

A theory of capital as value in progress

Cavalieri, Duccio

University of Florence

 $\mathrm{May}\ 2013$

Online at https://mpra.ub.uni-muenchen.de/47197/ MPRA Paper No. 47197, posted 25 May 2013 19:11 UTC

Duccio Cavalieri

(University of Florence) *

A THEORY OF CAPITAL AS VALUE IN PROGRESS

1. On the present state of capital theory.

The theory of capital is an intellectual adventure in the economics of value. It deals with the roles of time and money in production and represents an important chapter of the history of economic thought. Its relevance is unquestionable, but its exact frontiers are not well defined and its present state is controversial. The functional links of the theory of capital with the theory of value are not yet sufficiently cleared. In the literature on capital valuation there are ambiguities, lacunae and misunderstandings. We shall propose some remedies and discuss their implications.

Let us start our analysis from the 'surplus approach' to value and distribution, a backward oriented theoretical framework of ancient origin focused on past costs, where the social product and real wages are treated as exogenous variables, historically determined. The surplus emerges as a residuum: it is the excess of output over the necessary consumption of means of production and workers subsistence goods. It can be expressed either in physical terms, as a surplus product, or in money terms, as a value added ¹.

We shall hereafter distinguish four different versions of cost accounting and denote them as:

- a) the 'cost-of-production' version, in which commodities are priced on the basis of their direct or prime average costs (real cost, both variable and fixed, per unit of output);
- b) the 'cost-plus' variant, which includes the real cost per unit of output and the financial cost of investment per unit of output;
- c) the 'full-cost' variant, which includes the real cost of production, the financial cost of invested capital and other overhead costs (e.g., those for research, advertising, selling and administrative costs, insurance and mortgage payments);
- d) the 'full-cost-plus' variant, which is the sum of the full-cost and the mark-up that producers add on it.

Version a) is the result of the application of the standard cash flow accounting method to the costs and revenues of all operating activities. It does not include investment and financing activities. Version b), the 'cost-plus' variant, should be intended as referring to normal or natural prices (Marx's prices of production, or 'cost-prices') which include in addition to the real cost the financial cost of invested capital. Version c), the 'full-cost' variant, takes into account all the relevant costs, including the overhead costs incurred in operating a business. Version d), the 'full-cost-plus' variant, includes also a mark-up, that is a profit margin.

Corresponding to these distinct methods of accounting there are different concepts of a society's surplus product (a physically heterogeneous mass of commodities, which includes all output components less intermediate consumption and is unaffected by the price system) and surplus value (whose magnitude is affected by price changes). As those of operating surplus and of value added (at market prices and at factor cost, gross and net of depreciation of fixed capital, pre- and after-tax).

^{*} First version of an unpublished paper. The author is a retired full professor of Political Economy at the University of Florence (e-mail address: d.cavalieri@unifi.it).

¹ In contrast with the surplus approach, the neoclassical theories of capital provide a forward-looking subjective perspective, focused on the demand side of the market, that is on people's wants and needs. Capital is a distinct and quantifiable factor of production, reckoned in value terms.

The determinants of natural prices are the direct or prime costs incurred in production, for buying materials and hiring productive services, plus some indirect costs unrelated to the level of output, plus the financial cost of capital. All these elements must be accounted for in capital valuation.

An important problem which arises in this analytical context, and that up to now has been somewhat neglected in the accounting literature on the subject ², concerns the role of the opportunity-cost of invested capital, a virtual cost that expresses the minimum level of return required by the investor and is equal to the best alternative return opportunity foregone by making a determinate choice. How is this virtual cost related to normal profit, the minimum level of profit required by a firm to be competitive?

For an economist, normal profit is an implicit cost. He calls profit the difference between total revenue and the total cost of inputs and distinguishes normal profit from the quasi-rent, or producer surplus, that is from the revenue which exceeds the opportunity-cost of capital in the short-run, when sunk costs have already been incurred and are no longer reckoned by producers. Quasi-rent is the difference between total revenue and variable cost. It is a 'pure' or 'extra' profit, a residual which remunerates in the short-run the fixed factors of production, net of depreciation allowances; a temporary payment to a scarce factor, over and above its opportunity-cost, required to keep the factor in its current use. These notions of profit and quasi-rent are conceptually correct. The total payment to a factor is the sum of three distinct elements: fixed cost, variable cost, which should include normal profit, and a pure or extra profit, which may include a risk element. Fixed factors have an inelastic supply in the short-period, in which they cannot leave their present allocation, but they may have an elastic supply in the long-period, in which all factors become variable and quasi-rents disappear ³.

The professional accountant practice is different. Profit is not defined in balance sheets as the difference between total revenue and total cost, both fixed and variable, but as the difference between total revenue and explicit costs, that is real expenses. This difference includes 'normal' and 'pure' profit. The opportunity-cost of capital, which is an implicit or notional expense, is not considered a cost to be deduced.

The profitability of investment is therefore higher if the bookkeepers practice is followed and lower when the economist method of profit valuation is used. There can be accounting profit and no economic profit. At a firm break-even point, where economic profit is zero, accounting profit is generally positive.

Practitioners have still different views on the earnings of capital. In general, they prefer to rely on stock market valuations, rather than on financial accounting benchmarks.

We have therefore the following set of definitions of profit:

- *profit* (generic notion) = revenue minus cost,
- economic profit = revenue minus explicit and implicit (or opportunity) costs,
- accounting profit, or economic value added = revenue minus explicit costs,
- normal profit, an implicit or opportunity cost,
- pure or extra profit, a surplus (a residual).

This situation cannot be ignored. Distinguishing the economic from the accounting concept of profit is important, both in the theory of capital and in investment analysis. What is involved is a correct understanding of the nature of profit. Normal profit passes from a cost character to that of an

² See any reference manual on generally accepted accounting principles (they vary by country), as the *International Financial Reporting Standards* (*IFRS*), the *FASAB Handbook of Federal Accounting Standards*, or C.S. Chapman, A.G. Hopwood and M.D. Shields, eds., *Handbooks on Managerial Accounting Research*, published in two volumes by Elsevier.

³ Economists use also a concept of natural price, or price of production, which does not appear in glossaries of accounting terms. Natural prices include a normal profit margin, reckoned at a uniform rate, which can be regarded as a minimal required return element.

earning element. The concept of surplus is also affected; the producer surplus reckoned by an economist is less than the accountant surplus.

2. Basic relations between capital and value.

A second important remark that we want to make concerns the relationships between capital and value, capital and money and capital and labour.

Capital is value in progress. It is the valorising value of a heterogeneous stock of produced instrumental goods, or of a financial fund of abstract productive power ⁴. Money is a form and a measure of value, the usual form in which market values are expressed and the usual standard of prices.

Various concepts of capital are used in the economic literature. Let us recall some of them.

- i) A first one is the classical notion of 'real capital', a tangible collection of heterogeneous instrumental goods and intermediate products used in production (*real or productive capital*). This is the physical, or technical, concept of capital.
- ii) A second concept of capital is the neoclassical notion of a homogeneous and amorphous material substance; that of a single factor of production distinct from the 'original' factors labour and land, and rewarded by a specific category of income.
- iii) A third notion of capital is that of 'finance capital', a liquid fund of uncommitted purchasing power, measured in money units, though not necessarily held in the form of money. This notion of capital provides a direct conceptual link with the theory of interest ⁵.
- iv) A fourth notion of capital is the Austrian one of 'capital as time', in which capital is regarded as a revolving fund of stored-up services of the 'original factors' labour and land. Attention is focused on the vertical dimension, or time structure, of the production process ⁶.
- v) A fifth concept of capital is the Marxian one of a social relation of dominion, specific of a capitalist system ⁷.

The basic question that we must afford is the following: is capital, however defined, a source of economic value? The answer to this fundamental question is not univocal. For the classical economists, capital is such. It is a productive factor. For Marx, it is not. The only source of value and of capital formation, for Marx, is human labour: abstract human labour. That is the socially necessary amount of productive labour-time embodied in a commodity ⁸. Capital is simply stored up labour, abstract labour-time spent in producing.

For neoclassical economists value is not a substance. It has a subjective nature. It is explained by the common remark that things are desired by people. It results from utility and scarcity. Cost is not a good indication of value.

The reader will easily recognize the Marxian derivation of the theoretical vision underlying this essay. However, in the course of our analysis, the meaning of the Marxian notion of capital and of

⁴ "Value therefore now becomes value in process, money in process, and, as such, capital" (K. Marx, Capital, vol. I, ch. 4).

⁵ Examples of this type of capital are Marshall's 'free or floating capital', Menger's 'Dispositionskapital' and Böhm-Bawek's 'subsistence fund'. The notion of capital as a fund of purchasing power was further developed by J.B. Clark, G. Cassel and F.H. Knight.

⁶ The importance of the time dimension of capital had already been recognized by Ricardo, who correctly attributed to an inadequate treatment of the time element in production most of the difficulties he faced in his theory of value, where the presence of durable capital goods was an obstacle to proving the proportionality of normal prices, which depend on the rate of profit, and of labour values, which do not.

⁷ A further concept of capital is that of cognitive capital, an embodied knowledge of technical capabilities (the Marxian notion of *general intellect*).

⁸ For Marx, labour commanded – the quantity of labour-power that a commodity enables to purchase – is not a source of economic value. It is a measure of value in terms of labour, the real measure of exchangeable value.

its partition in constant and variable components will be critically reconsidered, with implications on the theory of capitalist exploitation of wage-labour. Our analytical treatment will confirm the logical necessity to abandon the pure labour theory of value, initially held by Marx. It will be argued that, contrary to Marx's thought, only a part of the value of the capital invested in material means of production – the part which is entirely consumed during the production process – is a constant flow of capital that is transferred unaltered in the value of output. The residual part of fixed capital is not constant and contributes actively to the creation of net value.

There is indeed a productive power of capital, as well as a productive power of labour. The difference between them is that capital has power in a real sense, whereas labour has not. Capital *is* power, it is the standard expression of a power of command over other productive resources.

Both value and capital express social relations between human beings. In a capitalist system these relations take a misleading appearance, the fetish form of material relations, that is of relations between things. In ancient times, when production was undertaken only for use, without the support of capital goods, things were different.

The basic question to afford is what determines value. Labour-time alone, or the entire cost of production of a commodity? And how should this cost be computed? As sum of the real outlays incurred by a producer to pay input services, or as a money sum, augmented of the financial cost of the invested capital? The latter is a notional charge, an opportunity-cost). And should this money sum be augmented also of the depreciation allowances of fixed capital, or not?

Human labour and capital are basic elements in production activity. They must always be combined to produce an output. Nothing can be produced with unassisted labour or with unassisted capital. Their quantities and their technical composition in the various branches of production are obviously relevant in capital theory. Differing technical proportions of present labour to past labour and of the ratio of fixed capital to circulating capital, or the ratio of constant capital to variable capita, imply different profit rates.

The nature of the functional connection between capital and labour is not uncontroversial. We may recall the idea of an alleged classical 'separation approach' to capital theory, implying a distinct determination of quantities and prices. With a prior determination of the real wage rate, of the level and composition of the social product and of the technical conditions of production – the analytical 'core' of the theory – and a separate determination of prices. This contrasts with the image of a structurally integrated economic system and does not reflect reality.

This interpretation is untenable when wages are paid in money and can be used to buy alternative bundles of consumption goods. It was initially assumed by Ricardo, in his corn model, in which capital and output were homogeneous quantities, wages were paid in kind and the rate of profit was reckoned as the ratio of two physical magnitudes. But it was soon rejected, in the third edition of the *Principles*, in the changed analytical framework of a multi-sector economic system. The criterion adopted to measure exchange values was there modified, after some criticisms raised by Malthus.

To avoid incurring into circular reasoning, the separation approach involves a prior determination of three basic elements: a given physical vector of the real wage, the level and composition of the social product and the technical conditions of production. These elements must be determined independently of the relative prices of commodities, of the profit rate and of the social distribution of income. Particular attention is focused on the long-period equilibrium position of a competitive economy, which is characterized by a uniform profit rate and towards which relative production prices should gravitate.

This interpretation is methodologically erroneous and historically questionable ⁹. The alleged 'core' of the system is a black box, whose inner structure is not examined. How is the initial factor endowment determined? Is the composition of capital affected by the demand conditions of the economy? What determines the rate of profit? Nothing is explained. What happens in the alleged

⁹ On this basic methodological and historico-critical point, see Cavalieri, 2009.

analytical core of the system is arbitrarily isolated from the 'extra-core' elements which regulate accumulation, growth and the distribution of income. After Ricardo's retraction, the separation approach fell rapidly out of use and was not retained by Marshall and by Walras. But it was later reproposed, in the second half of the 1900s, by Sraffa (though not explicitly ¹⁰) and by Garegnani, with the twofold purpose to reject the neoclassical theories of value and distribution and to revive the classical surplus approach ¹¹.

The refutation of the 'separation approach', so conceived, removes a serious obstacle to a correct analysis of capital theory. What is needed in this field of search is an integrated stock-and-flow dynamic approach to the subject, underlying the structural interdependence of commodities prices and quantities and between the financial and the real sector of the economy.

The true analytical separation to be considered is another one. It is that between the theory of value and the theory of the social distribution of income, denied by neoclassical economists. Capital theory is not independent from the theory of prices and from the distribution of income The distribution of income between wages and profits is the result of a social struggle and depends on the bargaining power of labour and capital.

3. On the role of money in capital theory.

A third point on which we shall focus is a correct understanding of the role played by money in capital theory, as an expression of abstract labour-time and as an element of formal mediation between labour values and prices.

Labour-time and money are two strictly intertwined dimensional elements in capital theory. Money, the universal equivalent, is not a factor of production, in technical sense, but in a market economy its initial availability is a necessary condition to undertake a production. It is a logical requirement, since production must be financed from its beginning and during the whole period that precedes the sale of output. Firms must dispose of a money fund, to be able to buy or hire what is needed by the production activity. And the provision of this initial fund of money implies for a firm the bearing of financial costs, in addition to the real costs incurred in production. Unfortunately, this point is not always recognized. Part of the literature considers the financial cost of capital a minimal required return element, not an implicit cost.

The integration of money in the theory of capital requires knowledge of the channels that link the financial sector to the real economy and provide the required feedback loops. The most important of them are the financial market, the bank lending channel and the financial accelerator. Financial assets, including money, provide alternative forms of storing value and holding wealth. What has to be established is the causal direction of the relationship between the financial and the real sector of the economy. Does causality run from the financial sector to the real one, through the bank-lending stimuli to the growth of the real sector? Or is the behaviour of the financial sector ultimately determined by the needs of the real sector? Which is the driving force at work in the system? Which the main cause of the systemic crises of capitalism?

We are inclined to believe in the existence of a bidirectional and asymmetrical causal nexus and to attribute to the supply of money a mixed nature, partly exogenous and partly endogenous. Its exogenous component is that of fiat money, legal tender issued by the monetary authority, or entered in the money circuit as a result of the acquisition and conversion of foreign currency. The endogenous component is that of credit money issued by commercial banks on demand of firms, in

¹⁰ See his 'price-equations method' of determination, where the real wage and the social product are given when the relative prices of commodities and the profit rate, inversely related to the real wage, are determined.

¹¹ For this conception, see Sraffa, 1960, appendix D, and Garegnani, 1970. For its criticism from a neoclassical point of view, see Bliss (1975) and Hahn (1982). For a critique from a different, non-neoclassical point of view, see Cavalieri, 2009 and 2010.

form of bank loans or overdraft facilities 12.

The determination of the prevalence of the endogenous or the exogenous component in the nominal supply of money is still an open problem in the literature. The nominal supply has indeed a controversial nature; whereas the real supply of money, which depends on the velocity of circulation (and thus, indirectly, on the demand for money), has certainly an endogenous nature. We recognize the interdependency of the supply and the demand for money and refuse to trace two distinct schedules for them. We do not regard the central bank as an accommodating price-maker and quantity-taker. That is as a lender of last resort who cannot refuse or limit the financing of the banking system.

Working in the banking school tradition, some post-Keynesians consider the supply of money infinitely interest-elastic at the current level of the interest rate established by the monetary authority (or taken as an inflation target) and unaffected by lending, a level represented by a horizontal line in the interest-money space. They are therefore called 'horizontalists' ¹³. They focus the attention on money flows, disregard the possibility of using money as a liquid store of wealth (a stock variable), assume a coincidence of supply and demand for credit money and downgrade the central bank to the role of a compliant lender of last resort. Their vision is shared by the 'monetary circuitists', who regard credit money as scriptural claims devoid of intrinsic value, created unlimitedly by banks to meet the demand by firms. They think that the causal order goes from bank loans to bank deposits and bank reserves, look at the monetary circuit as a closed loop and neglect the difficulties of its closure in equilibrium. They are opposed by the 'verticalists', heirs of the old view of money of the currency school, who believe in the quantity theory of money and in the logical priority of bank deposits over bank loans and pay much attention to the monetary base. They hypothesize a vertical money supply schedule, matched by a downward-sloping schedule of the demand for money, and assume that these curves intersect in the i-M space at the market interest rate.

An intermediate position is held by two groups of non-fundamentalist post-Keynesians: the 'structuralists', who consider the supply of money as represented by a positively sloped line, for the presence of institutional constraints, uncertainty and increasing financial risk, and the 'neo-chartalists', who emphasize the role of a peculiar type of State-created pay-token having legal tender, described by Knapp and supported by Keynes. They deny that the expansion of credit money could go on indefinitely over time and maintain that the commercial banks liquidity preference could affect negatively their responsiveness to the demand for credit. Our position is similar. We regard money supply and demand as two strictly interconnected variables.

The interdependence of money supply and demand should be recognized. Adequate attention should be paid to the money multiplier mechanism that make their interaction possible. It is not correct to trace two distinct curves for them in the quantity-price space.

Money capital is a necessary pre-requisite of any productive activity by firms. A money fund is needed to buy the services of the productive factors. Its financing implies an indirect cost, a user one that has to be added to the direct cost of producing commodities.

Special price indexes are required to convert labour-time into money, and vice versa. The labour cost of output can be reckoned at market prices in money terms by the 'monetary expression of value', MEV, the Marxian ratio of the net social product to the total amount of labour-time. This is

¹² See Cavalieri, 2004.

¹³ They maintain that their position matches Keynes's views on the direction of causality of the investment-saving macroeconomic relationship and on the relevance of the demand side in determining the short term condition of the economy. This point, however, is a controversial one. Keynes's opinions on the subject were first expressed in endogenous terms in the *Treatise on Money*, but were later modified in the *General Theory*, where the supply of money was regarded as exogenously determined by the monetary authority, to allow the closure of the underlying analytical model characterized by two liquidity-preference functions. Thus providing an awkward and contradictory 'pseudomonetarist' exogeneist position, not confirmed in some later writings.

a measure of the average unit cost of output in money terms for the economy as a whole ¹⁴. It is a valuation parameter that considers on the cost side both the present (living) and the past (dead) labour. As such, it must be distinguished from another valuation metric, the 'monetary expression of labour-time', MELT, commonly used by 'single-system' neo-Marxists who accept the labour theory of value, a parameter in which only the cost of living labour is reckoned.

Let us further consider these two indexes. MEV is a social cost index number: the ratio of the value of net revenue to that of net product. It expresses the quantity of money that corresponds to a unit of abstract labour. It is the sum of the money cost of productive services, which is a real cost, and the money cost of financial capital, a notional cost, and has the dimension of dollar per hour. If we call C_R the money cost of real capital, C_L the money cost of direct labour, C_K the money cost of all other inputs and r the opportunity-cost of capital, then the cost of financial capital may be written as r (C_R + C_L + C_K). Summing up the components of MEV, we have

$$MEV = C_R + C_L + C_K + r(C_R + C_L + C_K) = (C_R + C_L + C_K)(1 + r).$$

This is a proportionality factor that can be used to convert abstract labour values into money prices and is not affected by the particular type of monetary system. It is the ratio of the net product, or total money value added, at market prices, px, to the total amount of abstract labour-time L, which includes all labour, both living and dead, used in production. p is a price index of the net product and x a volume index.

4. Some value forms and their conversion.

MEV should not be confused with the monetary expression of living labour-time, MELT, a different value parameter, introduced in the literature by Duncan Foley. It is the ratio of the total money value added, reckoned at current market prices, to the living labour-time used in production. It is not a general expression of the money value of abstract labour-time. It reflects the money value of living labour, regarded as the only source of net value. In the literature there are two versions of this index: the 'simultaneist' (or atemporal) version and the 'temporalist' (or successivist) version. The former one, known as the NI-MELT, is the ratio of the value of net product to living labour. The other one, the TSSI-MELT, is the amount of money value of the net product which exchanges at current prices with a unit of living labour-time; i.e. the ratio of a unit of money to the amount of living labour-time that the unit commands, a ratio which can change during the production process. If we denote the first version by 'sigma' (σ) and the second one by 'tau' (τ), we have

$$\sigma = px/L^* = p(I-A) x/\ell x \quad \text{and} \quad \tau = py/L^* = p(x-c)/L^* = p(x-c)/\ell x.$$

Here L^* is living labour, A is the input-output matrix of technical coefficients, ℓ is the vector of living labour coefficients, x is a volume-index vector of gross product, y is the vector of net product, x is the vector of constant capital and y is the money price of net product, which depends from the quantity of money and the velocity of circulation.

The two versions of MELT can be derived only $ex\ post$, when the prices of commodities are known. They do not depend on the quantity of labour-time, but reflect the quantity and the velocity of circulation of money. σ is the money value of gross product at market prices per unit of living

¹⁴ 'Monetary expression of value' is the name used by Marx in *Value, Price and Profit*, in which he pointed out that "price, taken by itself, is nothing but the *monetary expression of value*". In *Capital* Marx made frequent use of this proportionality factor to transform labour values into money values. On this point, see Kristjanson-Gural, 2008.

labour; τ is the money price of net product per unit of living labour ¹⁵. In both of them Marx's 'new value equality' is dogmatically and erroneously presupposed. At the sector level, MELT can be decomposed in the product of MEV, the monetary expression of value, and VELT, the value expression of living labour-time. The latter parameter is the ratio of the value of net product to the amount of living labour-time. It depends on the quantity of labour and on labour productivity ¹⁶.

<u>Value-forms</u>	<u>Determination</u>	<u>Dimension</u>
Monetary expression of labour-value - MEV	R/L	Money value of net product/total labour-time
Monetary expression of living labour - MELT	R/L*	Money value of net product/living labour-time
Labour value of money - LVM	L/R	Total labour-time/ money value of net product
Living labour value of money – LEM	L*/R	Living labour-time/ money value of net product

As MEV is different from MELT, we must establish which of these two price indexes should be chosen. The money value of commodities reckoned at their current market prices, i.e. a variable that accounts for all explicit and implicit costs, including depreciation and the financial cost of capital? Or the money value of the living labour-time which commodities command at the current wage level. This is a variable which does not account for the financial cost of capital. A general principle of valuation must be established. What has to be cleared is how the search for a correct measure of the money value of capital goods, i.e. of their prices, has to be done. This search implies the separation of asset values into price and volume components and their combination into a single weighted index, with the rental prices or user costs of capital goods as weights. And this involves splitting changes in the values of capital assets of different kinds and ages into price and volume changes, measured in terms of effort units (labour-time) or of efficiency units (output) and adjusted for quality changes.

A 'weak separability' condition is required: commodities must be divisible into distinct homogeneous groups. A necessary condition which is not sufficient and is rarely satisfied.

5. Capital in equilibrium and in disequilibrium. Towards an integrated approach.

Let us start by considering the conventional stationary equilibrium framework of a structurally invariant economy – an imaginary steady-state or steady-growth framework in which all

¹⁵ Under certain circumstances σ can be negative, a state of things which would imply the possibility of positive profit with negative surplus-labour, and vice versa. In this case Morishima-and-Okishio controversial 'Fundamental Marxian Theorem' (FMT) – according to which in sufficiently general Leontief models with no joint products the existence of positive profits would imply the presence of surplus-labour and capitalist exploitation – does not hold. And this would have other important implications (as those concerning the tendency of the average rate of profit to fall). In the literature, it is disputed whether or not τ can be negative. If it cannot, FMT holds, under restrictive assumptions.

expectations are fulfilled and all individual plans are compatible and can be realized. Under such unrealistic conditions, the ontological characterization of capital as valorizing value would be lost and there would be no logical need for money. The theory of capital would therefore be much simplified. Real economies, however, are not in dynamic equilibrium. A disequilibrium approach is thus needed. This implies a consistent integration of the theory of capital with the theory of investment, that is a stock-and-flow approach to the problem, whose provision would go beyond the limited scope of this essay.

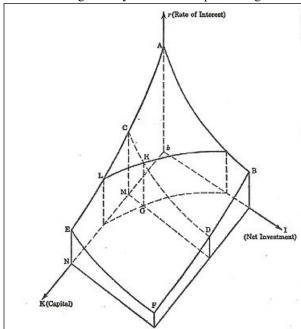
From an analytical point of view, some methodological difficulties are present, for the heterogeneous and multi-specific nature of capital goods, for the fact that their production takes time and that these goods last for a limited and uncertain period, for the low interest elasticity of the demand for capital goods and for the different time dimension of stocks and flows, which makes them incommensurable.

We shall begin our analysis by distinguishing the main functions in the theories of capital and investment by their stock or flow nature and by the side of the market they refer to. The demand and supply functions of real capital have a stock dimension, whereas the demand function for investment and the supply function of new capital goods have a flow dimension. These functions do not adjust instantaneously in the presence of excess demand or supply, as required by equilibrium.

The demand curve for capital goods cannot be identified with the savings curve, because savings may be partly hoarded. Likewise, the supply of capital goods cannot be identified with the investment function, because investments may be financed by dishoarding and bank credit. Together, the supply and demand for capital goods determine the prices of capital goods. Not the rate of interest, which is a monetary phenomenon.

Let us initially refer to a very simple economic system, one with a single capital asset, a single consumption good, homogeneous labour and stable expectations used by Lerner (1936-37) to configure a tri-dimensional geometrical treatment of the problem. In this model there are two horizontal axes, for the total stock of capital (K) and the rate of net investment (I), both reckoned as values. The origin of the diagram shows the inherited endowment of capital assets of the economy. The vertical axis measures the expected internal rate of return r(K, I) on the most convenient investment opportunity yet to be realized 17 . In the pair of perpendicular planes bounded by the

¹⁷ See Lerner, 1936-37, p. 29, and 1944, pp. 336-37 (and his 1953 correction). Here is Lerner's diagram. The horizontal plane supporting the tridimensional figure may be used as a phase diagram for a qualitative analysis of the



dynamic properties of the system.

vertical axis r and by either one of the horizontal axes K or I, there are two stock-variables, the demand and supply of real capital, and two flow-variables, the demand for investment and the corresponding supply flow per unit of time. In the stock-variables plane (r, K), corresponding to the stationary case of a zero level of net investment, the schedule of the internal rate of return (IRR) is represented. This is a demand function for a stock of real capital, inclusive of the implicit 'reservation demand' made by producers for the capital goods they use.

Lerner's marginal efficiency of investment (MEI) represents the demand function for investment. Keynes's marginal efficiency of capital (MEK) is the demand function for a stock of capital goods, obtained by dividing the marginal productivity of capital (MPK) by the cost of production of capital goods ¹⁸. MEI is therefore the flow-version of MEK. It ranks the investment projects according to their expected yields. Its construction requires a complex aggregation procedure.

In long-run equilibrium, MEI must equal both the rate of interest and MEK. Investment will take place up to the level at which equality between MEI and the rate of interest is attained. MEK is matched by a 'stock-augmented' supply curve of capital goods, SCK, an increasing function of the rate of return. This function represents the total stock of capital goods at a given point of time (i.e. the inherited stock, plus net current production) ¹⁹ and has different meanings in neoclassical and Keynesian models ²⁰.

This comparative-static model can provide a useful starting point for a further analysis of capital theory in a more dynamic context and in the presence of many kinds of capital goods. Let us begin by distinguishing the main analytical functions relevant in the theory of capital and investment, according to the side of the market and to their stock or flow nature. The stock relations will be: the demand for financial capital (investible funds), DFC; the supply of financial capital, SFC; the demand for a stock of capital goods (Keynes's marginal efficiency of capital), MEK; and the supply of a stock of capital goods, SCK. The corresponding flow relations will be: the demand for additional units of financial capital (marginal supply), MSF; the demand for an additional flow of capital goods (Lerner's marginal efficiency of investment), MEI; and the supply of additional units of capital goods, MCK.

We are now in the condition to describe the effects of an adjustment process implying net investment and a reshaping of the capital structure. Old capital goods are discarded and new capital goods enter the scene. In neoclassical models a pseudo-dynamic adjustment process will determine

¹⁸ It is wrong to assert that "the marginal productivity of capital is the marginal efficiency of investment when the rate of net investment is zero", as was done by Lerner (1944, p. 334). The correct relationship between MPK and MEI was shown by Lerner in another paper (1965), in which he introduced a further variable, the marginal productivity of investing (MPI), defined as the flow of extra capital produced per unit of time by diverting resources at the margin from consumption to investment. By multiplying MPK by MPI, one gets MEI.

¹⁹ As a norm, SCK should be a monotonically increasing function of the amount of capital, in accordance with the usual shape of a marginal cost curve. But in the presence of a reservation demand curve, the influence of demand in the determination of the price of capital goods can prevail over that of the flow-supply to such an extent that the stock-supply schedule can be nearly constant up to the level of output which corresponds to the given capacity of the capital goods industry.

¹⁵ In a neoclassical framework SCK shows the supply of savings, a real variable. Together with another real variable, IRR, the entrepreneurs demand for capital, SCK determines the equilibrium level of the rate of interest. In the Keynesian theory, where savings are a residuum and depend on the level of income, at any exogenously given rate of interest there are as many supply functions of capital as possible levels of income. Equilibrium in the capital goods market is brought about by changes in prices, rather than by changes in the rate of interest, separately determined in the money market. IRR and SCK determine the equilibrium stock of capital and the equilibrium output of capital goods. In equilibrium two conditions must be satisfied: the rate of return on capital must be high enough to induce firms to hold their endowment of capital goods, and the current supply of capital goods must be such to make the marginal cost of production equal to the market price of a unit of real capital. For each stock of capital goods, a different market price and a different rate of return are associated in equilibrium with each rate of output of capital goods and with each rate of investment.

the growth of the capital stock required to restore equilibrium. If K_t is the money value of the stock of capital at time t and δ the depreciation rate, net investment is $I_t = K_t - (1 - \delta) K_{t-1}$. Once the desired optimal size of the capital stock is specified, the flow of investment is determined. The speed of the adjustment process is not.

In a Keynesian model, where the investment is an autonomous macroeconomic variable which reflects the 'animal spirits' that urge entrepreneurs to action, the demand for investment is a function of effective demand, of the marginal efficiency of capital and of the conditions of the money market. At each level of the interest rate there will be a single optimal amount of capital desired by the entrepreneurs, i.e. a single point on the marginal rate of return (MRR) curve, in the (K, r) space, at which a MEI curve intersects the vertical axis at the given level of the rate of interest in the (I, r) space.

If the stock of capital held by the entrepreneurs at that point falls short of the optimal amount, there will be a net investment and this will cause an upward shift of the IRR schedule. This, in turn, will react on the MEI schedule and lower the level of investment. In neoclassical models the adjustment process will continue until a stationary equilibrium position is achieved ²¹. Any change in the demand of funds for investment will affect the supply of investible funds, via the level of income. But since the capital stock is assumed to adjust instantaneously in a disequilibrium situation, the investment function is practically eliminated.

Following Fisher, Keynes argued that investment is made until the present value of the expected future revenues equals at the margin the opportunity cost of capital. He called marginal efficiency of capital (MEK) Fisher's rate of return over cost, the discount rate corresponding to the average internal rate of return on the marginal investment ²².

In a two-sector neoclassical model, with distinct production functions for consumption and capital goods, capital can be treated as a malleable and homogeneous asset suitable to be used with various techniques in each of the two sectors (with 'putty' capital models, implying smooth factor substitutability both ex ante and ex post), or as a non-malleable and heterogeneous asset, that once has been installed in a productive sector is unsuited to be later utilized with a different technique ('clay' capital models, implying fixed and irreversible factor proportions). These models provide a variety of different frameworks for an analysis of capital theory. In some of them, capital-labour substitution may be possible ex ante, though not ex post. With a vintage capital structure capital goods belonging to different generations reflect different technologies and face different depreciation patterns. Investment can reduce the age of capital and increase its productivity.

In models of the 'neoclassical synthesis' type, the traditional and the Keynesian capital theories are integrated. Savings are then regarded as a direct function both of the level of income and of the rate of interest and the investment demand is inversely related to the rate of interest, but directly related to the current level of income, or to its changes. There are a whole family of investment demand schedules and a whole family of supply of investible funds schedules, and each of them is associated to a particular level of income.

6. Marx, from the labour theory of value to the law of value.

Let us now come back to Marx and explain how much we shall depart from him. Following the classical surplus approach, the young Marx conceived the economic surplus in real terms, as the

²¹ In equilibrium a 'three-fold margin of convenience' holds, implying the simultaneous equality of the rate of interest, the marginal productivity of capital and the social rate of return on investment. There is a balance of convenience between holding money as a security asset (the precautionary motive), spending money on consumption goods (the time preference motive) and investing money in the purchase of capital goods (the marginal productivity of capital argument).

²² On this point, see Cavalieri, 1963.

excess of the social product over what was required to replace the means of production and the means of workers subsistence consumed during the productive process. Profits, interests and rents were paid out of the surplus, so defined ²³.

It was not the same for the elder Marx. He considered the labour theory of value fully significant only at a high level of abstraction: that is only in a pre-capitalist economy, Smith's 'early and rude' primitive society in which commodities were produced by means of labour and rudimentary tools owned by their users. In more developed societies, the labour theory of value could not hold. It was substituted by a real cost of production theory of value, with a plurality of cost elements.

The labour theory of value was no longer valid at Marx's times. He realized that the classical labour theory of value could only be applied in the pre-capitalist system that preceded both the accumulation of stock and the appropriation of land', in which labour was the sole source of value and the whole product of labour belonged to the labourers. Then commodities could normally exchange at prices proportional to their values.

Unfortunately, in spite of Marx's unequivocal assertion that out of equilibrium prices cease to reflect the quantities of labour-time embodied in commodities, unless in unrealistic special cases characterized by an equal organic composition of capital in all sectors or by a zero profit rate, the pure labour theory of value was not abandoned. It remained central for most students working in a pseudo-Marxist intellectual tradition, who refused to admit its failure, due to the active productive role of capital goods.

There was however a problem. If the labour theory of value could no longer be used to explain the exploitation origin of profit, other justifications had to be found to legitimate Marx's idea that profits were the result of labour exploitation and to validate his appeal to a class struggle between wage labour and capital. The capitalist exploitation of labour had to be demonstrated.

This was gradually realized by Marx. In the third volume of *Capital*, he substituted the pure theory of labour with a more generic conception, that of a 'law of value' (*Wertgesetz*) regulating the determination of relative prices ²⁴. In that volume, Marx rejected the labour theory of value of classical lineage that he had previously endorsed. In chapter IX, he acknowledged that profits are distributed by competition in proportion to the prices of constant and variable capitals, and not in proportion to their values. He pointed out that "there is always the possibility of an error if the cost-price of a commodity in any particular sphere is identified with the value of the means of production consumed by it", before concluding that "our present analysis does not necessitates a closer examination of this point. It remains true, nevertheless, that the cost-price of a commodity is always smaller than its value". There Marx recognized that in the presence of different organic compositions of capital in production, and with a positive rate of profit, commodities prices are not proportional to labour values, but simply tend to gravitate around them. And he turned to a cost-of-production theory of value, in which both labour and capital were treated as directly productive agents.

There was therefore in Marx, in the middle 1840s, when he matured the leading thread of his economic thought, a fundamental change of theoretical perspective, an 'epistemological break', later pointed out by Louis Althusser and Alain Badiou. It signed the passage of Marx from ideology to science, from his former Hegelian metaphysical position to historical materialism. Marx's views on value and surplus-value thus evolved over time. He did not hold the labour theory of value for his lifetime, but took progressively the distance from it.

In the first two parts of volume I of Capital, the 'law of value' was present, but in a somewhat

²³ Marx used to group together fixed capital and raw materials. He called them 'constant capital' (c). The net product was for Marx the sum of the wage bill ('variable capital', ν) and 'surplus value' (s). Gross product was $c+\nu+s$, net product $\nu+s$ and the rate of profit $s/(c+\nu)$.

²⁴ This was not a return to Adam Smith's controversial adding-up theory of price, criticised by Ricardo. In Marx's 'simple merchant society' technologically primitive capital goods existed and workers owned them; whereas in Smith's early and rude imaginary state of society capital goods did not exist.

vague and indefinite form. It was supposed to hold at a macroeconomic level, for 'capital in general'; though not at a microeconomic level, for single commodities, where possible deviations from the law of value were mentioned. It was only in the third part of volume I, that Marx ceased to regard the law of value as a natural law and treated it as historically conditioned.

In 1857-58, in a famous passage of Grundrisse, the 'Fragment on Machines', Marx wrote: "In the machine, and even more in machinery as an automatic system, the use value, i.e. the material quality of the means of labour, is transformed into an existence adequate to fixed capital and to capital as such; and the form in which it was adopted into the production process of capital, the direct means of labour, is superseded by a form posited by capital itself and corresponding to it. In no way does the machine appear as the individual worker's means of labour. Its distinguishing characteristic is not in the least, as with the means of labour, to transmit the worker's activity to the object; this activity, rather, is posited in such a way that it merely transmits the machine's work, the machine's action, on to the raw material, supervises it and guards against interruptions. Not as with the instrument, which the worker animates and makes into his organ with his skill and strength, and whose handling therefore depends on his virtuosity. Rather, it is the machine which possesses skill and strength in place of the worker, is itself the virtuoso... The worker's activity, reduced to a mere abstraction of activity, is determined and regulated on all sides by the movement of the machinery, and not the opposite" (notebook VI, pp. 690-95, paragraphs 584-86). There is here an explicit acknowledgment that in an industrial society technologically advanced machines are directly productive of surplus-value. They are not constant capital. Yet for the economy as a whole, the labour theory of value continued to be valid, because the sum of values equalized the sum of production prices ²⁵.

In his later Manuscripts of 1861-63, Marx specified: "We have seen not only how capital produces, but how it is itself produced, and how it emerges from the production process as a relation changed in essence, how it develops in the production process... Since living labour is incorporated into capital - through the exchange between capital and the worker - since it appears as an activity belonging to capital, as soon as the labour process starts, all the productive powers of social labour present themselves as productive powers of capital, just as the general social form of labour appears in money as the quality of a thing. Thus the productive power of social labour, and the specific forms of it, now present themselves as productive powers and forms of capital, of objectified labour..." (MECW, vol. 34, XXI, 1317). Marx had definitely realized that in the age of industrial mechanization and automation the main force of production had become the social knowledge objectified in fixed capital. Labour was reduced to a living appendage of capital.

7. New directions of research.

The blow inflicted to the classical versions of the labour theory of value by Sraffa's 'physicalist' construction, that allowed to determine commodities relative prices without any recourse to labour values, seemed irretrievable. On logical grounds, the possibility of tracing back the origin of profit to capitalist exploitation of labour-power had to be dismissed ²⁶.

²⁵ This point was further explained by Marx in the *Results of the Immediate Process of Production* (1863), also known as the *Unpublished Chapter Six of Capital, Volume I*, where he described the passage from the phase of formal domination of capital and formal subordination of labour to the phase of real domination of capital and real subordination of labour.

²⁶ From a theoretical point of view, however, other virtual possibilities are available to recover the Marxian notions of surplus-value and exploitation. Surplus-value can be reinterpreted on neo-Sraffian lines as the difference between two distinct values of the real wage: the greatest theoretical value compatible with the existing technology (the value which corresponds to a zero rate of profit) and the real wage. The existence of surplus-value can also be deduced from the fact that production gives rise to a profit; or from the inverse relation which links the rate of profit to the real wage.

Since Sraffa had not cleared the origin of profit, a revival of the traditional neoclassical explanations of profit – as a reward for the productivity of capital, for waiting or for entrepreneurial risk taking, rather than as the result of the employers appropriation of unpaid wage labour – could be expected. It was however contrasted in the 1980s by the appearance of the so-called New Interpretation (NI) of Marxian theory, proposed by Foley and Duménil, in which the fundamental role of money as a form of value and an expression of social labour-time, underlined by Marx, was emphasized. Money was no longer treated simply as a numéraire, but was assigned the task of providing a formal mediation between values and prices. Value and capital could be measured either in labour-time units (by the effort involved in production) or in money units (i.e. in efficiency units). The money unit was purposely chosen in such a way that the money value of the net product equaled the labour embodied in it. The labour value of a commodity, however, was not measured by the amount of social labour embodied in it (as in the traditional version of the labour theory of value), but by the amount of social labour represented by the quantity of money that the owner of the commodity might obtain by selling it in the market. The basic assumption was that in each period of time the total money value added of the economic system could be considered the expression of the living labour employed, embodied in the net product. No theory of price formation was implied. Prices were determined independently of labour values and equaled the money value of abstract labour-time. The value of the labour-power was identified with the share of wages in the money value of the net product, not with a physical wage basket. The money value of net social output expressed the total abstract social labour employed in the economy. All was reckoned in money terms, in a labour-commanding perspective (that of the Rubin school's valueform). Values and prices did no longer pertain to different levels of analysis. They necessarily coincided. No problem of transformation of values into prices could thus arise. The law of value held as an average, for the economic system as a whole, though not at a lower level of abstraction, for single sectors of production.

A complete integration of the theory of value with the theory of money was achieved at the aggregate level. But the causality relations linking values and prices were not definitely cleared. It was not explained why money should be taken as a direct expression of labour value, in a world where commodities do not exchange at their values. And why the exchange-value of labour-power should be identified with the money wage, which is directly observable, as assumed by NI, rather than by the real wage, i.e. by the basket of wage goods consumed by workers, which is not. If the money wage is privileged, labour-commanded and labour-embodied measures of absolute value coincide, the value of labour-power is the share of money wages in the net output, and the equality of the sum of values and the sum of prices holds for the net product of the system.

What really matters, however, is the real wage. The exchange-value of money is known only when the price of money in terms of all commodities is established. In NI this does not occur. Money wages are identified with abstract labour-time. There is no intermediation process.

Two variants of NI should also be mentioned. One is the *Simultaneous Single System Interpretation* (SSSI), suggested by a group of Sraffian scholars (Wolff, Callari, Roberts and others), in which money is regarded as a form of labour value and all values are directly expressed in money terms, at production prices, interpreted as redistributed labour values. There is therefore no need to transform values into money prices. Input and output prices are simultaneously determined. They necessarily coincide. The social product reckoned in labour values and in prices of production is the same. A stationary equilibrium of the economy is assumed. This explains the name *Equilibrium Marxism* given to this approach by some critics.

A second variant of NI – the *Temporal Single System Interpretation* (TSSI), or *Marxian Disequilibrium Approach* – has also been proposed. This proposal came from fundamentalist Marxists (Kliman, Freeman, Carchedi and others) who regard production as a time consuming process in which inputs precede outputs in historical time, in a process in which first commodities are bought, then they enter production as inputs and finally they reappear as outputs. But since in

the meanwhile prices can change, there is no logical reason to think that commodities should have the same prices when they are considered as inputs or as outputs. This result is possible, but it is not supposed to be the norm. The labour theory of value is preserved. But prices are not deduced from values. Values and prices are seen as distinct and transformable expressions of a single system of interconnected variables. Neither of them is logically prior to the other. They should therefore be simultaneously and interdependently determined ²⁷.

What we cannot accept is the necessary logical premise of these interpretations: the alleged equivalence of new value and living labour, dogmatically and erroneously asserted by Marx.

8. The role of time in production and in capital theory.

We shall now consider some aspects of the historical background of capital theory and try to draw some lessons from them. Let us begin from the old-Austrian school of capital, which attributed to capital goods a complex physical, value and time dimension ²⁸. The productive process was conceived as a sequential transformation of primary inputs in final outputs, in the framework of a vertical system of structural relations ²⁹. Lower rates of interest were associated with higher values of the capital/labour ratio. This implied a rising capital/output ratio, for the decreasing marginal productivity of capital.

The Austrian theory of capital was not a unified theoretical construction. Menger's treatment of the subject had a marked subjectivist character. Capital was for Menger a fund of purchasing power, for Böhm-Bawerk a fund of productive capacity. An 'average production period' denoted the mean time which elapsed between the use of the services of the original factors labour and land and the reception of output. It was criticized because it was independent of the distribution of income, could work only under stationary equilibrium conditions and could not be applied to financial capital.

The old-Austrian theory of capital was opposed by a number of American neoclassical economists, who included J.B. Clark, Frank Knight and Irving Fisher. In Clark's doctrine of 'capital synchronization' the role of the time element was almost insignificant and could not affect the capital structure of the economy. His 'true' or 'pure' capital was a permanent and dematerialized fund of social wealth available for production. Any single part of this abiding fund could perish, but was immediately replaced by a new equivalent item, so that the fund could be regarded as a permanent entity in value terms.

Knight's productivity approach implied a theory of capital in which there was no need to refer to the quantity of capital. To avoid incurring in circular reasoning, the interest rate was determined by a flow-variable, the rate of investment.

In Fisher's theory of capital, which dealt with the inter-temporal choices between present and future consumption, the interest reflected people's subjective time preference, or 'impatience', which implied an underestimation of the future in the satisfaction of needs. Fisher did not consider capital a specific factor of production and took great care to separate the concepts of capital and

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²⁷ This contrasts with the *Simultaneous Dual System* interpretation-and-correction of Marx's production schemes advanced by Bortkiewicz and retained by Sweezy, Dobb and other Marxist scholars. We are in the presence of two different value paradigms.

²⁸ Hayek used his famous right 'triangles' to illustrate the relationship between the time structure of production (its stages) and the corresponding levels of consumption. His triangles had as base the production time and as height the output delivered to consumers. The slope of their hypotenuse was proportional to the interest rate. The triangle shape thus varied with the interest rate. Böhm-Bawerk depicted the production process by a set of concentric circles.

²⁹ Logical time, required for building a stock of capital goods, plays an essential role in the theory of capital. Once capital goods are installed in a production process, they last for some finite time, during which they are usually irreplaceable. Logical time must not be confused with historical time, which goes only one way, is linked to uncertainty and is irreversible.

income.

Knut Wicksell, another leading marginal productivity theorist, was sympathetic to the Austrian capital theory, but he did not regard time as a distinct productive factor. He realized that the concept of average period of production had to be abandoned and that capital had to be calculated in value terms, by the value of capital goods.

On the relative importance of productivity and time preference in the determination of the rate of interest, three lines of thought emerged. One of them regarded capital as a specific productive factor (a perpetual fund of value, suited to be reckoned as a single magnitude in value terms) and stressed the importance of the productivity element in the determination of the interest rate. Another one implied a time preference theory of interest. The third one was an eclectic position.

The debate on the theory of capital of the old-Austrian school changed the direction of value imputation, by shifting the attention from the historical costs of capital goods to their expected future earnings, and highlighted the fundamental role of entrepreneurs expectations. A revival of the Austrian theory of capital took place in the 1920s and 1930s. The 'new' Austrian school of Mises and Hayek abandoned psychologism and determinism in favor of a more pragmatic ('praxeological') attitude. The convergence of the economy towards a stationary equilibrium position was no longer taken for granted.

Then came Keynes. He was scarcely interested in the theory of value, that he regarded as a pedagogical tool devoid of practical relevance, and in the theory of capital. Yet he did not dismiss capital theory entirely. His theory was an expectations-augmented extension of that of Marshall, based on the principle of factor substitution, implying diminishing marginal productivities and sloping demand curves for single factors. Keynes did not focus his attention on the returns of past investments, but on the expected returns of new investments. That is on the marginal efficiency of capital. He correctly thought that what makes a capital asset desirable is its prospective yield.

In Keynes's theoretical system the demand for money was the result of two components: a passive one, the demand for cash, expressed by the liquidity preference schedule, and an active component, the demand for investible funds, made by firms for the finance motive. The rate of interest was independent of the marginal efficiency of capital. It was determined by the demand and supply of money, two stock-variables; not by the real forces of savings and investment, two flow-variables.

Keynes's work was criticized as lacking a coherent theory of capital. The debate with Hayek and some exponents of the LSE, that followed, raised two important questions. One concerned capital: whether the search for a measure of the quantity of capital had to be related or not to the period of production. The other question was on the relation of investment with the rate of interest and the level of final demand. The Cambridge Circus authors refused to treat capital as a homogeneous fund of value, in the typical neoclassical way, objecting that real capital was not a 'malleable' factor of production, but a collection of heterogeneous goods with a complex horizontal and vertical structure. They stressed the importance of financial capital and the role of monetary policy in disequilibrium. Their line prevailed ³⁰.

In the inter-war period, two significant additions to the theory of capital in equilibrium came from von Neumann and Hicks. Neumann wrote a seminal paper on capital accumulation and balanced growth at a maximal rate. Hicks, a neo-Walrasian, analysed the market process in a temporary equilibrium framework, in a forward-looking pseudo-dynamic perspective. He focused the analysis on growth equilibrium, using a methodology that allowed to make use of both flex-

³⁰ Keynes's treatment of capital and his refusal of the classical theory of interest as the reward for saving were criticized also by Ohlin, Hawtrey, Pigou, Robertson and Viner. In a harsh comment to Hayek's treatment of business cycles in *Prices and Production* (1931), Sraffa had pointed out the necessity of a substantial revision of the neoclassical theory of capital in equilibrium. He opposed to Hayek's concept of a real rate of interest the notion of the 'own rates of interest' of single commodities, which was then used by Keynes in *The General Theory*. The presence in the real world of multiple own rates of interest was for Sraffa a symptomatic sign of disequilibrium.

price (neoclassical) and fix-price (Keynesian) models. And he ultimately proposed a 'neo-Austrian' reformulation of the theory of capital, in which the time structure was the crucial variable.

Later on, in the early 1950s, a fundamental criticism of the neoclassical macroeconomic theory of capital came from Joan Robinson, who described the aggregate production function as a powerful instrument of economic miseducation, opened to severe logical objections, and raised two basic questions: what is the meaning of 'the quantity of capital' and what determines the rate of profit. She argued that there were neither a physical quantity nor a marginal product of capital and that the social distribution of income between wages and profits was not determined by the marginal productivities of factors ³¹.

The debate that followed in the 1960s, after the appearance of Sraffa's book on production of commodities by means of commodities, under the name of 'Cambridge controversy', shook the backwater of capital theory. It showed that the capital intensity of production is not a monotonic function of the rate of profit, that different capital goods cannot be aggregated into a single magnitude and that in the presence of heterogeneous capital goods 'Wicksell effects' can arise, involving changes in the methods of production and in the value of the capital stock associated with changes in the rate of interest incompatible with those assumed by the neoclassical theory ³². The debate confirmed that the amount of capital had to be reckoned in value terms and not in quantity terms, independently of the price system, and established that no measure of capital intensity is independent of the rate of interest and that there is the logical possibility of the insurgence of paradoxical phenomena, such as the 'reswitching of techniques' and the 'capital-value reversing'.

The controversy involved three theories: the classical labour theory of value, the neoclassical theory based on the aggregate production function and the neo-Ricardian and Sraffian theoretical construction (the 'Anglo-Italian' one), that prevailed. It was definitely established that the distribution of income is not governed by the opposing forces of market supply and demand and that there is no monotonic inverse relation between the amount of capital goods and their remuneration.

This was an important theoretical progress. The ultimate implications and the empirical relevance of the last Cambridge debate on the theory of capital, however, were contested. The Anglo-Italian theory was imputed to be a theory too abstract, a theory without measurement. Paradoxical behaviours continued to be regarded by neoclassical authors as simple anomalies: logical 'curiosa', or 'local puzzles', confined to a subset of price systems with constant interest rates. Aggregate production functions and neoclassical demand and supply curves of capital continued to be used. Much had been deconstructed, but little was reconstructed ³³.

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³¹ Leading 'neo-neoclassical' authors had made use of capital theory 'parables' and of a 'surrogate production function' implying an unrealistic one-commodity world or a single homogeneous and malleable capital good, no joint products and full substitutability of labour and capital, to show that an increase in capital intensity would have led to a lower marginal product of capital and to a lower level of the rate of interest. To preserve the inverse relation in demand theory between the price of any factor service and the quantity of the factor used in equilibrium, they maintained that the social distribution of income between capitalists and wage earners was determined by the technical parameters of the aggregate production function and that the labour/output ratio was a monotonically increasing function of the rate of interest.

³² Joan Robinson did not take an active part to the discussion, in that phase of the debate. She was interested in the working of capitalism in historical time, did not share Sraffa's emphasis on the long-period positions of the economy and regarded his approach to the theory of capital as too narrow in scope (half of a general equilibrium system, centered on the supply side of the economy).

The measurement of the value of capital goods is still done in a number of different ways: a) by capitalizing their historical or acquisition costs (the 'backward looking' classical method); b) by taking their demand price, which is equal to the present value of expected future earnings (a 'forward looking' method of valuation, used by Fisher and other neoclassical authors); c) by taking their supply price, or current replacement cost, a method which disregards the time structure of capital and emphasizes the role of living labour; d) by taking the current realizable value, or market resale price, of capital goods; e) by taking the value of the flow of capital services, reckoned at their rental prices. All

The really basic question to answer is how capital is created and valorised. Smith, Ricardo and Sraffa did not afford this fundamental problem directly. Marx did it. He was the first scholar to pose the question of the origin of profit. In our opinion, there is a need to start again from his work and to abandon his initial endorsement of the labour theory of value. A theory that is meaningless when capital is used in technical combination with human labour.

9. Towards a revision of the theory of capital.

Let us explain why the labour theory of value cannot hold in a capitalist society, using a simple model of production of Ricardian lineage, with homogeneous labour, circulating capital, linear technology of activity analysis type with a finite number of activities and no joint products. A matrix system of quantity equations

$$y = (I - A) x \tag{1}$$

where y is the net product vector denoting the activity levels, x is its quantity or volume size index and I is the identity matrix, describes the technological structure of the model and the commodity composition of the net product y required to satisfy at a certain moment a given final demand. Let A be a Leontief input-output matrix of technical coefficients of single product industries – that is the direct requirements matrix, derived from the inter-industry transactions matrix (in which the rows are producing sectors and the columns consuming sectors) by dividing each cell by its column total – and Ax be the circulating capital.

Let the system be viable. Then the matrix (I - A) can be written as a convergent geometric series with an infinite number of terms, $(I + A_1 + A_2 + A_3 + ... + A_n)$, where A_n tends to zero as n tends to infinity. Its inverse $(I - A)^{-1}$ describes the vertically integrated temporal structure of production and can be used to calculate the total production of commodities required to satisfy any possible composition of final demand.

The vector of production prices of commodities can be reckoned in money terms as a sum of profits and wages:

$$\mathbf{p} = \mathbf{p}A (1+r) + \mathbf{a}_n \mathbf{w}$$
 [2],

where r is the equilibrium rate of profit, equal to the rate of interest on money capital, a_n is the row vector of the labour inputs and w is the real wage rate. This is a system of n price equations, with n + 2 unknowns (n prices, r and w), that we can close by taking the price of a commodity as $num\acute{e}raire$ and by adding a further equation providing an exogenous value of r or w. Its general solution is the price vector

$$p = a_n w [I - (1 + r) A]^{-1}$$
 [3].

Commodity prices depend on the quantities of labour embodied in the commodities produced, on the technical conditions of production that determine the amount of the surplus and on the social distribution of income between wages and profits. The total purchasing power of the commodities produced in the economy, expressed in terms of labour commanded at prices p when the real wage is taken as *numéraire*, equals the sum of wages and profits. In the limit case in which w = 1 and r = 1

0, the labour theory of value hold in its pure form and the values coincide with the prices of production of commodities.

Let us proceed. Since the matrix within the square brackets can be expanded in an infinite Taylor series of convergent powers, in terms of the dated quantities of direct and indirect labour embodied in the various stages of production, vertically integrated, it is now possible to operate a reduction of commodities prices to dated and weighted labour quantities. The processes of production will thus be considered in a diachronic key, as sequences of labour inputs following each other over time up to the production of the commodities destined to the final uses.

To ensure convergence of the Taylor power series, the factor of capitalization at a compound rate $(1+r)^t$ must be less than the inverse of the eigenvalue with greatest modulus of matrix A. This condition is satisfied for any value of r lower than the greatest theoretical one, which corresponds to a subsistence wage. Relation [3] can then be re-written in the form

$$\mathbf{p} = \mathbf{a}_n w + (1+r) \, \mathbf{a}_n \, \mathbf{A} w + (1+r)^2 \, \mathbf{a}_n \, \mathbf{A}^2 w + (1+r)^3 \, \mathbf{a}_n \, \mathbf{A}^3 w + \dots$$
 [4],

in which production prices are shown as sums of wages and profits, each term is smaller than the previous one and $A^n \to 0$ as n tends to infinity. This is a matrix equation of 'reduction to dated quantities of labour', homogeneous in production prices. If it is satisfied by a particular price vector, any positive multiple of that vector satisfies it.

From this equation we can see that commodities prices are made of an infinite number of overlapping layers of wages and profits and depend on the proportions in which labour and capital are employed in the different stages of production and on the social distribution of income between wages and profits. A fundamental role is played by the vector a_n of the labour coefficients. This means that labour values are relevant to explain commodities prices, but they are not the only relevant element. With a different organic composition of capital in the single production processes, for every level of the rate of profit there is a given set of relative prices and a given real wage rate. Under such conditions, there cannot be a proportionality relation between prices and labour-values, unless in the unrealistic limit case of r = 0 and w = 1. This means that Marx's law of value does not hold when capital goods inputs are present and production is made for profit.

The price implications of this matrix system are a direct consequence of the assumption of constant returns to scale. Under such conditions, prices must equal the average reproduction-costs. To get the vector \mathbf{p} of commodities prices of production, one should add to the quantities of labour employed in the various stages of production the profits accrued in the course of time from the moment labour is employed to the moment output is sold. Denoting by λ the vector of labour-values, i.e. of the embodied labour coefficients, we have

$$p = \lambda [I - rA (I - A)^{-1}]^{-1}$$
 [5].

This expression allows to transform the vector of commodity labour-values λ into the corresponding vector of prices of production p. The matrix which determines the price solution is the transpose of the matrix A of the quantity solution. $(I - A)^{-1}$ is the Leontief inverse, or total requirements matrix. The sum of each column in this matrix is the output multiplier for the industry concerned. If we denote by T the inverse matrix in the square brackets, that describes the set of relations between final demand and production, the above condition requires that the price vector p should equal λT .

The algebraic problem of the transformation of labour-values into prices of production thus is formally solved. The column price-vector p is a linear combination of the row vectors of the transformation matrix T. The proportionality factor is λ , the vector of vertically integrated labour-values. In the particular case of a linear relationship between commodities labour values and market prices, λ is a constant. It can then be interpreted as the monetary expression of living labour-time (MELT). On the whole, these results confirm the importance of the role played by total labour-time,

that is by both living and dead labour, in the determination of production prices, and point out the special relevance assumed by living labour in the particular case of a linear relation.

The above input-output model approach is however subject to certain limitations, due to the assumptions of constant returns to scale and of a given technology matrix not affected by price changes. It is a supply-oriented price model entirely focused on the production side. The relevance of demand is limited to the determination of quantities.

We shall now introduce in this model the demand side. In a market economy the composition of the social surplus is the result of the technology of the system and of a free choice determined by the use-value and the scarcity of the different goods. We know the unit coefficients of direct need of intermediate goods in the different productive sectors of the economy. We can thus calculate the effect determined on the output of each sector by a given change in final demand. Hence, we can calculate the total output of each industry.

Denoting by d the column-vector of the demand for commodities, the equilibrium condition is

$$y = (I - A)x = d ag{6}.$$

The economy net output must thus be

$$x = (I - A)^{-1} d$$
 [7].

The items of each column of the inverse Leontief matrix $(I - A)^{-1}$ indicate the output of each sector which is required to satisfy the demand for the products of that sector. This allows to realize the passage from a closed to an open Leontief model. That is to a model in which the output of each sector depends on the relative output prices and the social surplus is distributed between the social classes in variable proportions.

The role played by the demand side in the Keynesian theory can be extended from the short to the long period, to derive a view of capital accumulation in which the size of the capital stock is itself determined by the demand for outputs. The productive capacity of the economy and its rate of utilization than become two dependent variables. They are influenced by the quantities of output that the market can absorb, and will react to any change in final demand.

10. Reconsidering capital in a 'full cost-plus' perspective.

Marx used to distinguish abstract labour-time, the internal measure of value, from the external measure, money. He saw a logical connection between these measures and used both of them. He called 'variable capital' the value of the part of money capital used to hire labour-power (i.e. to pay wages) and 'constant capital' the value of the means of production (materials, tools and machinery), which he assumed did not increase and yield a surplus. It could only decline over time, by depreciation due to wear and tear, and in the meantime be transferred to the product ³⁴. Marx called 'organic composition of capital' the ratio of constant to variable capital and used it as a measure of the capital intensity of production. But Marx's distinction between variable and constant capital is a wrong one. It does not clear whether capital should be regarded as a stock or a flow concept ³⁵.

³⁴ Textually: "That part of capital then, which is represented by the means of production, by the raw material, auxiliary material and the instruments of labour, does not, in the process of production, undergo any quantitative alteration of value. I therefore call it the constant part of capital, or, more shortly, *constant capital*" (Marx, *Capital*, vol. I, chapter 8th, p. 209).

³⁵ A better distinction is the classical one, criticized by Marx, between circulating capital – a flow concept applied to non-durable items (raw materials, energy, auxiliary substances and intermediate goods) used and consumed as inputs, which disappear in the course of a single production process – and fixed or durable capital, which is a stock concept.

As we have seen, the elder Marx recognized that living labour was not the single source of new value and that only a fraction of fixed capital is constant. In his more 'mature' writings, there is large textual evidence of this change of perspective. In his economic manuscripts of 1861-63, he wrote that "the value of a commodity is determined by the total labour time, past and living, which enters into it..., not only by the labour time contained in the fixed capital and the circulating capital" (1991 ed., p. 136). This was, for Marx, a fundamental doctrinal point.

In the real world, where a large number of commodities are produced, there are many kinds of capital goods whose production requires time, and they last for a limited time period. Our analysis can therefore provide only a starting point for the study of capital theory. More complex analytical tools are required. In more realistic multi-sector models Wicksell effects, of real and price nature, can occur and cause paradoxical capital behaviours. In steady state neoclassical equilibrium models the theory of capital is much simplified. Any act of saving implies an equal investment of capital and there is no logical need for money.

Let us now denote net social value of an economy the value of its net social output and redefine constant capital to include in social value only the part of the value of fixed capital that has been used and consumed in a single productive cycle and corresponds to the depreciation allowance. As in general this part is not directly observable, it has to be calculated by conventional methods. The residual is an hybrid. It does not measure the value of the net social output of the economy ³⁶.

It should be noticed that Marx's net social output includes the subsistence wages of workers. It is thus greater than the surplus-value, because it is net of the amortization quotas of fixed capital, but it is not, illogically, net of subsistence wages, 'the amortization quotas of human capital' that are destined to reintegrate the labour-power. As a result of using this spurious concept, conventionally adopted in modern national accounting, the net social output is wrongly reckoned by Marx as the sum of a net component, non-wage incomes, and a gross component, the wage bill. In this way profits are underrated and wages are overvalued. The social distribution of income is artificially altered and the reasons for social conflict become less evident.

We shall now make a further step forward and examine the behaviour of capital in a disequilibrium framework. This extension of our analysis implies the integration of the theory of capital with the theory of investment.

Investment demand has a derived nature. It is the result of a stock-adjustment mechanism of the 'flexible accelerator' type. The supply of loanable or investible funds at given rates of interest has a flow character and should obviously be distinguished from that of capital goods, with a stock nature, to which the Austrians were referring in their material conception of capital.

In neoclassical models the optimal size of the capital stock can be established so as to maximize the utility of a consumption stream or the internal rate of return on invested capital. To determine the flow of investment which is needed to adjust the capital stock, that is the rate of investment, specific assumptions about the speed of reaction of capital users are required. There will also be adjustment costs associated with the investment, whose entity will depend on how fast the adjustment of the capital stock is made ³⁷.

At any moment, the equilibrium point will be given by the intersection of the MEI curve corresponding to the existent stock of real capital with the equilibrium path showing the balanced growth of the inherited stock of capital goods. If the rate of interest lowers, new investments are

³⁶ This is obtained by subtracting constant capital, correctly defined, from gross social output. It measures the value of the material means of production, plus the value of the wage goods required for workers subsistence (Marx's variable capital), plus the surplus-value (defined as the excess of the value of the 'final' goods produced in the economy over the necessary re-integrations), minus the depreciation allowance of fixed capital.

³⁷ Strict convexity of the adjusting costs is a basic assumption of James Tobin's q-theory of investment, in which q is the average return on capital, measured by the ratio between the market value and the replacement cost or book value of physical assets, and the level of investment is explained by the firms valuation by the stock market relative to the book value of their capital goods.

made and the stock of capital grows. The economy will then move asymptotically, along the equilibrium path, from the short-period equilibrium position, implying equality of MEI and the rate of interest, towards the stationary state equilibrium position characterized by the three-fold margin of convenience between present and future consumption, real and financial investment, money and bonds, established by MRR, MEI and the rate of interest. If the inherited initial position is a point out of the equilibrium path, the system will first reach the stationary position and then move along the equilibrium path.

The dynamics of the capital stock can be analyzed by means of a phase diagram in the (K, λ) space, where K is a state variable and λ the value of an additional unit of capital, measured by the present value of the profit expected by the additional unit. The diagram shows the set of all possible trajectories or paths of the system considered (its 'phase portrait') and displays the evolution of a dynamic system from an initial moment. The equilibrium locus is a saddle-point. In equilibrium, the cost of the additional unit of capital installed must equal the value of that unit of capital. For $d\lambda/dt = 0$, the result is a downward sloping curve in the (K, λ) phase space. ...

Other disequilibrium approaches to the theory of capital and investment are those of the Austrian and the neo-Walrasian schools. The Austrian one is focused on changes in the capital time structure. It accounts for changes in the entrepreneurs expectations about the future, in a sequence of short-periods, in which temporary equilibria are established, but changes in the parameters can take place only in the passage between consecutive market periods. It lacks an autonomous investment function. The neo-Walrasian is a disaggregated general equilibrium approach, with initial endowments of the various types of capital and quantitative restrictions that prevent the normal working of the Walrasian *tatônnement* mechanism of adjustment and establish in the presence of rationing a prevalence of the 'short side' of the market. This approach to the problem can determine only a short-period equilibrium.

Chaos theory can also be tentatively applied to the study of non-linear economic dynamic systems of stochastic nature. It should allow to see the ultimate effects of a small change of a minor variable (as Lorenz's 'butterfly wing-beat') at thousand miles distance. But unpredictability of the future development of the economy seems in the case of non-linear dynamic systems to be the rule. The presence of chaotic attractors of random orbits having a fractal nature makes economic forecasting very difficult. Despite the initial enthusiasm about the possible relevance in economics, chaos theory does not appear to have changed significantly the methodology of scientific search in the fields of capital and investment.

11. Concluding remarks and policy implications.

It is time to make some final remarks and to point out the policy implications of our approach to capital theory. We have shown that there are significant interactions between the investment and the financing decisions of entrepreneurs, that the availability of money capital is a necessary prerequisite of productive activity, that time plays a fundamental role in capital theory and that investment decisions imply for a firm a financial cost covering the provision of invested capital. Unfortunately, the presence of this cost is disregarded in the accounting practice, which looks at it as a minimal required return element, rather than as an implicit cost. Normal profit loses in accounting its cost nature to achieve that of an earning element.

Economists call profit the difference between total revenue and total input cost. They distinguish normal profits from quasi-rents. Professional accountants and bookkeepers do not. They deduce from revenues only explicit expenses, thus failing to recognize that the value of a foregone opportunity represents a cost for the investor. This failure has important practical implications. It leads to an inefficient allocation of a society's resources, with negative effects on capital accumulation and growth policies.

Among the various methods of cost accounting used in capital and investment theory we have opted for a 'full-cost-plus' labour-and-capital monetary approach, suited to include the real cost of production, the financial cost of invested capital and a mark-up over cost. On this basis, we have exposed some guiding lines for the construction of a capital theoretical model, we have expressed our favor for a Marxian valuation metric, MEV, the monetary expression of labour-value, and we have refused the neo-Marxist price-index MELT, which regards as directly productive only living labour as directly productive. We do not accept the labour theory of value endorsed by the young Marx, a theoretical construction that presupposes a primitive society and cannot hold when capital goods are systematically used in production. We also refuse Marx's concept of constant capital. His distinction between variable and constant capital is logically unacceptable. But we do not object to his reference to a historically conditioned law of value regulating the determination of commodities relative prices.

Bibliographical references:

BLISS C. (1975), Capital Theory and the Distribution of Income, North-Holland, Amsterdam.

---- (2005), *The Theory of Capital: A Personal Overview*, Introduction, in C. Bliss, A.J. Cohen and G.C. Harcourt, eds., *Capital Theory*, Elgar, Cheltenham, 3 vols., vol. I, pp. xi-xxvi.

CAVALIERI, D. (1963), Sulla relazione tra il saggio di rendimento oltre il costo e l'efficienza marginale del capitale, "Giornale degli economisti e annali di economia", March-April, pp. 247-73.

---- (2004), On Some Equilibrium and Disequilibrium Theories of Endogenous Money: A Structuralist View, "History of Economic Ideas", vol. 12, n. 3, pp. 49-81.

---- (2009), Sull'inseparabilità delle strutture sintattiche nell'analisi classica del valore e della distribuzione, "Studi economici", vol. 64, no. 97, pp. 5-46.

---- (2010), Valore e costo reale di produzione. Un riesame del problema, dopo la "New Value Controversy", "Studi Economici", vol. 65, no. 102, pp. 5-47.

COHEN, A. J. and HARCOURT, G.C. (2003), Whatever Happened to the Cambridge Capital Theory Controversies?, "Journal of Economic Perspectives", vol. 17, no. 1, pp. 199-214.

FOLEY, D. K. (2000), *Recent Developments in the Labour Theory of Value*, "Review of Radical Political Economics", vol. 32, no. 1, pp. 1-39.

GAREGNANI, P. (1970), Heterogeneous Capital, the Production Function and the Theory of Distribution, "Review of Economic Studies", vol. 37, pp. 407-36.

---- (1984), Value and Distribution in the Classical Economists and Marx, "Oxford Economic Papers", vol. 36, pp. 291-325.

---- (1990), *Quantity of Capital*, from *The New Palgrave Dictionary of Economics Series: Capital Theory*, J. Eatwell, M. Milgate and P. Newman eds., Macmillan, London, pp. 1-78.

HAHN. F.H. (1982), *The Neo-Ricardians*, "Cambridge Journal of Economics", vol. 6, no. 4, pp. 353-74.

HARCOURT, G.C. (1976), *The Cambridge Controversies: Old Ways and New Horizons - or Dead End?*, "Oxford Economic Papers", vol. 28, pp. 25-65.

HAYEK, F.A. von (1941), The Pure Theory of Capital, Routledge & Kegan, London.

HICKS, J.R. (1939), Value and Capital, Clarendon Press, Oxford, 2nd ed. 1946.

---- (1973), Capital and Time: A Neo-Austrian Theory, Clarendon Press, Oxford.

KEYNES, J.M. (1973-83), *Collected Writings*, ed. by D. Moggridge, Macmillan, London, and Cambridge Univ. Press, Cambridge.

KLIMAN, A. (2007), Reclaiming Marx's Capital: A Refutation of the Myth of Inconsistency, Rowman and Littlefield, Lanham.

KRISTJANSON-GURAL, D. (2008), *Money is Time: The Monetary Expression of Value in Marx's Theory of Value*, "Rethinking Marxism", vol. 20, no. 2, pp. 257-72.

LERNER, A. (1936-37), *Capital, Investment and Interest*, "Proceedings of the Manchester Statistical Society", Report of Group Meetings, pp. 26-31, repr. in Lerner, *Essays in Economic Analysis*, Macmillan, London, 1953, pp. 347-53.

---- (1944), The Economics of Control: Principles of Welfare Economics, Macmillan, London.

---- (1965), On Some Recent Developments in Capital Theory, "American Economic Review", Papers and Proceedings, May, pp. 284-95.

MARX, K. (1867-1894), *Das Kapital: Kritik der politischen Ökonomie*, voll. 3, tr. *Capital*, Lawrence and Wishart, London, 1969-74, and *MECV*.

MAVROUDEAS, S. (2001), The Monetary Equivalent of Value and Certain Issues Regarding Money and the Value of Labour-Power, "Économie Appliquée", vol. 54, no. 1, pp. 3-34.

PASINETTI, L.L. *et alii* (1966), *Paradoxes in Capital Theory: A Symposium*, "Quarterly Journal of Economics", vol. 80, no. 4, November, pp. 503-83, with *A Summing Up* by P.A. Samuelson, pp. 568-83.

RIEU, D.-M. (2008), Estimating Sectoral Rates of Surplus Value: Methodological Issues, "Metroeconomica", vol. 59, no. 4, pp. 557-573.

SRAFFA, P. (1960), *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*, Cambridge Univ. Press, Cambridge.

TOBIN, J. (1969), A General Equilibrium Approach to Monetary Theory, "Journal of Money, Credit and Banking", no.1, pp. 15-29.

Abstract

This is a paper on the theory of capital. It deals with the role of capital in a cost-of-production theory of value in which both labour and capital are directly productive. The guidelines of an analytical method are proposed. Marx's 'monetary expression of abstract labour-value' (MEV) is used as price-index. It is preferred to the 'monetary expression of labour time' (MELT), exclusively focused on living labour, suggested by some neo-Marxist scholars during the 'New Value Controversy'. The author, a critical Marxist, develops the trace provided by Marx in his Grundrisse 'Fragment on Machines', where he pointed out the need to abandon the labour theory of value and to rely on a broader labour-and-capital monetary theoretical construction. Due attention is paid in this essay to the time and money dimensions of capital and to the roles of both real and financial capital.

JEL Classification: B13, B22, B51, E22, E41.