Bank Behavior in Oligopoly, Bank-Clients and Monetary Policy

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30 June 2013

Monetary Policy Implications on Banking Conduct and Bank-Clients Behavior

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Abstract

Using a two-stage Cournot game with scope economies, we examine the effects of monetary policy on the optimal bank behavior. The emphasis is given on the way the interest rate spread is influenced by the minimum reserve requirements. It is demonstrated that the sign of this effect depends on the kind of scope economies. Moreover, monetary policy implications on both the depositor's and borrower's behavior are presented. Assuming an overlapping generation context, we prove that minimum reserve requirements affect the optimal levels of bank-clients' consumption through the corresponding equilibrium interest rates.

Key Words: Bank Behavior, Scope Economies, Reserve Requirements, Substitution Effect, Income Effect

JEL Classification: C61, D11, D43, E52, G21, L13

1. Introduction

This paper investigates the impact of monetary policy on the optimal bank behavior under oligopolistic conditions. In addition, we attempt to extend this analysis in the sphere of bank-clients behavior, i.e. depositors and borrowers.

The relative literature focuses on the interbank rate as an instrument of monetary policy. The Klein-Monti model is a prototype model of the so-called Industrial Organization approach to banking, in which banks are considered as profit-maximizing firms that offer services to agents. Freixas & Rochet (2008) apply a traditional one-stage Cournot game, for the case of a finite

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number of banks that operate on the markets for loans and deposits. They show that, under the assumptions of symmetric costs and symmetric conduction, an increase in the interbank rate leads to an increase in the optimal interest rates on loans and deposits. The Klein-Monti model is described and compared to alternative models of banking in surveys by Baltensperger (1980) and Santomero (1984). Hannan (1991) shows that the model can be used to derive various empirical predictions. For this reason, it has been the (implicit) starting point for a number of empirical studies, as for instance for Molyneux et al. (1994), Neuberger and Zimmerman (1990) and Suominen (1994). Toolsema and Schoonbeek (1999) extend the previous model, introducing asymmetry in the cost function (Cournot game) or asymmetry in bank conduction (Stackelberg game). They conclude that the introduction of asymmetric cost leads to different results, regarding to the sign of the change in the individual volumes of loans and deposits of the bank with the lowest marginal cost of loans. Porras (2008) analyzes the effects of leadership in banking when oligopolistic competition exists in the market of deposits. The main findings suggest that there are private and social benefits associated to leaderships. Yamazaki & Miyamoto (2004) discuss a two-stage Cournot model with scope economies. It is demonstrated that in the case of large economies of scope, an increase in the interbank rate leads to an increase in the interest rate on loans and a decrease in the interest rate on deposits. Using the above model and assuming an overlapping generation context, Varelas (2007) proved that the interbank rate affects the optimal levels of bank-clients' consumption through the corresponding equilibrium interest rates. Kasya (1986) points out that economies of scope in banking are becoming more widespread with the development of monetary deregulation.

In this paper we concentrate on the way the minimum reserve requirements of commercial banks influence the optimal bank behavior in oligopoly. In particular, we are interested in the influence of this instrument of monetary policy on the interest rate spread. For this reason, we formulate a two-stage Cournot game with scope economies. In order to examine the effects of monetary policy on the optimal quantities and rates of both deposits and loans, comparative statics is implemented. We conclude that the sign of each change depends on the type of scope economies. Finally, treating an overlapping generation model as a guiding principle, we show that the minimum reserve requirements of commercial banks have an impact on the depositors' and borrowers' two periods' optimal level of consumption.

The paper is structured as follows. Section 2 presents the theoretical model of bank behavior and the relative effects of monetary policy. Section 3 examines the expansion of the previous model on the consumer field. Section 4 concludes.

2. Bank Behavior and Minimum Reserve Requirements

2.1 The Theoretical Model

Following Yamazaki, Miyamoto, (2004) model, we assume there are two banks, A and B, that operate both on the markets for deposits and loans. The total volumes of loans and deposits are:

$L = L_A + L_B$ **(1)** and $D = D_A + D_B$ **(2)**,

where L and D, are the total volumes of loans and deposits, respectively and $L_i, D_i,\ i = A, B$ are the individual amounts of loans and deposits.

The inverse demand function for loans has downward slope and its functional form has as follows:

$$r_L = r_L(L) = a_1 - bL, \text{ where } a_1, b > 0 \text{ and } r_L(L)' < 0 \quad \textbf{(3)}$$

while the mathematical form of the inverse supply function of deposits has as follows:

$$r_D = r_D(D) = \beta + \gamma D\ , \text{ where } \beta, \gamma > 0 \text{ and } r_D(D)' > 0 \quad \textbf{(4)}$$

The profit function of the individual bank is given by:

$$\Pi_i(L_i, D_i) = r_L(L)L_i + rM_i - r_D(D)D_i - C_i(L_i, D_i)\ , i = A, B \quad \textbf{(5)}$$

where:

$\Pi_i(L_i, D_i)$: the profit function of bank i, with $i = A, B$
M_i : the net position of bank i on the interbank market
$C_i(L_i, D_i)$: bank's i cost function
r : the exogenous interest rate on the interbank market

Under the assumption of a linear functional form, the net position of bank i is given by:

$$M_i = (1 - \alpha)D_i - L_i\ \ , i = A, B \quad \textbf{(6)}$$

where $\alpha \, \epsilon(0,1)$ denotes the fraction of the reserve requirements, which is determined exogenously by the Government or the Central Bank.

Moreover, the cost function of each bank is assumed to be non-linear and it incorporates the notion of scope economies between loans and deposits:

$$C_i(L_i, D_i) = \theta(D_i)L_i + \varphi D_i \, , i = A, B \tag{7}$$

where:
$\theta(D_i) > 0$: the marginal cost of loans & φ >0 : the constant unit cost of deposits

We assume that $C_i(L_i, D_i)$ is a continuous and differentiable function of any order. Moreover, the hypothesis that is made concerning the $\theta(D_i)$ function is that $\theta(D_i)'' = 0$. The sign of the cross derivative $\partial C_i(.)/\, \partial L_i D_i$ depends on the sign of the derivative $\partial\theta(D_i)/\partial D_i$ and it is related to scope economies. Scope economies exist when the joint offer of loans and deposits by a universal bank, that is both a commercial and an investment bank, is more efficient than the separable offer of loans and deposits by specialized banks.

Following Baumol (1982), we may distinguish between the following cases:

- If $\partial\theta(D_i)/\partial D_i < 0$, it holds $\partial C_i(.)/\, \partial L_i D_i$ < 0 and consequently there are economies of scope.
- If $\partial\theta(D_i)/\partial D_i > 0$, it holds $\partial C_i(.)/\, \partial L_i D_i$ > 0 and consequently there are diseconomies of scope.
- If $\partial\theta(D_i)/\partial D_i = 0$, it holds $\partial C_i(.)/\, \partial L_i D_i$ = 0 and consequently there are no economies of scope.

2.2 Solution of the Model

The typical form of bank's i maximization problem has as follows:

$$max\, \Pi_i(L_i, D_i) = r_L(L)L_i + rM_i - r_D(D)D_i - C_i(L_i, D_i)$$

If we substitute the relations (1), (2), (3), (4), (6) and (7) to (5), we get:

$$\max \Pi_i(L_i, D_i) = [a_1 - b(L_i + L_j) - r - \theta(D_i)]L_i + [r(1-a) - \beta - \gamma(D_i + D_j) - \varphi]D_i \quad (8)$$

where $i, j = A, B \; and \; i \neq j$

The profit maximization problem is described by a two stage game. In the first stage, the banks determine the volume of deposits simultaneously, while in the second stage they choose the volume of loans.

Each bank engages in a sequential portfolio problem. In order to solve the problem, the backward induction method is applied. We make two fundamental assumptions: a) the second stage constitutes a well-defined Nash equilibrium and b) subgame-perfect equilibrium is adopted as an equilibrium concept.

In the context of the game's second stage, the maximization problem of bank i has the following mathematical form:

$$\max_{L_i} \Pi_i(L_i, D_i) = [a_1 - b(L_i + L_j) - r - \theta(D_i)]L_i + [r(1-a) - \beta - \gamma(D_i + D_j) - \varphi]D_i \quad (9)$$

where $i, j = A, B \; and \; i \neq j$

The first order necessary and sufficient condition for an extremum has as follows:

$$\frac{\partial \Pi_A}{\partial L_A} = 0 \Rightarrow a_1 - 2bL_A - bL_B - r - \theta(D_A) = 0 \quad (10)$$

$$\frac{\partial \Pi_B}{\partial L_B} = 0 \Rightarrow a_1 - 2bL_B - bL_A - r - \theta(D_B) = 0 \quad (11)$$

The bank's i profit function will exhibit a maximum at the specified extremum if the second order condition is satisfied, that is if

$$\frac{\partial^2 \Pi_i}{\partial {L_i}^2} < 0, \qquad i = A, B$$

The solution of the first order conditions (10) and (11) with respect to L_A and L_B respectively leads to the extraction of the functional form of the equilibrium quantity of loans in the second stage subgame:

$$L_i = \frac{a_1 - r - 2\theta(D_i) + \theta(D_j)}{3b} \qquad (\mathbf{12})$$

where $i, j = A, B$ and $i \neq j$.

After making use of relation (12), it is very easy to show that the partial derivatives of the equilibrium quantities of loans (L_i) with respect to parameters a & b and variables D_i & D_j have as follows:

$$\frac{\partial L_i}{\partial a} = 0 \qquad \& \qquad \frac{\partial L_i}{\partial r} = -\frac{1}{3b} < 0 \quad (\mathbf{13})$$

$$\frac{\partial L_i}{\partial D_i} = -\frac{2\theta'(D_i)}{3b} \quad (\mathbf{14}) \qquad \& \qquad \frac{\partial L_i}{\partial D_j} = \frac{\theta'(D_j)}{3b} \quad (\mathbf{15})$$

The sign of the partial derivatives $\partial L_i / \partial D_i$ & $\partial L_i / \partial D_j$, as described by relations (14) and (15), is related to the sign of the first order derivatives $\theta'(D_i)$ & $\theta'(D_j)$ respectively. Given that the signs of $\theta'(D_i)$ & $\theta'(D_j)$ are differentiated with respect to the kind of the economies of scope that is in effect, we may distinguish between the following cases:

- If $\theta'(D_i) < 0$ & $\theta'(D_j) < 0$ (economies of scope), then

  $$\frac{\partial L_i}{\partial D_i} > 0 \text{ \& } \frac{\partial L_i}{\partial D_j} < 0.$$

- If $\theta'(D_i) = 0 = \theta'(D_j)$ (no economies of scope exist), then

  $$\frac{\partial L_i}{\partial D_i} = 0 = \frac{\partial L_i}{\partial D_j}.$$

- If $\theta'(D_i) > 0$ & $\theta'(D_j) > 0$ (diseconomies of scope), then

  $$\frac{\partial L_i}{\partial D_i} < 0 \text{ \& } \frac{\partial L_i}{\partial D_j} > 0.$$

The total equilibrium amounts of loans on the market is defined as the summation of L_A & L_B and its functional form, as this derived after making use of relation (12), is given by the following mathematical expression:

$$L = L_A + L_B = \frac{2a_1 - 2r - \theta(D_A) - \theta(D_B)}{3b}$$

Moving now into the first stage of the game, the optimization problem of bank i is related to the maximization of its profit function, as this is transformed after the substitution of the above relation and (12) in (9):

$$\max_{D_i} \Pi_i(L_i, D_i) = \frac{1}{b}\left[\frac{a_1 - r - 2\theta(D_i) + \theta(D_j)}{3b}\right]^2 + \left[r(1-a) - \beta - \gamma(D_i + D_j) - \varphi\right]D_i \qquad \mathbf{(16)}$$

where $i, j = A, B \; and \; i \neq j$

The first order condition for an extremum facing each bank in the context of its maximization problem is described respectively by the following equations:

$$\frac{\partial \Pi_A(.)}{\partial D_A} = 0 \Rightarrow \frac{-4}{9b}[a_1 + \theta(D_B) - 2\theta(D_A) - r]\theta'(D_A) + r(1-a) - \beta - \varphi - 2\gamma D_A - \gamma D_B = 0 \qquad \mathbf{(17)}$$

$$\frac{\partial \Pi_B(.)}{\partial D_B} = 0 \Rightarrow \frac{-4}{9b}[a_1 + \theta(D_A) - 2\theta(D_B) - r]\theta'(D_B) + r(1-a) - \beta - \varphi - 2\gamma D_B - \gamma D_A = 0 \qquad \mathbf{(18)}$$

The second order condition for a maximum is satisfied since:

$$\frac{\partial^2 \Pi_i(D_i)}{\partial D_i^2} = \frac{8}{9}[\theta'(D_i)]^2 - 2\gamma < 0 \qquad \mathbf{(19)}$$

where $i, j = A, B \; and \; i \neq j$

The equations (17) and (18) give the best response functions of the two banks in the deposits. They are denoted by $BR_A(D_B)$ and $BR_B(D_A)$ respectively. Their slope is downward, showing that deposits are strategic substitutes. In order to ensure the stability of the equilibrium, we make the following assumption:

$$\left|BR_i'(D_j)\right| = \left|BR_j'(D_i)\right| < 1, \text{ for } \quad i,j = A,B \text{ and } i \neq j \qquad \textbf{(20)}$$

Solving the system of equation (17) and (18), we obtain the equilibrium volumes of deposits. Their form has as follows:

$$D_i^* = D_i^*(r,a), i = A,B \qquad \textbf{(21)}$$

Substituting (21) in (12), we obtain the subgame-perfect equilibrium amounts of loans for each bank:

$$L_i^* = L_i^*\left[r,a,D_i^*(r,a),D_j^*(r,a)\right] \text{ for } \quad i,j = A,B \text{ and } i \neq j \qquad \textbf{(22)}$$

2.3 Monetary Policy Implications

Now, we examine the impact of changing the fraction of reserve requirements (α) on the equilibrium amounts D_i^* & L_i^* and the corresponding interest rates. In order to achieve this, we apply comparative statics by using the method of total differentials.

Taking the total differential of the first order conditions (17) and (18) (under the assumption that $\theta''(.) = 0$**),** we get the following system of equations in matrix form:

$$\Delta \begin{bmatrix} dD_A^* \\ dD_B^* \end{bmatrix} = \begin{bmatrix} r\,da - \left[\frac{4}{9b}\theta'(D_A) + (1-\alpha)\right] dr \\ r\,da - \left[\frac{4}{9b}\theta'(D_B) + (1-\alpha)\right] dr \end{bmatrix} \qquad \textbf{(23)}$$

where

$$\boldsymbol{\Delta} = \begin{bmatrix} \frac{8}{9b}(\theta'(D_A))^2 - 2\gamma & -\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right] \\ -\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right] & \frac{8}{9b}(\theta'(D_B))^2 - 2\gamma \end{bmatrix}$$

Under the assumption that $\left|BR_i'(D_j)\right| = \left|BR_j'(D_i)\right| < 1$, the determinant of matrix Δ, that is$|\boldsymbol{\Delta}|$, is positive definite:

$$|\boldsymbol{\Delta}| = \begin{vmatrix} \frac{8}{9b}(\theta'(D_A))^2 - 2\gamma & -\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right] \\ -\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right] & \frac{8}{9b}(\theta'(D_B))^2 - 2\gamma \end{vmatrix}$$

$$= \left\{\frac{8}{9b}[\theta'(D_A)]^2 - 2\gamma\right\}\left\{\frac{8}{9b}[\theta'(D_B)]^2 - 2\gamma\right\} - \left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]^2 > 0 \quad \textbf{(24)}$$

Proof See Appendix

Presuming that $dr = 0$ and $da \neq 0$ and applying the Cramer's Rule, we can determine the derivatives $\partial D_A{}^*/\partial a$ and $\partial D_B{}^*/\partial a$:

$$\frac{\partial D_A^*}{\partial a} = \frac{r}{|\boldsymbol{\Delta}|}\frac{8[\theta'(D_B)]^2 + 4\theta'(D_A)\theta'(D_B) - 9b\gamma}{9b} \quad \textbf{(25)}$$

&

$$\frac{\partial D_B^*}{\partial a} = \frac{r}{|\boldsymbol{\Delta}|}\frac{8[\theta'(D_A)]^2 + 4\theta'(D_A)\theta'(D_B) - 9b\gamma}{9b} \quad \textbf{(26)}$$

Let $D_A = D_B = D_S$ (symmetry). Then, relations (25) and (26) lead to:

$$\frac{\partial D_A^*}{\partial a} = \frac{\partial D_B^*}{\partial a} = \frac{\partial D_S^*}{\partial a} = \frac{r}{|\boldsymbol{\Delta}|}\frac{12[\theta'(D_S)]^2 - 9b\gamma}{9b} \quad \textbf{(27)}$$

Given that r, $|\boldsymbol{\Delta}|, b, \gamma > 0$ and from relation (27), it is proved that the sign of the effect of a change in a on D_S^* depends on the value interval of $\theta'(.)$.

Equations (4) and (27) imply:

$$\frac{\partial r_D^*}{\partial a} = \frac{\partial r_D^*}{\partial D}\frac{\partial D^*}{\partial a} = \frac{\partial r_D^*}{\partial D}\frac{\partial (D_A^* + D_B^*)}{\partial a} = \frac{\partial r_D^*}{\partial D}\frac{\partial (2D_S^*)}{\partial a} = 2\,\gamma\,\frac{\partial D_S^*}{\partial a} \quad \textbf{(28)}$$

Due to the fact that $\gamma > 0$, the equilibrium volume of deposits and the corresponding interest rate move towards the same direction after a change in the fraction of reserve requirements.

In order to investigate the effect of an infinitesimal change in a on the equilibrium values D_S^* and r_D^*, the equation $\partial D_S^*/\partial a = 0$ will be solved with respect to $\theta'(D_S)$ so as to determine the value intervals of $\theta'(.)$ for which the sign of $\partial D_S^*/\partial a$ alternates:

$$\frac{\partial D_s^*}{\partial a} = 0 \Rightarrow \frac{r}{|\boldsymbol{\Delta}|} \frac{12[\theta'(D_s)]^2 - 9b\gamma}{9b} = 0 \Rightarrow \frac{12[\theta'(D_s)]^2 - 9b\gamma}{9b} = 0$$

$$\Rightarrow \theta'(D_s) = \pm \frac{\sqrt{3b\gamma}}{2}$$

<u>Table 1</u>: Determination of the effects of a change in a on D_s and r_D. (The symmetric case)

	Economies of Scope		No Economies of Scope	Diseconomies of Scope	
θ' (.)	(-∞, ρ_1)	(ρ_1,0)	0	(0,ρ_2)	(ρ_2,+∞)
$\frac{\partial D_s^*}{\partial a}$	+	-	-	-	+
$\frac{\partial r_D^*}{\partial a}$	+	-	-	-	+
	$\frac{\partial D_s^*}{\partial a} = \frac{\partial r_D^*}{\partial a} = 0$ for $\theta'(D_s) = \rho_1 = -\frac{\sqrt{3b\gamma}}{2}$			$\frac{\partial D_s^*}{\partial a} = \frac{\partial r_D^*}{\partial a} = 0$ for $\theta'(D_s) = \rho_2 = +\frac{\sqrt{3b\gamma}}{2}$	

Table 1 depicts the direction of the effects of a change in a on the optimal amount of deposits and the corresponding interest rate, for different value intervals of $\theta'(.)$. We observe that:

- In the cases of large economies of scope [$\theta'(.) \in (-\infty, \rho_1)$] and large diseconomies of scope [$\theta'(.) \in (\rho_2, +\infty)$], an increase in the fraction of reserve requirements is followed by an increase in the equilibrium amounts and in the interest rate of deposits.
- When the value interval of $\theta'(.)$ is the (ρ_1, 0) (small economies of scope) or the (0,ρ_2) (small diseconomies of scope), an increase in the fraction of reserve requirements leads to a decrease in both the equilibrium amount of deposits and the corresponding interest rate.
- If economies of scope do not exist [$\theta'(.)=0$], D_s^* and r_D^* move toward the opposite direction from the change in a.
- If $\theta'(.)$ takes the value $\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ or the value $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$, monetary policy does not affect D_s^* and r_D^*

From equation (22), we can deduce the impact of a change in the fraction of reserve requirements on the equilibrium volume of loans of each bank $L_i^*,\ i = A, B$.

$$\frac{\partial L_i^*}{\partial a} = \frac{\partial L_i^*}{\partial a} + \frac{\partial L_i^*}{\partial D_i}\frac{\partial D_i^*}{\partial a} + \frac{\partial L_i^*}{\partial D_j}\frac{\partial D_j^*}{\partial a} = -\frac{2\theta'(D_i)}{3b}\frac{\partial D_i^*}{\partial a} + \frac{\theta'\left(D_j\right)}{3b}\frac{\partial D_j^*}{\partial a}$$

From relations (22), (13), (14) and (15) and the hypothesis of symmetry ($D_A = D_B = D_s$ and $L_A = L_B = L_s$), we have that:

$$\frac{\partial L_A^*}{\partial a} = \frac{\partial L_B^*}{\partial a} = \frac{\partial L_s^*}{\partial a} = \frac{\partial L_s^*}{\partial a} + \frac{\partial L_s^*}{\partial D_s}\frac{\partial D_s^*}{\partial a} + \frac{\partial L_s^*}{\partial D_s}\frac{\partial D_s^*}{\partial a} = -\frac{\theta'(D_s)}{3b}\frac{\partial D_s^*}{\partial a} \qquad (\mathbf{29})$$

Given that $b > 0$, it is inferred that the effect of a change in a on ${L_s}^*$ depends on the value interval of $\theta'(D_s)$. The same holds concerning the effect of a on the equilibrium value of $r_L$and the magnitude of the spread $(r_L^* - r_D^*)$.

From (3):

$$\frac{\partial r_L^*}{\partial \alpha} = -b\frac{\partial L_A^*}{\partial a} - b\frac{\partial L_B^*}{\partial a} = -2b\frac{\partial L_s^*}{\partial a} \qquad (\mathbf{30})$$

After the combination of relations (28) and (30), we take:

$$\frac{\partial (r_L^* - r_D^*)}{\partial \alpha} = \frac{\partial r_L^*}{\partial \alpha} - \frac{\partial r_D^*}{\partial \alpha} = -2\left[b\frac{\partial L_s^*}{\partial a} + \gamma\frac{\partial D_s^*}{\partial a}\right] \qquad (\mathbf{31})$$

The following table summarizes the signs of the impact of a change in α, in the symmetric case where $D_A = D_B = D_s$ and $L_A = L_B = L_s$ (Table 2):

Table 2: Determination of the effects of a change in $\boldsymbol{\alpha}$. The symmetric case ($\boldsymbol{D_A = D_B = D_s}$ & $\boldsymbol{L_A = L_B = L_s}$)

$\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$	**Economies of Scope**			**No Economies of Scope**	**Diseconomies of Scope**		
$\theta'(D_s)$	(-∞, ρ_1)	ρ_1	(ρ_1,0)	0	(0,ρ_2)	ρ_2	(ρ_2,+∞)
$\frac{\partial D_s^*}{\partial \alpha}$	+	0	-	-	-	0	+
$\frac{\partial L_s^*}{\partial \alpha}$	+	0	-	0	+	0	-
$\frac{\partial r_D^*}{\partial \alpha}$	+	0	-	-	-	0	+
$\frac{\partial r_L^*}{\partial \alpha}$	-	0	+	0	-	0	+
$\frac{\partial (r_L^* - r_D^*)}{\partial \alpha}$	-	0	+	+	?	0	?

We deduce that:

- In the case of economies of scope with $\theta'(.) \in (-\infty, \rho_1)$, an increase (decrease) in α leads to an increase (decrease) in L_s^*. The same holds in the case of small diseconomies of scope ($\theta'(.) \in (0, \rho_2)$).
- L_s^* remains constant if $\theta'(.)$ is equal to $\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ (economies of scope) or $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$ (diseconomies of scope) or 0 (economies of scope do not exist).
- If the value interval of $\theta'(.)$ is the (ρ_1,0) (economies of scope) or the (ρ_2,+∞) (diseconomies of scope), L_s^* decreases (increases) after an increase (decrease) in α.

However, the most considerable inference of the previous analysis is the effect of monetary policy on the spread ($r_L^* - r_D^*$). Table 2 shows that:

- When large economies of scope exist ($\theta'(.) \in (-\infty, \rho_1)$), restrictive monetary policy via α leads to a decrease of the spread ($r_L^* - r_D^*$).
- If θ′ (.) is equal to $\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ (economies of scope) or $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$ (diseconomies of scope), a change in α has no impact on the spread ($r_L^* - r_D^*$).
- In the case of small economies of scope ($\theta'(.) \in (\rho_1, 0)$, there is a positive relation among α and the spread ($r_L^* - r_D^*$). This is also the case

concerning the effect of α on $(r_L^* - r_D^*)$ when scope economies do not exist.

- If diseconomies of scope exist with $\theta'(.) \neq \rho_2$ ($\theta'(.) \in (0, \rho_2) \cup (\rho_2, +\infty)$), the sign of the effect of α on $(r_L^* - r_D^*)$ is undefined. In this case, further investigation is needed.

It is noteworthy that the roots $\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ and $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$ depend on the parameter of demand for loans b and the parameter of supply of deposits γ. Consequently, the effect of monetary policy via the reserve requirements on the optimal amount and interest rate on deposits and loans depends on bank-clients behavior.

In order to eliminate the indeterminacy of the sign of the effect of α on $(r_L^* - r_D^*)$ in the case of diseconomies of scope, we apply mathematical investigation.

Relations (29) and (31) imply:

$$\frac{\partial(r_L^* - r_D^*)}{\partial\alpha} = \frac{\partial r_L^*}{\partial\alpha} - \frac{\partial r_D^*}{\partial a} = -2\left[b\frac{\partial L_s^*}{\partial a} + \gamma\frac{\partial D_s^*}{\partial a}\right]$$

$$= \quad -2\left[-\frac{\theta'(D_s)}{3} + \gamma\right]\frac{\partial D_s^*}{\partial a}, \qquad \gamma > 0 \qquad \textbf{(32)}$$

In order to determine the value intervals of $\theta'(.)$ for which the sign of $\partial(r_L^* - r_D^*)/\partial\alpha$ alternates, we solve the equation $\partial(r_L^* - r_D^*)/\partial\alpha = 0$:

$$\frac{\partial(r_L^* - r_D^*)}{\partial\alpha} = 0 \Leftrightarrow -2\left[-\frac{\theta'(D_s)}{3} + \gamma\right]\frac{\partial D_s^*}{\partial a} = 0$$

$$\Leftrightarrow -2\left[-\frac{\theta'(D_s)}{3} + \gamma\right] = 0 \text{ or } \frac{\partial D_s^*}{\partial a} = 0$$

$$\Leftrightarrow \theta'(D_s) = 3\gamma > 0 \ \text{ or } \ \theta'(D_s) = \pm\frac{\sqrt{3b\gamma}}{2}$$

where the roots $\pm\frac{\sqrt{3b\gamma}}{2}$ are induced by setting the relation (27) equal to zero, as it was presented before.

We can distinguish between three cases[4]:

- In the case of $3\gamma > \sqrt{3b\gamma}/2 \Rightarrow b < 12\gamma$, the sign of the partial derivative $\partial(r_L^* - r_D^*)/\partial\alpha$, is presented in table 3a.

Table 3a: Determination of effect of a change in α on $(r_L^* - r_D^*)$. (The case of $3\gamma > \sqrt{3b\gamma}/2$)

$\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$	**Economies of Scope**		**No Economies of Scope**	**Diseconomies of Scope**		
$\theta'(D_s)$	$(-\infty,\rho_1)$	$(\rho_1,0)$	0	$(\mathbf{0},\boldsymbol{\rho_2})$	$(\boldsymbol{\rho_2},\mathbf{3\gamma})$	$(\mathbf{3\gamma},+\infty)$
$-2\left[-\frac{\theta'(D_s)}{3}+\gamma\right]$	-	-	-	**-**	**-**	**+**
$\frac{\partial D_s^*}{\partial\alpha}$	+	-	-	**-**	**+**	**+**
$\frac{\partial(r_L^* - r_D^*)}{\partial\alpha}$	-	+	+	**+**	**-**	**+**
	$\frac{\partial(r_L^* - r_D^*)}{\partial\alpha} = 0$ for $\theta'(D_s) = \rho_1$			$\frac{\partial(r_L^* - r_D^*)}{\partial\alpha} = 0$ for $\theta'(D_s) = \rho_2$ & $\theta'(D_s) = 3\gamma$		

We observe that if small diseconomies of scope exist ($\theta'(.) \in (0, \rho_2)$), an increase in the fraction of compulsory requirements (α) leads to an increase in the spread $(r_L^* - r_D^*)$. The same holds when the value interval of $\theta'(.)$ is the $(3\gamma, +\infty)$. Conversely, in the case of diseconomies of scope with $\theta'(.) \in (\rho_2, 3\gamma)$, restrictive monetary policy via α leads to a decrease in the spread $(r_L^* - r_D^*)$.

- If $3\gamma < \sqrt{3b\gamma}/2 \Rightarrow b > 12\gamma$, we obtain the following table:

[4] The relative tables present all the value intervals of $\theta'(D_s)$. The mathematical investigation is necessary to determine the sign of the partial derivative $\partial(r_L^* - r_D^*)/\partial\alpha$ when diseconomies of scope exist. However, this analysis confirms the respective signs that were discussed before, in the cases of economies of scope and no economies for scope.

<u>Table 3b</u>: Determination of effect of a change in $\boldsymbol{a}$ on $(\boldsymbol{r_L^*} - \boldsymbol{r_D^*})$. (The case of $\boldsymbol{3\gamma} < \sqrt{\boldsymbol{3b\gamma}}/\boldsymbol{2}$)

$\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$	**Economies of Scope**		**No Economies of Scope**	**Diseconomies of Scope**		
$\theta'(D_s)$	$(-\infty, \rho_1)$	$(\rho_1, 0)$	0	$\boldsymbol{(0, 3\gamma)}$	$(\boldsymbol{3\gamma}, \boldsymbol{\rho_2})$	$(\boldsymbol{\rho_2}, +\infty)$
$-2\left[-\frac{\theta'(D_s)}{3} + \gamma\right]$	-	-	-	-	+	+
$\frac{\partial D_s^*}{\partial a}$	+	-	-	-	-	+
$\frac{\partial(r_L^* - r_D^*)}{\partial \alpha}$	-	+	+	+	-	+
	$\frac{\partial(r_L^* - r_D^*)}{\partial \alpha} = 0$ for $\theta'(D_s) = \rho_1$			$\frac{\partial(r_L^* - r_D^*)}{\partial \alpha} = 0$ for $\theta'(D_s) = \rho_2$ & $\theta'(D_s) = 3\gamma$		

It can be clearly seen that when the value interval of $\theta'(.)$ is the $(0, 3\gamma)$ or the $(\rho_2, +\infty)$, the spread $(r_L^* - r_D^*)$ is positively related to changes in a. On the other hand if value interval of $\theta'(.)$ is the $(3\gamma, \rho_2)$, an increase in a is followed by a decrease in the spread $(r_L^* - r_D^*)$.

- Supposing $3\gamma = \sqrt{3b\gamma}/2 \Rightarrow b = 12\gamma$, table 3c shows the sign of the effect of a change in α on the spread $(r_L^* - r_D^*)$.

<u>Table 3c</u>: Determination of effect of a change in $\boldsymbol{a}$ on $(\boldsymbol{r_L^*} - \boldsymbol{r_D^*})$. (The case of $\boldsymbol{3\gamma} = \sqrt{\boldsymbol{3b\gamma}}/\boldsymbol{2}$)

$\rho_1 = -\frac{\sqrt{3b\gamma}}{2}$ $\rho_2 = +\frac{\sqrt{3b\gamma}}{2}$	**Economies of Scope**		**No Economies of Scope**	**Diseconomies of Scope**	
$\theta'(D_s)$	$(-\infty,\rho_1)$	$(\rho_1,0)$	0	$\boldsymbol{(0, 3\gamma)}$	$\boldsymbol{(3\gamma, +\infty)}$
$-2\left[-\frac{\theta'(D_s)}{3} + \gamma\right]$	-	-	-	**-**	**+**
$\frac{\partial D_s^*}{\partial a}$	+	-	-	**-**	**+**
$\frac{\partial (r_L^* - r_D^*)}{\partial \alpha}$	-	+	+	**+**	**+**
	$\frac{\partial (r_L^* - r_D^*)}{\partial \alpha} = 0$ for $\theta'(D_s) = \rho_1$			$\frac{\partial (r_L^* - r_D^*)}{\partial \alpha} = 0$ for $\theta'(D_s) = 3\gamma = \rho_2$	

It is inferred that when diseconomies of scope exist, there is a positive relation among a and the spread $(r_L^* - r_D^*)$.

<u>3. Monetary Policy Implications on Bank-Clients Behavior</u>

One of the main bank operations is the intermediation between the depositors and the borrowers. To carry out this operation, banks pay the interest rate on deposits (r_D) to the depositors and receive the interest rate on loans (r_L) from the borrowers. Therefore, it is understandable that monetary policy influence bank-clients behavior through deposit and lending rates.

Following the analysis of Varelas & Karpeti (2005), we will examine the effects of a change in the fraction of reserve requirements on the bank-clients behavior in an overlapping generation context, assuming that the individuals' lifespan is extended in two periods. In the beginning of the first period, the depositors make their deposits, while the borrowers raise a loan. In the beginning of the second period, the depositors withdraw their deposits, receiving the deposit rate, while the borrowers repay the full amount of the loan increased by the lending rate. Furthermore, it is assumed that the depositors do not inherit or bequeath any amount of money. They can invest only in interest-bearing deposits. Similarly, the borrowers do not inherit or dispute any debt. They cannot save any amount of money. Finally, it is assumed that the price level of consumer goods is stable.

Bank-clients' problem can be stated as maximization of their utility function subjected to their lifetime budget constraint. That is:

$$\max_{c_{1t}, c_{2t+1}} U(c_{1t}, c_{2t+1}) = c_{1t}^{z} c_{2t+1}^{1-z}$$

$$\text{s.t. } g(c_{1t}, c_{2t+1}) = (1+w)c_{1t} + c_{2t+1} = (1+w)y_1 + y_2 \qquad \textbf{(33)}$$

where:

$$z = \begin{cases} z_1, \text{in case of depositor} \\ z_2, \text{in case of borrower} \end{cases}, \quad w = \begin{cases} r_D^*, \text{in case of depositor} \\ r_L^*, \text{in case of borrower} \end{cases},$$

c_{1t} (c_{2t+1}): the individual's consumption that in period t (t+1) is in the first (second) period of their life & y_1 (y_2): the individual's income over the first (second) period of their life

The acquired optimal levels of consumption for the two periods have as follows:

$$c_{1t}^* = zy_1 + \frac{z}{1+w}y_2 \qquad (\mathbf{34})$$

&

$$c_{2t+1}^* = (1-z)[(1+w)y_1 + y_2] \qquad (\mathbf{35})$$

Moving now to the monetary policy implications, the effect of monetary policy via the infinitesimal change of reserve requirements on the equilibrium levels of consumption is given by:

$$\frac{\partial c_{1t}^*}{\partial \alpha} = \frac{\partial c_{1t}^*}{\partial w}\frac{\partial w}{\partial a} = -\frac{z}{(1+w)^2}y_2\frac{\partial w}{\partial a}$$

$$\frac{\partial c_{2t+1}^*}{\partial \alpha} = \frac{\partial c_{2t+1}^*}{\partial w}\frac{\partial w}{\partial a} = (1-z)y_1\frac{\partial w}{\partial a}$$

Since $w = r_D^*$ (r_L^*) in the case of the depositors (borrowers), the effects of a change in α on the equilibrium levels of depositors' (borrowers') consumption are given by relations (36) & (37) [(38)& (39)]:

$$Depositors\begin{cases}\dfrac{\partial c_{1t}^*}{\partial \alpha}=\dfrac{\partial c_{1t}^*}{\partial r_D}\dfrac{\partial r_D^*}{\partial a}=-\dfrac{z_1}{(1+r_D)^2}y_2\,\dfrac{\partial r_D^*}{\partial a} & (36)\\[2ex] \dfrac{\partial c_{2t+1}^*}{\partial \alpha}=\dfrac{\partial c_{2t+1}^*}{\partial r_D}\dfrac{\partial r_D^*}{\partial a}=(1-z_1)y_1\,\dfrac{\partial r_D^*}{\partial a} & (37)\end{cases}$$

$$Borrowers\begin{cases}\dfrac{\partial c_{1t}^*}{\partial \alpha}=\dfrac{\partial c_{1t}^*}{\partial r_L}\dfrac{\partial r_L^*}{\partial a}=-\dfrac{z_2}{(1+r_L)^2}y_2\,\dfrac{\partial r_L^*}{\partial a} & (38)\\[2ex] \dfrac{\partial c_{2t+1}^*}{\partial \alpha}=\dfrac{\partial c_{2t+1}^*}{\partial r_L}\dfrac{\partial r_L^*}{\partial a}=(1-z_2)y_1\,\dfrac{\partial r_L^*}{\partial a} & (39)\end{cases}$$

We infer that the influence of the changes in the fraction of reserve requirements on the equilibrium levels of depositors' and borrowers' consumption depends on the signs of $\partial r_D^*/\partial a$ and $\partial r_L^*/\partial a$ respectively. Hence, the direction of the change in c_{1t}^* and $c_{2t+1,}^*$ that is triggered by a change in α, depends on the value interval of $\theta'(.)$, i.e. the type of scope economies.

The following table shows the signs of the changes analytically, as these are obtained by relations (9), (10), (11) & (12).

Table 4: Determination of the effect of α on the equilibrium levels of consumption

$\rho_1=-\frac{\sqrt{3b\gamma}}{2}$ $\rho_2=+\frac{\sqrt{3b\gamma}}{2}$	**Economies of Scope**			**No Economies of Scope**	**Diseconomies of Scope**		
$\theta'(.)$	$(-\infty, \rho_1)$	ρ_1	$(\rho_1,0)$	0	$(0,\rho_2)$	ρ_2	$(\rho_2,+\infty)$
DEPOSITORS							
$\frac{\partial r_D^*}{\partial a}$	+	0	-	-	-	0	+
$\frac{\partial c_{1t}^*}{\partial \alpha}$	-	0	+	+	+	0	-
$\frac{\partial c_{2t+1}^*}{\partial \alpha}$	+	0	-	-	-	0	+
BORROWERS							
$\frac{\partial r_L^*}{\partial a}$	-	0	+	0	-	0	+
$\frac{\partial c_{1t}^*}{\partial \alpha}$	+	0	-	0	+	0	-
$\frac{\partial c_{2t+1}^*}{\partial \alpha}$	-	0	+	0	-	0	+

Relations (9), (10), (11) & (12) show that policy of reserve requirements influences the equilibrium level of consumption indirectly, via the interest rates of deposits and loans. However, the induced change in interest rates has two effects on consumption: the income effect and the substitution effect. In order to examine this issue, we apply comparative statics by using the method of total differentials. We assume that dc_{1t}^*, dc_{2t+1}^* and dq^* are the endogenous variables, while dw, dy_1 and dy_2 are the exogenous ones. After proper calculations, we result in the following relations:

$$\frac{\partial c_{1t}^*}{\partial w} = (c_{1t} - y_1)\frac{|\boldsymbol{H_{31}}|}{|\boldsymbol{H}|} + q\frac{|\boldsymbol{H_{11}}|}{|\boldsymbol{H}|} \quad (\mathbf{40})$$

$$\frac{\partial c_{2t+1}^*}{\partial w} = -(c_{1t} - y_1)\frac{|\boldsymbol{H_{32}}|}{|\boldsymbol{H}|} - q\frac{|\boldsymbol{H_{12}}|}{|\boldsymbol{H}|} \quad (\mathbf{41})$$

$$\frac{\partial q^*}{\partial w} = (c_{1t} - y_1)\frac{|\boldsymbol{H_{33}}|}{|\boldsymbol{H}|} + q\frac{|\boldsymbol{H_{13}}|}{|\boldsymbol{H}|} \quad (\mathbf{42})$$

where: $|\boldsymbol{H}|$: the Hessian determinant & $|\boldsymbol{H_{ij}}|$: the determinant that arises if we abstract the *i* row and the *j* column from the nominator's matrix in Cramer's Rule (it is identical to the determinant that arises if we abstract the *i* row and the *j* column from the Hessian matrix

Relations (40) and (41) show the *total effect* of a change in deposit and lending rate on the equilibrium levels of consumption. Proper calculations allow us to divide the total effect into two magnitudes: the substitution effect and the income effect. It can be demonstrated that the *substitution effect* is given by

$$\left.\frac{\partial c_{1t}^*}{\partial w}\right|_{U=\bar{U}} = q\frac{|\boldsymbol{H_{11}}|}{|\boldsymbol{H}|} \quad \text{and} \quad \left.\frac{\partial c_{2t+1}^*}{\partial w}\right|_{U=\bar{U}} = -q\frac{|\boldsymbol{H_{12}}|}{|\boldsymbol{H}|}$$

while the *income effect* is stated as

first period: $(c_{1t} - y_1)\frac{|\boldsymbol{H_{31}}|}{|\boldsymbol{H}|}$ and second period: $-(c_{1t} - y_1)\frac{|\boldsymbol{H_{32}}|}{|\boldsymbol{H}|}$

To the extent that the case of large economies of scope [θ' (.)$\in$ (-∞,$-\frac{\sqrt{3b\gamma}}{2}$)]

is concerned, an increase in a leads to an increase in $r_D{}^*$ and a decrease in $r_L{}^*$. We make the following assumptions: a) both periods' consumption are normal goods and b) the first periods' consumption price is equal to (1+w), while the second period's consumption price is the numéraire. Regarding the case of depositors, the final outcome is that the first period's consumption is affected negatively only if the substitution effect is greater than the income effect in absolute values, while the level of the second period's consumption is affected positively. On the other hand, in the case of borrowers, the final result is that the level of the first period's consumption is affected positively, while the impact on the second period's consumption is negative only if the absolute value of the substitution effect is greater than the absolute value of the income effect. The analysis is similar for the other cases of value intervals of $\theta'(.)$.

To conclude, the above analysis confirms the signs of table 3 if the substitution effect is greater than the income effect in absolute values.

4. Conclusion

In this paper we examined the way the optimal bank behavior is affected by the minimum reserve requirements under oligopolistic conditions. Firstly, we specified the inverse demand function for loans and the inverse supply function of deposits. Then, in a context of a two-stage Cournot game with scope economies, we solved the maximization problem of each individual bank. Applying comparative statics analysis, we showed the effects of monetary policy on the optimal levels of deposits and loans and on the corresponding interest rates. We concentrated on the change that is induced on the spread between the equilibrium rates on loans and deposits by a change in the fraction of reserve requirements. It is demonstrated that this effect is negative in the case of large economies of scope and positive if there are small economies of scope or economies of scope do not exist. However, in the case of diseconomies of scope, further investigation is needed.

Finally, we attempted to extent the model to the sphere of consumer behavior, i.e. depositors and borrowers. In a context of an overlapping generation model, we demonstrated that the minimum reserve requirements influence the optimal levels of bank-clients' consumption through the corresponding equilibrium interest rates. To achieve this, we started from the solution of the utility maximization problem in order to deduce the optimal level of consumption of each period of their lifetime horizon. After calculating the effect of reserve requirements on these levels, we applied comparative statics to determine the substitution effect and the income effect. We concluded that the direction of the aforementioned impact depends on the type of economies of scope.

Appendix

1. Proof of relation (24)

From (20): $\left|BR_i{}'(D_j)\right| = \left|BR_j{}'(D_i)\right| < 1,$ for $i,j = A,B \; and \; i \neq j \Rightarrow$

$$|BR_A{}'(D_B)| = |BR_B{}'(D_A)| < 1$$

In the case of Bank A:

$$|BR_A{}'(D_B)| < 1 \Rightarrow \left|\frac{dD_A}{dD_B}\right| < 1 \Rightarrow \left| -\frac{\frac{\partial^2 \Pi_A}{\partial D_A D_B}}{\frac{\partial^2 \Pi_A}{\partial {D_A}^2}} \right| < 1 \Rightarrow$$

$$\Rightarrow \left| -\frac{-\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]}{\frac{8}{9b}(\theta'(D_A))^2 - 2\gamma} \right| < 1 \Rightarrow \frac{-\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]}{\frac{8}{9b}(\theta'(D_A))^2 - 2\gamma} < 1 \Rightarrow$$

$$\Rightarrow -\frac{4}{9b}\theta'(D_A)\theta'(D_B) - \gamma > \frac{8}{9b}(\theta'(D_A))^2 - 2\gamma$$

(Because second order condition implies $\frac{8}{9b}(\theta'(D_A))^2 - 2\gamma < 0$)

$$\Rightarrow \frac{8}{9b}(\theta'(D_A))^2 - 2\gamma < -\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right] \qquad \textbf{(A.1)}$$

In the case of Bank B:

$$|BR_B{}'(D_A)| < 1 \Rightarrow \left|\frac{dD_B}{dD_A}\right| < 1 \Rightarrow \left| -\frac{\frac{\partial^2 \Pi_B}{\partial D_B D_A}}{\frac{\partial^2 \Pi_B}{\partial {D_B}^2}} \right| < 1 \Rightarrow$$

$$\Rightarrow \left| -\frac{-\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]}{\frac{8}{9b}(\theta'(D_B))^2 - 2\gamma} \right| < 1 \Rightarrow \frac{-\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]}{\frac{8}{9b}(\theta'(D_B))^2 - 2\gamma} < 1 \Rightarrow$$

$$\Rightarrow -\frac{4}{9b}\theta'(D_A)\theta'(D_B) - \gamma > \frac{8}{9b}(\theta'(D_B))^2 - 2\gamma$$

(Because second order condition implies $\frac{8}{9b}(\theta'(D_B))^2 - 2\gamma < 0$)

$$\Rightarrow \frac{8}{9b}(\theta'(D_B))^2 - 2\gamma < -\left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right] \quad \textbf{(A.2)}$$

We multiply the equations (A.1) & (A.2) by members

$$\left[\frac{8}{9b}(\theta'(D_A))^2 - 2\gamma\right]\left[\frac{8}{9b}(\theta'(D_B))^2 - 2\gamma\right] > \left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]^2 \Rightarrow$$

$$\left[\frac{8}{9b}(\theta'(D_A))^2 - 2\gamma\right]\left[\frac{8}{9b}(\theta'(D_B))^2 - 2\gamma\right] - \left[\frac{4}{9b}\theta'(D_A)\theta'(D_B) + \gamma\right]^2 > 0 \Rightarrow$$

$$|\Delta| > 0$$

Q.E.D.

2. Proof of Relations (40), (41) & (42)

The Lagrangian function is given by:

$$Q(c_{1t}, c_{2t+1}, q) = c_{1t}^{z} c_{2t+1}^{1-z} + q[(1+w)y_1 + y_2 - (1+w)c_{1t} - c_{2t+1}]$$

where q>0 denotes the Largange multiplier.

The first order conditions are described by the following relations:

$$\frac{\partial Q(.)}{\partial c_{1t}} = 0 \Rightarrow z c_{1t}^{z-1} c_{2t+1}^{1-z} - q(1+w) = 0 \quad \textbf{(A.3)}$$

$$\frac{\partial Q\ (.)}{\partial c_{2t+1}} = 0 \Rightarrow (1-z) c_{1t}^{z} c_{2t+1}^{-z} - q = 0 \quad \textbf{(A.4)}$$

$$\frac{\partial Q(.)}{\partial q} = 0 \Rightarrow (1+w)y_1 + y_2 - (1+w)c_{1t} - c_{2t+1} = 0 \quad \textbf{(A.5)}$$

The total differential of the first order conditions:

$$(z-1) z c_{1t}^{z-2} c_{2t+1}^{1-z}\, dc_{1t}^{*} + (1-z) z c_{1t}^{z-1} c_{2t+1}^{-z} dc_{2t+1}^{*} - (1+w)\, dq^{*} - q\, dw = 0$$

$$z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z}\,dc_{1t}^{*}+(-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1}dc_{2t+1}^{*}-dq^{*}-0dw=0$$

$$-(1+w)dc_{1t}^{*}-dc_{2t+1}^{*}+0dq^{*}+(y_1-c_{1t})dw+(1+w)\,dy_1+dy_2=0$$

We separate endogenous from exogenous variables:

$$(z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z}\,dc_{1t}^{*}+(1-z)zc_{1t}^{z-1}c_{2t+1}^{-z}dc_{2t+1}^{*}-(1+w)\,dq^{*}=q\,dw$$

$$z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z}\,dc_{1t}^{*}+(-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1}dc_{2t+1}^{*}-dq^{*}=0dw$$

$$-(1+w)dc_{1t}^{*}-dc_{2t+1}^{*}+0dq^{*}=-(y_1-c_{1t})dw-(1+w)\,dy_1-dy_2$$

The above system of equations can be written in matrix form as follows:

$$\begin{bmatrix}(z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & -(1+w)\\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & -1\\ -(1+w) & -1 & 0\end{bmatrix}\begin{bmatrix}dc_{1t}^{*}\\ dc_{2t+1}^{*}\\ dq^{*}\end{bmatrix}=\begin{bmatrix}q\,dw\\ 0dw\\ -(y_1-c_{1t})dw-(1+w)\,dy_1-dy_2\end{bmatrix}$$

where:

$$\mathbf{H}=\begin{bmatrix}(z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & -(1+w)\\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & -1\\ -(1+w) & -1 & 0\end{bmatrix}$$ the Hessian matrix

Let $dy_1=0, dy_2=0$ and $dw\neq 0$. Applying the Crammer's Rule, we obtain:

$$\frac{\partial c_{1t}^{*}}{\partial w}=\frac{\begin{vmatrix}q & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & -(1+w)\\ 0 & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & -1\\ (c_{1t}-y_1) & -1 & 0\end{vmatrix}}{\begin{vmatrix}(z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & -(1+w)\\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & -1\\ -(1+w) & -1 & 0\end{vmatrix}}=$$

$$=\ q(-1)^2\frac{|\boldsymbol{H_{11}}|}{|\boldsymbol{H}|}+(c_{1t}-y_1)(-1)^4\frac{|\boldsymbol{H_{31}}|}{|\boldsymbol{H}|}\Rightarrow$$

$$\frac{\partial c_{1t}^*}{\partial w} = (c_{1t} - y_1)\frac{|\boldsymbol{H_{31}}|}{|\boldsymbol{H}|} + q\frac{|\boldsymbol{H_{11}}|}{|\boldsymbol{H}|} \qquad \textbf{(40) Q.E.D.}$$

$$\frac{\partial c_{2t+1}^*}{\partial w} = \frac{\begin{vmatrix} (z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & q & -(1+w) \\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & 0 & -1 \\ -(1+w) & (c_{1t}-y_1) & 0 \end{vmatrix}}{\begin{vmatrix} (z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & -(1+w) \\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & -1 \\ -(1+w) & -1 & 0 \end{vmatrix}} =$$

$$= q\,(-1)^3\frac{|\boldsymbol{H_{12}}|}{|\boldsymbol{H}|} + (c_{1t} - y_1)(-1)^5\frac{|\boldsymbol{H_{32}}|}{|\boldsymbol{H}|} \Rightarrow$$

$$\frac{\partial c_{2t+1}^*}{\partial w} = -(c_{1t} - y_1)\frac{|\boldsymbol{H_{32}}|}{|\boldsymbol{H}|} - q\frac{|\boldsymbol{H_{12}}|}{|\boldsymbol{H}|} \qquad \textbf{(41) Q.E.D.}$$

$$\frac{\partial q^*}{\partial w} = \frac{\begin{vmatrix} (z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & q \\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & 0 \\ -(1+w) & -1 & (c_{1t}-y_1) \end{vmatrix}}{\begin{vmatrix} (z-1)zc_{1t}^{z-2}c_{2t+1}^{1-z} & (1-z)zc_{1t}^{z-1}c_{2t+1}^{-z} & -(1+w) \\ z(1-z)c_{1t}^{z-1}c_{2t+1}^{-z} & (-z)(1-z)c_{1t}^{z}c_{2t+1}^{-z-1} & -1 \\ -(1+w) & -1 & 0 \end{vmatrix}} =$$

$$= q\,(-1)^4\frac{|\boldsymbol{H_{13}}|}{|\boldsymbol{H}|} + (c_{1t} - y_1)(-1)^6\frac{|\boldsymbol{H_{33}}|}{|\boldsymbol{H}|} \Rightarrow$$

$$\frac{\partial q^*}{\partial w} = (c_{1t} - y_1)\frac{|\boldsymbol{H_{33}}|}{|\boldsymbol{H}|} + q\frac{|\boldsymbol{H_{13}}|}{|\boldsymbol{H}|} \qquad \textbf{(42) Q.E.D.}$$

3. Determination of the substitution effect

The substitution effect corresponds to a movement on the same indifference curve. When we are moving along the same indifference curve, it holds:

$$dU = 0 \Rightarrow zc_{1t}^{z-1}c_{2t+1}^{1-z}dc_{1t} + (1-z)c_{1t}^{z}c_{2t+1}^{-z}dc_{2t+1} = 0 \qquad \textbf{(A.6)}$$

From (A.3): $zc_{1t}^{z-1}c_{2t+1}^{1-z} = q(1+w)$

From (A.4): $(1-z)c_{1t}^{z}c_{2t+1}^{-z} = q$

Substituting (A.3) & (A.4) in (A.6):

$$q(1+w)dc_{1t} + qdc_{2t+1} = 0 \Rightarrow$$

$$q\left[(1+w)dc_{1t} + dc_{2t+1}\right] = 0 \overset{q>0}{\Longrightarrow}$$

$$(1+w)dc_{1t} + dc_{2t+1} = 0 \tag{A.7}$$

Calculating the total differential of budget constraint:

$$(1+w)dc_{1t} + dc_{2t+1} + c_{1t}\,dw = (1+w)\,dy_1 + dy_2 + y_1 dw \Rightarrow$$

$$(1+w)dc_{1t} + dc_{2t+1} = (1+w)\,dy_1 + dy_2 - (c_{1t} - y_1)\,dw \tag{A.8}$$

From (A.7) and (A.8) and due to the fact that $dy_1 = dy_2 = 0$, we obtain:

$$(c_{1t} - y_1)\,dw = 0 \Rightarrow c_{1t} - y_1 = 0 \tag{A.9}$$

Consequently, relations (40), (41) & (A.9) imply that:

<u>Substitution Effect</u>:

$$\left.\frac{\partial c_{1t}^{*}}{\partial w}\right|_{U=\bar{U}} = q\frac{|\boldsymbol{H}_{11}|}{|\boldsymbol{H}|} \quad \text{and} \quad \left.\frac{\partial c_{2t+1}^{*}}{\partial w}\right|_{U=\bar{U}} = -q\frac{|\boldsymbol{H}_{12}|}{|\boldsymbol{H}|}$$

<u>Income Effect</u>:

first period: $(c_{1t} - y_1)\dfrac{|\boldsymbol{H}_{31}|}{|\boldsymbol{H}|}dw$ and second period: $-(c_{1t} - y_1)\dfrac{|\boldsymbol{H}_{32}|}{|\boldsymbol{H}|}$

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