Regional Policy: An Alternative Approach

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Abstract

The regional policy problem is often conceived as a trade-off between aggregate efficiency and interregional equity. A policy to allocate investment across regions frequently causes a contradiction in the aims of regional policy, in the sense that it might lead to high rates of aggregate growth accompanied with an unequal distribution of income across regions. On the other hand, a policy to reduce regional inequalities may in fact be inefficient to promote growth of the economy as a whole. It is argued further that under certain conditions the contradiction between aims can be avoided.

KEYWORDS: Optimal Control Theory, Regional Disparities, Regional Policy
JEL: R10

1. Introduction

Recently, there has been an increasing interest in whether and to what extent regional policies may actually promote growth of the economy as a whole (e.g. Martin, 2008). Regional policy normally has both an 'efficiency' and 'equity' component – that is, it is concerned with stimulating growth in the economy as a whole and with narrowing interregional disparities. Indeed, most policy decisions attempt to promote both economic growth and redistribution in favour of less prosperous regions. These aims, however, are not entirely clarified and may contradict each other since maximising national income may do nothing towards reducing regional income differentials. Clarifying the objectives of regional policy, however, is only a first step. One then needs to look for optimal policies to achieve those objectives.

The strongest argument in favour of regional policies lies in the long-run persistence and even widening of interregional disparities. Just as an example, the general presumption is that policies to allocate investment should aim to reduce regional inequalities by focusing on poorer regions. However, the extent to which this should happen is far from clear. An allocation policy in favour of less prosperous regions, although improves regional equity, nonetheless, there is the possibility to reduce the growth rate of the economy as a whole. On the other hand, it is expected that allocating funds in the more productive regions will increase efficiency at the national level. Nevertheless, such allocation, very frequently, causes regional income disparities to increase.

In the light of the above example, it could be argued that there is a 'trade-off' (or 'substitution') between equity and efficiency. Prime facie, then, the aims of efficiency and equity seem to conflict each other. This conflict constitutes a kind of 'dilemma'. In regional economics, goal conflicts are the rule not the exception (Richardson, 1973). There is, however, an alternative possibility; that of complementary between efficiency and equity. This view accepts the argument that is possible to achieve both aims simultaneously. Chief interest, therefore, centres upon the detection of such cases. And so it becomes necessary to seek out the framework within which to examine this issue.

The inspiration for this paper comes from an early work by Intriligator (1964), which applies a well established in economics 'tool-kit', namely the theory of Optimal Control (hereafter OCT) in the problem of regional allocation of investment. Section 2 below lays out the basic model. To complete the discussion, a few words must be said about the 'switching' time of the allocation parameter. Section 3 is devoted to this issue. An attempt is made to examine some of the practical conclusions which

* The findings, interpretations and conclusions are entirely those of the author and, do not necessarily represent the official position, policies or views of the Ministry of Rural Development and Foods and/or the Greek Government.

emerge from this model in Section 4. The following section provides an extension of the analysis to an alternative model augmented with cases of 'compatibility' between equity and efficiency. A sixth section concludes the paper with policy recommendations and suggesting avenues for future research.

2. Regional Allocation of Investment

Rahman (1963) put forward the idea that it is possible to obtain maximum income for the economy as a whole by allocating investment across regions. The problem is to allocate total savings of the economy among two regions[1] at each point in time in such a way that the national economy acquires a predetermined level of total income. An obvious investment decision, therefore, is to allocate the funds in the more productive region. Nevertheless, interregional productivity differential is not the only 'investment-criterion' from optimality point of view. Investing in a low productivity region is also optimal, provided that this region exhibits a relatively high propensity to save. The analysis by Rahman (1963) runs in terms of a sequential discrete decision process, frequently referred to as '*Bellman's Principle of Optimality*'. According to this principle, if certain initial decisions are taken, the remaining decisions must be optimal with respect to the 'status' resulting from the initial decisions in order for the entire set of decisions to be optimal (Bellman, 1959).

Similar conclusions, however, can be derived following the principles of OCT. Indeed, advances in the literature of OCT offer the opportunity for a more sophisticated analysis by Intriligator (1964). Imagine an economy subdivided into two regions, labelled by 1 and 2. Each region produces a homogenous output (Y_i), which is proportional[2] to the regional capital stock (K_i). Thus,

$$Y_i = v_i K_i \text{, with } v_i > 0 \tag{1}$$

Equation (1) is a constant returns production function[3], where v_i is the (fixed) output-capital ratio (capital coefficient). Essentially, this approach draws upon the neoclassical model of growth. Some brief comments on the assumptions pertain this model will set the scene for what follows. To begin with, total (national) savings (S_N) are automatically invested (I_N): $I_N \equiv S_N$ while a constant proportion of output is saved: $S_i = s_i Y_i$, where s_i is the propensity to save. Assuming a constant and regionally invariant rate of depreciation, the rate of fixed capital formation ($\dot{K}_i \equiv dK_i / dt$) equals investment: $I_i \equiv \dot{K}_i$. Based on the assumption of identical regional production functions, then $S_N = s_1 v_1 K_1 + s_2 v_2 K_2$. Provided that $S_N \equiv I_N = \dot{K}_1 + \dot{K}_2$, then

$$\dot{K}_1 + \dot{K}_2 = \gamma_1 K_1 + \gamma_2 K_2 \text{, where } \gamma_i = s_i v_i \tag{2}$$

The term γ_i can be interpreted as the (constant) growth rate of each region. The investment fund for the two regions comes from the savings available to the economy as a whole. A final assumption is in order. Total savings are polled in a central agency and then allocated to only one region. This assumption can be encapsulated in 'allocation parameter' δ, defined as the proportion of savings allocated to region 1, leaving $(1-\delta)$ as the proportion allocated to region 2. Therefore,

$$\dot{K}_1 = \delta(\gamma_1 K_1 + \gamma_2 K_2) \tag{3}$$

$$\dot{K}_2 = (1-\delta)(\gamma_1 K_1 + \gamma_2 K_2) \tag{4}$$

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[1] Considering an economy with two regions is not uncommon in the relevant literature (e.g. Michel et al, 1983). Similar models, however, were developed in a multiregional context (e.g. Ohtsuki, 1971).

[2] The assumption of proportionality of output-capital implies absence of any technological progress; a not so unrealistic assumption if one adopts a short-run planning horizon.

[3] Equation (1) can be derived from a 'conventional' Cobb-Douglas production function: $Y_i = K_i^v L_i^{v-1}$, which can be expressed as $\bar{y}_i = \bar{k}_i^v$, where $\bar{y}_i = Y_i / L_i$ is labour productivity and $\bar{k}_i = K_i / L_i$ denotes the capital-labour ratio. In logarithmic terms this production function can be written as $\log \bar{y}_i = v \log \bar{k}_i$; an expression equivalent to equation (1). Constant returns are ensured by the assumption that the sum of the factor coefficients is equal to 1.

Equations (3) and (4), the 'equations of motion', describe the evolution of the 'state variables' (K_i) as a function of the 'decisions' taken at any point in time, reflected by the 'control variable', δ. Following the hypothesis that capital once placed in either region cannot shifted from the other region, then a continuous δ is implied for which $0 \leq \delta \leq 1$. Planners aim to obtain maximum national income at some terminal time, $Y_N(T) = Y_1(T) + Y_2(T)$. More formally, the problem is to maximise the 'objective function' $\underset{\delta}{Max} Y_N(T)$ given equations (3) and (4) and the restriction $0 \leq \delta \leq 1$. This problem can be solved by determining an optimal time path of δ, or alternatively, to choose a $\delta(t)$ sequence which maximizes the associated Hamiltonian function:

$$H = p_1 \dot{K}_1 + p_2 \dot{K}_2 \tag{5}$$

In equation (5) p_i denote the auxiliary (co-state) variables, which in the present context can be interpreted as the 'shadow' price of capital in each region or the price of one additional unit of capital in a region. Using equations (3) and (4), the Hamiltonian function is written as follows,

$$H = p_1 \delta(\gamma_1 K_1 + \gamma_2 K_2) + p_2(1-\delta)(\gamma_1 K_1 + \gamma_2 K_2) \tag{6}$$

Equation (6) can be expressed in alternative terms as,

$$H = [\delta(p_1 - p_2) + p_2](\gamma_1 K_1 + \gamma_2 K_2) \tag{7}$$

Due to the assumption of constant returns, then, at any point in time the optimal path allocates the fund to only one region. Given t the optimal solution is either $\delta^*(t) = 0$ or $\delta^*(t) = 1$[4]. If $\delta^*(t) = 1$, then $\dot{K}_2 = 0$ and region 1 receives the funds. Conversely, $\delta^*(t) = 0$ implies that $\dot{K}_1 = 0$ and the funds are allocated in region 2. In order to arrive at transparent conclusions, an investment-criterion is necessary. Specifically, $\delta^*(t) = 1$ if $p_1(t) - p_2(t) > 0$; $\delta^*(t) = 0$ if $p_1(t) - p_2(t) < 0$, i.e. funds are allocated to the region in which the 'shadow' price of capital is higher.

Following the 'Maximum Principle' the optimality conditions require that the ad-joint (co-state) equations, $\dot{p}_1 = -\partial H / \partial K_1$ and $\dot{p}_2 = -\partial H / \partial K_2$ must hold, and satisfy the transversality conditions: $p_1(T) = \partial Y_N(T) / \partial K_1(T)$ and $p_2(T) = \partial Y_N(T) / \partial K_2(T)$.

Lemma 1. If $p_i(t)$ is a decreasing function of time, then $\dot{p}_1 / \dot{p}_2 = \gamma_1 / \gamma_2$.

Proof: The ad-joint equations $\dot{p}_1 = -[\delta(p_1 - p_2) + p_2]\gamma_1$ and $\dot{p}_2 = -[\delta(p_1 - p_2) + p_2]\gamma_2$ imply that $\dot{p}_1 / \dot{p}_2 = \gamma_1 / \gamma_2$. Q.E.D.

Lemma 2. At t, $p_1(t) - p_2(t) = p_2(t)[(\gamma_1 - \gamma_2)/\gamma_2]$.

Proof: Given the state equation $\partial H / \partial \delta = [p_1(t) - p_2(t)](\gamma_1 K_1 + \gamma_2 K_2)$, it follows that $(\partial H / \partial \delta) / \partial t = (\dot{p}_1 - \dot{p}_2)(\gamma_1 K_1 + \gamma_2 K_2) + [p_1(t) - p_2(t)](\gamma_1 \dot{K}_1 + \gamma_2 \dot{K}_2)$. In steady-state $\dot{K}_i = 0$, implying $(\partial H / \partial \delta) / \partial t = (\dot{p}_1 - \dot{p}_2)(\gamma_1 K_1 + \gamma_2 K_2)$. Setting $(\partial H / \partial \delta) / \partial t = 0$ yields $(\dot{p}_1 - \dot{p}_2)(\gamma_1 K_1 + \gamma_2 K_2) = 0$ while $\partial H / \partial \delta = 0$ implies $[p_1(t) - p_2(t)](\gamma_1 K_1 + \gamma_2 K_2) = 0$. Since $\partial H / \partial \delta = 0$ and $(\partial H / \partial \delta) / \partial t = 0$, it follows that $[p_1(t) - p_2(t)](\gamma_1 K_1 + \gamma_2 K_2) = (\dot{p}_1 - \dot{p}_2)(\gamma_1 K_1 + \gamma_2 K_2)$. Consequently, $\dot{p}_1 - \dot{p}_2 = p_1(t) - p_2(t)$. By Lemma 1, $(\dot{p}_1 - \dot{p}_2) = \dot{p}_2[(\gamma_1 - \gamma_2)/\gamma_2]$. Provided that $\dot{p}_1 - \dot{p}_2 = p_1(t) - p_2(t)$, then $p_1(t) - p_2(t) = p_2(t)[(\gamma_1 - \gamma_2)/\gamma_2]$. Q.E.D.

Lemma 3. At $t = T$, $p_1(T) - p_2(T) = [(v_1 - v_2)/v_2]p_2(T)$.

Proof: The transversality conditions imply $p_1(T) = v_1$ and $p_2(T) = v_2$. Since $p_1(T)/p_2(T) = v_1/v_2$, then $p_1(T) - p_2(T) = [(v_1 - v_2)/v_2]p_2(T)$. Q.E.D.

Propositions 1 and 2 set out the maximising conditions over a given planning period, let $[0 \ldots T]$.

Proposition 1. At $0 \leq t < T$, $\delta^*(t) = 1$ if $\gamma_1 > \gamma_2$ while $\delta^*(t) = 0$ if $\gamma_1 < \gamma_2$.

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4 This solution is referred to as a typical 'bang-bang' control. See also Smith (1970).

Proof: By Lemma 2, if $\gamma_1 > \gamma_2$ then $p_1(t) - p_2(t) > 0$ implying $\delta^*(t) = 1$ while $\gamma_1 < \gamma_2$ implies $p_1(t) - p_2(t) < 0$, and $\delta^*(t) = 0$. Q.E.D.

Proposition 2. At $t = T$, $\delta^*(t) = 1$ if $v_1 > v_2$ while $\delta^*(t) = 0$ if $v_1 < v_2$.

Proof: According to Lemma 3, if $v_1 > v_2$ then $p_1(T) - p_2(T) > 0$ implying $\delta^*(t) = 1$ while $v_1 < v_2$ implies $p_1(T) - p_2(T) < 0$, and $\delta^*(t) = 0$. Q.E.D.

Suffice at this stage to recognise the policy that it ties in very closely with the problem of optimal regional allocation of investment. Concretely, the optimal allocation policy is to invest initially in the region with the higher growth rate and at the end of the planning period to allocate the funds only in the region with the higher output-capital ratio.

Assume that $\gamma_1 - \gamma_2 > 0$ and $v_1 - v_2 > 0$. In this case, $\delta^*(t) = 1$, $\forall t \in [0 \ldots T]$; an allocation policy consistent with the aim of efficiency. Suppose that $\gamma_1 - \gamma_2 < 0$, $v_1 - v_2 < 0$ and $s_1 - s_2 < 0$, then $\delta^*(t) = 0, \forall t \in [0 \ldots T]$.

Proposition 3. If $\gamma_1 / \gamma_2 > 1$ and $v_1 / v_2 > 1$, then $\delta^*(t) = 1, \forall t \in [0 \ldots T]$.

Proof: Proposition 1 implies that $\delta^*(t) = 1$ at $0 \le t < T$ if $\gamma_1 / \gamma_2 > 1$, while according to Proposition 2, $\delta^*(t) = 1$ at $t = T$ if $v_1 / v_2 > 1$. Q.E.D.

Proposition 4. If $\gamma_1 / \gamma_2 < 1$ and $v_1 / v_2 < 1$, then $\delta^*(t) = 0, \forall t \in [0 \ldots T]$.

Proof: According to Proposition 1, if $\gamma_1 / \gamma_2 < 1$, $\delta^*(t) = 0$ at $0 \le t < T$. Following Proposition 2 $\delta^*(t) = 0$ at $t = T$, if $v_1 / v_2 < 1$. Q.E.D.

Assume that $v_1 - v_2 < 0$ and $s_1 - s_2 > 0$, implying that $\gamma_1 - \gamma_2 > 0$. According to Proposition 1, $\delta^*(t) = 1$ at $0 \le t < T$ and region 1 receives the funds. At $t = T$, given the difference in capital coefficients $\delta^*(t) = 0$ and investment takes place in region 2. Irrespective of the productivity advantage of region 1, $\gamma_1 - \gamma_2 < 0$ and $s_1 - s_2 < 0$ ensures that $\delta^*(t) = 0$ at $0 \le t < T$. At $t = T$, $\delta^*(t) = 1$, given that $v_1 - v_2 > 0$ and funds are transferred to region 1. Overall, the 'switching' sequence of δ can be described by Propositions 5 and 6.

Proposition 5. If $\gamma_1 / \gamma_2 > 1$ and $v_1 / v_2 < 1$, then $\delta^*(t) = 1$ at $0 \le t < T$ and $\delta^*(t) = 0$ at $t = T$.

Proof: By Proposition 1, at $0 \le t < T$ if $\gamma_1 / \gamma_2 > 1$, then $\delta^*(t) = 1$ while Proposition 2 implies $\delta^*(t) = 0$ at $t = T$, if $v_1 / v_2 < 1$. Q.E.D.

Proposition 6. If $\gamma_1 / \gamma_2 < 1$ and $v_1 / v_2 > 1$, then $\delta^*(t) = 0$ at $0 \le t < T$ and $\delta^*(t) = 1$ at $t = T$.

Proof: By Proposition 1, if $\gamma_1 / \gamma_2 < 1$, then $\delta^*(t) = 0$ at $0 \le t < T$ while if $v_1 / v_2 > 1$, then $\delta^*(t) = 1$ at $t = T$, according to Proposition 2. Q.E.D.

A negative relation between $\Delta\gamma_{1,2} \equiv (\gamma_1 - \gamma_2)$ and $\Delta v_{1,2} \equiv (v_1 - v_2)$ results to a switch in δ. According to Propositions 5 and 6, which have been elaborated above this is entirely logical. This inverse relation carries important implications for the conflict (or compatibility) in aims. For the present purpose, though, there is another point that deserves special note. It is of particular interest to estimate the switching time of the allocation parameter. This is examined in Section 3.

3. The Switching Time

From what has been said in section 2, it is clear that OCT is applicable to the problem of regional allocation of investment[5]. A 'switch' in the allocation parameter is suggested. Nevertheless, an estimation of the time that this 'switch' takes place is not provided.

5 Rahman (1966), however, casts a sceptical view and claims that this is feasible only if $\delta = 1$.

This constitutes the departure point for a more elaborated analysis by Takayama (1967). Defining $\dot{p}_1(t)/\dot{p}_2(t)=[p_1(t)-p_1(T)]/[p_2(t)-p_2(T)]$ and using the transversality conditions yields

$$p_1(t)-p_2(t)=\theta p_2(t)+\rho(s_2-s_1)\text{, where } \theta=(\gamma_1-\gamma_2)/\gamma_2\text{, } \rho=v_1v_2/\gamma_2 \quad (8)$$

Given that $v_i>0$, then $\rho>0$. Several cases can be identified. If $\gamma_1>\gamma_2$ and $s_2>s_1$, then $p_1>p_2$, implying $\delta^*=1$. If $s_1=s_2$, investment takes place in the more productive region while $v_1=v_2$ implies that the funds are transferred to the region with the highest propensity to save. If $\gamma_1=\gamma_2$ and $s_2-s_1>0$, then $\theta=0$ and $\rho(s_2-s_1)>0$, implying $p_1>p_2$; hence, $\delta^*=1$. Assuming that $\gamma_1>\gamma_2$ and $s_2=s_1$, then $\theta>0$, $\rho(s_2-s_1)=0$ and $p_1>p_2$; in this case δ remains unchanged. If $\gamma_1>\gamma_2$ and $s_1>s_2$, then $p_2^*<0$. Suppose that $v_1=v_2$; hence, $p_2^*=v_2$, while if $v_2>v_1$, then $p_2^*>v_2$.

A switch in the allocation parameter takes place if $v_1<v_2$. If $v_2<v_1$, then $p_2^*<v_2$. But $v_1>v_2$ and $s_1>s_2$ imply $\gamma_1>\gamma_2$; beyond $p_1=p_2^*$, therefore, $p_1>p_2$ and $\delta=1$. During a given time period, $[t_0\ldots T]$, there is a point, let $t^*\in[t_0\ldots T]$, where $p_2(t)=p_2^*$. If $t_0<t^*$, then $\delta^*(t)=1$ at $t_0\le t<t^*$ and $\delta^*(t)=0$ at $t^*\le t\le T$.

Lemma 4. If $\gamma_1>\gamma_2$ and $s_1>s_2$, then $p_2^*=[(s_1-s_2)/(\gamma_1-\gamma_2)]v_1v_2$.

Proof: The conditions $\gamma_1>\gamma_2$ and $s_2-s_1<0$, imply $\theta>0$ and $\rho(s_2-s_1)<0$. Equation (8) implies $p_1-(\gamma_1/\gamma_2)p_2=\rho(s_2-s_1)$. Setting $p_1=p_2$, yields $p_2^*=[(s_1-s_2)/(\gamma_1-\gamma_2)]v_1v_2$. Q.E.D.

Proposition 7. A switch in δ occurs at $t^*=T-1/\gamma_2\log[(s_1-s_2)v_1/(\gamma_1-\gamma_2)]$.

Proof: By Lemma 1 $\dot{p}_2=-[\delta(p_1-p_2)+p_2]\gamma_2$. Setting $\delta=0$ yields $\dot{p}_2=-p_2\gamma_2$. This is a differential equation with the solution $p_2(t)=Ae^{\gamma_2(T-t)}$. At $t=T$, $p_2(T)=A$ and $p_2(t)=p_2(T)e^{\gamma_2(T-t)}$. By Lemma 3, $p_2(T)=v_2$. Thus, $p_2(t)=v_2e^{\gamma_2(T-t)}$. Setting $p_2(t)=p_2^*$ and using Lemma 4, it follows that $v_2e^{\gamma_2(T-t)}=[(s_1-s_2)/(\gamma_1-\gamma_2)]v_1v_2$. Solving for t yields the expression $t^*=T-1/\gamma_2\log[(s_1-s_2)v_1/(\gamma_1-\gamma_2)]$. Q.E.D.

4. Increasing 'Efficiency' or Improving 'Equity'?

Assume that planners decide to implement an allocation policy in order to promote efficiency, namely to increase national income. As previously, the economy consists of two regions. In addition assume a certain level of interregional inequalities, established in a period prior to the implementation of an allocation policy, $[0\ldots t_0)$. In order to have a concrete vocabulary define the interregional inequalities, or the 'gap' in regional incomes, as $G_{1,2}(t_0)>0$, where $G_{1,2}(t_0)\equiv\Delta Y_{1,2}(t_0)\equiv Y_1(t_0)-Y_2(t_0)$[6]. Let $\gamma_1-\gamma_2>0$ and $v_1-v_2>0$. As has been implied in section 2, the optimal policy, to be implemented at an initial time t_0, is $\delta^*=1$, $\forall t\in[t_0\ldots T]$ and investment takes place exclusively in region 1[7]. Arguably, while national income increases as $t\to T$, the 'gap' in regional incomes also follows a similar trend. The essence of this argument is illustrated in Figure 1.

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6 Assuming that $G_{1,2}(t_0)=0$ will not alter the main conclusions of the model.

7 Similarly, the funds are allocated to region 2, if $G_{1,2}(t_0)<0$, $v_1<v_2$ and $\gamma_1<\gamma_2$; in this case $\delta^*=0$, $\forall t\in[t_0\ldots T]$.

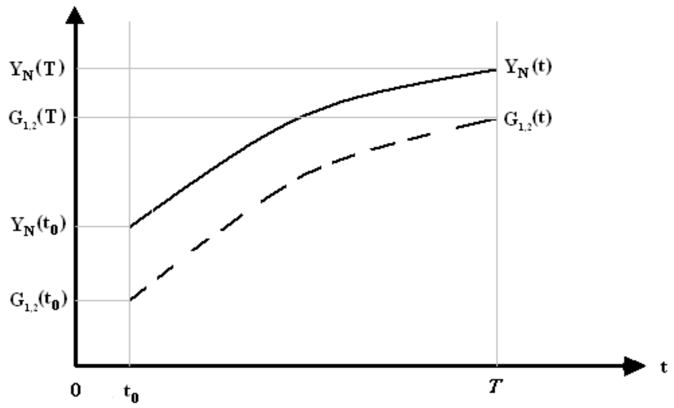

Figure 1: Trade-off between efficiency and equity

This allocation policy maximises national income, $Y_N(t) \to Y_N(T)$ with $Y_N(T) > Y_N(t_0)$. An increase, however, in regional income disparities is also illustrated in Figure 1, $G_{i,2}(t) \to G_{1,2}(T)$ and $G_{1,2}(T) > G_{i,2}(t_0)$. A by-product of this policy is that region 1 retains its advantage and grows at the expanse of the poor region. The reason for this resides into the fact that allocating funds in region 1 enhances the regional growth differentials once they established. This is a clear case of 'cumulative-causation'; a process perpetuating initial regional inequalities. Here lies the 'dilemma' of regional policy: growth of the economy as whole or reducing regional inequalities? This contradiction seems almost ineluctable; the objective function aims to maximise national income and a concern for regional inequalities is not included.

At first glance an allocation policy based on a single value of δ $\forall t \in [t_0 \ldots T]$ seems to sustain 'inherited' regional inequalities. It might be argued, however that persistent regional inequalities are nationally inefficient, since the underutilisation of productive capacity in the 'lagging' region indicates that prosperity of the economy as a whole is lower than it could otherwise be. In this light, implementing allocation policies in favour of the relatively poor regions may also promote national growth. Whether a switch in δ alters this situation or not is questionable and any conclusions are only tentative and circumscribed. There are two main questions here. First, might not the aims of efficiency and equity be complementary instead of competitive? And, second, is there a way to avoid a process of cumulative causation? Section 5 attempts to answer such questions by incorporating the two competitive aims into the ambit of single objective function and shed some further light on whether or not there is a trade-off in regional policy.

5. Compatibility between Equity and Efficiency

Maximising national income may not be entirely preferable by society. Based on the contention that a concern for interregional inequalities might reflect society's preferences, there is a need for an explicit incorporation of the 'equity' aim in the objective function. In this context the problem is how to define regional 'equity'. One obvious candidate is to consider absolute interregional equity at a terminal time, i.e. $\Delta Y_{1,2}(T) = 0$. This aim, however, might be unrealistic. A more pragmatic and feasible aim would be to implement such policies in order to minimise interregional income disparities over a planning period, $[t_0 \ldots T]$; that is $G_{1,2}(t) \to 0$, as $t \to T$. This aim can be specified further in terms of a 'tolerable' level of interregional disparities at the terminal time, $\overline{G}_{1,2}(T)$; $G_{i,2}(t) \to \overline{G}_{1,2}(T)$ and $\overline{G}_{1,2}(T) < G_{1,2}(t_0)$ accompanied by a certain level of national income; $Y_N(t) \to Y_N(T)$ and $Y_N(T) > Y_N(t_0)$.

Allocating funds in such a way as to maximise national income without exceeding a predetermined 'gap' in regional incomes is a rational way to tackle with the dilemma of regional policy, avoiding the process of 'cumulative causation'.

Once this knowledge is introduced, the next important step forward is to define an objective function encompassing 'efficiency' and 'equity'. In this context, it is reasonable to assume that planners pay at least some attention to interregional equity as well as national efficiency. It is possible to portray that consideration by attaching a 'weight' (ϕ) to the aim of interregnal equity. Thus, $\underset{\delta}{Max}[Y_N(T) - \phi G_{1,2}(T)]$. In this objective function the 'efficiency' concern is expressed in terms of national income at a terminal time while the 'equity' criterion is reflected by a negative value of the interregional income differential[8]. Using equation (1) the objective function appears as follows:

$$\underset{\delta}{Max}\{[v_1K_1(T) + v_2K_2(T)] - \phi[v_1K_1(T) - v_2K_2(T)]\}, \text{ where } 0 < \phi < 1 \qquad (9)$$

Given equations (3), (4) along with the restriction $0 \le \delta \le 1$, the conditions implied by the associated Hamiltonian function and the ad-joint equations are identical to those developed in Section 2[9]. The transversality conditions $P_1(T) = v_1(1-\phi)$ and $P_2(T) = v_2(1+\phi)$ imply

$$P_1(T) - P_2(T) = \{[(v_1 - v_2) - (v_1 + v_2)\phi]/[v_2(1+\phi)]\}P_2(T) \qquad (10)$$

Proposition 8. If $G_{1,2}(t_0) > 0$ and $\gamma_1 > \gamma_2$, then δ changes at $t = T$.

Proof: Bearing in mind Proposition 1, $\gamma_1 > \gamma_2$ implies $p_1(t) - p_2(t) > 0$. Hence $\delta^*(t) = 1$ at $t_0 \le t < T$. If $v_1 > v_2$, then $P_1(T) - P_2(T) < 0$. This inequality holds even if $v_1 < v_2$, provided that $\phi \to 1$. Therefore, $\delta^*(t) = 0$ at $t = T$. Q.E.D.

Proposition 9. If $G_{1,2}(t_0) > 0$ and $\gamma_1 < \gamma_2$, then δ remains unchanged $\forall\, t \in [t_0 \ldots T]$.

Proof: According to Proposition 1, $\gamma_1 < \gamma_2$ implies $p_1(t) - p_2(t) < 0$, hence $\delta^*(t) = 0$ at $t_0 \le t < T$. If $v_1 > v_2$, and given that $\phi \to 1$, then $P_1(T) - P_2(T) < 0$. The relation $P_1(T) - P_2(T) < 0$ holds even if $v_1 < v_2$. Therefore, $\delta^*(t) = 0 \quad \forall\, t \in [t_0 \ldots T]$. Q.E.D.

Propositions 8 and 9 indicate that a negative sign is always attached to the difference $P_1(T) - P_2(T)$. Provided that $\partial[P_1(T) - P_2(T)]/\partial\phi < 0$, then the sign of the difference $[P_1(T) - P_2(T)]$ is determined, essentially, by ϕ. This is of critical importance since the difference $P_1(T) - P_2(T)$ specifies which region will receive the funds at $t = T$. It is possible to detect two optimal policies based on the regional growth differentials, each in two variations (scenarios) according to the differentials in capital coefficients.

Tables 2 and 3 set out the optimality conditions implied by Propositions 8 and 9, respectively.

.

Table 1. Optimal Allocation of Investment: $G_{1,2}(t_0) > 0$ and $\gamma_1 > \gamma_2$

	$v_1 > v_2$, $s_1 > s_2$	$v_1 < v_2$, $s_1 > s_2$
$\delta^*(t)$ at $t_0 \le t < T$	**1**	**1**
$\delta^*(t)$ at $t = T$	**0**	**0**

Table 2. Optimal Allocation of Investment: $G_{1,2}(t_0) > 0$ and $\gamma_1 < \gamma_2$

	$v_1 > v_2$, $s_1 < s_2$	$v_1 < v_2$, $s_1 < s_2$
$\delta^*(t)$ at $t_0 \le t < T$	**0**	**0**
$\delta^*(t)$ at $t = T$	**0**	**0**

8 Achieving the 'equity' criterion is equivalent to minimise the objective function. This can be tackled by attaching a negative sign in the relevant component. See Sydsæter et al (2005).

9 Obviously, as $\phi \to 0$ a greater interest is placed upon economic efficiency.

Consider the case $\gamma_1 > \gamma_2$ and $v_1 > v_2$. Initially, the funds are transferred to the relative rich region, in accordance with the aim of efficiency. The weight attached to the aim of equity, however, leads to $P_1(T) - P_2(T) < 0$ [10]; hence $\delta^*(t) = 0$ at $t = T$. In this way, it is possible to achieve both aims at $[t^* \ldots T]$ and avoid perpetuating initial regional inequalities. It may be instructive to examine this argument schematically.

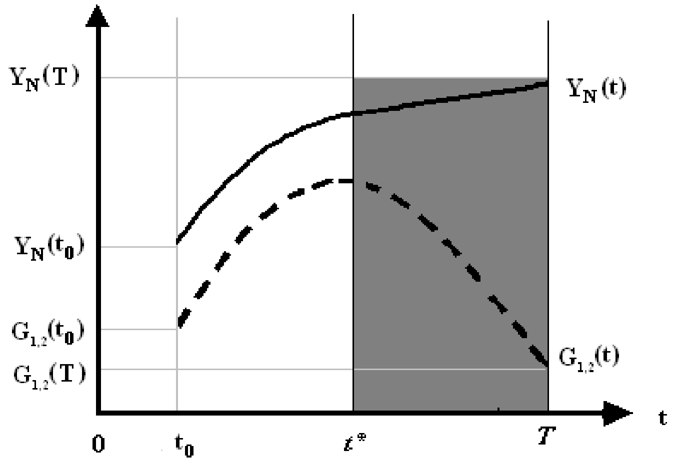

Figure 2: Optimal allocation when $\gamma_1 > \gamma_2$ and $G_{1,2}(t_0) > 0$

It is evident from Figure 2 that national income increases through the entire planning period ($\partial Y_N / \partial t > 0, \forall t \in [t_0 \ldots T]$). The gap in regional income also increases, $\partial G_{1,2} / \partial t > 0$, as $t \to t^*$. Following a 'switch' in the control variable at t^*, however, the gap begins to decline and at $t = T$ attains the predetermined limit or the 'acceptable' boundary set by policy makers. A conclusion is inescapable. Clearly, achieving both aims is feasible at $(t^* \ldots T]$ while the period $[t_0 \ldots t^*]$ corresponds to a conflict between efficiency and equity. The shaded area in Figure 2 corresponds to $\partial Y_N / \partial t > 0$ and $\partial G_{1,2} / \partial t < 0$, as $t \to T$. At $(t^* \ldots T]$ both aims are compatible since $Y_N(t_0) - Y_N(T) < 0$ and $G_{i,2}(t_0) - G_{1,2}(T) > 0$. Whereas an objective function concentrated exclusively on efficiency, in conjunction with the conditions $\gamma_1 > \gamma_2$ and $v_1 > v_2$ perpetuate the 'inherited' regional inequalities, setting a limit of interregional disparities and a weight in the aim of equity, reverse this situation and both aims are compatible. A similar situation can be detected if $\gamma_1 > \gamma_2$ and $v_1 < v_2$. Based on the optimality conditions in Table 1, the period before t^* can be considered as a temporary (or transitory) trade-off. The argument runs as follows. Introducing the weight ϕ alters the switching time. Thus,

$$t^*_\omega = T - 1/\gamma_2 \log[(s_1 - s_2)v_1(1-\phi)/(\gamma_1 - \gamma_2)] \qquad (11)$$

Comparing the expression for t^* in Proposition 7 and given that $v_1(1-\phi) < v_1$, it can be easily shown that $T - t^* > T - t^*_\omega$.

If $\gamma_1 > \gamma_2$ and $v_1 > v_2$, improving capital productivity and the saving behaviour in region 2 will reduce the transition period. If $\gamma_1 > \gamma_2$ and $v_1 < v_2$, then implementing policies or incentives to improve the propensity of save in region 2 has a similar impact upon the switching time.

A clear case to overcome the trade-off is when $\gamma_1 < \gamma_2$ (Figure 3).

10 Recall that $(v_1 + v_2) > 0$ implies $(v_1 - v_2) - (v_1 + v_2) < 0$.

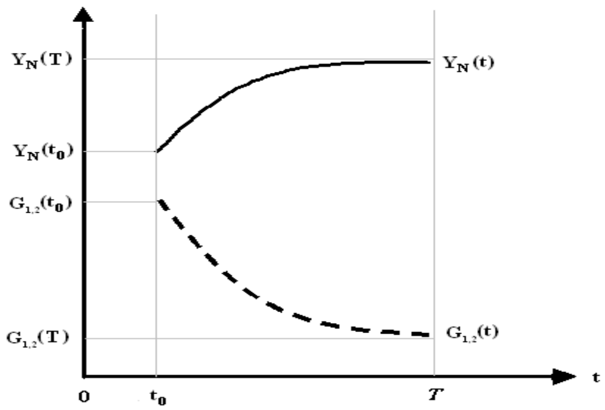

Figure 3: Optimal allocation when $\gamma_1 < \gamma_2$ and $G_{1,2}(t_0) > 0$

Figure 3 indicates that $\partial Y_N / \partial t > 0$ and $\partial G_{1,2} / \partial t < 0$, $\forall t \in [t_0 \ldots T]$ while at $t = T$, $Y_N(t_0) - Y_N(T) < 0$ and $G_{i,2}(t_0) - G_{1,2}(T) > 0$. Obviously both aims are obtainable. If $\gamma_1 < \gamma_2$, then $\delta^*(t) = 0$ $\forall t \in [t_0 \ldots T]$, irrespective of the sign attached to $(v_1 - v_2)$. The conclusion to drawn in that placing greater emphasis on the aim of equity prevents the possibility of widening interregional inequalities, implied by the policy $\delta^*(t) = 1$ at $t_0 \le t < T$ and $\delta^*(t) = 0$ at $t = T$. It is worthy to highlight here that such conclusions are valid as long as the value of ϕ retains the inequality $(v_1 - v_2) < (v_1 + v_2)$.

Assume that $G_{2,1}(t_0) > 0$. Given the objective function $\underset{\delta}{Max}[Y_N(T) - \phi \overline{G}_{2,1}(T)]$, with $0 < \overline{G}_{2,1}(T) < G_{2,1}(t_0)$, the equations of motion and the restriction $0 \le \delta \le 1$, at $t_0 \le t < T$ $\delta^*(t) = 1$ if $\gamma_1 > \gamma_2$, while $\delta^*(t) = 0$ if $\gamma_1 < \gamma_2$. The transversality conditions are modified as follows: $P_1(T) = v_1(1+\phi)$ and $P_2(T) = v_2(1-\phi)$. Hence,

$$P_1(T) - P_2(T) = \{[(v_1 - v_2) + (v_1 + v_2)\phi]/[v_2(1-\phi)]\}P_2(T) \tag{11}$$

Since, $(v_1 - v_2) < (v_1 + v_2)$, then $(v_1 - v_2) + (v_1 + v_2)\phi > 0$, irrespective of the sign attached to $(v_1 - v_2)$. Therefore, $P_1(T) - P_2(T) > 0$. Tables 3 and 4 set out the optimality conditions for $\gamma_1 > \gamma_2$ and $\gamma_1 < \gamma_2$, respectively.

Table 3. Optimal Allocation of Investment: $G_{2,1}(t_0) > 0$ and $\gamma_1 > \gamma_2$

	$v_1 > v_2$, $s_1 > s_2$	$v_1 < v_2$, $s_1 > s_2$
$\delta^*(t)$ at $t_0 \le t < T$	**1**	**1**
$\delta^*(t)$ at $t = T$	**1**	**1**

Table 4. Optimal Allocation of Investment: $G_{2,1}(t_0) > 0$ and $\gamma_1 < \gamma_2$

	$v_1 > v_2$, $s_1 < s_2$	$v_1 < v_2$, $s_1 < s_2$
$\delta^*(t)$ at $t_0 \le t < T$	**0**	**0**
$\delta^*(t)$ at $t = T$	**1**	**1**

Assume that $\gamma_1 - \gamma_2 > 0$. According to Table 3 the control variable remains unchanged, signifying exclusive investment in region 1. In this case the policy $\delta^*(t) = 1, \forall t \in [t_0 \ldots T]$ ensures the compatibility of the two aims, with both Y_N and $G_{2,1}$ following a path similar to that in Figure 2. Of particular interest is the case when $\gamma_1 - \gamma_2 < 0$. The optimal policy, then is $\delta^*(t) = 0$ at $t_0 \le t < T$ and $\delta^*(t) = 1$ at $t = T$. Here, there is a possibility of trade-off at $[t^* \ldots T]$ (Figure 4).

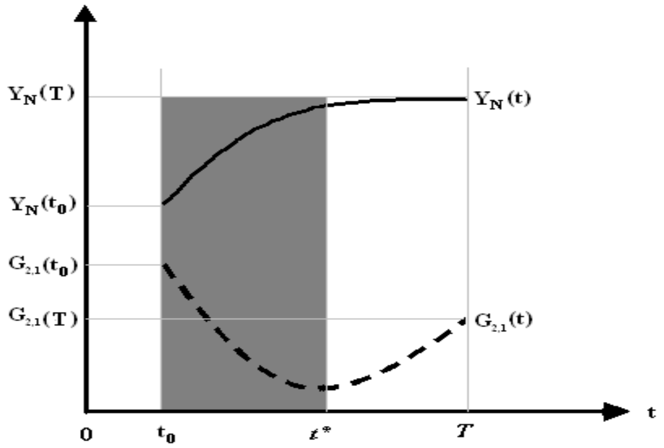

Figure 4: Optimal allocation when $\gamma_1 < \gamma_2$ and $G_{2.1}(t_0) > 0$

Comparing the optimal policies in Tables 1 and 4 it is evident that the distinctive feature is the period that the trade-off takes place. According to Table 1 and Figure 2, a trade-off between equity and efficiency emerges at $[t_0 \ldots t^*]$ while the optimality conditions in Table 4 imply a trade-off, after a 'switch' in δ. According to Figure 4 at $[t^* \ldots T]$ $Y_N(t_0) - Y_N(T) < 0$, while interregional inequalities follow an increasing tendency. It should be noted, however, that $G_{2.1}(t_0) - G_{2.1}(T) > 0$. In the light of the objective function, it could be argued, that this is the result of imposing a boundary in the terminal gap. The variations of policy in Table 4 imply that the trade-off period will be smaller if region 1 improves its propensity to save and capital coefficients.

Generally, introducing an 'equity' weight, the compatibility in aims is a possibility since the allocation policy always favours of the relatively poor region at $t = T$. Setting different values on ϕ, between zero and one, then a set of objective functions, essentially, a 'mix' of criteria, and, by extension, a set of optimal allocation policies.

6. Concluding Remarks

The primary contribution of this paper has been to provide an alternative aspect of policies allocating investment across regions using some of the key concepts relating to Optimal Control Theory. This approach, however, is by nature of restrictive character. As in any modelling situation, such exercise is, by its very nature, limited; it simplifies a complex reality. For example, it is almost an article of faith of regional economics that production is characterised by substantial localization and urbanization economies. These externalities justify policy intervention, especially, from an economic efficiency point of view. Spatial externalities are present in almost every activity, especially those related to knowledge and technology. Thus, the notion of 'efficiency' is an ambiguous concept. Income maximisation subject to given resource constraints is inadequate if spatial and technological externalities are taken into account. Incorporating such externalities in a planning model of regional allocation of investment opens up a promising avenue for future research and a point that should be taken into account by policy makers when they design regional policies and development projects.

If one considers only a reduction in interregional inequality, other aspects, above all the issue of equal distribution of achieved prosperity within a region, are ignored. Indeed, intraregional equity is one indicator which shows how equally the returns of investing in a specific region are distributed. Examination of the interaction between interregional and intraregional equity remains an important are for future research. In addition, there is the question of how policy-makers a 'tolerable' level of regional disparities. However, the analysis in this study provides a set of choices for the regional distribution of available resources, which provide a basis for the design of regional policy. The final selection and application of the models presented here is a challenge for policy making in different geographical and administrative levels.

The important point to grasp is that the analysis in this paper proposes a set of choices that can be described as 'compatibility between equity and efficiency'. Such knowledge assists policy-makers to design optimal regional policies in which the trade-off in aims can be avoided. Indeed, overcoming the trade-off is a difficult and ambitious task; nevertheless not unattainable. Application of the models discussed in this paper, constitute a challenge for policy-makers and practitioners, in different policy sectors and at different administrative levels. Hence, there is a need to rethink future regional policies along the lines of the implementation of more innovative and region-specific development strategies, based on the concept of optimality in decision-making. Thinking towards the future is an essential precondition for investigating where policies are necessary and how they should be shaped.

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