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The application of robust regression to a production function comparison – the example of Swiss corn

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Abstract: *The adequate representation of crop response functions is crucial for agricultural modeling and analysis. So far, the evaluation of such functions focused on the comparison of different functional forms. In this article, the perspective is expanded by also considering an alternative regression method. This is motivated by the fact that extreme climatic events can result in crop yield observations that cause misleading results if Least Squares regression is applied. We show that such outliers are adequately treated if and only if robust regression or robust diagnostics are applied. The example of simulated Swiss corn yields shows that the application of robust instead of Least Squares regression causes reasonable shifts in coefficient estimates and their level of significance, and results in higher levels of goodness of fit. Furthermore, the costs of misspecification decrease remarkably if optimal input recommendations are based on results of robust regression. We therefore recommend the application of the latter instead of Least Squares regression for agricultural and environmental production function estimation.*

Key words: *production function estimation, production function comparison, robust regression, crop response*

1 Introduction

The adequate representation of production or crop yield functions is crucial for modeling purposes in agricultural, environmental and economic analyses. The discussion and estimation of different functional forms has therefore gained much attention in both general and applied agricultural and environmental economics. The pertinent literature provides a variety of functional specifications to mathematically describe the technical relationships between the quantities of inputs employed and those of outputs produced [7]. Agricultural economists have devoted special attention to also taking natural processes into account in their mathematical representations of crop yield functions. Frequently applied are polynomial forms (e.g. the quadratic and the square root function) and functions based on the von Liebig idea (e.g. the linear/nonlinear von Liebig and the Mitscherlich-Baule production function).

Discussions on the theoretical appropriateness of different functional forms to agronomic problems can be found in Fuchs and Löthe [15], Heady and Dillon [19], Hexem and Heady [20], and, Keusch [27]. They primarily address the integration of agronomic processes into economic production functions. However, the distinction between different types of production functions is often negligible. The analysis of Frank et al. [11] suggests that no functional form dominates all other forms in every situation and therefore, crop yield functions must be reassessed for each location.

Various locations and functional forms have been considered so far in the literature. But, little attention has been given to estimate the impact of exceptional climatic events upon extreme variations in crop yields. This is particularly important since the estimation of a production function using the Ordinary Least Squares (OLS) fitting criterion can produce misleading results if data sets contain exceptional observations. Since the OLS method is not able to cope with a single outlier, one climatically exceptional year in the data set is sufficient to cause unreliable coefficient estimates. This phenomenon is not exclusive to agricultural and environmental issues. Rather, outliers are frequently observed in empirical data sets and particularly considered in the applied statistics and econometrics literature [16, 23].

Reliable results are provided by OLS if and only if robust regression methods or robust regression diagnostics are applied as well. The application of these methods ensures the non-inclusion or the appropriate down weighting of outliers in the analysis. Unlike Swinton and King [48], who applied robust regression methods to detect influential observations for trend estimation within yields, the focus of this analysis is on the estimation of a crop production function applying robust regression. This contrasts the standard procedure of excluding extreme climatic events by introducing dummy variables, such as Fuchs and Schanzenbächer [14], Fuchs and Löthe [15], and, Khan and Akbari [28]. Their procedure is based on a simple classification of climatic conditions to a certain dummy group. However, detecting outliers in multivariate regression analysis is much harder than in simple regression cases [22]. Therefore, classification based on informal procedures such as scatter plots is no longer sufficient [17]. Robust regression diagnostics have to be applied for outlier detection.

The aim of this analysis is to exhibit the vulnerability of usually applied production function estimation methods to extreme climatic events. It shows that inference based on usually applied methods can be misleading. The adequacy of certain models and even the significance of coefficient estimates can change, if outlying observations are adequately treated in the analysis. Besides relevance for crop production function estimation, this study is a general example for the efficient handling of climatic extreme events in environmental modeling, which cannot be attained with conventional approaches.

In this paper, the application of robust regression is combined with the evaluation of three types of production functions. The assessment of functional forms can be based on the coefficient of determination [2], residual distribution [4] and potential misspecification costs [11, 32]. We devote special attention to the cost of misspecification; that is the potential income loss that would arise from using OLS instead of robust regression methods or an improper specification of the production function.

The remainder of this paper is organized as follows. Sections 2 and 3 provide a brief presentation of the production functions and the data, respectively, that are used throughout our analysis. In Section 4, the estimation methodology is briefly introduced. The estimation results for corn (Zea mays L.) yields in Switzerland are presented and discussed in Section 5. Subsequently, optimal input levels and the cost of misspecification are investigated in Section 6. Finally, the advantage of applying robust regression techniques in production function estimation is discussed in the concluding Section 7.

2 Three Types of Production Functions

Three types of crop production functions are estimated in this study: two polynomial specifications (the quadratic and the square root function) and the Mitscherlich-Baule function. These functional forms are frequently used in the agricultural economic literature and proved to accurately capture the underlying relationships [3, 1, 5, 11, 14, 15, 32].

Being aware that corn yields are driven by numerous factors, we restrict our analysis to two crucial input factors: nitrogen fertilizer and irrigation water. Together with the concentration on three functional forms, this restriction serves the sake of clarity in our investigation. It is focused on the estimation methodology, rather than aimed at providing an approximation of the most appropriate form with an extended set of output determining factors.

The *quadratic form*, shown in equation (Eq. 1), consists of an additive composition of the input factors, their squared values, and an additional interaction term. The latter clarifies whether the input factors are independent of each other or not. The quadratic production function for a given crop is formally defined as follows:

$$Y = \alpha_0 + \alpha_1 \cdot N + \alpha_2 \cdot W + \alpha_3 \cdot N^2 + \alpha_4 \cdot W^2 + \alpha_5 \cdot N \cdot W \qquad (1)$$

Y denotes corn yield per area, N the amount of inorganic nitrogen applied, and W irrigation water applied. The α_i's are parameters that must satisfy the subsequent conditions in order to ensure decreasing marginal productivity of each input factor: $\alpha_1, \alpha_2 > 0$ and $\alpha_3, \alpha_4 < 0$. Furthermore, if $\alpha_5 > 0$ the two input factors are complementary. They are competitive if $\alpha_5 < 0$, while $\alpha_5 = 0$ indicates independence of the two input factors.

The *square root function* (Eq. 2) is very similar to the quadratic form but produces different shapes of the curves. The square root form is defined as follows:

$$Y = \alpha_0 + \alpha_1 \cdot N^{1/2} + \alpha_2 \cdot W^{1/2} + \alpha_3 \cdot N + \alpha_4 \cdot W + \alpha_5 \cdot (N \cdot W)^{1/2} \qquad (2)$$

To ensure decreasing marginal productivity of each input factor, the above given conditions for the parameters have to be satisfied here as well. Furthermore, the interpretation of the parameter α_5 is identical.

The *Mitscherlich-Baule function* (Eq. 3) is, according to Frank et al. [11] and Llewelyn and Featherstone [32], the most appropriate production function for corn yields. It allows for a growth plateau – i.e., maximum yield – which follows from the von Liebig approach to production functions (see Paris [37] for historical notes). Moreover, the Mitscherlich-Baule function is characterized by continuously positive marginal productivities of the input factors. It does not exhibit negative marginal productivities, as the above polynomial forms. Formally, the Mitscherlich-Baule function is given by

$$Y = \alpha_1 \cdot (1 - \exp(-\alpha_2 \cdot (\alpha_3 + N))) \cdot (1 - \exp(-\alpha_4 \cdot (\alpha_5 + W))) \qquad (3)$$

with α_1 representing the growth plateau, and α_3 and α_5 the natural factor endowments, respectively. These natural factors include nitrogen in the soil (α_3) and water endowments (α_5) such as soil moisture. The coefficients α_2 and α_4 describe the influence of the corresponding input factors on the yield. Unlike the classical von Liebig production function, the Mitscherlich-Baule function allows for factor substitution. It is not linear limitational in the input factors as the von Liebig function, i.e. the isoquants are not right-angled.

In each of the three production functions, the elasticity of substitution between the two input factors is by definition different from zero and not constant. These characteristics and the property of decreasing marginal productivity are important criteria of production functions in describing agricultural factor-yield relationships [27].

3 Data

Our analysis is based on corn yield data provided by the Agroscope Reckenholz-Tänikon Research Station ART in Zurich. The data is generated by the deterministic crop yield simulation model CropSyst [45, 46]. The model is calibrated to field trials [8, 9, 53] and sample data [10]. The comparability of simulated and observed yields is restricted because the simulations do not account for yield reducing events such as hail, disease and insect infestation. Details about the model setting and calibration used in this study are presented in Torriani et al. [49].

CropSyst is driven by weather data from six different locations on the Swiss Plateau for the years 1981 – 2003, as provided by the Swiss Federal Office of Meteorology and Climate (MeteoSwiss). The six locations are Berne-Liebefeld (46°56' N, 7°25' E), Lucerne (47°02' N, 8°18' E), Payerne (46°49' N, 6°57' E), Taenikon (47°29' N, 8°54' E), Wynau (47°15' N, 7°47' E) and Zurich (47°23' N, 8°34' E). These weather stations are located at elevation levels between 422 and 565 meter above sea level.

Compared to an approach with one single location, the use of observations from six different weather stations broadens the database and allows us to represent a large proportion of the entire Swiss corn producing acreage. This is important, since the data base is partly truncated by missing values. No weather data of 1981 is available for the stations Lucerne, Payerne, Taenikon and Wynau. Furthermore, there are missing values for the year 1984 in Berne, Taenikon and Wynau, as well as for 1987 in Taenikon.

Apart from agricultural inputs, the model is particularly driven by daily values of radiation, rainfall, wind-speed, temperature, and air moisture. Additional information requested by CropSyst concerns the atmospheric CO_2 concentration. Recorded values for the period 1981-2003, ranging from 339ppm to 379ppm [43], are considered in the CropSyst simulations.

The simulation and subsequent data analysis are restricted to one uniform type of soil for all locations (characterized by texture with 38% clay, 36% silt, 26% sand and soil organic matter content at 2.6% weight in the top soil layer (5 cm) and 2.0% in lower soil layers [49]), one type of irrigation (possible from day one after sowing to harvesting, never exceeding field capacity) and one type of fertilizer (inorganic nitrogen fertilizer). This approach avoids distortions due to non-uniform soil and management properties.

To generate a comprehensive data set, one simulation is conducted without application of fertilizer and irrigation for each location and each year. Furthermore, to enhance the variability in the corn yields, additional combinations of irrigation and fertilizer are applied randomly. Nitrogen fertilizer applications range from 0 to 320 kg/ha and irrigation water from 0 to 340 mm respectively. This leads to 212 different levels of nitrogen application to the plants and 60 different levels of irrigation.

The resulting dataset consists of 527 observations with different corn yields. Assuming a dry matter content of 85%, average yields for three different ranges of irrigation and fertilizer application, respectively, are shown in Table 1. This rough approximation of the average corn yields reveals a global yield maximum for $76 \le N \le 150$ and $71 \le W \le 140$. Simulated corn yields decrease if the amounts of irrigated water or applied fertilizer deviate from those input ranges.

Table 1: Average simulated corn yields 1981–2003

		Applied nitrogen in kg/ha		
		0–75	**76–150**	**151–320**
Applied irrigation water in mm	**0–75**	6 955	8 872	8 521
	76–140	7 293	9 717	9 100
	141–340	7 275	8 814	9 158

Source: CropSyst simulations

Furthermore, starting from zero inputs, Table 1 indicates a stronger response of the corn yield to increases in nitrogen fertilization than to the amount of irrigation water. An increase of irrigation water induces small changes of simulated corn yield, only. In contrast, enhanced nitrogen application increases the crop yield remarkably. Low water limitation in the Swiss Plateau may be the reason for this phenomenon, since precipitation is usually sufficient to ensure good crop yields.

4 Estimation Methodology

The estimation of the above production functions for Swiss corn yield data requires multivariate regression analysis. To this end, the methods of Least Squares and robust regression are applied. The Least Squares (LS) criterion selects the coefficient vector, which minimizes the sum of squared residuals. This estimation is conducted with the MODEL procedure of the SAS statistical package [41]. The vulnerability of this method to outlying observations has been demonstrated in various studies [18, 23, 39].

Outlying observations deviate from the (quasi) linear relationship described by the majority of the data. They can have a large influence on estimation results. In the case of OLS estimation, one outlier can be sufficient to move the coefficient estimates arbitrarily far away from the actual underlying values. To overcome this vulnerability to outliers, various approaches to robust regression analysis have been proposed [18, 23, 39]. The main idea of robust regression is to give little weight to outlying observations in order to isolate the true underlying relationship. The further away an observation is from the true relationship, the smaller is the corresponding weight of contribution to the regression analysis. The identification of the true relationship is a non-trivial challenge, in particular, if the situation exceeds the simple regression case.

In this context, the notation "true relationship" is restricted to an econometrical interpretation, while the excluded observations can be of particular interest from a scientific point of view. However, the inclusion of outliers in the analysis does not allow trustful regression inference. By contrast, separated analyses of outliers and inliers can lead to an information gain.

In this study, Reweighted Least Squares (RLS) regression is applied for the estimation of eqns. (1) and (2), using the ROBUSTREG procedure in the SAS statistical package. RLS is a weighted LS regression, which is based on an analysis of Least Trimmed Squares (LTS) residuals that gives zero weights to observations identified as outliers. If the robust estimated standardized residual exceeds the cutoff value of 2.5 [24], an indicator function generates a weight of zero for this particular observation. For a detailed description of the RLS method, see Rousseeuw and Leroy [39].

The nonlinear eqn. (3) is estimated using the NLIN procedure in the SAS software package. The solution of nonlinear problems requires iterative approaches, such as the Levenberg-Marquardt algorithm [33, 35, 42]. Robust regression is implemented in this case by using Iterative Reweighted Least Squares (IRLS). In contrast to the case above, the weights are generated by M-estimation using Tukey's biweight [18, 21, 47]. Using IRLS, the weights are re-estimated at every stage of the iteration process until convergence. Unlike the RLS regression, M-estimation using Tukey's biweight combines hard and soft rejection of outlying observations: very large residuals are given zero weights and for the remaining observations soft rejection applies. Continuous weights between zero and one are given to the observations. The larger a residual, the smaller is the weight of the corresponding observation within the subsequent iteration step. That is, influential observations are down-weighted or given zero weights as in RLS.

Besides the most important property of giving trustworthy coefficient estimates, robust regression provides detailed insight in the structure of the data. Observations identified as outliers reveal their origin and can exhibit inappropriateness of the employed model structure. Above all, the interpretation of outliers is indispensable. It should take not only statistical but mainly reasons from the subject matter science into account [17]. With regard to the functional relationship between nitrogen application, irrigation and corn yields, we particularly expect climatic conditions to be influential. For instance, the amount of rainfall can influence droughts or moisture built up, and thus indirectly restrict yield levels. Furthermore, the plants are expected to respond specifically to management under certain climatic conditions. The response to irrigation and fertilization changes under high and low water stress situations, respectively. Therefore, exceptional climatic years are supposed to have an extraordinary influence on plant's response to irrigation and fertilization. This has not adequately been taken into account so far. In the studies of Fuchs and Schanzenbächer [14], Fuchs and Löthe [15], and, Khan and Akbari [28], for instance, the influence of climate is filtered by the inclusion of dummy variables. The large number of observations and the three dimensionality of the problem hamper a simple allocation to a certain group of dummies. Robust regression diagnostics, as described above, should be applied instead.

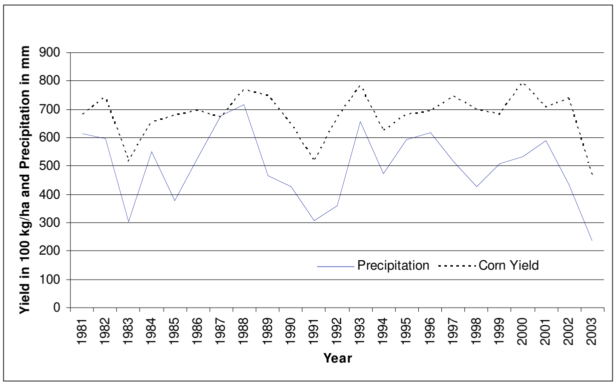
A further problem in estimating crop yields arises from the influence of the rates of nitrogen fertilizer application and irrigation upon the variance of the regression residuals, which causes heteroscedasticity [26, 30, 38]. Therefore, all estimated equations presented in this paper are corrected for heteroscedasticity using feasible generalized least squares regression [25].

5 Results and Discussion

In this section an outlier interpretation is provided, and estimation results are presented and discussed. As described above, the analysis of observations indicated as outliers is of particular relevance to exhibit the data structure. For simplicity and better interpretability, solely outliers are presented here that are indicated by the method of RLS for the estimation of the two polynomial forms.

About seven to eight percent of the 527 observations are indicated as outliers. Most observations in this set of outliers are identified in the years 1983, 1991 and 2003. High temperatures and low precipitation in the relevant seeding-to-harvest period characterize these years. The reason for the identification of outliers in these years is twofold. On the one hand, the dependent variable (yield) is affected and some input levels do not result in typical yields. On the other hand, the relationship between independent and dependent variables is affected by different reactions to input levels in situations where one of the inputs is a limiting factor. The example of different reactions to irrigation under different precipitation levels underlines this argument and the fact that rainfall is an important determinant for the corn yield [34]. Observed precipitation in the seeding-to-harvest period and simulated corn yields with zero fertilization and irrigation are plotted in Graph 1 for the station in Zurich and the years 1981 to 2003. The obvious relationship between yield and precipitation levels results in significant correlation coefficients of 0.7 (Pearson) and 0.55 (Spearman), respectively.

Table 2 and Table 3 present the estimation results for the quadratic and the square root production functions, respectively. It shows that each estimation coefficient has the correct (i.e. the expected) sign. However, the results for the example of Swiss corn differ from Llewelyn and Featherstone's [32] results for Western Kansas. Interestingly, in each of the four estimated polynomial functions, the coefficient α_5. (Applied Nitrogen* Irrigation Water) is not significantly different from zero (see Table 2 and 3). This reveals mutual independence of the two input factors irrigation water and nitrogen fertilizer in Swiss corn production. Rainfall is sufficient to ensure nitrogen uptake in normal years.



Source: CropSyst simulations and MeteoSwiss weather data.

Figure 1: Simulated corn yields and precipitation in Zurich 1981– 2003

Lower water limitation in the Swiss Plateau than in Western Kansas, where corn is frequently irrigated [36], is the reason for this difference. Even if summer precipitation would be lower in the next decades [12, 13], the results obtained for Western Kansas might not provide a prospect for future Swiss corn production, but indicate directions for future adaptation. Irrigation might become more important.

Table 2: Coefficient estimates for the quadratic production function

	Estimation Method	
Variable	OLS	RLS
Intercept	6638.265 (165.05)**	6661.421 (179.24)**
N	25.64327 (17.62)**	27.55239 (22.71)**
W	6.046902 (5.62)**	5.578582 (5.75)**
N2	-0.07104 (12.22)**	-0.07236 (14.94)**
W2	-0.01797 (3.87)**	-0.0162 (3.88)**
NW	0.007766 (1.51)	0.00373 (0.89)
adj. R^2	0.5680	0.7065

Note: Statistics in parentheses are t statistics
(**) – indicates significance at the 1% level
(*) – indicates significance at the 5% level

Table 3: Coefficient estimates for the square root production function

	Estimation Method	
Variable	**OLS**	**RLS**
Intercept	6589.997 (155.02)**	6601.924 (162.13)**
$N^{1/2}$	297.1821 (12.42)**	313.0936 (16.34)**
W1/2	75.09137 (4.26)**	67.1385 (4.17)**
N	-11.2156 (6.88)**	-10.544 (8.15)**
W	-3.03419 (2.40)*	-2.49922 (2.17)*
$(NW)^{1/2}$	1.46442 (1.43)	0.364377 (0.45)
adj. R^2	0.5834	0.7330

Note: Statistics in parentheses are t statistics
(**) – indicates significance at the 1% level
(*) – indicates significance at the 5% level

Moreover, the coefficients of determination are remarkably higher if RLS is used compared to OLS estimations. If exceptional observations are omitted in the analysis, the linear pattern formed by the remaining observations explains more of the variation in corn yields. We are aware of the fact that truncating the undesired observations is not a remedy for each estimation task. In particular, following sufficient truncation, many distributions are "normal" in the middle [50]. But in our opinion, extraordinary hot and dry years demand for a separately estimated production function. Only these specific years and the related yields should then be included in the analysis for climatically extreme years. If water is, unlike in normal years in the Swiss Plateau, a limiting factor for the plants, the yield response to irrigation water is expected to be much higher than usual. Furthermore, the plants' response to nitrogen also highly depends on water availability as nitrogen is taken up by the roots in a water solution [31]. Under dry climatic conditions, the interaction between fertilizer and irrigation water is expected to be more significant than currently. Unfortunately, the regression analysis fails to provide valuable results due to a lack of sufficient observations in the available dataset.

Table 4: Coefficient estimates for the Mitscherlich-Baule production function

	Estimation Method	
Variable	**LS (Levenberg-Marquardt)**	**IRLS (Levenberg-Marquardt)**
α_1	9180.6 (95.14)**	9410.3 (87.7)**
α_2	0.0288 (5.72)**	0.0266 (7.38)**
α_3	50.6952 (5.96)**	48.3036 (7.75)**
α_4	0.0598 (1.22)	0.0304 (2.95)**
α_5	45.1410 (1.24)	71.2249 (3.10)**
adj. R^2	0.736	0.809

Note: Statistics in parentheses are t statistics
(**) – indicates significance at the 1% level
(*) – indicates significance at the 5% level

In Table 4, the Mitscherlich-Baule production function estimates are presented. The Mitscherlich-Baule function using OLS estimation reaches higher goodness of fit than the OLS estimates of the quadratic and square root forms, again with coefficient estimates showing the expected signs. The two coefficient estimates for irrigation water and water endowment (α_4 and α_5) are not significantly different from zero at the level of five percent.

The Mitscherlich-Baule production function estimation based on robust regression (IRLS) explains more of the variation in corn yields than all other functions. In contrast to the OLS estimation, the coefficients α_4 and α_5 are significant at the one percent level if robust regression is used. Besides the change in the level of significance for α_5 , the coefficient estimate increased remarkably if robust regression is applied. Because mainly dry years are excluded or down-weighted, the soil moisture endowment is higher for the remaining observations.

Generally concluded, the use of robust regression methods leads to an increase of the goodness of fit for each of the analyzed production functions. Furthermore, levels of significance alter in a reasonable direction if robust regression methods are employed. In particular, modeling becomes more precise as coefficient estimates tend to have higher levels of significance. Therefore, the use of robust regression methods in yield production functions estimation is highly recommended. However, the decision on the most appropriate functional form can not exclusively be based on statistical measures, such as the goodness of fit. Hence, conclusions on the appropriateness of functional forms can be drawn if and only if the misspecification costs are also calculated and interpreted, such as in the subsequent section.

6 Optimal Input Levels and Costs of Misspecification

The knowledge of production functions is crucial for modeling purposes and economic analyses that are concerned with optimal resource allocation. This usually involves an assessment of optimal input and output levels, which is generally determined through maximization of a suitably defined profit function. For the purpose of our analysis, this is given by the subsequent definition

$$\pi = P_{Corn} \cdot f(W,N) - P_{Nitrogen} \cdot N - P_{Irrigation} \cdot W \qquad (4)$$

where the net return (or profit) per hectare π is equal to the gross return (crop price P_{Corn} times corn yield $f(W,N)$, minus total nitrogen costs (nitrogen price $P_{Nitrogen}$ times amount of nitrogen applied N) and total irrigation costs (irrigation price $P_{Irrigation}$ times amount of irrigation water W). For simplicity, other costs are assumed to be constant and therefore irrelevant for calculating the profit maximizing input combination, which follows from maximizing the profit function (Eq. 4). In this case, the optimal input levels are determined through the following first-order conditions:

$$P_{Nitrogen} = P_{Corn}\left(\partial f(W,N^*)/\partial N\right)$$

and

$$P_{Irrigation} = P_{Corn}\left(\partial f(N,W^*)/\partial W\right) \qquad (5)$$

Where N^* and W^* are the profit maximizing input levels of nitrogen application and irrigation respectively. In other words, efficiency in production requires employment and remuneration of all production factors according to their value marginal product. This is satisfied if, for each input factor, the input price equals the product of the crop price and the factor's marginal productivity.

In the further analysis, we set the corn price equal to CHF 0.642 kg^{-1}, the average annual value for the period 1981-2003 in Switzerland [44]. We assume a constant nitrogen price of CHF 1.6 kg^{-1} (extrapolated from ammonium nitrate 27.5 to pure nitrogen) at the 1993 level [29], and an estimated price for irrigation water of CHF 0.06 m^{-3}. The latter is based on the following assumptions. Since charges and fees for ground and surface water withdrawal in Switzerland are regulated at cantonal or municipal levels, we consider the situation in the Canton of Zurich where about CHF 0.01 is charged as variable costs for the withdrawal of one cubic meter surface water [6]. In addition, energy costs are about CHF 0.05 per cubic meter of irrigation water if electric power is used (assuming a standard pump and electricity costs for the canton of Zurich). Other water related costs are assumed to be constant in the short run. Thus, to get one mm of additional water, a farmer has to pay an irrigation price of CHF 0.60 per hectare. Using these data, the optimal input levels are calculated according to equation (5) and represented in Table 5.

Table 5: Optimal input levels, yield, and maximum net return

Functional Form-Estimation Method	Optimal amount of Nitrogen applied (kg/ha)	Optimal amount of irrigation Water applied (mm)	Optimal yield (kg/ha)	Maximum net return (CHF/ha)
Quadratic-OLS	172.8	179.6	9695	5840.32
Square Root-OLS	131.3	133.9	9180	5602.82
Mitscherlich-Baule-OLS	111.2	61.3	9078	5613.55
Quadratic-RLS	177.4	163.8	9859	5947.68
Square Root-RLS	147.7	108.6	9324	5684.56
Mitscherlich-Baule-IRLS	124.9	116.7	9286	5691.51

It shows that all optimal input levels are within the range of the simulation data. The lowest input use is recommended by the Mitscherlich-Baule (OLS) function, with 61.3 mm of irrigation water and 111.2 kg/ha of nitrogen. This goes along with the lowest yield (9078 kg/ha) and an estimated net revenue of 5613.55 CHF/ha. In contrast, the robust estimated quadratic function shows the highest yield (9859 kg/ha) and nitrogen use (177.4 kg/ha) and the highest profit (5947.68 CHF/ha), while the quadratic OLS function implies the highest optimal amount of irrigation water with 176.9 mm.

Our estimates for optimal input use are very low compared to those of Llewelyn and Featherstone [32] for Western Kansas that are in the following ranges: $238.5 < N^* < 305.3$ and $513 < W^* < 873$. This is mainly a consequence of the higher precipitation in Switzerland, and thus the substantially smaller amounts of irrigation water required on the Swiss Plateau. But the general results about the functional forms remain the same as in other studies. As in Ackello-Ogutu et al. [1], the polynomial functions recommend higher fertilizer use than the Mitscherlich-Baule functions. The quadratic form, in particular, results in a higher optimal use of nitrogen than all other functions. This is not surprising, since Anderson and Nelson [3] already gave evidence for the overestimation of optimal nitrogen amounts by the quadratic form.

Furthermore, the results in Table 5 show that the robust versions of production function estimates systematically lead to higher profit maximizing yields and higher profits than their non-robust counterparts. Moreover, for each functional form, the optimal amount of nitrogen fertilizer application increases if robust regression results are taken instead of OLS results. And, except for the case of the Mitscherlich-Baule function, robust regression leads to the expected adjustment towards lower use of irrigation water in the profit-maximizing situation. All in all, the use of robust estimation narrows the range of optimal input levels across the different functional forms.

For the final evaluation of production functions and estimation methods, we employ the concept of the relative costs of misspecification. Those are defined as the decrease in net return if optimal input levels of an incorrect function are used instead of those of the real underlying production function. The basic idea of this concept is to minimize the potential loss of a misspecification of the production function. Usually, the focus is on the potential loss due to the wrong functional form. In the following, we also consider the costs of applying the improper estimation technique.

Table 6: Relative Costs of Misspecification

	Cost of using optimal input levels based on:					
When the true function is:	Quadratic-OLS	Square Root-OLS	Mitscherlich-Baule-OLS	Quadratic-RLS	Square Root-RLS	Mitscherlich-Baule-IRLS
Quadratic-OLS	0	93.01	297.88	4.23	77.85	135.18
Square Root-OLS	30.61	0	39.83	32.13	8.41	2.01
Mitscherlich-Baule-OLS	113.22	41.38	0	109.97	41.86	27.34
Quadratic-RLS	3.77	104.65	296.39	0	68.59	145.23
Square Root-RLS	7.18	27.08	35.49	8.45	0	23.14
Mitscherlich – Baule-IRLS	57.52	54.08	3.11	51.85	9.86	0

Table 6 shows the relative costs of misspecification. The nine cells in the upper left-hand corner correspond to the traditional approach where only functional forms estimated with OLS are compared. If for instance the quadratic function would be the true underlying form, the use of the square root function induces a cost of misspecification of CHF 93.01. This increases to CHF 297.88 for the Mitscherlich-Baule function. The latter exhibits the highest costs of misspecification, while the square root function is the most appropriate if the misspecification-cost criterion is employed. The square root function is similar to the quadratic form, but flatter in its surface and comes therefore closer to the plateau approach of the Mitscherlich-Baule specification [1, 19]. Optimal input recommendations based on the square root function are correspondingly situated between those of the other two approaches we considered here.

Table 6 further reveals that, in most cases, the use of robust estimation methods results in lower costs of misspecification than the standard OLS approach, and that the square root specification performs better under this criterion than the other functional forms. This becomes obvious when comparing the top left-hand cells with the bottom right-hand ones, as well as from the comparison of the misspecification costs in the different lines of Table 6. Only in the cases where the square root specifications are assumed to be the true underlying functions does the quadratic OLS estimation show slightly lower costs of misspecification than its RLS counterpart. Furthermore, square root function estimation with OLS leads to a marginally lower decrease of the net profit than its robust counterpart if the Mitscherlich-Baule-OLS is assumed to be the underlying function.

Altogether, this supports the suggestion that the RLS estimation of the square root function is the best approximation of the real underlying crop response relationship. These findings further support the application of robust regression methods, besides the previously made recommendation from an econometrical point of view.

7 Summary and Conclusions

Simulated corn yield data for the Swiss Plateau are used for the estimation of crop production functions, with particular consideration of yield response to nitrogen fertilizer and irrigation water application. Three functional forms are considered: the quadratic, the square root, and the Mitscherlich-Baule function. In addition, the treatment of exceptional climatic events is investigated, and robust regression methods are used.

Observed yield data provide insufficient estimation possibilities due to a lack of variation within the data. In contrast, biophysical simulation generates an enlarged data base compared with field observations. For the present study, we used simulated corn yield data of the CropSyst model, which is widely used and validated (see Stöckle et al. [46] for a review of studies using CropSyst).

In the existing literature, several comparisons of corn yield production functions recommend flexible forms containing a growth plateau, such as the Mitscherlich-Baule function. In contrast, we found the square root function to be the most appropriate form to represent corn production in Switzerland. Furthermore, exceptional climatic events, such as the summer drought in 2003, prove to be the major source of misleading results if the Ordinary Least Squares criterion is used to estimate production function coefficients. Robust regression methods are recommended instead.

Since the yield response to irrigation water is expected to be much higher in years with extraordinary high water stress, it seems reasonable to analyze those years separately. In particular, it shows that the inclusion of those years in the OLS estimation of production functions can affect both the goodness of fit and the significance of coefficient estimates. An increase in the explained variance of the Swiss corn yield turns out for all functional forms if robust regression is employed. Furthermore, coefficient estimates and their level of significance change in reasonable directions. Thus, our investigation shows that, besides the functional form, the estimation method is important for production function comparisons. Furthermore, robust regression is a valuable tool for other agricultural and environmental modeling problems that face extreme climatic variability in data.

This conclusion is further supported by a comparison of the relative costs of misspecification. Using RLS instead of OLS generally results in lower costs of misspecification. Irrespective of the true underlying functional form, optimal input levels based on robust estimated functions reduce the maximum costs of misspecification compared to the counterparts estimated with OLS. We therefore recommend the application of robust regression methods for production function estimation.

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Appendix – Estimation techniques

Ordinary Least Squares. The general linear model is given by:

$$y = \alpha + \beta X + u \qquad \text{(A1)}$$

Where Y is the regressand vector, X is the regressor matrix, α an intercept, β the coefficient vector and u an error term vector. The Least Squares criterion selects the coefficient vector, which minimizes the sum of squared residuals:

$$\underset{\hat{\beta}}{Min} \sum_{i=1}^{n} u_i^2 \qquad \text{(A2)}$$

Thus, the vector of coefficient estimates, $\hat{\beta}$, is given by:

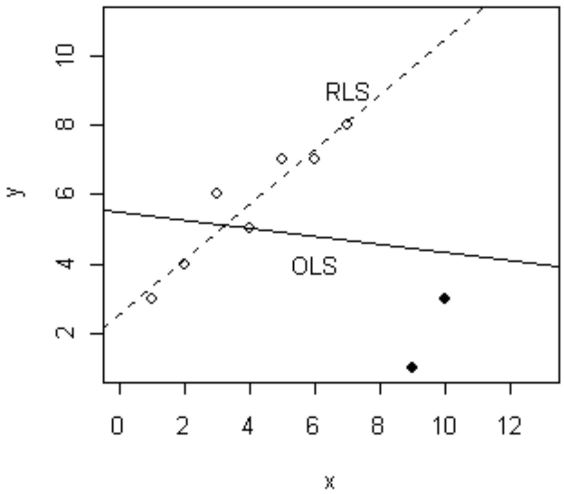
$$\hat{\beta} = (X'X)^{-1} X'Y \qquad \text{(A3)}$$

Following the Gauss-Markov theorem, Ordinary Least Squares (OLS) is the most efficient regression technique, i.e. provides the coefficient estimates with the smallest variance [25].

Robustness. An outlier is an observation that deviates from the relationship described by the majority of data. OLS cannot cope with a single outlier. The breakdown point concept is used to quantify robustness properties of an estimator. It is defined as the smallest amount of arbitrary (outlier) contamination, which can carry an estimator over all bounds [24]. The estimator becomes unreliable beyond this borderline. OLS possesses the lowest possible breakdown point of $1/n$[1]. Thus, one outlier can be sufficient to arbitrarily change OLS estimates [39]. A hypothetical example for OLS estimation in presence of outliers is given in Figure A1.

[1] n denotes the number of observations, in our analysis n=527.

Figure A1: OLS and RLS estimation for a contaminated data set



Note: Black dots indicate outlying observations.

Least Trimmed Squares. The method of Least Trimmed Squares (LTS) is a high-breakdown regression technique, i.e. it can possess the highest possible breakdown point of ½. Thus, LTS coefficient estimates are reliable up to an arbitrary contamination of fifty percent of the data. Based on the idea of trimming the largest residuals the LTS fitting criterion is defined as follows:

$$\underset{\hat{\beta}}{Min} \sum_{i=1}^{h} (r^2)_{i:n} \qquad \text{(A4)}$$

$(r^2)_{(i)}$ are the ascending ordered squared (robust) residuals and h is the so-called trimming constant. In our analysis, $h = [(3n + p + 1)/4]$ is employed[2] [40][3]. The computation of LTS coefficients is neither explicit (such as for OLS) nor iterative, but follows an algorithm described in Rousseeuw and Leroy [39]. Because the efficiency of LTS estimation is low, LTS results allow not for trustful infer-

[2] p denotes the number of coefficients that are estimated.

[3] Note: This choice of h yields in a smaller breakdown point than (for our model with p=5 and n=527: ≈ 0.25).

ence [39]. Thus, LTS estimation is only used as a data analytic tool for outlier identification. An observation is identified as an outlier if the absolute standardized robust residual ($|r_i / \hat{\sigma}|$) exceeds the cutoff value of 2.5 [24]. Where r_i is the (robust) LTS residual and $\hat{\sigma}$ is the (robust) LTS scale estimate [39].

Reweighted Least Squares. To provide both robust and efficient coefficient estimates, Reweighted Least Squares (RLS) regression is applied [39]. RLS regression is a weighted least squares regression with coefficient estimate being defined as follows:

$$\hat{\beta}_{RLS} = \left(X^{'}WX\right)^{-1} X^{'}WY \qquad \text{(A5)}$$

The diagonal elements of the weighting matrix ($W = diag\{w_1, \ldots, w_n\}$) are generated by an indicator function, $I_{Outlier}$:

$$w_i = I_{Outlier}\left[\left|\frac{r_i}{\hat{\sigma}}\right| \leq 2.5\right] \qquad \text{(A6)}$$

The indicator function generates weights of zero for observations that are identified as outliers and weights of one otherwise. RLS combines robustness and efficiency properties of LTS and OLS estimation, respectively. Therefore, this regression technique is highly suitable to ensure efficient estimation in presence but also in absence of outliers[4]. RLS regression is applied for coefficient estimation of the quasi linear functional forms (square root and quadratic).

Iteratively Reweighted Least Squares. Because LTS regression is not suitable for nonlinear problems, Iteratively Reweighted Least Squares (IRLS) regression is applied to estimate coefficients of the Mitscherlich-Baule function. A nonlinear model is, in general, defined as $Y = f(X, \beta) + u$. The functional part $f(X, \beta)$ is nonlinear with respect to the unknown parameters β_i. In IRLS regression, residuals are robustly weighted at each step of iteration until convergence. Tukey's biweight [18] is applied as weighting procedure and weights are generated as follows (following Hogg [21]):

$$w_i = \begin{cases} (1 - (r_i / \hat{\sigma} \cdot c)^2)^2, & |r_i / \hat{\sigma}| \leq c \\ 0, & |r_i / \hat{\sigma}| > c \end{cases} \qquad \text{(A7)}$$

r_i is the (robust) IRLS residual and $\hat{\sigma}$ the (robust) scale estimate and *c* a tuning constant. We employ the MAD (Median of absolute deviations from the median) for robust scale estimation and set the tuning constant to 5.0 (following Hogg [21]).

In contrast to LTS, IRLS is no high breakdown estimation technique. In order to validate results, we conduct sensitivity analysis of crucial factors such as starting values and tuning constant. The iterative least squares estimation uses the Levenberg-Marquardt algorithm (see Marquardt [33] and Moré [35] for details). Because coefficient estimates are highly correlated in our analysis, this algorithm ensures stable estimation compared with the Gauss-Newton algorithm [42].

[4] Due to these properties, RLS is chosen in our analysis in favor of other robust regression techniques (see Hampel et al. [18], for details).

Heteroscedasticity. Both nitrogen fertilizer application and irrigation cause heteroscedasticity [26]. Thus, the variance of the regression residuals is not constant for all residuals (i.e. $Var(u) \neq \sigma^2 I$), but is as follows:

$$Var(u) = \sigma^2 \Omega \qquad \text{(A8)}$$

Where *Var(u)* is the variance of residuals and *I* is the identity matrix. Furthermore, Ω is a positive definite matrix and is determined by the causal factors of heteroscedasticity (in this analysis: nitrogen and irrigation). Coefficient estimates remain unbiased and consistent but fail to be efficient if residuals reveal heteroscedasticity. If the residual variance is, for instance, proportional to a single input factor (X_i), $\sigma_i^2 = \alpha_1 X_i$, it follows that $\Omega = diag\{X_{i1}, \ldots, X_{in}\}$ [25]. Feasible generalized least squares (FGLS) regression has to be employed to ensure efficient estimation. The FGLS coefficient estimate is defined as follows:

$$\hat{\beta}_{FGLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \qquad \text{(A9)}$$

The White and the Breusch-Pagan test are used to test for heteroscedasticity [25]. All RLS and IRLS estimations in this study are corrected for heteroscedasticity.

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