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Abstract: Motivated by several empirical studies showing a positive relationship between residential density and vehicle fuel efficiency chosen by the residents, this paper presents a modified monocentric city model with endogenous vehicle-type choices. Consumers are assumed to explicitly consider driving inconvenience in the choice of vehicle sizes, and the resulting commuting cost is a function of residential density. This vehicle-type choice problem is embedded in an otherwise standard monocentric city model. A convenience-related advantage in less-dense areas makes our bid-rent curve flatter than that in the standard model. Comparative static analyses suggest that an increase in commuting cost per mile, especially from increased unit cost of driving inconvenience, may induce spatial expansion of the city. Since driving inconvenience is lower in less-dense suburbs, the increased unit cost of driving inconvenience pulls people toward suburbs, potentially leading to urban sprawl. Part of comparative static analysis shows how the city's vehicle fuel efficiency depends on the city characteristics such as population and agricultural rent.

Key words: Monocentric City Model, Vehicle Fuel Efficiency, Driving Inconvenience, Urban Sprawl

JEL Classification: R13, R14, R41

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1 Introduction

There have been growing concerns about the increased energy consumption potentially caused by urban sprawl, which has characterized the land development pattern in the US since 1950.[1] In response, many researchers attempted to estimate the causal influence of land-use patterns (mostly measured by population density) on automobile travel demand (Schimek (1996), Levinson and Kumar (1997), Boarnet and Sarmiento (1998), Boarnet and Crane (2001), Bento et al. (2005)). These studies suggest that land-use patterns have a statistically significant influence on household vehicle usage. Specifically, although the empirical magnitudes differ, the findings indicate that lower neighborhood densities at the residential location tend to induce a higher household vehicle usage (see Badoe and Miller (2000) for literature review).

Meanwhile, several recent studies document that land-use patterns affect not only vehicle usage but also the vehicle fuel efficiency chosen by residents (Fang (2008), Brownstone and Golob (2009), Kim and Brownstone (2012), Newman and Kenworthy (1999)). Along with automobile travel demand, vehicle fuel efficiency is an important determinant of total energy consumption. Indeed, fuel efficiency regulations have been viewed as the key instruments for reducing greenhouse gas emissions and the country's oil dependence. Corporate Average Fuel Economy (CAFE) standards, for example, impose fuel economy standards for new vehicles sold in the US, and there has been wide interest among researchers about the program's cost and effectiveness (see Klier and Linn (2011) for literature review). The motivation of this paper comes from these interests in fuel-efficiency control and its linkage with post war suburbanization in the US, as evidenced by the empirical relationship between fuel efficiency of the chosen vehicles and the resident's location choice. This paper presents a modified monocentric city model, which incorporates the link between the resident's location and land-consumption decisions and the decision on vehicle fuel efficiency.

[1] Kahn (2000) discusses the effects on energy usage and its environmental consequences of suburbanization. Perry et al. (2007) discuss negative externalities from increased energy consumption.

Several recent empirical findings provide evidence on the relationship between a particular neighborhood feature, population density, and the vehicle fuel efficiency chosen by the resident, controlling for the resident's other characteristics such as income. People residing in less dense suburban areas drive more than people in high density areas, and their fuel consumption shows a larger proportional increase than vehicle miles traveled (Brownstone and Golob (2009), Kim and Brownstone (2012)). This disproportionate increase in fuel usage suggests that households residing in less dense areas are more likely to use less fuel-efficient (bigger) vehicles than households in denser areas. Unlike Brownstone and Golob (2009), where vehicle choice is just implicit, Fang (2008) explicitly models vehicle choices, with residential density used as a key explanatory variable. Fang (2008) finds that when density increases, people tend to switch from trucks to cars and from large-size cars to small-size cars, which implies a positive correlation between residential density and vehicle fuel efficiency. Moreover, West (2004) finds that people choosing a less fuel-efficient bigger car tend to utilize the vehicle more and are usually located in the mid-west and south of the US, where residential density is lower. Reinforcing these findings, using a global sample of 32 cities, Newman and Kenworthy (1989) find a disproportionate increase in fuel consumption in cities with low density, where automobile dependency is high.

While fuel cost per mile is higher when density is lower, as suggested by the studies described above, other studies show that another factor, driving inconvenience, may work in the opposite direction, tending to lower commuting cost per mile in less dense suburbs. In an attempt to jointly estimate vehicle choice and vehicle utilization, several studies find that high density at the residential location yields lower marginal utility from driving, or a higher dislike of driving (Gillingham (2010), West (2004)). High density neighborhoods may reduce the speed of travel, presumably because of longer search time for parking and congestion (Levinson and Kumar (1997)). This finding suggests that a high neighborhood density may actually lead to a higher per mile cost of travel through increased time cost, despite the use of more fuel efficient vehicles.

In our model, the cost of driving inconvenience captures these incremental costs of travel in high density areas. The goal is to analyze the consumer's joint decision on land consumption and vehicle size (fuel efficiency) from an urban economics perspective, reflecting the empirical literature described above. As far as we know, no urban models exist that focus on this joint decision.[2] In the model, the consumer explicitly considers driving inconvenience, a kind of subjective commuting cost indicating how much the driver dislikes driving, in the choices of land consumption, residential location, and vehicle size. Driving inconvenience is assumed to be influenced by vehicle size (inverse of fuel efficiency) and residential density. A larger vehicle gives lower driving inconvenience, which reflects the greater comfort and safety offered by larger cars, holding neighborhood density fixed. We assume that high neighborhood density gives higher driving inconvenience, motivated by higher parking cost, congestion, and a worse driving environment in denser areas. Commuting cost per mile is then comprised of driving inconvenience and monetary costs related to vehicle size, including fuel costs. A resident chooses an optimal vehicle size that yields the lowest commuting cost per mile. The resulting minimized commuting cost per mile is expressed as a function of residential density.

This optimization problem is then imbedded in an otherwise standard monocentric city model. While residential densities in such a model ultimately depend on commuting cost, our framework also generates a reverse causal link running from density to commuting cost via driving inconvenience. The goal is to analyze the properties of the modified model taking this two-way linkage into account.

We first derive the modified bid-rent function to see how it differs from that in the standard model. Next, we carry out the kind of comparative static analysis originally done by Wheaton (1974) and elaborated by Brueckner (1987). In spite of the joint determination

[2]The limitation of using a monocentric city model, however, is that vehicle utilization is actually the same as the residential location choices. Once the consumers choose location, they commute the given length from the residential location to the central business district (CBD). Thus, there is nothing new about vehicle utilization in our model. The innovation in our model is the incorporation of the joint decision on residential location and vehicle fuel efficiency, which determines the cost per mile of commuting.

of density and commuting cost, the effects of increases in the urban population, agricultural rent, and consumer income on utility and city size are qualitatively the same as in the standard model. But, unlike the standard urban model, the influence on the city's spatial size of an increase in commuting cost per mile, especially from increased unit cost of driving inconvenience, is ambiguous. We can identify the source of this ambiguity as the better driving conditions that are more likely offered in less-dense suburbs. Because of the convenience-related benefits in less-dense suburbs, the increased unit cost of driving inconvenience actually pulls people toward the suburbs, where inconvenience is lower, potentially leading to urban sprawl. The main findings are confirmed by a numerical analysis.

This new result may overcome a limitation of the standard commuting-cost-based explanation for urban sprawl. The standard model suggests that a declining transport cost is a major source of urban sprawl, and this result is used in many studies to explain urban sprawl (Brueckner (2001), Glaeser and Kahn (2004), Baum-Snow (2007)). However, the pattern of declining transport cost is not entirely satisfactory as an explanation for urban sprawl because the time cost of travel, which is the largest portion of transport cost, would have increased over the same period because of rising incomes (Anas, Arnott, Small (1998)). Our model fills this gap by providing a way for the increased time cost of travel to generate urban sprawl.

Part of the comparative static analysis shows how vehicle size (or fuel efficiency) depends on city characteristics such as population and agricultural rent. Thus, we are able to represent the city's fuel efficiency as a function of the exogenous parameters characterizing the urban equilibrium, which would then provide empirical implications. Finally, as an extension of the model, we analyze the residential location and the vehicle choice patterns by heterogeneous consumers.

The existing literature on endogenous commuting costs in the monocentric city framework mostly focuses on policies for remedying the externalities caused by unpriced traffic congestion (Arnott (1979), Pines and Sadka (1985), Wheaton (1998), Brueckner (2007)). In these congested-city models, commuting cost per mile is influenced by the volume of traffic flows. In our model, the cost of commuting per mile is instead represented as a function of density at the residential location. The reason is that we want to tie our modeling with the empirical literature concerning the policy issue. As described above, the empirical literature is concerned about the effect of the built environment (mostly measured by density), which is the relevant policy instrument for urban planners, on vehicle usage and vehicle choice patterns.

The rest of the paper is organized as follows. Section 2 proposes the model. Section 3 implements the comparative static analysis. Section 4 presents numerical examples. In Section 5, we analyze the heterogeneity of households. Finally, section 6 concludes.

2 The model

2.1 Commuting cost function

Commuting cost per mile, denoted by t, is comprised of driving inconvenience and pecuniary cost, with $t = \alpha I + F$, where I is driving inconvenience, F is pecuniary costs, and α is the unit cost of driving inconvenience. The monetary commuting cost, F, captures vehicle size because bigger cars are less fuel-efficient and typically cost more to purchase. Driving inconvenience is a kind of subjective commuting cost, indicating how much the driver dislikes driving. Driving inconvenience may also capture time costs of travel.[3]

Driving inconvenience (I) is assumed to be a function of population density at the residential location (or residential density, denoted by D) and vehicle size (F). First, driving inconvenience (I) depends inversely on vehicle size (F) holding residential density fixed, reflecting the greater comfort, safety, and higher speed offered by larger cars. A consumer purchasing a more expensive, larger car is compensated by a lower inconvenience. Second,

[3]Note that we could put the subjective cost of driving inconvenience (I) in the utility function. However, following the traditional way of putting the subjective time cost of commuting (from sacrificing leisure) in the budget constraint, we treat this subjective cost as being paid out of income.

we assume that high neighborhood density gives higher inconvenience. As a result, driving inconvenience follows the relationship $I = \tilde{I}(D, F)$, with $\tilde{I}_D > 0$ and $\tilde{I}_F < 0$ (subscripts denote partial derivatives).

The link between driving inconvenience and population density could be motivated via the greater difficulty of parking, worse driving environment, and higher congestion (greater time cost) in a dense neighborhood. In the suburbs where land rent is cheaper and parking spaces are larger, consumers may have a private garage and cars are also easier to park. People may enjoy driving or dislike driving less when they drive in the less-dense suburbs because they expect fewer traffic lights, wider streets, and more highways, which provide a better driving environment. This intuition is evidenced by Gillingham (2010) and West (2004), where in an attempt to jointly estimate vehicle choice and vehicle utilization, the authors found that more-dense areas correspond to a lower marginal utility from driving, or higher dislike of driving. High density may also lower the speed of travel, presumably because of longer search time for parking and higher congestion in high density neighborhoods (Levinson and Kumar (1997)). Then, driving inconvenience may be interpreted as being associated with the time cost of travel.

It is also assumed that $\tilde{I}$ is twice-differentiable, with $\tilde{I}_{FF} > 0$, meaning that the convenience advantage of a bigger car increases at a diminishing rate. Also, the convenience disadvantage of a denser area increases as cars get larger, so that $\tilde{I}_{DF} > 0$. From the observation that a high density neighborhood tends to have smaller parking spaces, narrower streets and more congestion, the incremental inconvenience cost from a higher density will be higher for a bigger car than for a smaller car.

The consumer is then faced with a vehicle-type choice problem, choosing a vehicle size that gives the lowest commuting cost per mile. The consumer minimizes $t = \alpha\tilde{I}(D, F) + F$ by choice of F, which yields the optimal vehicle size as a function of D, giving $F = \tilde{F}(D)$. Totally differentiating the first-order condition, $\alpha\tilde{I}_F + 1 = 0$, with respect to D and F and using $\tilde{I}_{FF} > 0$ and $\tilde{I}_{DF} > 0$, it follows that $\partial\tilde{F}/\partial D < 0$. The empirical negative relationship between residential density and fuel cost per mile is confirmed in this way. This derivation reflects our view that the empirical evidence on the relationship between residential density and vehicle sizes is caused by households' behavior of adjusting their vehicle-types to the built environment around their residences.

Substituting the optimal vehicle size $F = \tilde{F}(D)$ into the commuting cost per mile yields the commuting cost function $t(D) = \alpha\tilde{I}(D, \tilde{F}(D)) + \tilde{F}(D)$, which gives the minimized commuting cost per mile. While we made it clear that commuting cost per mile is a function of neighborhood density, writing t as a function of $1/D$ is more convenient when we analyze the general equilibrium in the next section. Letting ℓ denote land consumption per person in the residential neighborhood, ℓ is equal to $1/D$ from the definitions. Let the commuting cost function be written as a function of ℓ $(= 1/D)$:

$$t(\ell) = \alpha I(\ell, F(\ell)) + F(\ell), \tag{1}$$

in which I and F are now written as functions of ℓ. Note that $\tilde{}$ in $\tilde{I}(D, F)$ and $\tilde{F}(D)$ are dropped with this replacement. Now the claim is that driving inconvenience is increasing in D ($\tilde{I}_D > 0$), or equivalently, decreasing in land consumption per person, ℓ, giving $I_\ell < 0$. In the same manner, the optimal vehicle size is decreasing in D ($\partial\tilde{F}/\partial D < 0$), or equivalently, increasing in ℓ, giving $\partial F/\partial\ell > 0$. Note that ℓ must be distinguished from land consumption "chosen" by the resident, denoted by q.[4] While ℓ captures density at the neighborhood level, q represents the individual's choice. Although $\ell = q$ will hold in equilibrium, the resident chooses q, not ℓ, a distinction that is made clearer in the next section.

Using the envelope theorem, the derivative of commuting cost per mile with respect to ℓ equals

$$t_\ell = \alpha I_\ell < 0. \tag{2}$$

[4]Since housing production is suppressed in this model, housing consumption is equivalent to land consumption.

Thus, commuting cost per mile is an increasing function of residential density, or equivalently, a decreasing function of land consumption per person, ℓ. While fuel cost per mile (F) is lower in denser areas, (2) shows that the cost of driving inconvenience (I) leads to an overall commuting cost per mile (t) that is higher in denser areas.

2.2 Incorporating vehicle choice in the monocentric model

In this section, we add the vehicle choice framework to the monocentric city model. The city is circular and contains the central business district (CBD) at its center. Each resident in the city commutes to the CBD to earn income y, using a radial road network. Commuters rely on only car travel, and commuting cost per mile is $t(\ell)$, as derived in the previous section. The disposable income of a consumer at distance x is then $y - t(\ell)x$. Consumer utility depends on land consumption, q, and a composite non-housing good, c. The rental price per unit of land is p and the price of the non-housing composite good is normalized to unity. The budget constraint is then $c + pq = y - t(\ell)x$. Consumers have the common quasi-concave utility function, $v(c, q)$, and elimination of c allows utility to be written as $v(y - t(\ell)x - pq, q)$. The consumer maximizes this expression by choice of q subject to the budget constraint.

Note that commuting cost per mile (t) in the housing consumption choice problem is a function of residential density (or $\ell = 1/D$), not a function of the resident's housing consumption (q). In effect, each resident in the city takes residential density (D) and consequently commuting cost per mile (t) as fixed when she decides q, neglecting the influence of her land consumption on neighborhood density. Hence, q is chosen viewing t as fixed, leading to the first order condition, $v_q(y - t(\ell)x - pq, q) = v_c(y - t(\ell)x - pq, q)p$. But, $\ell = q$ must hold in equilibrium. Although each individual resident takes residential density as fixed, the aggregated q choices will determine residential density, so that $\ell = q$ holds in equilibrium. Therefore, the commuting cost function, $t(\ell)$, must be replaced by $t(q)$ in writing the consumer equilibrium condition, which becomes

$$v_q(y - t(q)x - pq, q) = v_c(y - t(q)x - pq, q)p. \tag{3}$$

An additional equilibrium condition requires that the identical consumers in the city attain the same utility level u. Spatial variation in p provides the key to achieving equal utilities throughout the city. Using $\ell = q$, the equal-utility condition is written

$$v(y - t(q)x - pq, q) = u. \tag{4}$$

The simultaneous equations (3) and (4) yield solutions for p and q as functions of location and the other exogenous parameters. The solution for p as a function of x, $p(x)$, gives the 'bid-rent' function for housing.

Our bid-rent function is comparable with that in the standard model. To derive the slope of the bid-rent function, (4) is totally differentiated with respect to x using $v_q = v_c p$, which yields

$$\frac{\partial p}{\partial x} = -\frac{t(q)}{q} - \frac{1}{q} t_q x \frac{\partial q}{\partial x}, \tag{5}$$

where t_q denotes the partial derivative of t with respect to q ($\equiv 1/D$). Given $q = \ell$, we have $t_q = t_\ell = \alpha I_\ell < 0$ from (2). The second term in (5) accounts for the change in commuting cost from an increase in x, and it does not appear in the standard model, where the bid-rent slope is given by $-t/q$. After totally differentiating (3) with respect to x and solving for $\partial q/\partial x$, substituting it into (5) yields the following result:[5]

$$\frac{\partial p}{\partial x} = -\frac{t(q)}{q} \left[\frac{\eta}{\frac{t_q x}{q} + \eta} \right] < 0, \tag{6}$$

[5] Totally differentiating (3) with respect to x gives $\left[t_q x(pv_{cc} - v_{qc}) + v_{qq} - pv_{qc} - pv_{cq} + p^2 v_{cc}\right] (\partial q/\partial x) = (v_{qc} - pv_{cc})t(q) + (v_c - pv_{cc}q + v_{qc}q)(\partial p/\partial x)$.

where $\eta \equiv (v_{qq} - 2pv_{cq} + p^2 v_{cc})/v_c$. By substituting $p = v_q/v_c$ into η, it is easily seen that $\eta = \partial MRS/\partial q|_{v=u}$, where $MRS \equiv v_q/v_c$. The convexity of indifference curves implies $\eta < 0$. Recalling $t_q < 0$, $\partial p/\partial x$ is then negative, meaning that an increase in x leads to a utility-equalizing decline in p. Since utility is fixed, the increase in q with respect to x is exactly the substitution effect of the decrease in p. Moreover, $\partial q/\partial x = (\partial p/\partial x)(1/\eta)$ (> 0) holds as in the standard model, implying a decline in density as x increases.

Note that the bid-rent slope (6) contains an additional term, $\eta \left(t_q x/q + \eta\right)^{-1}$, which does not appear in the standard urban model, where the bid-rent slope is given by $-t/q$. Also note that the new factor, $\eta \left(t_q x/q + \eta\right)^{-1}$, is between zero and one.[6] Since the fall in density implies a benefit from lower driving inconvenience as x increases, we can say that additional distance from the CBD (and thus extra commuting costs) can be compensated by a smaller decrease in p than in the standard model. In other words, less of a decrease in p is needed for compensation in this model since there is a convenience-related benefit from a decrease in density as x increases.

The other partial derivatives ($\partial p/\partial \theta$ and $\partial q/\partial \theta$, where $\theta = y,\ \alpha,\ u$) hold less intrinsic interest. But, these partial derivatives are needed in the comparative static analysis in the next section. The partial derivative signs are as follows (see Appendix A):

$$\frac{\partial p}{\partial y} > 0, \quad \frac{\partial p}{\partial \alpha} < 0, \tag{7}$$

and the corresponding q derivatives have the opposite signs. The signs and the formulas match those of the standard model, except for the presence of the extra term in (6), $\eta \left(t_q x/q + \eta\right)^{-1}$. So, the rule that makes the price derivatives smaller in absolute value than in the standard model is applied here again. While $\partial q/\partial u > 0$ holds unambiguously, a suf-

[6] A direct comparison between the magnitudes of the bid-rent slopes in the present model and that in the standard model is not available since q and t would differ across models. Nevertheless, we can say that a different rule is applied in this model.

ficient condition is needed to determine the sign of $\partial p/\partial u$ (see Appendix A). In particular,

$$\frac{\partial p}{\partial u} < 0 \quad if \quad p + t_q x \geq 0. \tag{8}$$

The expression $p + t_q x$ gives the marginal cost of an increase in q when the effect on driving inconvenience is taken into account. If this expression were negative, then (taking general equilibrium effects into account) the consumer could acquire more housing for a lower overall cost. The implausibility of this outcome makes this assumption that $p+t_q x \geq 0$ a reasonable one. In addition, $p + t_q x < 0$ would imply $\partial p/\partial u > 0$, so that a higher utility would be implausibly associated with a higher housing price. Thus, the comparative static analysis is carried out with the presumption that $p + t_q x \geq 0$, and consequently $\partial p/\partial u < 0$. We check whether $p + t_q x \geq 0$ holds in the numerical examples below.

3 Comparative static analysis

We carry out a comparative static analysis to explain the intercity differences in spatial structures among cities. Through the analysis, we investigate how the qualitative and quantitative properties of the standard urban model are modified under the current framework. Part of the comparative static analysis will show how vehicle fuel efficiency depends on city characteristics such as population and agricultural rent.

As usual, the comparative static analysis requires two additional conditions that characterize the overall equilibrium of the city. The first equilibrium condition requires that the urban population, denoted by L, exactly fits inside the urban fringe, which is denoted by $\bar{x}$. The equilibrium condition is written as

$$\int_0^{\bar{x}} \frac{2\pi x}{q(x, y, \alpha, u)} dx = L. \tag{9}$$

The second equilibrium condition requires that urban land rent equals the exogenous agricultural rent p_A at the urban fringe. This condition is written as

$$p(\bar{x}, y, \alpha, u) = p_A. \tag{10}$$

Under the closed city assumption, which makes u and $\bar{x}$ endogenous, we analyze the effects of the exogenous variables, L, p_A, y, and α, on u and $\bar{x}$, using these two equilibrium conditions as well as (3) and (4). Here, the income level is exogenous, which means that the urban rent is paid to absentee landlords. This assumption is distinguished from the case of a fully closed city, where the rent is redistributed to the urban residents.[7] We also investigate the nature of dependencies of vehicle size (F) on the exogenous variables.

3.1 The effects of increases in population and agricultural rent

The standard derivation of the effects of L and p_A on u and $\bar{x}$ only makes use of the signs of the p and q derivatives. These signs in the current model are the same as in the standard model (see Appendix A). So, the standard proof applies, yielding

$$\frac{\partial u}{\partial L} < 0, \quad \frac{\partial \bar{x}}{\partial L} > 0, \quad \frac{\partial u}{\partial p_A} < 0, \quad \frac{\partial \bar{x}}{\partial p_A} < 0. \tag{11}$$

Thus, an increase in the city population reduces the utility level while inducing expansion of the city. An increase in the agricultural rent reduces both the utility level and the city size.

The urban population and the agricultural rent influence the vehicle fuel efficiency chosen by residents by affecting population densities in the city. Since L and p_A are not direct arguments of F, L and p_A have only an indirect effect on F, which operates through ℓ ($= 1/D$). Since $\ell = q$ holds in equilibrium, this effect is given by

$$\frac{dF}{d\lambda} = \frac{\partial F}{\partial q}\frac{dq}{d\lambda}, \quad \lambda = L,\ p_A, \tag{12}$$

[7] Pines and Sadka (1986) carry out the same kind of comparative static analysis under the fully-closed-city assumption. Their comparative static results are quite consistent with that in Wheaton (1974) and Brueckner (1987), where landlords are absentee, except for the influence of the city population.

where $\partial F/\partial q > 0$ holds as shown in Section 2.1. Since L and p_A are not direct arguments of q, there is only an indirect influence on q, which operates through u. The following holds:

$$\frac{dq}{d\lambda} = \frac{\partial q}{\partial u}\frac{\partial u}{\partial \lambda}, \quad \lambda = L,\ p_A. \tag{13}$$

Using the partial derivative signs shown above, the following comparative static signs are derived:

$$\frac{dF}{dL} < 0, \quad \frac{dF}{dp_A} < 0. \tag{14}$$

Thus, vehicle sizes get smaller (or fuel efficiency increases) throughout the city as L increases. An increase in population leads to higher densities at all locations in the city, and people adjust their vehicles toward smaller and more fuel efficient ones to fit their higher neighborhood density. This conclusion is consistent with Newman and Kenworthy (1999), where the authors found that big and more populous cities tend to have higher average fuel efficiency. In the same manner, an increase in the agricultural rent, by inducing higher densities throughout the city, improves vehicle fuel efficiency throughout the city.

3.2 The effects of an increase in income

An increase in income raises the utility level and induces expansion of the city. By totally differentiating the equilibrium conditions (9) and (10), and using the partial derivative expressions given in Appendix A, we get the following comparative static signs (Appendix B shows the derivation):

$$\frac{\partial u}{\partial y} > 0, \quad \frac{\partial \bar{x}}{\partial y} > 0. \tag{15}$$

Although the above partial derivative signs are unambiguously determined and the same as in the standard model, the current model has the additional terms in the partial derivatives (see (6) and (22) - (25) in Appendix A), which would modify quantitative influences of an increase in y on $\bar{x}$. Indeed, our new commuting cost function will accentuate the urban-sprawl force of an increase in y (i.e., $\partial\bar{x}/\partial y$) because there is a convenience-related advantage from an increase in q in our model. Since an increase in q has an additional benefit in our model through lower driving inconvenience from a lowered density, the incremental demand for housing from an increase in y will be larger in our model than in the standard model, which would cause the city expand further in our model. The numerical examples below show that the increase in $\bar{x}$ from an increase in y is higher at higher α values, implying that y's urban sprawl force is accentuated as α increases. Thus, the presence of driving inconvenience in our model, interacted with rising incomes, may help to explain post war suburbanization in the US.

Consumer incomes in the city influence the optimal vehicle size at each location. Since y is not a direct argument of F, y affects F only through the induced change in ℓ $(= q)$.[8] But, the influence of y on q is not immediate since an increase in y has both a direct effect on q and the effect operating through u. The total derivative of F with respect to y is written as

$$\frac{dF}{dy} = \frac{\partial F}{\partial q}\frac{dq}{dy} = \frac{\partial F}{\partial q}\left(\frac{\partial q}{\partial y} + \frac{\partial q}{\partial u}\frac{\partial u}{\partial y}\right). \tag{16}$$

Appendix D shows that dq/dy is ambiguous throughout the city, which leads to ambiguous F changes at all locations.[9] Our numerical examples below show, however, that q rises and population density falls as y increases at all locations under reasonable parameter values, leading to larger vehicles throughout the city.

The comparative static results discussed so far are summarized as follows:

Proposition 1 *The effects of increases in population, agricultural rent, and consumer in-*

[8]The vehicle-type choice problem in section 2.1 does not involve consumer income.

[9]In the standard model, q rises at central locations as y increases, but the change in q in response to the increase in y is ambiguous in the other parts of the city. In our model, however, the q change is ambiguous even at the city center because the convenience-related benefit from a higher q drives up the bid-rent further than in the standard model, which results in ambiguous p and q changes at the city center.

come on utility and city size are qualitatively the same as in the standard model. Increases in population and agricultural rent, by making the city denser, reduce vehicle size and thus raise fuel efficiency throughout the city. An increase in consumer income has an ambiguous effect on vehicle fuel efficiency at all locations.

3.3 The effect of an increase in unit cost of driving inconvenience

We now investigate the effects of an increase in commuting cost per mile. In our model, the increased commuting cost per mile comes from an increase in α, a component of $t = \alpha I(\ell, F(\ell)) + F(\ell)$. As easily predicted, a higher α value is associated unambiguously with a lower utility level, so that $\partial u/\partial \alpha < 0$. However, the effect of an increase in α on the city's spatial size is ambiguous, unlike in the standard urban model, where the city shrinks spatially in response to an increase in commuting cost per mile.

We formally show below how an increase in α can actually lead to spatial expansion of the city. But here is a brief interpretation. The increase in α creates two opposite forces that operate through the choices of location and land consumption. First, an increase in commuting cost per mile makes commute trips of any given length more expensive, with the result that the city center becomes more attractive while the suburbs become less attractive. Furthermore, the household's disposable income decreases at any given location. Therefore, the resident would choose to consume less space in response.[10] Consequently, the city tends to shrink in response to an increase in commuting cost per mile.

However, there is an opposite force of the increase in α. Recall that commuting cost per mile decreases with q through the convenience-related advantage in a less dense area ($t_q = \alpha I_\ell < 0$). The parameter α determines the magnitude of this convenience-related advantage. Specifically, as α increases, the convenience-related advantage in the less dense

[10]But, not all residents decrease their space consumption. Based on the analysis of the standard urban model (Brueckner (1987)), an increase in commuting cost per mile raises the rent at central locations while lowering the rent at more distant locations, inducing a clockwise rotation of the bid-rent curve. Consequently, housing consumption decreases in the central city, but the effect on land consumption in the suburbs is ambiguous. In our model, the effect of an increase in α on bid-rent at the old $\bar{x}$ is ambiguous, so the effect on land consumption in the outer part of the city is still ambiguous.

area gets larger. As α increases, the higher q in the suburbs and the resulting better driving condition becomes more important to the resident in the choice of location. This is another effect of the increase in α, which makes consumers prefer the wider spaces and pulls them toward the suburbs. If this second effect of α dominates, the city size may actually increase in response to an increase in α.[11]

The effect of α on $\bar{x}$ depends on how rapidly driving inconvenience falls moving toward the suburbs. Note that dI/dx is in fact negative, which is easily seen by $dI/dx = (I_\ell + I_F F_\ell)(\partial q/\partial x) < 0$. The claim is that the city's spatial expansion with the increase in α ($\partial \bar{x}/\partial \alpha > 0$) is more likely to occur when driving inconvenience falls faster as x increases, so that the gap between driving inconvenience at any given location and at the urban fringe ($I - \bar{I}$) becomes larger.

We now turn to analysis of the effect of an increase in α on vehicle sizes. The parameter α has a direct influence on F as well as an indirect influence, which works through q $(= \ell)$. Moreover, an increase in α has both a direct effect on q and the effect operating through u. The total derivative of F with respect to α is given by

$$\frac{dF}{d\alpha} = \frac{\partial F}{\partial \alpha} + \frac{\partial F}{\partial q}\frac{dq}{d\alpha} = \frac{\partial F}{\partial \alpha} + \frac{\partial F}{\partial q}\left(\frac{\partial q}{\partial \alpha} + \frac{\partial q}{\partial u}\frac{\partial u}{\partial \alpha}\right). \tag{17}$$

Appendix E shows that the total effect of an increase in α on F is ambiguous throughout the city, as was the effect of y.

The comparative static results with respect to α are summarized as follows:

Proposition 2 *Higher commuting cost from an increase in the unit cost of driving inconvenience reduces the utility level. However, its effect on the city's spatial size is ambiguous.*

[11]By the convenience-related benefit in less-dense areas, we mean that "per mile" cost of commuting declines moving toward the less-dense suburban location, which is seen by $dt/dx = t_q(\partial q/\partial x) < 0$. The direction of the total commuting cost, $d(tx)/dx = t + t_q(\partial q/\partial x)x$, is analytically ambiguous. But, according to our numerical results, the total commuting cost increases with x, but at a diminishing rate due to the property of $dt/dx < 0$. Indeed, the property that "per mile" cost of commuting declines with an increase in x is sufficient to generate the second force. The direction of the total commuting cost is not related to the second force.

The city may spatially expand with the increase in α when the gap between driving inconvenience at any given location and at the urban fringe ($I-\bar{I}$) is large. The effect of an increase in α on vehicle fuel efficiency is ambiguous at all locations.

Proof. See Appendix C. ■

Thus, the model's notable difference from the standard model is the effect of α on city size. An increase in α heightens the convenience advantage of the suburbs, with the gain related to the magnitude of $I - \bar{I}$. A large gain in the suburbs (large $I - \bar{I}$) pulls people toward the suburbs more strongly, raising the possibility that $\bar{x}$ rises with α.

The potential positive influence of α on $\bar{x}$ suggests that the increase in α could help explain the post war suburbanization in the US. A declining commuting cost per mile, the standard explanation for suburbanization, has drawbacks as an explanation for the post war pattern. The reason is that the time cost of commuting, the largest portion of transport cost, would have increased over this period as a result of secular wage increases, perhaps causing commuting cost to rise rather than fall, despite investment in transportation infrastructure (Anas, Arnott, Small (1998)). Our model overcomes this challenge faced by the standard model by suggesting that an increased unit cost of driving inconvenience, which may be associated with the increased time cost of travel, would contribute to urban sprawl.

However, our claim about the effect of an increase in α on city size may face a challenge. Our commuting cost function depends on density only at the residential location, so it cannot fully account for driving inconvenience along the whole route. In effect, our framework uses population density at the residential location as a proxy for the average density along the commute route. Denoting the average density the consumer encounters along the commute route by D_a, a possible representation for the total commuting cost would then be $t(D_a)x$.[12] The use of $t(D)x$ as the total commuting cost, instead of $t(D_a)x$, simplifies the analysis, allowing derivation of results. But, the property that induces urban sprawl by pulling consumers to the suburban location would be the same regardless of whether $t(D)x$ or $t(D_a)x$ is

[12] Alternatively, the total commuting cost would be gotten by integrating t over the entire route, which equals $\int_0^x t(D)dx$. But, this representation would be much harder to handle analytically.

used. The reason is that commuting cost per mile, $t(D_a)$, also falls with x because average density the consumer encounters along the route will also fall as the consumer resides farther from the center. This property, i.e., $dt/dx < 0$, is sufficient to generate the second force (see footnote 11).

Nevertheless, the convenience benefit from residing farther toward the suburban location may still be over-stated under our framework. Our commuting cost function, however, may have some support from empirical perspective. According to Baum-Snow (2010), for example, commute trips from suburban residences to other suburban areas have increased drastically while commute trips from suburbs to city centers have declined. This change in commute patterns has been associated with employment decentralization since 1950, with polycentric cities becoming a more relevant feature of modern urban landscapes. Accounting for this recent pattern of job decentralization, density at the residential location may properly capture the driving inconvenience the consumer encounters.

4 Numerical Examples

Through the numerical analysis, we first investigate the effect of an increase in α on $\bar{x}$, which was ambiguous in general. Specifically, we want to see whether an increase in α can actually lead to the spatial expansion of the city. The results below show that an increase in α may lead to the increase in $\bar{x}$ depending on the parameter values. Second, recall that the effects of increases in y and α on F were analytically ambiguous at all locations. To clear up the ambiguities, we draw F gradients, which plot the optimal F values at each x, and see how the F gradients shift as y or α changes. Finally, we investigate the responsiveness of $\bar{x}$ to an increase in y, and see how the responsiveness changes as α changes.

4.1 Functional form assumptions

4.1.1 Commuting cost function

Driving inconvenience, $\tilde{I}(D,F)$, is assumed to take a parametric form, $b - d(\sqrt{F})/D$. Note that this functional form satisfies all the maintained assumptions about $\tilde{I}$ ($\tilde{I}_D > 0$, $\tilde{I}_F < 0$, $\tilde{I}_{FF} > 0$, $\tilde{I}_{FD} > 0$). Then, the commuter minimizes $\alpha \left[b - d(\sqrt{F})/D\right] + F$ by choice of F. Solving this problem yields the following optimal vehicle size:

$$F = \frac{\alpha^2 d^2}{4}\frac{1}{D^2} = \frac{\alpha^2 d^2}{4} q^2. \tag{18}$$

We can observe that the optimal F is a decreasing function of D and thus an increasing function of q. Substituting the optimal F into the commuting cost per mile yields $t(q) = \alpha b - \alpha^2 k q^2$, where $k \equiv d^2/4$. Since $t_\alpha = I > 0$ must hold, when we choose values of α in $t(q)$, we limit the choice of α to those values where $t(q)$ is increasing in α. In other words, α values that generate t_α $(= b - 2\alpha k q^2) < 0$ at any x are excluded. Note that t_q $(= -2\alpha^2 k q)$ is negative.

4.1.2 Utility function

Consumers are assumed to have Cobb-Douglas preferences over land, q, and the non-housing good, c, with the utility function given by $v(c,q) = c^{1-\gamma} q^{\gamma}$. The budget constraint is $c + pq = y - t(\ell)x$. Since the influence of the q choice made by each resident is neglected, commuting cost per mile is fixed at t. Maximizing utility and substituting for t using $t(q) = \alpha b - \alpha^2 k q^2$, the demand for q is then given by

$$q = \frac{\gamma\left[y - (\alpha b - \alpha^2 k q^2)x\right]}{p}. \tag{19}$$

The second equilibrium condition, which requires that all residents in the city attain the same utility level, is written

$$\left[y - (\alpha b - \alpha^2 k q^2)x - pq\right]^{1-\gamma} q^{\gamma} = u. \tag{20}$$

Substituting p from (19) into (20) yields

$$(1-\gamma)^{1-\gamma}\left[y - (\alpha b - \alpha^2 k q^2)x\right]^{1-\gamma} q^{\gamma} = u. \tag{21}$$

If all exogenous parameters are given, q is implicitly determined in (21) and the determined q uniquely determines p from (19).

4.1.3 Parameter values

The numerical examples shown below rely on exogenous parameter values, given as follows. Following Brueckner (2007), the housing exponent γ in the Cobb-Douglas utility is set at 0.15.[13] Income (y) is set at \$50000 per year, a figure approximately equal to the sample average from the 2001 National Household Travel Survey. Agricultural rent (p_A) is set at \$4 per square foot per year, which is equivalent to \$500 per month for 1500 square feet of housing. The urban population (L) is set at 1 million.

The initial parameter values in the commuting cost function are given as follows. The baseline value of α is set at 0.012. The value of b is 100000 and k is set at 1.2. According to this initial parameterization, the household living in 3 miles away from the CBD and living in 1650 square feet of housing will spend about \$1400 for fuel consumption per year. The corresponding driving inconvenience (αI) amounts to about \$800 per year. With these parameters, the commuting cost per mile is $t = 1200 - 0.0001728q^2$.

[13] We also used a γ value of 0.2, and the results were qualitatively the same as in the case of $\gamma = 0.15$.

4.1.4 Finding the equilibrium

The procedure for finding the equilibrium works as follows. The city is divided into narrow, discrete rings indexed by i. The continuous distance measure x is replaced by x_i using the ring subscript i. Each ring has a small value of width ϵ, yielding the relationship, $x_i = \epsilon i$. Using this discrete distance measure, the endogenous variables, $p(x)$ and $q(x)$, can also be replaced by p_i and q_i.

First, we generate land consumption at each x_i, denoted by q_i, from the equilibrium condition (21). We can also generate p_i at each x_i from the equilibrium condition (19) using the generated q_i. Then, we find x value where the entire population fits in the city (i.e., $\bar{x}$). From the equilibrium condition (9), we find a value i^* such that for i^*, $\sum_{i=1}^{i^*} \left(\frac{x_i}{q_i}\right) \epsilon \leq L/2\pi$ and for i^*+1, $\sum_{i=1}^{i^*+1} \left(\frac{x_i}{q_i}\right) \epsilon > L/2\pi$, indicating that the city population just fits inside x_{i^*}. In this way, we find $\bar{x}$, which is equivalent to $x_{i^*} = \epsilon i^*$.

Note that the endogenous utility level, u, was predetermined before this numerical calculation was implemented. If the calculated $\bar{x}$ does not satisfy the land-rent-equality condition (10), the predetermined u is adjusted until we get a value where the equilibrium condition (10) is satisfied. In this way, we can find a pair of equilibrium values of u and $\bar{x}$, which correspond to given parameter values.

4.2 Numerical results

First, we check whether the sufficient condition for negative $\partial p/\partial u$, i.e., $p + t_q x \geq 0$, is satisfied at all x. As seen later in Table 3, $p + t_q x$ is consistently greater than zero at all α values used, satisfying the sufficient condition in our numerical examples.

4.2.1 Effects of income and unit cost of inconvenience on vehicle size

Figure 1 shows the F gradients (F as a function of x) drawn at different y values to see how the F gradient moves as y increases. The influence of income on vehicle size is analytically ambiguous at all locations, as shown in Proposition 1. But, from our numerical results, we observe that vehicle size (F) increases at all locations in response to an increase in income (y). The figures are drawn from the baseline parameter values mentioned above. But, to check for the possibility that the other parameters affect the result, we used different sets of parameter values, and the result was robust. Thus, the numerical results indicate that an increase in consumer income induces lower densities (higher q) throughout the city, which leads to higher vehicle sizes at all locations.

Figure 2 shows the F gradients drawn at different α values, with the other parameters being fixed. The influence of α on vehicle size is also analytically ambiguous. Under our reasonable parameter ranges, however, vehicle size (F) increases at all locations as α increases. The positive direct effect ($\partial F/\partial\alpha > 0$) thus dominates the indirect effect, which operates through density (see Appendix E).

[Figure 1, 2 about here]

4.2.2 Effects of population, agricultural rent, and income on city size

Table 1 shows the effects of increases in L and p_A on u and $\bar{x}$, and the numerical results confirm the analytical results derived above. Table 2 shows the effects of an increase in y on $\bar{x}$, evaluated at different α values. We can see that an increase in $\bar{x}$ from an increase in y is larger at higher α values, suggesting that the urban sprawl force of y is accentuated as α increases. From this finding, the presence of driving inconvenience in our model, interacted with rising incomes, may be claimed as a potential source of urban sprawl.

[Table 1, 2 about here]

4.2.3 Effects of unit cost of inconvenience on city size

Table 3 shows the comparative statics with respect to α. The values of the parameter α range from 0.01000 to 0.01350 in increments of 0.00025, so that we have 15 observations.[14]

[14]The α values beyond 0.01350 are excluded because those α values generate the negative driving inconvenience at some x. Likewise, α values below 0.01000 are unlikely since the corresponding driving inconveniences are too high compared to monetary costs.

Our first finding is that the utility level (u) globally decreases as α increases over the included range (i.e., $a \in [0.01000, 0.01350]$). This is consistent with the model prediction.

Next, we find that $\bar{x}$ decreases as α increases until α reaches 0.01200. But, $\bar{x}$ increases as α increases over the range $\alpha \in [0.01225, 0.01350]$. Figure 3 shows the urban size (i.e., $\bar{x}$) that corresponds to each α value. Accordingly, the curve in Figure 3 is U-shaped. The corresponding αI, F, and t $(= \alpha I + F)$, evaluated at $x = 3$, are presented in Table 3. The relative magnitudes of αI and F are reasonable over the range $\alpha \in [0.01225, 0.01350]$, where $\partial \bar{x}/\partial \alpha$ is positive.

Table 3 also presents the gap between driving inconvenience evaluated at $x = 0$, denoted by $I(0)$, and driving inconvenience at $\bar{x}$, denoted by $\bar{I}$. We observe that $I(0) - \bar{I}$ is higher over the range $\alpha \in [0.01225, 0.01350]$, where $\partial \bar{x}/\partial \alpha$ is positive, compared to when $\partial \bar{x}/\partial \alpha$ is negative. Thus, the numerical results confirm the analytical prediction that urban sprawl in response to an increase in α is more likely to occur when the gap between I at any given location and $\bar{I}$ is larger.

[Figure 3, Table 3 about here]

5 Heterogeneity of households

The analysis so far assumes that households are homogeneous in all aspects such as income and driving inconvenience. In this section, we ask which part of the city would be occupied by different types of households. The standard theory of location by heterogeneous households focuses on income differences across households and suggests that the relative magnitudes of the income elasticity of land demand and the income elasticity of the time cost of commuting determine location patterns (Wheaton (1977), Glaeser, Khan, Rappaport (2008)). In this model, we focus on the role of the unit cost of inconvenience, α, on the location pattern. We might expect that the importance of driving inconvenience, as captured by α, may differ across heterogeneous households. While the comparative static analysis in Section 3 helps to explain the role of α in determining intercity differences in urban spatial structure, the analysis of heterogeneous households focuses on the role of α in explaining the residential location pattern within the city.

Suppose that there are two groups of households, who have different α values, with $\alpha^H > \alpha^L$.[15] Let $p_H(x)$ denote the bid-rent for the α^H group and $p_L(x)$ denote the bid-rent for the α^L groups. Then, the group with a higher bid-rent at a given location will occupy the land at that location. Let $\hat{x}$ denote the boundary between the areas, satisfying $p_H(\hat{x}) = p_L(\hat{x})$. Then, the usual approach is to compare the slopes of the bid-rent functions evaluated at $\hat{x}$. If $p'_H(\hat{x}) < p'_L(\hat{x})$, then the bid-rent curve for the α^H group is steeper, implying that the α^H group occupies locations with $x < \hat{x}$ and the α^L group occupies locations with $x > \hat{x}$. If $p'_H(\hat{x}) > p'_L(\hat{x})$, the residential pattern is reversed. Due to the complexity of the bid-rent slope in this model (see (6)), however, we cannot analytically compare the magnitudes of the bid-rent slopes for the two groups.[16] Thus, we carry out a numerical analysis.

Even though the closed city model has been used in the analysis so far, the simpler open-city model, where utility is exogenous, can be used to investigate the group location pattern. The lessons learned will also apply to a closed-city model with two income groups, where the equilibrium is harder to compute. Under the open-city assumption, the endogenous population automatically satisfies the population-fits-inside-city condition (9) once u is given. But, the utility values for the two groups must be chosen to make sure that the two groups coexist in the city, with each being the highest bidder for some of the land. We adjust the utilities so that each group's population is around the half of the entire population.

The parameters are set at the baseline values shown in Section 4.1.3. Figure 4 shows the bid-rent curves with α values of 0.009 and 0.012.[17] As can be seen, the bid-rent curve of

[15]Since $t_\alpha = I > 0$, $t^H > t^L$ follows.

[16]The reason is that the q choices for the two groups cannot be analytically ordered. Other components in (6) further complicate ordering of the bid-rent slopes.

[17]The utility for the group with $\alpha = 0.009$ is set at 25230, and the utility for the group with $\alpha = 0.012$ is set at 25105.

the α^H group is steeper than that of the α^L group. So, consumers with the higher α will occupy the central part of the city, where $x < \hat{x}$ while consumers with the lower α occupy the suburbs, where $x > \hat{x}$ (in Figure 4, $\hat{x}$ is 1.86). This result is natural since a higher α value means a higher commuting cost per mile. The source of a higher commuting cost per mile, which is α here, is different from that in the standard model. Nevertheless, the city center is more attractive to households with the higher commuting cost per mile, so they occupy it.

However, a different location pattern can emerge for a different range of α values. Figure 5 depicts the pattern when the parameters are set at $\alpha^L = 0.012$ and $\alpha^H = 0.0135$.[18] This α range contains the urban-sprawl-inducing α values, i.e., α values inducing $\partial\bar{x}/\partial\alpha > 0$, in Section 4.2.3. For α's in this range, consumers with the higher α value both locate at the city center and at the edge of the city while consumers with the lower α locate in middle part of the city. So, there are two boundaries, with $\hat{x}_1 = 1.45$ and $\hat{x}_2 = 3.368$ in Figure 5. So, the α^H group occupies both locations where $x < 1.45$ and locations where $x > 3.368$ while the α^L group occupies the intermediate locations, where $1.45 < x < 3.368$.

Since commuting cost per mile is still higher for consumers with the higher α, they outbid the low-α households at the city center. As before, consumers with the higher unit cost of driving inconvenience want to reduce commute time by living near the city center. But, these same consumers also flock to the suburban area, where inconvenience cost is lower because of low densities, outbidding the low-α consumers for suburban locations. Therefore, higher-α consumers are found both near the city center and in the suburbs when the α's lie in this higher range.

[Figure 4, 5 about here]

Finally, we investigate the vehicle choice pattern in a city where households have heterogeneous α values. Once the residential pattern is determined as shown above, households adjust their vehicles to their residences. Figure 6 shows the F gradient when the parameters

[18]The utilities for the α^L group and for the α^H group are set at 25105 and 25065, respectively.

are set at $\alpha^L = 0.009$ and $\alpha^H = 0.012$. Vehicle size (F) tends to increase with x because density falls as x is higher. But, the α^H group, who occupies the central locations, tends to choose bigger vehicles, reflecting the direct effect of the higher α (see Appendix E). So, the F gradient is discontinuous at $\hat{x}$, as shown in Figure 6. Figure 7 shows the F gradient when the parameters are set at $\alpha^L = 0.012$ and $\alpha^H = 0.0135$. It is analytically clear that the α^H group occupying suburban locations where $x > \hat{x}_2$ would choose bigger vehicles than any other households in the city. Since the residential pattern changes twice, the F gradient is discontinuous both at $\hat{x}_1$ and at $\hat{x}_2$, as shown in Figure 7.

[Figure 6, 7 about here]

6 Conclusion

This paper has proposed a modified monocentric city model that incorporates the consumer's vehicle choice problem. The interdependency between residential density and commuting cost enables us to represent a city's vehicle fuel efficiency as a function of exogenous characteristics such as population and agricultural rent. Comparative static analyses suggest that the qualitative properties of the present model are quiet similar to those of the standard model. One notable difference from the standard model comes in the effect of an increase in commuting cost per mile. Unlike the standard model, where an increase in commuting cost per mile leads to shrinkage of the city, our model suggests that an increase in commuting cost per mile, especially from increased unit cost of driving inconvenience, may cause the city to expand. This finding may overcome a limitation of the standard commuting-cost-based explanation for suburbanization.

We finally offer some comments about the efficiency implications of our model. In the model, each resident takes residential density and consequently commuting cost per mile as fixed. Since an individual household disregards the potential positive external effect of consuming more space on commuting cost per mile, individual land consumption tends to be inefficiently small compared to the socially optimal level.[19] However, there is an opposing externality that leads to the reverse kind of inefficiency. In particular, the use of less fuel-efficient vehicles generates higher social costs through greater pollution, an effect that is ignored by consumers and not explicitly present in the model. Since people tend to choose less fuel-efficient vehicles when their neighborhood density is lower, additional space consumption may then generate unpriced social costs. With these offsetting effects present, the direction of inefficiency in our model is unclear.

A related ambiguity arises in the model Riley (1974). In his model, a road-congestion externality is present, but he also discusses an additional pollution externality. Because greater space consumption causes the city to spread out and makes commutes trips longer, it raises the amount of pollution generated. However, Riley argues that the difference between the city where this externality is corrected by a location-specific pollution tax on each driver and one where it is left uncorrected is unclear. Shorter trips would beneficially reduce pollution, making greater centralization optimal, but Riley's assumption that pollution damage depends on local traffic density, which is lower in the suburbs, might make it optimal for city to be less rather than more centralized.

Although this ambiguity involves the spatial effects of correcting a single externality (pollution), the ambiguity in our model is different, being due to the interaction of two different externalities, the external effect of land consumption on driving inconvenience, and effect of vehicle size on air pollution. Regardless, it seems worthwhile to research this topic further, following Riley's lead. Specifically, we may compare the equilibrium urban spatial structure in our model to the optimal structure, both with and without a pollution externality. This exercise would enable us to discuss the welfare implications of recent fuel efficiency regulations, such as the Corporate Average Fuel Economy (CAFE) standards.

[19] As a similar view, Brueckner and Largey (2008) point out that, because the household would fail to consider the external effect of consuming more land, the resulting density externality involving social interaction gives rise to inefficiently high density, which leads to a less social interaction than the socially desirable level.

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A The influences of y, α, and u on p and q

Totally differentiating (3) and (4) with respect to each parameter and rearranging the terms provides the necessary partial derivatives. The expressions of $\partial p/\partial \theta$, where $\theta = y$, α, are given as follows:

$$\frac{\partial p}{\partial y} = \frac{1}{q}\left[\frac{\eta}{\frac{t_q x}{q} + \eta}\right] > 0, \tag{22}$$

$$\frac{\partial p}{\partial \alpha} = -\frac{t_\alpha x}{q}\left[\frac{\eta}{\frac{t_q x}{q} + \eta}\right] < 0, \tag{23}$$

where $t_\alpha = I > 0$, $t_q = \alpha I_\ell < 0$, and $\eta \equiv \partial MRS/\partial q|_{v=u} < 0$. The effects of the parameters on q are exactly the substitution effect of the associated changes in p, with $\partial q/\partial \theta = (\partial p/\partial \theta)(1/\eta)$, where $\theta = y$, α.

Next, $\partial q/\partial u$ and $\partial p/\partial u$ are as follows:

$$\frac{\partial q}{\partial u} = \left[-\frac{1}{q v_c} - \frac{\partial MRS}{\partial c}\frac{1}{v_c}\right]\left[\frac{1}{\frac{t_q x}{q} + \eta}\right], \tag{24}$$

$$\frac{\partial p}{\partial u} = -\frac{1}{q v_c}\left[\frac{\eta - \frac{\partial MRS}{\partial c} t_q x}{\frac{t_q x}{q} + \eta}\right]. \tag{25}$$

$\partial q/\partial u > 0$ holds given $\partial MRS/\partial c > 0$ under the normality of q. For $\partial p/\partial u$ to be negative, as in the standard model, we need $\eta - (\partial MRS/\partial c) t_q x < 0$. To check whether $\eta - (\partial MRS/\partial c)\, t_q x < 0$ holds, η is written as

$$\eta \equiv \left.\frac{\partial MRS}{\partial q}\right|_{v=u} = \frac{\partial MRS}{\partial q} - \frac{\partial MRS}{\partial c} MRS. \tag{26}$$

Using (26) and the first order condition ($MRS = p$), we have

$$\eta - \left(\frac{\partial MRS}{\partial c} t_q x\right) = \frac{\partial MRS}{\partial q} - \frac{\partial MRS}{\partial c}\left(p + t_q x\right). \tag{27}$$

Since $\partial MRS/\partial q < 0$ and $\partial MRS/\partial c > 0$ under the normality of q and c, (27) is negative if $p + t_q x \geq 0$, making $\partial p/\partial u$ negative, as in the standard model. The sufficient condition $p + t_q x \geq 0$ is thus the key for the consistency (i.e., $\partial p/\partial u < 0$) with the standard model. This sufficient condition is reasonable, as explained earlier.

B Proof of $\partial u/\partial y > 0$ and $\partial \bar{x}/\partial y > 0$

Totally differentiating (9) with respect to y gives

$$\frac{\partial \bar{x}}{\partial y}\frac{\bar{x}}{\bar{q}} + \int_0^{\bar{x}} \left\{\left(-\frac{x}{q^2}\right)\frac{\partial q}{\partial u}\frac{\partial u}{\partial y} + \left(-\frac{x}{q^2}\right)\frac{\partial q}{\partial y}\right\} dx = 0. \tag{28}$$

Totally differentiating (10) with respect to y, and evaluating at $\bar{x}$ gives

$$\frac{\partial \bar{x}}{\partial y} = -\left(\frac{\partial \bar{p}}{\partial x}\right)^{-1}\frac{\partial \bar{p}}{\partial u}\frac{\partial u}{\partial y} - \left(\frac{\partial \bar{p}}{\partial x}\right)^{-1}\frac{\partial \bar{p}}{\partial y}. \tag{29}$$

Substituting (29) into (28), and rearranging terms yields the following result:

$$\frac{\partial u}{\partial y} = -\frac{\left(\frac{\partial \bar{p}}{\partial x}\right)^{-1}\left(\frac{\partial \bar{p}}{\partial y}\right)\frac{\bar{x}}{\bar{q}} + \int_0^{\bar{x}} \frac{x}{q^2}\frac{\partial q}{\partial y} dx}{\left(\frac{\partial \bar{p}}{\partial x}\right)^{-1}\left(\frac{\partial \bar{p}}{\partial u}\right)\frac{\bar{x}}{\bar{q}} + \int_0^{\bar{x}} \frac{x}{q^2}\frac{\partial q}{\partial u} dx} > 0. \tag{30}$$

Next, by multiplying (29) by $-\partial \bar{p}/\partial x$, we can see that $\partial \bar{x}/\partial y$ has the same sign as

$$\frac{\partial \bar{p}}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial \bar{p}}{\partial y}. \tag{31}$$

Then, by substituting (30) into (31) and multiplying it by the denominator of (30), which is positive, we can see that $\partial\bar{x}/\partial y$ has the same sign as

$$\int_0^{\bar{x}} \frac{x}{q^2}\left(\frac{\partial q}{\partial u}\frac{\partial \bar{p}}{\partial y} - \frac{\partial q}{\partial y}\frac{\partial \bar{p}}{\partial u}\right) dx. \tag{32}$$

Then, by substituting (22), (24), and (25) into (32), we can see that $\partial\bar{x}/\partial y$ has the same sign as the following:

$$\int_0^{\bar{x}} \frac{\bar{\eta}}{q}\left(\frac{1}{\bar{v}_c} - \frac{1}{v_c}\right) dx + \int_0^{\bar{x}} \left(-\frac{\partial MRS}{\partial c}\frac{1}{v_c}\bar{\eta} - \frac{1}{q\bar{v}_c}\frac{\partial \bar{M}RS}{\partial c}\bar{t}_q\bar{x}\right) dx. \tag{33}$$

The second term in (33) is positive under the normality of q and convex indifference curves. The first term in (33) is also positive since $\partial v_c/\partial x$ is positive. Thus, we have $\partial\bar{x}/\partial y > 0$.

C Proof of Proposition 2

Totally differentiating (9) with respect to α gives

$$\frac{\partial\bar{x}}{\partial\alpha}\frac{\bar{x}}{\bar{q}} + \int_0^{\bar{x}} \left\{\left(-\frac{x}{q^2}\right)\frac{\partial q}{\partial u}\frac{\partial u}{\partial\alpha} + \left(-\frac{x}{q^2}\right)\frac{\partial q}{\partial\alpha}\right\} dx = 0. \tag{34}$$

Totally differentiating (10) with respect to α and evaluating at $\bar{x}$ yields

$$\frac{\partial\bar{x}}{\partial\alpha} = -\left(\frac{\partial\bar{p}}{\partial x}\right)^{-1}\frac{\partial\bar{p}}{\partial u}\frac{\partial u}{\partial\alpha} - \left(\frac{\partial\bar{p}}{\partial x}\right)^{-1}\frac{\partial\bar{p}}{\partial\alpha}. \tag{35}$$

Substituting (35) into (34) and rearranging terms yields

$$\frac{\partial u}{\partial\alpha} = -\frac{\left(\frac{\partial\bar{p}}{\partial x}\right)^{-1}\left(\frac{\partial\bar{p}}{\partial\alpha}\right)\frac{\bar{x}}{\bar{q}} + \int_0^{\bar{x}}\frac{x}{q^2}\frac{\partial q}{\partial\alpha}dx}{\left(\frac{\partial\bar{p}}{\partial x}\right)^{-1}\left(\frac{\partial\bar{p}}{\partial u}\right)\frac{\bar{x}}{\bar{q}} + \int_0^{\bar{x}}\frac{x}{q^2}\frac{\partial q}{\partial u}dx} < 0. \tag{36}$$

By multiplying (35) by $-\partial\bar{p}/\partial x$, we can see that $\partial\bar{x}/\partial\alpha$ has the same sign as

$$\frac{\partial\bar{p}}{\partial u}\frac{\partial u}{\partial\alpha}+\frac{\partial\bar{p}}{\partial\alpha}. \tag{37}$$

By substituting (36) into (37) and multiplying it by the denominator of (36), we can see that $\partial\bar{x}/\partial\alpha$ has the same sign as

$$\int_0^{\bar{x}}\frac{x}{q^2}\left(\frac{\partial q}{\partial u}\frac{\partial\bar{p}}{\partial\alpha}-\frac{\partial q}{\partial\alpha}\frac{\partial\bar{p}}{\partial u}\right)dx. \tag{38}$$

By substituting (23), (24), and (25) into (38), and rearranging the terms, we can see that $\partial\bar{x}/\partial\alpha$ has the same sign as

$$\int_0^{\bar{x}}\frac{\bar{\eta}}{q}\frac{\bar{x}}{v_c}\left[\bar{t_\alpha}-t_\alpha\left(\frac{x/\bar{v}_c}{\bar{x}/v_c}\right)\right]dx+\int_0^{\bar{x}}\left(\bar{t_\alpha}\bar{x}\bar{\eta}\frac{\partial MRS}{\partial c}\frac{1}{v_c}+t_\alpha x\frac{1}{q\bar{v}_c}\frac{\partial\bar{MRS}}{\partial c}\bar{t_q}\bar{x}\right)dx. \tag{39}$$

The second term in (39) is clearly negative under the conditions of normality of c and convex indifference curves. However, the first term in (39) is potentially positive because $\bar{t_\alpha}$ $(=\bar{I})$ is smaller than t_α $(=I)$ at all x. Recall that $dI/dx<0$ holds, making $\bar{I}<I$ at all x. Although the first term in (39) is ultimately ambiguous because $(x/\bar{v}_c)/(\bar{x}/v_c)$ is between zero and one, the claim is that, as the gap between $\bar{I}$ and I at any given x is larger, making $\bar{I}-I$ strongly negative, the first term in (39), which is potentially positive, may dominate, tending to make $\partial\bar{x}/\partial\alpha>0$.

D Derivation of the sign of dq/dy

To investigate the influence of y on q, it is helpful to look at the change in p in response to an increase in y by computing the following:[20]

$$\frac{d\tilde{p}}{dy}=\frac{\partial\tilde{p}}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial\tilde{p}}{\partial y}, \tag{40}$$

[20] Investigating the sign of dq/dy directly leads to the same conclusion.

where $\tilde{}$ indicates that the variable is evaluated at $\tilde{x}$ satisfying $0 < \tilde{x} < \bar{x}$.

Substituting $\partial u/\partial y$ from (30) into (40) and multiplying it by the denominator of (30), which is positive, $d\tilde{p}/dy$ has the same sign as

$$-\left(\frac{\partial \bar{p}}{\partial x}\right)^{-1} \frac{\bar{x}}{\bar{q}} \left(\frac{\partial \tilde{p}}{\partial u}\frac{\partial \bar{p}}{\partial y} - \frac{\partial \tilde{p}}{\partial y}\frac{\partial \bar{p}}{\partial u}\right) + \int_0^{\bar{x}} \frac{x}{q^2} \left(\frac{\partial q}{\partial u}\frac{\partial \tilde{p}}{\partial y} - \frac{\partial q}{\partial y}\frac{\partial \tilde{p}}{\partial u}\right) dx. \tag{41}$$

To investigate the sign of (41), by substituting (22), (24), and (25) into (41) and evaluating the expression both at $\tilde{x} = 0$ and $\tilde{x} = \bar{x}$, we observe that (41) is ambiguous at $\tilde{x} = 0$ and positive at $\tilde{x} = \bar{x}$. Thus, the change in p in response to an increase in y is ambiguous at $\tilde{x} = 0$, but p increases as y increases at $\tilde{x} = \bar{x}$. Note that $dq/dy > 0$ holds at any location where p falls while the sign of dq/dy is ambiguous at locations where p rises, the standard result that continues to hold in this model (see Brueckner (1987)). Therefore, dq/dy is ambiguous both at $\tilde{x} = 0$ and at $\tilde{x} = \bar{x}$. By continuity, the sign of dq/dy is ambiguous at any intermediate locations. Therefore, the total influence of y on q is ambiguous at all locations in the city.

E Derivation of the sign of $dF/d\alpha$

First, to determine the sign of the direct effect, i.e., $\partial F/\partial\alpha$, totally differentiating the first-order condition for the vehicle choice problem, $\alpha\tilde{I}_F + 1$, with respect to α and F yields

$$\frac{\partial F}{\partial \alpha} = -\frac{\tilde{I}_F}{\alpha \tilde{I}_{FF}} > 0. \tag{42}$$

This partial derivative sign implies that holding density fixed, a higher unit cost of driving inconvenience leads to a higher F, which would reduce driving inconvenience.

Second, to determine the sign of $dq/d\alpha$, we investigate the influence of α on p by computing:

$$\frac{d\tilde{p}}{d\alpha} = \frac{\partial \tilde{p}}{\partial u}\frac{\partial u}{\partial \alpha} + \frac{\partial \tilde{p}}{\partial \alpha}, \tag{43}$$

where ˜ indicates that the variable is evaluated at $\tilde{x}$ satisfying $0 < \tilde{x} < \bar{x}$.

Substituting $\partial u/\partial \alpha$ (36) into (43) and multiplying it by the denominator of (43), which is positive, $d\tilde{p}/d\alpha$ has the same sign as:

$$-\left(\frac{\partial \bar{p}}{\partial x}\right)^{-1} \frac{\bar{x}}{\bar{q}} \left(\frac{\partial \tilde{p}}{\partial u}\frac{\partial \bar{p}}{\partial \alpha} - \frac{\partial \tilde{p}}{\partial \alpha}\frac{\partial \bar{p}}{\partial u}\right) + \int_0^{\bar{x}} \frac{x}{q^2} \left(\frac{\partial q}{\partial u}\frac{\partial \tilde{p}}{\partial \alpha} - \frac{\partial q}{\partial \alpha}\frac{\partial \tilde{p}}{\partial u}\right) dx. \tag{44}$$

To evaluate the sign of (44), substituting (23), (24), and (25) into (44), $d\tilde{p}/d\alpha$ is positive at $\tilde{x} = 0$ and $d\tilde{p}/d\alpha$ is ambiguous at $\tilde{x} = \bar{x}$. Note that q falls and thus D rises at any location where p rises, which then tends to reduce F. But, together with the direct influence of α on F from (42), the total effect of an increase in α on F is ambiguous at $\tilde{x} = 0$. At $\tilde{x} = \bar{x}$, given the ambiguity of the indirect influence of α on q, the total effect of an increase in F is also ambiguous. By continuity, the sign of $dF/d\alpha$ is ambiguous at any intermediate locations. Therefore, the total effect of an increase in α on F is ambiguous throughout the city.

Table 1: Effects of increases in L and p_A on u and $\bar{x}$

		u			$\bar{x}$		
Variables		From	To	Δ	From	To	Δ
L	1 million $\rightarrow$ 1.5 million	25106	24760	-346	4.2893	5.1622	0.8730
p_A	\$4 $\rightarrow$ \$4.5	25106	24520	-586	4.2893	3.9978	-0.2915

[a] *Other parameters are set at $\alpha = 0.012$, $b = 100000$, $k = 1.2$, and $y = 50000$.*

Table 2: Effect of an increase in y on $\bar{x}$, evaluated at varying α

	$\bar{x}$		
Income (y) increases from \$50000 to \$52000	From	To	Δ
Effect evaluated at $\alpha = 0.0100$	4.2969	4.4069	0.1100
Effect evaluated at $\alpha = 0.0110$	4.2911	4.4053	0.1142
Effect evaluated at $\alpha = 0.0120$	4.2893	4.4079	0.1187
Effect evaluated at $\alpha = 0.0130$	4.2917	4.4148	0.1231
Effect evaluated at $\alpha = 0.0135$	4.2944	4.4201	0.1256

[a] *Other parameters are set at $b = 100000$, $k = 1.2$, $p_A = 4$, and $L = 1$ million.*

Table 3: The effect of an increase in α on endogenous variables

α	0.01000	0.01025	0.01050	0.01075	0.01100	0.01125	0.01150	0.01175	0.01200	0.01225	0.01250	0.01275	0.01300	0.01325	0.01350
Radius ($\bar{x}$)	4.2969	4.2951	4.2937	4.2923	4.2911	4.2902	4.2895	4.2893	4.2893	4.2895	4.2899	4.2907	4.2917	4.2931	4.2944
Utility (u)	25189	25175	25163	25151	25140	25130	25121	25114	25106	25101	25096	25093	25090	25089	25088
F	324	340	357	374	391	409	427	446	466	486	506	527	548	571	593
αI	352	345	336	327	318	307	295	282	269	254	238	221	203	184	164
$\alpha I + F$ [b,c]	676	685	693	701	709	716	723	729	734	739	744	748	752	754	757
$I(0) - \bar{I}$ [d]	35030	36124	37215	38291	39355	40413	41464	42501	43537	44534	45528	46509	47477	48421	49340
$p + t_q x$ at $x = 1$ [e]	4.87	4.86	4.85	4.84	4.83	4.82	4.80	4.79	4.77	4.75	4.73	4.70	4.68	4.65	4.62
$p + t_q x$ at $x = 3$	3.20	3.14	3.07	3.01	2.95	2.88	2.81	2.74	2.67	2.59	2.51	2.43	2.35	2.27	2.19
$p + t_q x$ at $\bar{x}$	2.17	2.08	1.99	1.89	1.79	1.69	1.59	1.49	1.38	1.27	1.16	1.04	0.92	0.80	0.68

[a] *Other parameters are set at* $b = 100000$, $k = 1.2$, $y = 50000$, $p_A = 4$, *and* $L = 1$ *million.*

[b] *The variables are evaluated at* $x = 3$.

[c] *Fuel consumption* (F) *and driving inconvenience* (αI) *are dollar values of per mile and per year.*

[d] $I(0)$ *denotes driving inconvenience evaluated at* $x = 0$, *and* $\bar{I}$ *denotes driving inconvenience at* $\bar{x}$.

[e] *This is presented to check* $p + t_q x \geq 0$, *the sufficient condition for* $\partial p / \partial u < 0$.

Figure 1: *F* gradients with varying *y*

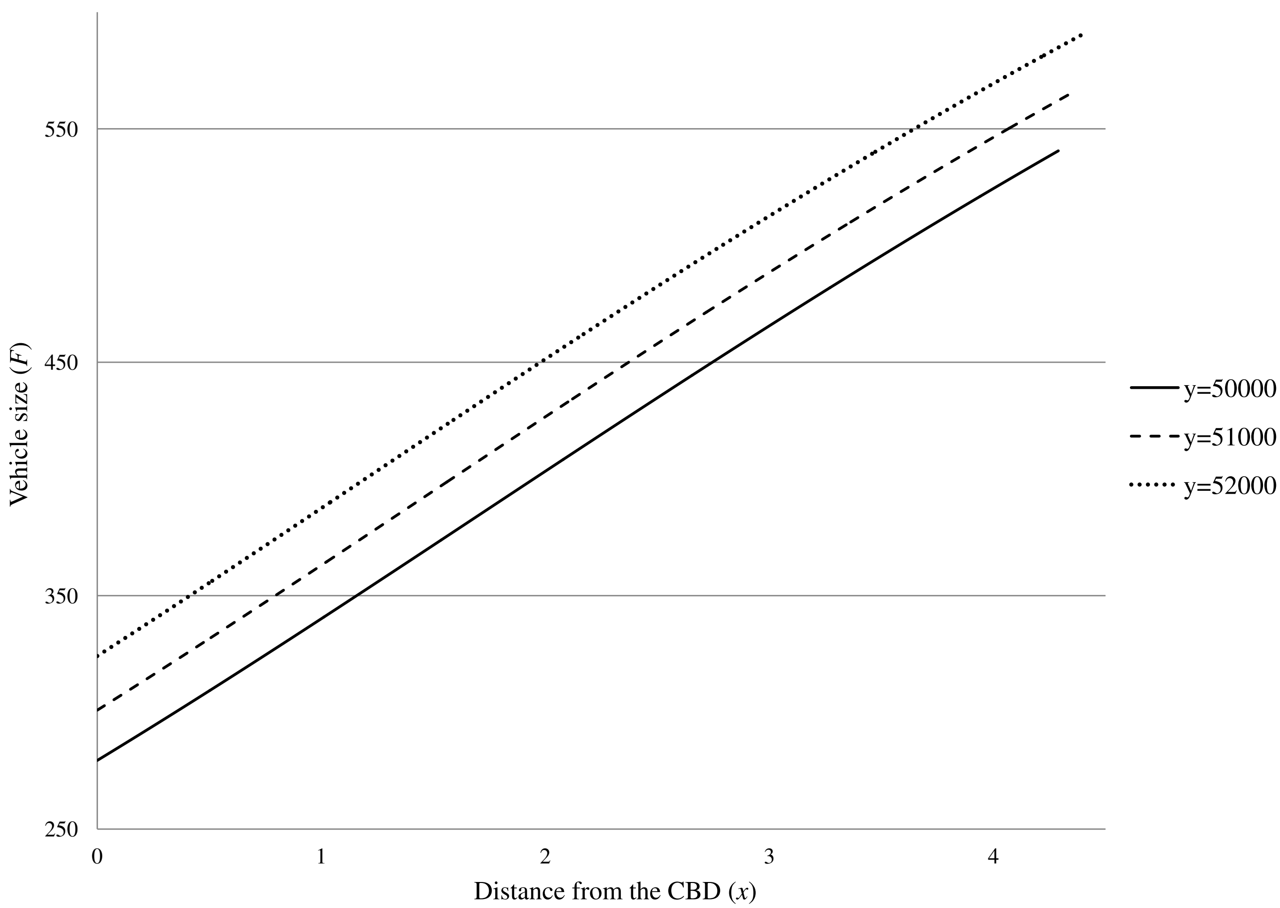

Figure 2: *F* gradient with varying *alpha*

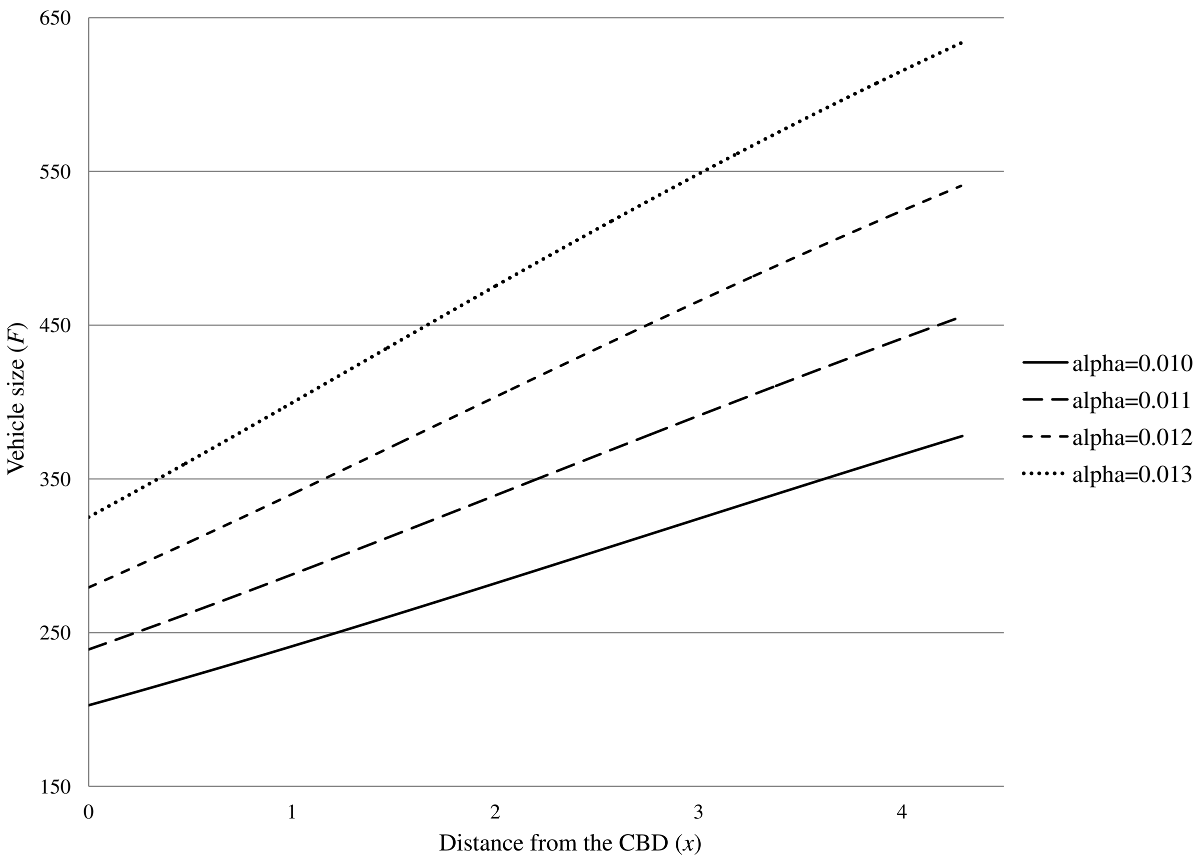

Figure 3: Radius of city with varying *alpha*

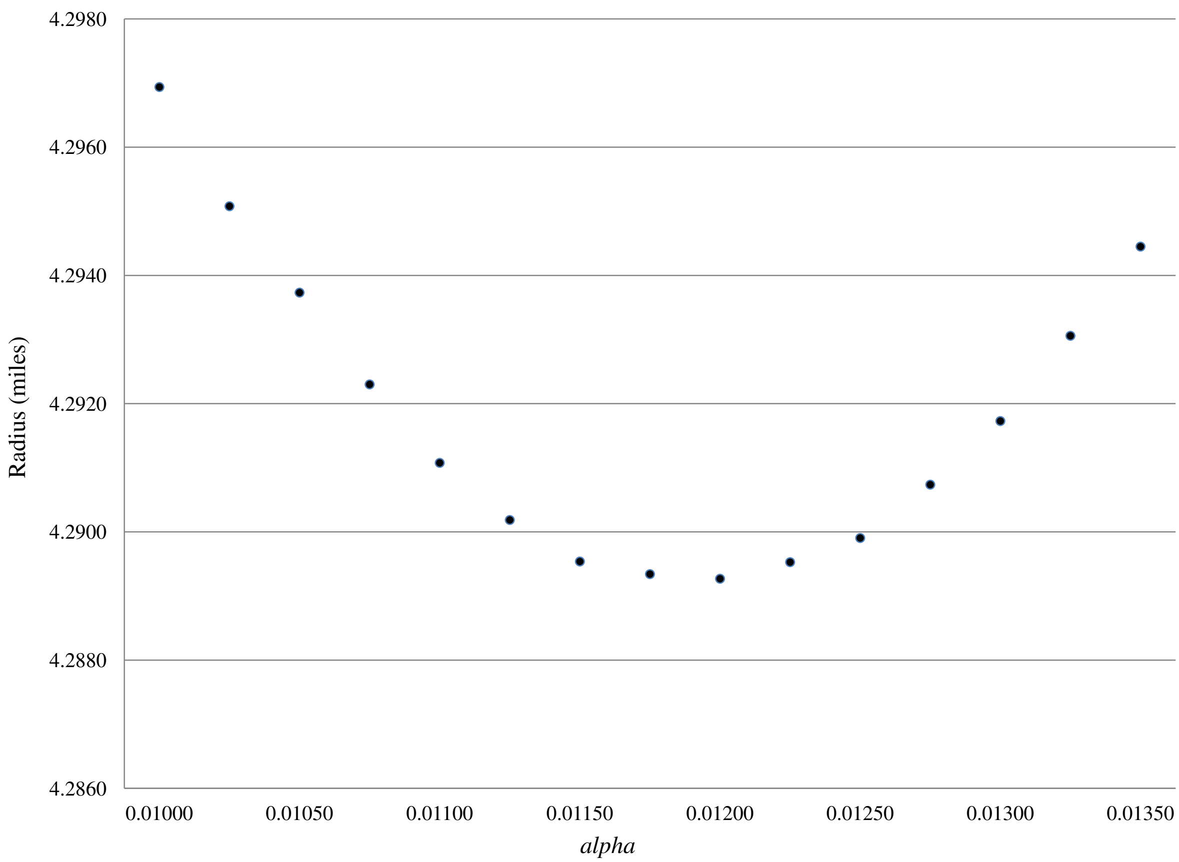

Figure 4: Bid-rent functions with two *alpha*-groups

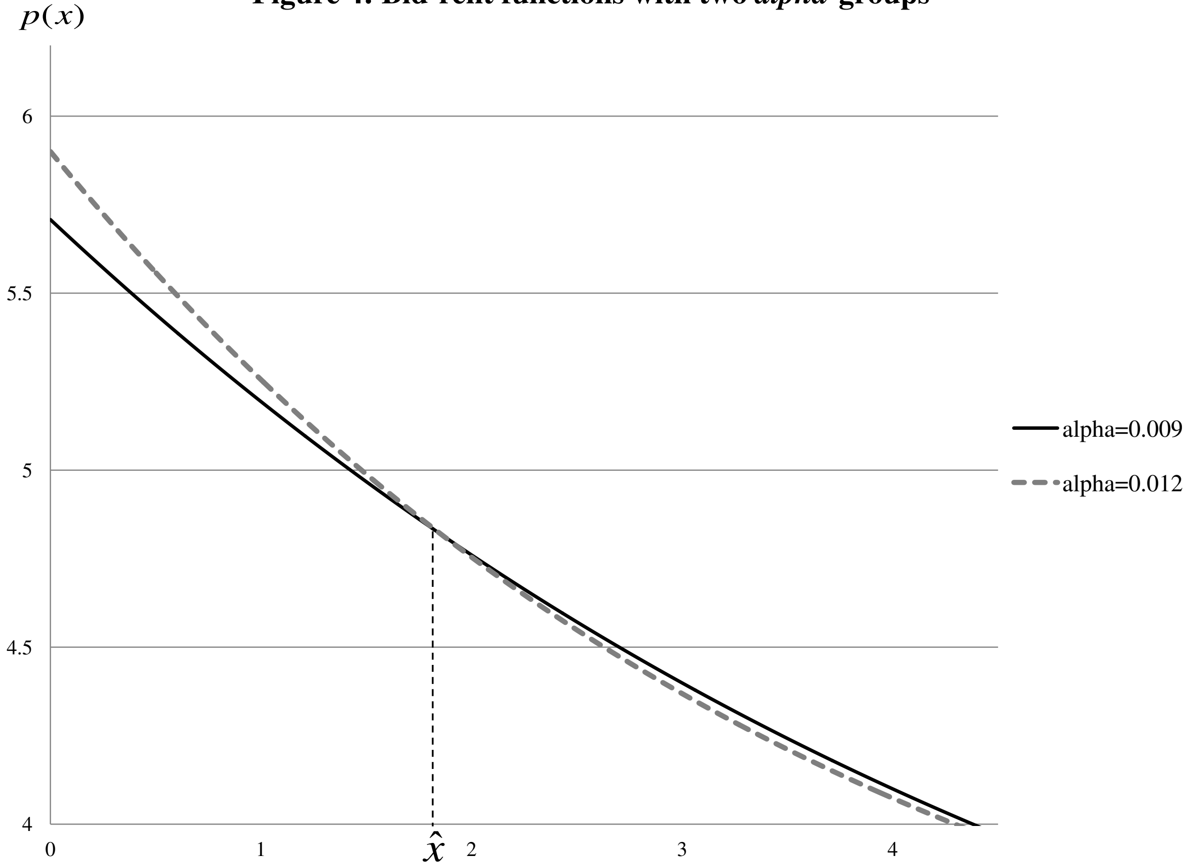

Figure 5: Bid-rent functions with multiple intersections

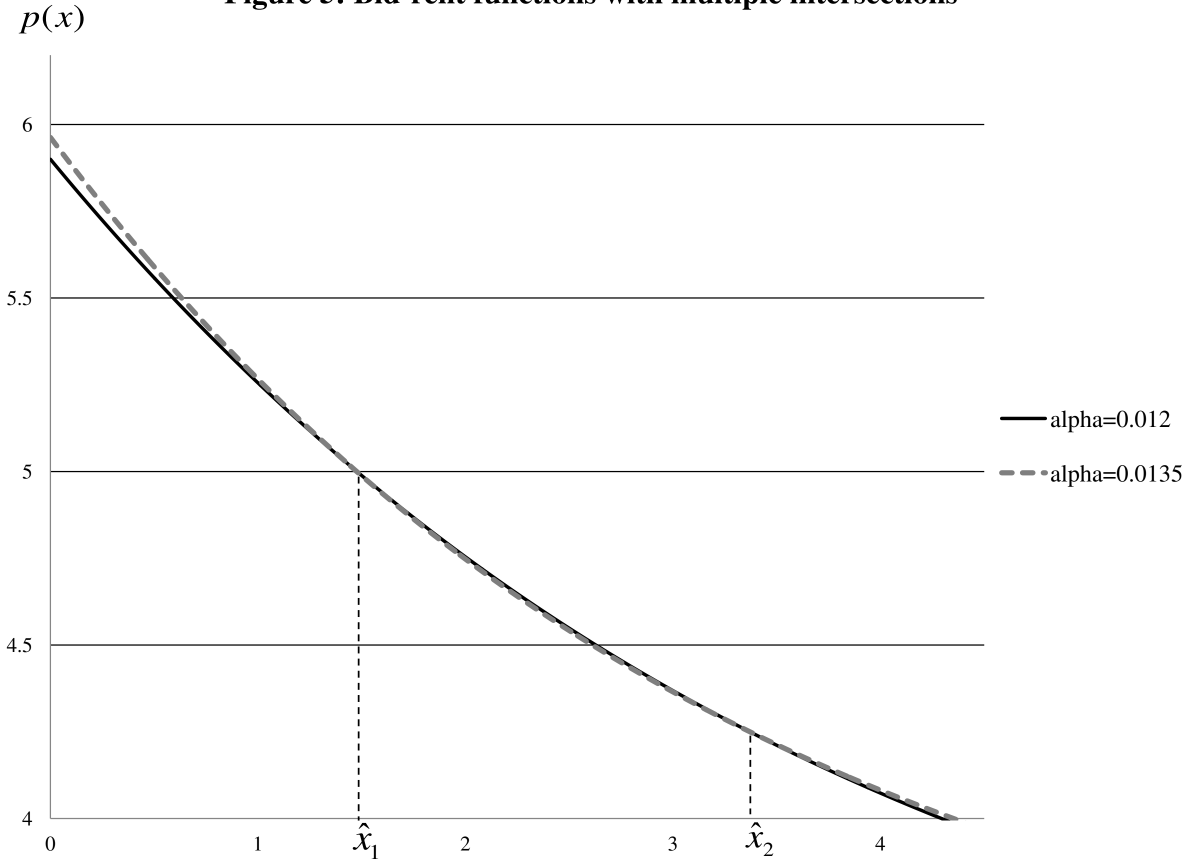

Figure 6: *F* pattern in heterogeneous city

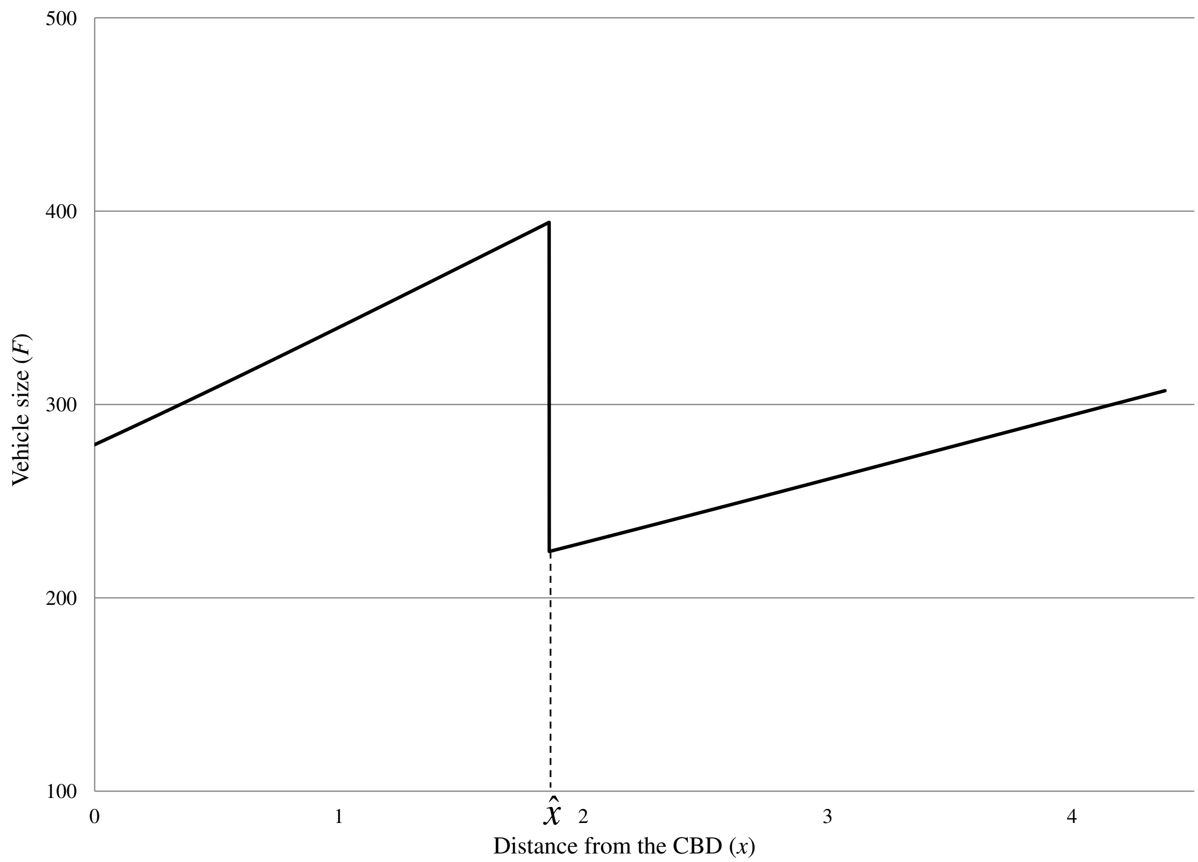

Figure 7: *F* pattern with multiple bid-rent intersections