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The views expressed in this paper are those of the authors only, and do not necessarily represent those of the Bank of Israel.

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The Information Content of Inflationary Expectations Derived from Bond Prices in Israel

David Elkayam and Alex Ilek*

Abstract
In this paper we analyze the information content of data on inflationary expectations derived from the Israeli bond market. The results indicate that these expectations are unbiased and efficient with respect to the variables considered. In other words, we cannot reject the hypothesis that these expectations are rational.

The existence of continuous data of this type, which is unique to the Israeli economy, enables us to test a number of hypotheses concerning the nature of price adjustment. The study found that expected inflation is a primary factor in the explanation of current inflation. This result is in agreement with the neo-Keynesian approach according to which the adjustment of prices is costly and as a result price increases in the present are determined primarily by expectations of future price increases. It was also found that inflation in Israel is better explained by the neo-Keynesian approach than by the Classical approach or the 'lack of information' approach according to which current inflation is determined by past, rather than current, inflationary expectations.

Another issue examined in this study is whether inflationary inertia existed in Israel during the 1990s. From conventional estimation of an inflation equation (i.e. using future inflation as proxy for expectations) one can get the impression that there was strong inflationary inertia during this period. However, when data on inflationary expectations from the bond market were used in the estimation, this inertia (i.e. lagged inflation) became negative (and insignificant). This finding raise the possibility that inflationary inertia that is found elsewhere is not a structural phenomenon but an outcome of lack of reliable data on inflationary expectations.

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1. Introduction

During the last decade, the neo-Keynesian approach has become increasingly popular. According to this approach, inflationary expectations are a central factor in the determination of current inflation.\(^1\) It holds that prices are not totally flexible in the price adjustment process due to the fact that it is costly for sellers of goods and services to continually adjust prices. Therefore, it is not optimal for them to adjust their prices instantaneously to equilibrium levels. When adjusting prices, a seller takes into account the inflation expected in future periods when he may refrain from raising prices. Thus, price increases in the present are influenced by price increases expected in the future. The practical difficulty in the empirical analysis and application of this theory lies in the lack of time series data on expected price increases.

One of the accepted methods of estimation is to assume that expectations are rational, to use future rates of inflation as a proxy for expectations and to estimate the equation using the instrumental variables method.\(^2\) The disadvantage of this approach is the difficulty in choosing appropriate instrumental variables and the need to assume that expectations are rational. Another possibility is to use data from surveys of inflationary expectations.\(^3\) However, this approach is also problematic. First, it is difficult to find a continuous series of survey data. Second, those surveyed are often reluctant to make the effort required to forecast inflation. In Israel, however, there exist unique data on inflationary expectations derived from the bond market.\(^4\) The advantages of this data are as follows:

1. The data are continuous.
2. The data are not dependent on the answers given in a survey but are derived from the behavior of the public as it is reflected in the outcome of market transactions.
3. It is not necessary to assume that expectations are rational and moreover, the nature of the expectations can be tested statistically.

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\(^1\) See, for example, Walsh (2003), C.G.G. (1999) and McCallum (1999).
\(^2\) Or maximum likelihood methods (if there is a model). See, for example, Gali and Gertler (1999), Roberts (1995), Fuhrer and Moore (1995) and Rudebusch (2002).
\(^3\) See, for example, Roberts (1995, 1997).
\(^4\) These expectations are calculated as the difference between the (nominal) yield to maturity on non-indexed bonds one year from maturity and the real yield to maturity on indexed bonds one year from maturity. See Yariv (1990).
These expectations, derived from bond market data, constitute an important indicator for the Bank of Israel in its evaluation of inflationary pressures in the economy. It is sometimes claimed that the use of this data by the central bank is misguided or at least that the importance attributed to these data is exaggerated since the public's expectations are primarily adaptive, that is, they reflect inflation which has already occurred and only to a much smaller extent inflation expected in the future. One of the objectives of this study is to test this claim. In what follows, we will try to analyze the nature of expectations data from the bond market and in particular we will try to test whether, in constructing these expectations, the public efficiently uses the information available to it. As part of the analysis, we will try to clarify the point that the existence of strong correlation between inflationary expectations and past or present inflation in itself does not prove that expectations are formed adaptively and furthermore that such correlation is compatible with, and in fact can be derived from, the neo-Keynesian approach which assumes rational expectations.

An additional objective of this study is to test a number of hypotheses regarding the nature of price adjustment which can only be done using this type of data (see below for more details). In particular, we will use the data on inflation expectations, as derived from the bond market, to test the neo-Keynesian theory of price adjustment in the Israeli economy.

An important and controversial issue in the literature is whether inertia exists in the rate of inflation and what is its source. The importance of this question relates to the cost involved in attempts to reduce inflation (or to correct a deviation of inflation from a target level). If there exists considerable inertia in inflation and its source is structural, then any attempt to reduce inflation will be costly even if it is totally credible. Roberts (1997) showed that if some of the public have rational expectations and those of the rest are adaptive, then the empirical specification of the inflation equation will contain an inertial component which will be identical to that which would result from structural factors in a rational model. In most of the empirical studies done thus far (using US or European data) it was found that the rate of inflation has a significant inertial component. However, in most of the studies, researchers did not possess data on expected inflation and were forced to use future

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5 As reflected, for example, in the monthly announcements of the central bank interest rate and in the Bank of Israel Inflation Report.
inflation (ex post) as a proxy for expected inflation. In what follows we will try to determine whether the existence of data on inflationary expectations can shed light on this issue for the Israeli economy. Specifically, we will estimate an inflation equation first using future inflation (ex post) and then using expectations from the bond market. As we shall see, in the first case a significant positive inertial component is found to exist and the coefficient on the output gap is small and insignificant. In the second case, when expectations from the bond market are used, this component becomes negative and insignificant and the coefficient of the output gap increases and becomes significant (as predicted by the new-Keynesian theory).

As stated above, according to the neo-Keynesian approach, inflation in the present period is determined primarily by expectations of future inflation. This is in contrast to the Classical approach developed by Phelps (1967) and Freidman (1968) or the 'lack of information' approach developed by Ball, Mankiw and Reis (2003) according to which inflation is influenced by past, rather than current, inflationary expectations. In what follows, we will use the data on expectations to test which of these theories best explains inflation in the Israeli economy.

In the next section, we will analyze the question of whether expectations from the bond market are unbiased and efficient. As we will see, it will not be possible to reject the hypothesis that expectations from the bond market are rational. In the third section, we shall use another method to test whether there is an adaptive element in these expectations. In the fourth section, we shall use the data on inflationary expectations to test whether inflationary inertia is present in the Israeli economy and in the fifth section we shall test whether the development of inflation in Israel can be better explained by the neo-Keynesian approach or by the Classical Phillips curve approach. The sixth section will be devoted to discussion and conclusions.

2. Are expectations from the bond market rational?

In this section we will try to determine whether inflationary expectations derived from bond market data are rational. In other words, are they unbiased and does the public make efficient use of the relevant information available in the formation of expectations?

Before we present the results of the various tests, we will briefly describe the data which consist of monthly observations of expected inflation for the subsequent
12 months. In order to reduce noise we transformed the data into a quarterly series by averaging over three months. In other words, we use quarterly data of expected inflation for the subsequent four quarters. By the nature of the data, there is an overlap between quarters with regard to the period for which the expectations are formed. This overlap creates an econometric problem that requires special attention. Using annual data (for example, the end of the fourth quarter for each year) solves the problem but significantly reduces the number of observations. In what follows, we will carry out the various tests using both quarterly and annual data.

2a. Testing of unbiasedness and efficiency using quarterly data

The starting point for our tests is the following equation:

\[ (p_t - p_{t-4}) = \alpha + \beta \text{Exp}_{t-4}(p_t - p_{t-4}) + u_t, \quad t = 1, \ldots, T \]

where:
- \( p_t - p_{t-4} \) - rate of inflation during the four quarters prior to (and including) quarter \( t \).
- \( \text{Exp}_{t-4}(p_t - p_{t-4}) \) - expectations of inflation (from the bond market) formed in quarter \( t - 4 \) for the next four quarters.
- \( u_t \) - residual.

The hypothesis of unbiasedness is tested through the joint hypothesis \( \alpha = 0 \) and \( \beta = 1 \). If the expectations are unbiased then the residual \( u_t \) represents unexpected inflation during the period from \( t - 4 \) to \( t \). If expectations are also efficient then the residual should be uncorrelated with any variable that was known when the expectations were formed (that is, in period \( t - 4 \) and all previous periods).

As already mentioned, the data are quarterly but the period they relate to is one year. Therefore, when we move from one quarter to the next, there is an overlap of three quarters since expectations and inflation relate to the four subsequent quarters. In addition, since the monthly Consumer Price Index is published with a lag of two weeks following the end of the relevant month, the correlation can reach up to a lag of four quarters.\(^7\) The problem of serial correlation in the residuals requires

\(^7\) See, for example, Brown and Maital (1981) and Rich (1989).
special attention. It is important to mention that this correlation arises from the overlap of periods and does not say anything about the character of the data.

Despite the existence of serial correlation in the residuals, least squares estimates of the equation are unbiased (and consistent) although statistical tests calculated in the standard manner will be biased. One way of overcoming this difficulty is to use annual data. However, this involves the loss of a large number of observations. (Our data is for the period 1989:1 until 2006:4, i.e. 72 quarterly observations. The move to annual data leaves us with only 18 observations.) Nevertheless, in order to test the robustness of the results we will carry out the various tests with annual data as well. An alternative method (using quarterly data) is to use ordinary least squares but to modify the statistical tests. The standard modification involves the application of Newey-West (1987)’s method. In what follows we will also present the tests using this method. However, a more elegant way of carrying out the tests is demonstrated by Rich (1989). He applies the Hansen (1982) GMM method and simultaneously tests the hypotheses of unbiasedness and efficiency. Below we will briefly describe the method and present the tests.

First, we will briefly explain the idea of testing the efficiency of expectations with regard to any variable $X$ which was known to the public when it formed its expectations. If the public used $X$ in an efficient manner in the formation of its expectations, then any influence of $X$ on future inflation has already been taken into account. Therefore, there should be no systematic link between unexpected inflation and the values of $X$ as known at the time the forecast was made. More specifically, let $u_t$ be unexpected inflation from period $t-4$ to $t$(with expectations formed at time $t-4$). Let us assume that at that point in time, the public knew the value of $X_{t-4}$. If $X$ was used efficiently, then we would expect no correlation to exist between $u_t$ and lagged values of $X$ starting from the fourth lag.

Rich (1989)’s idea is as follows: The estimation of equation (1) using the GMM method with $Exp_{t-4}(p_t - p_{t-4})$ as the instrumental variable gives identical estimates to those of ordinary least squares (but with the correct standard deviations). Suppose that we wish to test whether a group of variables $Z$ is orthogonal to the error of the equation. ($Z$ can include lagged unexpected inflation, lagged inflation or any

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8 Hansen and Hodrick (1980) showed that estimation using all the observations while making the appropriate modification to the standard deviation is more efficient than moving to annual data.
other variable known at the time when the expectations were formed.) The method suggested by Rich (1989), which is based on Hansen (1982), is to estimate equation (1) using the GMM method with $Z$ serving as instrumental variables, in addition to $\text{Exp}(\hat{y}_t - \hat{y}_{t-4})$. Let $J(\text{Exp},Z)$ be the resulting $J$ statistic. The hypothesis that $Z$ is orthogonal to the residuals can be tested using the statistic $Q = T \times J(\text{Exp},Z)$ which has (asymptotically) a $\chi^2$ distribution with degrees of freedom equal to the number of variables in $Z$ where $T$ is the number of observations.\(^9\)

The results of the tests carried out using this method are presented in Tables A and A1 (in Appendix 1). We will first look at Table A. The estimated equation is always equation (1) (estimated by the GMM method, with quarterly data for the period 1989:1 to 2006:4). In the first column we number the equations. The second column contains the instrumental variables that were used. The third and fourth columns contain the values of the constant and slope that were obtained (and in brackets the $t$ statistics). The fifth column contains the $p$ value of Wald’s test of the hypothesis of unbiasedness. The sixth column contains the value of the $Q$ statistic for testing the exogeneity of the additional variables ($Z$). In brackets is the $p$ value for testing the hypothesis that the contribution of these variables is negligible. (For simplicity we shorten the expression $\text{Exp}(\hat{y}_t - \hat{y}_{t-4})$ to $\text{Exp}_{t-4}$.)

The first row of Table A present the GMM estimates where the explanatory variables are the constant and expected inflation (the estimates are identical to OLS).\(^{10}\) As seen from the table, the constant is not significantly different from zero, the intercept is close to one and even at a significance level of 62 percent the hypothesis of unbiasedness cannot be rejected. In the second row, we test the hypothesis that the fifth lag of unexpected inflation is orthogonal to the residuals of the equation. As shown in the sixth column of that row, even at a 56 percent significance level the hypothesis cannot be rejected. The third row presents a similar test of the fifth to eighth lags of unexpected inflation. As seen from the table (the sixth column of that same row) the $Q$ statistic is not significant even at a significance level of 61 percent. In the fourth and fifth rows of the table we test the contribution of the fifth to eighth lags of actual quarterly inflation. Here as well we do not find a significant influence.

\(^9\) For a clear explanation of this test, see pages 217-221 of Hayashi (2000).

\(^{10}\) The $R^2$ of this equation is equal to 0.71
Table A – Testing of unbiasedness and efficiency of inflationary expectations with respect to lagged inflation and lagged unexpected inflation
(All tests are performed on equation (1) for the period 1989:1 to 2006:4, T=72)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Instrumental Variables</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$p$ value $\alpha = 0, \beta = 1$</th>
<th>$Q$ statistic $(T \times J_{statistic})$ and in brackets the $p$ value</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$l, Exp_{t-4,t}$</td>
<td>0.432</td>
<td>0.914</td>
<td>0.62</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>(2)*</td>
<td>$l, Exp_{t-4,t}$ $u_{t-5}$</td>
<td>0.455</td>
<td>0.903</td>
<td>0.52</td>
<td>0.34 (0.56)</td>
<td>1</td>
</tr>
<tr>
<td>(3)**</td>
<td>$l, Exp_{t-4,t}$ $u_{t-5 \to -8}$</td>
<td>0.361</td>
<td>0.886</td>
<td>0.30</td>
<td>2.68 (0.61)</td>
<td>4</td>
</tr>
<tr>
<td>(4)</td>
<td>$l, Exp_{t-4,t}$ $\Delta p_{t-5}$</td>
<td>0.422</td>
<td>0.921</td>
<td>0.55</td>
<td>0.00 (0.99)</td>
<td>1</td>
</tr>
<tr>
<td>(5)</td>
<td>$l, Exp_{t-4,t}$ $\Delta p_{t-5 \to -8}$</td>
<td>0.068</td>
<td>0.945</td>
<td>0.55</td>
<td>1.21 (0.88)</td>
<td>4</td>
</tr>
</tbody>
</table>

* period 1990.2-2006.4.

We can summarize the results in Table A as follows:

1. The hypothesis of unbiasedness of the expectations cannot be rejected (at any reasonable level of significance).
2. Unexpected inflation is orthogonal to its lagged values. In other words, there is no autocorrelation in the unexpected inflation series (apart from the correlation arising from the overlap of periods).
3. Expectations of inflation are efficient with respect to lagged inflation. In other words, when the forecast of inflation is made, the public efficiently uses the information contained in past inflation.

The above results indicate that the expectations from the bond market have the characteristics of rational expectations. In addition, it appears that they are not
adaptive. (In theory, a situation is possible in which adaptive expectations are also rational. In the next chapter we shall provide additional tests for the possibility of the existence of adaptive elements in the expectations.) In what follows we will expand the testing to additional variables that may affect inflation.

In Table A1 in Appendix 1 we similarly test whether, at the time expectations are formed, the public efficiently uses variables that may influence inflation. Following are the variables tested:

\( \Delta i \) - Changes in the Bank of Israel interest rate
\( \Delta e \) - Rate of change in the exchange rate (shekel/dollar)
\( ygap \) - The output gap\(^{11}\)
\( \Delta y \) - Rate of change in output
\( \Delta g \) - Rate of change in local government consumption
\( \Delta m1 \) - Rate of change in the money supply
\( \Delta ur \) - Change in unemployment
\( ugap \) - Unemployment gap
\( \Delta pf \) - Rate of change of world prices of imports
\( \pi^T \) - The inflation target

In order to test the robustness of the findings, we also carried out the tests with respect to the above variables using the more traditional approach, that is, estimation using ordinary least squares with a modification of standard deviations in accordance with the Newey-West method. The findings were similar and are presented in Tables A2 and B2 in Appendix 2.

2b. Testing of unbiasedness and efficiency using annual data

In what follows, we will test the hypothesis that the expectations are rational using annual data. The advantage in the use of annual data is the elimination of the problem of overlap; the disadvantage is of course a smaller number of observations. Nonetheless, this type of test can provide support for the results in the previous section.

\(^{11}\) The output gap and the unemployment gap were estimated by using the HP filter.
In principle, any one of the quarters can be selected for the move to annual data. In other words, the testing can be done with four different samples. However, due to the overlap problem, the tests are clearly not independent. The dependence is greater for consecutive quarters and therefore we chose to carry out the test with two samples - one containing the fourth quarter of each year and one containing the second quarter of each year.

We will first examine the sample containing the fourth quarter data for each year. The data run from the fourth quarter of 1989 until the fourth quarter of 2006 (a total of 18 observations).

Define:

dpy\textsubscript{t} - rate of inflation during year \textit{t};

\text{Exp}_{t-1}(\text{dpy}\textsubscript{t}) – expected inflation during year \textit{t} formed at the end of year \textit{t} – 1;

The estimation of equation (1) with these data gave the following results:

\begin{align*}
(1a) \quad & dpy\textsubscript{t} = 0.058 + 0.908 \text{Exp}_{t-1}(\text{dpy}\textsubscript{t}) \\
& (0.0) \quad (6.4)
\end{align*}

\begin{align*}
R^2 = 0.717 \quad DW = 2.29 \quad s = 3.80 \quad T = 18 \quad (1989 - 2006) \\
F_{\alpha=0.05,\beta=1} = 0.52 \quad (pvalue = 0.61)
\end{align*}

It can be seen that the residuals are not serially correlated and that even at a significance level of 61 percent, the hypothesis of unbiasedness cannot be rejected.

When the second quarter of each year is chosen, the estimation of equation (1) yields the following results:

\begin{align*}
(1b) \quad & dpy\textsubscript{t} = 0.125 + 1.037 \text{Exp}_{t-1}(\text{dpy}\textsubscript{t}) \\
& (0.1) \quad (7.3)
\end{align*}

\begin{align*}
R^2 = 0.779 \quad DW = 1.86 \quad s = 2.88 \quad T = 17 \quad (1990 - 2006) \\
F_{\alpha=0.05,\beta=1} = 0.06 \quad (pvalue = 0.95)
\end{align*}

\textsuperscript{12} The numbers in the parenthesis (below the coefficients) are \(t\) statistics.
It can be seen that a good fit is obtained; the residuals are clean of serial correlation and in addition the hypothesis of unbiasedness cannot be rejected even at a significance level of 95 percent.

In Table B, the results of the test for the efficiency of the inflationary expectations using annual data (the fourth quarter of each year) are presented. It can be seen that the results are similar to those obtained with quarterly data. In addition, none of the variables were found to be significant at a significance level of less than 5 percent. In Table C2 in Appendix 2 we expand the testing of efficiency to additional variables that may affect inflation.

**Table B – Testing for efficiency of inflationary expectations with respect to lagged inflation and unexpected inflation**

(The tests were performed on equation (1) with annual data for the period 1989 to 2006, T=18)

<table>
<thead>
<tr>
<th>The variables in the information group</th>
<th>$p$ value for the F-test of significance of variables in the information group</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{t-1}$</td>
<td>0.52</td>
</tr>
<tr>
<td>$u_{t-1 \text{to} -2}$</td>
<td>0.35 (T=16)</td>
</tr>
<tr>
<td>$dpy_{t-1}$</td>
<td>0.28</td>
</tr>
<tr>
<td>$dpy_{t-1 \text{to} -2}$</td>
<td>0.55</td>
</tr>
</tbody>
</table>

3. Are there adaptive elements in expectations?

The results of the previous section indicate that we cannot reject the hypothesis that expectations are rational. On the other hand, inspection of the data on expectations and inflation reveals the existence of high correlation between expectations (of future inflation) and inflation during the recent past. The claim is often made that this is proof of the adaptivity of expectations. However, the existence of high correlation between expectations and inflation in the recent past is likely to be a result of the possibility that inflation expectations influence prices (and inflation) in the present, as
predicted by the neo-Keynesian approach. In other words, such a correlation is consistent with the neo-Keynesian approach and the rationality of expectations.

In what follows, we will try to identify adaptive elements in expectations using the approach presented in Roberts (1998). Roberts assumes the possibility that a certain proportion \( \alpha \) of the public form their expectations adaptively and the rest \( (1 - \alpha) \) form their expectations rationally. He estimates the parameter \( \alpha \) and finds that according to the data which he analyzed, some 40 percent of the American public form their expectations adaptively while some 60 percent form them rationally. We shall apply this approach to the expectations data from the bond market. The findings will indicate that we cannot reject the hypothesis that 100 percent of the Israeli public form their expectations rationally.

Before presenting the tests, we will examine Diagram 1 and 2 below. Diagram 1 presents two time series: Inflation during the previous four quarters \( (p_t - p_{t-4}) \) and inflation that was expected for those four quarters \( \text{Exp}_{t-4}(p_t - p_{t-4}) \). Diagram 2 presents actual inflation during the previous four quarters and expected inflation for the subsequent four quarters\(^{13}\) \( \text{Exp}_t(p_{t+4} - p_t) \). As can be seen, the correlation between the two series in Diagram 2 is much greater than that between the two series in Diagram 1. This is consistent with the possibility that the expectations derived from the bond market are adaptive. Another possibility is that the development of inflation in the present is influenced primarily by expectations of future inflation, as suggested by the neo-Keynesian approach. The goal of this subsection is to test which of those two possibilities is more likely.

\(^{13}\) Actually, in this diagram and in the relevant regressions to come, we used a 4 quarter moving average of expected inflation.
Diagram 1 – Actual Inflation during the Previous Four Quarters and the Expectation of Inflation at the Beginning of that Period

Diagram 2 – Actual Inflation during the Previous Four Quarters and the Expectation of Inflation during the Subsequent Four Quarters

\[ \text{Exp}(t-4)[p(t)-p(t-4)] \quad \text{and} \quad p(t)-p(t-4) \]
Following Roberts (1998) we assume that expectations are partly adaptive and partly rational. In other words we assume the following equation:

\[ \text{Exp}_{t-4}(p_t - p_{t-4}) = \alpha (p_{t-4} - p_{t-8}) + (1 - \alpha)RAT_{t-4}(p_t - p_{t-4}) \]

where \( \alpha \) is the proportion of people who have adaptive expectations and \( RAT_{t-4}(p_t - p_{t-4}) \) are rational expectations. We will define \( u_t \) as unexpected inflation under rational expectations. Thus,

\[ u_t = (p_t - p_{t-4}) - RAT_{t-4}(p_t - p_{t-4}) \]

Substituting (3) into (2) yields:

\[ \text{Exp}_{t-4}(p_t - p_{t-4}) = \alpha (p_{t-4} - p_{t-8}) + (1 - \alpha)(p_t - p_{t-4}) - (1 - \alpha)u_t \]

In this equation, current inflation and the residual of the equation are correlated. In order to facilitate estimation, we move current inflation to the left side and obtain the following equation:

\[ (4') \ (p_t - p_{t-4}) = \frac{1}{1 - \alpha} \text{Exp}_{t-4}(p_t - p_{t-4}) - \frac{\alpha}{1 - \alpha}(p_{t-4} - p_{t-8}) + u_t \]

As already mentioned, the parameter \( \alpha \) is the “extent of adaptivity” in the expectations. The variables on the right side of the equation are pre-determined and therefore the least squares estimator of the parameter \( \alpha \) using non-linear regression is consistent. However, as a result of the overlap in the quarterly data, the standard deviations of the estimate must be modified. The estimation was performed both with quarterly and annual data.

The estimation of equation (4’) using quarterly data (sample period 1990:1-2006:4) yielded an estimate of –0.065 for the value of the parameter \( \alpha \) with standard deviation (modified using the Newey-West method) of 0.291. Estimation using annual data (sample period 1989-2006) yielded a value of 0.218 for \( \alpha \) and a standard deviation of 0.284. These results indicate that the hypothesis that 100 percent of the public form their expectations rationally cannot be rejected, at any reasonable level of significance.
If we add an additional lag of inflation to equation (2), thus allowing expectations to be influenced by inflation lagged twice, similar results are obtained and neither of the coefficients of lagged inflation are significant.\textsuperscript{14}

4. The use of market expectations to estimate a new Keynesian Phillips curve and test for the existence of inflation inertia

A central element in the neo-Keynesian theory is the assumption of rigidity in price adjustment. The theory assumes that there is a cost to adjusting current prices to equilibrium levels. On the other hand, there is also a cost in maintaining prices that are not at equilibrium levels. The sellers in this type of market are aware of both of these costs, and when determining current prices, they take into account the possibility that in the future they will refrain, for a certain period of time, from adjusting prices to equilibrium levels. As a result, a central factor in deciding to what extent to adjust prices is the price that is expected in the future. In other words, due to the existence of costs to price adjustments, the level of prices expected in the future becomes a central factor in the determination of the extent of adjustment in the present.

There are a number of approaches in the literature to modeling the phenomenon of price rigidity. The outcome of these various approaches is an equation of the following form:\textsuperscript{15}

\[
(5) \quad dp_t = E_t(dp_{t+1}) + \gamma y_t + \varepsilon_t
\]

where:

- \( dp_t \) – inflation in period \( t \).
- \( E_t(dp_{t+1}) \) - inflation expected in time \( t \) for period \( t+1 \).
- \( y_t \) - the output gap (deviation of output from its potential).
- \( \varepsilon_t \) – residual.

The residual \( \varepsilon_t \) can be white noise (as in Rotemberg (1987) and Calvo (1983)) or may

\textsuperscript{14} Using quarterly data yields coefficients of 0.16 and 0.03 with standard deviations of 0.32 and 0.03, respectively. Using annual data yields coefficients of –0.22 and –0.03 with standard deviations of 0.28 and 0.21, respectively.

\textsuperscript{15} For a brief survey of the subject see Roberts (1995) and Roberts (2001).
possess a serially correlated or moving average structure (as in Taylor (1979)). The parameter $\gamma$ is a function of the cost of adjusting prices.

Testing the validity of this equation and/or estimating its parameters is not simple since all the variables appearing on the right side of the equation are unobservable. With regard to expectations, one possibility is to assume rational expectations, to substitute future inflation for expectations and to estimate the equation using the instrumental variable method. Another possibility is to use survey data or data from the bond market, if they exist. With regard to the output gap, the conventional method is to estimate potential output by some filter (usually the Hodrick-Prescot filter).

Many of the attempts at applying this equation to the US economy have been unsuccessful. Fuhrer and Moore (1995) and Fuhrer (1997a, 1997b) showed that if we add lagged inflation to the equation, the fit of the regression improves significantly. In order to explain the influence of these lags, two types of solution have been suggested in the literature: The first is structural, in other words, assumptions are made as to the micro-structure of price adjustment.\(^{16}\) According to this approach, lagged inflation will appear in the equation even when expectations are rational. The implication of this solution is that the declaration and implementation of a program to reduce inflation will be costly even if it is totally credible. In this case, expectations will adjust themselves immediately but lagged inflation will still have an influence. Therefore, the rate of inflation will decline only gradually. This is called inflationary inertia in the literature. (In this situation, it is said that not only prices are sticky but also inflation is sticky.) Another type of solution involves the Roberts (1997) approach which assumes that one part of the public has rational expectations while the other has adaptive expectations. This situation yields an equation which is similar to (5) and, in addition to expectations, includes lagged inflation. Roberts (1997) uses expectations from survey data and suggests ways of testing for the source of inflationary inertia.

The tests in the previous sections have shown that expectations from the bond market are rational and therefore if inflationary inertia is found to exist in the Israeli

---

economy, then we can conclude that it is a result of structural factors and not adaptive elements in expectations.

Much of the empirical work which has tested the neo-Keynesian theory of price adjustment has used the following equation for estimation:

\[
(6) \quad dp_t = \beta_{lead} E_t(dp_{t+1}) + (1 - \beta_{lead}) dp_{t-1} + \lambda y_t + \varepsilon_t
\]

where the parameter \(1 - \beta_{lead}\) is a measure of the extent of inflationary inertia. Rudebusch (2002) estimated an equation similar to (6) using data on the US economy and found that this parameter is equal to 0.7. Gali and Gertler (1999) claim that the use of the output gap as an approximation of marginal cost is an error and instead suggest using unit labor cost. They estimated the equation with data on both the US and European economies. The European data yielded an estimate close to 0 for the coefficient of lagged inflation while the US data yielded estimates of 0.25-0.4.

In all the above studies (apart from Roberts (1997)) the researchers assumed rational expectations. When estimating the equation they substituted future inflation for expectations and estimated using the GMM method or maximum likelihood. It is well known that the success of this kind of estimation is dependent, among other things, on the validity of the instrumental variables. The existence of higher quality data for inflationary expectations is likely to improve the estimation and the understanding of the relationship between the variables. In what follows we will demonstrate these points on the Israeli economy using expectations from the bond market.

For purposes of estimation, taking into account the character of our data and modifying the model to open economy,\(^{17}\) equation (7) was modified as follows:

\[
(7) \quad dpX_t = \beta_{lead} \text{Exp}X_t(dp_{t+4}) + (1 - \beta_{lead}) dpX_{t-4} + \beta_{y} yX_t + \varepsilon_t
\]

where:

\[
dpX_t \text{ -- rate of change in prices during the previous four quarters.}
\]

\[
deX_t \text{ -- rate of change in the foreign exchange rate during the previous four quarters.}
\]

\(^{17}\) See Appendix 3 for details.
$dpX_t^*$ – rate of change of the world's price (in dollar terms) of imported consumer goods during the previous four quarters.

$ExpX_t(dp_{t+4})$ – moving average of inflation expectations during the previous four quarters formed (in each quarter) for the inflation during the next four quarters.

$yX_{t-4}$ – moving average of the output gap during the four quarters previous to quarter $t-4$.

Equation (7) was estimated in three alternative forms, as will be described below, using the GMM method. $dpX_{t+4}$ and $deX_t$, and $deX_{t+4}$ were treated as endogenous variables. The following group of instrumental variables was used in all the estimations: $iX_{t-4}$, $iX_{t-8}$, $yX_{t-4}$, $yX_{t-8}$, $dpX_{t-4}$, $dpX_{t-8}$, $ExpX_{t-4}$, $deX_{t-4}$, $deX_{t-8}$, $dpX^*_{t-4}$, $dpX^*_{t-8}$ and $dpX^*_t$. These variables have already been defined above except for $iX_{t-4}$ which is the moving average of the change in the Bank of Israel interest rate during the four quarters previous to quarter $t-4$.

The estimates obtained for the various alternatives are presented in Table D. The first column presents the conventional variables included in an equation of this kind where future inflation replaces expectations. As can be seen from the results the coefficient on future inflation is significantly less than 1 and thus the coefficient on lagged inflation is significantly positive. These results indicate that when future inflation is used as a proxy for expectations, the results obtained are “reasonable” in the sense that they are similar to those obtained for other economies, although it turns out that the coefficient of the output gap is small and insignificant. In any case, the results indicate a strong and significant influence of lagged inflation.

The second column presents the results for the case in which future inflation is replaced by expectations from the bond market. As can be seen from the table, this replacement leads to a drastic change in the estimates. The coefficient of expectations increases to 1.197 and is not significantly different from 1, which also means that the coefficient of lagged inflation becomes negative (-0.197) and not significant. These results hints that the bond market expectations contain essential information about current inflation and their inclusion obviates the need to include either future or lagged inflation. Notice also that the coefficient for the output gap has increased and has become significant (different from zero).
In the third column we show the results when we drop lagged inflation. As can be seen, the coefficient of market expectations becomes closer to 1 and the rest of the coefficients hardly change.

**Table D – Estimation of equation (7) in various configurations using the GMM method**

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Exp} X_t (dp_{t+4}) )</td>
<td>-</td>
<td>1.197 (9.7)</td>
<td>1.054 (22.2)</td>
</tr>
<tr>
<td>( dpX_{t+4} )</td>
<td>0.608 (6.6)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( dpX_{t-4} )</td>
<td>0.392 (=1-0.608) (4.3)</td>
<td>-0.197 (=1-1.197) (1.6)</td>
<td>-</td>
</tr>
<tr>
<td>( yX_{t-4} )</td>
<td>0.107 (0.8)</td>
<td>0.491 (4.2)</td>
<td>0.387 (4.8)</td>
</tr>
<tr>
<td>( \beta_{\text{lead}} (deX + dpX^<em>)<em>{t+4} - (1 - \beta</em>{\text{lead}}) (deX + dpX^</em>)_{t-4} )</td>
<td>0.179 (5.1)</td>
<td>0.208 (4.3)</td>
<td>0.201 (4.2)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.812</td>
<td>0.887</td>
<td>0.901</td>
</tr>
<tr>
<td>( T \times J \text{ statistic} )</td>
<td>6.7 (0.67)</td>
<td>7.2 (0.62)</td>
<td>8.8 (0.46)</td>
</tr>
</tbody>
</table>

* Rows 1-5 contain the values of the estimated parameters and their \( t \) statistics.

* Row 7 contains the value of the \( J \) statistic multiplied by the number of observations. The number of degrees of freedom appears beside this figure in brackets and the \( p \) value of the significance test according to the chi squared distribution appears in brackets below this figure.

* Estimation period 1989.1 – 2005.3 (T=67)

Comparing the results in columns (2) and (3) to those in column (1) shows that the exclusion of market expectations is liable to lead to an erroneous conclusion with regard to the existence of positive inflationary inertia.\(^{18}\) The fact that in other economies data of this type on inflationary expectations do not exist prevents us from determining whether these findings are unique to the Israeli economy.

\(^{18}\) Or, in other words, a lack of data on inflationary expectations may lead to the erroneous conclusion that inflationary inertia is in fact present.
5. The use of market expectations data to distinguish between the Classical theory and the neo-Keynesian theory of price adjustment

During the late 1960s, following the appearance of Friedman (1968) and Phelps (1967), the Classical theory of price adjustment began to evolve. The theory assumes that in every period a certain proportion of prices is determined for the subsequent period and that there is no overlap between periods. With a few additional assumptions, a price adjustment equation of the following form is obtained: 19

\[ dp_t = E_{t-1}(dp_{t-1}) + \gamma y_t + \varepsilon_t \]

where \( dp_t \) is the change in prices during period \( t \), \( y_t \) is the output gap in period \( t \) and \( E_{t-1}(dp_{t-1}) \) are inflationary expectations for period \( t \) which existed in period \( t-1 \).

It is worth emphasizing that the expectations which appear in equation (8) are those which existed in period \( t-1 \) for inflation in period \( t \). The equation implies that only the unexpected portion of inflation has an effect on the deviation of output from its potential. In other words, if expectations are assumed to be rational, then only surprises in monetary policy can affect real economic activity. This theory was widely criticized during the 1970s and early 1980s and as a result the neo-Keynesian theory began to develop in the mid-1980s.

The inflation equation which is derived from the neo-Keynesian theory is very similar to equation (8) with the main difference being that expectations are formed in period \( t \) for period \( t+1 \). In other words, according to the neo-Keynesian theory current inflation is determined by current expectations of inflation in the future, while according to the Classical theory the main factor determining current inflation is past expectations of current inflation. This may appear to be a minor difference but in fact is a critical one. According to the new approach, changes in monetary policy affect real economic activity in the short run even if they were expected.

In what follows, we will perform tests to determine which approach better explains data from the Israeli economy. In order to do so, we formulate an equation which incorporates both theories:

\[ dp_t = E_{t-1}(dp_{t-1}) + \gamma y_t + \varepsilon_t \]

---

19 See Woodford (2003) for a detailed analysis of the foundations of both the Classical and the neo-Keynesian theories.
\[ (9) \quad dp_i = \phi E_{i-1}(dp_i) + (1-\phi)E_i(dp_{i+1}) + \gamma_i + \epsilon_i \]

If the parameter \( \phi \) is close to 0, then the new theory fits the data better, while if it is close to 1, then the Classical theory is a better fit. It is worth pointing out that in order to perform this test, we require data on expectations of inflation. If such data are not available, the test cannot be performed even if we assume that expectations are rational.

In order to modify the model for an open economy and to take into account the characteristics of our data (i.e., quarterly data with overlap), we formulate the following equation:

\[ (10) \quad dpX_i = \phi \beta_{exp} E_{i-4}(dp_i) + (1-\phi)\beta_{exp} E_{i}(dp_{i+4}) + b_{\gamma}Y_{i-4} + b_{\epsilon}[(deX + dpX^*)]_{t} - (deX + dpX^*)_{i+4} + \epsilon_i \]

Equation (10) was estimated using the GMM method with the same instrumental variables used to estimate equation (7) above. The results of the estimation are presented in Table E for three alternatives: under the assumption \( \phi = 1 \), under the assumption \( \phi = 0 \) and with no restrictions on \( \phi \).

As can be seen from the results appearing in the first row of Table E, if we assume \( \phi = 1 \) (in accordance with the Classical theory), we obtain a reasonable fit \( (R^2 = 0.76) \) and a significant coefficient of expectations (which is not significantly different from 1). The second row of the table presents the results when we assume that \( \phi = 0 \) (in accordance with the neo-Keynesian theory). In this case, we obtain similar coefficients but a much better fit \( (R^2 \text{ increases to } 0.90) \). The third row presents the results for estimation with no restrictions on the parameter \( \phi \). In this case, the estimate of \( \phi \) is negative and not significantly different from 0. It can be concluded from these results that the neo-Keynesian theory provides a better explanation of the data for the Israeli economy than does the Classical theory.
Table E – Estimation of equation (10) under alternative assumptions using the GMM method*

<table>
<thead>
<tr>
<th>Estimated Parameters</th>
<th>$\beta_{exp}$ $(1)$</th>
<th>$b_y$ $(2)$</th>
<th>$b_e$ $(3)$</th>
<th>$\phi$ $(4)$</th>
<th>$R^2$ $(5)$</th>
<th>$T \times J_{statistic}$ $(6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative A:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classical Theory</td>
<td>0.974 (18.4)</td>
<td>0.400 (2.8)</td>
<td>0.295 (4.4)</td>
<td>1.0</td>
<td>0.757</td>
<td>11.4 (0.25)</td>
</tr>
<tr>
<td>$(\phi = 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative B:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neo-Keynesian Theory</td>
<td>1.054 (22.2)</td>
<td>0.387 (4.8)</td>
<td>0.201 (4.2)</td>
<td>0.0</td>
<td>0.901</td>
<td>8.8 (0.46)</td>
</tr>
<tr>
<td>$(\phi = 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alternative C:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$ unrestricted</td>
<td>1.065 (21.1)</td>
<td>0.381 (4.9)</td>
<td>0.196 (3.9)</td>
<td>-0.122</td>
<td>0.902</td>
<td>8.2 (0.41)</td>
</tr>
</tbody>
</table>

* Columns 1-4 contain the estimated parameter and the t-statistics of the test of significance.
* Column 6 presents the value of the J statistic multiplied by the number of observations. Beside it (in brackets) is the number of degrees of freedom and under it (in brackets) appears the $p$ value of the test for significance according to the $\chi^2$ distribution.

Estimation period 1989.1- 2006.3 (T=71)

6. Discussion and conclusions

In this paper we have analyzed the nature of the information contained in the data on inflationary expectations derived from the Israeli bond market. The findings indicate that these expectations are unbiased and efficient with regard to the variables considered. In other words, we cannot reject the hypothesis that these expectations are rational.

The existence of a data series of this type, which is unique to the Israeli economy, enables the testing of a number of hypotheses concerning the nature of price adjustment. Thus, it was found that current expectations (for future inflation) are a central factor in the determination of current inflation, as predicted by the neo-Keynesian approach to price adjustment. It was also found that the development of inflation in Israel is better explained by the neo-Keynesian theory (price rigidity) than the Classical theory or the 'lack of information' theory according to which current
Inflation is determined by past expectations (for current inflation) rather than by current expectations of future inflation.

We also examined the issue of whether inflationary inertia can be identified in the development of inflation in Israel during the 1990s. Estimation of the inflation equation in the conventional manner, i.e., not using expectations from the bond market, supports the existence of strong inflationary inertia. However, when expectations from the bond market are utilized in the estimation, the coefficient of lagged inflation becomes negative and insignificant. This result raises the possibility that a lack of good data on inflationary expectations might lead to an erroneous conclusion regarding the existence of inflationary inertia.

An important issue not dealt with here and which is deserving of study is the risk premium contained in the data on inflationary expectations derived from bond transactions. However, since we used expectations for a period of only one year, the risk component should not have been of critical importance. Nonetheless, this issue should be given the attention it requires. Another point deserving additional attention is a possible switch to monthly data. As discussed earlier, the raw data contain monthly observations (in fact the raw data are daily data) and therefore all the tests which were performed using quarterly data can be done using monthly data. In this case, the problem of overlap will become more serious but on the other hand the number of observations will increase significantly.
Bibliography
Appendix 1 – Testing of efficiency of the inflation expectations using GMM

In the body of the article we tested unbiasedness and efficiency of inflation expectations with respect to lagged inflation and lagged unexpected inflation. In this appendix we expand this test by considering additional economic variables. Following are the variables tested:

- $\Delta i$ - Changes in the Bank of Israel interest rate
- $\Delta e$ - Rate of change in the exchange rate (shekel/dollar)
- $y_{\text{gap}}$ - The output gap\(^{20}\)
- $\Delta y$ - Rate of change in output
- $\Delta g$ - Rate of change in local government consumption
- $\Delta m$ - Rate of change in the money supply
- $\Delta u$ - Change in unemployment
- $u_{\text{gap}}$ - Unemployment gap
- $\Delta pf$ - Rate of change of world prices of imports
- $\pi^T$ - The inflation target

As can be seen from Table A1, the tests of efficiency resulted in a $p$ value of less than 15 percent for four variables - the fifth lag of the output gap, the unemployment gap, the money growth and the inflation target. However, Table B2 in Appendix 2, which presents the efficiency test results using OLS, gives support for the significance of only first two variables listed above - the output gap and unemployment gap (the lags of the last two variables were not significant). However, since data on output are published with a lag of several months, the values of $y_{\text{gap}}_{t-5}$ and $u_{\text{gap}}_{t-5}$ were not yet known when the expectations were formed, i.e., at time $t-4$, and therefore it is possible that the above result is the outcome of the lag in publication of the data. This is supported by the fact that the sixth to eighth lags were not significant (at a significance level of 10 percent). Another reason for the significance of the fifth lag of the output gap and unemployment gap may stem from the difficulty of the public in identifying the potential output and the natural unemployment from the new published data.

\(^{20}\) The output gap and the unemployment gap were estimated by using the HP filter.
In contrast to most of the variables which are published with a lag, there are several variables which the public already know during the same quarter in which expectations were formed. These variables include the Bank of Israel interest rate which is published about a week before the beginning of each month and the exchange rate which is known and accessible to the public on a continuous basis. However, because the Consumer Price Index is published with a lag of two weeks from the end of the relevant month, at the time expectations are formed for a certain quarter, the inflation during that quarter is not yet known. Therefore, the existence of correlation between variables which were known at the time the forecast was made and the error in that same forecast is not proof of a lack of efficiency. Thus, in order to test for the efficiency of expectations of inflation as they appear in Table A1, these variables also appear only from the fifth lag.

### Table A1 – Testing of unbiasedness and efficiency of inflationary expectations with respect to other variables

(All tests are performed on equation (1) for the period 1989.1 to 2006.4, T=72)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Instrumental Variables</th>
<th>α</th>
<th>β</th>
<th>pvalue</th>
<th>Q statistic ((T \times J_{statistic})) and in brackets the pvalue</th>
<th>Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(1, \text{Exp}<em>{t-4, t} ) (\Delta i</em>{t-5})</td>
<td>0.530</td>
<td>0.895</td>
<td>0.48</td>
<td>0.60 (0.44)</td>
<td>1</td>
</tr>
<tr>
<td>(2)</td>
<td>(1, \text{Exp}<em>{t-4, t} ) (\Delta i</em>{t-5 \ to \ -8})</td>
<td>0.652</td>
<td>0.881</td>
<td>0.37</td>
<td>6.16 (0.19)</td>
<td>4</td>
</tr>
<tr>
<td>(3)</td>
<td>(1, \text{Exp}<em>{t-4, t} ) (\Delta e</em>{t-5})</td>
<td>0.340</td>
<td>0.941</td>
<td>0.72</td>
<td>0.47 (0.49)</td>
<td>1</td>
</tr>
<tr>
<td>(4)</td>
<td>(1, \text{Exp}<em>{t-4, t} ) (\Delta e</em>{t-5 \ to \ -8})</td>
<td>0.562</td>
<td>0.939</td>
<td>0.70</td>
<td>1.74 (0.78)</td>
<td>4</td>
</tr>
<tr>
<td>(5)</td>
<td>(1, \text{Exp}<em>{t-4, t} ) (ygap</em>{t-5})</td>
<td>0.609</td>
<td>0.921</td>
<td>0.73</td>
<td>2.08 (0.14)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Expression</td>
<td>Value 1</td>
<td>Value 2</td>
<td>Value 3</td>
<td>Value 4</td>
<td>Value 5</td>
</tr>
<tr>
<td>---</td>
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<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>(6)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (y</em>{g \text{ap}_{t-5 \to -8}})</td>
<td>0.262</td>
<td>0.977</td>
<td>0.95</td>
<td>5.70 (0.22)</td>
<td>4</td>
</tr>
<tr>
<td>(7)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta y</em>{t-5})</td>
<td>0.405</td>
<td>0.934</td>
<td>0.76</td>
<td>0.60 (0.44)</td>
<td>1</td>
</tr>
<tr>
<td>(8)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta y</em>{t-5 \to -8})</td>
<td>0.524</td>
<td>0.957</td>
<td>0.90</td>
<td>3.89 (0.42)</td>
<td>4</td>
</tr>
<tr>
<td>(9)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta g</em>{t-5})</td>
<td>0.322</td>
<td>0.993</td>
<td>0.91</td>
<td>0.15 (0.70)</td>
<td>1</td>
</tr>
<tr>
<td>(10)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta g</em>{t-5 \to -8})</td>
<td>0.483</td>
<td>0.974</td>
<td>0.86</td>
<td>2.48 (0.65)</td>
<td>4</td>
</tr>
<tr>
<td>(11)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta m</em>{t-5})</td>
<td>0.452</td>
<td>0.902</td>
<td>0.53</td>
<td>2.16 (0.14)</td>
<td>1</td>
</tr>
<tr>
<td>(12)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta m</em>{t-5 \to -8})</td>
<td>0.413</td>
<td>0.887</td>
<td>0.34</td>
<td>3.71 (0.45)</td>
<td>4</td>
</tr>
<tr>
<td>(13)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta u</em>{t-5})</td>
<td>0.309</td>
<td>0.981</td>
<td>0.95</td>
<td>0.49 (0.48)</td>
<td>1</td>
</tr>
<tr>
<td>(14)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta u</em>{t-5 \to -8})</td>
<td>0.347</td>
<td>0.983</td>
<td>0.92</td>
<td>1.37 (0.85)</td>
<td>4</td>
</tr>
<tr>
<td>(15)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (u</em>{g \text{ap}_{t-5}})</td>
<td>0.259</td>
<td>0.960</td>
<td>0.93</td>
<td>2.03 (0.15)</td>
<td>1</td>
</tr>
<tr>
<td>(16)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (u</em>{g \text{ap}_{t-5 \to -8}})</td>
<td>0.123</td>
<td>0.970</td>
<td>0.95</td>
<td>3.04 (0.55)</td>
<td>4</td>
</tr>
<tr>
<td>(17)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta p</em>{t-5})</td>
<td>0.219</td>
<td>1.02</td>
<td>0.85</td>
<td>1.42 (0.23)</td>
<td>1</td>
</tr>
<tr>
<td>(18)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\Delta p</em>{t-5 \to -8})</td>
<td>0.571</td>
<td>0.985</td>
<td>0.74</td>
<td>2.48 (0.65)</td>
<td>4</td>
</tr>
<tr>
<td>(19)</td>
<td>(1, {\text{Exp}<em>{t-4J}}) (\pi</em>{t-5}^T)</td>
<td>0.337</td>
<td>0.779</td>
<td>0.11</td>
<td>2.78 (0.11)</td>
<td>1</td>
</tr>
</tbody>
</table>
Appendix 2 – Testing of efficiency of the inflation expectations using OLS

In the body of the article, results were presented of tests of the efficiency of inflationary expectations using the Rich (1989) approach. In order to further strengthen the results, we also performed the tests using OLS with quarterly data. As described above, the estimated model is the following:

\[
(1) \quad (p_t - p_{t-4}) = \alpha + \beta \text{Exp}_{t-4}(p_t - p_{t-4}) + u_t
\]

Once we have accepted the hypothesis that inflationary expectations are unbiased (in other words \(\alpha = 0\) and \(\beta = 1\)) the model can be rewritten as follows:

\[
( p_t - p_{t-4} ) = \text{Exp}_{t-4}( p_t - p_{t-4} ) + u_t
\]

and therefore,

\[
u_t = ( p_t - p_{t-4} ) - \text{Exp}_{t-4}( p_t - p_{t-4} )\]

In order to test the efficiency of expectations we ran the regression of \(u_t\) on its lagged values starting from the fifth lag and estimated the standard deviations using the Newey-West method. In order to test orthogonality we ran a regression of \(u_t\) on all variables known at time \(t - 5\) which may have been relevant in the formation of expectations.

As can be seen from Tables A2 and B2, the results are similar to those obtained using Rich’s approach.

**Table A2 - Testing of efficiency of inflation expectations with respect to lagged inflation**

<table>
<thead>
<tr>
<th>Type of variables in the information group</th>
<th>(p) value of the (\chi^2) test for significance of the variables in the information group</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_{t-5})</td>
<td>0.60</td>
</tr>
<tr>
<td>(u_{t-5 \ to \ -8})</td>
<td>0.32</td>
</tr>
<tr>
<td>(\Delta p_{t-5})</td>
<td>0.85</td>
</tr>
<tr>
<td>(\Delta p_{t-5 \ to \ -8})</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Table B2 – Testing of efficiency of inflation expectations with respect to other variables

<table>
<thead>
<tr>
<th>Type of variables in the information group</th>
<th>$p$ value of the $\chi^2$ test for significance of the variables in the information group</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta i_{t-5}$</td>
<td>0.07</td>
</tr>
<tr>
<td>$\Delta i_{t-5 \to -8}$</td>
<td>0.09</td>
</tr>
<tr>
<td>$\Delta e_{t-5}$</td>
<td>0.46</td>
</tr>
<tr>
<td>$\Delta e_{t-5 \to -8}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$y_{gap_{t-5}}$</td>
<td>0.04</td>
</tr>
<tr>
<td>$y_{gap_{t-5 \to -8}}$</td>
<td>0.22</td>
</tr>
<tr>
<td>$\Delta y_{t-5}$</td>
<td>0.72</td>
</tr>
<tr>
<td>$\Delta y_{t-5 \to -8}$</td>
<td>0.49</td>
</tr>
<tr>
<td>$\Delta g_{t-5}$</td>
<td>0.59</td>
</tr>
<tr>
<td>$\Delta g_{t-5 \to -8}$</td>
<td>0.54</td>
</tr>
<tr>
<td>$\Delta m_{t-5}$</td>
<td>0.47</td>
</tr>
<tr>
<td>$\Delta m_{t-5 \to -8}$</td>
<td>0.24</td>
</tr>
<tr>
<td>$\Delta ur_{t-5}$</td>
<td>0.50</td>
</tr>
<tr>
<td>$\Delta ur_{t-5 \to -8}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$u_{gap_{t-5}}$</td>
<td>0.16</td>
</tr>
<tr>
<td>$u_{gap_{t-5 \to -8}}$</td>
<td>0.42</td>
</tr>
<tr>
<td>$\Delta pf_{t-5}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\Delta pf_{t-5 \to -8}$</td>
<td>0.45</td>
</tr>
<tr>
<td>$\pi_{t-5}$</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table C2 presents result of the test for the efficiency of the inflationary expectations using annual data. As can be seen from Table C2, none of the variables in the information set is significant ($p$ value less than 15 percent). The lowest $p$ values were obtained for the first lag of the output gap (0.19).
Table C2 – Testing for efficiency of inflation expectations with respect to other variables
(The tests were performed on equation (1) with annual data for the period 1989 to 2006, T=18)

<table>
<thead>
<tr>
<th>The variables in the information group</th>
<th>$p$ value for the F-test of significance of variables in the information group</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta i_{t-1}$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\Delta i_{t-1 \to -2}$</td>
<td>0.56</td>
</tr>
<tr>
<td>$\Delta e_{t-1}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$\Delta e_{t-1 \to -2}$</td>
<td>0.45</td>
</tr>
<tr>
<td>$ygap_{t-1}$</td>
<td>0.68</td>
</tr>
<tr>
<td>$ygap_{t-1 \to -2}$</td>
<td>0.53</td>
</tr>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>0.31</td>
</tr>
<tr>
<td>$\Delta y_{t-1 \to -2}$</td>
<td>0.19</td>
</tr>
<tr>
<td>$\Delta g_{t-1}$</td>
<td>0.34</td>
</tr>
<tr>
<td>$\Delta g_{t-1 \to -2}$</td>
<td>0.62</td>
</tr>
<tr>
<td>$\Delta m_{t-1}$</td>
<td>0.31</td>
</tr>
<tr>
<td>$\Delta m_{t-1 \to -2}$</td>
<td>0.49</td>
</tr>
<tr>
<td>$\Delta ur_{t-1}$</td>
<td>0.61</td>
</tr>
<tr>
<td>$\Delta ur_{t-1 \to -2}$</td>
<td>0.71</td>
</tr>
<tr>
<td>$ugap_{t-1}$</td>
<td>0.80</td>
</tr>
<tr>
<td>$ugap_{t-1 \to -2}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$\Delta pf_{t-1}$</td>
<td>0.41</td>
</tr>
<tr>
<td>$\Delta pf_{t-1 \to -2}$</td>
<td>0.67</td>
</tr>
<tr>
<td>$\pi^T_{t-1}$</td>
<td>0.94(T=14)</td>
</tr>
</tbody>
</table>
Appendix 3 – Derivation of equation (7) (the new Keynesian Phillips curve for an open economy)

The starting point is the common assumption (for an open economy) that the Consumer Price Index (CPI) is an aggregate of two components as follows:

\[(A1)\quad p_t = w p_t^f + (1 - w) p_t^h\]

where:

- \(p\) - the (log of) general price level
- \(p^h\) - the (log of) price of locally produced goods
- \(p^f\) - the (log of) price of imported goods
- \(w\) - the weight of the imported goods in the CPI

In terms of inflation rates equation (A1) takes the form:

\[(A2)\quad dp_t = w dp_t^f + (1 - w) dp_t^h\]

For the inflation of the locally produced goods we assume equation (6), that is:

\[(A3)\quad dp_t^h = \beta_{lead} E_t(dp_{t+1}^h) + (1 - \beta_{lead}) dp_{t-1}^h + \lambda y_t,\]

where:

- \(y\) - the output gap

Now we express \(dp_t^h\) in terms of \(dp\) and \(dp^f\) from equation (A2) and substitute it into equation (A3). After some algebra we get:

\[(A4)\quad dp_t = \beta_{lead} E_t(dp_{t+1}) + (1 - \beta_{lead}) dp_{t-1} + (1 - w) \lambda y_t + w[dp_t^f - \beta_{lead} E_t(dp_{t+1}^f) - (1 - \beta_{lead}) dp_{t-1}^f]\]

For the inflation of the imported goods we assume:

\[(A5)\quad dp_t^f = de + dp^*\]

where:

- \(de\) - exchange rate changes
- \(dp^*\) - world inflation

Equation (A5) represents the assumption of complete and immediate exchange rate pass-through.

After substitution of equation (A5) into (A4) we get the final equation for CPI inflation, in terms of measurable variables:

\[(A6)\quad dp_t = \beta_{lead} E_t(dp_{t+1}) + (1 - \beta_{lead}) dp_{t-1} + (1 - w) \lambda y_t,
\[+ w[(de + dp^*) + \beta_{lead} E_t(de + dp^*_t)_{t+1} - (1 - \beta_{lead})(de + dp^*)_{t-1}]\]
<table>
<thead>
<tr>
<th>Year</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999.01</td>
<td>M. Beenstock, O. Sulla</td>
<td>The Shekel’s Fundamental Real Value</td>
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<td>2004.03</td>
<td>D. Elkayam</td>
<td>The Information Content of Inflationary Expectations Derived from Bond Prices in Israel</td>
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Y. Hecht, H. Pompushko – Normality, Modal Risk Level, and Exchange-Rate Jumps

M. Beenstock, A. Ilek – Wicksell’s Classical Dichotomy: Is the Natural Rate of Interest Independent of the Money Rate of Interest?

Z. Wiener, H. Pompushko – The Estimation of Nominal and Real Yield Curves from Government Bonds in Israel

E. Azoulay, M. Brenner, Y. Landskroner – Inflation Expectations Derived from Foreign Exchange Options

A. Ilek – Aggregation versus Disaggregation - What can we learn from it?

D. Elkayam, A. Ilek – The Information Content of Inflationary Expectations Derived from Bond Prices in Israel

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