysics, based on statistical physics' methods, which can be named **statistical econophysics**.

On the other hand, econophysics implies including in the research resources of other domains of physics, especially from phenomenological physics, as chapters of thermodynamics, electricity, solid state physics, nuclear or optical physics etc. Indeed in the last years many researchers included in their papers some models based on analogies between the economical phenomena or processes and laws or processes from other fields of physics such as thermodynamics, electricity or optical physics etc.

---

* Bucharest Hyperion University, 169 Calea Călăraşilor, St., Bucharest, 030615, Romania.

[15-20]. In this way we can talk about a new facet of econophysics, named fundamental or **phenomenological econophysics**. Both types and directions of econophysics are equally useful for modelling the economic processes or phenomena, resorting to the methods and laws of general physics or especially statistical physics.

In this paper we attempt to introduce an econophysics approach to modelling the investment process using the electrical field flow concept (the Gauss' Law) from electrostatic chapter of applied physics.

2. The Law of the Electric Field Flow (Gauss' Law) [21]

As it is known from the applied physics, the electric or electrostatic field (for idle electric charges) represents a special physical entity that characterizes the interactions between electrical charges.

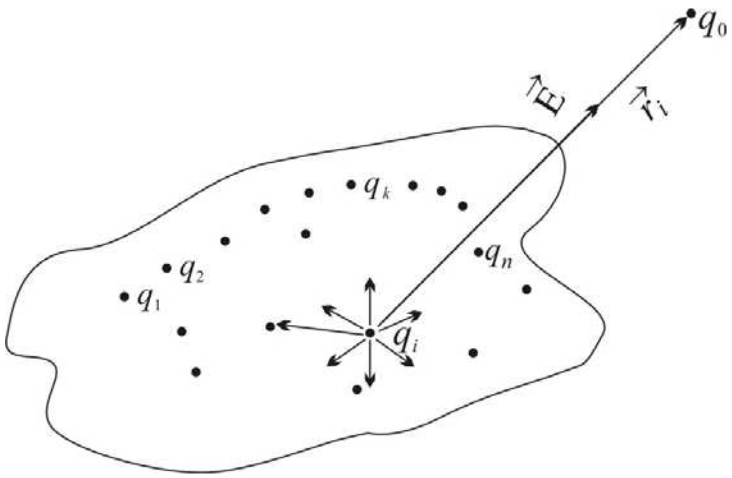

Figure 1. Electrostatic field created by electric charges q_1, q_2, ..., q_n.

In the case of the charges system q_1, q_2 … q_n in Figure 1, which is investigated (tested) with a very small charge q_0 named test or trial charge, the force F that results by summing the interactions of each charge q_i with the charge q_0, is given by the Coulomb's law, which describe the interactions between electrical charges [21]:

$$F = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_0 q_i}{r_i^2}\cdot\vec{e}_i = q_0\sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\cdot\vec{e}_i. \quad (1)$$

where $\vec{e}_i$ is the unitary vector $\vec{r}_i/r_i$, and ε_0 is the vacuum electrical permittivity.

In equation (1), the constant q_0 appears as a proportionality factor with the term $\sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_i}{r_i^2} \cdot \vec{e}_i$ that depends only on the initial structure (without q_0) of the charges system q_1, q_2 … q_n and on the point's position (x_0, y_0, z_0) defined through the vector $\vec{r}_i$ (with the origin in charge q_i and the peak in the point where is q_0). This vectorial size, noted with $\vec{E}$ represents the electric field strength (or simply the electric field) generated by the charges q_1, q_2 … q_n, which are the field's sources:

$$\vec{E}(x_1, y_1, z_1) = \sum_{i=1}^{n} \frac{q_i}{4\pi\varepsilon_0 r_i^2} \cdot \vec{e}_i. \tag{2}$$

Taking into account (2), the equation (1) is written as:

$$\vec{F} = q_0 \vec{E}. \tag{3}$$

Resulting that:

$$\vec{E} = \frac{\vec{F}}{q_0}. \tag{4}$$

If in (4) is considered q_0=+1, then the electric field will coincide in direction, size and sense with the force $\vec{F}$. Thus the electric field's strength $\vec{E}$ (x_0, x_0, z_0) in a random point in space is a physical quantity numerically equal to the force exerted by the electric field (produced by other charges) on a standard charge, equal to the unitary positive charge located in that point and having the orientation/direction of this force F.

The definition (4) admits an idealization, namely that the point like charge q_0 is the test-charge (sample-charge) which highlights the force $\vec{F}$, has to have a very small value, so that its presence doesn't disturb the investigated electric field.

The electric field can be created also by a single charge (macroscopic) with a size q (Fig. 2). In this case the electric field intensity in a point P_1 is given by a formula similar with (2) (without the part which contains the summation symbol $\sum$):

$$\vec{E}(x_0, y_0, z_0) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot \frac{\vec{r}}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot \vec{e}. \tag{5}$$

where ε_0 is the vacuum dielectric permittivity.

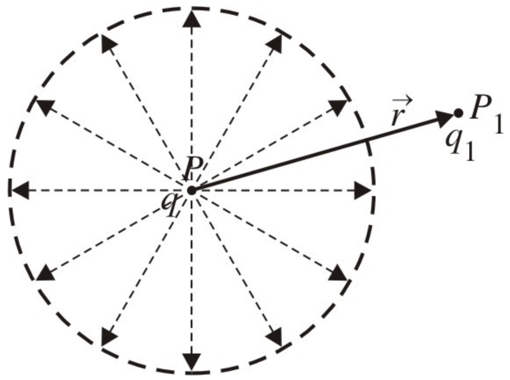

Figure 2. The electric field of an isolated charge *q*.

Here the charge q is the only source of the field E and $\vec{r}$ is the position vector of the point $P_1(x_1, y_1, z_1)$, where the field is calculated, its origin being considered in the point $P(x, y, z)$ where is located the charge q.

The electric field strength E characterizes the action exerted by the source-charge q on a small body with a punctual charge q_1. This action is exerted in all the directions competing in the point P, having the same intensity in all the points equally distant by P i.e. located on the surface of a sphere containing in its centre the charge q; then it's usual for this action to be inversely proportional to the sphere's area through which propagates. This is why in the Coulomb Law (see the equation (1)), the proportionality factor were chosen equal to $1/4\pi$, characteristic for the spherical symmetry effects. The same, this factor appears and in the formula of the electric field strength (see (2) or (5)) and in every formula in electricity referring to a phenomenon that presents spherical symmetry.

We can conceive a visual representation of the electric field by an image that emphasize the field's sources and the areas where are manifesting the interaction forces with other charges. To this end, for each point from space where the field manifests we can associate a vector determining the field's direction. Adopting a simplified representation (in plane) of the points where we can draw the origins of the field vectors $\vec{E}$ in the given plan, are considered the so called field lines whose tangents in any point coincide with the electric field direction in that point (Fig. 3,a). In this case, if the source-charges are considered fixed/stationary, and the test-charge will be let free, it will move on the trajectories that constitute the electric field lines and whose sense coincides, by convention, with that in which would move the positive unitary charge. The field lines of an isolated point charge are straight lines that start radial from the charge if the charge is positive (Fig. 3,b) or come radial to it, if it is negative (Fig. 3,c). In the case where there is a two charges system that can interact, the field lines begin in the positive charge(s) and ends in the negative charge(s) (Fig. 3,d).

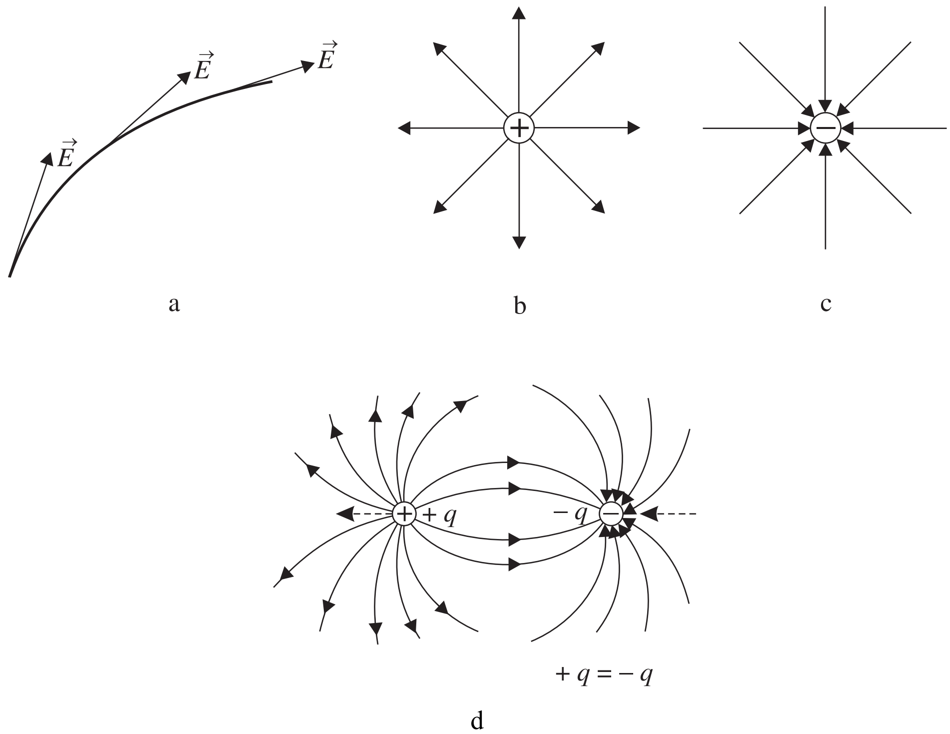

Figure 3. a) field line; b) the electric field of an isolated positive charge $+q$; c) the electric field of an isolated negative charge $-q$; d) the electric field of two electric charges of contrary sings located close one to another and $+q = -q$.

Conventionally we may introduce as a measure for the electric field intensity by the **number** of field lines drawn in a given area, so by the field lines density. Therefore, if in the way of the field lines shall be perpendicularly placed a surface of a value equal with the unit (Fig. 4,a), the higher the electric field intensity E is, the larger will be the number of lines N crossing the surface.

It is conventionally considered that through the surface S perpendicular to the direction of the electric field, we can draw a number N

of field lines so that the number of field lines passing through the surface to be equal to the value of electric field intensity from the surface area (Fig. 4,a) i.e.:

$$\frac{N}{S} = E. \tag{6}$$

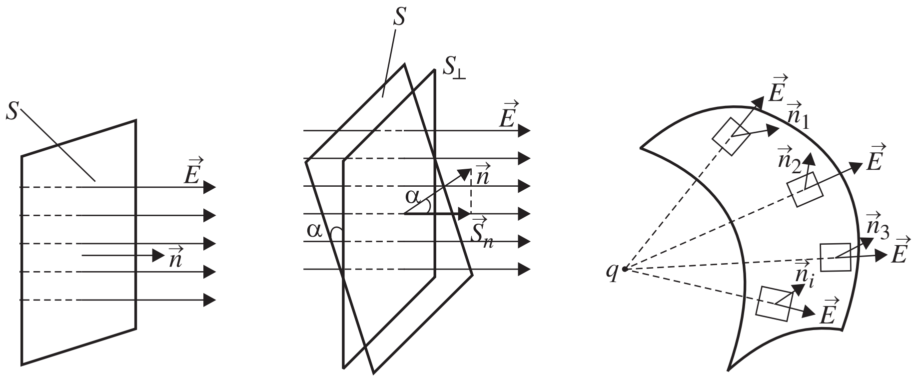

Figure 4. The definition of the electric field flow.

In this way, the field's intensity E value is related to the density of field lines. The equation (6) allows the introduction of a new physical quantity named **electrical flow** (or **electric field flow**) defined by the total number N of field lines that cross a surface S perpendicular to the direction of a uniform electric field $\vec{E}$ (Fig. 4,a); from equation (6) we can see that Φ (=N) is given by:

$$\Phi\ (=N) = ES. \tag{7}$$

In the case when the considered surface makes an angle α with the direction of the field lines, to define the electrical flow we can introduce the concept of oriented surface, given by the versor's sense $\vec{n}$ of the normal (Fig. 4,b) so:

$$\vec{S} = S\vec{n} \tag{8}$$

where $\vec{n}$ is the normal (unitary-vector) to the surface.

The flow through the surface inclined with an angle α to the field direction can be calculated either with the help of the projection of the surface S on the plane that is perpendicular to $\vec{E}$ (Fig. 4,b):

$$S_\perp = S\cos\alpha \tag{9}$$

or with the projection of $\vec{E}$ on the normal direction to the plan:

$$E_n = \vec{E}\vec{n} = E\cos\alpha. \tag{10}$$

In this last case, for the expression of electric field flow results:

$$\Phi = ES_{\perp} = ES\cos\alpha = \Phi_0\cos\alpha = E_n S, \tag{11}$$

where:

$$\Phi_0 = ES, \tag{12}$$

represents the maximum value of the flow, which would pass through a surface perpendicular to the field lines. (Fig. 4,a)

In case the surface is not plane, it is divided in small surface/area elements (elementary surfaces/areas) ΔS_i (Fig. 4,c) where the field's intensity varies very slightly, so that the field can be considered uniform within each elementary domain. The flow of the field lines through the entire finite area S is given by the algebraic sum of elementary flows (see equation 7):

$$\Phi = \sum_{i=1}^{n}\Delta\Phi_i = \sum_{i=1}^{n} E_{n_i}\Delta S_i. \tag{13}$$

The surface/area elements $\Delta\vec{S}_i = \vec{n}_i\Delta S_i$ must be taken infinitely small, so that the elementary flow dΦ that pass through dS will be:

$$\mathrm{d}\Phi = E_n\mathrm{d}S = E\mathrm{d}S\cos\alpha. \tag{14}$$

By integrating over the entire surface shall be obtained the total flow:

$$\Phi = \int_S E\mathrm{d}S\cos\alpha = \int_S \vec{E}\overrightarrow{\mathrm{d}S}, \tag{15}$$

where the surface $\overrightarrow{\mathrm{d}S}$ is considered directed after the normal $\vec{n}$ to the concerned element:

$$\overrightarrow{\mathrm{d}S} = \vec{n}\mathrm{d}S = \mathrm{dS}\cos a. \tag{16}$$

The flow of the field lines is positive when the field $\vec{E}$ has the same sense as the normal $\vec{n}$ and negative when the two vectors are directed in opposite senses.

Using the equations (5)-(7) we can calculate the electric flow of a point-charge q through a sphere surface of radius r centred in the point where the charge is (Fig. 5).

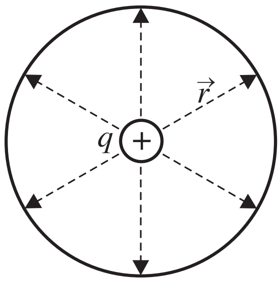

Figure 5. The electric field of a punctual charge located in the centre of a sphere with radius *r*.

Being a radial field and depending of the sphere's radius *r*, in all the points of the sphere surface will have a constant value (see equation (5)):

$$E = \frac{1}{4\pi\varepsilon} \cdot \frac{q}{r^2}. \qquad (17)$$

Such as the normal $\vec{n}$ in each point coincides with the sphere radius, the flow calculation is done directly:

$$\Phi = ES = \frac{q}{4\pi\varepsilon r^2} \cdot 4\pi r^2 = \frac{q}{\varepsilon} \qquad (18)$$

where ε represents the environment's permittivity[1].

The equation (18) is valid for any type of charge *Q* included within any closed area *E*, because every type of closed surface can be included in a spherical surface as the one in figure 5, to which are applied the formulas (12) and respectively (17) and (18).

For closed surfaces that aren't containing charges *Q*, the electric field flow will be equal to zero. These considerations underlying the Gauss'Law, which reads as follows [21]:

The total flow of the field lines through a closed area is equal to zero if the field sources are outside the volume enclosed by the surface S and equal to $\frac{Q}{\varepsilon_0}$ *when the charges Q are inside of the considered surface (Fig. 5) i.e.:*

$$\Phi = \oint_S \vec{E}\, \overrightarrow{\mathrm{d}S} = \begin{cases} 0, & \text{if } Q \text{ is outside of volume enclosed by the surface } S; \\ \dfrac{Q}{\varepsilon_0}, & \text{if } Q \text{ is inside of volume enclosed by the surface } S. \end{cases} \qquad (19)$$

[1] In the case of the air is considered that ε has the value of the vacuum permittivity noted with ε_0.

If inside of the volume the charge Q_{int} is continuously distributed with a density $\rho(x,y,z)$ then Gauss' Law should be written as:

$$\Phi = \oint_S \vec{E}\,\mathrm{d}\overrightarrow{S} = \frac{1}{\varepsilon_0}\int_V \rho(x, y, z)\mathrm{d}v. \qquad (20)$$

3. The econophysics model for investments based on microcredits or subscription of shares

In the proposed model the electric charges inside the enclosed surface being the sources of the electric field E are assimilated with financing sources, which can be banks or individuals etc., able to provide capital, i.e. the amount of money as microcredits or subscription of shares, in order to establish a company or any investment, which manifests similar to a electrical field (Fig. 1), named in the present case **investment field.** The field lines that start from the financial sources can be assimilated with microcredits for investments, so that the flow of the investment field Φ_i is defined by the total number N of microcredits similarly with the number N of electrical field lines in equation (6). If the enclosed surface S containing inside the financial sources from which start the microcredits lines, is considered as money source (money offer), which different investors may do in economic boom periods, then for the investment I can be written an equation similar to equation (6):

$$\frac{N\,(\text{microcredits})}{S\,(\text{money offers})} = I(\text{investment}), \text{ similarly to } \vec{E}. \qquad (21)$$

In this way, the value of the investment field strength (i.e. the investments) is related to the density of the field lines (the microcredits flow). The equation (21) allows the introduction of an econophysics notion named **investment flow** (equal to the sum of total microcredits) similarly to the electric field flow, defined by the total number of field lines N (the total number of microcredits) that cross a surface S (the money offer) perpendicularly to the direction of a uniform electric field E (here equal with I = investments) (Fig. 4,a); then from the equation (21) we can see that $\Phi(=N)$ is given by:

$$\Phi\,(\textit{total microcredits}) = I(\textit{investments}) \cdot S(\textit{money offer}). \qquad (22)$$

As was pointed out, in the case when the surface isn't plane, this is divided in smaller elementary surface elements ΔS_i (Figure 4,c) where the field's intensity varies slightly, so that the field can be considered uniform between the limits of each elementary domain. The investment field flow (total microcredits) through the entire finite surface S (money offer) is given by the algebraic sum of the elementary microcredits flows according to equation (12) and for the analysis of different situations the equations (12)-(20) are used.

Considering the Gauss'Law (see (19)) and eq. (22), the equation (21), expressing the value I of the investment can be written as:

$$I = \frac{\Phi}{S} = \frac{Q}{\varepsilon_0 S} = kQ \tag{23}$$

were the proportionality constant:

$$k = \frac{1}{\varepsilon_0 S},$$

depends on the money offer (through the factor $\frac{1}{S}$) and on the conditions for credit granting (the interest rate, payment terms, guarantees/securities etc.) expressed by the factor $\frac{1}{\varepsilon_0}$ where ε_0 represent a financial permittivity similar to environment's permittivity from (18).

4. Conclusions

Using the analogy between the electric field flow and the investment supply flow with microcredits, a new econophysics model for the investments is introduced and analyzed.

It shows that the investment value is determined by the volume of money sources Q and the money offer, but also by the concrete crediting conditions (guarantees, interest rates, payment terms etc.).

The equation (23) expresses a fact confirmed by the economic practice, namely that the investment value I clearly depends by the nature and value of the investment sources (noted by Q). The confirmation of this economic law is enhanced by a mathematical equation (see equation (23)) deducted based on the laws of physics, which is an exact science and at the same time are laws of nature science. Therefore, the confirmation to that economic law (for investments) has the support of a high precision (specific for the exact sciences) and the validity of these equations being deducted based on the laws of physics in accordance with the laws of nature; this fact supports the economic law for investments showing that, as has been done so far, it is the most appropriate law for the analyzed economic process.

REFERENCES

[1] Y. Liu, P. Gopikrishman, P. Cizeau, M. Meyer, C. Peng and H. E. Stanley, *Statistical Properties of the Volatility of Price Fluctuations*, Phys. Rev. E **60**, 1390-1400 (1999).

[2] H. E. Stanley, P. Gopikrishman, V. Pleron, L.A.U. Amaral, *Quantifying Fluctuations in Economic Systems by Adapting Methods of Statistical Physics*, Physica A **287** 339-361 (2000).

[3] R. N. Mantegna and H. E. Stanley, *An Introduction to Econophysics*, Cambridge University Press, Cambridge, 2000.

[4] A. A. Drăgulescu and V. M. Yakovenko, *Statistical Mechanics of Money*, Eur. Phys. J. B **17**, 723-729 (2000);

[5] A. A. Drăgulescu and V. M. Yakovenko, *Evidence for the Exponential Distribution of Income in the USA*, Eur. Phys. J. B. **20**, 585-589 (2001).

[6] Tomoya Suzuki, Tohru Ikeguchi and Masuo Suzuki, *A model of complex behavior of interbank exchange markets*, Physica A: Statistical Mechanics and its Applications **337** (1-2), 196-218 (2004).

[7] A. A. Drăgulescu, V. M. Yakovenko, *Probability Distribution of Return in the Heston Model with Stochastic Volatility,* Quantitative Finance, vol. **2,** 443-453 (2002).

[8] R. N. Mantegna, Z. Palagyl, H. E. Stanley, *Applications of Statistical Mechanics to Finance*, Physica A **274**, 216 (1999).

[9] F. Lillo and R. N. Mantegna, *Variety and Volatility in Financial Markets*, Phys. Rev. E **62**, 6126-6134 (2000).

[10] A. A. Drăgulescu and V. M. Yakovenko, *Exponential and Power-Law Probability Distributions of Wealth and Income in the United Kingdom and The Unite States*, Physica A **299**, 213-221 (2001).

[11] Araceli Bernabe, Esteban Martina, Jose Alvarez-Ramirez and Carlos Ibarra-Valdez, *A Multi-model Approach for Describing Crude Oil Price Dynamics*, Physica A: Statistical Mechanics and its Applications **338** (3-4), 567-584 (2004).

[12] Terence C. Mills, *Statistical Analysis of Daily Gold Price Data*, Physica A: Statistical Mechanics and its Applications **338** (3-4), 559-566 (2004).

[13] Chiara Cornalba and Paolo Giudici, *Statistical Models for Operational Risk Management*, Physica A: Statistical Mechanics and its Applications **338** (1-2), 166-172 (2004).

[14] M. Ausloos, J. Miskiewicz, M. Sanglier, *The Duration of Recession and Prosperity: does their Distribution Follow a Power or an Exponential Law?* in arXIV: cond-mat/0403143 VI 4 martie (2004).

[15] Wang Yougui, WU Jinshan, DI Zengru, *Physics of Econophysics*, in arXiv: cond-mat/ 0401025, v**1**, 4 Jan (2004).
[16] Anca Gheorghiu, I. Spânulescu, *New Econophysics Approches and Models*, The Hyperion Scientific Journal, **4,** 73-91 (2004).
[17] A. Khrennikov, *Financial Heat Machine*, arXiv: cond-mat/0408560 (2005).
[18] I. Spânulescu, Anca Gheorghiu, *Economic Amplifier – A New Econophysics Model*, arXiv: 0707.3703 (2007).
[19] Anca Gheorghiu, *Econophysics of Investments* (in Romanian), Victor Publishing House, Bucharest (2007).
[20] A. Johansen, O. Ledoit, and D. Sornette, *Crashes as Critical Points,* International Journal of Theoretical and Aplied Finance **3** (1), 1 (2000).
[21] I. Spânulescu, *Electricity and Magnetism (in Romanian),* Victor Publishing House (1978).