On the Welfare Cost of Inflation: The Case of Pakistan

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1. Introduction

Inflation generally defined as sustained increase in price levels is viewed as having widespread implications for an economy on different accounts. Inflation creating several economic distortions stifles government's efforts to achieve macroeconomic objectives. In principle, price stability is considered a necessary condition for lessening income fidgets and disparities. Further, in the long-run, price stability is not only considered a necessary but also a sufficient condition for growth stability. Several studies provide empirical evidence that growth declines sharply during a high inflation crisis (see, for example, Bruno and Easterly (1996)). Since high inflation creates uncertainty, distorts investment plans and priorities, and reduces the real return on financial assets, it discourages savings, and hence affects growth negatively. Furthermore, higher inflation adversely affects economic efficiency by distorting market signals. All these costs are associated with the unanticipated inflation and have received considerable attention in the literature. Most of these costs involve transfer of resources from one group to another and the losses and gains tend to offset each other. However, it is widely agreed that most of unexpected inflation-related costs can be avoided if the inflation is correctly anticipated. Though, inflation, even when fully anticipated, results in a loss to the society in terms of the net loss of the valuable services of real money balances.

Inflation is one of the major macroeconomic problems for Pakistan's economy, particularly in the recent years. During the second half of 2000s, inflation has increased from previous levels of less than 5% annually to double digits by the end of the decade. The first significant acceleration occurred in 2005 when inflation rose to 9.3% year, following excessive money flows towards the public and private sectors. To put a bar on the rise in demand and pressure on prices, State Bank of Pakistan (SBP) took action by using tight monetary policy, and thus, inflation was somewhat reduced to 7.8% for the next two years. However, in the end of 2008, inflation increased to double-digit levels (12% per year). The double-digit inflation is evident that the tight monetary policy in the form of raising discount rate and cash requirement on demand and time deposits was ineffective in reducing inflation. Perhaps, excessive government borrowings from the SBP during this period were one of the major causes of the ineffectiveness of tight monetary policy.

Governments of most of the developing countries are too weak to enact adequate tax programs and to administer them effectively. Rather, issuing money is considered as an easy way of raising revenue to cover their expenditures or financing deficits. Likewise, in Pakistan monetary policy has not been formulated to achieve price stability. Expansions in high powered money and bank credit are determined by the government's borrowing needs for budgetary support. This painless mode of financing government's deficits is inefficient and appealing as resources move from the private sector to the public sector. At the aggregate level, this method of financing has always been disastrous for the economy. Sustained deficits not only result in high inflation but also a persistent increase in the rate of inflation. A monetary regime with such a built-in inflationary bias definitely sends signals of high inflation in future.

Under inflationary environment, people anticipate inflation and accordingly adjust the ratio of real balances to income to the opportunity cost of holding money.[1] Since, there is no close substitute for real balances, and since an unavoidable cost of holding money is its opportunity

[1] Inflation resulting from this process imposes a tax on cash balances and a loss in terms of non-optimal holding of money.

cost – the nominal interest rate, the nominal rates reflect the expected inflation. So, according to Fisher hypothesis, the cost of holding real balances increases with an increase in anticipated inflation.

Beginning with Bailey (1956), the welfare cost of inflationary finance is treated as the deadweight loss of inflation tax which is calculated by integrating the area under the money demand curve (Harberger Triangle). Traditional analyses of welfare costs of inflation have emphasized that these costs depend on the form of money demand function (see, for example, Bailey 1956). Models based on a Cagan-type semi-logarithmic demand and double-log money demand functions have extensively been employed in the literature for calculating the welfare cost of inflation. The two different types of demand specifications are very likely to give very different estimates of welfare cost. This difference mainly exists due to the behavior of the two demand curves towards low inflation (see, for details, Lucas 2000).

Empirical literature on the welfare cost of inflation suggests that money stock should be defined in the narrowest form representing the true liquidity services provided to the society. Money stock is taken in its narrow form as monetary base and M1. In some of the cases M1 tends to overstate the welfare cost because when treated as a single aggregate (currency only economy) welfare integral runs from zero to the positive nominal interest rate. To accommodate for the interest bearing demand deposits component of M1, recent studies calculate welfare cost in currency-deposit framework. [2]

Traditional studies on hyperinflation countries estimated the welfare cost of inflation against Friedman's deflation rate as under hyperinflation real interest rate is zero and deflation rule implied zero inflation. However, in applying this method to a relatively developed country with stable prices and positive real interest rates, researchers evaluate the welfare cost of a positive inflation against both zero inflation and deflation policies. All these issues regarding formulation of monetary model, definition of monetary aggregates, and optimal inflation and interest rate policies are equally important areas of inquiry.

Empirical studies on inflation in Pakistan have mainly focused on exploring the significant derivers of inflation (see, for example, Qayyum 2006, Khan and Schimmelpfennin 2006, Kemal 2006, Khan et al. 2007). A general consensus of these studies is that the monetary factors have played dominant role in recent inflation. Moreover, some of the studies have emphasized the role of State Bank of Pakistan to implement independent monetary policy with the objective of attaining price stability.[3] To best of our level, we do not find even a single study assessing the cost borne by the society due to a positive inflation in Pakistan.

Given this background, this paper attempts to comprehensively investigate the welfare cost of inflation for Pakistan. Thus, this study endeavors to bridge the gap in empirical literature on inflation in Pakistan. We use time-series data over the period 1960 to 2007 for monetary aggregates, namely, monetary base, M1, currency and demand deposits, Gross Domestic Product (GDP), and nominal interest rates to estimate both semi-log and double log (aka log-log/log-lin model) money demand functions. Our paper is also very different from the existing literature on money demand function with regard to estimation technique used in

[2] Distinct role of currency and deposits is emphasized in Marty (1999), Bali (2000), and Simonsen and Rubens (2001).

[3] See, for example, Hussain (2005), Mubarik (2005), and Khan and Schimmelpfennig (2006) giving some threshold levels of inflation.

earlier studies.[4] Specifically, we employed the Autoregressive Distributed Lag Model (ARDL) developed by Pesaran et al. (2001) in our empirical estimation. Major advantage of ARDL modeling is that it does not require any precise identification of the order of integration of the underlying data. In addition to that, this technique is applicable even if the explanatory variables are endogenous.

After estimating the parameters of the long run demand functions for narrow money, we assess quantitatively the welfare losses associated with different rates of inflation. The policy issue of reducing inflation to zero and further reducing it to Friedman's deflation rule is addressed by computing and comparing welfare loss across different money demand specifications and monetary aggregates.

The rest of the paper proceeds as follows. Section 2 reviews the literature on the welfare cost of inflation and money demand function in Pakistan. Section 3 explains the theoretical model specifications, describes the estimation technique, discusses the data used in our analysis, and presents the definition of the variables included in our empirical models. Section 4 reports the estimation results and the welfare cost calculations based on the estimated models, while Section 5 contains the conclusions.

2. Literature Review

In this section, we provide a brief review of prior empirical studies on the estimation of welfare cost of inflation. We also review the literature related money demand functions as the welfare cost of inflation crucially depends on the behavior of money demand function.

2.1 The Welfare Cost of Inflation

The issue of welfare cost of inflation is addressed under both partial equilibrium (traditional) and general equilibrium (neo-classical) frameworks. Bailey (1956) is the first to study the welfare implications of public sector inflationary finance. He shows that open (anticipated) inflation costs members of the society more than the revenue which accrues to the government. The dead weight loss associated with this implicit tax is the difference between the cost to money holders and the transfer to the government. Inflation acts like an excise tax on money holding and the dead weight loss of anticipated (open) inflation is the welfare cost of inflation.

Reviewing the literature, we find that neoclassical non-monetary models have been extended in three ways to allow for a role of money: (i) Money-in-the-Utility Function model (MIU), money directly yields utility and is treated like a consumer good (Sidrauski 1967), (ii) Cash-in-Advance model (CIA), some transactions require cash and transactions or illiquidity costs create demand for money (Clower 1967; Kiyotaki and Wright 1989), and (iii) Overlapping Generation model where money is used for the intertemporal transfer of wealth (Samuelson 1958).

Welfare cost of inflation in its magnitude depends on the bench-mark inflation rate. That is, what should be the desirable or optimal rate of inflation? Optimal inflation rate in some of the studies is taken as zero-inflation or price stability and in others as the Friedman's deflation

[4] See Section 2 on Pakistan-specific literature on the empirical estimation of money demand functions.

rate. Bailey (1956) measuring welfare cost of inflation for hyperinflation countries used zero-inflation rate as the benchmark, which was also equivalent to Friedman's deflation rule because in hyperinflation the real rate of interest is zero. However, later studies show that the welfare loss function is lowest when Friedman's optimal deflation rule is applied (Friedman (1969), Barro (1972), Lucas (2000)). Friedman's deflation rule is based on Pareto optimality condition where socially efficient level of production of a commodity is the one where marginal cost is equal to marginal benefit (later being the price of the commodity). Marginal cost of producing money is nearly zero for the monetary authority but the social cost is the nominal interest rate, the opportunity cost of holding cash. To minimize the cost of holding money, the nominal interest rate should be brought to zero which requires deflation equal to the real interest rate.

The traditional partial equilibrium model does not take into account the fact that the receipts from inflation tax can be used for the production of government capital and can contribute to economic growth. This aspect of inflationary finance was developed by Mundell (1965) and was later extended in Marty (1967) in welfare cost of inflation context. Marty (1967) using Cagan's and Mundell's money demand specifications for Hungary shows that the traditional measure of welfare is close to the measure of welfare cost in the model where inflation induces growth. The welfare cost of 10% inflation is 0.1% of income and 15.84% of government budget. Welfare cost estimates of Bailey (1956) and Marty (1967) are based on the average cost of revenue collection through money creation but Tower (1971) measures it as a marginal cost. For a hypothetical economy "Sylvania" the average and marginal costs are compared. The rate of inflation at which the average cost of inflationary finance is 7% corresponds to marginal cost of 15%.

Anticipated inflation raises the transaction costs as the individuals raise the frequency of transactions and results in increased velocity of money (Bailey (1956)). However, another cost of inflation arises when individuals facing high inflation employ alternative payments media with higher transaction costs. Barro (1972) is the first to identify the role of substitute transaction media. Using partial equilibrium model for Hungary, the welfare costs of high, hyperinflation and unstable hyperinflation are calibrated. He finds that welfare cost of 2-5% monthly inflation rate is between 3-75%. He also shows that welfare cost increases sharply for inflation rate above 5% per month.

Fisher (1981) studied the distortionary costs of moderate inflation and applied the partial equilibrium analysis to the U.S. economy. Welfare loss is measured by consumer surplus measure that incorporates the production and taxation through portfolio choice decision. Using high-powered money as the monetary asset the welfare loss of 10% inflation is estimated to be about 0.3% of GNP. Using Bailey's (1956) consumer surplus formula Lucas (1981) calculated welfare cost of inflation for the U.S., defining money as M1. The welfare gain is estimated to be 0.45% of GNP as the economy move from 10% inflation to zero inflation.

Cooley and Hansen (1989) estimated the costs of anticipated inflation in real business model where money demand arises from cash-in-advance (CIA) constraint. In this model anticipated inflation operates as inflation tax on activities involving cash (consumption) and individuals tend to substitute the non-cash activities (leisure) for the cash activities. Welfare cost is measured as a reduction in consumption as a percentage to GNP. Using quarterly data of U.S. over the period from 1955:3 to 1984:1 for macroeconomic aggregates and using parameters of microeconomic data studies, the model is calibrated. The simulation results show that the estimates of welfare loss are sensitive to the definition of money balances and to the length of time households are constrained to hold cash. For moderate annual inflation rate of 10% welfare loss is about 0.39% of GNP where money is taken as M1 and the individual holds cash for one quarter. But this cost is substantially reduced to 0.1% for the monetary base and further when the individual is constrained to hold cash for one month.

Extending Cooley and Hansen (1989) CIA model, the revenue and welfare implications of different taxes are analyzed in Cooley and Hansen (1991). Calibration and simulation techniques are used and the study reaches the conclusion that the presence of distortionary taxes (taxes on capital and labor) doubles the welfare cost of a given steady-state inflation policy. Permanent zero-inflation policy with other distortionary taxes held at their benchmark level improves welfare by 0.33% of GNP. Another type of zero-inflation policy that is assumed to be permanent, and where the lost revenue from inflation tax is replaced by raising distortionary taxes, the welfare cost is higher than the original policy with 5% inflation. Moreover, a temporary reduction of inflation rate to zero makes the economy worse-off due to intertemporal substitutions.

Cooley and Hansen (1989) measure the welfare cost under the assumption of cash only economy. However, in Cooley and Hansen (1991), the availability of costless credit is taken into account. Gillman (1993) introducing the Baumol (1952) exchange margin allows the consumer to decide to purchase good from cash or credit with further assumption of costly credit. Consumers in making decision weigh the time cost of credit against the opportunity cost of cash. The interest rate elasticity and welfare loss from costly credit setup is compared with the cash-only and costless credit economies. Using U.S. average annual data from 1948 to 1988, the study shows that both interest elasticity and welfare cost in costly credit economy are greater than the cash-only and costless credit settings. The cost associated with 10% inflation is 2.19% of income compared to 0.58% and 0.10% for cash-only and costless credit economies respectively.

Eckstein and Leiderman (1992) in addition to Cagan semi-log model use Sidrauski-type money-in-uitility (MIU) model to study seigniorage implications and welfare cost of inflation for Israel. Parameters of the intertemporal MIU model are estimated by using Generalized Methods of Moments (GMM), on quarterly data from 1970:I to 1988:III. The simulation results show that inflation rate of 10% has welfare loss of about 1% of GNP. Degree of risk aversion is identified as an important determinant of welfare cost and loss of lower inflation rates predicted by the intertemporal model is higher than that calculated from the Cagan-type model. The welfare cost estimates from intertemporal model are more reliable as it produced national income ratios and seigniorage ratios much closer to the actual values.

López (2000) following Eckstein and Leiderman (1992) intertemporal model studies the seigniorage behavior and welfare consequences of different inflation rates in Columbia. For the period 1977:II to 1997:IV the parameters of the model are estimated using GMM. Welfare loss due to increase in inflation from 5% to 20% is 2.3% of GDP, and 1% of GDP when inflation increases from 10% to 20%. Eckstein and Leiderman's (1992) model with some modifications is employed in Samimi and Omran (2005) to study the consumption and money demand behavior from intertemporal choice. The welfare cost of inflation is calculated using annual data from 1970 to 2000 for Iran. Welfare cost is found to be a positively related to the inflation rate. While the welfare cost of 10 % is 2% of GDP, the cost is 4.37% of GDP for an inflation rate of 50%.

Several studies, including Bailey (1956), Wolman (1997), and Eckstein and Leiderman (1992), have pointed out that the estimates of welfare cost depend largely on the money demand specification. Lucas (1994, 2000) estimates the double log money demand function in explaining the actual scatter plot than the semi-log functional form for the period 1900-1994. Bailey's consumer's surplus formulae are derived and used to compute the welfare cost of inflation for both semi-log and log-log money demand functions. Based on log-log demand curve the welfare gain from moving from 3% to zero interest rate is about 0.01% of real GDP, while for semi-log estimates it is less than 0.001%.

Simonsen and Rubens (2001) theoretically extended Lucas (2000) transactions technology model to allow for the interest bearing assets. Simonsen and Rubens (2001) reach the conclusion that with interest earning assets included, the upper bound lies between Bailey's consumer surplus measure and Lucas' measure of welfare cost. Bali (2000) using different monetary aggregates calculated welfare cost using two approaches, Bailey's welfare cost measure and compensating variation approach. Error Correction and Partial Adjustment models are applied to find the long interest elasticities and semi-elasticities. For the quarterly data ranging from 1957:I to 1997:II, the empirical results show that constant elasticity demand function accurately fit the actual U.S data. Loss to welfare associated with 4% inflation turned out to be 0.29% of income (benchmark to be zero nominal interest rate) and the welfare gain in moving from 4% to zero inflation is 0.11% of income with currency-deposit specification, welfare cost is around 0.18% of GDP when monetary base is used whereas with M1 the loss is much higher than the earlier two cases and is approximately 0.55% of GDP.

Serletis and Yavari (2003) calculate and compare the welfare cost of inflation for two North American economies, namely Canada and the United States, for the period 1948 to 2000. Following Lucas (2000), they assume a constant interest elasticity of money demand function. They show 0.22 interest rate elasticity for Canada, while 0.21 for the USA, much lower than 0.5 assumed by Lucas (2000). Welfare cost is measured using traditional Bailey's approach and Lucas' compensating variation approach. Welfare gain of interest rate reduction from 14% to 3% (consistent with zero inflation) for U.S is equivalent to 0.45% increase in income. Reducing the nominal interest rate further to the optimal deflation rate yields an increase in income by 0.18%. For Canada the distortionary costs are marginally lower, reducing rate of interest from 14% to 3% increases real income by 0.35%, and further reducing to Friedman's zero nominal interest rate rule resulted in a gain of 0.15% of real income.

Serletis and Yavari (2005) estimate the welfare cost of inflation for Italy. Estimating a long-horizon regression, they find that interest elasticity is 0.26. Using the same approaches of calculating welfare cost of inflation as in Serletis and Yavari (2003), they show that lowering the interest rate from 14% to 3% yield a benefit of about 0.4% of income. Same analysis was extended in Serletis and Yavari (2007) to calculate direct cost of inflation for seven European countries, Ireland, Australia, Italy, Netherlands, France, Germany, and Belgium. The welfare cost estimates of these countries showed that the cost is not homogeneous across these countries and is related to the size of the economy. Welfare cost was lower for Germany and France than the small economies.

The welfare costs of anticipated inflation are the distortions in the money demand brought about by the positive nominal interest rate so the major emphasis of studies after Lucas (2000) is first to check for the proper money demand specification. Ireland (2007) finds that

Cagan-type semi-log money demand function is better description of post 1980 U.S data. For the quarterly data from 1980 to 2006 the semi-elasticity is estimated to be 1.79 and welfare cost of inflation is measured using consumer's surplus approach of calculating the area under the money demand curve. For 2% inflation rate the welfare cost is 0.04% of income and 0.22% of income for the 10% inflation rate. Price stability is taken as benchmark instead of Friedman's optimal deflation policy.

Gupta and Uwilingiye (2008) measure the welfare cost of inflation for South Africa. Double log and semi-log money demand functions are estimated using Johansen's cointegration method and the long-horizon regression method. The study apart from estimating the proper money demand function analyzes whether time aggregation affects the long run nature of relationship or not. Interest elasticity and semi-elasticity estimates are used to measure welfare cost of inflation using Bailey's traditional approach and Lucas compensating variation approach. Estimation results show that for the period 1965:II to 2007:I, compared to the cointegration technique, the long-horizon approach gives a more consistent long-run relationship and welfare estimates under the two time aggregation sampling methods. The welfare cost of target inflation band of 3 to 6% lies between 0.15 and 0.41% of income.

In sum, the review of literature shows that the welfare cost of inflation has found its initial application in hyperinflation countries. In Bailey (1956), Marty (1967) and Barro (1972), and in many other studies, the welfare cost is measured mainly for the developed countries with stable inflation like U.S and now it is extended to European countries and South Africa. This issue also needs to be addressed for developing countries where inflation rate is primarily determined by money supply. For policy-makers to conduct an effective monetary policy, it is very much important to estimate the welfare cost of inflation based on a stable estimated money demand function. At the best of our knowledge, this article is one of the first to calculate the welfare cost of inflation in Pakistan.

Second, there is also a transition from partial equilibrium analysis to general equilibrium analysis to calculate welfare cost of inflation. To provide general equilibrium rationale for holding money, we will use the Money-in-the Utility Function model. Other general equilibrium models like Cash-in-Advance and transaction time technology models are relatively more sophisticated approaches but we cannot apply these due to two main reasons. First, the underlying assumptions of the models regarding distinction among the cash and credit goods do not effective in developing countries' market environment. Secondly, the studies employing CIA constraint in Real Business Cycle (RBC) model use the calibration technique which makes use of the results of studies using microeconomic data. For most of the developing countries in general and specifically for Pakistan the data on non-durable (cash) goods and durable (credit) goods is not available. Similarly, the impact of inflation on the marginal decisions like working hours, capital accumulation and investment decisions at micro level have not been addressed for Pakistan.

2.2 Pakistan-Specific Empirical Money Demand Studies: A Review

Welfare cost estimates are highly sensitive to the specification of money demand function. In this section we therefore provide a review of the recent development on this issue in Pakistan. From a theoretical prospective, the main determinants of money demand are the opportunity cost variables and the scale variable proxied by income. Mangla (1979) was the first who tests the empirically validity of these variables for Pakistan. In particular, using both GNP and permanent income as proxies for scale variables and both annual yield on government bonds and call money rate as a proxy for the opportunity cost of holding money, Mangla estimates the real and money demand for M1 over the period from 1958-1971. He finds that the income elasticity of nominal demand for money is significantly greater than 1 and interest elasticity ranged from -0.04 to -0.16 for call money rate, while for the bonds yield it ranged from -0.31 to -0.96. He shows that while the income elasticity is greater than one, the interest elasticity turns out to be low, -0.02 to 0.02, for call money rate and positive for bonds yield.

Khan (1980) estimates the demand for money and real balances by defining money as M1 and M2 for the period 1960 to 1978. The main objective of his study was to identify the correct scale variable – current or permanent income – for the money demand function. Applying ordinary least squares, he finds that the income elasticity for both nominal and real money demand functions is significantly greater than one, implying diseconomies of scale. He further argues that both permanent and current income give approximately similar results, lending no superiority to one measure over the other. For nominal money demand functions (M1 and M2), he reports that the interest elasticity is insignificant but for the real money demand it has the expected negative sign.

Similar analysis of finding appropriate scale and opportunity cost variables for the money demand function was carried out in Khan (1982). The scale variables were taken to be permanent income and measured income, while opportunity cost variables were the interest rate (call money rate, interest on time deposits) and the expected and actual inflation rates. Using Cochrane-Orcutt technique the demand functions of M1 and M2 are estimated for the six Asian developing countries (Pakistan, India, Malaysia, Thailand, Sri Lanka, and Korea) for the period 1960 to 1978. For Pakistan with M1 definition of money, he finds that there is no difference between permanent and measured income elasticities. His estimates provide evidence that income elasticity is greater than one, representing diseconomies of scale. Money demand is significantly explained by interest on time deposits and interest elasticity ranged from -0.42 to -0.44. For broader money (M2), he reports that the income elasticity is greater than for M1, and interest elasticity ranges from -0.37 to -0.39. For Pakistan inflation and expected inflation tend to affect money demand but the magnitude (-0.05) is much less than the coefficient of interest rate. Khan (1982) also reaches the same conclusion as Khan (1980) that interest rate is the proper opportunity cost variable in money demand function.

Nisar and Aslam (1983) estimate the term structure of time deposits and substitute the parameters in the money demand function, using data over the period from 960 to 1979. They find that the coefficient of term structure for both M1 and M2 monetary aggregates is negative and has a smaller magnitude for the M2 definition of money ranging from -0.51 to -0.73. They also show that time deposits are positively related to interest rate (representing own rate of return), whereas interest rate has negative effect on currency so overall the magnitude of interest elasticity is low for M2 due to the inclusion of time deposits. Consistent with Khan (1982), they conclude that the money demand is elastic with respect to the scale variable. While coefficient of inflation rate bears a positive sign and is statistically not significant. Secondly the study compares the stability of money demand function estimated using term structure against the conventional money demand function with simple average interest rate (call money rate). The covariance analysis shows that the term structure money demand function remained stable while conventional function does not pass the stability test.

Developing countries like Pakistan lack sophisticated financial system. Currency constitutes a large proportion of total monetary assets. Qayyum (1994) using data from 1962:I to 1985:II estimates the long-run demand for currency holding. He shows that currency demand is determined by interest rate defined as bond rate, rate of inflation and income. Coefficient of income is approximately unity, and money-income proportionality hypothesis is tested. Further, he argues that Money-income proportionality holds and imposing this restriction, the steady state demand for currency turns out to be related to inflation and bonds rate. The coefficients of inflation and interest are negative and significant showing that people can substitute between currency and real goods, and also between currency and financial assets.

Hossain (1994) estimates the money demand for both the real narrow (M1) and broad money (M2) balances for the two sub-periods ranging from 1951 to 1991 and 1972 to 1991. Double log specification of money demand function is used with income, interest rate (govt. bond yield, call money rate) and inflation rate as the explanatory variables. The results for the sample period from 1972 to 1991 are more encouraging where the income elasticity for broad money is around unity and about 0.86 for the narrow money. Interest elasticity in absolute terms is greater for narrow money (-0.54) than for M2 (-0.05). The results for both the sample periods show that real money balances are not cointegrated with inflation rate and that the narrow money demand is more stable than the broad money demand function.

Financial sector reforms of 1980s increased the interest in money demand function. Khan (1994) and Tariq and Mathew (1997) investigate the impact of financial liberalization on money demand. In particular, Khan (1994) examines the effect of these reforms on stability of money demand. Engle-Granger two-step method of cointegration is used to estimate the money demand function using quarterly data starting from 1971:III to 1993:II. The results of cointegration analysis for double-log money (nominal M1 and M2) demand function show that demand for broader money is determined by real income, nominal interest rate of medium term maturity real interest rates, and the inflation rate. While for M1 definition the cointegral relationship holds for all the arguments except short term and medium term nominal interest rates.

The second study in the same line of addressing the effects of financial reforms is Tariq and Matthews (1997) that investigated the impact of deregulation on the definition of monetary aggregates. In this study divisia monetary aggregates are compared to the simple monetary aggregates in finding the stable money demand function. Conventional money demand function is estimated with scale and opportunity cost variable, opportunity cost is taken as differential of interest on an alternative asset and own rate of return on the given monetary aggregate. Cointegration analysis shows that demand for all the four monetary aggregates, M1, M2, Divisia M1 and Divisia M2 is positively related to the scale variable and negatively to opportunity cost variable. Income elasticity is greater than unity implying that velocity has decreasing trend. Error correction model (ECM) is used to estimate the short run dynamic money demand function, which shows that all the four monetary aggregates are equally good in explaining the money demand function and there is no superiority of divisia aggregates over the simple-sum monetary aggregates.

There is difference between the money demand behavior of household and business sector and the sectoral money demand has been studied in developed countries. Its first application in Pakistan is Qayyum (2000) who studies the demand for money by business sector. Owing to the difference in the behavior of business sector, total sales is taken as the scale variable instead of income. He shows that the long run demand for M1 is determined by sales and inflation rate. The sales/ transactions elasticity of business sector's demand for real balances is unitary. In the long run the demand for money is not determined by the interest rate but the short run dynamic ECM shows that money demand is determined by changes in the return on saving deposits, changes in inflation rate, and movements in the previous money holding.

Qayyum (2001) estimates the money demand function at aggregate level and for both the household and business sectors using quarterly data from 1959:III to 1985:II. He finds that all the three money demand functions are sensitive to income, inflation rate and interest rate. He concludes that bond rate is the relevant opportunity cost variable in aggregate and household money demand functions. For the business sector the appropriate interest rate representing opportunity cost is the rate of interest on bank advances. Money-scale variable proportionality holds in all the money demand functions. Scale variable is defined as income/ real GDP for the aggregate and household money demand function while for the business sector it is real sales. Business sector demand for real balances is explained by own rate of return and inflation rate and further the money-sales proportionality holds in the long run. The results from ECM show that in the short run interest rate is an important variable determining the aggregate demand for real balances and liquidity demand of the business sector.

Another study by Qayyum (2005) estimates the demand for broader monetary aggregate M2 at aggregate level for the annual data frm 1960 to 1999. This study reaches similar conclusion as Qayyum (2001) that the major determinants of money demand are own rate of return (call money rate) and opportunity cost variables (inflation rate and the Govt. bond yield) and income. However, the magnitude of coefficients is high for both the interest rates.

Using annual data from 1972 to 2005 Hussain et al. (2006) estimated the demand for money; money is defined as monetary base, M1 and M2. The study finds that there is no cointegration and unit root in the data series. They model the demand for all the three monetary aggregates as a function of the real GDP, inflation rate, financial innovation and the interest rate on time deposits. They find that the long run income elasticity ranges from 0.74 to 0.779 and interest elasticity ranges from -0.344 to -0.464. Of all the three definitions of money M2 is found to better explain the long run stable money demand function.

Ahmad et al. (2007) estimated the long run money demand function using error correction model. The conventional money demand function with income and call money rate is estimated for the period 1953 to 2003. The results show that both the arguments of money demand function have theoretically correct signs for M1 and interest semielasticity is -0.012. Interest rate coefficient is positive and insignificant for real M2. For both narrow money M1 and broad money M2 the money-income proportionality does not hold.

In this study we want to calculate the welfare cost of inflation based on the estimated parameters of a stable money demand function. The studies on welfare cost of inflation suggest that we have to define money in the narrowest form, like monetary base or M1 so that the interest rate is the opportunity cost of holding money[5]. Estimating the demand for broader monetary aggregate (M2) is not relevant for our analysis because it includes some interest bearing assets, interest coefficient in most of the studies turned out to be positive or insignificant showing that interest rate is own rate of return rather than an opportunity cost variable for M2.

[5] Hussain (1994) argues that narrow money demand in Pakistan is more stable than broad money demand.

Welfare cost is a steady state analysis for which the money-income proportionality is assumed to hold. Following the social welfare loss of inflation analysis we need to (newly) re-estimate the money demand function taking the ratio of money balances to income (scale variable) as the dependent variable with a single argument; the nominal interest rate. We estimate demand functions defining money as Monetary Base, Narrow Money M1 and disintegrating M1 into its constituent components and estimate the demand functions of demand deposit and currency.

3. Estimation Methods

Following Bali (2000), we estimate the currency-deposit model to analyze the welfare cost of inflation in Pakistan. The rationality of employing currency-deposit model is that both currency and deposits have different opportunity costs. The implicit cost of holding currency is the nominal interest (i), while that of demand deposit is the difference between the nominal interest rate (i) and the interest on deposits (i_d). The studies that lump both the currency and demand deposits together as non-interest bearing assets are likely to overstate the true cost of inflation (see, for details, Lucas 1994 and 2000). Another advantage of this disintegrated asset model is that the single monetary asset models are the nested model of this broader model.

When estimating the model, we ignore uncertainty and labour-choice, focusing on the implications of the model for money demand and welfare cost of inflation. Further, we assume that the representative household derives utility from a consumption good (c_t) and flow of services from the real money balances that consist of currency (m_t) and demand deposits (d_t). In particular, the utility function takes the following form:

$$\sum_{t=0}^{\infty}(1+\rho)^{-1}U(m_{t,}c_t,d_t) \qquad \textbf{(Eq. 1)}$$

where ρ is the subjective rate of time preference. In equation (Eq. 1), the utility function is assumed to be increasing in all the three arguments, strictly concave and continuously differentiable. Economy-wide budget constraint of household sector, in real units, is given by

$$(1+\pi_{t+1})m_{t+1}+(1+\pi_{t+1})d_{t+1}+k_{t+1}+(1+r_{t+1})^{-1}b_{t+1}=m_t+d_t(1+i_d(t))+k_t(1-\delta)+b_t+f(k_t)+c_t+h_t$$

(Eq. 2)

The budget constraint indicates that the household can transfer resources from one period to the next by holding nominal currency (m_t), demand deposits (d_t), bonds (b_t), and physical capital (k_t). Given the current income $f(k_t)$, its assets and any net transfer (h_t) from the government sector the household allocates its resources among current consumption (c_t) and savings (left side of equation (Eq. 2)). The real rate of return on bonds $(1+r_{t+1})$ is equal to $(1+i_{t+1})/(1+\pi_{t+1})$, where i_{t+1} denotes the nominal return on bonds held from t to $t+1$, whereas $(1+i_d)$ is the return on demand deposits.

Household maximizes its utility (equation (Eq. 1)) subject to budget constraint (equation (Eq. 2)). Solving the optimization problem for two periods, t and t-1, yields the following first-order Euler equations:

$$\frac{u_m(c_t,m_t,d_t)}{u_c(c_t,m_t,d_t)} = -1+(1+\rho)(1+\pi_t)\frac{(1+r_t)}{(1+\rho)} = i_t \qquad \text{(Eq. 3)}$$

$$\frac{u_d(c_t,m_t,d_t)}{u_c(c_t,m_t,d_t)} = -1(1+i_d)+(1+\rho)(1+\pi_t)\frac{(1+r_t)}{(1+\rho)} = i_t - i_d(t) \qquad \text{(Eq. 4)}$$

Euler equations (Eq. 3) and (Eq. 4) indicate that the marginal rate of substitution between money and consumption and between deposits and consumption is equal to the opportunity costs of respective assets. These first order Euler equations are the implicit form of asset demand functions which can be estimated by assuming some specific form of utility function.

In order to derive the implications of the model for welfare cost of inflation using Lucas compensation variation approach, the following CES isoelastic utility function is used:

$$u(c_t,m_t,d_t) = \frac{\left\{\left[\gamma_1^{1/\theta} c_t^{(\theta-1)/\theta} + \gamma_2^{1/\theta} m_t^{(\theta-1)/\theta} + \gamma_3^{1/\theta} d_t^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)}\right\}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad \text{(Eq. 5)}$$

where $\theta > 0$ is the elasticity of intertemporal substitution. Substituting the marginal utilities from equation (Eq. 5) into Euler equations (Eq. 3) and (Eq. 4) gives the following real currency and real deposit demand functions:

$$m_t = \left(\frac{\gamma_2}{\gamma_1}\right) i_t^{-\theta} c_t \qquad \text{(Eq. 6)}$$

$$d_t = \left(\frac{\gamma_3}{\gamma_1}\right) (i_t - i_d(t))^{-\theta} c_t$$

Steady state analysis of welfare cost of inflation requires that the proportion of income held as cash, should be independent to the growth in real income. This implies that velocity remains constant.[6] Under steady state we write the money demand function as the ratio of real

[6] Velocity becomes function of the interest rate and it is transformed to money demand function which is integrated under Bailey's approach to get welfare cost as a proportion of scale variable.

$$u(c_t,m_t,d_t) = \left(\frac{1}{1-\omega}\right)\left[\left(\frac{1}{1-\gamma}\right)(c_t^{1-\gamma}-1)+\left(\frac{\varphi}{1-\alpha}\right)(m_t^{1-\alpha}-1)+\left(\frac{\phi}{1-\beta}\right)(d_t^{1-\beta}-1)\right]^{1-\frac{1}{\omega}}$$

$$m_t = \varphi^{1/\alpha} i_t^{-1/\alpha} c_t^{\gamma/\alpha} \qquad d_t = \phi^{1/\beta}(i_t - i_d(t))^{-1/\beta} c_t^{\gamma/\beta}$$

where γ/α and γ/β are the scale elasticities of demand for real currency and real deposits and $1/\alpha$ and $1/\beta$ are elasticities of currency and deposits with respect to their respective opportunity costs. Unitary scale elasticities require that $\alpha = \beta = \gamma$ must hold, which implies that the assets demand functions have same opportunity cost elasticities.

money balances to the real scale variable. It further requires that under both currency and demand deposits have same interest elasticities (θ). Recall that cost of holding money defined as demand deposits, is the disparity between the yield on other assets (i_t) and interest on deposits (i_d). When the banks are operating at zero profit condition, $i_d = (1-\mu)i_t$, where μ is the reserve ratio. The zero profit condition implies that opportunity cost of holding deposits is $(i_t - i_d) = \mu i$. Below equation (Eq. 7) presents the demand function for demand deposits.

$$d_t = \left[\frac{\gamma_3}{\gamma_1}\right](\mu i_t)^{-\theta} c_t \qquad \text{(Eq. 7)}$$

For the single monetary asset the utility function in money-in-utility (MIU) framework takes the form as:

$$\sum_{t=0}^{\infty}(1+\rho)^{-1} U(m_{t,} c_t,)$$

Solving the optimization problem with changing the budget constraint without role of demand deposits gives the money demand function equivalent to the currency demand function presented in equation (Eq. 6).

3.1 Money Demand Specification

To compute the welfare cost function we estimate both the double log and semi-log money demand functions.

3.1.1 Double-log Money Demand Function

To calculate the welfare cost of inflation we are interested specifically of the effect of opportunity cost (nominal interest rate) on money holding. The demand for real balances is given by

$$\left(\frac{M_t}{P_t}\right) = L(i_t, y_t)$$

where left side in the above equation is the ratio of money stock to price level showing the demand for real balances as function of nominal interest rate i_t, and y_t is the real income. In the long run liquidity demand function takes the following form.

$$L(i\ , y) = m(i)y \qquad \text{(Eq. 8)}$$

Equation (Eq. 8) indicates that money demand is proportional to income. It is evident that the estimates of the income elasticity of money demand (i.e., M1, M2 and currency) obtained for Pakistan tend to be around unity (Qayyum (1994), (2000), (2001) and (2005)). Therefore, the unitary scale (income) elasticity restriction is imposed which enables us to estimate the money demand function (*m(i)*) defined as the ratio of real money balances to real income with single argument defined as the opportunity cost of holding money.

$$m / y = m(i) \qquad \text{(Eq. 9)}$$

Equations (Eq. 6) and (Eq. 7) are in the form of (Eq. 8) and dividing by the scale variable can be converted into the final form of demand function required for the welfare cost analysis.

$$\left(\frac{m_t}{c_t}\right) = \left(\frac{\gamma_2}{\gamma_1}\right) i^{-\theta}$$

$$\left(\frac{d_t}{c_t}\right) = \left(\frac{\gamma_3}{\gamma_1}\right) (\mu i)^{-\theta}$$

These equations take the form of equation (Eq. 9) and can be written in the following double-log form:

$$m(i) = e^{\alpha_0} i^{-\alpha_1} \qquad \text{(Eq. 10)}$$

$$d(\mu i) = e^{\alpha_2} (\mu i)^{-\alpha_1} \qquad \text{(Eq. 11)}$$

where the dependent variables are taken as ratio to scale variable and welfare cost is expressed as the percentage of GDP.

3.1.2 Semi-log Money Demand Function

Standard utility functions mostly yield double-log money demand function, but semi-log models have gained great application in money demand literature for its seigniorage implications. Number of studies, such as Lucas (2000), Bali (2000) and Gupta and Uwilingiye (2008), have estimated both the double-log and semi-log money demand functions and compared welfare costs associated with both the specifications. Following these studies, we also estimate the semi-log money demand function along with the log-linear function and judge the sensitivity of the estimated welfare cost for the two models towards low interest rates.

To compare the semi-log model with the derived double log currency-deposit model we restrict both currency and demand deposits to have same interest semi-elasticity. The demand functions for currency and deposits under semi-log specification are given as follow:

$$m(i) = e^{\alpha_0 - \alpha_1 i} \qquad \text{(Eq. 12)}$$

$$d(i) = e^{\alpha_2 - \alpha_1 (\mu i)} \qquad \text{(Eq. 13)}$$

After estimating the steady state money demand functions the welfare cost will be computed using both Bailey's and Lucas' measures of welfare cost. What follows below, we present a brief discussion of these welfare cost measures.

3.2 Welfare cost of inflation and money demand function

3.2.1 Bailey's Consumer Surplus Approach

The first attempt to measure the welfare cost of anticipated inflation is credited to Bailey (1956) wherein the nominal interest rate is the opportunity cost of holding money. Inflationary finance/ anticipated inflation is excise tax on real cash holding; and welfare cost is the loss in consumer surplus and is measured as area under the money demand curve. Changes in inflation rate are related to changes in nominal interest rate through Fisher hypothesis that holds for Pakistan (Hassan (1999)). Thus, welfare cost is measured as loss in consumer surplus not compensated by total revenue. This can be described as follows:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_{0}^{r} m(x)dx - rm(r) \qquad \text{(Eq. 14)}$$

where $m(r)$ is money demand function and $\psi(x)$ is the inverse demand function. m is defined as ratio of money to income, welfare function w is function of income therefore, welfare loss is defined as proportion of income.

3.2.1.1 Welfare Cost of Inflation for Semi-log Money Demand function

Bailey (1956), Marty (1967), Friedman (1969) and Tower (1971) have used Cagan semi-log money demand function. All these studies were based on hyperinflation economies, and welfare gain for this specification comes largely by moving from high interest to low interest rates, while for the interest rate approaching zero the solution is trivial.

A. Single Monetary Asset Model

When monetary stock is taken to be monetary base or M1 (single monetary asset model) the semi-log money demand function is given as follows:

$$m(r) = e^{\alpha_o - \alpha_1 i}$$

Substituting the money demand function in (Eq. 14) gives the following welfare cost measure

$$\int_0^r e^{\alpha_0 - \alpha_1 x} dx - i\left(e^{\alpha_0 - \alpha_1 i}\right)$$

$$WC = \frac{e^{\alpha_0}}{\alpha_1}\left[1 - e^{-\alpha_1 i}(1 + i\alpha_1)\right] \qquad \text{(Eq. 15)}$$

where α_o is the intercept in money demand function and α_1 is the interest rate related semi-elasticity of money demand.

B. Currency-Deposit Model

For the modified money-in-utility function which allows for the distinct role of currency and demand deposits, the welfare cost takes the following form:

$$WC = \int_0^i f(x)dx - if(i) + \int_0^{\mu i} g(x)dx - \mu i g(\mu i)$$

$$WC^{Semi-\log} = \frac{e^{\alpha_0}}{\alpha_1}\left[1 - e^{-\alpha_1 i}(1+\alpha_1 i)\right] + \frac{e^{\beta_0}}{\beta_1}\left[1 - e^{-\beta_1 \mu i}(1+\beta_1 \mu i)\right] \qquad \text{(Eq. 16)}$$

where demand for currency is $f(x) = e^{\alpha_0 - \alpha_1 i}$ and semi-log demand function for deposits is $g(x) = e^{\beta_0 - \beta_1 \mu i}$. First term in equation (Eq. 16) represents the dead weight loss accruing from currency while the second term is the dead weight loss measured for demand deposit. For currency the integral runs from zero to positive nominal interest (i) and for demand deposits it runs from zero to opportunity cost of holding demand deposits (μi). Under restricted model, where both currency and deposits are restricted, the semi-elasticities should have to be same, $\alpha_1 = \beta_1$.

3.2.1.2 Welfare Cost of Inflation for Double log Money Demand Function

A. Single Monetary Asset Model

The double log money demand specification for a single monetary asset (i.e., monetary base or M1) is given as follows:

$$m(i) = e^{\alpha_0} i^{-\alpha_1}$$

So, the welfare cost formula is derived by substituting the money demand function in (Eq. 14), which is presented as follows:

$$WC = e^{\alpha_0} i^{1-\alpha_1}\left[\frac{\alpha_1}{1-\alpha_1}\right] \qquad \text{(Eq. 17)}$$

where α_o and α_1 are the intercept and slope coefficient of double log money demand function respectively.

B. Currency-Deposit Model

For the double-log demand for currency and deposits given in equation (Eq. 14) becomes

$$WC^{double-\log} = \left(\frac{\alpha_1}{1-\alpha_1}\right) e^{\alpha_o} i^{1-\alpha_1} + \left(\frac{\beta_1}{1-\beta_1}\right) e^{\beta_o} (\mu i)^{1-\beta_1} \qquad \text{(Eq. 18)}$$

The welfare cost formula shows that the cost is entirely in terms of α_o, α_1, β_o and β_1, parameters of the estimated asset demand functions and their opportunity costs.

3.2.2 Lucas Compensating Variation Approach

Lucas in arriving at a welfare measure starts with the assumption that two economies have similar technology and preferences; the only difference is in the conduct of monetary policy. In one of the economies Friedman's zero interest rate policy is adopted whereas in the other economy interest rate is positive. He defines the welfare cost of inflation as compensation in income (defined as percentage of income) required to leave the household (living in the second economy) indifferent to live in either of the two economies. The left side of equation (Eq. 19) shows the welfare in second economy with a positive interest rate and right hand side is the characterization of the first economy operating at deflation policy. *w(i)* is the measure of income compensation or the welfare cost of inflation.

$$u[1+w(i), \overline{m}(i), \overline{d}(\mu i)] = u[1, \overline{m}(0), \overline{d}(0)] \qquad \text{(Eq. 19)}$$

Lucas has given two measures of welfare cost for the two specifications of long-run money demand function due to their different behavior at low interest rates; (a) Square-Root Formula and (b) Quadratic Approximation.

3.2.2.1 Welfare Cost of Inflation for Semi-log Money Demand function and Quadratic Approximation

Semi-log money demand specification originally due to Cagan (1956) and Bailey (1956) gives rise to quadratic formula for welfare cost of inflation. Under this specification there is the existence of satiation in money demand and the quadratic formula derived for this specification is sensitive to high interest rate. Wolman (1997) and Bakhshi (2002) show that for semi-log model there is satiation in asset holding *m(0)* and *d(0)* in (Eq. 19), represent maximum currency and demand deposits holdings at zero interest rate. The above mentioned studies also showed that under satiation welfare gain of moving from positive inflation to zero inflation is higher compared to the gains of moving further to Friedman's zero interest rules.

To derive the quadratic formula from equation (Eq. 19) the second-order Taylor series expansion is applied to the welfare function around zero interest rate.

$$w(i) = w(i)\big|_{i=0} + w'(i)\big|_{i=0}(i-0) + \frac{1}{2}w''(i)\big|_{i=0}(i-0)^2 = \frac{1}{2}i^2\left[-\overline{m}'(0) - \mu^2\overline{d}'(0)\right] \qquad \text{(Eq. 20)}$$

A. Single Monetary Asset Model

For the single-monetary-asset model, equation (Eq. 19) takes the following form:

$$u\left[1 + w(i), \overline{m}(i)\right] = u\left[1, m(0)\right]$$

and welfare cost of inflation is $w(i) = \frac{1}{2}\overline{m}(0)\eta i^2$ (Eq. 21)

where η is the semi-elasticity of demand for M1 or monetary base with respect to interest rate.

B. Currency-Deposit Model

Assuming that demand deposits and currency have same semi-elasticity (restricted case) welfare loss formula (20) is transformed as follows: [7]

$$w(i) = \frac{1}{2}\eta i^2\left[\overline{m}(0) + \mu^2\overline{d}(0)\right] \quad \text{(Eq. 22)}$$

Given the semi-log demand functions $\overline{m}(i) = \overline{m}(0)e^{-\eta i}$, $\overline{d}(\mu i) = \overline{d}(0)e^{-\eta(\mu i)}$, $\overline{m}(0)$ and $\overline{d}(0)$ initial conditions are calculated by assuming $\overline{m}(i)$ and $\overline{d}(i)$ functions pass through the values of currency holdings, deposits, interest rates observed at the end of sample period. Semi-elasticity η is measured from long-run semi-log asset demand functions.

3.2.2.2 Welfare Cost of Inflation for Double log Money Demand Function and Square-Root Formula

Square-Root formula is applicable if double-log is the proper specification of money demand function. Under this specification as nominal interest rate approaches zero, demand for real balances become arbitrarily large (Ireland 2007), and equation (Eq. 19) takes the form as: $u\left[1 + w(i), \overline{m}(i), \overline{d}(\mu i)\right] = u\left[1, \infty, \infty\right]$.

A. Single Monetary Asset Model

Welfare cost formula for a single monetary aggregate (Monetary base or M1) without assigning distinct roles to currency and deposits the welfare function is given as:

$$w(i) = \left[1 - \left(e^{\alpha_o}\right)i_t^{1-\alpha_1}\right]^{\alpha_1/(\alpha_1 - 1)} - 1 \quad \text{(Eq. 23)}$$

where $\boldsymbol{\alpha_1}$ is slope (interest elasticity) and $\boldsymbol{\alpha_o}$ is the intercept term in log-linear model with single monetary aggregate.

B. Currency-Deposit Model

For currency-deposit welfare cost is calibrated by employing estimated parameters from log-log specification of the demand deposits and currency demand functions.

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[7] For the unrestricted model that allows for different semi-elasticities for currency and deposits the welfare cost formula is written as $w(i) = \frac{1}{2}\left[\eta\overline{m}(0) + \varepsilon\mu^2\overline{d}(0)\right]$ where η is the semi-elasticity of currency and ε is the semi-elasticity of demand deposits.

$$w(i) = \left[1 - \left(e^{\alpha_o}\right) i_t^{1-\alpha_1} - \left(e^{\alpha_2}\right) \mu_t^{1-\alpha_1} i_t^{1-\alpha_1}\right]^{\alpha_1/(\alpha_1-1)} - 1 \qquad \text{(Eq. 24)}$$

This model is derived from CES utility function where α_1 is the interest elasticity for both the assets demand functions. Welfare cost of inflation is measured by empirically estimating the money demand function parameters. Welfare cost is measured as the value of welfare measures evaluated at different nominal interest rates.

3.3 Estimation Procedure and Empirical Technique

The main objective of the study is to estimate the stable money demand function for Pakistan and to compute the welfare cost of inflation. Cointegration technique is used to determine the long run relationship between different time series. Specifically, we use Autoregressive Distributed Lag (ARDL) model to estimate the long-run interest elasticity and semi-elasticity of money demand function. This approach has an advantage that it provides long-run coefficients even for small data set and it does not require all the regressors to be integrated of same order that is I(1). It can be applied in case where the regressors have mixed order of integration; only restriction is that none of the variable should be I(2) or integrated of order greater than 1. Further, the problem of endogneity also does not affect the bounds test for cointegration.

To apply the bounds test for cointegration the Unrestricted Error Correction Model (UECM) representation of double log money demand function: $m(r) = e^{\alpha_0} i^{-\alpha_1}$ takes the following form:

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \log(i_{t-i}) + \lambda u_{t-1} + \varepsilon_t \qquad \text{(Eq. 25)}$$

In this equation m_t is real money balances taken ratio to real GDP, i_t is the interest rate, βo is the intercept, β_1 and β_2 are the slope coefficients and λ is the coefficient of error correction term u_{t-1}, this term shows the correction of model towards the long-run equilibrium. If the error correction term is replaced by the lagged variables we get the ARDL model incorporating short-run and long-run information.[8]

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} \Delta \log(i_{t-i}) + \beta_3 \log(m_{t-1}) + \beta_4 \log(i_{t-1}) + u_t \qquad \text{(Eq. 26)}$$

Similarly to estimate the interest semi-elasticity of money demand the ARDL model takes the following form

$$\Delta \log(m_t) = \beta_0 + \sum_{i=1}^{p} \beta_{1i} \Delta \log(m_{t-i}) + \sum_{i=0}^{p} \beta_{2i} (i_{t-i}) + \beta_3 \log(m_{t-1}) + \beta_4 (i_{t-1}) + u_t \qquad \text{(Eq. 27)}$$

We will employ ARDL two-step method of Bahmani-Oskooee and Bohal (2000) and find the maximum lag length (p), the order of UECM and check the existence of long-run relationship. The null hypothesis of no cointegration implies that coefficients of lagged level variables β_3 and β_4 are simultaneously zero. ARDL approach of Pesaran and Shin (1998) can

[8] Long-run elasticity can be derived directly as – (β_4/β_3).

be applied by OLS method and the test is based on comparing the F value (joint significance of lagged levels of variables) of the model with the critical bound values given in Pesaran et al. (2001). It reports the two asymptotic critical bounds values under two conditions (i) lower bounds assuming all the regressors to be I(0) and (ii) upper bound taking all the regressors to be I(1). If the calculated F statistics is less than the lower bound it shows that there is no long-run relationship, if F value falls between the lower and the upper bound it means we enter the indecisive region, its only when F value is greater than the upper bound that there exists the cointegration relationship. After identifying the existence of long-run relationship and maximum lag length, we proceed to the second step find the optimal lag length based on Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC)[9], and $\bar{R}^2$, and calculate the long-run coefficients of the model. Finally diagnostic tests are applied to check that the model passes the functional form stability, heteroscedasticity and serial correlation tests.

4. Data Description

We use annual data on different monetary aggregates. The sample covers the period from 1960 to 2007. We use income measured by gross domestic product (GDP) as a scale variable in our empirical investigation. Monetary aggregates that we use in this paper are M1, monetary base, currency, and demand deposits. Both GDP and monetary assets are deflated by the consumer price index (CPI) to get real balances to real income ratio.

For single monetary aggregates and currency, we use the nominal interest rate (call money rate) as a proxy for the opportunity cost. For the demand deposit model, long term interest rate, the relevant opportunity cost variable is defined as the difference between interest rate offered on other assets (long-term assets) minus own rate of return (the rate of return on current and other deposits).

Data on deposits rates excluding current and other deposits are compiled by the State Bank of Pakistan since 1990. Using State Bank's definition we have calculated it for the period 1960 to 1990 as weighted average of the interest rates on the individual longer term components of time deposits, with weights being quantity shares of these deposits. The calculation of the welfare cost of inflation requires the information of the average reserve ratio for the entire sample period. Reserve Ratio is measured as the reserves taken as ratio to deposits.[10] Data on nominal GDP, monetary stocks, CPI and call money rate are obtained from the International Financial Statistics (IFS) database. The data for rate of return on long-term maturity deposits, however, are taken from State Bank of Pakistan Annual Reports.

5. Estimation Results

5.1 Testing for Unit Root

We begin our examination by checking the stationarity of the data using the Augmented Dickey Fuller (ADF) unit root test. To select the appropriate lag order for the ADF equations,

[9] Computation of ARDL procedure in Microfit 4.0 selects the optimal lag on the basis of maximum values of AIC and SBC.
[10] Following Agenor and Montiel (1996) reserve ratio is measured as (Reserve Money – Currency)/(M1 + Quasi Money – Currency).

we started with zero lag and continued adding lags until the Breusch Godfrey LM test, applied to the residual of the ADF regression, showed no serial correlation. Whether the ADF regression has an intercept only or an intercept along with trend, ADF general to specific method was used as suggested in Enders (2004). Starting with the general form which includes both the constant and deterministic trend, the significance of the trend coefficient based on the t-test is checked. If it is significant and hypothesis of unit root is not rejected, we conclude that the test includes constant and trend.

Table 1 presents the results of the ADF test. The coefficient on liner-time trend term appears statistically significant for only log (demand deposits/GDP) variable. The estimates provide strong evidence that all the variables are nonstationary in their level, while their first differences are stationary. This is, all the series are I(1). As none of the series is integrated of order greater than one, we can apply ARDL bounds test for cointegration.

Table 1: Unit root test results

Variable	Levels			First Differences		
	Lags	Model	τ – value	Lags	Model	τ- value
Log(M1/GDP)	1	constant	-2.6890	0	constant	-5.9508**
Log(Mo/GDP)	0	constant	-2.4537	0	constant	-6.7867**
Log (Currency/GDP)	2	constant	-2.0391	0	constant	-5.4555**
Log (Demand Deposits/GDP)	0	Const & Trend	-2.6438	0	Const & Trend	-7.1718**
Interest Rate	0	constant	-2.4830	0	constant	-6.7077**
Log (Interest Rate)	0	constant	-2.6760	1	constant	-5.7319**
Deposit Rate	0	constant	-1.7544	0	constant	-7.5469**
Log (Deposit Rate)	1	constant	-2.3256	0	constant	-5.3193**

Notes: ADF regression equation : $\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-1} + u_t$

The null and alternative hypotheses for ADF test applies on the coefficient of the first lag of dependent variable γ. Under null hypothesis $\gamma = 0$ or the series is nonstationary and under alternative hypothesis of stationarity $\gamma < 0$. γ has non-standard distribution so τ-value is compared to McKinnon (1991) critical values. Critical values at 5% level of significance are -2.9266 and -3.51074 for the constant only and constant & trend models respectively. ** indicates that the series are stationary at 1% level of significance.

5.2 Estimation of Money Demand Function and Calculation of Welfare Cost of Inflation for Monetary Base

5.2.1 Estimating Demand Function for Monetary Base

As we mention above, we apply two-step ARDL approach. Specifically, in first step, we test the existence of the long-run relationship, using the bound test. After confirmation of the presence of the long-run relationship, the ARDL framework proposed by Pesaran and Shin (1999) is used to estimate the long-run estimates of the underlying variables. We estimate two different specifications of money demand function, namely semi-log and double log demand function, based on monetary base.[11] The F-statistics to testing for the existence of cointegration are sensitive to order of lag in the model, therefore the ARDL (1, 0) is selected based on the Akaik Information Criterion (AIC) and Schwarz Information Criterion (SIC) for both semi- and double-log money demand functions with money taken to be monetary base.

[11] As unit root test showed that monetary base and interest rate series had drift only so the ARDL equation does not include trend term.

Besides to this, several tests are applied to selected model to confirm the volatility of the estimated model. The results are presented in Table 2.

The estimated F-statistics given in Panel A of the table provide evidence of the presence of long-run association between the variables. Specifically, as we can see form the table, the value of F-statistic is greater than the upper critical bounds, indicating the rejection of the null hypothesis of no cointegration. This implies that the variables included in the model have a stable long-run equilibrium relationship. This holds for both semi- and double-log models.

Table 2: ARDL results for monetary base

Panel A: F-statistics for testing the existence of long-run relationship		
No. of optimal lags	**Semi-log demand function**	**Double log demand function**
1	55.744***	53.920***
Panel B: Long-run coefficients		
Regressor	**Coefficient**	**Coefficient**
Constant	-1.366***	-2.665***
Interest rate	-0.054***	-0.331***
$\bar{R}^2$	0.786	0.779
DW-statistic	1.990	1.949
F-statistic	85.263***	82.218***
Panel C: Diagnostic tests		
$\chi^2_{SC(1)}$	0.002[0.960]	0.017[0.894]
$\chi^2_{FF(1)}$	1.343[0.246]	1.102[0.294]
$\chi^2_{N(2)}$	25.125[0.000]	0.159[0.690]
$\chi^2_{Het(1)}$	0.240[0.624]	22.717[0.000]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: unrestricted intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.
For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

The long-run estimates of the models are given in Panel B of Table 2. Both the parameters of the model are significant regardless of whether the model is estimating in semi-log or double-log form. The interest rate semi-elasticity of monetary base (α1) shows that a 1% increase in nominal interest rate lowers the demand for monetary base by 5.4%. The value of adjusted R2 (0.79) shows that the ARDL specification (1, 0) is a quite good fit.

The long-run estimates from the double-log demand function have expected signs and are statistically significant. Interest rate elasticity of the demand for monetary base is 0.33 almost same to those (0.34) estimated in Hussain et al. (2006). The value of adjusted R^2 is 0.77 which shows the goodness of fit of the model. The results of diagnostic tests reported in Panel C of the table provide evidence that both of the models are well-specified and free from the specification errors. Specifically, diagnostic tests indicate that there is no problem of serial correlation, heteroscedasticity and functional form mis-specification in the selected models.

5.2.2 Calculating Welfare Cost of Inflation for Monetary Base

In this subsection we calculate the welfare cost of inflation using Lucas compensating variation measure and consumer's surplus measure for both semi-log and double log models. The results are presented in Table 3.

Welfare cost is measured both as moving from Friedman's optimal inflation rate to some positive inflation rate (from zero nominal interest rate to positive interest rate) and moving from zero inflation (stable price) to positive inflation rate. Real interest rate is approximately 2% for 200712 therefore, i = 0.02 is the benchmark value of nominal interest rate under zero inflation. When i = 0.08 it means inflation rate is 6%, and for i = 0.10 inflation rate is 8%. Table 3 shows the welfare cost as percent of GDP associated with increasing interest rate from zero to a positive rate. Welfare cost entry against each interest rate is the loss in welfare for deviating from the Friedman's Deflation rule.

Table 3: Welfare cost of inflation for monetary base

	Semi-log Model		**Double log Model**	
Interest Rate	Compensation variation approach	Consumer's surplus approach	Compensation variation approach	Consumer's surplus approach
0.00	0.000	0.000	0.000	0.000
0.01	0.007	0.006	0.159	0.159
0.02	0.028	0.026	0.254	0.253
0.03	0.064	0.056	0.333	0.332
0.04	0.115	0.097	0.404	0.402
0.05	0.180	0.146	0.470	0.467
0.06	0.259	0.203	0.531	0.527
0.07	0.353	0.266	0.590	0.584
0.08	0.461	0.336	0.645	0.639
0.09	0.583	0.411	0.699	0.691
0.10	0.720	0.490	0.750	0.742
0.20	2.882	1.395	1.200	1.179
0.30	6.484	2.276	1.583	1.546
0.40	11.528	2.991	1.928	1.874
0.50	18.012	3.523	2.249	2.175
0.60	25.938	3.899	2.552	2.457
0.70	35.304	4.156	2.841	2.724
0.80	46.112	4.327	3.118	2.978
0.90	58.360	4.439	3.387	3.222
1.00	72.050	4.511	3.648	3.457

For Semi-log Model

Compensation variation approach: $w(i) = 0.7205i^2$

Consumer's surplus approach: $WC = \frac{e^{-1.3668}}{5.4999}\left[1 - e^{-5.4999i}(1 + 5.499\,i\ )\right]$

For Double log Model

Compensation variation approach: $w(i) = \left[1 - 0.0696 i_t^{0.66814}\right]^{-0.4967} - 1$

Consumer's surplus approach: $WC = 0.4967e^{-2.665} i^{0.66814}$

Second column of the table shows the welfare cost using compensating variation approach. Welfare cost of 5% nominal interest (3% inflation) is 0.15% of GDP against zero inflation,

[12] Following Gillman (1993) $i = 0.093$ and $\pi = 0.0721$ giving the value of r approximately equal to 0.02.

while comparing with zero nominal interest rate (optimal deflation rule) the cost is approximately 0.18% of GDP. Keeping in view the end of sample period inflation rate of 7% (i = 0.09) the welfare gain of moving towards zero inflation (i = 0.02) is 0.55% of GDP (the difference between the welfare costs at 9% and 2% nominal interest, 0.583 and 0.028 respectively) and further moving to the deflation rate results in an additional gain of 0.028% of GDP.

The welfare cost based on consumer's surplus approach is given in column 3 of Table 3. The welfare cost of 5% nominal interest rate is 0.12% of GDP against price stability and slightly higher at 0.14% of GDP when compared to zero nominal interest rate. Similarly the welfare cost of 9% inflation is 0.41% of against the deflation rate, which is less than the 0.58% of GDP calculated under Lucas (2000) approach form. We find that for all the nominal interest rates the welfare cost is higher under compensating approach than under Bailey's approach.

The costs under two approaches are comparable for the single digit nominal interest rates and the difference widens for the higher interest rates. Deviating from Friedman's Deflation rule, cost of 20% nominal interest rate is 2.8% of income under Lucas' approach, while for the Bailey's approach the cost is 1.4%. The difference between the calculated welfare losses from the two approaches is due to the quadratic nature of the compensating variation formula, in which the nominal interest rate appears in the quadratic form.

For the log-log money demand function, the estimated welfare costs are given in last two columns of Table 3. The welfare cost of 5% nominal interest rate is 0.47% of real income. The welfare cost of 5% inflation against the bench mark of zero inflation is 0.21% of income. The cost of 9% nominal interest rate the call money rate at the end of sample period costs about 0.7% of real output. Reducing nominal interest rate from 9 to 2% (under zero inflation) yields welfare gain equivalent to an increase in income by 0.44%.

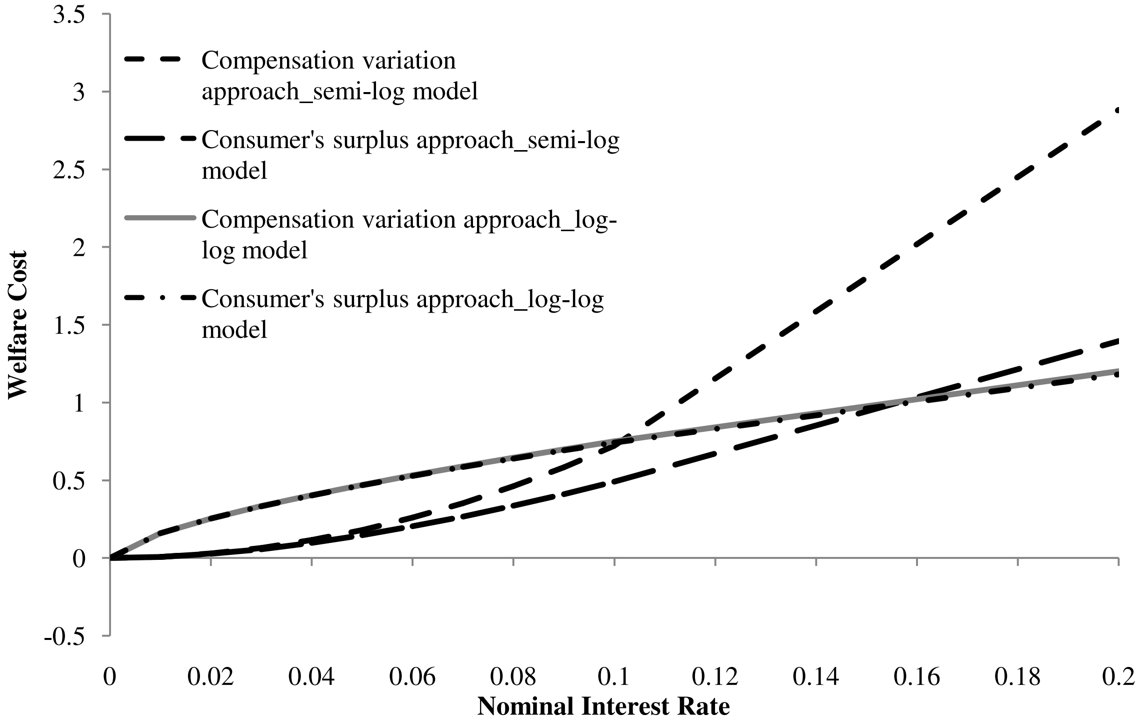
Similar to the case of semi-log money demand function, the welfare costs estimated based consumer's surplus approach are lower than the welfare costs estimated using compensation variation approach. However, for log-log money demand model, the difference is minor.

Comparing the estimated welfare costs across both specifications of demand for money, we find that the welfare cost of inflation for moderate inflation under semi-log money demand function is relatively small compared to that under log-log model. Moving from zero inflation to deflation rule results in welfare gain of only 0.02% of GDP in semi-log model compared to a substantial gain of 0.25% of GDP for the double log model.[13]

The welfare losses (relative to deflation rule) at different nominal interest rates are plotted in Figure 1. Nominal interest rate up to 20% is taken on the horizontal axis. The two approaches give almost same welfare loss calculations for low inflation/ interest rates but they tend to diverge for higher interest rates.

[13] Wolman (1997) and Ireland (2007) show that towards low interest rate the semi-log model shows satiation in money holding but for the double log model the money holdings take asymptotic trend as nominal interest rate approaches zero.

Figure 1: Welfare cost of inflation for monetary base



5.3 Estimation of Money Demand Function and Calculation of Welfare Cost of Inflation for M1

5.3.1 Estimating Demand Function for M1

Table 4 presents the ARDL results for M1. Similar to the case of monetary base, two specification of demand for money, namely semi-log model and log-log model, are estimated. The estimated F-statistic indicates that there is a level relationship (cointegration) between the variables for both semi- and log-log models. The long-run coefficients of money demand function given in Panel B of Table 4 have theoretically correct signs and are statistically significant. The interest rate semi-elasticity of M1/GDP ratio is -3.172, the interest rate elasticity of money demand from log-log model is -0.208. Both models generally satisfy all diagnostic tests.

5.3.2 Calculating Welfare Cost of Inflation for M1

The estimated welfare costs of inflation for both semi-log and log-log models of M1 are presented in Table 5. Specifically, column 2 of the table gives the value of welfare loss against different nominal interest rates based on compensating variation approach. Welfare loss of 3% inflation corresponding to 5% nominal rate of interest is 0.21% of GDP against zero interest rate, while it reduces to 0.17% against price stability. Welfare loss associated with inflation rate of 7% is 0.64% of income compared to a zero inflation rate, while reducing the inflation further to deflation rate results in additional gain of 0.03% of GDP or total gain of 0.67% of GDP. It should be noted that the welfare cost of inflation associated with higher interest rates/inflation rates is substantially high than the welfare cost at lower inflation rates.

It should also be noted that the welfare cost of inflation based on compensation variation approach is higher than the welfare cost of inflation based on consumer's surplus approach through the range of interest rates used in the estimation. However, the difference is more profound at higher interest rates.

Table 4 : ARDL results for M1

Panel A: F-statistics for testing the existence of long-run relationship		
No. of optimal lags	**Semi-log demand function**	**Double log demand function**
3	24.018***	19.311***
Panel B: Long-run coefficients		
Regressor	**Coefficient**	**Coefficient**
Constant	-1.013***	-0.844***
Interest rate	-0.031***	-0.208***
$\bar{R}^2$	0.785	0.744
DW-statistic	2.011	2.019
F-statistic	24.018***	19.311***
Panel C: Diagnostic tests		
$\chi^2_{SC(1)}$	0.027[0.869]	0.081[0.776]
$\chi^2_{FF(1)}$	0.049[0.823]	4.598[0.032]
$\chi^2_{N(2)}$	1.090[0.296]	0.245[0.884]
$\chi^2_{Het(1)}$	0.008[0.996]	1.122[0.289]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: unrestricted intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.
For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

Specifically, we observe that the welfare loss as a proportion of GDP based on consumer's surplus approach rises from 0.02% when the rate of interest is 2% (inflation rate is zero) to over 0.38% at a rate of interest of 9%. Difference between these two welfare costs (0.38-0.02 = 0.36) gives the welfare loss of 7% inflation rate against zero inflation. The welfare cost of 3% inflation is 0.13% of income and moving to zero interest rate yields welfare gain of 0.02% of GDP.

By comparing the welfare cost of inflation under both semi-log and log-log models for M1, we find that welfare cost from semi long money demand function gives higher cost for higher interest. This holds regardless of whether the welfare cost is estimated by using compensation variation approach or consumer's surplus approach. Further, the estimated welfare cost based on the double-log money demand indicates that both the approaches give almost same measure of welfare loss from deviating from the deflation rule. The welfare gain of moving from higher to lower nominal interest rate is almost same for the log-log model but for the semi-log model the welfare gain is associated with the rate of interest. Under the semi-log model one% decrease in nominal interest rate for higher interest rate results in more benefit compared to one percent decrease in nominal interest rate at the lower end of the curve. The welfare costs are plotted in Figure 2.

As in Wolman (1997) we are interested in the apportionment of the total gain of moving form positive interest rate to the deflation rate. This gain has two parts; first gain comes in moving from positive nominal interest rate to price stability and second from moving from zero inflation to the deflation policy. Owing to the sensitivity of the demand curves to low interest rates we find that for the semi-log model larger benefit accrues as the economy move towards zero inflation but further moving to deflation rate has very small gain. Figure 2 shows that for semi-log money demand function under consumer surplus the welfare gain of moving from 12% interest rate to deflation rate is equal to 0.64% of GDP while for double log the gain is 0.81% of income. The proportion of gain from moving from zero inflation to deflation is only 5.15% of the total gain for the semi-log model and for the double log it is 24.2%.

Table 5 : Welfare cost of inflation for M1

	Semi-log Model		Double log Model	
Interest Rate	Compensation variation approach	Consumer's surplus approach	Compensation variation approach	Consumer's surplus approach
0.00	0.000	0.000	0.000	0.000
0.01	0.008	0.005	0.113	0.113
0.02	0.033	0.022	0.197	0.196
0.03	0.074	0.048	0.272	0.270
0.04	0.133	0.084	0.342	0.339
0.05	0.208	0.129	0.409	0.405
0.06	0.299	0.182	0.473	0.468
0.07	0.408	0.243	0.536	0.529
0.08	0.532	0.311	0.596	0.588
0.09	0.674	0.386	0.656	0.645
0.10	0.832	0.467	0.714	0.701
0.20	3.330	1.526	1.251	1.214
0.30	7.493	2.820	1.744	1.673
0.40	13.326	4.144	2.214	2.101
0.50	20.810	5.386	2.669	2.506
0.60	29.976	6.492	3.115	2.895
0.70	40.794	7.444	3.555	3.271
0.80	53.284	8.245	3.990	3.635
0.90	67.446	8.906	4.423	3.990
1.00	83.260	9.445	4.854	4.337

For Semi-log Model

Compensation variation approach: $w(i) = 0.8326i^2$

Consumer's surplus approach: $WC = 0.1145\left[1 - e^{3.1718}\left(1 + 3.1718\, i\right)\right]$

For Double log Model

Compensation variation approach: $w(i) = \left[1 - 0.1643 i_t^{0.7911_1}\right]^{-0.2641} - 1$

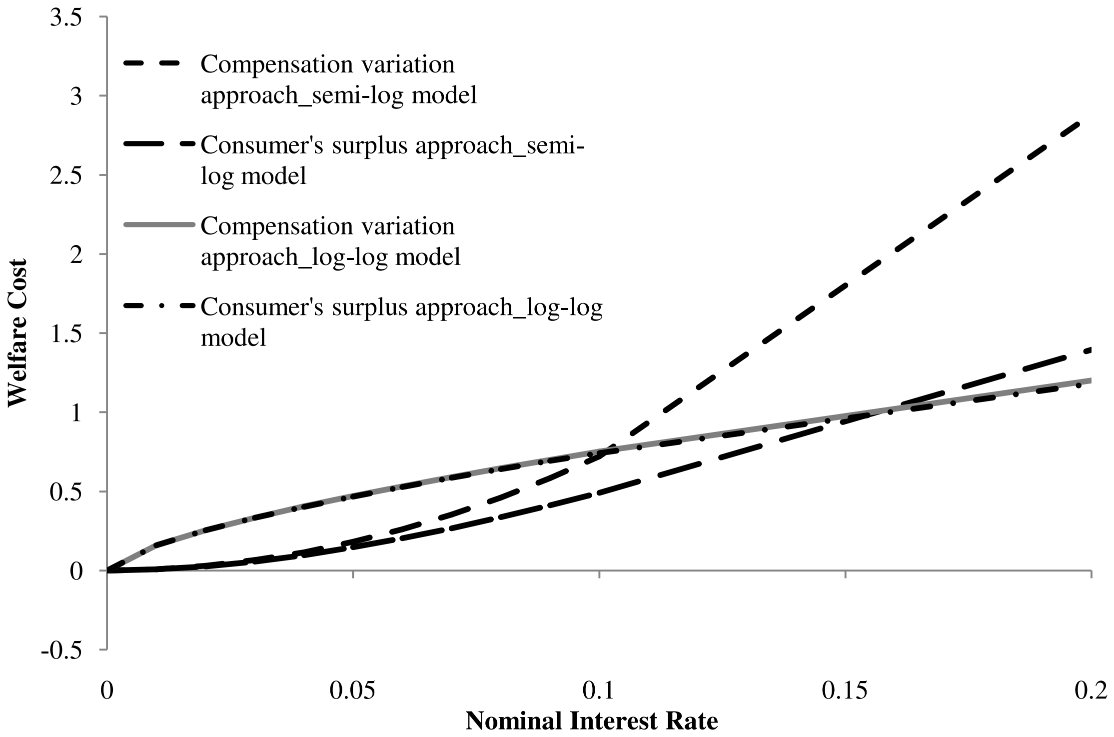
Consumer's surplus approach: $WC = 0.2640 e^{-1.8060} i^{0.7911}$

The difference between the estimates of welfare loss is reduced when the cost is measured relative to zero inflation nominal interest rate. For the present study the end of period real interest rate is 2% which under price stability is equivalent to the nominal interest rate. As shown in Figure 2 the welfare cost of non-optimal policy with positive inflation rate has same welfare loss under the three cases the semi-log model- the lighter colored overlapping lines (drawn under compensating variation and traditional approaches) and the double-log model with consumer surplus approach shown by the dark solid line14. The gain of moving from 12% interest rate to stable prices ranges from 0.61 to 0.63% of income. For both the money demand specifications for M1 the welfare loss is almost same for low and moderate inflation

[14] Lucas (2000) showed that the welfare gain of moving towards price stability is same for both the log-log and semi-log versions.

rates. The welfare loss line drawn for double log model under compensating variation approach diverges from the rest of the three cases as interest rate rises above 10%.

Figure 2: Welfare cost of inflation for M1



5.4 Estimation of Demand Function and Calculation of Welfare Cost of Inflation for Currency-Deposit Model

5.4.1 Estimating Demand Function for Currency-Deposit Model

After estimating the demand function and the welfare costs for the single money stocks, Mo and M1 we estimate welfare loss for the Currency-Deposit Model. We disintegrate the two components of M1 for the reason that both currency and demand deposits don't have the same opportunity cost. Hand-to-hand used currency offers no return; its opportunity cost is the yield on other financial assets while banking system offers interest rate on the demand deposits, opportunity cost of holding demand deposits is the difference between the yield on alternative assets and the return on deposits. This difference requires that both currency and demand deposit demand functions should be estimated separately with their own opportunity costs. This also requires some modification in the welfare cost formulae. We apply the ARDL approach on bivariate models separately for currency and demand deposits using both semi-log and log-log specifications. The optimal lag selected based AIC and SBC is one for all four models. The results are given in Table 6.

The results indicate the existence of long-run relationship for both currency and demand deposits demand functions regardless of whether the model is estimated in semi-log and log-log form. The long-run coefficients from all four models are reported in Panel B of the table. The lower panel of the table shows that the estimated models do not have serial correlation, heteroscedasticity and that the regression passes the functional form mis-specification and the normality tests.

Table 6: ARDL results for currency-deposit model

Panel A: F-statistics for testing the existence of long-run relationship				
	Semi-log demand function		**Double log demand function**	
No. of optimal lags	**Currency**	**Demand Deposit**	**Currency**	**Demand Deposit**
1	149.347***	24.793***	53.921***	23.629***
Panel B: Long-run coefficients				
Regressor	**Coefficient**	**Coefficient**	**Coefficient**	**Coefficient**
Constant	-1.576***	-2.083***	-1.340***	-2.088***
Interest rate	-0.063***	-0.045**	-0.037**	-0.151*
Trend		0.016***		0.015***
$\bar{R}^2$	0.906	0.674	0.903	0.663
DW-statistic	1.640	1.871	1.549	1.737
F-statistic	149.347***	24.793***	144.860***	23.629***
Panel C: Diagnostic tests				
$\chi^2_{SC(1)}$	1.563[0.211]	0.051[0.821]	2.477[0.116]	1.001[0.317]
$\chi^2_{FF(1)}$	0.343[0.558]	0.431[0.511]	0.206[0.649]	0.754[0.385]
$\chi^2_{N(2)}$	3.893[0.143]	0.311[0.856]	5.695[0.058]	0.217[0.897]
$\chi^2_{Het(1)}$	0.262[0.608]	0.647[0.421]	0.069[0.791]	0.553[0.457]

Notes: Asymptotic critical value bounds are obtained from Pesaran et.al.(2001) Table F in appendix C, Case III: unrestricted intercept and no trend for k=1, at 1% level of significance lower bound = 6.84 and upper bound = 7.84, at 5% level of significance lower bound = 4.94 and upper bound = 5.73.
Case V: intercept and trend for k=1, at 1% level of significance lower bound = 8.74 and upper bound = 9.63, at 5% level of significance lower bound = 6.56 and upper bound = 7.30.
For diagnostic test the Lagrange Multiplier statistics $\chi^2_{SC(1)}$, $\chi^2_{FF(1)}$, $\chi^2_{Het(1)}$, and $\chi^2_{N(2)}$ are the distributed with chi-square values for serial correlation, functional form mis-specification, Heteroscedasticity, and Normality test with degrees of freedom in parentheses.

The semi-elasticity of currency-to-GDP ratio is -6.36 which is higher than for any other money stock. On the other hand, the corresponding figure with respect to deposit rate for demand deposits is -4.5. From log-log specification, the interest rate elasticity is -0.037 and -0.151 for currency-to-GDP ratio and demand deposits, respectively.

5.4.2 Calculating Welfare Cost of Inflation for Currency-Demand Model

Using the estimates given in Table 6, we calculate the welfare costs of inflation for unrestricted and restricted models. The results are presented in Table 7. As one can see from column (2) of the table, the welfare gain of moving from 9% nominal interest rate to zero inflation (2% nominal interest rate) is 0.48% of GDP and further moving to deflation rate results in additional gain of 0.18% of GDP. Based on the Bailey's approach, 10% inflation costs equivalent to a reduction of output by 0.38 percent. Under the log-log currency-deposit model the gain in moving from price stability to Friedman's optimal rule of deflation is 0.13% of GDP. The welfare estimates based on both the consumers' surplus and compensating variation approach tend to give similar costs of inflation.

Table 7 : Welfare cost of inflation for currency-demand model

Interest Rate	Semi-log Model: Compensation variation approach: Restricted model	Semi-log Model: Compensation variation approach: Unrestricted model	Semi-log Model: Consumer's surplus approach: Restricted model	Semi-log Model: Consumer's surplus approach: Unrestricted model	Double log Model: Calibration: Double log model	Double log Model: Consumer's surplus approach: Restricted model
0.00	0.000	0.000	0.000	0.000	0.000	0.000
0.01	0.004	0.006	0.004	0.006	0.056	0.074
0.02	0.016	0.027	0.016	0.024	0.102	0.134
0.03	0.036	0.060	0.035	0.052	0.143	0.188
0.04	0.065	0.108	0.061	0.089	0.183	0.240
0.05	0.102	0.168	0.093	0.134	0.221	0.290
0.06	0.147	0.243	0.130	0.185	0.258	0.338
0.07	0.200	0.331	0.172	0.242	0.295	0.385
0.08	0.261	0.432	0.218	0.304	0.330	0.431
0.09	0.331	0.547	0.268	0.370	0.365	0.475
0.10	0.408	0.675	0.322	0.439	0.399	0.520
0.20	1.634	2.702	0.973	1.196	0.724	0.932
0.30	3.677	6.079	1.677	1.881	1.029	1.311
0.40	6.537	10.808	2.315	2.403	1.324	1.670
0.50	10.215	16.887	2.848	2.772	1.612	2.016
0.60	14.709	24.318	3.274	3.027	1.897	2.350
0.70	20.021	33.099	3.607	3.203	2.180	2.676
0.80	26.150	43.232	3.865	3.329	2.461	2.995
0.90	33.096	54.715	4.066	3.424	2.742	3.307
1.00	40.860	67.550	4.225	3.501	3.023	3.614

For Semi-log Model

Compensation variation approach

Restricted model: $w(i) = 0.4086i^2$

Unrestricted model: $w(i) = 0.6755i^2$

Consumer's surplus approach:

Restricted model: $WC = 0.04184\left[1 - e^{-4.52i}(1 + 4.52i)\right] + 0.0274\left[1 - e^{-0.5537i}(1 + 0.5537\,i)\right]$

Unrestricted model: $WC = 0.0325\left[1 - e^{-6.36i}(1 + 6.36i)\right] + 0.0274\left[1 - e^{-0.5537i}(1 + 0.5537\,i)\right]$

For Double log Model

Calibration: $w(i) = \left[1 - (0.15432)i_t^{0.84913}\right]^{-0.1777} - 1$

Consumer's surplus approach: $WC = 0.03614i^{0.84193}$

After estimating the money demand functions and calculating the welfare cost for the three models we draw the following conclusions regarding the welfare cost and its sensitivity to the selection of money demand function, approaches to calculate welfare loss and the definition of money.

(i) Comparing the two approaches to measure welfare loss we find that across all monetary assets under semi-log model, Lucas' quadratic formula gives bigger values of the loss function for higher interest rates. On the other hand, for double log model, the two approaches give approximately the same loss in welfare.

(ii) Welfare cost of inflation is sensitive to the money demand specification. For all the monetary aggregates the welfare gain of moving from price stability to zero interest rate under double log model ranges from 0.10% to 0.25% of GDP, while for semi-log model the gain is trivial and ranges from 0.01% to 0.03% of GDP.

(iii) Bailey and Lucas welfare cost formulae are based on the elasticity and semi-elasticity of money demand function. Long run estimates of both the semi log and double log models show that for all the four money stocks the elasticities and semi-elasticities are different.

(iv) Comparing M1 and the Currency-Deposit model which calculates welfare loss based on different opportunity costs of the constituent components of M1, we find that the welfare cost for currency-deposit model is less than the loss measured using M1. These findings are in line with the empirical literature on welfare cost, that as currency and deposits are lumped together in M1 and cost evaluated at the same market rate of interest for both currency and demand deposits (treating deposits as non-interest bearing asset) exaggerates the true cost.[15]

(v) The welfare cost of inflation is sizable for Pakistan in comparison to the developed countries. Welfare gain of moving from 14% to 3% nominal interest rate is 0.65% of GDP, which is greater than estimated gains for the U.S, Canada and the European countries (for double log specification using Lucas compensating variation approach).[16] Similarly the cost computed from semi-log model and using consumer surplus approach yields welfare loss of 0.06% and 0.62% of GDP as moving from 2% and 10% inflation rates to price stability. This cost is greater than computed for the U.S. which ranges from 0.04 to 0.21% of income under similar settings.[17]

6. Conclusions

In this study we quantified the welfare cost of inflation from the estimated long-run money demand functions for Pakistan for the period 1960-2007 using cointegration approach. Demand functions for four monetary aggregates; monetary base, narrow money (M1), currency and demand deposits taken as ratio to income against their respective opportunity costs are estimated. The welfare cost of inflation calculated for constant interest elasticity specification is compared to the constant semi-elasticity specification for two types of monetary asset models. For the single monetary asset model money stock is defined as monetary base and narrow money M1, while in the currency-deposit model M1 is disintegrated into currency and deposits based on the return on each of its constituent components. In calculating the welfare loss we have employed the traditional approach proposed by Bailey (1956) where loss due to inflation is measured by area under the money demand curve and Lucas (2000) compensating variation approach.

The empirical results show that all the monetary aggregates are negatively related to the interest rate. The welfare gain of moving from positive inflation to zero inflation is approximately same under both money demand specifications but the behavior of the two

[15] Distinct role of currency and deposits is emphasized in Marty (1999), Bali (2000), and Simonsen and Rubens (2001).
[16] See Serletis and Yavari (2003, 2005, 2007).
[17] Ireland (2007).

models is different towards low interest rates. Moving from zero inflation to zero nominal interest rate has substantial gain under log-log form compared to the semi-log function. Compensating variation approach for the semi-log model gives higher welfare loss figures compared to the Bailey's approach due to the quadratic nature of nominal interest rate in the Lucas (2000) welfare measure. However, the two approaches yield approximately similar the welfare cost of inflation for the log-log specification.

The findings of this study suggest that the society bears a substantial loss due to inflation and positive nominal interest rate. This is the first attempt to break new grounds for measuring the welfare cost of inflation for Pakistan. However, limitation of this study is that the welfare cost analysis is based on the direct cost of inflation, not addressing other channels through which inflation results in inefficient allocation of resources. The direct cost of inflation under-states the actual cost of inflation, as inflation tends to distort marginal decisions by altering work-leisure choice and interact with the tax structure of the economy. The actual cost of inflation is much greater than estimated in this study. The State Bank of Pakistan should opt for an independent monetary policy. For the last two years the government has financed its expenditures by borrowing heavily from the SBP, which contradicted the bank's tight monetary policy stance and passed on the signal of rising inflation in the economy. Furthermore, the Taylor Principle driven raising nominal interest rate contribute to inflation through cost side. Best policy contribution to sustain growth and welfare will be to maintain price stability.

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