On the Modeling of Exchange Rate: Some Evidence from Pakistan

Rashid, Abdul and Husain, Fazal

International Institute of Islamic Economics, Pakistan, PIDE, Pakistan

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Abdul Rashid
International Institute of Islamic Economics (IIIE),
International Islamic University (IIU), Islamabad, Pakistan
Email: ch.arahmad@yahoo.com

and

Fazal Husain
Pakistan Institute of Development Economics (PIDE),
Islamabad, Pakistan
Phone No: +9251 9248060
Email: fazal.husain1960@gmail.com

Biographical notes:

Abdul Rashid holds an MPhil (Financial Economics) degree from Institute of Business Management, Pakistan. Having completed his study, he has been involved in research and teaching at International Islamic University Islamabad, Pakistan (IIU). He has published more than 25 articles in both national and international journals. He also acts as paper reviewer in international journals. His research interests lie in the areas of exchange rate arrangements and firms’ capital structure. Currently, he is a PhD student at the University of Sheffield, UK.

Fazal Husain is a chief of research at Pakistan Institute of Development Economics, Pakistan. He authored over 30 academic publications. His research is mainly on capital markets and monetary economics topics. He completed his PhD in economic at The Catholic University of America (America).
On the Modeling of Exchange Rate: Some Evidence from Pakistan

Abstract

This paper tests the interconnected form of PPP and UIP while allowing the random component of exchange rate in the specification. We find a significant long-run association among exchange rates, price and interest rate differentials. Besides the PPP and UIP conditions, the previous period exchange rate plays an important role in explaining exchange rate variability. The coefficient of error correction term reveals substantial convergence towards long-run equilibrium. These findings are interesting because they explicate the dilemma of PPP and UIP and illustrate the significance of the joint modelling of these parity conditions in explaining the convergence towards equilibrium exchange rates.

JEL classification: C51, F31
Keywords: purchasing power parity, uncovered interest rate parity, random walks, exchange rate regimes
1 - Introduction

Since the collapse of the Bretton Woods agreement, researchers have devoted much exertion to formalize the association between exchange rates and economic fundamentals, such as interest rates and inflation. Particularly, they have emphasized on whether the equilibrium exchange is determined in context of the PPP or UIP condition. While most of the prior empirical studies have been failed to provide strong evidence in favor of the validity of PPP or/and UIP conditions when they examine them in isolation\(^1\), there has been an increasing trend for the joint modeling of the PPP and UIP conditions. Since goods and capital markets are interrelated with each other, both the parities, indeed, jointly restore the equilibrium whenever exchange rate deviates from the PPP or UIP condition due to other factors such as productivity differential, the time-varying risk premium and trade barriers.

Looking at the recent strand of literature on exchange rates, we find that following the seminal work by Johansen and Juselius (1992), a voluminous number of studies including Juselius (1995), Camarero and Tamarit (1996), Caporale, Kalyvitis and Pittis (2001), Miyakoshi (2004) and Rashid (2009) have documented that both the parities (PPP and UIP) simultaneously play an important role in the determination of exchange rates. These studies generally attribute the rejection of PPP and UIP individually to the disregard for the conceivable associations between the current and capital accounts. Thus, the omission of price levels or interest rates from cointegration mechanism for UIP and PPP, respectively, is one of the possible reasons why so many previous studies have failed to find a co-movement among exchange rates, price differentials and interest rate gaps in the long run.

Theoretical rationales also endorse for the joint modelling of the PPP and UIP conditions. It is well-established in the literature that the development in both goods and capital markets affects exchange rate arrangements and therefore the two parity conditions may not be independent of each other at least in the long run. In addition, since the capital account has been used to finance any disparity in current account, shocks in one market have considerable effects on the other. The joint modelling of the PPP and UIP conditions not only outperforms the models where both conditions are in insulation but also enables one to compare the role of PPP and UIP in exchange rate convergence\(^2\).

Since there has been an increasing trend of financial reforms and trade liberalizations in developing and emerging economies over the past few decades, it is of particular interest to examine whether the liberalization of interest rate and removal of trade barriers affect exchange rate movements. Moreover, the validity of exchange rate models such as monetary approach to exchange rate and monetary approach to exchange rate pressure crucially depends on the existence of the PPP condition. The movements of real exchange rate can be used as a gauge to examine the competitiveness of a country in world trade. Exchange rate also plays an important role in maintaining parity in balance of payments.

\(^1\)See, for instance, Meese and Rogoff (1988), Rogoff (1996), Mark and Wu (1996) and Bahmani-Oskooee and Goswami (2005), among many others.
\(^2\)See, for further details, Pesaran, Shin and Smith (2000) and Rashid (2009).
These implications rouse the interest of policymakers and researchers to examine the behavior of exchange rates.

Since early 1990s, both financial reforms and trade liberalization are at great concern of economic policy in Pakistan. During the last two decades, a number of positive development including the decline in interest rates, the removal of economic sanctions, and the trade concessions changed the environment altogether and led to liberalize and globalize the economy. Regarding exchange rate regimes, the significant measures have been taken to adopt the floating exchange rate system. Pakistan pursued a fixed exchange rate policy until January 1982 when it shifted to a managed floating rate. In order to minimize the adverse effects of economic sanctions, Pakistan moved to a dual exchange rate system in July 22, 1998.\(^3\)

The dual exchange rate system was replaced with managed floating unitary exchange rate system in May 19, 1999. In July 21, 2000, however, the unified exchange rate system was also replaced with free-floating exchange rate regime. Overall, the relaxations in foreign exchange restrictions, trade liberalization and changes in exchange rate regimes have increased the importance of exchange rate dynamics in Pakistan. Thus, it is worthwhile to examine whether exchange rates are determined in the context of PPP or UIP and to what extent the liberalization of interest rate affects the exchange rate movements.

Differing from the existing studies that combine PPP and UIP, this study formalizes the interaction between the two-arbitrage conditions (PPP and UIP) in conjunction with the random walk component of exchange rate into a single framework. The rationale for considering the random walk component is that since the studies by Roll (1979), Adler and Lehman (1983), Froot and Rogoff (1995), Froot, Kim and Roggof (1995), Sánchez-Fung and Prazmowski (2004) and Rashid (2006) have documented that exchange rates follow random walk (RW), the mean-reverting behavior of exchange rate may one of the reasons behind the failure of PPP and UIP. The joint modeling of all three parities enable us to examine to what extent PPP and UIP converge.

This study differs from the work by Rashid (2009), who test the combined PPP-UIP for Pakistan in two ways. First, we utilize monthly data over the period ranging from January 1991 to December 2009 and, instead of splitting the sample, use dummy for managed and pure floating exchange rate regimes. Whereas, Rashid has covered the sample period from only 1999 to 2006, which may indeed be considered too short to test the PPP and UIP hypotheses as these parities seem relatively a long-run phenomenon. Second, the present study utilizes a relatively more compatible approach viz. autoregressive distributed lag model (ARDL) to recover the long-run estimates and the bounds testing procedure for cointegration while Rashid applied the standard cointegration method based on full information maximum likelihood technique.

\(^3\)Under this exchange rate regime there existed two exchange rates, namely the inter-bank floating rate and the composite rate. Market forces of demand and supply determined the inter-bank floating rate and the State Bank of Pakistan determines the official exchange rate. The composite rate was the weighted average of official exchange rate and the inter-bank floating rate.
The empirical results of the study are mainly consistent with the prior studies that modeled the PPP and UIP conditions jointly and report a stationary long-run relationship among exchange rates, price differential and interest rate differential. The derived long-run estimates based on ARDL are statistically significant and have signs which are in line with the underlying theories. The ARDL estimation results indicate that besides the PPP and UIP conditions, the previous period observed value of exchange rate is highly significant in explaining the short-run dynamics of exchange rates. The estimated coefficient of the error correction term indicates that the exchange rate drifts, with a speed of adjustment 33.8%, towards restoring the PPP and UIP conditions whenever disequilibrium occurs in response to the temporary shocks of the previous period.

The sketch of the remainder of the paper is as follows. Section 2 briefly reviews the PPP and UIP conditions and presents the framework to combine these theories with the random component of exchange rate. Section 3 describes the econometric methodology for estimating the specified empirical model and data sources. Section 4 covers the estimation. Finally, Section 5 concludes the study.

2 - Theoretical Background

2.1 Money Demand Function

As in Obstfeld, Rogoff and Wren-Lewis (1996), the real money demand function using the nominal interest rate instead of expected inflation and assuming that output is exogenous can be presented as follows:

\[ m_t - p_t = \lambda y_t - \xi i_t, \quad t = 1, \ldots, T \]  

(1)

where \( m_t = \log \text{of nominal demand for domestic currency at time } t \), defined as the number of domestic currency units required to purchase goods and services.

\( p_t = \log \text{of domestic price level at time } t \)

\( i_t = \text{domestic interest rate at time } t \)

\( y_t = \log \text{of domestic output at time } t \)

Although a precise log-version of the above money demand function would imply \( -\xi \left(1 + i_t^d\right) \) … on the right-hand side of Equation (1). However, the study ticks the time-notation which implies \( i_t \), rather than \( i_t + 1 \).

2.2 The PPP Condition

The origin of the PPP hypothesis lies from the “Law of One Price (LOP)”, which states that, in the absence of transaction and transportation costs, freely internationally traded identical commodities should have the same price everywhere. In practice, many factors such as trade barriers, relative importance of the tradable and non-tradable products,
technological gaps and growth differentials may drive exchange rate way from PPP. Thus, the relative form of PPP is more suitable for empirical analysis as it allows a permanent wedge caused by these factors. Specifically, PPP in its relative form can be expressed in the following way

\[ e_t = a + b(p_t - p^*_t) + \varepsilon_t \quad t = 1, \ldots, T \]  

(2)

where \( e_t \) = log of nominal exchange rate for domestic country at time \( t \), defined as the number of domestic currency units required to purchase one unit of foreign currency.

- \( p_t \) = log of domestic price level at time \( t \)
- \( p^*_t \) = log of foreign price level at time \( t \)
- \( \varepsilon_t \) = trade shock with zero mean and finite variance

\( a \) is a constant, representing the permanent variant from absolute PPP due to factors such as growth and technological differentials and trade barriers. \( T \) refers to the number of observations over time. The strong or absolute form of PPP can be obtained by imposing the restriction that \( a = 0 \) and \( b = 1 \) (symmetry and proportionality conditions). The cointegration of variables in the system defining the parities with unitary coefficients (or the stationarity of real exchange rates \( e_t + p^*_t - p_t - a = q \approx I(0) \)) can be interpreted evidence supporting the PPP hypothesis.

The PPP condition has been extensively tested in exchange rate literature; however, the empirical evidence is mixed at best. Surveys by MacDonal (1995), Froot and Rogoff (1995), Breuer (1994), and Schotman (1989) provide a comprehensive literature review of the evidence for long-run PPP. There are ample alternative explanations of the conflicting results for the validity of the PPP hypothesis. The most common explanations for the failure of PPP are imperfect market structures, the choice of price indices, information and transport costs, trade barriers and relative growth differentials. Besides this, as suggested by many observers, non-linear dynamics, the low power of the conventional unit-root tests particularly over short-time spans of data and temporal aggregation are among the empirical modelling issues that are responsible for the mixed results.

2.2 The UIP Condition

The UIP hypothesis states that interest rate differential between domestic and foreign country is equal to the expected change in the nominal spot exchange rate. Particularly, on can define the UIP as follows

\[ \text{According to UIP, if interest rate in domestic country is higher than similar interest rate in foreign country, then foreign investors have more incentive to purchase domestic country’s assets, driving the domestic country’s spot rate down (the domestic currency appreciates).} \]

\[ \text{For convincing evidence on this issue, see Brook and Hargreaves (2001).} \]

\[ \text{See, for instance, Taylor and Sarno (1998), Lathian and Taylor (1996) and Sarno and Taylor (2001).} \]

\[ \text{According to UIP, if interest rate in domestic country is higher than similar interest rate in foreign country, then foreign investors have more incentive to purchase domestic country’s assets, driving the domestic country’s spot rate down (the domestic currency appreciates).} \]
$E_t(e_{t+1}) - e_t = \gamma + \eta (i_t - i^*_t) + u_t, \quad t = 1, \ldots, T$ (3)

where

- $i_t = \log$ of domestic interest rate at time $t$
- $i^*_t = \log$ of an equivalent foreign interest rate at time $t$
- $\gamma$ = constant, which captures the fixed effects
- $E_t(\cdot) = \text{the expectations operator conditional upon information available at time } t$

$u$ is the risk premium associated with holding domestic currency assets (see, for details, Svensson (1992)). $\eta$ is the interest rate differential’s coefficient and the estimated value of it is expected to be negative. The null hypothesis of UIP can be expressed as $H_o: \gamma = 0, \eta = 1$.

A large number of studies have been done to examine the validity of the UIP condition. The results of these studies are, however, inconclusive. The findings of Flood and Rose (2002), Chinn and Meredith (2000), MacDonal and Nagayasu (2000), and Chinn and Meredith (2004) provided evidence to support the Uncovered Interest Rate Theory. Whereas, some empirical studies have rejected UIP (see, for instance, Meese and Rogoff (1988), Edison and Pauls (1993), and Mark and Wu (1996), among others).

### 2.3 Random Walk (RW)

In context of random walk with a drift, the exchange rate expectations can be expressed as follows:

$$E_t(e_{t+1}) = \alpha + j e_t + \varsigma_t,$$  (4)

where $e_t$ is as defined above and $\varsigma_t$ is a white noise residual term. It would be expected that $j$ would be equal to unity. It implies that tomorrow’s exchange rate (future exchange rate) is equal to today’s exchange rate (spot exchange rate) plus a random shock.

### 2.3 Formalizing the Interactions of PPP, UIP and RW

In this subsection, we formalize the interactions among PPP, UIP and RW in a single equation to capture the role of interest rate and price differentials in exchange rate dynamics while allowing the portion $n$ of the exchange rate behaving randomly. It is assumed that PPP forms the basis of expectations in the UIP condition. Algebraically, this relationship is obtained by bringing all the three conditions together. Specifically, inserting $p_t$ and $i_t$ into equation (1) according to equation (2) and equation (3), yielding

$$m_t - (1/b)(e_t + bp_t^* - a - e_t) = \lambda y_t - (\xi / \eta)(i_t^* + E_t(e_{t+1}) - e_t - \gamma - u_t)$$  (5)
Plugging $m_i$ into equation (5) and rearranging

$$E_i(e_{i+1}) = \omega_0 + \omega_1(p_t - p_t^*) + \omega_2(i_t - i_t^*) + \omega_3e_t + \zeta_i$$

(6)

where $\omega_1$, $\omega_2$ and $\omega_3$ are weights on the PPP, UIP and RW, respectively, which depend on the underlying structural parameters. $\zeta_i$ is a residual term with zero mean and constant variance. The random walk term of equation (6) differentiates this paper from the prior studies that only combine the PPP and UIP conditions. Finally, by substituting (6) into (3), yielding

$$(1 - \omega_3)e_t + (1 - \omega_2)(i_t - i_t^*) - \omega_1(p_t - p_t^*) + \omega_o = \Omega_t$$

(7)

The parities will have an empirical meaning if and only if interest rate differentials $(i_t - i_t^*)$, price differentials $(p_t - p_t^*)$ and nominal exchange rate $(e_t)$ follow the same order of integration, and $\Omega_t \approx I(0)$, implying that deviation from the underlying equilibrium correction mechanism (ECM) are stationary.

3 - Methodology and Data

The study aims to investigate that to what extent the variations in exchange rate are attributed to PPP or/and UIP and what is the intensity of the persistence of the exchange rates dynamics. ARDL approach is used for testing cointegration between exchange rates, price differentials and interest rate differential and to recover the long-run estimates for exchange rate model. We prefer ARDL over the conventional cointegration tests to attain the following advantages. Unlike the residual based test such as Engle-Granger (1987) and the maximum likelihood based test such as Johansen (1991 and 1995) for testing the long-run association, the ARDL approach does not require that the series included in system should have same order of integration. Another advantage of this approach is that the model takes sufficient number of lags to reduce the intensity of serial correlation of residuals in a general to specific modelling framework. Furthermore, a dynamic error correction model (ECM) can be derived from ARDL through simple linear transformation. The ECM emerges the short-run dynamics with the long-run stable equilibrium without losing long-run information.

The ARDL regression yields a test statistic which can be compared to two asymptotic critical values (upper and lower critical values). If the test statistic is above an upper critical value at the given level of significance, the null hypothesis of no long-run relationship is rejected regardless whether the orders of integration of the variables are one or zero. Alternatively, if the calculated test statistic is below the lower critical value at given level of significant, the null hypothesis of no long-run relationship is accepted. However, if the test statistic falls between upper and lower bounds, the result is inconclusive. Another advantage of this approach is that an appropriate specification of the ARDL equation helps to fix the problems of endogenous variables and residual serial
correlation. Finally, it performs better than Engle-Granger (1987) and Johansen (1990 and 1995) cointegration tests even in case of small samples.

Specifically, the model with \( k \) exogenous \( I(1) \) variables, \( x_j, j = 1, \ldots, k \), in error correction framework is expressed as:

\[
\Delta y_t = a_0 + a_1 t + b y_{t-1} + \sum_{i=1}^{m} a_i \Delta y_{t-i} + \sum_{j=1}^{k} \sum_{i=0}^{n} \beta_{ji} \Delta x_{j,i-t} + \sum_{j=1}^{k} \beta_{j} x_{j-1} + \delta D_t + u_t
\]  

(8)

where \( y \) is the endogenous \( I(1) \) variable, \( a_0 \) is the constant term, \( t \) is a linear time trend, \( D_t \) is a dummy has value one for pure float exchange rate regime and zero otherwise and \( u_t \) is the error term. The model (8) is just a reparameterization of a general autoregressive distributed lag model in the levels of \( y \) and \( x \) with \( m \) lags on \( y \) and \( n \) lags on \( x_j, j = 1, \ldots, k \).

“Long-run equilibrium” in the model (8) is usually defined as a state where \( \Delta x_j = u_t = 0 \) for all \( j \) and \( t \), and \( \Delta y_t = 0 \). The long-run equilibrium relation is thus given by

\[
y_t = \theta_0 + \sum_{j=1}^{k} \theta_j x_{j,t} \quad \text{where} \quad \theta_0 = -b_0 / a \quad \text{and} \quad \theta_j = -\beta_{j} / a, j = 1, \ldots, k.
\]  

(9)

Many empirical studies such as Delong, Nankervis and Whiteman (1989) and Diebold and Rudebusch (1991) have reported that the classical unit-root tests, e.g., ADF and PP tests are not very powerful against relevant alternatives and are biased to the null of non-stationary. To avoid this problem, the present study uses the KPSS test proposed by Kwiatowski et al. (1992) and the PP test developed by Phillips and Perron (1988) to identify the order of integration of the variables. Under the KPSS method, the null hypothesis is stationary instead of non-stationary. Unlike the ADF test, this test ensures that the alternative will be accepted (null rejected) only when there is strong evidence for (against) it. The KPSS test statistic is defined as follows:

\[
\hat{\eta} = T^{-2} \sum \frac{S_i^2}{s^2(I)}
\]  

(10)

where \( S_i \) is the partial sum of the residuals \( \xi_i \) obtained from a regression of the respective variable on only an intercept in case of level stationary, and on an intercept and a linear time trend in case of trend stationary; that is defined as

\[
S_t = \sum_{j=1}^{t} \xi_j \quad \text{and} \quad s^2(I) = T^{-1} \sum_{j=1}^{T} \xi_j^2 + 2T^{-1} \sum_{m=1}^{l} w(m, l) \sum_{j=m+1}^{T} \xi_j \xi_{j-m}
\]  

(11)

\footnote{Critical values of the LM test statistic are based upon the asymptotic results presented in KPSS (1992, Table 1, p. 166).}

\footnote{For details on this, see Laurenceson and Chai (2003).}
where \( w(m,l) \) is an optional weighting function; this is, \( w(m,l) = 1 - m/(1 + l) \), where \( l \) is the maximum lag-length.

Using monthly data for nominal exchange rates, consumer price indices and market interest rates the proposed model in this paper is estimated for Pakistan. The study covers the period from January 1991 to December 2009. All the variables are obtained from the International Financial Statistics (IFS) databases prepared by International Monetary Fund (IMF). The nominal exchange rate is defined as the nominal bilateral exchange rates against the United States. The regime dummy is used, which has value zero for the managed floating exchange rate period spanning from January 1991 to June 1999 and one for the pure floating period ranging from July 1999 to December 2009 in our case. In empirical analysis of this paper, all the variables are transformed with natural logarithm with exception of interest rates that are already in percentage form.

4 - Empirical Investigation

We begin our empirical analysis by illustrating the time series plots of the dynamics of exchange rates, price and interest rate differentials for the managed floating and pure floating exchange rate regimes in Figures 1 and 2, respectively. According to the figures, there is a significant difference in the pattern of fluctuations across both the exchange rate regimes. Both the nominal exchange rate and interest rate differential seem less volatile during the free floating exchange rate regime as compared to the managed one. In addition, as we can see from Figure 1, there are significant spikes in exchange rates during the managed exchange rate regimes, whereas, such dramatic dynamics cannot be observed over the free-floating regime.

This observation is in line with the findings of Calvo and Reinhard (2000), who find the low variability of exchange rate for the countries that allow their exchange rate to float freely. They further argue that even the countries that claim for fully free-floating exchange rate are actively involved in stabilizing exchange rate at the cost of interest rate or/and price fluctuations. Regarding price levels, the fluctuations of price gaps between the home and foreign country are roughly same across both the exchange rate regimes.

Finally, the illustration of the figures shows that the fluctuations of exchange rates, interest rate differential and price gaps are interconnected with one another. The variations in exchange rate seem more associated with ups and downs of price levels, particularly, over the free-floating exchange rate regime. Thus, it is useful looking into whether the relationship among exchange rates, price and interest rate differentials are same across both the exchange rate regimes. Instead of estimating the model of exchange rate on sample of managed- and free-floating period we use the regime dummy to capture the impact of regime switching.
The results from unit-root tests for levels and first differences are given in Table 1. All the PKSS and PP tests regressions are estimated with a constant term and both a constant and a linear trend term for each variable. The appropriate lag lengths are selected based on Modified Akaike Information Criterion (MAIC). The results indicate that all the variables are non-stationary at their levels. The findings of both the tests are consistent with each other for all the variables except interest rate differential which is mean-
reverting according to the KPSS test\(^9\). However, the de-trended series of the interest rate differential appears non-stationary as the estimated KPSS statistic is greater than the critical values at the 5\% levels indicating the rejection of the null of stationary. Since all the series are stationary at their first differences, each of the variable is integrated of order one\(^{10}\).

<table>
<thead>
<tr>
<th>Variables</th>
<th>KPSS Test</th>
<th>PP Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Constant and trend</td>
</tr>
<tr>
<td>Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>1.961* (11)</td>
<td>0.386* (11)</td>
</tr>
<tr>
<td>Price Differential</td>
<td>1.835* (6)</td>
<td>0.251* (6)</td>
</tr>
<tr>
<td>Interest Rate Differential</td>
<td>0.269 (10)</td>
<td>0.248* (10)</td>
</tr>
<tr>
<td>First Difference</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal Exchange Rate</td>
<td>0.196 (6)</td>
<td>0.114 (6)</td>
</tr>
<tr>
<td>Price Differential</td>
<td>0.315 (9)</td>
<td>0.314 (9)</td>
</tr>
<tr>
<td>Interest Rate Differential</td>
<td>0.050 (8)</td>
<td>0.043 (8)</td>
</tr>
</tbody>
</table>

* and ** denote the significant (rejection of null hypothesis) at the 5\% and 10\% levels, respectively. Numbers in the parentheses are optimal lag lengths selected by Modified Akaike Information Criterion (MAIC). The null of the KPSS test is to test for stationary, whereas the null of the PP test is to test for nonstationary.

To derive the long-run estimates for exchange rate model, the ARDL procedure developed by Pesaran, Shin and Pesaran (2001) is utilized. There are two steps in carrying out this technique. First, the bounds test (F-statistic) is used for testing the existence of a long-run relationship among exchange rates, price differential and interest rate gaps. In particular, the F-statistics are calculated by estimating the conditional model of exchange rate as described in Equation (8) using the OLS with and without a linear time trend for 4, 6 and 8 lag lengths\(^{11}\). As mentioned prior all the models are estimated over the entire sample period spanning from January 1991 to December 2009 using a dummy variable for managed- and free-floating exchange rate regimes.

Table 2 presents the estimates of the F-statistics (denoted \(F_C, F_{CT},\) and \(F_N\)) for testing a long-run association among the variables under three different cases subject to whether the exchange rate model is estimated with a linear trend or without a trend term and

\(^9\)Indeed, the interest rate differential is trend stationary.

\(^{10}\)As the interest rate is mean-reverting at its levels, I use the bound tests for testing the long-run associations between exchange rates, price and interest rate differentials which, unlike the standard cointegration tests, does not require the assumption of the same order of integration of the variables in the system.

\(^{11}\)Although the Modified Akaike Information Criterion suggests the appropriate lag length 6 for model with and without a liner time trend, I use the two other lag lengths (4 and 8) to confirm the robustness of the existence of the long-run relationship.
whether the trend coefficients are restricted or not. The upper and lower bounds critical values for $F_C$, $F_{CT}$, and $F_N$ at the 5% levels are (4.87, 5.58), (5.17, 6.15) and (3.79, 4.85), respectively.

### Table 2
F-statistics for Testing the Existence of a Long-Run Association for the Exchange Rate Model

<table>
<thead>
<tr>
<th>No. of Lags</th>
<th>Model with Deterministic Trend</th>
<th>Model without Deterministic Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_C$</td>
<td>$F_{CT}$</td>
</tr>
<tr>
<td>4</td>
<td>3.747*</td>
<td>4.455*</td>
</tr>
<tr>
<td>6</td>
<td>6.239***</td>
<td>5.897**</td>
</tr>
<tr>
<td>8</td>
<td>5.125**</td>
<td>5.562**</td>
</tr>
</tbody>
</table>

*, **, *** denote the acceptance, the inconclusiveness and the rejection of the null of no long-run association at the 95% levels, respectively.

The values of F-statistics for lag length 4, namely $F_C$, and $F_{CT}$, are significantly less than the lower critical bounds indicating the acceptance of the null hypothesis of no long-run relationship. Similarly, for lag length 8, the estimates, irrespective of whether the zero restrictions are imposed only on the coefficients of the lagged level variables or both on the trend term and the coefficients, are inconclusive. However, for lag length 6 selected by the modified AIC, the F-statistics ($F_C = 6.239$) significantly lies outside the upper critical value bounds indicating the rejection of the null hypothesis of no long-run association.

When the exchange rate model is estimated without the deterministic trend, the values of F-statistics (namely $F_N$) are significantly higher than the upper critical value bounds at the 5% levels providing strong evidence of the existing of the long-run association between exchange rates, interest rates and price differentials. As we can observe from the table, the hypothesis that there exists no long-run relationship for the exchange rate model is rejected regardless of whether the exchange rate model is estimated using 4, 6 or 8 lags. This insensitivity of long-run association to different lag lengths indicates the robustness of the findings. In summary, there is significant evidence in support of the existence of the long-run association for the exchange rate model when the model is estimated without the deterministic trend. Therefore, to pursue the second step of the analysis, we estimate the model without a trend term with 6 lags.

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$F_C$ is the F-statistic for testing zero restriction only on the coefficients associated with lagged level variables in the exchange rate model. $F_{CT}$ is the F-statistic for testing zero restriction on both the trend term and the coefficients associated with lagged level variables in the model. $F_N$ is the F-statistic for testing zero restrictions on the coefficients associated with the lagged level variables when the model is estimated without the trend term.
After confirming the existence of the long-run relationship for the exchange rate model, the model specified in Equation (8) is estimated using ARDL approach to derive the long-run estimates and the short-term dynamics. Specifically, the model is estimated setting the maximum lag order equal to 6 without a trend but with a dummy variable for exchange rate regimes. To seek the well-defined parsimonious model, the Schwarz Bayesian Criterion (SBC) is used, which selects the order of the lag ARDL (2, 0, 0). Table 3 gives the results of the ARDL estimation for the short-run exchange rate model along with the estimates of diagnostic tests.

The results of the diagnostic tests indicate that the short-run model is well-specified and does not exhibit any problem of serial correlation and heteroskedasticity. All the estimates of the coefficients for the short-run model have expected signs and are statistically significant. It is interesting to notice that the first and the second lagged values of the exchange rate are highly significant indicating the strong persistence of the exchange rate. This suggests that besides the PPP and UIP conditions, the prior observed value (at least of the past two periods) of the exchange rates plays an important role in explaining exchange rate variability. This finding is in line with the idea that a portion of exchange rate dynamics behaves randomly and cannot be projected by fundamentals of exchange rate (i.e., the PPP and UIP conditions).

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{t-1}$</td>
<td>0.458</td>
<td>0.063</td>
<td>0.000</td>
</tr>
<tr>
<td>$e_{t-2}$</td>
<td>-0.396</td>
<td>0.067</td>
<td>0.000</td>
</tr>
<tr>
<td>$(i - i^*)_t$</td>
<td>-0.018</td>
<td>0.009</td>
<td>0.014</td>
</tr>
<tr>
<td>$(p - p^*)_t$</td>
<td>0.048</td>
<td>0.012</td>
<td>0.000</td>
</tr>
<tr>
<td>Regime Dummy</td>
<td>0.421</td>
<td>0.465</td>
<td>0.366</td>
</tr>
<tr>
<td>Constant</td>
<td>0.162</td>
<td>0.037</td>
<td>0.000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SE of regression</td>
<td>0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>31725</td>
<td></td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Table 3**

ARDL Estimation Results for the Exchange Rate Model selected based on Schwarz Bayesian Criterion

<table>
<thead>
<tr>
<th>Diagnostic Tests</th>
<th>F-test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial Correlation</td>
<td>0.235</td>
<td>0.856</td>
</tr>
<tr>
<td>Functional Form</td>
<td>1.014</td>
<td>0.256</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>0.083</td>
<td>0.635</td>
</tr>
</tbody>
</table>

Table 3 provides another considerable finding that is, the estimate of the coefficient of regime dummy appears statistically insignificance. This implies that the change in exchange rate regime does not have significant effect on the association of exchange
rates, price differential and interest rates differential\textsuperscript{13}. Finally, the estimates on the price differential and interest rates gaps provide evidence that the short-run exchange rate dynamics are relatively more sensitive to the price differential between the home and the foreign country. This, somehow, implies that there may a significant pass through from exchange rate swings to domestic price levels.

Next, the error correction form of the exchange rate model is estimated in ARDL framework and the results are presented in Table 4. The estimate on the error correction term has the right (negative) sign and is statistically significant confirming the long-run relationship among the variables in the model. The magnitude of the coefficient of the error correction term is -0.338 implying that the deviation caused by the short-run shocks of the prior period converges towards the long-run equilibrium with the speed of adjustment 33.8%. Thus, the full convergence is achieved in approximately 3-month periods. To check the stability of the coefficients of the error correction regression, the plots of the cumulative sum (CUSUM) based on the recursive residuals are illustrated in Figure 3. The plot does not pass through the lines representing critical bounds at the 5\% levels illustrating the stability of the estimated parameters.

\begin{table}[h]
\centering
\caption{Error Correction Form of the Exchange Rate Model selected based on SBC, Dependent variable: $\Delta e_t$}
\begin{tabular}{llll}
\hline
Regressor & Coefficient & Standard Error & p-value \\
\hline
$ECM_{t-1}$ & -0.338 & 0.009 & 0.000 \\
$\Delta e_{t-1}$ & 0.396 & 0.062 & 0.000 \\
$\Delta(i - i^*)$, & -0.018 & 0.009 & 0.014 \\
$\Delta(p - p^*)$, & 0.048 & 0.012 & 0.000 \\
Regime Dummy & 0.421 & 0.465 & 0.366 \\
Constant & 0.162 & 0.037 & 0.000 \\
R-squared & 0.652 & & \\
\hline
\end{tabular}
\end{table}

Finally, the derived long-run estimates are presented in Table 5. The long-run estimates for both price differential and interest rates differential have the expected signs which are in line with the PPP and UIP conditions and are statistically significant. This implies that the long-run equilibrium of exchange rate is determined in context of both the PPP and UIP conditions simultaneously. Furthermore, the significance of the long-run estimates for both PPP and UIP supports the joint modelling of the PPP and UIP conditions. The findings indicate that the prior empirical studies that reject the PPP or/and UIP conditions while testing in isolation may provide misleading results. Indeed, the developments in both capital and goods markets have significant influence on exchange rate dynamics.

\textsuperscript{13}We also estimate the exchange rate model using the interactions term between the regime dummy and the price differential and the interest rate differentials. The estimates for both the interaction terms were insignificant. The results are not reported here to economize the space, however, are available on request.
Thus, the interconnected form of PPP and UIP is likely to outperform the individual parity conditions in determining exchange rate equilibrium.

Table 5
Long-Run Estimates of the Exchange Rate Model
selected based on SBC, Dependent variable: Exchange Rate ($e_t$)

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(i - i^*)_t$</td>
<td>-0.024</td>
<td>0.012</td>
<td>0.048</td>
</tr>
<tr>
<td>$(p - p^*)_t$</td>
<td>1.527</td>
<td>0.301</td>
<td>0.000</td>
</tr>
<tr>
<td>Regime Dummy</td>
<td>0.095</td>
<td>0.174</td>
<td>0.543</td>
</tr>
<tr>
<td>Constant</td>
<td>4.214</td>
<td>0.059</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Figure 3: Plot of CUSUM of Recursive Residuals (ARDL)

The straight lines represent critical bounds at 5% significance level

5 - Conclusions

This paper formalizes the interactions of PPP, UIP and RW and tests this interconnected form of these parity conditions for Pakistan in ARDL framework. After confirming the order of integration of the variables the bounds tests are used for exploring the existence of the long-run relationship among exchange rates, price differential and interest rate differential. The empirical analysis covers the period from January 1991 to December 2009. Instead of splitting the sample for managed- and free-floating exchange rate period a regime dummy variable is used to examine the regime switching effects.

It is found that there is a stationary long-run relationship for the exchange rate model. Moreover, it is found that the co-movement in the long run is robust for different lag lengths. These results are consistent with the prior studies that tested both PPP and UIP
jointly, whereas, they are in contrast with the results of those studies, which tested these parity conditions in isolation and reported that there is no a long-run relationship in these variables.

The estimates for the short-run dynamics indicate that besides the price and interest rate differentials, the past value of exchange rate are important in explaining the exchange rate variability. From the estimation of the error correction form of the exchange rate model, it is observed that the coefficient of error term is highly significant confirming the existence of the cointegration relationship among the variables. In addition, the significance of estimate on the error term provides evidence of the convergence of exchange rate to the long-run equilibrium.

Finally, the signs of the long-run estimates for the price and interest rate differentials are in line with the economic theory and the estimates are statistically significant indicating the validity of the PPP and UIP conditions. Since the paper explains the puzzle of PPP and UIP and indicates the significance of the joint modelling of these parity conditions in explaining how exchange rate drifts in direction of restoring PPP and UIP, the findings are of significance for policy-makers and market participants alike.

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References


