The Struggle to Survive in the RD Sector: Implications for Innovation and Growth

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May 2013

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Abstract

By allowing for investment activities by research and development (R&D) firms to prevent product obsolescence, we show that if legal patent protection is too strong, a higher R&D subsidy rate delivers insufficient investments for survival in the R&D sector, depressing innovation and growth in the long run.

JEL classification: O31, O34, O41
Keywords: Firm survival, R&D subsidy, patent breadth, endogenous growth

*The author acknowledges the hospitality and support of Simon Fraser University, where some of the work in this paper was completed. The author is grateful to Angus C. Chu and Tsuyoshi Kawasaki for their very helpful comments and suggestions. The author also gratefully acknowledges the partial financial support of a Grant-in-Aid for Young Scientists (B) #23730198.

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1 Introduction

The essential role of the entry, exit, and survival of firms has been emphasized in growth theory. In Schumpeterian growth models,[1] the economy grows through survival cycles commencing with the entry of a research and development (R&D) firm inventing a new high-quality technology and ending with the exit of the firm by destruction of its rents once a newer technology is introduced. Recent research stresses endogenous survival of firms engaging in private rent protection and examines the consequences for innovation and long-run growth (Dinopoulos and Syropoulos 2007, Eicher and García-Peñalosa 2008).[2] In line with these studies, this note examines the effects of R&D policies on firm survival, innovation, and growth.

The struggle to survive in the real world typically requires that firms make dynamic decisions.[3] We highlight this aspect, using a variety-based growth model with product obsolescence (Lai 1998). In doing so, we model R&D firms engaging in investments with the aim of increasing their probability of survival against obsolescence by means of a dynamic programming approach provided by Akiyama, Furukawa, and Yano (2011).[4] This approach results in a tractable equilibrium behavior of surviving firms, which is analogous to the equilibrium behavior in Dinopoulos and Syropoulos's (2007) quality-ladder model.

The main finding of this paper is that if R&D firms invest in their intertemporal survival, R&D policies may reduce innovation and long-run growth. Specifically, if patent protection is too strong,[5] a higher R&D subsidy rate delivers insufficient investments for the survival of R&D firms, depressing innovation and growth in the long run. This contrasts with the property of the standard R&D-based growth model whereby R&D subsidies promote innovation and growth, which holds in the Dinopoulos and Syropoulos model. In addition, the policy implication of our result is new to the literature[6] in suggesting a substantial interdependence between the two R&D policy instruments of R&D subsidies and patent protection. This note extends this line of research by showing that R&D subsidies can interact with patent policy to have a negative effect on innovation and growth.

[1] See Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

[2] See Grieben and Sener (2009), Radhakrishnan (2011), and Davis and Sener (2012) for quality-ladder models based on Dinopoulos and Syropoulos's setting. See Akiyama and Furukawa (2010) for a North–South analysis. Another related work is Thoenig and Verdier (2003), who use a quality-based model to argue that a firm can endogenously avoid obsolescence by using a defensive, more tacit-knowledge-intensive technology. More broadly, our basic framework may be related to the market quality theory of Yano (2008, 2009), in which institutions are considered endogenous.

[3] This is the common view in a variety of fields including industrial organization, marketing, and technology management. See, for example, Agarwal and Gort (2002).

[4] The present study differs from Akiyama, Furukawa, and Yano (2011) in two respects. First, we focus on product obsolescence in a closed economy, whereas they considered imitation of products in a North–South setting (where no product becomes obsolete). Second, we analyze the effects of R&D subsidies and patent breadth and show an interdependence between R&D policy levers.

[5] Following Li (2001) and many others, we measure the strength of patent protection by *patent breadth.*

[6] See, for example, Segerstrom (2000), Li (2001), Goh and Olivier (2002), Chu (2009, 2011), Chu, Cozzi, and Galli (2012), Chu, Pan, and Sun (2012), and Iwaisako and Futagami (2013).

2 The Model

We consider a variety expansion model of endogenous growth à la Romer (1990) and Grossman and Helpman (1991). We assume discrete time because it is useful to model endogenous survival activities of firms in a variety expansion model by means of the dynamic programming approach (Akiyama, Furukawa, and Yano 2011). There is an infinitely lived representative consumer who inelastically supplies L units of labor in each period. This consumer is endowed with the utility function $U = \sum_{t=0}^{\infty} \beta^t \ln C_t$, where $\beta \in (0,1)$ is the time preference rate and the consumption C_t is defined as a constant elasticity of substitution (CES) function on the continuum of differentiated goods: $C_t = \left(\int_0^{N_t} x_t(j)^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)}$, where $\sigma > 1$ is the elasticity of substitution; $x_t(j)$ is the amount of differentiated good j; and N_t is the number of goods available in period t. It is well known that the corresponding dynamic optimization problem has a solution that yields the Euler equation:

$$\frac{E_{t+1}}{E_t} = \beta(1 + r_t), \tag{1}$$

where r_t is the interest rate and $E_t = \int_0^{N_t} p_t(j)x_t(j)dj$ represents the consumer's spending in period t with the price $p_t(j)$ of final good j. The static demand function for good j is given by $x_t(j) = E_t \left(p_t(j)\right)^{-\sigma} / \left(P_t\right)^{1-\sigma}$, where P_t is the price index defined by $P_t = \left(\int_0^{N_t} p_t(j)^{1-\sigma} dj \right)^{1/(1-\sigma)}$. Assume that a unit of each good j can be manufactured from a unit of labor. If good j survives up until period t, it is manufactured by the monopolistic firm (patent holder).

To allow for a role for patent policy, we consider an upper-bound $\mu \in (1, \sigma/(\sigma-1)]$ in the markup.[7] Therefore, the equilibrium price becomes $p_t(i) = \mu w_t$, where w_t is the wage rate. As in the existing literature,[8] we interpret μ as patent breadth (i.e., a measure for the strength of patent protection). In this setting, a larger patent breadth μ means a higher markup in accordance with the seminal vision of Gilbert and Shapiro (1990) on "breadth as the ability of the patentee to raise the price." This pricing gives rise to the following demand and profit functions:

$$x_t(j) = x_t = \frac{E_t}{\mu w_t N_t} \quad \text{and} \quad \pi_t(j) = \pi_t = \left(\frac{\mu - 1}{\mu} \right) \frac{E_t}{N_t}. \tag{2}$$

2.1 R&D and Survival

There are a number of perfectly competitive potential R&D firms. A potential R&D firm can innovate one new technology to produce a new intermediate good in period t by investing $1/\left(\kappa N_{t-1}\right)$ units of labor in period $t-1$, where the standard assumptions regarding knowledge spillover are assumed. Here, $\kappa \in [0, \infty)$ denotes the productivity of R&D. We denote $s \in [0,1)$ as a subsidy rate for innovation, so that the unit cost of R&D is equal to $(1-s)\, w_{t-1}/\kappa$.[9]

[7] To allow for a sufficiently large patent breadth μ, we consider that σ is sufficiently small. To verify that sufficiently large patent breadths are not empirically too restrictive, we can provide a calibration result; see Section 4.

[8] See also Li (2001), Goh and Olivier (2002), Chu (2011), and Iwaisako and Futagami (2013) for a similar formulation in the dynamic general equilibrium model.

[9] This subsidy is financed by a lump-sum tax.

A firm that successfully innovates a new product, j, manufactures product j monopolistically, thereby earning a monopolistic rent in period t, π_t. This rent continues through subsequent periods. At an endogenous probability of $1-\iota_t(j)$, where $\iota_t(j) \in [0,1]$ stands for the probability of survival at the end of period t, we assume that an innovated good j becomes obsolete and the R&D firm innovating good j has to leave the market. This assumption is based on Lai's (1998) assumption of product obsolescence over the endogenously expanding variety of differentiated goods.[10]

We consider that the R&D firm engages in a struggle to avoid obsolescence and survive. To incorporate this, we follow Akiyama, Furukawa, and Yano (2011) by assuming that the firm can increase the probability of survival $\iota_t(j)$ by investing $z_t(j)/N_t$ units of labor in period t.[11] Specifically, $\iota_t(j) = (z_t(j))^\alpha$, in which $z_t(j) \in [0,1]$ denotes the intensity of survival investment and $\alpha \in (0,1)$ is a technological parameter.[12] An active R&D firm's value is the expectation of the net present discounted value of profits. Given that $\pi_t(j) = \pi_t$ in (2), we have $z_t(j) = z_t$ and $\iota_t(j) = \iota_t$ for all j in equilibrium. The R&D firm's behavior can be described as the following Bellman equation:

$$V_t^* = \max_{z_t \in [0,1];\ \iota_t = (z_t)^\alpha} \left[\pi_t - \frac{w_t z_t}{N_t} + \iota_t \frac{V_{t+1}^*}{1+r_t} \right]. \tag{3}$$

The solution to (3) gives rise to the following policy function:

$$z_t^* = \min \left\{ \left(\frac{\alpha V_{t+1}^*/(1+r_t)}{w_t/N_t} \right)^{1/(1-\alpha)},\ 1 \right\}.^{13} \tag{4}$$

This is essentially analogous to the equilibrium condition on rent protection activities in Dinopoulos and Syropoulos (2007) and other studies based on their approach. However, as discussed later, the policy implications of our model are different.

Before proceeding, it is important to consider more specifically the survival investment against product obsolescence. If we followed Ethier's (1982) interpretation that the differentiated goods were intermediate goods used for producing the consumption good C_t through the CES production function, then we would suppose that an intermediate product becomes obsolete as a result of the introduction of new, more high-tech intermediate goods. The survival investment would be made to update/upgrade the invented intermediate product to catch up with cutting-edge standards. In this note, we interpret the differentiated goods as consumption goods. The survival investment of a firm is made to update/upgrade the product and keep the consumer interested in its innovated consumption good; this is more akin to the vision of Lai (1998) that a consumption good becomes obsolete owing to the "introduction of more sophisticated goods" for the consumer with a "love of sophistication." For either interpretation, our point is that the incumbent firms invest in their survival against product obsolescence.

[10] Whereas his focus is on gradual obsolescence, we consider that product obsolescence is stochastic and discrete. We leave for future research the task of analyzing firm survival against gradual obsolescence.

[11] We also assume the knowledge spillover effect for the survival investment.

[12] For simplicity, we adopt the simplest function for survival probability $\iota_t(j)$, but we obtain qualitatively the same results using a more general form of the survival probability such as $\iota_t(j) = (z_t(j))^\alpha + \phi$ or $(\gamma (z_t(j))^\alpha + (1-\gamma)(\phi)^\alpha)^{1/\alpha}$, where $\phi \in (0,1)$ and $\gamma \in (0,1)$ are parameters that capture market or institutional attributes for firm survival.

[13] Clearly, $z_t = 0$ is not an equilibrium choice because $d\iota_t/dz_t \to \infty$ as $z_t \to 0$. Noting $\iota_t \leq 1$, the usual Karush–Kuhn–Tucker solution leads to (4). Note that the transversality condition is satisfied, because ι_t^* is uniformly bounded in the present model.

2.2 Market Equilibrium

Free entry into the R&D market ensures that the discounted value of an innovation is equal to the cost, so that we have:

$$\frac{V_{t+1}^*}{1+r_t} = \frac{(1-s)\,w_t}{\kappa N_t}. \tag{5}$$

From (4) and (5), in market equilibrium, the intensity of survival investment and the probability of survival, z_t^* and ι_t^*, respectively, are independent of time: $z_t^* = z^*$ and $\iota_t^* = \iota^*$ for all t. Specifically,

$$z^* = \begin{cases} (\alpha(1-s)/\kappa)^{\frac{1}{1-\alpha}} & \text{if } \alpha(1-s)/\kappa < 1 \\ 1 & \text{if } \alpha(1-s)/\kappa \geq 1 \end{cases}, \tag{6}$$

$$\iota^* = \begin{cases} (\alpha(1-s)/\kappa)^{\frac{\alpha}{1-\alpha}} & \text{if } \alpha(1-s)/\kappa < 1 \\ 1 & \text{if } \alpha(1-s)/\kappa \geq 1 \end{cases}. \tag{7}$$

Note that the lifetime (ι^*) becomes shorter when the R&D subsidy rate s increases. This is because the firm responds to large R&D subsidies by investing more in innovation than in survival.

Now we can close the model by considering two conditions. First, the number N_t of consumption goods changes over time, which increases with an innovation and decreases with the exit of firms. Then, we have:

$$N_{t+1} = \iota^* N_t + M_t, \tag{8}$$

where M_t denotes the inflow of innovation made in period t and $\iota^* N_t$ is the number of firms that survive at the end of period t. Second, the labor market clearing condition is given by:

$$L = N_t x_t + \left(\frac{1}{\kappa N_t}\right) M_t + \left(\frac{z^*}{N_t}\right) N_t, \tag{9}$$

in which the right-hand side denotes the three labor demands: $N_t x_t$ for production, $\left(\frac{1}{\kappa N_t}\right) M_t$ for innovation, and $\left(\frac{z^*}{N_t}\right) N_t$ for survival.

By (1), (2), (3), (5), (8) and (9), we can characterize the long-run equilibrium of the model with the following theorem.

Theorem 1 *In the initial period* 0, *the economy jumps into a unique balanced growth path that is characterized by the following long-run rate of economic growth:*

$$1 + g^* = \frac{\beta}{1-s+\beta(\mu-1)}\left(\kappa L(\mu-1) + (\mu-s)\,\iota^* - \mu\kappa z^*\right), \tag{10}$$

where $g^* = (N_{t+1} - N_t)/N_t$ for all $t \geq 0$. The equilibrium investment and probability of survival, z^* and ι^*, are given by (6) and (7).

Proof. See Appendix A. ■

3 Effects of R&D Subsidies and Patent Protection

We will examine the effects of two R&D policy levers: subsidies and patent protection (i.e., patent breadth in our analysis). To do this, we have the following preliminary result: *The higher the R&D subsidy rate s is, the lower the probability of survival of firms ι^* is.* This result is intuitive: the firms respond to an increased R&D subsidy rate by engaging more in R&D than they do in survival activities.

Taking into account this effect, we first use (10) together with (6) and (7) to verify that, when no firm exit is to take place in equilibrium ($\iota^* = 1$ as $\alpha(1-s)/\kappa \geq 1$), the R&D subsidy only has the usual growth-enhancing effect: g^* increases with s.[14] However, the effect may be different for a more realistic case where some firms leave the market in each period ($\iota^* < 1$ as $\alpha(1-s)/\kappa < 1$). Differentiating (10) with respect to s, we have the following proposition.[15]

Proposition 1 *In the presence of firm exit (when $\alpha(1-s)/\kappa < 1$), the effect of an increase in the R&D subsidy rate s on growth g^* is negative if the patent breadth μ is sufficiently large.*

Proposition 1 shows an interdependence between these two policies— subsidies and patent breadth—suggesting that whether the R&D subsidy enhances growth depends on the patent breadth. The intuition for this policy interdependence is as follows. A higher R&D subsidy rate s results in a decrease in the expectation of the R&D firm values, by reducing the probability of survival ι^* (preliminary result). The decrease in the probability of survival ι^* has a much more serious and damaging effect on the expected value of R&D firms (consisting of the future profits) when the future profits are larger because of a larger patent breadth μ. Therefore, as μ is large, the effect of a larger R&D subsidy rate on innovation and growth tends to be negative.

Although our model and the Dinopoulos and Syropoulos model have similar equilibrium behaviors of R&D firms in survival, R&D subsidies always increase innovation in the Dinopoulos and Syropoulos model, in contrast to ours. This difference comes from the fact that, in the Dinopoulos and Syropoulos model, endogenous growth is driven by quality improvement and the survival activity (rent protection) makes further research difficult. In this sense, the survival investment hurts future innovation. In our model, while the growth engine is variety expansion and the mechanics of obsolescence are different, the survival activity does not discourage future research. Rather, the survival of existing R&D firms, as well as the entry of new R&D firms, can encourage long-run growth in our model. Therefore, too many R&D subsidies may decrease innovation by losing the balance between R&D and survival in the resource distribution.

4 Quantitative Analysis

To see whether real-world patent protection results in a positive or negative effect on R&D subsidies, we calibrate the model containing a square-root survival function by normalizing $\alpha = 0.5$. Consider the set of variables, $\{\beta, \kappa, \mu, s, g^*\}$. We set the time preference rate β to a standard value of 0.97. As for patent breadth (i.e., the measure for

[14] See Appendix B for the formal proof.
[15] See Appendix B for the formal proof.

patent protection), we consider two polar levels of the markup from the realistic range, $\mu \in \{1.6,\ 2.5\}$.[16] We work on the entire range of the subsidy rate $s \in (0,1)$. Using a plausible rate of survival, 0.925,[17] we calibrate the R&D productivity κ. Finally, we take a realistic growth rate $g^* = 0.016$ as the benchmark.[18]

Numerical calculations show that, for the large patent breadth case ($\mu = 2.5$), the growth effect of R&D subsidies s is negative above a very low threshold, $s \simeq 0.08$ (about 8 percent). Even for the small patent breadth case ($\mu = 1.6$), the threshold level goes up to $s \simeq 0.18$ (about 18 percent). Given the real-world average rates of R&D subsidies (approximately 10 percent for the US, 20 percent for the UK, 30 percent for Canada, and 40 percent for France),[19] our calculations suggest that, in countries with a high R&D subsidy rate such as Canada and France, the current level of patent breadth may have a negative effect of R&D subsidies on innovation and economic growth because of the decreased survival of R&D firms.

References

[1] Agarwal, R., and Gort, M., 2002. Firm and product life cycles and firm survival. *American Economic Review Papers and Proceedings*, 92, 184–190.

[2] Aghion, P., and Howitt, P., 1992. A model of growth through creative destruction. *Econometrica*, 60, 323–351.

[3] Akiyama, T., and Furukawa, Y., 2009. Intellectual property rights and appropriability of innovation. *Economics Letters*, 103, 138–141.

[4] Akiyama, T., Furukawa, Y., and Yano, M., 2011. Private defense of intellectual properties and economic growth. *International Journal of Development and Conflict*, 1, 355–364.

[5] Chu, A., 2009. Effects of blocking patents on R&D: A quantitative DGE analysis. *Journal of Economic Growth*, 14, 55–78.

[6] Chu, A., 2011. The welfare cost of one-size-fits-all patent protection. *Journal of Economic Dynamics and Control*, 35, 876–890.

[7] Chu, A., Cozzi, G., and Galli, S., 2012. Does intellectual monopoly stimulate or stifle innovation? *European Economic Review*, 56, 727–746.

[8] Chu, A., Pan, S., and Sun, M., 2012. When does elastic labor supply cause an inverted-U effect of patents on innovation? *Economics Letters*, 117, 211–213.

[9] Davis, L., and Şener, F., 2012. Private patent protection in the theory of Schumpeterian growth. *European Economic Review*, 56, 1446–1660.

[16] See the estimates in Hall (1986) and a calibration analysis based on these estimates in Kwan and Lai (2003).

[17] Note that, on average, 90–95 percent of firms survive in a single year (Agarwal and Gort 2002). We take the average of 90 and 95 to obtain 0.925.

[18] See Kwan and Lai (2003).

[19] See Parsons (2011).

[10] Dinopoulos, E., and Syropoulos, C., 2007. Rent protection as a barrier to innovation and growth. *Economic Theory*, 32, 309–332.

[11] Eicher, T., and García-Peñalosa, C., 2008. Endogenous strength of intellectual property rights: Implications for economic development and growth. *European Economic Review*, 52, 237–258.

[12] Ethier, W., 1982. National and international returns to scale in the modern theory of international trade. *American Economic Review*, 72, 389–405.

[13] Gilbert, R., and Shapiro, C., 1990. Optimal patent length and breadth. *RAND Journal of Economics*, 21, 106–112.

[14] Goh, A.-T., and Olivier, J., 2002. Optimal patent protection in a two-sector economy. *International Economic Review*, 43, 1191–1214.

[15] Grieben, W. H., and Şener, F., 2009. Globalization, rent protection institutions, and going alone in freeing trade. *European Economic Review*, 53, 1042–1065.

[16] Grossman, G., and Helpman, E., 1991. Quality ladders in the theory of growth. *Review of Economic Studies*, 58, 43–61.

[17] Hall, R., 1986. Market structure and macroeconomic fluctuations. *Brookings Papers on Economic Activity*, 2, 285–322.

[18] Iwaisako, T., and Futagami, K., 2013. Patent protection, capital accumulation, and economic growth. *Economic Theory*, 52, 631–668.

[19] Kwan, Y., and Lai, E., 2003. Intellectual property rights protection and endogenous economic growth. *Journal of Economic Dynamics and Control*, 27, 853–873.

[20] Lai, E. L.-C., 1998. Schumpeterian growth with gradual product obsolescence. *Journal of Economic Growth*, 3, 81–103.

[21] Li, C.-W., 2001. On the policy implications of endogenous technological progress. *Economic Journal*, 111, 164–179.

[22] Parsons, M., 2011. Rewarding innovation: Improving federal tax support for business R&D in Canada. *C.D. Howe Institute Commentary*, 334, 1–23.

[23] Radhakrishnan, R., 2011. Endogenous IPR protection expenditure and Economic growth. *Mimeo*, available at: home.wlu.edu/~radhakrishnanr/Paper3.pdf

[24] Romer, P., 1990. Endogenous technological progress. *Journal of Political Economy*, 98, S71–S102.

[25] Segerstrom, P., 2000. The long-run growth effects of R&D subsidies. *Journal of Economic Growth*, 5, 277–305.

[26] Segerstrom, P., Anant, T. C. A., and Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. *American Economic Review*, 80, 1077–1091.

[27] Thoenig, M., and Verdier, T., 2003. A theory of defensive skill-biased innovation and globalization. *American Economic Review*, 93, 709–728.

[28] Yano, M., 2008. Competitive fairness and the concept of a fair price under Delaware law on M&A. *International Journal of Economic Theory*, 4, 175–190.

[29] Yano, M., 2009. The foundation of market quality economics. *Japanese Economic Review*, 60, 1–32.

Appendix A:

By (5) $w_t = \frac{V_{t+1}^*}{1+r_t} \frac{\kappa N_t}{1-s}$. With (1), this implies

$$w_t = \frac{\beta E_t V_{t+1}^*}{E_{t+1}} \frac{\kappa N_t}{1-s}. \tag{A1a}$$

By substituting (2), (8), and (A1a) into (9), we can obtain:

$$\frac{N_{t+1}}{N_t} = \frac{\kappa L + \iota^* - \kappa z^*}{\frac{1-s}{\beta\mu} + \frac{V_{t+1}^* N_{t+1}}{E_{t+1}}} \frac{V_{t+1}^* N_{t+1}}{E_{t+1}}. \tag{A1b}$$

Noting (1), (2), and (A1a), (3) can become in equilibrium

$$V_t^* = \left(\frac{\mu-1}{\mu}\right) \frac{E_t}{N_t} + \left(\iota^* - \frac{\kappa z^*}{1-s}\right) \frac{\beta E_t V_{t+1}^*}{E_{t+1}}. \tag{A1c}$$

By multiplying both sides of (A1c) by N_t/E_t, with (A1b), we can rewrite (A1c) as

$$\begin{aligned} \frac{V_{t+1}^* N_{t+1}}{E_{t+1}} &= \frac{\kappa L + \iota^* - \kappa z^*}{\beta\left(\iota^* - \frac{\kappa}{1-s} z^*\right)} \left(\frac{V_t^* N_t}{E_t}\right) \\ &\quad - \frac{1}{\beta} \left(\frac{\kappa L \left(1-\mu^{-1}\right) + \left(1 - s/\mu\right) \iota^* - \kappa z^*}{\iota^* - \frac{\kappa z^*}{1-s}}\right). \end{aligned} \tag{A2}$$

The steady state v^* satisfying that $\frac{V_{t+1}^* N_{t+1}}{E_{t+1}} = \frac{V_t^* N_t}{E_t} = v^*$ for any t is given by

$$v^* = \frac{\kappa\left(1 - (1/\mu)\right) L + \left(1 - s/\mu\right) \iota^* - \kappa z^*}{\kappa L + (1-\beta)\, \iota^* + \frac{\beta - (1-s)}{1-s} \kappa z^*}. \tag{A3}$$

By (A2), by means of a usual phase diagram analysis, we can show that only a path starting from v^* is consistent with the transversality condition and dynamic optimization; $\frac{V_t^* N_t}{E_t} = v^*$ for all $t \geq 0$ (saddle-path stability). By substituting (A3) into (A1b) implies (10). To ensure $g^* > 0$, we assume the labor force is sufficiently large to meet:

$$(\mu-1)L - \mu z^* > \frac{(1-\beta)(1-s)}{\kappa\beta} + (1-\iota^*) \frac{\mu - s}{\kappa} > 0, \tag{A4}$$

which implies $(\mu - 1)L > \mu z^*$.

Appendix B:

When $\alpha(1-s)/\kappa < 1$, by differentiating (10) with respect to s, with (6), we obtain

$$\begin{aligned} &\frac{d}{ds}(1+g^*) \\ &= \frac{\beta}{\mu-1} \frac{\kappa L + (1-\beta)\phi + \left(\frac{\alpha(1-s)}{\kappa}\right)^{\frac{\alpha}{1-\alpha}} \left(s + (1-\alpha)(1-s)\frac{\mu}{\mu-1} - (1-s+\beta(\mu-1))\left(1 + \frac{\alpha s(\mu-1)}{(1-\alpha)(1-s)}\right)\right)}{\left((\sigma-1)(1-s)+\beta\right)^2}. \end{aligned} \tag{B1}$$

As μ goes to ∞, the first two terms in the right-hand side go to 0 while the third term goes to $-\infty$. As $\mu \to 1$, the right-hand side of (B1) goes to $+\infty$. When $\alpha(1-s)/\kappa \geq 1$, by (6), (7), and (10), we have

$$\frac{d}{ds}(1+g^*) = \beta \frac{1 - \beta + \frac{\kappa}{\mu-1}\left((\mu-1)L - \mu z^*\right)}{\left((1-s) + \beta(\mu-1)\right)^2}, \tag{B2}$$

which is strictly positive as $(\mu-1)L > \mu z^*$ must hold for a positive growth rate, $g^* > 0$; see Appendix A.