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Shuanping Dai

IINO, University of Bremen

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# The Emergence of Efficient Institutions and Social Interactions*

Shuanping Dai†

IINO, University of Bremen, Germany

## Abstract

Institutions are the equilibrium states of games, and the emergence of institutions is an evolutionary, stochastic, and (social) structural dependence process of interactions among agents. In this paper, we address the relationship between the institutional emergence and the structure of social interactions under the context of (network) coordination games. The model here shows when the agents are socially restricted, and individual decision-making is based on mutual agreements, inefficient institutions will be the stable states in the long run, say, institutions are locked-in inefficiently. When the agents are not restricted socially, the institutional stability will wander between two states. The efficient institutions can emerge only as the agents are facing strong cost constraints and, are in the contexts with relative high certainties, for instance, as the interactive population size is becoming smaller.

**Keywords:** Institutional Emergence, Coordination Games, Stochastically Stable Equilibrium, Network Formation, Social Distance

**JEL Classification:** B15, B52, C73

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† Shuanping Dai, Ph.D. student at the University of Bremen, Germany. Email:shuanping.dai@uni-bremen.de

# 1 Introduction

In a human interaction context, the issues of emergence and processes of economic systems should be at the heart of economic analysis. The concept of emergence is ubiquitous and, frequently is used in a substantial number of scientific areas and disciplines, for some literature clues see e.g. De Haan (2006), Harper and Lewis (2012) and Harper and Endres (2012). For example, from the system perspective, the phenomena of emergence occur in systems that are generated, where the whole system's behavior cannot be obtained by summing the isolated individual agents' behavior, and is unlikely to be predicted precisely. For a classical instance, Schelling (1971, 1978) demonstrates how the phenomenon of segregation emerges in a theoretical setting of the local interactions ('micro-motivation') generate global patterns ('macro-behavior'). Here, we consider emergent phenomena as collective outcomes of agent-based interactions intentionally or unintentionally.

The problem of the emergence of institutions is the starting point of classic economics (Elsner, 1986, 1989), also is the fundamental issue of the modern institutional economics. It is difficult to neglect that, some economists, mainly of self-deemed new institutionalists, argue that institutions, strictly are inanimate objects, can be designed artificially and therefore exogenously given. Actually, such so-called institutions are apparently designed, but they are just the external expressions of the existed results of social interactions that are endogenous in economy and related activities. For example, the establishments of laws always are the results of artificially designed, written and published by some experts and official organizations, but it is not the whole story of the emergence of laws indeed. In general, it is required by the objective needs of real worlds in which the people and some lawsuits have been harmed since lacking of proper laws or, of the strict implement of the laws. Moreover, it is natural to image some people have tried to find suitable law cases or legal provisions to remedy those problems, but it is not adequate to cover all potential loopholes. Hence, a lot of uncertainties and gambling will be emerged due to the imperfection of the legal system. As we are at the legislation, the different parties with own interests will definitely intervene the establishment of specific provisions, and finally, the established law will be a result of compromise, coordination as well as cooperation, or say, of series of games. Therefore, institutions have always been considered as the device to reduce uncertainties, but their emergence are frequently sketched as the results of strategic interactions of agents (Elsner, 1989, Lesourne et al., 2006), hence, a crucial question arises here is "why and in what ways do individuals coordinate their behavior (Elsner, 1989, p.190)" in the process of something new comes out.

Game theory is the most powerful branch of applied mathematics in dealing with the behaviors in interacting strategic situations, which interprets an institution as an equilibrium state consciously or unconsciously reached by several agents (Walliser, 2006, and moreover, for the pioneer works, see Schotter, 1981/2008 and Axelrod, 1984/2006 for instance). Walliser (2006) concludes that there are three explanatory approaches, say, and the process of collective choice, deduction and evolution, to be considered for any emergent phenomenon in the game theoretical contexts. In a collective way, the institutions are considered as the result of a voluntary choice process, and are directly introduced in the decision process. For instance, in the situation of coordination failure, a planned institution aim at solving the problem can be determined in a favorable way by some prior agents, i.e. they can make some forbidden provisions on the specific possibilities. Hence, the institutions are exogenously given. Nevertheless, in some situation, the institutions will be treated as stemming as an equilibrium state from some games. In such deductive process, the agents embedded in games can achieve instantaneously an equilibrium state perfectly conceptualized through reasoning and computing. Nevertheless, the deductive approach requires that the agents are full cognitive rationality and to reason at the same time and it cannot offer an efficient way to deal with the situation with multiple equilibria. The evolutionary approach with more unrestricted behavioral assumptions is proposed to deal with the emergence of specific institution (particularly convention) among possible ones (also see Sugden, 1995; Binmore and Samuelson, 2006), for example, the approach of evolutionary stable states, and stochastically stable equilibrium will be discussed below.

Sociologists, for instance Granovetter (1985), have criticized economists for neglecting the roles of social networks in economic transactions, because all agents are involved in different kinds of social networks, and their behaviors naturally are influenced by the neighbors. For instance, the decision of an agent to install new document processing software is usually influenced by the choices of her or his colleagues and co-workers. A large volume of theoretical and empirical studies has investigated the role of networks in game theoretical models.Currently, such criticize is accepted widely by economists, for example, a recent investigation by Aoki (2011). The social networks where interacting agents embedded, bring some novel highlights to the game theoretical approach (for the fundamental issues, see e.g. Goyal (2007) and Jackson (2008), and for recent reviews, see e.g. Galeotti et al., 2010; Jackson and Zenou, 2014), correspondingly, to the emergence of institutions. Hence, in sum, the emergence of institutions is an evolutionary, stochastic, and (social) structural dependence process of interactions among agents.

The remainders of the paper are organized as following. In the next section, the basic modeling background and the approaches of multiple equilibrium selection will be introduced. The section 3 gives the models of institutional emergence. In this section, we mainly frame the effects of network dynamics, like links, locations, and agent distance, in this issue. Finally, it concludes.

## 2 Coordination Games and the Emergence of Institutions

### 2.1 Coordination Games and Multiple Equilibrium Selection

Coordination games, for instance, adoption of new technology, common technology standards, traffic rules, choice of languages or software, as well as social norms, have numerous distinct characteristics, which make them widely applied and interesting in many contexts of modeling with multiple Nash equilibria when players choose the same or corresponding strategies.

The typical instance for coordination games is the traffic problems of choosing the side for driving, which can be presented by the possibility if $a = b$, $c = d$, and $a > c$ , see table 1.

**Table 1: The Basic Structure of Coordination Games**

| | | Agent 2: A | Agent 2: B |
|---|---|---|---|
| Agent 1 | A | a, a | c, d |
| | B | d, c | b, b |

In a simplified sense, a most basic rule should be established to avoid colliding head on when two drivers meet on a narrow road. If they can drive coordinately, that is to say, on the action combination $(A, A)$ or $(B, B)$ in the payoff matrix, and follow the same bidirectional traffic rules, both of them are following left-hand traffic rule or right-hand traffic rule, therefore can benefit from coordinating their actions. Formally, such traffic rules can be considered as the equilibria of coordination games, and the rules we selected to follow are the results of equilibrium selection. In a general sense, we cannot distinguish the advantages and disadvantages of these two kinds of rules. At the very beginning, each country therefore specifies a uniform traffic rule without any idiocratic reason in reality, after many shifts, currently people in around $2/3$ countries drive in right-hand, and the others oppositely. Furthermore, with a substantial probability, this proportion is still continuously changing. Such of kinds of issues of equilibrium selection, like the dynamic selection mechanism of traffic rules, therefore should be investigated deeply, why do not all countries follow the same traffic rules, and how do they coordinate in the same way across most countries? At first glance, the individuals will be better off from coordinating with others, and we identify such process to be coordinated as the process of emergence of basic social institutions. For example, the individual can realize the benefits from obeying the coordinated institutional behavior of the other individuals.

Comparing to the problems of the selection of traffic rules, coordinating issues in the real world are more complicated, for instance, when agents have their own preferences on the specific actions, or some restricted coordination possibilities, so it is necessary to extend the analysis of coordination games through assuming various of payoffs numerically. Generally, we can distinguish such kinds of games by whether the agents have interest conflicts for the purpose of coordination. If we assume that $a > b$ and $a - d < b - c$, such situation can be termed as a Stag Hunt game, which was briefly told by the French philosopher, Jean Jacques Rousseau, in A *Discourse on Inequality* (originally published by Marc-Michel Rey in 1755 in Holland).

This game describes a situation in which two hunters are hunting together, each of them can individually choose to hunt a stag or a hare without communications. If one hunter chooses to hunt a stag, he must cooperate with the other one, both of them can obtain 10 as a result of cooperation, otherwise, he cannot success in hunting a stag and obtain therefore 0. Any individual hunter can hunt a hare separately, and obtain 8 as the other hunter tries to hunt a stag, or obtain 7 in a coordination case of hare, which is less for the individuals since a stag is much more valuable than a hare, however the overall payoff is better off than in the dis-coordination case. Here the optimal outcome can be reached only if they can work together on hunting stag coordinately, however, both of them are in the risk of dis-coordination. See the table 2.

**Table 2: Stag Hunt Game**

| | | Agent 2 | |
|---|---|---|---|
| | | Stag | Hare |
| Agent 1 | Stag | 10, 10 | 0, 8 |
| | Hare | 8, 0 | 7, 7 |

Hence, we have learned that, there are two pure strategy equilibria, $(A, A)$ is Pareto efficient and payoff dominant, $(B, B)$ is risk dominant in the terminology of Harsanyi and Selten (1988) in coordination games.

Hereby we investigate the processes and mechanisms of emergences of institutions applying the coordination problem devices with various payoff structures, the corresponding properties of institutions exhibit certain distinct varieties. In this current article, we define the emergence of institutions as the result of equilibrium selection of coordination games (Stag-hunt game, specifically). Such an equilibrium achieved, in other word, can be considered as the conventional way to play the game (Young, 1993). Young (1996) argues that a convention can be established through two ways. One is by the central authority, for example, a new revolutionary government prone to change the existed governing rules; the other way is by the gradual accretion of precedent, such avenue is much more common than the former one, since most conventions emerge as local custom, and spread across regions over generations. In reality, these two ways are mutually inclusive. Hence, the focus here is to elucidate how the selective mechanism works, or to which of the equilibria of an unperturbed process converges. Even following this analysis way, there are also distinct approached to define what a convention is, for instance, in the landmark textbook, Weibull (1995, p.34) argues that an Evolutionary Stable Strategy (ESS, hereafter) may be thought of as a convention, nevertheless, similarly, the absorbing state of a Markov chain is termed as a convention in Young (1993). Intuitively, the emergence of any convention cannot be predicted in advance, instead, as the interactions are repeated over time and resolved through chance. This leads to more agents hearing about and applying it, then consequently, the way of behaving formed after a consequence of positive feedback loops. It will be showed that the selection of a convention is not because of its superiorities over others, but due to some historical circumstances, for instance, someone made mistakes.

### 2.2 The Stochastic Stable Equilibrium

A substantial number of scientific inquiries in evolutionary game theory have attempted to elaborate the issues of equilibrium selection in coordination games. In contrast to the traditional approaches to equilibrium refinements, which cannot provide a precise selection mechanism from multiple Nash equilibria, the evolutionary theoretical methods apply some feasible tools, for example, ESS, stochastically strategy equilibrium (SSE, hereafter), etc., to such issues.

In general, the combination of two dynamic mechanisms, say, mutation mechanism that generates variety and selection mechanism that highlights some varieties over all, formalizes an evolutionary process. Here, the focus is the later one and to find the mechanism of the spreading of an equilibrium strategy within a population. The core concept in evolutionary game theory, that is, ESS, was initiated and defined by the seminal works of Maynard Smith and Price (1973) and Maynard Smith (1974), and in the extensive form games by Selten (1983) for dealing with the stability and robustness of strategies in a population, and a normal form game developed in Maynard Smith (1982). It has significantly drawn the attention of the research about the equilibrium selection to the evolutionary perspective. The ESS is a strategy (can be pure or mixed), which is adopted by a population of agents once, cannot be invaded by any other small mutant strategies. The agents playing an ESS fare better than the mutants with invasion strategies in the population do and therefore the whole population will converge to an evolutionary stable state in the long run. It implies that any low frequent biological mutation or economic experimentation, such as one time shock, cannot live or exist over an extended period.

For Example, in the Stag Hunt game in the Table 2 shows, we can calculate the ESS of this game, when the population state is at the interval $x \in (0,7/9)$, it will converge to the stable state $x^* = 0$, which means all agents will choose the strategy Hare. When $x \in (7/9,1)$, it will converge to the stable state $x^* = 1$, which means all agents will choose strategy Stag. Hence, both $x^* = 0$ and $x^* = 1$ are ESS, but $x^* = 7/9$ is not an ESS.

However, in the conceptual context of ESS, we cannot predict which stable state will be converged when at the unstable points, for example, at the point $x^* = 7/9$. Similarly, the prediction of an ESS mainly depends on the initial conditions of population configuration. Hence, we can conclude that the ESS is a locally and dynamically stable equilibrium, not determined, and both of equilibria have same probability to be reached regardless of the initial population distribution.

In the ESS approach, the evolutionary process is driven by the power of switch to the best response against the current strategy configuration. However, Foster and Young (1990) argue that the concept of ESS does not capture the effects of stochastic perturbations, i.e., the dynamic of population frequencies, of the evolutionary systems over the long run, since it treats any deviation from equilibria as an isolated event. Therefore, any small invasion of mutant strategies will only live shortly and the initial equilibrium state will be restored. In reality, the evolutionary process is repeatedly perturbed by random mutations, for example, births and deaths, which shows that such successive perturbation processes much more significant selection properties, say, it can make selections among strict Nash equilibria, while the ESS cannot. For a recent comparative comment, see e.g., Sandholm (2008) and Young (2011). Thus a concept of SSE, which is different from both the traditional ESS and the strange attractor of a dynamical system, in a context of stochastic dynamical system by virtue of the mathematical techniques by Freidlin and Wentzell (1984), is developed by Foster and Young (1990). For instance, in the case of symmetric payoff structure, a desirable way to coordinate is going to follow the mass, and the ESS of this game therefore may easily be identified. However, if we introduce a small noise term, i.e. the unstable proportion of agents playing specific strategies, the environmental changes, and mutations as well, this situation will be complicated in the stochastic selection processes, and the asymptotic behavior of the system might be changed.

Let us describe the evolutionary behavior via a discrete-time, irreducible, recurrent, aperiodic Markov chain[1] $\{X_i^{N,\varepsilon}\}_{i=0}^{\infty}$ on the state space $X^N$, where $\varepsilon > 0$ represents the noises or mutations in the system, or say, some of agents have a probability to drop the previous strategies and simply switch to a new one without any appropriate reason. When the system is buffeted constantly by such small random shocks, in the context of ESS, the distribution of stable strategies will be restored repeatedly. However, an ESS only is stable for the continuous dynamic, but is false in a context of discrete dynamics, under a non-degeneracy condition (Taylor and Jonker, 1978); furthermore, it cannot predict the unique long run equilibrium as the games with multiple stable equilibria. However, if we take into account the experimentation, for given $\varepsilon > 0$, let $P(\varepsilon)$ be the transition matrix[2] of the perturbed process and $\mu$ be the limit invariant distribution[3], then, we have $\mu = \mu P(\varepsilon)$ which always is known as *Global Balance Condition*, which gives the long run trend of the Markov chain. The notion of SSE can be defined as follows.

**Definition 1** *The population state* $(x \in X^N)$ *is stochastically stable if it has positive weight in the limit invariant distributions*$(\mu^{N,\varepsilon})$ *of Markov Chains, i.e.* $\mu_t(i) = \Pr(X_t = i)$ *, when the mutation* $\varepsilon$ *goes arbitrarily small or zero:*

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[1] A Markov chain is collection of random variables $\{X_n\}$ with the Markov property that the future state $(i_0, i_1 \cdots i_n)$ is conditionally independent of the past states, given the current state. Formally,

$$\mathrm{P}(X_{n+1} = i | X_1 = i_1, X_2 = i_2, \ldots, X_n = i_n) = \mathrm{P}(X_{n+1} = j | X_n = i) = P_{ij},$$

[2] The transition matrix is the represent of transitions between the states in the state space, for example, $P_{ij}$ can be interpreted as the probability of transition from state $i$ to state $j$.

[3] Such distribution, or say, equilibrium vector, gives the long run trend of the Markov chain. It shows that the random process will come closer and closer to a stationary state, independent of the initial distribution, under the power of transition matrix.

$$\lim_{\varepsilon \to 0} \mu^{N,\varepsilon}(x) > 0.$$

A stochastically stable population is a limit distribution vector as $\varepsilon \to 0$ and $t \to \infty$. Intuitively, when the noise goes very small, we can ignore the intermediate states (or say, transient states) and ensure that the system will converge to one of stable states in the long run.

**Definition 2*:*** *Absorbing State: a state $\sigma^*$ of a Markov Chain is absorbing if* $\Pr(\sigma^*|\sigma^*) = 1$.

If the population dynamic is an absorbing Markov chain, the selection of the final stable state with probability one highly depends on the initial state, rather, in the case of no absorbing state, namely a regular Markov chain, the stationary distribution is concentrated on one of pure equilibria with a probability closed to one, as the noise is small. Furthermore, in the latter case, the final state is independent on the initial state and the dynamic processes (Young, 1993). For some applications, we will see the tight connections between the risk dominant strategy and the SSE, or say, no intermediate strategies are selected in the long run (i.e. Kandori *et al*. (1993), Ellison (1993), and Ellison (2000) as well). Hence, the main idea of SSE is that the vanishing mutation rate is a selection mechanism which choose the comparative stable absorbing states of the multiple equilibria, since the number of mutation (or the cost of switching, or the length of path) determines the final states selected, that is, the states can be achieved through minimum mutations. The highlight of this approach is able to select from strict Nash equilibria comparing to some of other refinements. However, it should be pointed out that if the agents make mistakes than can be thought, in this case, the process cannot converge to absorbing states, or say, the SSE only can be observed when the noise goes to zero.

Thus, the basic issue here is to design an algorithm for the limit distribution of the population. In the case of Young (1993), he applies a method of shortest path, and shows the risk dominant equilibrium is the unique SSE in $2 \times 2$ coordination games.

**Theorem 1** *(Young (1993), p.72) let $\Gamma$ be a $2 \times 2$ matrix game with two strict Nash equilibria in pure strategies. The generically stable equilibria are the weakly risk dominant Nash equilibria.*

Considering the table 2 as a numerical example, by the best reply rule, $\Pi(Stag|n) > \Pi(Hare|n)$, then we have the critical mass $n > n^* = [N(b-c) + a - b]/(a + b - c - d)$, where $n$ is the number of playing strategy stag, $N$ is the whole population, say $n^* = (7N - 3)/9 \approx 4.44$ as $N = 6$. Hence, the transition of $0 \mapsto N$, it needs at least 5 mutations, in contrast, the transition of $N \mapsto 0$ can be done within no more than 2 mutations, thus the potential state of playing Hare needs more smaller cost to be stable, say, the risk dominant strategy, say Hare, is the only SSE.

In summary, the evolutionary process will be perturbed from the equilibrium or leaving the path of best response from an absorbing state if agents make mistakes, and furthermore, it is not predictable as the system is not determined, none can predict the conventional equilibrium always being played. However, the stochastically stable convention, of which limit stationary distribution is close to a specific convention, will be observed in the long run when the probability of noise goes to zero or very small. We can observe that the final convergence has nothing to do with the payoff structure of coordination and initial states, but the length of path to the stable is co-determined by the payoff structure and the population size.

# 3. The Model of Institutional Emergence

## 3.1 The Basic Elements of the Model

### 3.1.1 The Networks

We consider a finite set of agents, or say, a population, is denoted by $N$, with members $i = 1,2, \cdots n,$ and $n \geq 2$, connected in a network $g$, and assume that the agents are bounded rational and partially informed of the game being played. Formally, let $l_{ij} = \{ij| \, i,j \in N, and \; i \neq j\}$ be the link between the agent $i$ and $j$, and the links represents the interactions (can be directly or indirectly) among the agents. We suppose that $l_{ij} = \{0,1\}$, and if $l_{ij} = 1$ say that the agent $i$ has a link with agent $j$, obviously, one agent cannot connect with himself, say $l_{ii} \notin g$. If $l_{ij} = l_{ji}$, the network $g$ is undirected, conversely, the network $g$ is directed if $l_{ij} \neq l_{ji}$, for any agents $i$ and $j$.

The neighbors of agent $i$ are $j \in N$ who connect. The set of neighbors of agent $i$ is denoted by $N_i(g) = \{j \in N| \, ij \in g\} = k_i$. In addition, the degree $(d_i(g))$ of an agent $i$ can be presented by the number of its neighbors, say, $d_i(g) = N_i(g) = k$. The degree distribution $(P(k))$ of a network is a description of the relative frequencies of agents that have different degrees, and is the fundamental characteristic of the network (Jackson, 2008), thus $\sum P\big(d_i(g)\big) = 1$. The average degree in network $g$ can be defined as $\sum P(k)k$. For example, in a star network, the core agent has $n-1$ neighbors, or say, has degree $n-1$, while the other peripheral agents having degree 1. Hence, the degree distribution of this network is $P(1) = (n-1)/n$, and $P(n-1) = 1/n$ , and the average degree is $2 - 2/n$ . An attractive concern in the study of networks is about a kind of networks whose degree distribution approximately follows a power law, called scale-free network. Many networks are scale free, for instance, World Wide Web, professional collaborative networks, *etc*. (see, e.g. Barabási and Albert (1999), Barabási (2009), among others, for details).

We denote $d_{ij}$ as the distance, which is the length of the shortest path between $i$ and $j$ in the network, between agents $i$ and $j$. Hence, we can define the set of neighbors, which have distance $d$ from agent $i$ as $N_i^d(g)$, for example, in a complete network, for any agent $i$, we have $\sum_{d=0}^{d=\infty} N_i^d(g) = n-1$, since the distance is 1 between any agents. If there is no path between the agents, then the distance is infinite by convention.

**Assumption 1**: *The agents in networks meet each other with probabilities* $\rho_{ij}$*:*

*(1) For each agent i, the probability is* $\rho_{ii} = 0$*;*

*(2) For the agents* $i$ *and* $j$*, the probabilities* $\rho_{ij} \geq 0$*, and* $\sum_{i=1}^{n} \rho_{ij} = 1$*, for all i*

This assumption imposes that the "self-meeting" is not a possibility, or say oneself do not necessarily coordinate himself. In addition, the sub-assumption 2 shows that the agents can meet each other completely as long as the network is complete and the agents interact globally. In the context of global interactions, $\rho_{ij} = 1/(n-1)$, for any $i \neq j$.

### 3.1.2 The Game Setting

A game on a network $g$ can be defined as an interaction situation within a population of $N$ agents. The payoff of agent $i$ is the average payoff in $g$ over her or his $N_i(g)$ neighbors on the same network. The set of actions or strategies $S$: $s_i \in S$, say, for example $S = \{A, B\}$ in the $2 \times 2$ coordination game, which are given in the Table 2 above. For the convenience, we remind here again of the payoff structure, $a > b$ and $a - d < b - c$, hence, $(A, A)$ is the efficient equilibrium, and $(B, B)$ is the risk dominant equilibrium. Furthermore, we suppose that an efficient equilibrium represents an efficient institution, similarly, the risk dominant equilibrium selected means inefficient institutions emerge. The average payoff function of agent $i$ in the network game is then given by,

$$\pi_i = \frac{\sum_{j \in N_i(g)} \pi_i(s_i, s_j)}{k_i}$$

**Definition 3:** *A population state* $n$ *is a Nash equilibrium, if for the agent* $i \in N$*, and each strategy* $s_i \in S$ *such that* $n_i > 0$ *,*

$$\forall\, s_j \in S \quad \Pi_{si}(n) \geq \Pi_{sj}(n)$$

**Assumption 2:** *We apply the myopic best reply rule in the consideration of strategic switches. In a population state* $N$ *for both strategies* $A$ *and:*

*(1) An agent playing strategy* $A$ *switches strategy when*$\Pi(A \mid n) < \Pi(B|n-1)$*, analogously,*

*(2) An agent playing strategy* $B$ *switches strategy when* $\Pi(B \mid n) < \Pi(A \mid n-1)$.

Such assumption implies when the current payoff from games is lower than the potential payoff after switching strategies, the agents will change their strategies, i.e., if the payoff to agent when $n$ of $N$ agents (including the objective agent himself) playing $A$ is less than the payoff he would obtain if he were switch to $B$, when $n-1$ agents (excluding himself) still play the strategy $A$. Alternatively, simply say, the agent only need to compare his current payoff to the expected immediate payoff, because they cannot have much knowledge and computing abilities to form the payoff expectation of the very far future.

## 3.2 The Static Analysis of Institutional Emergence

We start our analysis by considering the social networks are fixed, which means the agents, links, and topology of networks, except the strategies playing, do not change in the process of games. The purpose of this sub-section is to examine the distribution of the stable state when the agents are restricted in the networks.

***Proposition 1:*** *When there are immobile* $N$ *agents,*

*(i) in a complete network* $g^N$*, when* $N < 2(c-d)/(a-d-b+c)$ *, the SSE will be the risk dominant strategy; on contrary, when* $N > 2(c-d)/(a-d-b+c)$ *, the SSE will be the payoff dominant strategy.*

*(ii) in a star network* $g^S$*, there are two SSEs if the central agent adjust strategy firstly; the situation will be same with the case (i) above if the peripheral agents start to change the strategies.*

*(iii) in a circle network* $g^C$*, if the scope of interaction* $m < 2(b-a)/(a-d-b+c)$ *, the ESS will be the risk dominant strategy* $B$ *in the long run, otherwise, the SSE will be the payoff dominant strategy* $A$ *.*

First, we consider a situation where all $N$ agents are fully linked in the network, or say which can be defined a complete network $g^N$. There are $n(n-1)/2$ links in whole networks and $n-1$ links each agent has. We start from a situation where all agents ($N = n$) are playing strategy A, and suppose some of agents will adjust their strategies from A to B unintentionally, denoted as $l_{\varepsilon}^{AB}$. Hence, if any agent $i$ tries to adjust the strategy at least it should meet the myopic best reply rule $\Pi(A \mid n) < \Pi(B \mid n-1)$.We have following, if all agents are playing strategy $A$ initially:

$$\Pi_{iA} = (n - 1 - l_\varepsilon^{AB})a + l_\varepsilon^{AB} c,$$
$$\Pi_{iB} = (n - 2 - l_\varepsilon^{AB})d + (l_\varepsilon^{AB} + 1)b.$$

Then, $\Pi_{iB} > \Pi_{iA}$, we have,

$$l_\varepsilon^{AB} > p^* \frac{(n-1)(a-d)+d-b}{b-c}, \tag{3.1}$$

Analogously, if all agents are playing B initially, then we have,

$$l_\varepsilon^{BA} > p^* \frac{(n-1)(b-c)+c-a}{b-c}, \tag{3.2}$$

Following the logic of SSE, we need to compare the relative cost of strategic adjustment ($l_\varepsilon^{AB}$and $l_\varepsilon^{BA}$), then we get the critic point of population involved is $2(c-d)/(a-d-b+c)$. If the payoff structure meets the condition of Stag-Hunt game, for the simplicity, we apply the numerical example in the Table 2, that is, a=10, b=7, c=0, d=8. Then the inequalities (3.1) and (3.2) can be written respectively as $l_\varepsilon^{AB} > (2n-1)/9$ and $l_\varepsilon^{BA} > (7n-17)/9$. When $n \le 3$ ($n$ is positive integer), then $l_\varepsilon^{AB} > l_\varepsilon^{BA}$, hence, the cost of $n \longmapsto 0$ is greater than that of $0 \mapsto n$, the SSE will converge to risk dominant strategy B in the long term. In contrast, say, when $n \ge 4$, in the large population, the SSE will converge to the payoff dominant one in the long run. This conclusion is opposite to that of Jackson and Watts (2002) which does not consider the population size, and they argue the only equilibrium state is the risk dominant one.

Second, consider a case where the $N$ agents form a star network $g^S$, in which only one agent is at the core, and connect with the other $n-1$ peripheral separated agents. There are $n-1$ links in whole networks and $n-1$ links the central agent has and only one link each peripheral agent has. If the central agent adjusts his strategy first, the strategies of agents in whole network will be changed, and both adjustments just only take one tremble, hence, both of these equilibria are the stable strategy in the long run, also see the cases in Jackson and Watts (2002). Actually, once the central agent is supposed to be the first mover, such situation is equivalent to the issue of two agents' game. If the peripheral agents adjust the strategies first, in which the only investigation is its influence on the central agent. Again, we start from a situation where all agents ($N = n$ ) are playing strategy $A$, then the critical point for the central agent is where the payoff will be improved if he switches to the strategy $B$. The adjustments of strategy of the central agent will lead to the switches of whole network, according to the myopic best reply rule. which have same dynamic mechanism with the complete network above since the central agent is completed connected, the situation of SSE will be same as the complete network in the long run.

Third, consider a case where the $N$ agents form a circle network $g^C$, in which the agents linked from head to tail. There are $n$ links in whole networks and two direct links each agent exactly has. In this situation, the scope of interactions should be taken into account. We suppose each agent $i$ interacts with $m$ neighbors ($m \le n-1$), and some of agents will adjust their strategies, denoted as $l_\varepsilon^{AB}$. Following the same reply rule, we start from the situation where $m+1$ agents playing $A$, then have:

$$\Pi_{iA} = (m - l_\varepsilon^{AB} + 1)a + l_\varepsilon^{AB} c,$$
$$\Pi_{iB} = (m - l_\varepsilon^{AB})d + (l_\varepsilon^{AB} + 1)b.$$

Then,

$$l_\varepsilon^{AB} > p^* \frac{(m+1)(a-d)+(d-b)}{b-c} \tag{3.3}$$

Analogously, if all agents are playing B, then we have,

$$l_{\varepsilon}^{BA} > p^* \frac{(m+1)(b-c)+(c-a)}{b-c} \tag{3.4}$$

Following the logic of SSE, we obtain the critic point determined by $2(b-a)/(a-d-b+c)$ via comparing the cost of adjustment. In the case of Stag-Hunt game shown in Table 2 we rewrite the inequities (3.3) and (3.4) as $l_{\varepsilon}^{AB} > (2m+3)/9$ and $l_{\varepsilon}^{BA} > (7m-3)/9$ respectively. Following the logic of SSE, we need to compare the relative cost of strategic adjustment. Since only $m = 1$ ($m$ is positive integer), $2m+3 > 7m-3$, which means when the agent only interact with one neighbor, the SSE will be the risk dominant strategy $B$ in the long run, otherwise, the SSE will be the payoff dominant strategy $A$.

Hence, in summary, we have learned that the distribution of stable strategy is determined by the payoff structure of games and the topology of networks where the agents interact with each other. In the case of immobility, the agents lose the opportunities to choose the network and only can adjust their strategies to reach an improved situation. For example, if they are fully connected in a large network, a payoff dominant strategy can be feasible in the long run. In addition, if the connectivity is limited, in a circle for instance, a possible way for achieving a better solution is to expand the interaction scope. In words, we suggest that in some societies under strict restrictions, a possible way to break through an inefficient barrier is to make the social connections closer. In this case, the people will have more possibilities to communicate and exchange information, and opinions than that they can before. Therefore, it would be expected that the payoff dominant strategy could be achieved if the social networks are open and the agents can freely leave from or stay at, or move on the networks.

## 3.3 The Dynamic Analysis of Institutional Emergence

### 3.3.1 Link Dynamics

In this subsection, we consider the influence of link dynamics on the equilibrium selection. The decision of links is an obvious feature of social interaction comparing to the classical game theory. Besides the strategic adjustments, the agents also can decide to serve or establish links with his interactive connections at the previous rounds. We propose a new term $K_e - K_m$ as the total cost of link adjustments, $K_e$ is the cost of establishing links, and $K_m$ is the cost of maintain links. At each round, the agents will be informed of the payoffs and the distribution of strategies, and then they can decide to change the strategies, or to serve links with those who playing opposite strategy, or to adjust links and strategies simultaneously. In addition, we also classify the situations whether the links are based on the mutual agreements of agents, and compare the effects of between unilateral links and bilateral links.

**Proposition 2**

*(1) Under strong cost constraints, say,* $a > k_e > k_m > b > d > c$, *the only stable strategy is the payoff dominant one;*

*(2) Under weak cost constraints, say,* $a > b > k_e > k_m > d > c$ *or* $a > b > d > k_e > k_m > c$. *If the* agents *only can adjust the links, whenever the links are unilateral or bilateral, two stable states coexists. If the agents can adjust the links and strategy simultaneous, when the links are unilateral, the payoff dominant strategy will be the stable state, if the trembling level meets* $l_{\varepsilon}^{AB} > l_{\varepsilon}^{BA*}$ or $l_{\varepsilon}^{BA} < l_{\varepsilon}^{AB*}$; *when the links are bilateral, it converges to the risk dominant one*

For the first case, when we are facing strong cost constraints, $a > k_e > k_m > b > d > c$, it is relatively straightforward. Here there is only one possible situation that the agents have no choice to play the strategy $B$, or mis-coordinate due to the cost constraints, they have to stay the payoff dominant strategy for a positive result, furthermore, the adjustment process independents on the network structure. When the agents form links, whatever unilaterally or bilaterally, including establish one and sever one. The agents make strategic decisions on removing or adding links based on the computation of payoff following the assumption of the myopic best reply rule. Given a state where $N$ agents complete connected playing strategy $B$, all of them will lose at least $k_m - b > 0$ at each stage. In such situation, the agents, in one way, can keep the network complete disconnected for avoiding of lose; in the other way, for maintaining the networks and keep profitable for all connections, the agents have to adjust the strategy (with probability $\lambda = 1$) to the payoff dominant one (strategy $A$). Hence, intuitively, in a situation with higher cost to keep the social interactions, the agents will be forced to find the only profitable strategy; otherwise, they will face more uncertainties as in the case of isolation. Conversely, in a situation where all agents are playing the strategy $A$, no agent would change strategy if some agents adjust strategy mistakenly, even some one unintendly leaves from the payoff dominant strategy, and he or she will come back immediately.

Now we move to how agents establish and sever links, and how they adjust the strategies based on the mutual agreements. In the situation, the process of strategic adjustment is a based on the mechanism of link formation. We suppose that agents $i$ and $j$ are linked, say, $l_{ij} = 1$. If the links are unilateral, the agents can freely make the link decisions based on personal payoff at each period. Simply, if we face very strong cost constraints on the link adjustments as above, the network formation will be obviously at the payoff dominant state, and no two agents have incentive to leave from the stable states.

Secondly, under weak cost constraints conditions, the cost of maintaining and establishing the links cannot have a determine influence on the selection of the multiple equilibria. If the links are unilateral, the phenomena of mis-coordination will be avoided due to the weak cost constraints, we thus only need to address the problem of multiple equilibria selection. Now, the agents will have three options to adapt the adjustments of strategies, first of all, they can switch the strategies and keep their network structure, then this situation will be same to the static analysis we have addressed before, and the factors of link adjustments will be meaningless. Secondly, the agents can consider trying to connect new neighbors without any strategic changes. Thus, the formation of network will be continuously changed as the adjustments of the strategies. To see this, we suppose that all agents initially are playing the strategy $A$ and completely connect with each other. Some agents unintentionally switch to play the strategy $B$, and then the agents playing $A$ would like to sever links with those agents at once to avoid of more losses. Hence, the agents playing $B$ are isolated and the former network is collapsed, say, two clusters of agents playing strategy $A$ and $B$ will appear as a temporary network structure. So to speak, as long as the mistakes cannot be avoided, the formation of networks will converge to two stable states with quiet high speed, however, the stable states are not easily kept since the relative lower cost of link adjustments. Furthermore, it is assumed that the agents cannot change their strategies, hence, there are not issues of equilibrium selections, and the structure of distribution of strategies is highly dependents on the initial states of strategies.

However, in contrast to the situation with unilateral links above, we have to discuss the possibilities of mis-coordination since the links are bilaterally. Because the overall payoff of an agent is based on the interacting partners, once this agent changed his strategy in order to adapt some partners, the remaining partners will be trapped into mis-coordination. For example, an agent playing strategy $A$ switches his strategy B in order to his growing interacting partners playing $B$, then, his partners playing A will be exploited by him, but cannot sever the links with him without his permission. As the case of unilateral links formation, we will not discuss the situation where the agents only can adjust strategy. Then, we immediately move to the case of adjusting links. Intuitively, if the benefit of links is zero or negative for both, this link can be severed. We start from a situation where all agents ($n\ of\ N$) playing strategy $A$, some of them ($l_\varepsilon^{AB}$) mistakenly switch to $B$, if the other agents ($n - l_\varepsilon^{AB}$) do not adjust the strategy, the $l_\varepsilon^{AB}$ agents have no incentive to sever these links because they can exploit the other agents playing opposite strategy. Similarly, if we start from a situation where all playing $B$, the remaining ($n - l_\varepsilon^{BA}$) will benefit from the trembles of $l_\varepsilon^{BA}$ and prefer to stay in such state. In words, the networks will be at two stable states if the agents cannot adjust strategies.

Thirdly, still under weak cost constraints conditions, the agents can adjust the links and strategy simultaneously, and then the network is formed endogenously. Suppose that any agents can adjust their strategies and connections as they are under the situation of mis-coordination, thus, the expected payoff function will be depend on the individual strategy (with probability $\lambda > 0$ to change strategy). The probability of meeting the specific strategic species (with probability $\rho_{ij} > 0$, we assumed the fraction of new linked agents playing same strategy is $\rho_{ij}$ after the strategy adjustment). We start from the condition of all agents playing strategy $A$, and $l_\varepsilon^{AB}$ agents make mistakes. Thus, we have the expectation payoff function,

$$\Pi_{iA} = (n - 1 - l_\varepsilon^{AB})a + l_\varepsilon^{AB}c,$$

$$\Pi_{iB} = (1 - \rho_{ij})l_\varepsilon^{AB}d + \rho_{ij}l_\varepsilon^{AB}b + (n - 1 - l_\varepsilon^{AB})b.$$

If the agents want to adjust the strategy, that is, $\Pi_{iB} > \Pi_{iA}$, then,

$$l_\varepsilon^{AB} > \frac{(n-1)(a-b)}{[a+d-b-c+\rho_{ij}(b-d)]} = l_\varepsilon^{AB*}. \tag{3.5}$$

Suppose that $N$ agents playing strategy $B$ at the initial state,

$$\Pi_{iB} = (n - 1 - l_\varepsilon^{BA})b + l_\varepsilon^{BA}d,$$

$$\Pi_{iA} = (1 - \rho_{ij})l_\varepsilon^{BA}c + \rho_{ij}l_\varepsilon^{BA}a + (n - 1 - l_\varepsilon^{BA})a$$

If the agents want to adjust the strategy, that is, $\Pi_{iA} > \Pi_{iB}$, then,

$$l_\varepsilon^{BA} < \frac{(n-1)(a-b)}{[a+d-b-c-\rho_{ij}(a-c)]} = l_\varepsilon^{BA*}. \tag{3.6}$$

Now, in order to compare the relative cost of strategic adjustment, we should compare $l_\varepsilon^{AB*}$ and $l_\varepsilon^{BA*}$. Then, we have, $l_\varepsilon^{AB*} - l_\varepsilon^{BA*} = \frac{\rho_{ij}(n-1)(b-a)(a+b-c-d)}{[a+d-b-c+\rho_{ij}(b-d)][a+d-b-c-\rho_{ij}(a-c)]} < 0$, that is, $l_\varepsilon^{AB*} < l_\varepsilon^{BA*}$. Because the inequalities (3.5) and (3.6) are not parallel, the space will be divided into three domains, which are confined by $l_\varepsilon^{AB*}$ and $l_\varepsilon^{BA*}$, say, we face three possibilities to discuss the convergence states:

I: if $l_\varepsilon^{AB} > l_\varepsilon^{BA*}$, it converges to the payoff dominant one in the long run, since $l_\varepsilon^{AB} > l_\varepsilon^{BA}$;

II: if $l_\varepsilon^{BA} < l_\varepsilon^{AB*}$, it converges to the payoff dominant one in the long run too, since $l_\varepsilon^{AB} > l_\varepsilon^{BA}$;

III: if $l_\varepsilon^{AB*} < l_\varepsilon^{AB} \lesseqgtr l_\varepsilon^{BA} < l_\varepsilon^{BA*}$, in this sub-domain, the comparison between the relative cost is ambiguous. Therefore, both two states are possible.

Now, the issue of the value of trembling is becoming crucial. Obviously, if the values of $l_\varepsilon^{AB*}$ and $l_\varepsilon^{BA*}$ are closer, the possibility of $l_\varepsilon^{AB} > l_\varepsilon^{BA}$ will be higher. Let $l_\varepsilon^{AB*} = l_\varepsilon^{BA*}$, then, we will have $b - d = a - c$, that is, the risk of strategy adjustment is same for both sides. In general, under the Stag hunt games, such ambiguous situations will not disappear. However, the population size and probability meeting same strategy have strong impacts on the achievement of the payoff dominant strategy. As the population size is increasing, the probability of meeting same strategy agents is decreasing, accordingly, the critical mass for the payoff dominant strategy will be increasing. In short, the payoff dominant strategy will be the stable state under certain trembling level, say, $l_\varepsilon^{AB} > l_\varepsilon^{BA*}$ or $l_\varepsilon^{BA} < l_\varepsilon^{AB*}$, in addition, in the ambiguous sub-domain, we cannot tell the stable solution, and as the population size is becoming lager, this ambiguous area will become larger, and it is more difficulty to reach the payoff dominant stable state.

When the links are bilateral, the link adjustment is based on the mutual agreements. The requirements for adjusting the links and strategy simultaneously therefore cannot be satisfied, since the exploiters do not have incentives to sever links, as the links are unilateral shown before. The exploiters will not disappear until all agents are playing the strategy $B$. Hence, as the mutation rate goes to zero, the network will be stable at the risk dominant one.

### 3.3.2 Location Dynamics

Imagine in a city where a number of residents live in different areas and houses. Every day, the most residents are involved in several interactions, i.e. shopping, working, recreation, etc. of which some can be interpreted as some kinds of coordination games. It is nature that we suppose that some of residents are uncomfortable in the activities of interactions, say, in the inefficient situations or mis-coordination. For a better context, they can move to a new location freely, for example, find a new job, move to a new apartment, or just go to a new supermarket for pleasure shopping environment. However, with relative lower possibilities, they move to another city or go to a supermarket 100 kilometers away from home. Hence, we can argue that most interactions take place locally in most environments, i.e., places of work, social communities, *etc*. Consequently, the problem of space limitation or competition may arise. In this subsection, we will address such issues of what roles of location dynamics play in the coordination games.

At the initial state, the whole population $N$ randomly locates on the social networks. Let $\mathcal{L} = \{\ell_1, \ell_2, \cdots, \ell_I\}$ represent the set of possible locations ($\mathcal{L} \geq N \geq 2$) on the networks. In such context, the connections between agents depend on those of locations, or say, the agents are the belongings to the locations, which are neighboring relations. The agents can choose their neighborhoods through moving between locations, after the immigration, the agents have the chances to adjust their strategies to the one that gained highest payoff in the previous period with a positive probability according to the best reply rule.

**Assumption 2:** *at every period* $t$ *, each agent receives the opportunity, (1) to adjust his strategy with probability* $\lambda > 0$*; and (2) to change his location with probability* $\theta > 0$.

We assume that the agents will consider whether change the location each period, after randomly location at the very beginning of the games. The probability $\theta$ and $\lambda$ may depend on the distribution of strategies in the networks and the expectation payoffs. In addition, for simplicity, we do not consider the heterogeneity of agents, locations, and periods. Moreover, we assume that strategy adjustment occurs with greater possibility than to move to new locations, say, $\lambda > \theta$, such assumption also can be found in Ely (2002). At every period, the agents only can observe the average payoff of agents by both strategies at their locations $\ell_i$, rather than the average payoff of the individual strategies. Therefore, the movements of agents only depend on the judgment of location preferences or neighborhood structure at each period. Furthermore, they also need to observe the average payoffs by both strategies at other locations $\ell_j$ and, to calculate their expected payoff after moving to the new locations.

Formally, at each period of plays, the agents will observe the average payoffs of strategies, then, they make decision of strategy and location adjustment sequentially. Before going deeper, we should clarify the differences between link dynamics and location dynamics. On the surface, when an agent is leaving from a location, it means he or she breaks the links with the connections at the same time. It is unnecessary to address their dynamics separately. However, the links between agents are not equivalent to the locations completely. For example, in a complete network, leaving from a location, the agent maybe leave the whole network; conversely, as breaking a link or some links, the agent is still in the network as long as the population $N > 2$. For another example, when a network contains of some (location) clusters, which are formed based on heterogeneous strategies, the agents will make the decision of leaving or staying in the current locations depends on the average payoff of both strategies of their local neighbors. As the Figure 3 shows, each location stands for a strategy and the strategy number stands for its payoff under coordination, and when mis-coordination, the strategy with smaller number will obtain all, hence the payoff structure satisfies the condition of coordination game, and the strategy 1 is the most risk dominant one, the strategy 3 is the most payoff dominant one. Meanwhile, it shows us some pure strategy clusters and strategy mixture clusters. Now, if the agents can calculate the average payoff within two paths, say, they are locally informed or can observe. Considering the agent $A$, $B$, and $C$ as objects respectively, for the agent $A$, all his local neighbors are playing the strategy 1 and have same gains except the agent B, yet, he has no information of other strategies, so he will stay and remain the strategy. For the agent B, his local neighbors have different gains, so he will consider move to a new location, or adjust strategy, or both. For agent $C$, his local neighbors with three kinds of strategy have different gains, and he will find that strategy 2 has highest average payoff and adjust to it with higher probability.

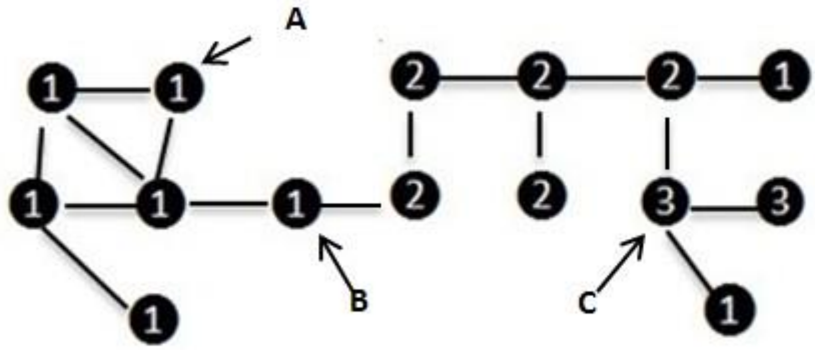


**Figure 3: The Location Dynamics**

After the moves, the agents who have not moved adjust their strategies. However, one point should be clear that the probability ($\lambda$) of strategy adjustment is independent on the strategic mutation rate above. Such probability of adjustment is determined by, for the location instance, the capacity of networks, relative payoffs, and the probability of location adjustment. For example, if the agents can change their locations freely, it is reasonable that they can adjust strategies with a positive probability. However, if they cannot move, and will definitely ($\lambda = 1$) change strategies for keeping connected. Therefore, we will discuss three combinations: (1) moving and adjusting, (2) moving but not adjusting, and (3) not moving but adjusting, of dynamics of locations in each scenario below. In summary, we will discuss,

$$\begin{cases} (1)\ max\{\pi_s(\ell_i, \ell_j)\} \\ (2)\ max\{\pi\ (l_i)\} \\ (3)\ max\{\pi_s(\ell_i)\} \end{cases}$$

Suppose that the agents are fully connected, the payoff function will be as follows, under the conditions as in the Table 1. If the agent at $\ell_i$ is playing strategy $A$, then,

$$\pi_A(\ell_i) = n_{A,i}a + n_{B,i}c$$

Analogously, if the agent at $\ell_i$ is playing strategy $B$, then,

$$\pi_B(\ell_i) = n_{B,i}b + n_{A,i}d$$

, where $n_{A,i}$ is the number of playing strategy $A$ in the neighbors of agents at $\ell_i$, then the number of playing $B$ is $n_{B,i}$.

Hence, at the location $\ell_i$, given the distribution of strategies and the formation of networks, the agent will choose his strategy through comparing the payoffs $\pi_A(\ell_i)$ and $\pi_B(\ell_i)$. Thus, we have,

$$\pi_A(\ell_i) - \pi_B(\ell_i) = (n_{A,i}a + n_{B,i}c) - (n_{B,i}b + n_{A,i}d)$$

, then,

$$\pi_A(\ell_i) - \pi_B(\ell_i) = n_{A,i}(a - d) - n_{B,i}(b - c)$$

, if $\pi_A(\ell_i) > \pi_B(\ell_i)$, it will be required that $n_{A,i}/n_{B,i} > (b - c)/(a - d)$.

Because $a - d < b - c$, from the perspective of population distribution, the neighbors with same strategy will be required, when the efficient strategy is selected. For example, in the Stag-Hunt game as the Table 2 shows, the relative ratio of the agents playing different strategies at least is $n_{A,i}/n_{B,i} > 7/2 = 3.5$, so to speak, the risk dominant strategy can be satisfied by a relative lower heterogeneous network structure.

### 3.4 Social Distance

Formally, the interactive agents are involved in various social relationships in different forms, economically or politically for instance. Furthermore, within the social interactions, the social functions of the agents may therefore be differentiated since the distinct properties of social structure they embedded. We will introduce here the concept of social distance between agents to describe the changing social structure, and investigate its impacts of on the emergence of institutions.

Social distance can be explained in various forms in terms of its underlying problems. Firstly, in general, social distance will be geographical (Akerlof, 1997), for example, each agent has a proprietary location, which represents where the agent is. In a star network, the distance between the core agent and the peripheral agents is 1, and the distance between the peripheral agents is 2. The close topics to such sense of social distance are the research on segregation (Schelling, 1969, 1971, 1978), and on social mobility (Beshers and Laumann, 1967). When we consider social structure as a network, the agents with distinct characteristics, e.g. race, age, gender, etc., may move based on their corresponding demands on the networks. Specifically, in the case of social mobility, i.e. occupational mobility, we may take the path and distance of the flows of the mobility into account, then the differentiation or the gap between them would become significant in terms of the formation and dynamics of the social networks.

Secondly, in practice, social distance also can be psychological, which is subjectively defined on several dimensions, for instance, emotional, mental, and condition contingent, *etc.*

(Trope and Liberman, 2010). Furthermore, it highly depends on the formations and conditions of the social interactions in the specific environments in terms of the underlying organizational structure and civilization level. As Dai *et al.* (2011) argue that the psychological distance will be larger than its expected normal level in the society or environment without relative high-generalized trust level. For example, under the context of military confrontation, the leaders from all sides can sit together, say, the geographical distance is close, however, they will try best to maximize their own interests, even to kill each other deliberately, say, meanwhile the psychological distance among them is quite far. Hence, it is obvious that such kinds of distance cannot be measured precisely.

Now let us reformulate the payoff function considering the distance between interacting agents. Now, we have the baseline payoff function.

$$\Pi_i = \sum \frac{\pi_i(a,l)}{d_{ij}(g)}$$

Before we go further, one point should be pointed out, that is, we only consider the effects of social distance under a static situation. Such simplifying setting can immediately find the role of social distance in individual decisions, and can separate the effects of movements from the investigation. Now we suppose that all agents are playing $A$ initially, and then we have the expected payoff as follows:

$$\Pi_{iA} = (n-1)a + l_\varepsilon^{AB}(c-a)/d_{ij},$$

$$\Pi_{iB} = (n-2)d + b + l_\varepsilon^{AB}(b-d)/d_{ij}$$

If the agents want to adjust the strategy, that is, $\Pi_{iB} > \Pi_{iA}$, then,

$$l_\varepsilon^{AB} > d_{ij}\, p^* \frac{(n-1)(a-d)+(d-b)}{b-c}, \tag{3.7}$$

Analogously, if all agents are playing B initially, then we have,

$$l_\varepsilon^{BA} > d_{ij} p^* \frac{(n-1)(b-c)+c-a}{b-c}, \tag{3.8}$$

Then we compare the relative cost of population transition, the critic point of population involved is $2(c-d)/(a-d-b+c)$. Such result is same to that of in the static situations. Hence, both population size and payoff structure here are the determine factors, the stable equilibrium states are codetermined by these factors, see the cases in the static analysis above.

Now, interestingly, the social distance among agents does not have significant influence on the equilibrium selection. However, the absolute level of the number of trembles in the single decision will be influenced by the social distance. Simply say, the influences of trembles from some agents who locate in distanced places of networks will be smaller than that from closer agents, which can be easily found in the inequalities (3.7) and (3.8). Therefore, if we are trying to approach to an efficient equilibrium, the distant agents should devote more efforts on that, on the contrary, the agents, who have more close neighbors, impose much significant influence on the entire dynamics. However, the stable states still is fundamentally determined by the payoff structure.

In summary, under the dynamic situations, the efficient equilibrium can be the only stable state, when the agents are facing strong adjustment cost constraints. This is an external selection mechanism. When such constraints are becoming weaker or the agents can neglect its impacts, the emergence of efficient equilibrium will depends on the other factors, for instance, when the agents are facing some kinds of certainties, i.e., interactive population size is relative smaller, the efficient equilibrium can be the stable state. Nevertheless, when the agents are freely to adjust strategies and to move on the networks, they will be locked-in the inefficient equilibrium. In terms of institutions, the emergence of efficient institutions can emerge in some conditions, for example, when the inefficient institutions cannot offer enough benefits, and force the agents would try to establish better institutions. However, in general, when the external constraints are not strong as that, the (social) payoff structure will be the fundamental factor, which defines the distribution of benefits of current society, say, if the payoffs to the different human behaviors can be modeled as a Stag-hunt game, then the agents will be locked in, and governed by, the inefficient institutions.

# 4 Conclusion

In this paper, we define institutions are the equilibrium states of games and, consider the emergence of institutions is an evolutionary, stochastic, and (social) structural dependence process of interactions among agents. We found that when the agents are restricted socially and lose the opportunities to choose the network and only can adjust their strategies to reach an improved situation. A possible way to break through an inefficient barrier is to make the social connections closer. In this case, the people will have more possibilities to communicate and exchange information, and opinions than that they can before. When the agents are not restricted socially, say, they can interact with the agents as they want and, move on the networks freely, then, the institutional stability will wander on the two states. Only when the agents are facing some kinds of certainties, the efficient equilibrium can be emerge. However, the most important determinative is the payoff structure of the social games, which define the social distribution of benefits. For example, social distance among agents has significance on the individual decision-making, but it cannot determine the final stable state. In addition, an external mechanism also is found in this paper, that is, when the external cost constraints are very strong, it will force the interactive agents to improve the situation, and efficient institution can emerge.

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