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Abstract

This paper studies the optimal behavior of labor-income taxation in a simple model with credit frictions. Firms' borrowing to pay their wage payments in advance is constrained by the value of their collateral at the beginning of the period. The labor-income tax rate and the shadow value on the credit constraint lead to a wedge between the marginal product of labor and the marginal rate of substitution between labor and consumption. This paper suggests that while the notion of "static wedge smoothing" is carried over to this environment, it is achieved only through a volatile labor-income tax rate. As the shadow value on the financing constraint varies over the business cycle, tax volatility is needed in order to counter this variation and, thus, allow for "wedge smoothing". In particular, the optimal labor-income tax rate is lower when the credit market is more tightened and higher when it is less tightened. Therefore, when firms are more credit-constrained and the demand for labor is reduced, optimal fiscal policy calls for boosting labor supply by lowering the labor-income tax rate. It is also shown that the optimal behavior of the labor-income tax rate that is discovered in this study is consistent with its historical behavior in the U.S.

Key Words: Labor tax smoothing; Credit frictions; Borrowing constraints; Labor wedge.
JEL Classification: E44, E62, H21, H30.

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1 Introduction

A classic result in optimal fiscal policy is that the labor-income tax rate should be virtually constant over the business cycle ("labor tax smoothing"). This paper studies the optimal behavior of the labor-income tax rate in a simple growth model in which firms borrow to pay factors of production in advance and borrowing is constrained by their beginning-of-period collateral. The paper suggests that the labor-income tax rate should vary over the business cycle; if firms are more constrained in hiring labor, the labor-income tax rate should be lowered to boost labor supply, thus increasing labor in equilibrium. The optimal behavior of the labor-income tax rate suggested by this paper is consistent with the historical behavior of the labor-income tax rate versus a measure of credit market tightness in the U.S.; the average income tax rate displayed a strong negative correlation with the corporate credit spread over the business cycle.

Besides the government, the baseline setup assumes two types of agents in the economy: households and a representative firm. The firm hires labor from households in a neoclassical labor market at a given real wage. The firm borrows in order to pay at least part of the wage bill at the beginning of the period ("working capital"). Borrowing, in turn, is constrained by the firm's value of real estate. This corresponds to the usual limited enforcement problem as in Kiyotaki and Moore (1997).

The basic intuition behind the main result of the paper is a follows. Because of the binding credit constraint, labor demand is inefficiently low and it depends on the tightness of the credit constraint. When the credit constraint tightens more, labor supply should be encouraged by reducing the labor tax rate. When the credit constraint is less tightened, the labor tax rate is relatively higher. In either case, the labor tax rate is lower than in otherwise model with no credit frictions. The labor tax rate thus moves in opposite to the tightness of the credit constraint in order to prevent excessive volatility in labor, hence output and consumption. This result calls for an active fiscal policy that uses the labor tax rate as a stabilizing tool in the face of exogenous shocks to the macroeconomy.

An alternative way of viewing the result is by considering the implications of the collateral constraint. Due to the binding credit constraint, the firm hires labor so that the marginal product of labor exceeds the real wage rate, thus generating a "markup". Optimal policy thus aims for offsetting this markup (at least partially) by "subsidizing" labor supply. An increase in labor supply lowers the before-tax wage rate and leads to a higher quantity of labor. Also, the labor tax rate and the shadow value on the credit constraint generate a “labor wedge”, thus breaking down the full mapping between the labor wedge and the labor tax rate. I show that the time-varying labor-income tax rate allows for complete smoothing of the labor wedge when credit frictions are present. Therefore, even though the labor-income tax rate is not completely smoothed, the notion of “static wedge smoothing” remains optimal in this environment.

The idea that the labor-income tax rate should be virtually constant over time is well-known in the literature since the partial-equilibrium complete- markets analysis of Barro (1979). Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991, 1994) show that this result holds in a general-equilibrium setup that assumes neoclassical labor markets. In an economy with incomplete markets and no capital, Aiyagari et al. (2002) partially affirm the results of Barro (1979). Schmitt-Grohe and Uribe (2004a) show that the volatility of the labor tax rate is very small in a model with flexible prices (with and without perfect competition in the product market), but significantly higher if prices are sticky. Andersen and Dogonowski (2004) suggest that the optimal tax rate should be procyclical to smooth leisure. Recently, Arseneau and Chugh (2012) have shown that the result of labor tax smoothing does not hold in a model with labor market frictions: labor tax rate volatility is optimal to induce efficient fluctuations in the labor market by keeping distortions (or wedges) constant over the business cycle.

The economic events of recent years call for studying the effects of various aspects of financial frictions on optimal policies. In particular, the difficulties of firms in obtaining sufficient credit during the last recession raise questions about the optimal policies that governments should follow during this type of economic episodes. This is essentially addressed in this paper in the context of the optimal cyclical behavior of the labor-income tax rate.

The remainder of the paper proceeds as follows. Section 2 outlines the model and defines the private-sector equilibrium. Section 3 presents the problem of the social planner and section 4 discusses the problem of the Ramsey planner. Section 5 presents some analytical results. The calibration and the solution methodology of the model are described in Section 6. Section 7 presents the main quantitative results of this paper. Section 8 presents some robustness analysis. Section 9 presents the co-movement of the average U.S. income tax rate and a measure of the credit spread over the period 1979-2009. Section 10 concludes.

2 The Model

The economy is populated by households, a representative firm and the government. Households consume and supply labor to the firm on spot markets. The firm needs to pay (at least part of) its input costs before production takes place, thus giving rise to borrowing from households. Borrowing is constrained by the value of real estate that the firm owns. This is the source of the credit friction in the baseline model.

2.1 Households

In each period *t,* the representative household purchases consumption c_t, supplies labor l_t, purchase real estate h_t (in the form of housing) and lends b_t^f to the firm at the beginning of the period at an intra-period gross real interest rate of R_t^f. The household also has access to a standard one-period real government bond b_t that pays a gross real interest rate of R_t.

Households maximize their expected discounted lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, l_t), \tag{1}$$

where E_0 is the expectation operator, $\beta < 1$ is the subjective discount factor and $u(c_t, h_t, l_t)$ is the period utility function from consumption, labor and real estate. This function satisfies: $\frac{\partial u(\cdot)}{\partial c} > 0$, $\frac{\partial^2 u(\cdot)}{\partial c^2} < 0$, $\frac{\partial u(\cdot)}{\partial h} > 0$, $\frac{\partial^2 u(\cdot)}{\partial h^2} < 0$, $\frac{\partial u(\cdot)}{\partial l} < 0$ and $\frac{\partial^2 u(\cdot)}{\partial l^2} < 0$.

Maximization is subject to the sequence of budget constraints of the form:

$$(1-\tau_t^l) w_t l_t + (1-\tau_t^\pi)\Pi_t + q_t h_t + R_{t-1} b_t + R_t^f b_t^f = c_t + q_t h_{t+1} + b_{t+1} + b_t^f, \tag{2}$$

where c_t denotes consumption, w_t is the real wage rate, q_t is the market price of housing τ_t^l and is the labor-income tax rate, Π_t denotes lump-sum profits from the ownership of the firm and τ_t^π is the tax rate on those profits.

The optimal choices of consumption, bonds, lending to firms, labor supply and real estate yield:

$$R_t^f = 1, \tag{3}$$

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t \,, \tag{4}$$

$$u_{c,t} = \beta R_t E_t(u_{c,t+1}) \,, \tag{5}$$

$$q_t u_{c,t} = \beta E_t(u_{h,t+1} + q_{t+1}u_{c,t+1}) \,, \tag{6}$$

where $u_{c,t}$ is the marginal utility of consumption in period *t*, $u_{h,t}$ is the marginal utility of housing in period *t* and $u_{l,t}$ is the marginal disutility of supplying labor in period *t*. Equation (3) governs the lending of households to firms. As is in Carlstrom and Fuerst (1998), households are basically passive suppliers of credit to the firm. Equation (4) is the standard labor-supply condition, equation (5) is the standard consumption Euler equation and condition (6) is an asset pricing-type condition. This condition states the marginal utility from consumption is equalized to the marginal gain from real estate. The latter has two components- a direct utility from real estate and the possibility to expand future consumption by the realized resale value of real estate.

2.2 The Firm

The representative firm hires labor and uses real estate to produce a homogenous good using the following production function:

$$y_t = z_t f(x_t, l_t) \,, \tag{7}$$

where y_t is output, z_t is total factor productivity and x_t denotes the stock of real estate of the firm at the beginning of the period.

Due to a mismatch between the timing of the realization of revenues and wage payment, at least part of labor costs are paid before the realization of revenues, which requires the firm to borrow at the beginning of period *t*. This assumption has some similarity with the assumption of Carlstrom and Fuerst (1998), but with differences in the specifics of the model. Borrowing, however, is constrained by the value of the firm's assets, which are entirely held in the form of real estate. Therefore, the firm's collateral is equal to the beginning-of-period market value of its real estate.

Assuming that firms use real estate as collateral is common in the literature: for example, Kiyotaki and Moore (1997) assume that borrowing is tied to the value of land and Iacoviello (2005) assumes that entrepreneurs use housing as collateral. Chaney, Sraer and Thesmar (2011) show that, for U.S. firms over 1993-2007, appreciation in the real estate values of firms led to increases in investment, which is mainly financed through additional debt issuance. This effect is particularly strong for credit-constrained firms. I, therefore, follow those studies and use the value of real estate as the firm's collateral.

As shown in Appendix A, the firm's problem with credit frictions can be reduced to the following maximization problem:

$$Max[z_t f(x_t, l_t) - w_t l_t], \tag{8}$$

subject to

$$\phi w_t l_t \leq \kappa q_t x_t, \tag{9}$$

where x_t is the firm's beginning-of-period stock of real estate, κ is the share of assets that can be used as collateral (or the loan-to-value ratio), ϕ is the fraction of factor payments that has to be paid in advance. Clearly, if $\phi = 0$, then the model collapses to the standard model with neoclassical labor markets.

Denoting the Lagrange multiplier on (9) by μ_t, profit maximization gives the following labor demand condition:

$$z_t f_{l,t} = (1 + \phi \mu_t) w_t, \tag{10}$$

The firm thus hires labor so that its marginal product is a "markup" over the real wage. The net markup is given by $\phi \mu_t$ and it arises only due to the external financing needs of the firm. This result is similar to the result in the "output model" of Carlstrom and Fuerst (1998). In their model, agency costs, which arise due to the monitoring activity of lenders, induce differences between the marginal products of labor and capital and their respective factor prices. The use of the term "markup" in this paper is borrowed from their own study.

In the second part of the period, the firm chooses the next-period real estate taking into account the role of real estate as collateral and subject to the budget constraint $z_t f(x_t, l_t) - w_t l_t + q_t x_t = q_t x_{t+1} + \Pi_t$. The left-hand side is the total resources of the firm after production takes place, and they are equal to the sum of operating profits $z_t f(x_t, l_t) - w_t l_t$ and the market value of assets $q_t x_t$. Those resources are first used to finance the purchasing of next-period real estate x_{t+1}.Then, any remaining profits (or resources), denoted by Π_t, are remitted to households in a lump-sum fashion.

I also assume that in the process of accumulating assets, the firm is more impatient than households (one may think about this firm as being managed by an entrepreneur who is more impatient that households). For this reason, the firm's stochastic discount factor is $\gamma\Xi_{t,t+1}$, where $\Xi_{t,t+1} = \beta \frac{u_{c,t+1}}{u_{c,t}}$ is the households' stochastic discount factor and $\gamma < 1$. The parameter γ is introduced to avoid self financing by the firm.

The assumption that the borrower (firm/entrepreneur in this case) is less patient than the lender is standard in this class of models (see, for example, Carlstrom and Fuerst 1997, 1998). In addition, assuming that profits are transferred to households simplifies the optimal policy problem as it reduces the objective function of the Ramsey planner to only the utility function of households. This formulation also allows for better comparisons with the standard neoclassical model.

With this characterization of the firm's problem, the choice of x_t gives the following dynamic equation in the price of real estate:

$$q_t u_{c,t} = \gamma\beta E_t\left[u_{c,t+1}[z_{t+1} f_{x,t+1} + (1+\kappa\mu_{t+1})q_{t+1}]\right], \tag{11}$$

which makes explicit the roles of the credit friction and the additional discount factor.

Since profits Π_t are transferred to households, one could alternatively assume that the objective function of the firm is to choose labor and real estate in order to maximize $\Pi_t = z_t f(x_t, l_t) - w_t l_t + q_t x_t - q_t x_{t+1}$. In this case, the firm's problem is to maximize $E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{0,t} \Pi_t$ subject to the financing constraint (9). The first-order conditions with respect to labor and real estate of this problem are exactly the same as (10)-(11). In this respect, both approaches are identical.

2.3 The Government

The government collects labor-income and profit taxes and issues real debt to finance an exogenous stream of real government expenditures g_t. The government budget constraint in period t is thus given by:

$$\tau_t^l w_t l_t + \tau_t^\pi \Pi_t + b_{t+1} = g_t + R_{t-1} b_t \tag{12}$$

2.4 Market Clearing

In equilibrium, the resource constraint of the economy reads:

$$z_t f(x_t, l_t) = c_t + g_t, \tag{13}$$

and the market for real estate clears:

$$h_t + x_t = 1. \tag{14}$$

which, as in Kiyotaki and Moore (1997) and Iacoviello (2005), implies a fixed supply of real estate.

2.5 The Private Sector Equilibrium

Definition 1: *Given the exogenous processes* $\{z_t, g_t, \tau_t^l, \tau_t^\pi\}$, *the private-sector equilibrium is a state-contingent sequence of allocations* $\{c_t, l_t, h_t, x_t, w_t, q_t, R_t^f, R_t, b_t, \mu_t\}$ *that satisfy the equilibrium conditions (3)-(6) and (9)-(14).*

3 Efficient Allocations

It is useful to consider the optimal tax results that emerge as a solution to the social planner's problem in order to better understand the results of the Ramsey planner later. I refer to the allocations of the social planner as the "efficient allocations" or the "first-best allocations", interchangeably. Those are the allocations the planner will choose when lump-sum taxes are available.

Definition 2: *Given the exogenous processes* $\{z_t, g_t\}$, *the problem of the social planner is to choose consumption, labor and real estate to maximize (1) subject to (13)-(14).*

As Appendix B shows, the choice of labor and consumption yield:

$$-\frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t}, \tag{15}$$

which state that the social planner chooses consumption and labor so that the marginal rate of substitution between labor and consumption is equalized to the marginal product of labor. This is the standard efficiency condition in this class of models. Notice also that this condition holds regardless of whether or not the model includes physical capital. The same holds for all the analytical results that are presented in what follows. See Appendices D-F for more details about the model with capital.

4 Optimal Labor Taxation- The Ramsey Problem

In this section, I present the solution to the second-best labor taxation problem using the standard Ramsey approach (maximizing the utility of households subject to the private-sector equilibrium conditions and the resource constraint). Following Lucas and Stokey (1983) and Chari and Kehoe (1999), I use the primal approach, in which the government only chooses allocations after prices and taxes have been substituted out using the private-sector equilibrium conditions. To do so, I derive the present-value implementability constraint (PVIC) by substituting the equilibrium conditions into the households' budget constraint. Differently from standard Ramsey models, however, the PVIC in this paper does not capture all of the equilibrium conditions of the private sector (in addition to the resource constraint, of course). Therefore, the Ramsey problem will be enlarged beyond just maximizing utility subject to the PVIC and the resource constraint.

As shown in Appendix C, the PVIC in this problem reads:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{c,t} c_t + u_{l,t} l_t + \beta u_{h,t+1} h_{t+1} - u_{c,t} (1 - \tau_t^{\pi}) \Pi_t \right] = u_{c,0} R_{-1} b_0 + u_{c,0} q_0 h_0 , \tag{16}$$

Definition 3: *Given the exogenous processes* $\{z_t, g_t\}$, *the Ramsey planner chooses sequences of allocations* $\{c_t, l_t, h_t, x_t, q_t, \mu_t\}$ *to maximize (1) subject to (11), (13)-(14) and (16).*

I assume that $\tau_t^{\pi} = 1$ (which is the standard assumption in this class of models). Confiscating all profits has the advantage of generating tax revenues that allow for reducing distortionary taxes without influencing households' decisions at the margin. Setting $\tau_t^{\pi} = 1$, the solution to Ramsey problem yields:

$$-\frac{u_{l,t} + \zeta (u_{l,t} + u_{ll,t} l_t + u_{cl,t} c_t + u_{hl,t} h_t)}{u_{c,t} + \zeta (u_{c,t} + u_{cc,t} c_t + u_{lc,t} l_t + u_{hc,t} h_t)} = z_t f_{l,t} , \tag{17}$$

with ζ being the Lagrange multiplier on the PVIC and $u_{xy,t}$ being the second derivative of u with respect to any two arguments. Comparing (17) with (15), the solutions to the Ramsey problem and the social planner problem coincide if ζ =0 as the problem of the Ramsey planner is essentially reduced to the problem of the social planner.

Finally, the combination of labor supply (4) and labor demand (10) gives:

$$-\frac{u_{l,t}}{u_{c,t}} = \left(\frac{1-\tau_t^l}{1+\phi\mu_t}\right) z_t f_{l,t}, \qquad (18)$$

which suggests that the labor tax rate and the credit friction drive a wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor. We may refer to this wedge as the "labor wedge".

5 Analytical Results

The main analytical results about the optimal labor-income tax rate are presented in this section. I start by describing the solution to the social planner's problem and then turn to the solution of the Ramsey planner's problem.

5.1 First-Best Labor-Income Taxation Policy

In this subsection, I show that the market solution with a constant labor tax rate is not efficient. Comparing condition (18) to condition (15), the market allocation is efficient only if

$$\tau_{FB,t}^l = -\phi\mu_t, \qquad (19)$$

with $\tau_{FB,t}^l$ being the first-best labor tax rate. Condition (19) suggests that for the market allocation to be efficient, labor income should be subsidized by the size of the credit friction. More importantly, the size of this subsidy is not constant as μ_t varies over the business cycle. Clearly, this subsidy is not needed when credit frictions are absent; in this case, the first-best labor tax rate is zero in all dates and states.

5.2 Second-Best Labor-Income Taxation Policy

In order to provide an analytical solution to the Ramsey taxation problem, I assume the following separable period utility function:

$$u(c_t, h_t, l_t) = logc_t + \psi logh_t - \chi l_t, \qquad (20)$$

with ψ and χ being parameters that measure the relative weights on real estate and the disutility from labor, respectively. Given this functional form, condition (17) reads:

$$-\frac{u_{l,t}}{u_{c,t}} = \frac{z_t f_{l,t}}{1+\zeta}, \qquad (21)$$

which differs from the solution of the Ramsey planner only due to the shadow value on the PVIC.

The combination of (18) and (21) gives the following optimal labor tax rate:

$$\tau^{l}_{SB,t} = \frac{\zeta - \phi\mu_t}{1+\zeta}, \tag{22}$$

where $\tau^{l}_{SB,t}$ is the optimal (second-best) labor-income tax rate that is chosen by the Ramsey planner. Equation (22) is the key expression characterizing the optimal labor tax rate in this section.

The main insights that come out of this condition can be summarized as follows:

Proposition 1: *In an economy with no credit frictions (ϕ =0), the optimal labor-income tax is constant over the business cycle.*

Proof: Setting ϕ =0 in condition (22), we have $\tau^{l}_{SB,t} = \frac{\zeta}{1+\zeta}$, which is completely constant. This is re-affirmation to the classical result in optimal labor taxation with neoclassical labor markets. *QED.*

Proposition 2: *In an economy with credit frictions (ϕ >0), the optimal labor-income tax rate is non constant. Moreover, the optimal labor tax rate is decreasing in the degree of tightness of the credit constraint.*

Proof: When ϕ >0, condition (22) suggests that, to the extent that μ_t is time varying, the labor tax rate is time varying as well. Clearly, the labor tax rate is lower whenever the shadow value on the financing constraint is higher, and vice versa. *QED.*

By reducing the labor tax rate more in periods of tighter credit markets, optimal policy in this setup "leans against the wind". The optimal labor tax rate is lower than in an otherwise model with no credit imperfections; the Ramsey planner sets a lower labor tax rate to boost labor supply whenever labor demand is reduced due to the binding credit constraint.

Finally, the assumption that the credit constraint is always binding does not alter the main insights of this subsection. If the constraint was assumed to only occasionally bind, μ_t will be either zero or positive, hence not constant. In turn, the labor-income tax rate will not be constant. In order to simplify matters and to make the computational solution more tractable, I do not consider this case here.

6 Computational Strategy and Calibration

6.1 Parameterization and Functional Forms

The time unit is a quarter and hence the discount factor β is set to 0.99, implying an annual interest rate of roughly 4 percent. I also assume the following period utility function for households:

$$u(c_t, h_t, l_t) = \log c_t + \psi \log h_t - \chi \frac{l_t^{1+\theta}}{1+\theta}. \tag{23}$$

The parameter θ is set to zero, implying a linear disutility function of labor. The implied labor supply elasticity helps in capturing the volatility of total hours in a model with no extensive margin, as is the case in this paper. The parameter χ is calibrated so that the steady state value of l is 0.33 and ψ is set so that the steady state of h is 0.8.

Firms produce using the Cobb-Douglas production function:

$$f(x_t, l_t) = x_t^{\alpha} l_t^{1-\alpha}, \tag{24}$$

with α, the share of real estate in the production function, being of 0.03, in line with Iacoviello (2005) .

Total factor productivity is governed by the following AR(1) process:

$$log(z_t) = (1-\rho_z) log(z) + \rho_z log(z_{t-1}) + u_t, \tag{25}$$

with the innovation term u_t being normally distributed with zero mean and a standard deviation of σ_u. The coefficient ρ_z is set to 0.95 and the standard deviation σ_v is set to 0.0075, in line with the literature. The deterministic steady state value of z_t is normalized to 1.

Similarly, government expenditures evolves according to the following AR(1) process:

$$log(g_t) = (1-\rho_g) log(g) + \rho_g log(g_{t-1}) + v_t, \tag{26}$$

where v_t is normally distributed with zero mean and a standard deviation of σ_v and g is set so that the deterministic steady state value of government spending is 20 percent of deterministic steady state output (which is the average government-GDP ratio over 1960-2007). In line with the literature, ρ_g and σ_v are set to 0.90 and 0.018, respectively.

The steady state value of b is obtained so that b/y is 0.36. This is the average of the gross federal debt held by the public as percentage of GDP over the period 1960-2007 (see Table B79 of the *2011 Economic Report of the President*). I choose 2007 as the final year of the sample because this ratio has increased dramatically in the last three years; including those years in the sample may only bias my results without adding any further insights.

The additional discount factor γ is set to 0.99, implying an annual discount rate of about 0.98 for the firm, in line with Iacoviello (2005). I set ϕ set to 1 in the benchmark calibration of the model, but I also consider other values of this parameter in the robustness analysis section. I set the loan-to-value ratio κ to 0.89, which equals the entrepreneurial loan-to-value ratio as reported in Iacoviello (2005).

6.2 Solution Methodology

The decision rules that solve this problem are obtained through a second-order approximation to the optimality conditions of the Ramsey planner around the non-stochastic steady state of the model. I apply the second-order approximation procedure that was developed by Schmitt-Grohe and Uribe (2004b).

7 Quantitative Results

This section presents the main numerical results regarding the optimal labor-income tax rate.

7.1 Second-Best Labor Taxation Policy

Table 1 presents the mean and the second moments of the labor tax rate following shocks (of one standard deviation size) to total factor productivity only, government expenditures only and to simultaneous shocks to TFP and government expenditures.

With credit frictions, optimal policy calls for a time-varying path of the labor tax rate. The standard deviation of the labor tax rate is significantly high (and higher than models with neoclassical labor markets usually predict). In all cases considered, the standard deviation of the labor tax rate is more than twice as large as the standard deviation of output. Interestingly, this high volatility of the labor tax rate is observed even though the volatility of output is empirically plausible. The case of simultaneous shocks suggests a relatively high volatility of output, in which case the *relative* volatility of the labor tax rate to the volatility of output is perhaps a better indicator for the non-constant path of the labor-income tax rate. The labor tax rate also displays little persistence over the business cycle as a result of the borrowing constraint. As the shadow value of the binding borrowing constraint changes, the labor-income tax rate fluctuates as well, leaving little room for persistence in the labor tax rate.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
		Government Expenditures and TFP shocks		
τ^l	0.1740	0.0551	0.1062	-0.9590
y	0.3285	0.0230	0.1990	1.0000
l	0.3346	0.0223	0.1240	0.9479
		TFP Shock		
τ^l	0.1955	0.0360	0.1397	-0.9260
y	0.3160	0.0168	0.2729	1.0000
l	0.3210	0.0143	0.0916	0.8933
		Government Expenditures Shock		
τ^l	0.1882	0.0483	0.0827	-0.9949
y	0.3247	0.0186	0.2435	1.0000
l	0.3297	0.0195	0.2445	0.9806

Table 1: Optimal fiscal policy with credit frictions. The standard deviation of the U.S. GDP over 1964:1-2007:4 is 0.0152.

The labor-income tax rate falls in recessions and rises in booms. Fiscal policy thus "leans against the wind". The fall in the labor tax rate following a negative shock to government expenditures is not surprising- the planner cuts the labor-income tax rate accordingly. In this paper, the fall in the labor tax rate is also due to the binding credit constraint, as discussed above.

The volatility of the labor tax rate in this paper allows for the more general result of "wedge smoothing", which is a very central result in optimal taxation, to hold. In the lack of credit frictions, labor taxation is the only source of the labor wedge (see condition 18). Therefore, smoothing the labor wedge is equivalent to smoothing the labor tax rate. In this model, however, the credit friction is another source of the labor wedge and complete smoothing of the labor tax rate is not translated into complete smoothing of the wedge. Keeping the labor tax rate constant in the face of exogenous shocks induces variations in the wedge over the business cycle. Under the optimal tax policy characterized in section 4 and numerically evaluated in Table 1, the labor wedge is completely smoothed following exogenous shocks. We thus conclude that even though labor tax smoothing is not optimal in this setup, smoothing distortions over the business cycle remains optimal.

7.2 Impulse Responses

Figure 1 displays the response of the economy to a one standard deviation shock to TFP (for illustration purposes, I only consider a negative shock). A negative TFP shock reduces the demand of labor and the real wage, but at the same time the real price of real estate and the demand of the entrepreneur for real estate fall as well. The overall effect is an increase in the shadow value on the credit constraint μ, which in turn leads to a fall in the labor tax rate (of about 2 percent). The fall in the equilibrium amount of labor and the fall in real estate held by entrepreneurs lead to a fall in output and consumption.

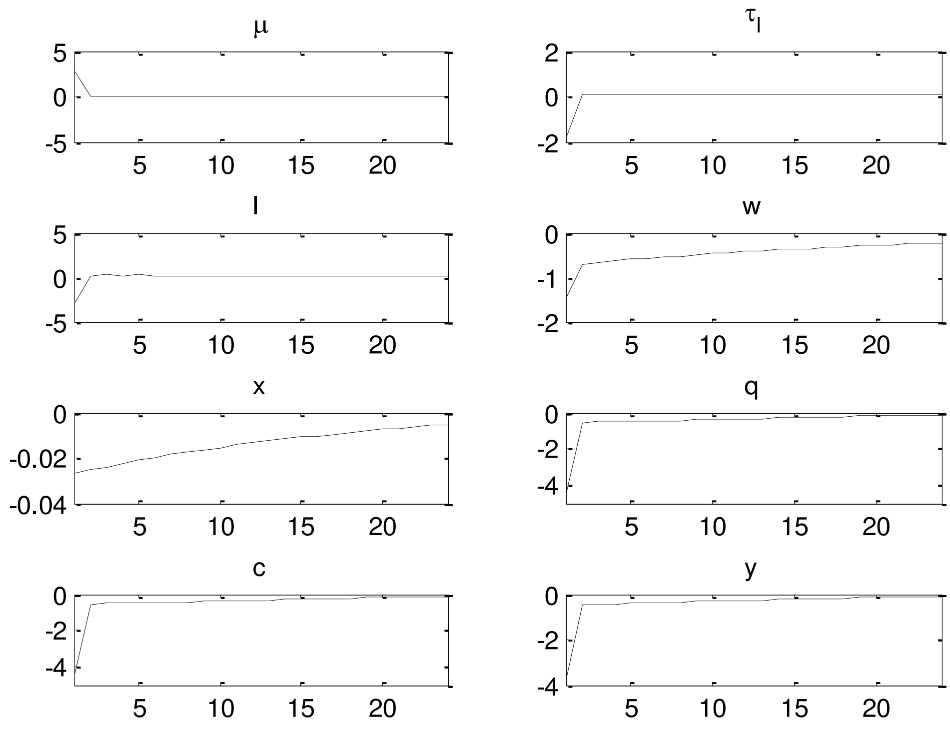

Figure 1: Response to a TFP shock (percentage deviations from SS levels).

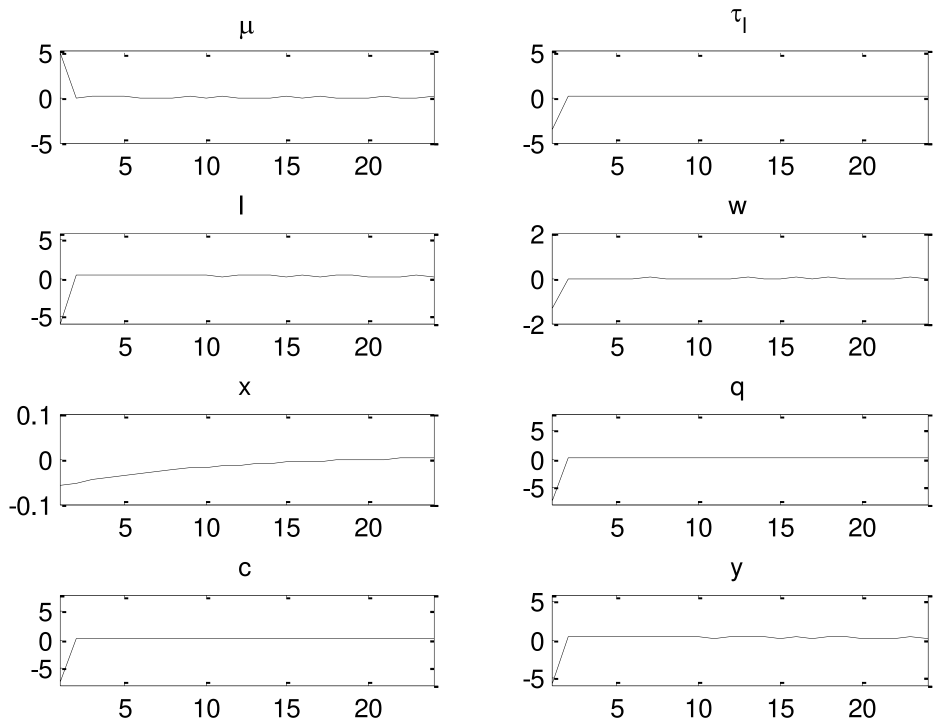

Figure 2: Response to a government expenditure shock (percentage deviations from SS levels).

The fall in the labor tax rate is bigger following a fall in government expenditures (Figure 2). In this case, the fall in government expenditures and the increase in the tightness of the credit constraint lead to a stronger fall in the labor tax rate. Other variables display similar patterns as in Figure 1, but with different magnitudes.

8 Robustness Analysis

I first show the volatility of the labor tax rate for different values of the parameter ϕ. I then show the results under a finite elasticity of labor supply and study the case when profits are not taxed at the optimal rate of 100 percent. Finally, I study the case when capital and capital taxation are introduced into the model.

8.1 Changing the Value of the Parameter ϕ

Figure (3) presents the standard deviation of the labor tax rate for various values of ϕ between 0.2 and 1 following all types of shocks considered in Table 1. For illustration purposes, the results for ϕ=0 are not presented, but with the note the labor tax rate is completely constant in this case.

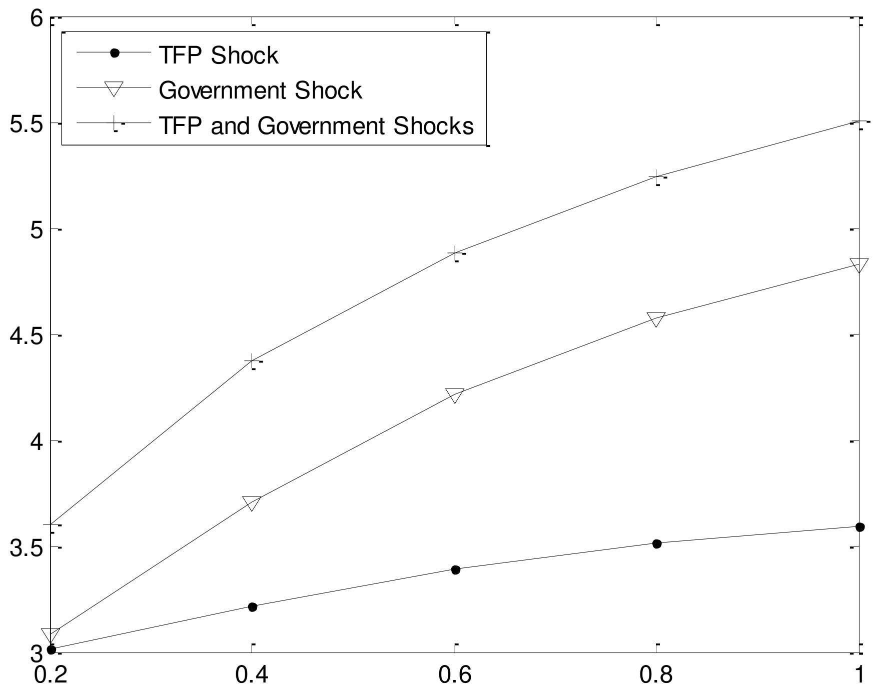

Figure 3: The standard deviation of the labor-income tax rate for various values of ϕ (in percentage terms).

The main observations can be summarized as follows. First, following all types of shocks, the standard deviation of the labor tax rate is significantly meaningful even if ϕ is relatively low. For example, for ϕ=0.2, the standard deviation of the tax rate is roughly 3 percent following TFP and government shocks and more than 3.5 percent following simultaneous shocks. Second, the volatility of the labor tax rate is increasing in the value of ϕ, but at a lower rate. Third, the differences between the volatilities of the tax rate following different types of shocks are increasing in ϕ. We can better understand this result by first considering the case with no credit frictions- the volatility is zero following all shocks. As ϕ becomes positive but remains low, the volatilities remain highly similar. However, the differences start to increase when this parameter increases more as the type of the shock becomes more important for the behavior of the labor tax rate.

8.2 A Lower Elasticity of Labor Supply

The analysis so far assumed that $\theta=0$, implying infinite labor supply elasticity, to give numerical predictions to support the analytical results of section 5. In this subsection, I consider a lower labor supply elasticity since the size of this elasticity can matter for the volatility of labor and the volatility of the labor-income tax rate.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
	Government Expenditures and TFP shocks			
τ^l	0.1823	0.0514	0.2958	-0.8497
y	0.3277	0.0186	0.4938	1.0000
l	0.3324	0.0191	0.3103	0.8277
	TFP Shock			
τ^l	0.1967	0.0345	0.5012	-0.6293
y	0.3184	0.0133	0.7392	1.0000
l	0.3228	0.0127	0.4899	0.5759
	Government Expenditures Shock			
τ^l	0.1908	0.0421	0.2641	-0.9926
y	0.3250	0.0158	0.5264	1.0000
l	0.3298	0.0168	0.5316	0.9723

Table 2: Optimal fiscal policy with credit frictions and $\theta=1$.

Table 2 shows the results for unitary labor supply elasticity. I keep using a relatively high labor supply elasticity to better account for the volatility of total hours in this setup. In general, the results of this subsection support my earlier findings about the volatility of the labor tax rate despite a slight decrease in the volatility of the labor tax rate compared to the results reported in Table 1. The labor tax rate remains highly volatile and significantly more volatile than output (which also becomes less volatile given the same magnitudes of shocks). Therefore, the choice of the labor supply elasticity behind the results of Table 1 is not significant for the main result of this paper: credit frictions induce, optimally, a high volatility in the labor-income tax rate over the business cycle.

8.3 Zero Taxation of Profits

The analyses above assumed that the government confiscates all profits (i.e. $\tau_t^{\pi} = 1$). Since after-tax profits do not affect households' decision at the margin, it is optimal to tax them on the rate of 100 percent and thus allow for other taxes to be reduced. In this subsection, I show the results when this assumption is relaxed. Specifically, I consider the other pillar case of $\tau_t^{\pi} = 0$.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
		Government Expenditures and TFP shocks		
τ^l	0.1948	0.0625	0.5581	-0.9482
y	0.3163	0.0237	0.5447	1.0000
l	0.3207	0.0226	0.4494	0.9348
		TFP Shock		
τ^l	0.2046	0.0481	0.6147	-0.9290
y	0.3125	0.0187	0.6974	1.0000
l	0.3171	0.0170	0.5166	0.8816
		Government Expenditures Shock		
τ^l	0.1973	0.0572	0.5114	-0.9885
y	0.3152	0.0200	0.4390	1.0000
l	0.3193	0.0210	0.4406	0.9753

Table 3: Optimal fiscal policy with credit frictions and $\tau_t^{\pi} = 1$.

The results, obtained under the benchmark calibration of the model, are presented in Table 3. Labor tax rate volatility remains optimal in this case. In fact, the volatility of the labor tax rate in this case is considerably higher than in the benchmark case presented in Table 1. In addition, the average of the labor tax rate is higher in all cases considered since the lack of profit taxation requires heavier taxation of labor income to generate sufficient government revenues. Zero taxation of profits also requires the planner to vary the labor tax rate even more over the business cycle, thus reducing the degree to which the labor tax rate can be smoothed.

8.4 A Model with Capital

In what follows, I show that the main result of the paper is not affected if capital-income taxation is added to the set of government instruments. To do so, I assume that households accumulate physical capital k_t, rent it to firms at a price of r_t and they pay a capital-income tax rate of τ_t^k. In this setup, all firm's factor payments are subject to the collateral requirement (see Abo-Zaid, 2013 for a similar setup). The main reason for this exercise is to verify whether, having another source of tax revenue, optimal policy will use capital taxation to allow for complete smoothing of the labor-income tax rate. It is also important quantitatively (i.e. to experiment on the size of the labor-income tax rate suggested by the above analyses).

The model with capital is derived in Appendix D and the corresponding PVIC is shown in Appendix E. I start by discussing the analytical result that is obtained in this setup (Appendix F): clearly, in the model with capital, the labor-income tax rate and the Lagrange multiplier on the credit constraint move in opposite directions over the business cycle and, therefore, the labor-income tax rate is not completely smoothed over time. This result re-affirms the finding in subsection 5.2.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
		Government Expenditures and TFP shocks		
τ^l	0.2271	0.0667	0.0664	-0.9657
y	0.8614	0.0227	0.6892	1.0000
l	0.3274	0.0277	0.6929	0.9336
		TFP Shock		
τ^l	0.2311	0.0372	0.0713	-0.9420
y	0.8603	0.0160	0.7179	1.0000
l	0.3273	0.0118	0.6874	0.9797
		Government Expenditures Shock		
τ^l	0.2405	0.0516	0.0642	-0.9708
y	0.8620	0.0161	0.6610	1.0000
l	0.3279	0.0251	0.6941	0.9953

Table 4: Optimal fiscal policy with credit frictions- the model with capital.

Table 4 summarizes the numerical results in this case. Interestingly, the size of the labor-income tax rate is higher than in Table 1. The reason for this result is that, due to the credit friction, optimal fiscal policy calls for *subsidizing* capital income.[1] To finance this subsidy, the long-run labor-income tax rate is higher relative to the case with no capital. In particular, in all cases considered, the average labor-income tax rate is increased by about 5 percent, which is very similar to the size of the capital subsidy (roughly 5.5 percent, on average).

More importantly, the labor-income tax rate is again far from being fully smoothed over the cycle. We thus conclude that the benchmark results are robust to inclusion of capital taxation in the model. Therefore, the main result of the paper is not due to the lack of policy instruments on the part of the government. Instead, periods of tightened credit markets that reduce labor demand (thus constraining firms' projects) should be countered by encouraging labor supply.

9 The Average Tax Rate and the Credit Spread in the U.S.

In this section, I test the results of the paper versus the observed U.S. data. To do so, I obtain data on the average income tax rate and a measure of credit market tightness. I use the credit spread between the Moody's Seasoned Corporate Bond Yield (BAA) and the 1-Year Treasury Constant Maturity Rate (GS1). Since the available tax data are annual, I present the evidence on what follows using annual data. The average tax rate data are taken from the Tax Policy Center and the original data on the BAA and GS1 are available in the FRED database of the Federal Reserve Bank of St. Louis.

To better account for the business cycle behavior of both variables, I calculate the HP-filtered cyclical component of each data series. Figure 4 presents the behavior of the cyclical component of each variable over the period 1979-2009 (for which data on the average income tax rate are available). The figure shows clear negative correlation between the average income tax rate and the credit spread over the business cycle (the correlation coefficient is -0.81). Periods with above-trend credit spread are associated with below-trend tax rates and vice versa. In other words, the correlation does not only hold during periods of stress, but also in expansions. This figure supports the findings

[1] In a model similar to this one, Abo-Zaid (2012) shows that the optimal long-run capital-income tax rate is *negative* unless the degree of depreciation allowance is very high (which is not the case here) and that the subsidy depends on the size of the credit friction $\phi\mu$.

of this paper; facing tighter credit markets, the labor-income tax rate is reduced. In addition, it suggests a countercyclical labor-income tax rate, which essentially implies that the labor-income tax rate is not completely smoothed over time.

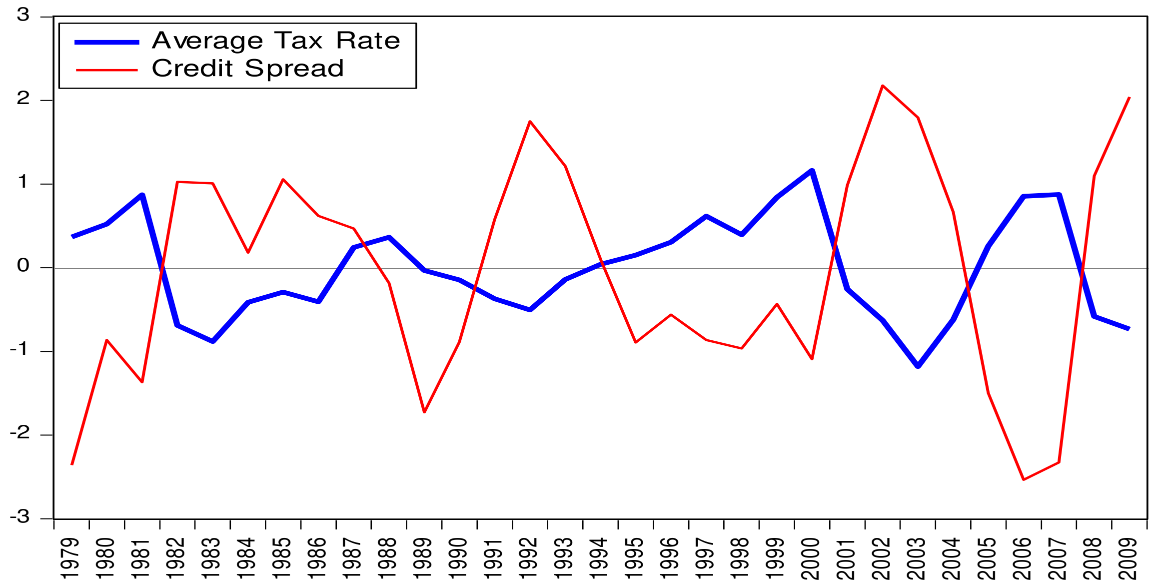

Figure 4: The cyclical component of the average income tax rate and the cyclical component of the credit spread, 1979-2009.

10 Conclusions

This paper studies the optimal behavior of the labor-income tax rate over the business cycle in a model with credit frictions. Firms' borrowing to finance the hiring of labor at the beginning of the period is constrained by the value of their collateral. The credit constraint induces an inefficiently low demand for labor. In this environment, complete smoothing of the labor-income tax rate is not optimal. When the credit constraint tightens more, it is optimal to hold a relatively lower labor-income tax rate in order to boost labor supply. When the credit constraint is less tightened, the optimal labor tax rate should be relatively higher. The labor-income tax rate thus moves in the opposite direction of the tightness of the credit constraint.

The paper also shows that this optimal behavior of the labor-income tax rate is consistent with the behavior of the average income tax rate in the U.S. over the period 1979-2009; periods with tighter credit markets (as measured by the corporate credit spread) are characterized by reduced income tax rates. Specifically, periods with above-trend credit spreads are associated with below-trend tax rates and vice versa.

Quantitatively, the volatility of the labor tax rate is considerably higher in a model with credit frictions than in an otherwise model with frictionless credit markets and significantly higher than the (empirically-plausible) volatility of output. The volatility of the labor tax rate allows for stabilizing labor (thus output) over the business cycle, which on itself induces a smoothed path of consumption.

Since the borrowing constraint induces inefficiently low demand for labor, the firm hires labor so that the marginal product of labor is a “markup” over the real wage rate. The tax reduction in more tightened credit markets helps in offsetting this markup, thus positioning the economy closer to the efficient allocation.

This paper is part of the very timely line of research that studies the implications of credit frictions for macroeconomic policies in general, and for optimal taxation in particular. Credit frictions are proven as important factors in shaping macroeconomic policies in addition to their traditional role in magnifying the effects of exogenous shocks on the macroeconomy. Essentially, the theoretical and empirical results of this paper call for an active fiscal policy over the business cycle. The aim of such a policy is to counter the effects of distortions (here, in the form of credit frictions) that may have bigger impact on economic activity otherwise. In other words, governments should use labor-income taxation as a stabilizing tool in the face of exogenous shocks.

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Appendix: Mathematical Derivations

A The Firm's Problem

The firm chooses labor and loans to maximize:

$$z_t f(x_t, l_t) + b_t^f - w_t l_t - R_t^f b_t^f \,, \tag{A1}$$

subject to

$$b_t^f - \phi w_t l_t \geq 0 \,, \tag{A2}$$

and,

$$\kappa q_t x_t - b_t^f \geq 0 \,, \tag{A3}$$

Letting υ_t and γ_t denote the Lagrange multipliers on the constraints (A2) and (A3), respectively, the optimality condition with respect to b_t^f reads:

$$\gamma_t = R_t^f + \upsilon_t - 1 \,. \tag{A4}$$

Similarly, the first order condition with respect to l_t yields:

$$z_t f_{l,t}(x_t, l_t) = (1 + \phi \upsilon_t) w_t \,, \tag{A5}$$

Recalling that $R_t^f = 1$, equation (A4) becomes:

$$\upsilon_t = \gamma_t \,. \tag{A6}$$

Alternatively, conditions (A2) and (A3) can be combined to get:

$$\kappa q_t x_t - \phi w_t l_t \geq 0 \,, \tag{A7}$$

which is condition (9) in the text. Furthermore, substituting $R_t^f = 1$ in (A1), the profit function is now given by:

$$z_t f(x_t, l_t) - w_t l_t \,, \tag{A8}$$

which is condition (8) in the text. Therefore, the optimization problem of the firm is to maximize (A8) subject to (A7). Letting μ_t be the Lagrange multiplier on (A7), the demand function of labor reads:

$$z_t f_{l,t}(x_t, l_t) = (1 + \phi \mu_t) w_t \,, \tag{A9}$$

which is condition (10) in the text. This condition coincides with equation (A5) when $\upsilon_t = \gamma_t = \mu_t$.

B Efficient Allocations

The social planner chooses consumption, labor and real estate for the next period to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, h_t), \tag{B1}$$

subject to the sequence of resource constraints:

$$z_t f(1-h_t, l_t) = c_t + g_t. \tag{B2}$$

Letting η_t be the Lagrange multiplier associated with (B2), the first-order conditions with respect to c_t, l_t and h_{t+1}, respectively, read:

$$u_{c,t} = \eta_t, \tag{B3}$$

$$u_{l,t} + \eta_t z_t f_{l,t} = 0. \tag{B4}$$

and

$$\beta E_t u_{h,t+1} - \beta E_t \left[\eta_{t+1} z_{t+1} f_{h,t+1}\right] = 0. \tag{B5}$$

Combining (B3) and (B4) yields

$$-\frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t}(x_t, l_t), \tag{B6}$$

and hence efficiency requires the marginal rate of substitution (the left hand side of condition (B6)) to be equal to the marginal product of labor (given by the right-hand side of condition (B6)).

Similarly, combining (B3) and (B5) gives

$$\frac{u_{h,t}}{u_{c,t}} = z_t f_{h,t}, \tag{B7}$$

which is another efficiency condition when households derive utility from real estate and firms produce using real estate.

C The Present- Value Implementability Constraint

I show here the derivation of the PVIC for the Ramsey problem. Recalling that $R_t^f = 1$, the households' budget constraint becomes:

$$(1-\tau_t^l) w_t l_t + (1-\tau_t^\pi)\Pi_t + q_t h_t + R_{t-1} b_t = c_t + q_t h_{t+1} + b_{t+1}. \tag{C1}$$

By introducing $E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t}$ to (C1) and rearranging, we have:

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^l)w_t l_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^{\pi})\Pi_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_t \\
&+E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}b_{t+1} = 0
\end{aligned}
\quad . \tag{C2}
$$

Recall that, from the solution to the households' problem, we have:

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t\,, \tag{C3}$$

$$u_{c,t} = \beta R_t E_t(u_{c,t+1})\,, \tag{C4}$$

$$u_{c,t}q_t = \beta E_t(u_{h,t+1} + u_{c,t+1}q_{t+1})\,, \tag{C5}$$

Substituting (C3) in the first term of (C2), (C5) in the sixth term of (C2) and (C4) in the last term of (C2) yield:

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left(-\frac{u_{l,t}}{u_{c,t}}\right)l_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^{\pi})\Pi_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_{tt} + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}R_{t-1}b_t \\
&-E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t - E_0\sum_{t=0}^{\infty}\beta^t \beta E_t(u_{h,t+1}+u_{c,t+1}q_{t+1})h_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t \beta R_t u_{c,t+1}b_{t+1} = 0.
\end{aligned}
\quad . \tag{C6}
$$

Combining the third and sixth terms of (C6) yield:

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_{tt} - E_0\sum_{t=0}^{\infty}\beta^t \beta E_t u_{h,t+1}h_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t \beta E_t(u_{c,t+1}q_{t+1}h_{t+1}) \\
&= u_{c,0}q_0h_0 - E_0\sum_{t=0}^{\infty}\beta^{t+1}u_{h,t+1}h_{t+1}
\end{aligned}
\quad . \tag{C7}
$$

Similarly, combing the fourth and seventh terms of (C6) gives

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t \beta R_t u_{c,t+1}b_{t+1} = u_{c,0}R_{-1}b_0 + E_0\sum_{t=1}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t \beta R_t u_{c,t+1}b_{t+1} \\
&= u_{c,0}R_{-1}b_0
\end{aligned}
\quad . \tag{C8}
$$

Also, the combination of the first and fifth terms of (C6) gives:

$$E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left(-\frac{u_{l,t}}{u_{c,t}}\right)l_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t = -E_0\sum_{t=0}^{\infty}\beta^t u_{l,t}l_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t\,. \tag{C9}$$

Finally, substituting (C7)-(C9) into (C6) yield:

$$E_0\sum_{t=0}^{\infty}\beta^t u_{l,t}l_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t + E_0\sum_{t=0}^{\infty}\beta^{t+1}u_{h,t+1}h_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^{\pi})\Pi_t - u_{c,0}R_{-1}b_0 - u_{c,0}q_0h_0 = 0$$

or,

$$E_0\sum_{t=0}^{\infty}\beta^t\left[u_{c,t}c_t + u_{l,t}l_t + \beta u_{h,t+1}h_{t+1} - u_{c,t}(1-\tau_t^{\pi})\Pi_t\right] = u_{c,0}R_{-1}b_0 + u_{c,0}q_0h_0\,, \tag{C10}$$

which is condition (16) in the text.

D The Model with Capital

Households maximize their expected discounted lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, l_t), \tag{D1}$$

subject to the sequence of budget constraints:

$$\begin{aligned}&(1-\tau_t^l) w_t l_t + \left[1-\delta + (1-\tau_t^k) r_t\right] k_t + (1-\tau_t^{\pi})\Pi_t + q_t h_t + R_{t-1} b_t + R_t^f b_t^f \\ &= c_t + k_{t+1} + b_{t+1} + b_t^f + q_t h_{t+1}\end{aligned}, \tag{D2}$$

where r_t is rental rate of capital, δ denotes the depreciation rate of capital and τ_t^k is the capital-income tax rate. All other variables and parameters are as in the text.

The optimal choices of consumption, labor supply, capital, real estate, lending to firms and bond holdings yield the following optimization conditions:

$$R_t^f = 1, \tag{D3}$$

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l) w_t, \tag{D4}$$

$$u_{c,t} = \beta R_t E_t(u_{c,t+1}), \tag{D5}$$

$$u_{c,t} = \beta E_t\left[u_{c,t+1}\left(1-\delta + r_{t+1}(1-\tau_{t+1}^k)\right)\right], \tag{D6}$$

$$q_t u_{c,t} = \beta E_t(u_{h,t+1} + q_{t+1} u_{c,t+1}), \tag{D7}$$

with Equation (6) being the standard capital supply condition and all other conditions are as in the main text.

The representative firm hires labor, rents capital from households and uses real estate to produce a homogenous good using the following production function:

$$y_t = z_t f(k_t, x_t, l_t), \tag{D8}$$

with y_t, z_t and x_t being output, total factor productivity and the firm's real estate, respectively. The problem of the firm now is

$$Max\left[z_t f(k_t, x_t, l_t) - w_t l_t - r_t k_t\right], \tag{D9}$$

subject to

$$\phi(w_t l_t + r_t k_t) \le \kappa q_t x_t. \tag{D10}$$

Denoting the Lagrange multiplier on (D10) by μ_t, profit maximization gives:

$$z_t f_{l,t}(k_t, x_t, l_t) = (1+\phi\mu_t) w_t, \tag{D11}$$

$$z_t f_{k,t}(k_t, x_t, l_t) = (1+\phi\mu_t) r_t, \tag{D12}$$

The firm thus hires labor and rents capital so that the marginal product of each of the two inputs is a "markup" over its factor price.

In the second part of the period, the firm chooses the next-period real estate taking into account the role of real estate as collateral and subject to the budget constraint $z_t f(k_t, x_t, l_t) - w_t l_t - r_t k_t + q_t x_t = q_t x_{t+1} + \Pi_t$. The left-hand side is the total resources of the firm after production takes place; they are equal to the sum of operating profits, $z_t f(k_t, x_t, l_t) - w_t l_t - r_t k_t$, and the market value of assets, $q_t x_t$.

With this characterization of the firm's problem, the choice of x_{t+1} gives the following dynamic equation in the price of real estate:

$$q_t u_{c,t} = \gamma \beta E_t \left[u_{c,t+1}[z_{t+1} f_{x,t+1} + (1+\kappa\mu_{t+1}) q_{t+1}] \right], \tag{D13}$$

which makes explicit the roles of the credit friction and the additional discount factor.

Since profits Π_t are transferred to households, one could alternatively assume that the objective function of the firm is to choose labor, capital and real estate in order to maximize $\Pi_t = z_t f(k_t, x_t, l_t) - w_t l_t - r_t k_t + q_t x_t - q_t x_{t+1}$. In this case, the firm's problem is to maximize $E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{0,t} \left[z_t f(k_t, x_t, l_t) - w_t l_t - r_t k_t + q_t x_t - q_t x_{t+1} \right]$ subject to (D10). It is straightforward to show that the first-order conditions with respect to labor, capital and next-period real estate are exactly the same as (11)-(13). In this respect, both approaches are identical.

The government collects capital-income taxes, labor-income taxes, profit taxes and obtains real debt to finance an exogenous stream of real government expenditures g_t. The government budget constraint in period *t* is thus given by:

$$\tau_t^l w_t l_t + \tau_t^k (r_t - \rho\delta) k_t + \tau_t^\pi \Pi_t + b_{t+1} = g_t + R_{t-1} b_t \tag{D14}$$

This condition combined with households budget constraint gives the following resource constraint:

$$z_t f(k_t, x_t, l_t) + (1-\delta) k_t = c_t + k_{t+1} + g_t. \tag{D15}$$

Finally, real estate is in a fixed supply:

$$h_t + x_t = 1. \tag{D16}$$

E The Present- Value Implementability Constraint with Capital

The derivation of the Implementability Constraint (IC) for the Ramsey problem with capital is presented in what follows. Recalling that $R_t^f = 1$, the households' budget constraint becomes:

$$(1-\tau_t^l)w_t l_t + \left[1-\delta+(1-\tau_t^k)r_t\right]k_t + (1-\tau_t^\pi)\Pi_t + q_t h_t + R_{t-1}b_t = c_t + k_{t+1} + b_{t+1} + q_t h_{t+1}. \tag{E1}$$

By introducing $E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}$ to (D1) and rearranging, we have:

$$\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^l)w_t l_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left[1-\delta+(1-\tau_t^k)r_t\right]k_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^\pi)\Pi_t \\
&+ E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t \\
&- E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}k_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}b_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_{t+1} = 0
\end{aligned} \quad . \tag{E2}$$

Recall that, from the solution to the households' problem, we have:

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t\,, \tag{E3}$$

$$u_{c,t} = \beta R_t E_t(u_{c,t+1})\,, \tag{E4}$$

$$u_{c,t} = \beta E_t\left[u_{c,t+1}\left(1-\delta+r_{t+1}(1-\tau_{t+1}^k)\right)\right], \tag{E5}$$

$$u_{c,t}q_t = \beta E_t(u_{h,t+1} + u_{c,t+1}q_{t+1})\,, \tag{E6}$$

Substituting (E3) in the first term of (E2), (E4) in the eighth term of (E2), (E5) in the seventh term of (E2) and (E6) in the last term of (E2) yield:

$$\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left(-\frac{u_{l,t}}{u_{c,t}}\right)l_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left[1-\delta+r_t(1-\tau_t^k)\right]k_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^\pi)\Pi_t \\
&+ E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t \\
&- E_0\sum_{t=0}^{\infty}\beta^t \beta u_{c,t+1}\left[1-\delta+r_{t+1}(1-\tau_{t+1}^k)\right]k_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t \beta R_t u_{c,t+1}b_{t+1} \\
&- E_0\sum_{t=0}^{\infty}\beta^t \beta E_t(u_{h,t+1} + u_{c,t+1}q_{t+1})h_{t+1} = 0
\end{aligned} \quad . \tag{E7}$$

Combining the second and seventh terms of (E7) yield:

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left[1-\delta+(1-\tau_t^k)r_t\right]k_t - E_0\sum_{t=0}^{\infty}\beta^t\beta u_{c,t+1}\left[1-\delta+(1-\tau_{t+1}^k)r_{t+1}\right]k_{t+1}\\
&=u_{c,0}\left[1-\delta+(1-\tau_0^k)r_0\right]k_0 + E_0\sum_{t=1}^{\infty}\beta^t u_{c,t}\left[1-\delta+(1-\tau_t^k)r_t\right]k_t\\
&-E_0\sum_{t=0}^{\infty}\beta^{t+1}u_{c,t+1}\left[\left(1-\delta+r_{t+1}-(r_{t+1}-\rho\delta)\tau_{t+1}^k\right)\right]k_{t+1}\\
&=u_{c,0}\left[1-\delta+r_0-\tau_0^k(r_0-\rho\delta)\right]k_0
\end{aligned}
\quad , \qquad \text{(E8)}
$$

where the last two terms of (E8) were canceled using summation rules.

Similarly, combing the fifth and eighth terms of (E7) gives

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t\beta R_t u_{c,t+1}b_{t+1} = u_{c,0}R_{-1}b_0 + E_0\sum_{t=1}^{\infty}\beta^t u_{c,t}R_{t-1}b_t - E_0\sum_{t=0}^{\infty}\beta^t\beta R_t u_{c,t+1}b_{t+1}\\
&=u_{c,0}R_{-1}b_0
\end{aligned}
\quad . \qquad \text{(E9)}
$$

Combining the fourth and last terms of (E6) yield:

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}q_t h_t - E_0\sum_{t=0}^{\infty}\beta^t\beta E_t u_{h,t+1}h_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t\beta E_t(u_{c,t+1}q_{t+1}h_{t+1})\\
&=u_{c,0}q_0h_0 - E_0\sum_{t=0}^{\infty}\beta^{t+1}u_{h,t+1}h_{t+1}
\end{aligned}
\quad . \qquad \text{(E10)}
$$

Also, the combination of the first and sixth terms of (E7) gives:

$$
E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}\left(-\frac{u_{l,t}}{u_{c,t}}\right)l_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t = -E_0\sum_{t=0}^{\infty}\beta^t u_{l,t}l_t - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t\,. \qquad \text{(E11)}
$$

Finally, substituting (E8)-(E11) into (E7) yield:

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t u_{l,t}l_t + E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}c_t + E_0\sum_{t=0}^{\infty}\beta^{t+1}u_{h,t+1}h_{t+1} - E_0\sum_{t=0}^{\infty}\beta^t u_{c,t}(1-\tau_t^{\pi})\Pi_t\\
&-u_{c,0}R_{-1}b_0 - u_{c,0}\left[1-\delta+r_0(1-\tau_0^k)\right]k_0 - u_{c,0}q_0h_0 = 0
\end{aligned}
$$

or,

$$
E_0\sum_{t=0}^{\infty}\beta^t\left[u_{c,t}c_t + u_{l,t}l_t + \beta u_{h,t+1}h_{t+1} - u_{c,t}(1-\tau_t^{\pi})\Pi_t\right] = A_0\,, \qquad \text{(E12)}
$$

where, $A_0 = u_{c,0}R_{-1}b_0 + u_{c,0}\left[1-\delta+r_0(1-\tau_0^k)\right]k_0 + u_{c,0}q_0h_0$

and $\Pi_t = z_t f(k_t,x_t,l_t) - \dfrac{z_t f_{k,t}}{1+\phi\mu_t}k_t - \dfrac{z_t f_{l,t}}{1+\phi\mu_t}l_t + q_t x_t - q_t x_{t+1}\,.$

F Optimal Policy in the Model with Capital

Definition: *Given the exogenous processes* $\{z_t, g_t\}$, *the Ramsey planner chooses sequences of allocations* $\{c_t, k_t, l_t, h_t, x_t, q_t, \mu_t\}$ *to maximize (D1) subject to (D13), (D15)-(D16) and (E12).*

Assuming $\tau_t^{\pi} = 1$, the solution to Ramsey problem yields:

$$-\frac{u_{l,t} + \zeta(u_{l,t} + u_{ll,t}l_t + u_{cl,t}c_t + u_{hl,t}h_t)}{u_{c,t} + \zeta(u_{c,t} + u_{cc,t}c_t + u_{lc,t}l_t + u_{hc,t}h_t)} = z_t f_{l,t}\,, \tag{F1}$$

with ζ being again the Lagrange multiplier on the PVIC. Combining the labor supply (D4) and labor demand (D11) gives:

$$-\frac{u_{l,t}}{u_{c,t}} = \left(\frac{1-\tau_t^l}{1+\phi\mu_t}\right) z_t f_{l,t}\,, \tag{F2}$$

which, again, suggests that the labor tax rate and the credit friction generate a “labor wedge” .

To obtain analytical results to the Ramsey taxation problem, I assume the following separable period utility function:

$$u(c_t, h_t, l_t) = log c_t + \psi log h_t - \chi l_t\,. \tag{F3}$$

With this functional form, condition (F1) reads:

$$-\frac{u_{l,t}}{u_{c,t}} = \frac{z_t f_{l,t}}{1+\zeta}\,, \tag{F4}$$

which is exactly as in the model with no capital (but with different values of course).

Combining (F2) and (F4) gives the following optimal labor tax rate:

$$\tau_{SB,t}^l = \frac{\zeta - \phi\mu_t}{1+\zeta}\,, \tag{F5}$$

Which is the same result we obtained in the main text. The negative correlation, thus between, the labor-income tax rate and the tightness of the credit constraint is robust to the inclusion of capital in the model. The inclusion of capital may only matter quantitatively.

Finally, the market allocation is efficient only if

$$\tau_{FB,t}^l = -\phi\mu_t\,, \tag{D6}$$

with $\tau_{FB,t}^l$ being the first-best labor tax rate. This is the same result we obtained before.