Electricity prices and cross-border trade: volume and strategy effects

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Abstract

In this paper we derive equilibrium bid functions in isolated domestic electricity markets and then analyse their modifications when cross-border trade among them is managed using the implicit auction method. We show that cross-border trade can induce price convergence across countries and thereby reallocate gains and losses as a result of two concomitant effects: a "volume" effect due to the mere increase/decrease of demand and supply in each market and a "bid effect" due to the modifications of bid functions brought about by interconnection. The latter effect can either contrast or reinforce the former. We derive conditions affecting the net result.

Keywords: Electricity markets, implicit auctions, cross-border trade

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1. Introduction

Since the early 1990s electricity markets throughout the world have been subject to radical transformations. These reforms have introduced new institutional frameworks intended to ease competitive entry, to provide incentives for efficiency in the generation, transmission,

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distribution and retailing of output, and to reduce tariffs and permit direct access. Following the example of the England & Wales electricity market regulation introduced in the UK in 1990, governments in many countries have implemented mechanisms supposedly designed to enable efficient price-quantity exchange at wholesale level. These mechanisms have taken the form of a pool-based market for the physical supply of bulk electricity. Two pricing rules have been implemented so far: the uniform (or marginal) price and the discriminative (or, pay-as-bid) price. The former is the most common following its initial application in the English Pool. However, the replacement of the Pool with the NETA (which works with a pay-as-bid rule) shows that the choice of pricing rule for electricity auctions is still being debated. There is an extensive literature dealing with: a) how competition in electricity markets can be appropriately modelled; and b) how efficient the outcomes of the game are. This stream of literature, starting with Green and Newbery (1992), (see also Newbery 1998), applies the supply function equilibrium model of Klemperer and Meyer (1989) to electricity markets, whereas the stream of literature following von der Fehr and Harbord (1993), (see also Garcia-Diaz and Marin (2003) and Fabra et al., (2004)), models electricity markets as multi-unit procurement auctions. In the latter approach the electricity exchange is modelled as a game of perfect information about agents' costs with the possibility of considering asymmetric firm capacities. Garcia-Diaz and Marin (2003) in particular, found that under price-inelastic demand the Cournot model overestimates pool prices determined applying a uniform-price rule. Fabra et al. (2004) contrasts the prices emerging from two different auction formats, namely discriminatory and uniform price rules. Results indicate that the uniform price rule (also known as the system marginal price rule) yields higher average prices than the discriminative rule. Parisio and Bosco (2003) have more explicitly called into question the efficiency results of competitive electricity auctions, in a model that departs from the above literature in that imperfect information about agents' costs is assumed. They derived a family of flat equilibrium bid functions characterised by the presence of mark-ups that vary with the amount of energy sold.

A related but less well investigated set of issues is represented by the regulation of the exchange of electricity among countries rather than within national markets. The main issues considered were initially represented by the externalities associated with the loop flow phenomenon existing in an electric power network and by the concern that they constitute a significant barrier to the formation of efficient markets for electricity and transmission services. Chao and Peck (1996) found that the separation of energy markets achieves a social optimum and that it coincides with nodal pricing complemented by Transmission Systems Operators (TSOs) in the presence of complete and competitive markets with no uncertainty and full information. The removal of some of those assumptions permitted Bushnell (1999) to demonstrate that generators can exercise market power by withholding physical transmission contracts, and Smeers and Jnhg-Yuan (1997) show that if only a limited number of traders arbitrage prices between nodes, then they exercise market power. More recently, Joskow and Tirole (2000) and Neuhoff (2003) analysed a situation in which the transmission rights are held by generators. Neuhoff (2003) examined cases where either transmission rights over constrained links are auctioned and traders can arbitrage separate energy spot markets, or transmission rights and energy markets are integrated through a system operator that clears several energy spot markets using available transmission capacity between these markets. Neuhoff (2003, 17) found that in the former case (separation) the effect of traders submitting quantity bids is that the amount of energy transmitted between markets is not directly influenced by the output decision of strategic generators, but by the aggregate bid that traders submit to both energy spot markets. All these findings, and particularly the latter, imply that the efficiency of a domestic pool cannot be analysed without explicit reference to the trading of electricity across markets, since trade opportunities may influence domestic bidding strategies.

On the empirical side, even casual observation shows that transmission activity across markets has become increasingly important in all industrial countries. Given the existence of technical restrictions on available cross-border transmission capacity, however, the import and export of electricity first needs to be subjected to some form of rationing and then, and more importantly, to be co-ordinated with the national markets operating at each transmission node. Experience, especially in Europe, shows that this rationing activity can be managed under different legal arrangements whose actual implementation may lead to quite different efficiency results. However, following the incorporation of the Electricity Directives (from 1996 onwards) in national legislations, the European Electricity Market still looks more like a mere juxtaposition of weakly connected, individual markets than an integrated and normally functioning regional commodity market (Finon, 2001). This situation makes the regulation of European cross-border trades even more crucial.

In this paper we try to bridge across the above mentioned sets of issues and discuss how bidding behaviour and auction outcomes in domestic markets are affected by interconnection. In particular, we analyse the case in which cross-border interconnection is congested, and congestion is managed by the so called implicit auction method. This method consists of a pricing rule and an allocation rule for cross-border capacity, both based on the results realised in the domestic markets existing at each transmission node. Specifically, the use of transmission capacity is charged as the difference between the two domestic closing prices (pricing rule) and the rights to use this capacity (allocation rule) are reserved to the bidders included in the domestic merit orders generated by the two auctions. As a result of various meetings of the Florence Forum, the implicit auction is the recommended method for managing cross-border congestion and, for this reason, we analyse domestic bidding behaviour under the assumption that the use of capacity for cross-border trade is priced and allocated according to that system. We pay particular attention to the phenomenon of demand shift across countries and to modifications to the number of despatched generating units brought about by cross-border trade, and discuss whether and how interconnection amplifies the distortions between bids and true generation costs in each domestic market. Our results indicate that the asymmetry of information about bidders' costs and the strategic opportunities offered domestically by the uniform price rule, combined with application of the implicit auction for capacity, affect the outcomes of domestic auctions by modifying the mark-up components of bids over costs. In particular, interconnection determines an increase in the bid submitted by all infra-marginal bidders. This is not always the case for the bid made by the marginal unit and we derive condition for this latter bid to increase after interconnection. Finally, we discuss the welfare implications of price changes induced by interconnection: the reallocation of gains and losses across countries rewards consumers in the high cost countries and producers in the low cost countries. This effect is not a mere consequence of the operation of a demand-supply scheme since it can be either amplified or hampered by interconnection depending upon the bidding strategies of domestic generators.

This paper is organised as follows. In section 2 we present the general assumptions of the model. In section 3 we model domestic Power Exchanges as a decentralised method to allocate electricity under asymmetric information and derive expressions for the equilibrium bids. In section 4 we discuss the alternative methods for managing cross-border exchanges of electricity and particularly the so-called implicit auction method. In section 5 we study the modification of bid functions in domestic markets when the two Power Exchanges are interconnected and the congestion is managed by the implicit auction method, and discuss the main results obtained. In the section 7 we provide some concluding comments.

2. Assumptions

We assume that there are two countries in which electricity is both produced and consumed. The two countries can engage in cross-border trade. For each country we make the following general assumptions about industry structure, costs, demands and information.

A1 The industry There are n firms operating in each national market. Each firm $i \in N = \{1,...,n\}$ produces electricity using two types of plants: base-load and peak-load. As a result, there are *2n* plants/generators in each market. We use α to indicate base-load plants, and β to indicate peak-load plants. We assume symmetric firms with base-load capacity and peak-load capacity indicated by $\overline{q}_\alpha$ and $\overline{q}_\beta$, respectively.

A2 Costs The base-load and peak-load plants of each firm *i*, are assumed to have constant marginal costs of generation c_α^i and c_β^i respectively, with $c_\alpha^i < c_\beta^i, \forall i \in N$ and $\max\left\{c_\alpha^1,...,c_\alpha^n\right\} < \min\left\{c_\beta^1,...,c_\beta^n\right\}$.

A3 Pool Markets Exchange of electricity is organised via a centralised market mechanism which is managed by an independent operator acting as an auctioneer. Each firm reports a menu bid to the Market Operator, which specifies any possible quantity level up to capacity and the associated (constant) asking price. Assume that portfolio bidding is not allowed, so that each firm *i* submits (menu) bids (b_α^i, b_β^i) for base-load and peak-load production separately. The Market Operator calculates the merit order of bids and, based on demand, allocates energy production among units in a cost minimising way. The market closing price corresponds with the last accepted bid and is called the System Marginal Price (SMP) to be paid to all the despatched units. Accordingly, Pool markets can be seen as First Price Uniform Auctions.

A4 Information We assume cost uncertainty. Thus, costs represent the "type" of each plant and each multi-plant firm observes its own costs of production. The market operator does not observe any cost level. Costs are assumed to be independently drawn from a commonly known distributions $F_\alpha\left(c_\alpha\right)$ and $F_\beta\left(c_\beta\right)$ for base and peak plants, respectively. They have strictly positive and finite densities $f_\alpha\left(.\right)$ and $f_\beta\left(.\right)$ on the intervals $\left[\underline{c}_\alpha, \overline{c}_\alpha\right]$ and $\left[\underline{c}_\beta, \overline{c}_\beta\right]$. Since in A2 it was assumed that $c_\alpha^i < c_\beta^i, \forall i \in N$ this requires $\overline{c}_\alpha < \underline{c}_\beta$. A2 and A4 implies that $\left[1 - F_\beta\left(c_\beta^i\right)\right]$ indicates the probability of observing a (peak) cost level which is both higher than c_β^i and c_α^i.

A5 Demand We assume that in each country the Market Operator announces the total demand before the opening of the day-ahead bidding session. In particular, calling D_t the total demand at hour t we assume that $n\overline{q}_\alpha < D_t < n\left(\overline{q}_\alpha + \overline{q}_\beta\right)$. This implies that all the n base-load units would be the first to be despatched. Consequently, demand exceeding $n\overline{q}_\alpha$ must be matched by peak supply. We assume price-taking behaviour on the part of consumers, as in Garcia-Diaz and Marin (2003: 203), and, in order to simplify the model, we also assume totally inelastic demand. At the present time, the latter assumption appears to resemble the reality of many electricity markets, for instance the Italian and the Spanish, where the eligibility threshold is still quite high.

A6 Cross-border trade The two countries have an industry structure as in A1. The grids are connected by means of a network that permits trade between them. We assume that costs are systematically different in the two countries: one country (*L*) is supposed to have base and peak costs lower than the corresponding costs of the other country (*H*) for any level of capacity. In *L* and *H* costs are distributed in such a way that in *L* the survival function of costs, $\left[1-F_L^\alpha(c_\alpha)\right]$, First Order Stochastically Dominates the survival function of country *H*, for base and peak generation:

$$\left[1-F_H^\alpha(c_\alpha)\right] \geq_{FSD} \left[1-F_L^\alpha(c_\alpha)\right]$$
$$\left[1-F_H^\beta(c_\beta)\right] \geq_{FSD} \left[1-F_L^\beta(c_\beta)\right]$$

The rules governing the allocation of cross-border transmission capacity are discussed below.

A1 and **A2** approximate the typical structure of the electricity supply. Indeed, base-load plants are normally those generating units with very low marginal costs and high start-up costs. Nuclear power plants and hydro plants are typical examples of base-load units. Peak-load plants, on the other hand, are characterised by low fixed costs and high marginal costs. Base-load plants are therefore expected to operate almost 24 hours a day throughout the year (with some allowance for maintenance), whereas peak-load plants are expected to operate only when demand exceeds total base-load supply.

A4 (cost uncertainty), is motivated by the recognition that generators frequently belong to multi-utilities providing similar services often characterised by scope and scale economies (Fraquelli et al., 2004, among others). The cost of generation therefore can vary across firms because firms can exploit production diversities in ways that are not perfectly observable by competitors. Accordingly, costs are random variables representing the "type" of each bidder whose distribution is assumed to be common knowledge, as in standard auction theory. **A4** implies that firms are ex-ante symmetric in all characteristics relevant to the model. However, it will be shown that despite this symmetry, multi-plant firms are able to exploit market power through "bidding externalities".

A5 incorporates into the model a general feature of electricity demand. By looking at the data registered in a load duration curve in a typical market we can see that there is a minimum level of demand that must be matched for the 8,760 hours in a year, and there are a number of plants which are always operating. In our model, the n units that we call base-load, are the low marginal cost units, which match the above minimal level of demand. They are despatched with certainty and never fix the SMP. The assumption that demand is price-insensitive can be justified by the fact that the great majority of consumers pay regulated tariffs, whereas only a small proportion of the market demand is represented by large (industrial) consumers who are able to set price/quantity bids.

3. Domestic markets working in isolation

Day-ahead pool markets typically work like a multi-unit simultaneous auction in which given price-inelastic demand firms compete on the supply side. Considering multi-unit bidders on the demand side, Ausubel and Cramton (2002) show that they tend to "shade" their bids for units demanded after the first and they also show that bid shading increases with the number of units demanded. The intuitive explanation for this is that when a bidder wants multiple units of the good being auctioned, there is a positive probability that its bid for a second or later unit will be pivotal, thus determining the price that the bidder pays for the units she wins. Given this, the bidder has an incentive to bid less than true value for later units in order to reduce the price it will pay for the earlier units. In our context **A5** implies that base-load plants are never pivotal and therefore bidding at cost for base capacity is a weakly dominant strategy for all bidders as well as, in Ausubel and Cramton (2002), bidders bid truthfully for the first unit. In electricity markets there is an additional reason for not overbidding for base-load units. If a base-load plant bid is higher than costs there is a chance it will not be despatched and therefore the plant will be temporarily switched off, which will increase generating cost. We can therefore maintain that each firm $i, i \in N$, bids at cost for its base-load unit, namely $b_\alpha^i = c_\alpha^i$, as in a second price (unit) auction. We believe therefore that a strategy must be designed for bids submitted for peak-load quantities and that this optimal strategy should maximise the total expected profit to the firm from both types of plants.

Consider now the bidding process taking place for delivery of electricity at hour *t* of the next day. By virtue of **A5** D_t strictly exceeds total base-load capacity and $1 \le m \le n$ peak-load units are necessary:

$$D_t = n\overline{q}_\alpha + (m-1)\overline{q}_\beta + q_\beta^i$$

Since capacities are of equal size and demand is known at the time of bidding, the number *m* of despatched units is also known when bids are solicited. According to **A3** and taking previous considerations into account, each firm *i* submits a menu bid given by $\left(c_\alpha^i, b_\beta^i\right)$ to the market operator who uses them to compute a merit order for that hour:

$$c_\alpha^{(1:n)} \le \ldots \le c_\alpha^{(n:n)} < b_\beta^{(1:n)} \le \ldots \le b_\beta^{(m-1:n)} \le b_\beta^{(m:n)} \le \ldots \le b_\beta^{(n:n)}$$

The marginal plant selected (the mth out of the ordered n peak units) is the one that submits the highest accepted bid, i.e. the plant that serves the last portion of demand. Consequently, the total quantity served by firm *i* can be:

$$Q_i\left(\mathbf{b}, D_t\right) = \begin{cases} \overline{q}_\alpha + \overline{q}_\beta \text{ when } b^i_\beta < b^{(m:n)}_\beta \text{ and so the bidder is infra-marginal} \\ \overline{q}_\alpha + q^i_\beta \text{ when } b^i_\beta = b^{(m:n)}_\beta \text{ and so the bidder is marginal} \\ \overline{q}_\alpha \text{ when } b^i_\beta > b^{(m:n)}_\beta \text{ and so the bidder is extra-marginal} \end{cases}$$

where $\mathbf{b}$ is the vector of bids. When the bidder is infra-marginal, it sells its entire capacity at a price higher than its bid. When the bidder is marginal, it sets the SMP for the market and sells its base-load capacity plus the residual demand, namely $q^i_\beta = \min\left\{\overline{q}_\beta, D_t - n\overline{q}_\alpha - (m-1)\overline{q}_\beta\right\}$. When the bidder is extra-marginal it sells base-load capacity only at a price lower than b^i_β.

The probability of the three events can be calculated as follows. Let c^m_β be the cost of the marginal plant. Then the probability of firm *i* being infra-marginal is obtained using the distribution of the $(m-1)th$ order statistics from a sample of $(n-1)$ observations: $\left[1 - F_{(m-1:n-1)}\left(c^i_\beta\right)\right]$ where $F(.)$ is the parental distribution of costs defined in **A4**. Assume that a symmetric equilibrium bid function $b_\beta(\cdot)$ exists and that it is monotonically increasing with an inverse $\sigma\left(b^i_\beta\right) = c^i_\beta$. Assume also that all bidders different from *i* follow $b_\beta(\cdot)$ then the probability of being infra-marginal in the equilibrium is:

$$H_I\left(b^i_\beta\right) = \left[1 - F_{(m-1:n-1)}\left(\sigma\left(b^i_\beta\right)\right)\right]$$

and the density of the SMP when firm *i* is infra-marginal can be obtained differentiating H_I.

In the same manner, let H_m indicate the probability of being marginal:

$$H_m\left(b_\beta^i\right)=F_{(m-1:n-1)}\left(\sigma\left(b_\beta^i\right)\right)-F_{(m:n-1)}\left(\sigma\left(b_\beta^i\right)\right)=\frac{(n-1)!}{(m-1)!(n-m)!}F\left(\sigma\left(b_\beta^i\right)\right)^{m-1}\left[1-F\left(\sigma\left(b_\beta^i\right)\right)\right]^{n-m}$$

Finally, the probability of being extra-marginal, namely to have costs higher than the *m.th* among the $(n-1)$ competitors, is written as.

$$H_E\left(b_\beta^i\right)=F_{(m:n-1)}\left(\sigma\left(b_\beta^i\right)\right)$$

Then the expected profit of firm *i* is given by the sum of three components and can be written as follows:

$$\begin{aligned}
E\left[\Pi_i\left(b_\beta^i\right)\right]=&\,E\left[\Pi_i\left(b_\beta^i\right)\middle| i \text{ is infra-marginal}\right]+\\
&E\left[\Pi_i\left(b_\beta^i\right)\middle| i \text{ is marginal}\right]+\\
&E\left[\Pi_i\left(b_\beta^i\right)\middle| i \text{ is extra-marginal}\right]\\
&=\left(\overline{q}_\alpha+\overline{q}_\beta\right)\int_{b_\beta^i}^{\overline{c}}\tilde{b}\,dH_I\left(\sigma\left(\tilde{b}\right)\right)-\overline{q}_\beta c_\beta^i H_I\left(\sigma\left(b_\beta^i\right)\right) \qquad (1)\\
&+\overline{q}_\alpha b_\beta^i\left[H_m\left(\sigma\left(b_\beta^i\right)\right)\right]+q_\beta^i\left(b_\beta^i-c_\beta^i\right)\left[H_m\left(\sigma\left(b_\beta^i\right)\right)\right]\\
&+\overline{q}_\alpha\int_{\underline{c}}^{b_\beta^i}\tilde{b}\,dH_E\left(\sigma\left(\tilde{b}\right)\right)-\overline{q}_\alpha c_\alpha^i
\end{aligned}$$

The first order condition of problem (1) generates (to shorten notations we put $q_\beta^i=q_\beta$):

$$\begin{aligned}
&\frac{db_\beta^i}{dc_\beta^i}-b_\beta^i\left[\left((m-1)\frac{\left(\overline{q}_\beta-q_\beta\right)}{\left(\overline{q}_\alpha+q_\beta\right)}\frac{f\left(c_\beta^i\right)}{\left[F\left(c_\beta^i\right)\right]}\right)+(n-m)\frac{\left(q_\beta\right)}{\left(\overline{q}_\alpha+q_\beta\right)}\frac{f\left(c_\beta^i\right)}{\left[1-F\left(c_\beta^i\right)\right]}\right]= \qquad (2)\\
&-c_\beta^i\left[\left((m-1)\frac{\left(\overline{q}_\beta-q_\beta\right)}{\left(\overline{q}_\alpha+q_\beta\right)}\frac{f\left(c_\beta^i\right)}{\left[F\left(c_\beta^i\right)\right]}\right)+(n-m)\frac{\left(q_\beta\right)}{\left(\overline{q}_\alpha+q_\beta\right)}\frac{f\left(c_\beta^i\right)}{\left[1-F\left(c_\beta^i\right)\right]}\right]
\end{aligned}$$

Call $\Omega_1=(m-1)\frac{\left(\overline{q}_\beta-q_\beta\right)}{\left(\overline{q}_a+q_\beta\right)}$ the ratio between the peak plant's capacity that remains idle and the total capacity sold when firm *i* turns out to be the marginal seller and call

$\Omega_2 = (n-m)\dfrac{q_\beta}{(\overline{q}_\alpha + q_\beta)}$ the ratio of the peak share to the total quantity sold when firm *i* is marginal. Using the above definitions, and the standard boundary condition in symmetric auction models, $b(\overline{c}) = \overline{c}$, the differential equation (2) has a solution given by[1]:

$$b^i_\beta = c^i_\beta + \frac{\left[F\left(c^i_\beta\right)\right]^{\Omega_1}}{\left[1-F\left(c^i_\beta\right)\right]^{\Omega_2}} \int_{c^i_\beta}^{\overline{c}} \left[\frac{\left[1-F(\tilde{c})\right]^{\Omega_2}}{\left[F(\tilde{c})\right]^{\Omega_1}}\right] d\tilde{c} \qquad (3)$$

Function (3) represents a family of symmetric bid functions, i.e. a menu of flat bids, one for each possible level of quantity despatched[2]. It can be seen that function (3) incorporates many of the properties of the bid of single-object procurement auctions. In particular, it is increasing in own costs as well as in $\overline{c}$. Specifically, as $q_\beta \to \overline{q}_\beta$ and $\overline{q}_\alpha \to 0$, bid function (3) coincides with a typical equilibrium bid made in a first price procurement auction. Observe that, $\lim_{\substack{m\to n \\ q_\beta \to \overline{q}_\beta}} b^i_\beta = \overline{c}$. In this case, generators make strictly positive profits from selling base and peak quantities, whereas the highest cost generator makes profits on the base-load unit only. This is analogous to Garcia-Diaz and Marin (2003:205), for a SMP equal to $p^{\max}$.

A series of observations can be made about the optimal bid in equation (3). However, since many of the comparative static properties of the equilibrium will become relevant in the presence of cross-border trade, for this reason the analysis is conducted in section 5 rather than in this section.

At this stage we can emphasise the effect of the industry productive mix ($\overline{q}_\alpha, \overline{q}_\beta$) on the bid function. It is easy to show (see the Appendix and section 5 for the derivation of results) that the

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1 Given the probabilities defined in the text:

$$\frac{h_l}{H_m} = (m-1)\frac{f(\cdot)}{F(\cdot)} \text{ and } \frac{\frac{dH_m}{dc^i_\beta}}{H_m(c^i_\beta)} = (m-1)\frac{f(\cdot)}{F(\cdot)} - (n-m)\frac{f(\cdot)}{[1-F(\cdot)]}$$

Substituting the above expressions into the F.O.C. of problem (1) and solving the differential equation we obtain (3).

bid function (3) increases with both $\overline{q}_\alpha, \overline{q}_\beta$. The SMP rule induces a "positive bidding externality": increasing the bid of the (potentially) marginal peak unit may increase the SMP and hence the profit to all units despatched. This explains why the bid function increases with the size of the base-load unit. Nevertheless, in our symmetric model, this does not alter the relative position of plants in the merit order, thereby maintaining the efficiency selection properties of the mechanism. However, it permits firms to obtain higher mark-ups with respect to the case in which $\overline{q}_\alpha = 0$. Given the above externality effect, our electricity auction model shares some of the characteristics of Ausubel and Cramton's (2002) multi-unit auction model, and thus we can state that the SMP rule allows multiple-plant firms to exploit market power, even in a symmetric context, by optimally shading their bids. This market power emerges as a result of a Nash-Bayes equilibrium in which all bidders expect the opponents to exploit the externality between peak and base units and find it optimal to follow them. Does cross-border trade reduce or amplify this possibility of exploiting market power? In order to discuss this issue we now explicitly introduce into the model cross-border trade and its regulation.

4. Methods for managing cross-border congestion

In Europe the restrictions on the cross-border trade of electricity induced by the presence of some capacity constraints have led to the use of price and non-price allocation methods to manage cross-border congestion (see, for example, Laurent and Percebois, 2001). These methods are aimed at solving the congestion problems in the inter-connectors (i.e. the system of direct transmission lines between pairs of countries) since half of these lines are permanently or at least frequently congested while only less than a third are seldom or never congested, see Table 1.

Insert Table 1 here

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2. Flat bid functions are also obtained in other market models like Fabra et al (2004).

The methods that can be used to solve the congestion problems vary considerably and include market and non-market methods. Among the latter we include pro-rata assignments, queues (i.e. first come first served) and retention. Market methods imply some competition for the use of capacity, which can be organised in the form of either explicit or implicit auctions. When there is a lack of any systematic congestion pattern and the network topology makes it difficult to use auctions, Transmission System Operators (TSO) operating in connected local areas can coordinate cross-border trade by re-despatching or counter trading. From Table 1 we can see that the total transmission capacity along European lines is equal to 73.305 MW whereas the freely available capacity after considering the long term contracts still in operation is equal to 65.351 MW. Note that about half the free capacity is assigned by means of market methods, either explicit or implicit auctions. This is more or less the same proportion as is allocated using market methods along permanently or frequently congested lines. The allocation frequency also differs across borders, although the daily base seems to be the most commonly used system. However,, the *use it or lose it* principle also seems to have been widely adopted.

With retention, capacity is reserved to vertically integrated utilities with long term contracts. Under the pro-rata system importers make requests for capacity to the national TSO which then curtails total demand on a pro-rata basis in order to exhaust the available import quantity. If a first come first served rule is applied, total demand is curtailed on the basis of the timetable given by requests. Clearly, none of these systems assign capacity on the basis of users' willingness to pay and therefore do not provide market signals to operators.

Acknowledging the above shortcomings of non-market methods, and building on the recommendations that emerged during the Meetings of the Florence Forum, the EU Commission issued European Regulation 1228/2003 which recommends the adoption of market-based mechanisms to solve cross-border congestion problems. However, the regulation does not indicate a specific market method and this leaves open the issue of proper design of the mechanism to be adopted.

Explicit auctions were the first market methods to be considered and implemented. They work as competitive auctions for segments of the existing capacity, with the closing price corresponding to the last accepted bid price to be paid by all winning bidders. After nomination, options become obligations and flows in opposite directions can be netted out. This system is quite widespread and (understandably) is adopted by countries connected by lines that are never or only rarely congested (see Table 1). Initially, separate auctions were generally held by each TSO independently; however, they have evolved towards a jointly held auction format. Explicit auctions send price signals to operators, but have been alleged to lead to inefficient use of capacity in the presence of meshed networks. In fact, the optimal utilisation of meshed networks would require a more centralised and integrated mechanism.

The theoretical optimal method is nodal pricing; its implementation has not yet been considered due to the presence of national TSOs that are responsible for the security of local networks. Coordinated market methods, which have received more attention in recent debates, are all variants of the implicit auction, and are also referred to as market splitting, which is the allocation mechanism that has been used by North European countries since 1993. Ehrenmann and Smeers (2005, 142) rightly emphasise that market splitting is a market-based method and hence fulfils the requirement of EU regulation. At the same time market splitting can be used in different sub-markets with some flexibility since it requires only a minimum common design and can be adapted to specific conditions because it allows for some diversity across sub-markets, such as the management of intra-zone congestion, intra-day trading, balancing, etc. In implicit auctions, the available capacity is allocated simultaneously with the physical energy in the domestic markets. The physical energy bought is charged at the SMP prevailing in the local market, whereas the use of cross-border capacity is remunerated by the difference between the two domestic prices realised in each national market either side of the inter-connector. TSOs retain this price difference as compensation for the intermediation activity after netting, i.e. once the opposite flows are taken into account and compensated for. More recently, EuroPEX and

ETSO (2004) presented a joint proposal for flow-based market coupling. Local market clearing is obtained in isolation and net export curves are derived as a function of price differential. A central auctioneer then determines the efficient use of available capacity, using the information provided by local markets and resorting to an iterated procedure.

The market coupling hypothesis is based on the price differential in the two national markets, which remunerates the use of capacity provided by the connector. In a situation of perfect information the price differential would correspond to the cost differential and this would guarantee an efficient outcome. Brunekreeft et al. (2005) recently emphasised that market coupling is superior to joint explicit auctions because it limits the market power of generators located in frequently congested areas. This is because markets clear simultaneously and the TSOs can make all the transmission capacity available. Winning at explicit auctions, on the other hand, gives market power to the capacity holders, a fact that may distort bidding and nodal prices in the subsequent energy markets.

Another problem is related to the way in which bilateral contracts are integrated into the above scheme, since national spot markets and bilateral trade compete to use segments of the transmission networks. Different ways of handling this problem were suggested in ETSO (2002:12).

A second problem brought about by the use of market methods to allocate transmission capacity is related to the feedback to domestic bidding strategies of cross-border trade. In any of the systems described above the difference between local equilibrium prices plays a crucial role when the interconnecting network is congested. In the market splitting case, in particular, domestic prices determine the lower bound for the transfer price. With perfect information this bound would correspond to the cost differentials across the firms operating in the connected markets. Under asymmetric information it becomes important to analyse how the bidding strategy in each market is affected by the interconnection as the winning bids in the domestic markets play a sort of double role in determining not only the spot price in each market, but also the bound of the capacity fee.

5. Volume and strategy effects on prices with interconnection

In this section we analyse how bidding behaviour in a domestic market is affected by interconnection with another market. Recalling **A6** we take into consideration two local markets whose electricity trade is managed using the implicit auction method. As mentioned, the domestic auctions simultaneously allocate electricity to each market and the right to transfer it through the lines. More specifically, eligible buyers from the external market, that enter the merit order of accepted bids, obtain energy and transfer rights up to the interconnection capacity. The participation of new buyers in the exporting market can be modelled as an exogenous increase in electricity demand. Correspondingly, in the importing market there will be an inward shift in electricity demand. This ultimately implies an increase in the number of plants despatched in the low cost country and a decrease in the number of plants despatched in the high cost country.

The effects of demand shifts can be analysed using the bidding functions introduced in section 3. Differentiating b_β^i with respect to a generic argument, represented as *z*, of Ω_1 and Ω_2 we can explore the properties of the above family of bid functions. Hence:

$$\frac{\partial b_\beta^i}{\partial z} = H(c)\left\{ \int_{c_\beta^i}^{\bar{c}} -\Lambda(\tilde{c}) \log \frac{\left[F\left(\tilde{c}_\beta^i\right)\right]^{\partial\Omega_1/\partial z}}{\left[1-F\left(\tilde{c}_\beta^i\right)\right]^{\partial\Omega_2/\partial z}} d\tilde{c} + \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log \frac{\left[F\left(c_\beta^i\right)\right]^{\partial\Omega_1/\partial z}}{\left[1-F\left(c_\beta^i\right)\right]^{\partial\Omega_2/\partial z}} \right\} \quad (4)$$

where

$$H(c) = \frac{\left[F\left(c_\beta^i\right)\right]^{\Omega_1}}{\left[1-F\left(c_\beta^i\right)\right]^{\Omega_2}} \quad \text{and} \quad \Lambda(c) = \frac{\left[1-F\left(c_\beta^i\right)\right]^{\Omega_2}}{\left[F\left(c_\beta^i\right)\right]^{\Omega_1}}.$$

Applying the rule of integration by parts to the term inside the curly bracket (see the Appendix for details), one can see that $\partial b_\beta^i / \partial z \leq 0$ if

$$\frac{\partial}{\partial c_{\beta}^{i}}\left[\log\left(\frac{\left[1-F\left(c_{\beta}^{i}\right)\right]^{\partial\Omega_2/\partial z}}{\left[F\left(c_{\beta}^{i}\right)\right]^{\partial\Omega_1/\partial z}}\right)\right]\leq 0$$

When in (4) z is replaced by either n, m, $\overline{q}_{\beta}, q_{\beta}, \overline{q}_{\alpha}$, we obtain:

$$\frac{\partial b_{\beta}^{i}}{\partial n}\leq 0, \frac{\partial b_{\beta}^{i}}{\partial \overline{q}_{\beta}}\geq 0, \frac{\partial b_{\beta}^{i}}{\partial \overline{q}_{\alpha}}\geq 0 \tag{5}$$

This means that the bid decreases with the number of bidders *n,* whereas it increases with both capacity levels $\overline{q}_{\beta}, \overline{q}_{\alpha}$. As a consequence, the larger the firms the higher the expected mark-ups. On the other hand, we can see that:

$$\frac{\partial b_{\beta}^{i}}{\partial q_{\beta}}\geq 0 \text{ if } \left[F\left(c_{\beta}^{i}\right)\right]\leq \frac{\left(m-1\right)\left(\overline{q}_{\alpha}+\overline{q}_{\beta}\right)}{\left(n-1\right)\overline{q}_{\alpha}+\left(m-1\right)\left(\overline{q}_{\beta}\right)} \tag{6}$$

$$\frac{\partial b_{\beta}^{i}}{\partial m}\geq 0 \text{ if } \left[F\left(c_{\beta}^{i}\right)\right]\geq \frac{\left(\overline{q}_{\beta}-q_{\beta}\right)}{\overline{q}_{\beta}} \tag{7}$$

Inequality (6) represents a necessary and sufficient condition for the bid to increase with the quantity supplied. The sign of the derivative depends upon the cost level of the firm (idiosyncratic element) and upon symmetric variables related to market structure: capacities, number of bidders and number of despatched units (level of demand). Then, although firms are assumed to be symmetric in their capacities, the way they are expected to adjust their bids with respect to the quantity they are required to sell is influenced by the parameters of the market structure. For given values of parameters $m, n, \overline{q}_{\alpha}$ and $\overline{q}_{\beta}$ there is a cost level beyond which supply functions decrease with quantity. This effect is similar to the analysis of a common value environment in Bulow and Klemperer (2003). On the other hand, increasing supply functions are expected for the same set of parameters when costs are below the above threshold. Condition (6) is obviously satisfied for $m = n$.

As for condition (7), we observe that the bid function may increase or decrease with the number of despatched units. Specifically, it is always satisfied by infra-marginal bidders for which $q_\beta = \overline{q}_\beta$. They are therefore expected to increase their bids when more units are required. In contrast, for the marginal unit, which usually does not sell all its capacity, we stress that condition (7), for a given $q_\beta < \overline{q}_\beta$, is more likely to be satisfied by high cost bidders. To analyse the above effect in more details, let m_H, m_L be the number of despatched units in the two countries before the demand shifts and $\hat{m}_H, \hat{m}_L$, with $\hat{m}_H < m_H, \hat{m}_L > m_L$, be the number of despatched units after these shifts. Consider now the supply function of the exporting country L. Infra-marginal bidders, i.e. plants ranging from 1 to $\left(\hat{m}_L - 1\right)$ in the merit order, now submit the following equilibrium bid:

$$\hat{b}^i_\beta = c^i_\beta + \frac{\int_{c^i_\beta}^{\overline{c}} \left[1 - F_L\left(\tilde{c}\right)\right]^{\left(n-\hat{m}_L\right)\overline{q}_\beta/(\overline{q}_\beta+\overline{q}_\alpha)} d\tilde{c}}{\left[1 - F_L\left(c^i_\beta\right)\right]^{\left(n-\hat{m}_L\right)\overline{q}_\beta/(\overline{q}_\beta+\overline{q}_\alpha)}} \tag{8}$$

An increase in the number of despatched units determines an increase in the bids submitted by all infra-marginal bidders with respect to the case of an isolated country, making $\hat{b}^i_\beta > b^i_\beta$. We also notice that new and more costly plants are now called into operation, namely plants ranging from position m_L to $\hat{m}_L$ in the merit order. Consider the bid function of the (new) marginal bidder *j* from the exporting country after the demand shift:

$$\hat{b}_\beta^j = c_\beta^j + \frac{\left[F_L\left(c_\beta^j\right)\right]^{\Omega_1}}{\left[1-F_L\left(c_\beta^j\right)\right]^{\Omega_2}} \int_{c_\beta^j}^{\bar{c}} \left[\frac{\left[1-F_L\left(\tilde{c}\right)\right]^{\Omega_2}}{\left[F_L\left(\tilde{c}\right)\right]^{\Omega_1}}\right] d\tilde{c}$$

$$\Omega_1 = \left(\hat{m}_L - 1\right)\frac{\bar{q}_\beta - q_\beta}{\bar{q}_\alpha + q_\beta} \tag{9}$$

$$\Omega_2 = \left(n - \hat{m}_L\right)\frac{q_\beta}{\bar{q}_\alpha + q_\beta}$$

After the shift, Ω_1 increases whereas Ω_2 decreases. Once again, the final effect on the bid function depends upon condition (7). We can see, however, that for any probability distribution, as more costly units are called into operation, condition (7) for $\partial b_\beta^j / \partial m \geq 0$ is more likely to be satisfied for any given quantity level. Therefore, though we cannot establish in general whether the new SMP will be further driven up by the strategic behaviour of bidders, the previous analysis indicates that it is more likely that bidders will increase their bid when demand increases. Therefore, the closing price for the exporting country will increase for both the above mentioned reasons: on the one hand, because of the despatch of more costly units, and on the other hand, because bidders incorporate into their bidding strategies the increase in the despatched units from m_L to $\hat{m}_L$ and increase their bid shading to obtain more mark-up. Since the SMP remunerates all energy transactions, this fact has important implications for the distribution of welfare among consumers and producers in this country.

Consider now the results for the SMP in the importing country where m_H decreases. Following the above line of reasoning, we would expect first a decrease in the bids of the infra-marginal units, since $\hat{m}_H < m_H$. Secondly, since high cost generators are excluded from despatching, we expect that the bids made by the marginal unit will decrease in line with the SMP in the importing country. Once again this affects the welfare distribution between consumers and producers.

Hence, as expected, interconnection modifies relative prices across countries. Given that energy is exchanged at the SMP in both markets, surplus modifications reflect the variations that occur in SMPs. For the importing country, we can see that cross-border trade leads to a reduction in the SMP and to replacement of the supply of some domestic plants by imports. There is a positive change in the consumers' surplus and a reduction in the total domestic producers' rent led by the exclusion from supply of the most inefficient plants, previously despatched. Moreover, due to modifications in bidders' strategies, the reduced demand in the importing market makes the supply function flatter, which further increases the consumers' gain.

In contrast, in the exporting country the increased SMP reduces the consumers' surpluses and increases the producers' rents. However, since strategy changes with m_L, the aggregate supply function is modified in a way that might increase the consumers' losses and increase the producers' rents.

In all, the demand shifts enabled by interconnection modify the SMP in both countries via a pure "volume" effect (demand reduction/increase) and via a "strategy" effect. The latter may amplify the adverse effects of interconnection on welfare in the low cost market and accentuate the favourable effects in the high cost one.

5.1 Numerical simulations

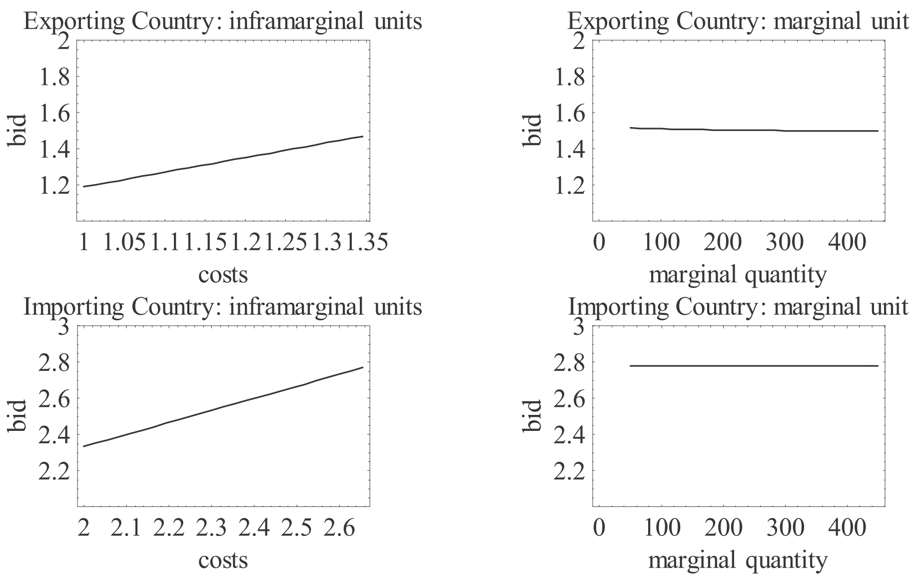
In this subsection we present some numerical results obtained from simulations of bid functions before and after interconnection. We consider two connected markets characterised by systematically different cost levels such that country L is the exporter and country H is the importer. The assumptions about variables and parameters are reported in Table 2.

Table 2: Numerical values for simulation

	$F(c_\beta)$	$\overline{q}_\alpha$	$\overline{q}_\beta$	n	D_t isolation	D_t interconnection
Low Cost Country	Uniform (1,2)	1,350MWh	450MWh	28	42,525	48,825
					($m_L = 11$)	($\hat{m}_L = 25$)
High Cost Country	Uniform (2,3)	1,350MWh	450MWh	28	46,575	40,275
					($m_H = 20$)	($\hat{m}_H = 6$)

Using the data presented in Table 2 to simulate bid function (3) we can calculate the aggregate supplies when the two markets are isolated. These are shown in Figure 1.

Figure 1: Supply before interconnection

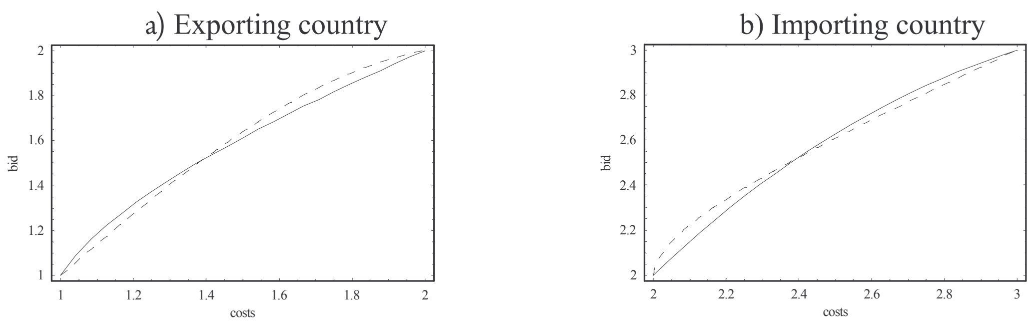


Given D_t before interconnection, there are 10 infra-marginal peak units in the exporting country and the marginal bidder's expected cost, $\left(E\left(c_{(11:28)}\right)\right)$ is equal to 1.38. Consequently the interval of expected costs of the 10 infra-marginals is $[1,1.345]$. The bidding behaviour of infra-marginal operators, each selling its entire capacity, determines the aggregate "infra-marginal" supply, which is depicted in the top left panel of Figure 1. As expected, this supply increases linearly in the cost domain corresponding to the above defined interval. The expected bid of the marginal operator, which sells the residual quantity $q_{\beta}^{11} = 225$ is depicted in the top right panel. We note that its bid function is almost flat with respect to the marginal quantity. Given the above results obtained from bid function (3), the expected SMP in the exporting country before interconnection is equal to 1.503, and the mark-up gained by the marginal operator is equal to 8.9%.

The two bottom panels of Figure 1 plot the same functions simulated for the importing country where $m_H = 20$ (isolation). The left panel depicts the supply of the 19 infra-marginal units that sell their entire capacity and have expected costs ranging from 2 to 2.65. The right panel depicts the marginal unit that sells $q_{\beta} = 225$ and has expected costs $E\left(c_{(20:28)}\right) = 2.69$. In the isolated high cost country the expected SMP is equal to 2.79 with a margin of 3.7% over costs for the last unit in the merit order.

The difference in SMPs realised in the two isolated markets leads to cross-border exchange of electricity. Based on the data in Table 2 we assume that interconnection determines opposite demand shifts in the two markets so that now $\hat{m}_H = 6$ and $\hat{m}_L = 25$. Let us first consider how the new marginal units adjust their bids after interconnection.

Figure 2: The effect of interconnection on marginal units



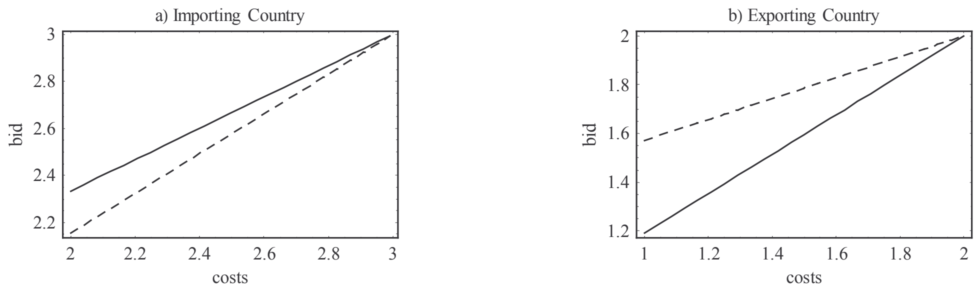
In both panels of Figure 2 the dotted lines describe the supply function of all units in the sample when $q_{\beta} = 225$. The solid lines describe the bidding behaviour of the units before interconnection. The new marginal bidders can be identified in the two graphs taking the points corresponding to the expected cost of the $\hat{m}_L^{th}$ and $\hat{m}_H^{th}$ operator, respectively. Specifically, $E\left(c_{(25:28)}\right)=1.86$ for country L and $E\left(c_{(6:28)}\right)=2.21$ for country H. The "bid effect" is evident from the distance between the two curves. In the left panel the difference between the two bid functions, before and after interconnection, is positive for high cost units, and negative for low cost units. The reverse occurs in the right panel. This result accords with condition (7) which requires the bid to increase with *m* for high realisations of costs, and vice versa.

In the exporting country the interconnection leads to an increase in the expected SMP for two reasons. In the first place more costly units are called into operation (so-called volume effect). In this connection we notice that the expected cost differential between the marginal units before and after interconnection is equal to $\left(E\left(c_{(25:28)}\right)-E\left(c_{(11:28)}\right)\right)=\left(1.86-1.38\right)=0,48$ which corresponds to the portion of price increase due to the volume effect. After interconnection, the new expected SMP is equal to 1.94 with a difference of 0.437 with respect to the SMP before interconnection. We notice that since the bid function is concave for costs, the increase in price is lower than the increase in the expected costs. On the other hand, from the left panel of Figure 2 we can see that the new marginal unit is located in the region where the "bid effect" is positive. Taking the appropriate co-ordinate on the graph, we notice that the new SMP, in the absence of any bid effect, would have been equal to 1.899. Taking the effect of interconnection on demand into account, the new marginal operator is able to increase its mark-up and to realise larger profits.

In the case of the importing country we can see that the most efficient units remain operational after interconnection, but contrary to naïve expectation, they increase their bids (see the right panel of Figure 2). The expected cost of the marginal unit is 2.69 before interconnection ($m_H=20$), whereas it is equal to 2.21 after interconnection ($\hat{m}_H=6$). In the case of the high cost country the interconnection shifts the equilibrium point towards the region of low cost realisation and this again may imply a positive bid effect that reduces the expected decrease in the SMP. In fact, our simulation shows that the final SMP decreases from 2.79 to 2.35 with a difference of 0.48 which is less than the expected cost difference of 0.44. In the absence of any bid effect the new marginal operator would bid 2.30 and the difference between the two values of the SMP would correspond to the cost differential.

To complete the simulation, in the two panels of Figure 3 we illustrate how the supply function changes for the infra-marginal plants, i.e. for plants that sell their entire capacity $q_\beta = \overline{q}_\beta$.

Figure 3: The effect of interconnection on infra-marginal bids



In Figure 3.a) (importing country) we represent the supply function before interconnection ($m_H = 20$)with a solid line and after interconnection ($\hat{m}_H = 6$) with a dotted line. Infra-marginal operators adjust their bids downwards after interconnection, as indicated by condition (7) for any cost level. In panel b) we conduct the same experiment for country L. We can see that the effect of interconnection (increase from $m_L = 11$ to $\hat{m}_L = 25$) is to make the supply function of all infra-marginal units flatter, with an increase in the bid. However, since none of the infra-marginal units fixes the SMP the above results do not influence the SMP itself.

6. Concluding comments

In this paper we discussed some implications of cross-border trade on the efficiency of domestic electricity Pool markets operating at transmission nodes. We have shown that demand shifts due to interconnection modify domestic bidding strategies. Specifically, cross-border trade can induce price convergence across countries and thereby reallocate gains and losses correlated with market power as a result of two concomitant effects: a “volume” effect, due to the mere increase/decrease of demand in each market, and a “bid effect”, corresponding to the modifications of bid strategies induced by the increased/decreased number of despatched generators. The volume effect induces price modifications in line with standard expectations about trade, whereas the "bid effect" can either reinforce or contrast price convergence across countries according to demand, quantities and cost conditions. Consumers' advantages brought about by interconnection in the high cost country, occur at the expense of consumers located in the low cost country whereas rent reductions for producers operating in the importing country are counterbalanced by rent increases for producers in the exporting country. This reallocation of gains and losses across countries might explain some of the resistance in the EU to a full integration of electricity markets.

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Appendix 1

We show the derivation of results (5) and (6). Differentiation of (3) with respect to the specific variables yields:

$$\frac{\partial b_\beta^i}{\partial n} = H(c)\left\{ \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) \log\left[1 - F\left(\tilde{c}_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha + q_\beta\right)}} d\tilde{c} - \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log\left[1 - F\left(c_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha + q_\beta\right)}} \right\}$$

$$\frac{\partial b_\beta^i}{\partial m} = H(c)\left\{ \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) \log \frac{\left[1 - F\left(\tilde{c}_\beta^i\right)\right]^{\frac{-q_\beta}{\left(\bar{q}_\alpha + q_\beta\right)}}}{\left[F\left(\tilde{c}_\beta^i\right)\right]^{\frac{\left(\bar{q}_\beta - q_\beta\right)}{\left(\bar{q}_\alpha + q_\beta\right)}}} d\tilde{c} - \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log \frac{\left[1 - F\left(c_\beta^i\right)\right]^{\frac{-q_\beta}{\left(\bar{q}_\alpha + q_\beta\right)}}}{\left[F\left(c_\beta^i\right)\right]^{\frac{\left(\bar{q}_\beta - q_\beta\right)}{\left(\bar{q}_\alpha + q_\beta\right)}}} \right\}$$

$$\frac{\partial b_\beta^i}{\partial \bar{q}_\beta} = H(c)\left\{ \int_{c_\beta^i}^{\bar{c}} -\Lambda(\tilde{c}) \log\left[F\left(\tilde{c}_\beta^i\right)\right]^{\frac{m-1}{\left(\bar{q}_\alpha + q_\beta\right)}} d\tilde{c} + \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log\left[F\left(c_\beta^i\right)\right]^{\frac{m-1}{\left(\bar{q}_\alpha + q_\beta\right)}} \right\}$$

$$\frac{\partial b_\beta^i}{\partial q_\beta} = H(c)\left\{ \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) \log \frac{\left[1 - F\left(\tilde{c}_\beta^i\right)\right]^{\frac{(n-m)\bar{q}_\alpha}{(\bar{q}_\alpha + q_\beta)^2}}}{\left[F\left(\tilde{c}_\beta^i\right)\right]^{-\frac{(m-1)(\bar{q}_\alpha + \bar{q}_\beta)}{\left(\bar{q}_\alpha + q_\beta\right)^2}}} d\tilde{c} - \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log \frac{\left[1 - F\left(c_\beta^i\right)\right]^{\frac{(n-m)\bar{q}_\alpha}{(\bar{q}_\alpha + q_\beta)^2}}}{\left[F\left(c_\beta^i\right)\right]^{-\frac{(m-1)(\bar{q}_\alpha + \bar{q}_\beta)}{\left(\bar{q}_\alpha + q_\beta\right)^2}}} \right\}$$

$$\frac{\partial b_\beta^i}{\partial \bar{q}_\alpha} = H(c)\left\{ \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) \log \frac{\left[1 - F\left(\tilde{c}_\beta^i\right)\right]^{-(n-m)\frac{q_\beta}{\left(\bar{q}_\alpha + q_\beta\right)^2}}}{\left[F\left(\tilde{c}_\beta^i\right)\right]^{-(m-1)\frac{\left(\bar{q}_\beta - q_\beta\right)}{\left(\bar{q}_\alpha + q_\beta\right)^2}}} d\tilde{c} - \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log \frac{\left[1 - F\left(c_\beta^i\right)\right]^{-(n-m)\frac{q_\beta}{\left(\bar{q}_\alpha + q_\beta\right)^2}}}{\left[F\left(c_\beta^i\right)\right]^{-(m-1)\frac{\left(\bar{q}_\beta - q_\beta\right)}{\left(\bar{q}_\alpha + q_\beta\right)^2}}} \right\}$$

To evaluate the sign of each derivative we apply the rule of integration by parts to the first term inside the curly brackets and this generates the results reported in the text. For instance, in the case of $\partial b_\beta^i / \partial n$ the term inside the curly bracket is:

$$\left\{ \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) \log\left[1-F\left(\tilde{c}_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} d\tilde{c} - \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log\left[1-F\left(c_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} \right\}$$

then, given that $\frac{\mathrm{d}}{\mathrm{d}\tilde{c}}\left(\int_{\tilde{c}}^{\bar{c}} -\Lambda(s) ds \right) = \Lambda\left(\tilde{c}\right),$ we obtain:

$$\int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) \log\left[1-F\left(\tilde{c}_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} d\tilde{c} = \int_{c_\beta^i}^{\bar{c}} \Lambda(\tilde{c}) d\tilde{c} \log\left[1-F\left(c_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} +$$

$$-\int_{c_\beta^i}^{\bar{c}} \left(-\int_{\tilde{c}}^{\bar{c}} \Lambda(s) ds \right) \left(\frac{d}{dc_\beta^i} \log\left[1-F\left(c_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} \right) d\tilde{c}$$

Substituting back yields:

$$\frac{\partial b_\beta^i}{\partial n} = \int_{c_\beta^i}^{\bar{c}} \left(\int_{\tilde{c}}^{\bar{c}} \Lambda(s) ds \right) \left(\frac{d}{dc_\beta^i} \log\left[1-F\left(c_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} \right) d\tilde{c}$$

Since $\frac{d}{dc_\beta^i} \log\left[1-F\left(c_\beta^i\right)\right]^{\frac{q_\beta}{\left(\bar{q}_\alpha+q_\beta\right)}} \leq 0,$ then the derivative $\partial b_\beta^i / \partial n$ is always negative.

Table 1: *Methods implemented for the management of cross-border congestion*

Country (Exp.)	Country (Imp.)	Capacity MW winter 2003-04	Allocation Method	Long term contracts (2001)	Congested
CH	IT	2800	Retention / PR	29%	always
FR	IT	2650	PR	69%	always
AT	IT	220	FCFS / PR	50%	always
FR	DE	2250	FCFS	13%	frequently
DE	NL	3900	Joint EA*	46%	frequently
FR	BE	2250	FCFS	72%	frequently
FR	UK	2000	Joint EA*		frequently
DK-W	DE	1200	Joint EA *		frequently
FR	ES	1400	FCFS / PR	40%	frequently
DK-W	NO	950	IA		frequently
DK-E	DE	550	Joint EA *	100%	frequently
SE	DE	460	Retention / FCFS	100%	frequently
UK	IE	330	EA *		seldom
NO	SE	3350	IA		occasionally
SE	NO	3100	IA		occasionally
SE	FI	2050	IA		occasionally
DK-E	SE	1700	IA		occasionally
FI	SE	1400	IA		occasionally
SE	DK-E	1300	IA		occasionally
NO	DK-W	1000	IA		occasionally
ES	PT	850	PR / IA		occasionally
DE	DK-W	800	Joint EA		occasionally
PT	ES	700	PR		occasionally
DK-W	SE	490	IA		occasionally
SE	DK-W	460	IA		occasionally
DE	DK-E	550	Joint EA *	36%	occasionally
DE	SE	370	Retention / FCFS	100%	occasionally
FR	CH	5350	FCFS		frequently
UK	FR	2000	Joint EA *		frequently
BE	NL	2450	Joint EA	18%	seldom
NL	BE	2350	Joint EA		seldom
NL	DE	2700	Joint EA		seldom
ES	FR	600	PR / IA		seldom
BE	FR	2750	FCFS	16%	never
IT	CH	3100	FCFS /PR		never
CH	FR	3000	Retention		seldom
DE	FR	4600	FCFS		never
IT	FR	2200	PR		never
AT	CH	2000	FCFS / PR		never
CH	AT	2000	Retention / FCFS		never
CH	DE	3500	Retention / FCFS		never
DE	CH	3000	FCFS		never
DE	AT	1200	FCFS / PR		never
AT	DE	1500	FCFS / PR		never
GR	IT	350	PR		never
IT	GR	500	EA		occasionally
IT	AT	220	PR		never
IE	UK	70	EA *		seldom

FCFS = First Come First Served; PR = Pro rata; IA = Implicit Auction /Market Splitting; EA = Explicit Auction; * indicates the use of a reserve price

Source: ETSO, 2002 and EFET, 2004