Uncertainty in optimal pollution levels: Modeling the benefit area

Halkos, George

Department of Economics, University of Thessaly

June 2013

Uncertainty in optimal pollution levels: Modeling the benefit area

By

George E. Halkos and Dimitra C. Kitsou

Laboratory of Operations Research

Department of Economics, University of Thessaly, Volos Greece

Abstract

This paper identifies the optimal pollution level under the assumptions of linear, quadratic and exponential damage and abatement cost functions and investigates analytically the certain restrictions that the existence of this optimal level requires. The evaluation of the benefit area is discussed and the mathematical formulation provides the appropriate methods, so that to be calculated. The positive, at least from a theoretical point of view, is that both the quadratic and the exponential case obey to the same form of evaluating the benefit area. These benefit area estimations can be used as indexes between different rival policies and depending on the environmental problem the policy that produces the maximum area will be the beneficial policy.

Keywords: Benefit area; damage cost; abatement cost; pollution.

JEL Classifications: C02; C62; Q51; Q52.

This research has been co-financed by the European Union (European Social Fund – ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Heracleitus II. Investing in knowledge society through the European Social Fund.

Co- financed by Greece and the European Union

1. Introduction

Rationality in the formulation and applicability of environmental policies depends on careful consideration of their consequences on the nature and on the society. For this reason it is important to quantify the costs and benefits in the most accurate way. But the validity of any cost benefit analysis (hereafter CBA) is ambiguous as the results may have large uncertainties. Uncertainty is present in all environmental problems and this makes clear the need for thoughtful policy design and evaluation. We may have uncertainty over the underlying physical or ecological processes, as well as over the economic consequences of the change in environmental quality.

These sources of uncertainty and their impact on policy formulation may be represented by the non-linear nature of the damage and abatement cost functions. Damage or external costs can be estimated by an analysis of the chain of pollution emissions, their dispersion and "transportation" (in cases of transboundary pollution like the acid rain problem), their effect measured among others with a dose-response function and their final (if feasible) monetary valuation. A similar picture is realized when referring to abatement costs, which may be less uncertain, compared to damage costs, but they are quite severe. The main problem in this case is related to technological change which may be essentially difficult to predict or sometimes even to characterize.

Uncertainty is obvious not only in the parameters' estimation, but also in the choice of the appropriate model that "fits" the problem. To make parameters' uncertainty clearer, we may thing in terms of the fitted model assumed for the damage and abatement curves in a regression analysis that "lies" between the upper and the lower bound of a

95% confidence interval. That is there are two curves creating an interval of values for the fitted model and in this way uncertainty due to variation of the estimated coefficients.

As uncertainty may be due to the lack of appropriate abatement and damage cost data, we apply here a method of calibrating non-existing damage cost estimates relying on individual country abatement cost functions. In this way a “calibrated” Benefit Area (BA^c) is estimated. Specifically, we try to identify the optimal pollution level under the assumptions of linear, quadratic and exponential abatement and damage cost functions. As far as the parameters are concerned the first two are linear while the third is a non-linear function. That is we improve the work of Halkos and Kitsos (2005) extending the number of different model approximations of abatement and damage cost functions and thus the assumed correct model eliminates uncertainty about curve fitting. The target of this paper is to develop the appropriate theory whatever the model choice is.

The structure of the paper is the following. Section 2 discusses the background of the problem and reviews the relative existing literature. Section 3 identifies analytically the intersection of the marginal abatement cost curve (hereafter MAC) with the marginal damage cost curve (hereafter MD), in order to examine when and if an optimal pollution level exists. The existence of the intersection, despite the general belief, is not always true and the conditions are analytically examined here. In section 4 an empirical application for a sample of European countries, with different industrial structure, is presented. For these countries, the “calibrated” Benefit Area (BA^c) is evaluated explicitly, provided there is an intersection of the MD and MAC functions. The last section concludes the paper and comments on the policy implications related to this analysis providing evidence useful to researchers and policy makers.

2. Background to the problem

Abatement and damage cost functions are highly non-linear and the precise shapes of the functions are unknown. At the same time, environmental policies are related with significant irreversibilities, which usually interact in a very complex way with uncertainty. This complexity becomes worse, if we think of the very long time periods that characterize environmental problems (Pindyck, 2007).

The damage cost function relates pollution and emissions of a specific pollutant. Damages are measured as the effect of these emissions on health, monuments, recreational activities, lakes, buildings etc. Efforts to measure the existence or other indirect use values are made with the help of contingent valuation and other methods (Halkos and Jones, 2012; Halkos and Matsiori, 2012; Bjornstad and Kahn, 1996; Freeman, 1993). As expected, the accurate measurement of damage is significant but also difficult due to many practical problems as presented in Farmer *et al.* (2001), Georgiou *et al.* (1997) and Barbier (1998).

Uncertainties in the functions of damage and control costs influence the policy design in a number of ways (Halkos, 1996). The first effect is in terms of the choice of the appropriate policy instrument. Weitzman (1974), in his seminal paper, showed that in the presence of uncertainty in cost functions, the instrument choice depends on the slopes of the curves. In certainty conditions either instrument will be equally effective but in uncertainty the choice is important and depends on the slopes of the marginal damage and abatement cost curves. In the case of steep marginal damage and flat marginal control cost curves, quantity-based instruments are more adequate; while in the case of steep marginal abatement cost and flat marginal damage curves, a price-based instrument is to be chosen (Halkos, 2000).

A number of studies have extended the Weitzman's thesis and showed that in the case of uncertainty "hybrid" policies of combining both instruments will dominate to the single instrument (Roberts and Spence, 1976; Weitzman, 1978; Pizer 2002; Jacoby and Ellerman, 2004).

It is worth mentioning that uncertainty may also affect the optimal timing of policy implementation if there are sunk costs in the implementation of that policy or the environmental damage from the lack of any policy is at least partly irreversible. The consequences of irreversibility have been studied extensively in the literature (Pindyck 2000, 2002; Fisher and Hanemann 1990; Gollier *et al*., 2000; Ulph and Ulph, 1997; Kolstad, 1996).

As mentioned, damage and abatement cost functions seem to have a large curvature and in many cases to be non-linear functions. A number of studies have tried to assess the cause and extent the uncertainty over the benefits from emissions reduction[1]. Rabl *et al*. (2005) compare damage and abatement costs for a number of air pollutants. They distinguish between discrete and continuous policy choices. Setting a limit for sulphur dioxide emissions from power plants is an example of a continuous choice while the decision to demand a specific abatement method associated with a constant rate of emissions may be considered as a case of a discrete choice. They have also focused on uncertainties not only in damage costs but also in abatement costs claiming that the extent

[1] The cost functions may not behave well or may not satisfy the conditions of convexity or concavity. In the case of the damage cost function this may take place by threshold effects as well as by any irreversibility where pollution reaches a critical point at which the receptor (rivers, lakes, etc) is damaged completely and cannot sustain any life. If one or both of the cost functions are not well behaved then our results will be different. At the same time the distinction between flow and stock pollutants is important as for stock pollutants the persistence has to be taken into consideration due to the accumulation (and decay) of pollutant(s) in time (Perman *et al*., 2011). As an example, we may consider the case of F-Gases with the very high global warming potentials (Halkos, 2010).

of uncertainties in the control costs are similarly important. They find a remarkable insensitivity to uncertainties for continuous choices claiming that for NO_X and SO_2 an error by a factor of 3 will raise total social cost by a maximum of 20%. For dioxins and CO_2 uncertainties of damage costs are larger compared to the case of NO_X and SO_2. Similarly in the case of discrete choices there is no general conclusion and there is a cost penalty only in the case of wrong choice with the difference between abatement and damage costs to be higher than the uncertainties.

Exploration of resources, extraction and processing may cause environmental damages like habitat disruption from exploration and drilling activities, oil spills, water and air pollution etc (Litman, 2013). In the case of US oil spills cleanup and damage-compensation costs were around \$300 per barrel in the case of 1979 Ixton I spill in the Gulf of Mexico and around \$25,000 per barrel for the 1980 Exxon Valdez spill in Alaska (Cohen, 2010a). These costs are undervalued as a number of ecosystem services are difficult to be estimated. According to various surveys wildlife damages were almost \$3 billion compared to around \$1.0 billion in total wildlife cleanup and compensation costs (Cohen, 2010a). This implies that total damage costs and willingness to pay of the society to avoid damages are a lot higher (2 to 5 times) compared to the financial costs imposed by the oil industry (Cohen, 2010b).

Resources exploration, extraction and processing together with distribution may cause health problems to people like accidents or pollution related health illness. In 2006 workers in petroleum production faced almost 21 fatalities per 100,000 workers, a number much higher compared to the typical service industry but lower than the other heavy industries like coal miners (almost 50) and loggers (around 87.5) (Bureau of Labor Statistics, 2007).

Table 1 presents the number of cases in Europe and Denmark related to the impacts of Danish anthropogenic emissions for the year 2000 as presented in Brandt *et al.* (2013). The total health-related external costs with respect to impacts in the whole of Europe due to Danish emissions and for the year 2000 are estimated to be 4.9 bn € per year, of which 817 million € per year account for the external costs in Denmark alone. It is also shown that the major Danish contributors are agriculture (43%), road traffic (18%), power production (10%), non-industrial (domestic) combustion plants including wood combustion (9%) and other mobile sources (8%).

Table 1: Number of cases of different health impacts related to Danish anthropogenic emissions for the year 2000

Health impact	Number of cases (in Europe)	Number of cases (in Denmark)
Chronic Bronchitis	4350	802
Restricted Activity Days	4 440 000	820 000
Respiratory Hospital Admissions	234	44
Congestive Heart Failure	324	69
Lung Cancer	666	123
Bronchodilator Use Children	128 000	21 600
Bronchodilator Use Adults	851 000	157 000
Cough Children	441 000	74 600
Cough Adults	876 000	162 000
Lower Respiratory Symptoms Children	170 000	52 800
Lower Respiratory Symptoms Adults	316 000	58 300
Infant mortality	5	1
Total	7277220	1355951

Source: Modified from Brandt *et al.* (2013)

These figures are compared to a higher difference of approximately 15% (impact in Europe) and 19% (impacts in Denmark), that is costs of 5.68 billion € and 971 million € per year, when the sum of individual assessments is considered. This difference may be justified from the atmospheric chemical regime in the region of interest. In general and in cases where non-linear processes are involved, the sum of individual processes is expected to be different compared to all processes together (Brandt *et al.*, 2013).

Litman (2013) provides estimates of US external costs of petroleum production, importation and distribution of $90-160 per barrel and a total of $635-1.080 billion yearly. This implies that the internal cost of every $ spend by consumers on petroleum imposes $0.63-1.08 in terms of external costs. In these external cost the estimated costs of uncompensated environmental damage (ground and surface water pollution, production, loss and aesthetic degradation) range from $10 - 30 billions.

Concerning air pollution and in the case of GHGs the first CBA was carried out by Nordhaus (1991). As found in Tol (2013) there are 16 studies and 17 estimates of the global welfare impacts of climate change (Nordhaus 1994a,b, 2006, 2008, 2011; Fankhauser, 1994, 1995; Tol, 1995, 2002a,b; Bosello *et al.*, 2012; Maddison, 2003; Mendelsohn *et al.*, 2000a,b; Maddison and Rehdanz, 2011; Rehdanz and Maddison, 2005). The welfare effect of doubling the atmospheric concentration of GHGs is relatively small (just a small percentage of GDP).

Tol (2013) presents a list of 75 studies with 588 estimates of the social cost of carbon emissions. He applies a kernel density estimator to the 588 observations expressed in 2010 US$ and pertaining to emissions in 2010. Tol finds that the mean marginal cost of carbon estimated value in these studies equals to $196 per metric tone of carbon with a mode estimate of $49/tC. This implies a high asymmetry due to some very high estimates explained mainly by the use of different pure rates of time preference. A higher rate of time preference indicates that the costs of climate change taking place in the future have a lower present value. Tol extracts a mean social cost of carbon for the studies with 3% and 0% rate of time preference equal to $25/tC and $296/tC respectively.

Recently, the Department of Energy and Climate Change (2009) estimated the social cost of carbon (including control of emissions costs) in a range between £35 and £140 per tonne of CO_2. Miyoshi and Mason (2013) present the estimated marginal damage cost of CO_2 for Manchester Airport. Specifically, in the case of the central price of carbon (£51/t), the total damage cost caused by carbon dioxide produced by passengers' surface access is almost £11 m (in 2009 prices) while the cost of CO_2 due to 'drop off and pick up' users is almost £0.72 per passenger compared with £0.62 for 'taxi', £0.41 for 'car and parking' and £0.77 for 'minicab' users. Given that the average number of occupants is around 3.8, this implies a damage cost of CO_2 per trip for 'drop off and pick up' users equal to around £2.74 compared with £0.87 for 'car and parking', £3.23 for 'minicab' and £2.48 for 'taxi' users.

Damage costs estimates can be also found among others in three well-known integrated assessment models (IAMs): the Dynamic Integrated Climate and Economy (DICE), the Policy Analysis of the Greenhouse Effect (PAGE), and the Climate Framework for Uncertainty, Negotiation, and Distribution (FUND). For emission changes taking place in 2010, the value of the central social cost of carbon (hereafter SCC) is $21/t of CO_2 emissions increasing to $26/t of CO_2 in 2020 (Greenstone *et al*., 2013).

Each of the three models is given equal weight in the SCC values developed by the interagency group. A number of simplifying assumptions and judgments underlie the three IAMs, reflecting the modellers' best attempts to synthesize the available scientific and economic research. Although the frameworks of other IAMs may better reflect the complexity of the science, they do not link physical impacts to economic damages, an essential step for estimating the social cost of carbon.

Nordhaus (1994a) presents estimates of the percentage loss in gross world product, while Roughgarden and Schneider (1999) relying on Nordhaus' survey, together with other surveys, constructed confidence intervals for a damage function. Similarly Heal and Kriström (2002) and Pizer (2006) assess uncertainty using subjective analysis and the opinions of experts. Specifically Pizer (2003) modified the DICE model developed by Nordhaus (1994b) replacing the original quadratic relationship between damage and temperature change with a more complex function. The main conclusion from the empirical studies so far is that although there is a level of uncertainty we are unable to quantify it.

In terms of marginal damages of pollutants, Nordhaus (2008) presents a range of between \$6 and \$65/t carbon with a central estimate of \$27. Interagency Working Group on Social Cost of Carbon (2010) provides a three model mean cost of \$21/t, as well as a \$65/t in the 95th% estimate. These are in line with the estimates by Nordhaus (2008).

3. Determining the optimal level of pollution

Economic theory suggests that the optimal pollution level occurs when the marginal damage cost equals the marginal abatement cost. Graphically the optimal pollution level is presented in Figure 1 where the marginal abatement (MAC=g(z)) and the marginal damage (MD=φ(z)) are represented as typical mathematical cost functions. The point of intersection of the two curves, $I=I(z_o, k_o)$, reflects the optimal level of pollution with k_0 corresponding to the optimum cost (benefit) and z_0 to the optimum damage restriction. It is assumed (and we shall investigate the validity of this assumption in the sequence) that the curves have an intersection and the area created by these curves (region AIB) is what we define as Benefit Area (Kneese, 1972, among others).

Our previous analysis (Halkos and Kitsos, 2005) examined only three cases for the abatement cost function MAC=g(z):

- Linear [$MAC(z)=\beta_0+\beta_1 z$, $\beta_1\neq 0$];
- Quadratic [$MAC(z)=\beta_0+\beta_1 z+\beta_2 z^2$, $\beta_2 > 0$];
- Exponential [$MAC(z) = \beta_0 e^{\beta_1 z}$, $\beta_1 \neq 0$]

and linearity for the marginal damage cost function ($MD(z)=\varphi(z)= \alpha+\beta z_o$). We briefly review it, so that the extensions to be more clear, and cover all the possible, in practice, cases.

Under these assumptions the extracted benefit areas were calculated. We shall examine in the sequence of this paper how this crucial benefit area can be evaluated, providing an index, when different areas are investigated (like countries or provinces), adopting different rival models and policies as are expressed by the two curves under consideration.

Let A and B be the points of the intersection of the curves MD and MAC (see Figure 1) with the 'Y-axis". Obviously we are restricted to positive values. For these points A=A(0, a) and B=B(0, b) the values of a and b are the constant terms of the assumed curves that approach MD and MAC respectively. For the linear case we have

A=A(0, α) and B=B(0, β_0) (see Figure 1) and it is assumed that $\alpha > \beta_0$ in Figure 1, provided that MAC is an increasing function and MD is a decreasing one.[2]

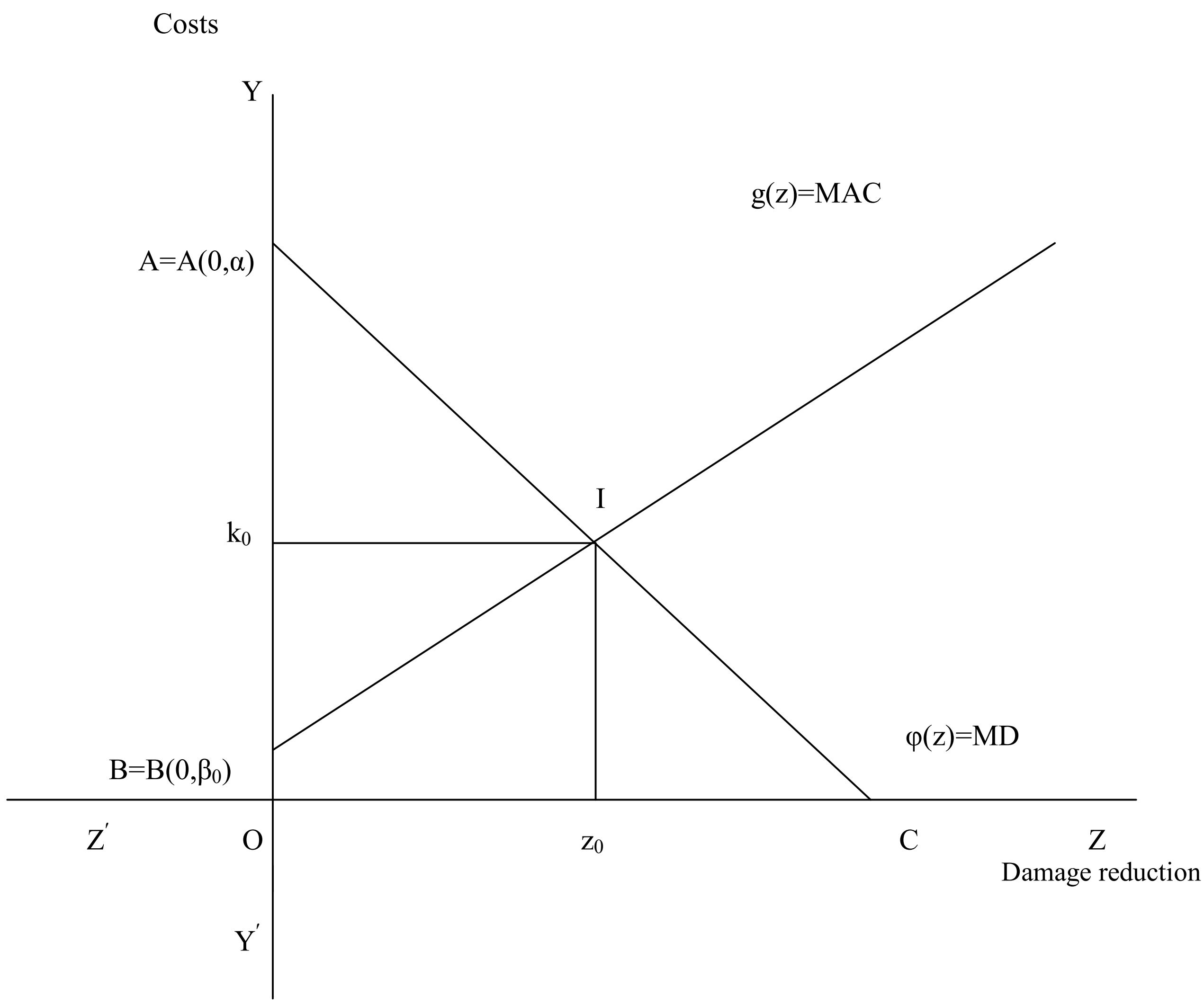


Figure 1: Graphical presentation of the optimal pollution level (general case)

Specifically, we are now considering a number of cases in order to examine under what restrictions the two curves have an intersection, which is presented as I =I(z_0, k_0). That is the equivalent mathematical problem is: what are the values of the points z_0 and k_0 in order to have the optimal damage restriction and the corresponding value of the

[2] This is clear as if it is assumed that $\alpha < \beta_0$ there is no intersection (no benefit area) and if we let $\alpha = \beta_o$ the benefit area coincides with the point, namely A=B=I, that is a one point area is created.

optimal cost respectively. It is clear that, in principle, the intersection satisfies that $MAC(z_0)=g(z_0)=\varphi(z_0)=MD(z_0)$, with z_0 being the optimal restriction in damages. We are emphasizing that the coefficients of the abatement cost functions (β_0, β_1, β_2) can be estimated by applying the OLS method to the appropriate dataset (Hutton and Halkos, 1995). Let us examine each assumption and case in turn.

Case 1: MD and MAC functions are both linear

In the case of linearity of both MAC and MD the intersection $I=I(z_0, k_0)$ satisfies the following relationship: $\beta_0+\beta_1 z_o=\alpha+\beta z_o$ and therefore z_0 can be evaluated as:

$$z_0 = \frac{\beta_0 - \alpha}{\beta - \beta_1} = -\frac{\beta_0 - \alpha}{\beta_1 - \beta} \tag{1}$$

Now we are asking for z_0 to be positive, i.e. to lie on the right half of $z'z$ axis as in Figure 1. If both g(z) and φ(z) are linear the intersection exists at z_0 as in (1) if $\beta_1>\beta$ as already $\alpha>\beta_0$. The corresponding optimal cost or benefit values should be equal for both curves.

$$k_0 = \phi\left(\frac{\beta_0 - \alpha}{\beta - \beta_1}\right) = \alpha + \beta\frac{\beta_0 - \alpha}{\beta - \beta_1} \qquad \text{or} \qquad k_0 = g\left(\frac{\beta_0 - \alpha}{\beta - \beta_1}\right) = \beta_0 + \beta_1\frac{\beta_0 - \alpha}{\beta - \beta_1}$$

This is true as their difference is zero. The benefit area, BA, is evaluated, in principle, through the following relation:

$$BA = (ABI)=(AIz_0 0)-(BIz_0 0) \tag{2}$$

Where in parenthesis are the corresponding evaluated areas of Figure 1.

The benefit area for the case linear-linear (LL), BA_{LL}, can be evaluated as the area of the triangle ABI, namely:

$$BA_{LL} = (ABI) = \frac{(AB)(Ik_0)}{2} = \frac{(\alpha - \beta_0)(0z_0)}{2} = \frac{(\alpha - \beta_0)^2}{2(\beta_1 - \beta)} \tag{3}$$

Similarly, using (2) the area can be evaluated by subtraction of the areas of the two trapezoidals ending up to expression (3) (for details see Halkos and Kitsos. 2005).

Case 2: MD linear and MAC quadratic functions

Let us consider now the case of a quadratic abatement cost function, which is the most likely case, i.e. $g(z)=MAC(z)=\beta_0+\beta_1 z+\beta_2 z^2$. It is assumed that $b=MAC(0)=\beta_0>0$ and $\frac{dg(z)}{dz}=\beta_1+2\beta_2 z>0$, i.e. positive marginal abatement cost, that is $z>-\frac{\beta_1}{2\beta_2}$, which also means that g is increasing. The intersection point with the marginal damage can be evaluated as:

$$MAC(z_0) = MD(z_0) \Rightarrow \beta_0+\beta_1 z_0+\beta_2 z_o^2 = \alpha+\beta z_0 .$$

Recall that for the points A=A(0, a), B=B(0, b), and $a=\alpha$ and $b=\beta_o$, we assume that $\alpha>\beta_0$, therefore we have

$$(\beta_0-\alpha)+(\beta_1-\beta)z_0+\beta_2 z_0^2 = 0 \tag{4}$$

If we set $K=\beta_0-\alpha$, $L=\beta_1-\beta$ then (4) becomes: $\beta_2 z_0^2+Lz_0+K=0$, with roots :

$$z_0=\frac{-L\pm\sqrt{D}}{2\beta_2}, \qquad D=(\beta_1-\beta)^2-4\beta_2(\beta_0-\alpha)\geq 0 \tag{5}$$

Actually the negative D has no economical meaning, so a zero D, leads from (4) to a double or unique optimal restriction of damages of the form:

$$z_0=-\frac{L}{2\beta_2}=-\frac{\beta_1-\beta}{2\beta_2} \tag{6}$$

When assuming $\beta_1<\beta$ the value of z_0 is positive, as β_2 has been assumed positive already. Thus the corresponding k_0 value, for the evaluated z_0 is:

$$k_0=\varphi(z_0)=\alpha-\beta\frac{\beta_1-\beta}{2\beta_2} \quad \text{or} \quad k_0=g(z_0)=\beta_0+\beta_1\left[-\frac{\beta_1-\beta}{2\beta_2}\right]+\beta_2\left[-\frac{\beta_1-\beta}{2\beta_2}\right]^2$$

Under the assumptions of $\beta_0-\alpha<0$ and $\beta_2>0$ the quantity $-4\beta_2(\beta_0-\alpha)>0$ and therefore the value of the determinant is D>0. This is true because the sum of the roots (equals to $2z_0$) is positive, while the product of the roots (equals to $[(\beta_0-\alpha)/\beta_2]$) is negative. We are interested for at least a positive root z_0 in (4), which under the assumption $\alpha>\beta_0$ can be evaluated only when $\beta_1<\beta$ and eventually from (5) we choose the positive z_0.

The corresponding benefit area for linear MD and quadratic MAC case, BA_{LQ}, is evaluated through the general form (2) subtracting from the trapezoidal AIz_o0 the area BIz_00, namely:

$$BA_{LQ} = \frac{(OA)+(Iz_0)}{2}(0z_0) - \int_0^{z_0} g(z)dz = \frac{\alpha + g(z_0)}{2} z_0 - \left[G(z_0) - G(0)\right] \qquad (7)$$

with $$G(z) = \beta_0 z + \beta_1 \frac{z^2}{2} + \beta_2 \frac{z^3}{3}$$

which implies that G(0)=0. So (7) is reduced to:

$$BA_{LQ} = \frac{\alpha + g(z_0)}{2} z_0 - G(z_0), \qquad g(z)=MAC(z) \qquad (8)$$

The value of z_0 as in (6) and the assumptions $\beta>\beta_1$ and $\alpha>\beta_0$. That is as in (3) a general form for the benefit area was produced, when a linear marginal abatement cost $\varphi(z)$ was examined.

Case 3: MD linear and MAC exponential functions

Let us consider the case of an exponential MAC function, i.e. $MAC(z)=\beta_0\exp(\beta_1 z)$. In such a case $b=MAC(0)=\beta_o$ with $a=\alpha$ and the general line of thought for the intersection leads to:

$$\beta_0 e^{\beta_1 z_0} = \alpha+\beta z_0 \Leftrightarrow \exp(\beta_1 z_0)=\alpha^* + \beta^* z_0, \quad \text{with} \quad \alpha^* = \frac{\alpha}{\beta_0},\ \beta^* = \frac{\beta}{\beta_0} \text{ with } \beta_0 \neq 0.$$

This results to (Halkos and Kitsos, 2005):

$$\beta_1 z_0 = \ln(\alpha^* + \beta^* z_0) \Leftrightarrow z_0 = \frac{1}{\beta_1}\ln(\alpha^* + \beta^* z_0) = F(z_0) \qquad (9)$$

Now equation (9) is of the form $z_0=F(z_0)$, can be only solved adopting numerical analysis techniques, through the fixed-point theorem (see Ortega and Rheinbolt, 1970; Halkos and Kitsos, 2005 for details). That is the iteration formed as:

$$z_{0,n+1} = \frac{1}{\beta_1}\ln(\alpha^* + \beta^* z_{0,n}) \qquad n=0,1,2\ldots \qquad (10)$$

converges to z_0, i.e. $\lim z_{0,n+1} \to z_0 = F(z_0)$. Specifically, the optimal restriction of damages level, z_0, in the exponential case of MAC only approximately can be evaluated and therefore the corresponding optimal cost or benefit level is approximately evaluated too.[3]

The corresponding benefit area (AIB) for the linear-exponential case, BA_{LE}, is then evaluated through (2) as:

$$BA_{LE} = (AIz_0 0) - \int_0^{z_0} g(z)dz = \frac{\alpha + g(z_0)}{2} z_0 - [G(z_0) - G(0)] \qquad (11)$$

With: $$G(z_0) - G(0) = \int_0^{z_0} \beta_0 e^{\beta_1 z} dz = \frac{\beta_0}{\beta_1}\int_0^{z_0} e^{\beta_1 z} d(\beta_1 z) = \frac{\beta_0}{\beta_1}(e^{\beta_1 z_0} - 1) \qquad (11.1)$$

i.e. $G(0)=1$, while (7) still holds, providing an index for the benefit area for both quadratic and exponential cases, but with $G(z_0)$ as in (11.1) and z_0 approximated as in (10).

[3] Practically that results the value of the difference $MAC(z_0)$-$MD(z_0)$ is not zero, but close to zero, with a certain accuracy $\pm\zeta$, say $\zeta=10^{-3}$ or 10^{-6}.

Case 4: **MD quadratic and MAC linear functions**

Let us now consider that

$$\text{MAC} = g(z) = \beta_0 + \beta_1 z \quad , \quad \beta_1 \neq 0 \qquad \text{and} \quad \text{MD} = \varphi(z) = \alpha z^2 + \beta z + \gamma, \quad a > 0$$

Here the values of a and b, the intersections of MD and MAC with the Y-axis are b=MAC(0)=β_o and a=MD(0)=γ, see Figures bellow. To insure that an intersection between MAC and MD occurs we need the restriction $0 < \beta_0 < \gamma$. Considering $\alpha > 0$ three cases can be discussed, through the determinant of $\varphi(z)$, say D, $D = \beta^2 - 4\alpha\gamma$. If $D=0$ (see Figure 2), $D>0$ (see Figure 3), while the case $D<0$ is with no economical interest (due to the complex roots). Therefore the two cases are discussed below, while for the dual case $\alpha<0$ see case 4c.

Case 4a: $\alpha > 0, \quad D = \beta^2 - 4\alpha\gamma = 0.$

In such a case there is a double real root for MD(z), say $\rho = \rho_1 = \rho_2 = -\dfrac{\beta}{2\alpha}$. We need the root $\rho>0$ and hence is required that $\beta<0$. To identify the optimal pollution level point $\mathrm{I}(z_0, k_0)$ the evaluation of point z_0 is the one that

$$\begin{aligned} MD(z_0) = \varphi(z_0) = g(z_0) = \mathrm{MAC}(z_0) \;&\Leftrightarrow\; \alpha z_0^2 + \beta z_0 + \gamma = \beta_0 + \beta_1 z_0 \\ &\Leftrightarrow\; \alpha z_0^2 + (\beta - \beta_1) z_0 + (\gamma - \beta_0) = 0 \end{aligned} \tag{12}$$

In order relation (12) to provide the unique (double) solution we need the quantity the determinant of (12) $D = (\beta - \beta_1)^2 - 4\alpha(\gamma - \beta_0)$ to be zero i.e. $D=0$ which is equivalent to

$$z_0 = -\frac{\beta - \beta_1}{2\alpha} = \frac{\beta_1 - \beta}{2\alpha} \tag{13}$$

As z_0 is positive and $\alpha > 0$ we conclude that $\beta_1 > \beta$. So for $\alpha > 0$, $\beta_1 > \beta$, $0 < \beta_0 < \gamma$ we can easily calculate

$$k_0 = MAC(z_0) = \beta_0 + \beta_1 \frac{\beta_1 - \beta_0}{2\alpha} > 0 \tag{14}$$

and therefore $\mathrm{I}(z_0, k_0)$ is well defined. The corresponding Benefit Area ($\mathrm{BA_{QL}}$) in this case is

$$\mathrm{BA}_{QL} = (\mathrm{ABI}) = \int_0^{z_0} (\varphi(z) - g(z))dz = \int_0^{z_0} (az^2 + (\beta - \beta_1)z + (\gamma - \beta_0))dz$$

$$= \left[\alpha \frac{z^3}{3} + (\beta - \beta_1)\frac{z^2}{2} + (\gamma - \beta_0)z \right]_0^{z_0} = \alpha \frac{z_0^3}{3} + (\beta - \beta_1)\frac{z_0^2}{2} + (\gamma - \beta_0)z_0 \tag{15}$$

Case 4a

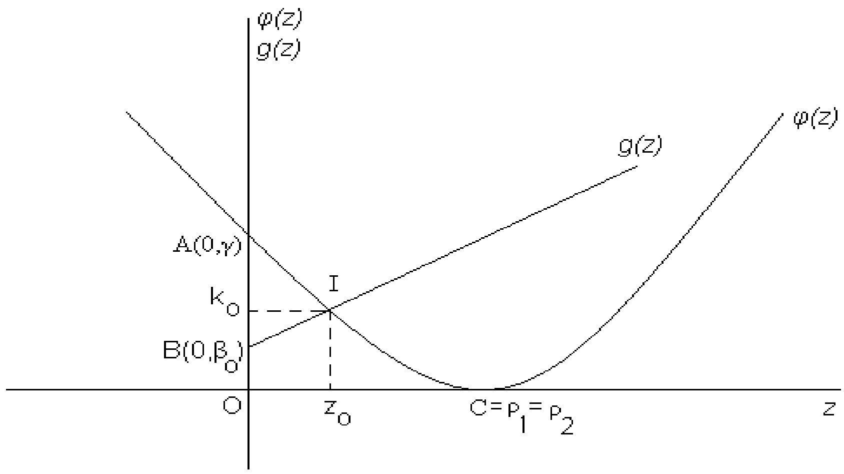


Figure 2: $\mathrm{C} = \mathrm{C}\left(-\frac{\beta}{2\alpha}, 0\right), \quad \alpha > 0$

Case 4b: $\alpha > 0, \quad D = \beta^2 - 4\alpha\gamma > 0.$

In such a case for the two roots ρ_1, ρ_2 it is $|\rho_1| \neq |\rho_2|$, $\varphi(\rho_1) = \varphi(\rho_2) = 0$ and we suppose $0 < \rho_1 < \rho_2$, see Figure 3. The fact that *D>0* is equivalent to $0 < a\gamma < \left(\frac{\beta}{2}\right)^2$, while the minimum value of the MD function is $\varphi\left(-\frac{\beta}{2\alpha}\right) = \frac{4\alpha\gamma - \beta^2}{4\alpha}$

Proposition 1: The following order for the roots and the value which provides the minimum (see Figure 3) is true under the relation $\beta<0<\alpha\gamma<\left(\frac{\beta}{2}\right)^2$ (16)

Proof

The order of the roots $0<\rho_1<\rho_2$ is equivalent to the set of relations:

$$D>0, \qquad \alpha\varphi\left(-\frac{\beta}{2\alpha}\right)<0, \qquad \alpha\varphi(0)>0, \qquad 0<\frac{\rho_1+\rho_2}{2}. \tag{16.1}$$

The first is valid, as we have assumed $D > 0$. For the imposed second relation from (16.1) we have $\alpha\varphi\left(-\frac{\beta}{2\alpha}\right)<0 \;\Leftrightarrow\; \alpha\frac{4a\gamma-\beta^2}{4\alpha}<0 \;\Leftrightarrow\; \Delta>0$,

which holds. As both the roots are positive $\rho_1,\rho_2>0$, then the product $\rho_1\rho_2>0$ therefore $\frac{\gamma}{\alpha}>0 \;\Leftrightarrow\; \alpha\gamma>0$ which is valid as $0<a\gamma<\left(\frac{\beta}{2}\right)^2$. The third relation $\alpha\varphi(0)=\alpha>0,$ in (16.1) is true already and $0<\frac{\rho_1+\rho_2}{2}\Rightarrow 0<-\frac{\beta}{2\alpha}$ equivalent to $\beta<0$.

Therefore we get $\beta<0<a\gamma<\left(\frac{\beta}{2}\right)^2$.

We can then identify the point of intersection z_0: $\mathrm{MAC}(z_0)=MD(z_0)$ as before. Therefore under (16) and $\beta_1>\beta_0$ we evaluate k_0 as in (14) and the Benefit Area BA_{QL} can be evaluated as in (15).

Case 4b

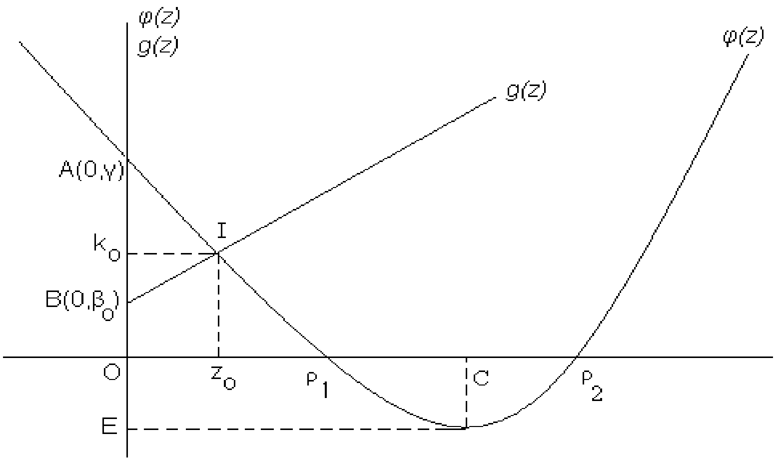


Figure 3: $C=C\left(-\frac{\beta}{2\alpha},0\right)$, $E=E\left(0,\varphi\left(-\frac{\beta}{2\alpha}\right)\right)$, $\varphi\left(-\frac{\beta}{2\alpha}\right)=\min\ \varphi(z)$, $\alpha>0$

Let us now consider the case *α < 0.* Under this assumption the restriction *D = 0* is not considered, as the values of *φ(z)* have to be negative. Consider Figure 2, the graph of *φ(z)* will be symmetric around the point C to this in Figure 2. Therefore we consider the following case.

Case 4c: $\alpha < 0, \quad D = \beta^2 - 4\alpha\gamma > 0$

Under the assumption of Case 4c, the value $\varphi\left(-\frac{\beta}{2\alpha}\right)=\frac{4\alpha\gamma-\beta^2}{4\alpha}$ corresponds to the maximum value of *φ(z)*. We consider the situation where $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ (see Figure 4) while the case $0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2$ has no particular interest (it can be also considered as in case 4b).

Case 4c

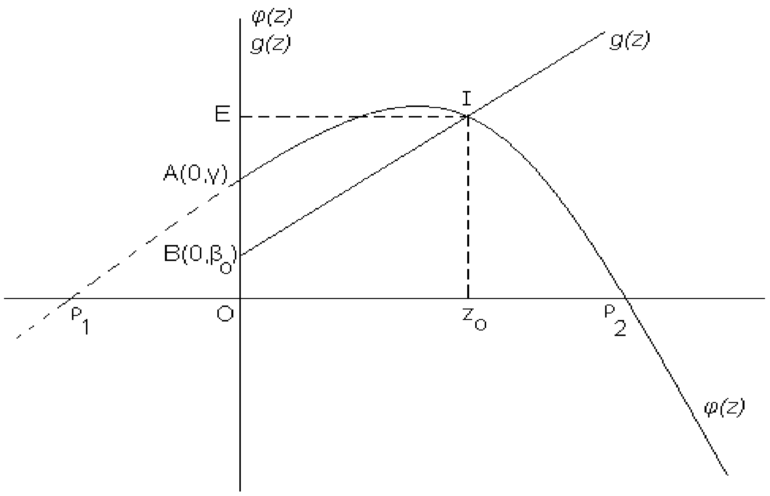


Figure 4: $C = C\left(-\frac{\beta}{2\alpha}, 0\right)$, $E = E\left(0, \varphi\left(-\frac{\beta}{2\alpha}\right)\right)$, $\varphi\left(-\frac{\beta}{2\alpha}\right) = \min\ \varphi(z)$, $\alpha < 0$

Proposition 2:

For the case 4c as above holds: $\rho_1 < 0 < -\frac{\beta}{2\alpha} < \rho_2$ when $\alpha\gamma<0$.

Proof

The imposed assumption is equivalent to $\alpha\varphi(0) < 0 \Leftrightarrow \alpha\gamma < 0$ true as $\rho_1\rho_2 < 0$

$\alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0 \Leftrightarrow \alpha\gamma < \left(\frac{\beta}{2}\right)^2$. Therefore the imposed restrictions are $\alpha\gamma < 0 < \left(\frac{\beta}{2}\right)^2$

Actually $\alpha\gamma < 0$. In case 4c it is now asked $\beta_0 < \gamma$ and $\beta_1 > 0$. To calculate z_0 we proceed as in (12) and z_0 is evaluated as in (13) with $\alpha < 0$, therefore $\beta_1 - \beta < 0$ i.e. $\beta_1 < \beta$. Thus for $\beta_1 < \beta$, $\alpha\gamma < 0$, the BA as in (15) is still valid.

Therefore the following analysis is now considered, extending case 4 of both MD and MAC to be quadratic functions.

Case 5: MD and MAC functions both quadratic

Let us consider that both curves, MAC and MD are of the second order as

$$\text{MAC} = g(z) = \beta_0 + \beta_1 z + \beta_2 z^2 \qquad \beta_2 \neq 0 \text{ (with determinant d)}$$

$$\text{MD} = \varphi(z) = \alpha z^2 + \beta z + \gamma, \qquad a \neq 0 \text{ (with determinant D)}$$

In this case the intersections of MAC and MD with the Y-axis are b =MAC(0)=β_o and a=MD(0)=γ. The following two substances are investigated below.

Case 5a: $D = \beta^2 - 4\alpha\gamma = 0, \quad a > 0$ and $d = \beta_1^2 - 4\beta_2\beta_0 \leq 0, \quad \beta_2 > 0,$ see Figure 5.

For the identification of the optimal pollution level point $\mathrm{I}(z_0, k_0)$ we need to estimate z_0 such that

$$\begin{aligned} \varphi(z_0) = g(z_0) \quad &\Leftrightarrow \quad \alpha z_0^2 + \beta z_0 + \gamma = \beta_0 + \beta_1 z_0 + \beta_2 z_0^2 \\ &\Leftrightarrow \quad (\alpha - \beta_2) z_0^2 + (\beta - \beta_1) z_0 + (\gamma - \beta_0) = 0 \end{aligned} \tag{17}$$

When the determinant of this equation δ is zero,

$$\delta = (\beta - \beta_1)^2 - 4(\alpha - \beta_2)(\gamma - \beta_0) = 0$$

the unique root of (17) equals

$$z_0 = -\frac{\beta - \beta_1}{2(\alpha - \beta_2)}. \tag{18}$$

Recall case 2, relation (6). It is asked not only z_0 to be positive but to be less than the (double) root of MD (see Figure 5), i.e.

$$0 < z_0 = -\frac{\beta - \beta_1}{2(\alpha - \beta_2)} < -\frac{\beta}{2\alpha},$$

Therefore we have the restriction $\qquad \dfrac{\beta}{\alpha} < \dfrac{\beta - \beta_1}{\alpha - \beta_2}.$

Thus the corresponding to the optimal restriction in damage z_0, optimal cost k_0 point is

$$k_0 = \varphi(z_0) = \alpha \frac{(\beta-\beta_1)^2}{4(\alpha-\beta_2)^2} - \beta_0 \frac{\beta-\beta_1}{\alpha-\beta_2} + \gamma \tag{19}$$

Thus the corresponding Benefit Area, BA_{QQ}, can be evaluated as

$$\mathrm{BA}_{QQ} = \int_0^{z_0} (\varphi(z) - g(z))dz = (\alpha-\beta_2)\frac{z_0^3}{3} + (\beta-\beta_1)\frac{z_0^2}{2} + (\gamma-\beta_0)z_0 \tag{20}$$

Case 5a

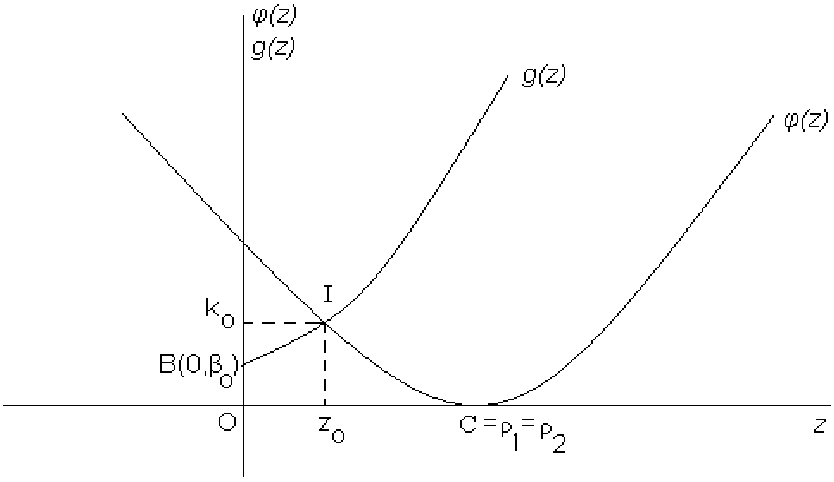


Figure 5: $C=\rho_1=\rho_2 = C\left(-\frac{\beta}{2\alpha}, 0\right), \quad \varphi\left(-\frac{\beta}{2\alpha}\right) = 0, \quad a > 0$

Another interesting case is the following.

Case 5b: $D = \beta^2 - 4\alpha\gamma > 0, \quad a > 0$ and $d = \beta_1^2 - 4\beta_2\beta_0 \leq 0, \ \beta_2 > 0$ (see Figure 6).

In this case we need the order of the points to be $0 < z_0 < \rho_1 < -\frac{\beta}{2\alpha} < \rho_2$. Recall also case 4b (see Figure 6). Therefore we need the following conditions

$\alpha\varphi\left(-\frac{\beta}{2\alpha}\right) < 0,$ which holds and $D > 0, \qquad \alpha\varphi(z_0) > 0, \qquad z_0 < \frac{\rho_1+\rho_2}{2} = -\frac{\beta}{2\alpha}.$

Eventually is needed to have the restrictions

$$\alpha\varphi(z_0) > 0 \quad \text{and} \quad z_0 < -\frac{\beta}{2\alpha}.$$

The point z_0 is eventually as in (18) and the corresponding k_0 as in (19) so $\mathrm{I}(z_0, k_0)$ is identified. The BA is evaluated as in (20).

Case 5b

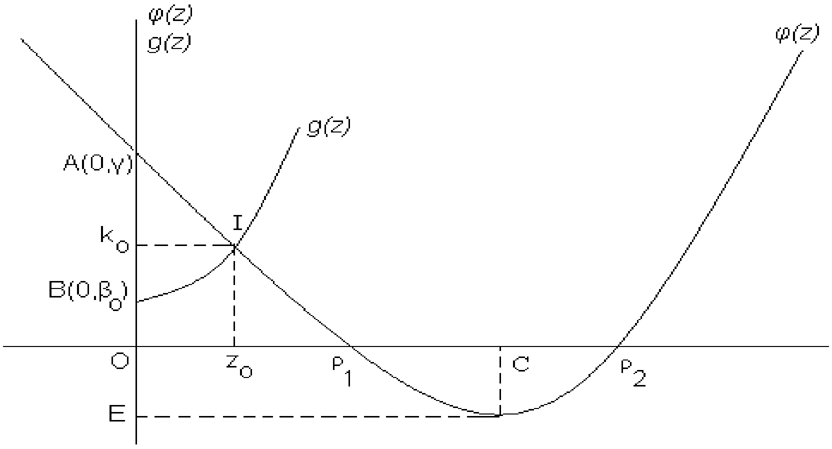


Figure 6: $C = C\left(-\frac{\beta}{2\alpha}, 0\right)$, $E = E\left(0, \varphi\left(-\frac{\beta}{2\alpha}\right)\right)$, $\varphi\left(-\frac{\beta}{2\alpha}\right) = \min \varphi(z)$, $a > 0$

$0 < z_0 < p_1 < -\frac{\beta}{2\alpha} < p_2$

Case 6: MD quadratic and MAC exponential functions

We assume now that MD is quadratic function and MAC is exponential, that is

$$MAC(z) = g(z) = \beta_0 e^{\beta_1 z}, \ \beta_0 > 0 \qquad MD(z) = \varphi(z) = \alpha z^2 + \beta z + \gamma, \qquad a > 0$$

Here b=MAC(0)=β_o , and a=MD(0)=γ. In particular we are investigating the following sub-cases, which are within our target.

Case 6a: For MD it is $D = \beta^2 - 4\alpha\gamma > 0$, see also Figure 7.

Evaluating I(z_0, k_0), z_0 has to obey to the relationship:

$$\varphi(z_0) = g(z_0) \iff \alpha z_0^2 + \beta z_0 + \gamma = \beta_0 e^{\beta_1 z_0} \iff \alpha z_0^2 + \beta z_0 + \gamma - \beta_0 e^{\beta_1 z_0} = 0 \qquad (21)$$

This equation (21) is non-linear so we have to prove that there is one solution, which can be evaluated numerically.

Case 6

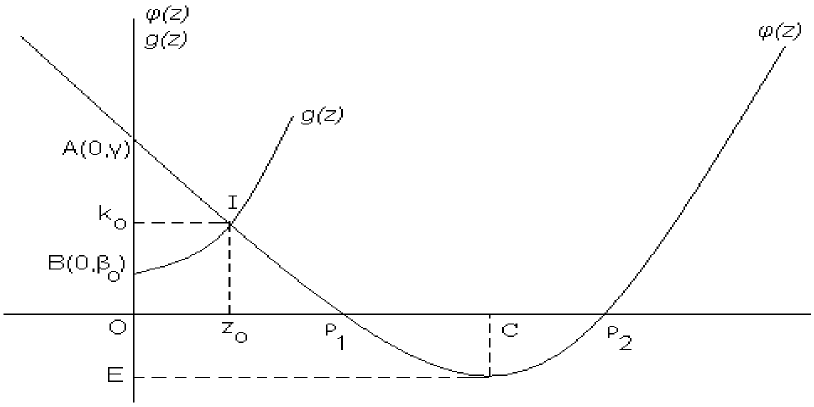


Figure 7: $C = C\left(-\dfrac{\beta}{2\alpha}, 0\right)$

Proposition 3: There is a $z_0 \in (0, \rho_1)$: $MD(z_0)=MAC(z_o)$ function

Proof

If we consider the equation $F(z) = \alpha(z-\rho_1)(z-\rho_2) - \beta_0 e^{\beta_1 z}$ then under the imposed restrictions $F(0) = \gamma - \beta_0 > 0$ as $\rho_1\rho_2 = \dfrac{\gamma}{\alpha}$ and $F(\rho_1) = -\beta_0 e^{\beta_1\rho_1} < 0$. Therefore as $F(0)F(\rho_1) < 0$ there exists a number $z_0 \in (0, \rho_1) : F(z_0) = 0$ and therefore (21) is true that is there exists a real solution (root) z_0 for it.

The evaluation of the root z_0 of (21) can be numerically calculated and one way is by adopting the Bisction method. The corresponding $k_0 = g(z_0) = \beta_0 e^{\beta_1 z_0}$ can be easily evaluated, as the corresponding Benefit Area (BA_{EQ}) is calculated as

$$\mathrm{BA}_{EQ} = \int_0^{z_0} (\varphi(z) - g(z))dz = \alpha \frac{z_0^3}{3} + \beta \frac{z_0^2}{2} + \gamma z_0 - \frac{\beta_0}{\beta_1}\left(e^{\beta_1 z_0} - 1 \right) \tag{22}$$

The last case is to have both MAC and MD functions exponential as follows.

Case 7: **MD and MAC both exponential functions**

In this case it is assumed that both MB and MAC are exponential of the form

$$MAC(z) = g(z) = \beta_0 \mathrm{e}^{\beta_1 z}, \;\; \beta_0 > 0 \qquad MD(z) = \varphi(z) = \theta_0 \mathrm{e}^{\theta_1 z}, \quad \theta_0 > \beta_0 > 0$$

The target is to evaluate the z_0 point of the optimal pollution level as

$$\varphi(z_0) = g(z_0) \;\Leftrightarrow\; \beta_0 \mathrm{e}^{\beta_1 z_0} = \theta_0 \mathrm{e}^{\theta_1 z_0} \;\Leftrightarrow\; \frac{\beta_0}{\theta_0} = \mathrm{e}^{(\theta_1 - \beta_1) z_0} \;\Leftrightarrow\; (\theta_1 - \beta_1) z_0 = \ln \frac{\beta_0}{\theta_0}$$

$$\Leftrightarrow\; z_0 = \frac{1}{(\theta_1 - \beta_1)} \ln \frac{\beta_0}{\theta_0} = \frac{1}{(\beta_1 - \theta_1)} \ln \frac{\theta_0}{\beta_0}$$

From the above relation it is clear that there is no intersection when $\theta_1 = \beta_1$. The corresponding Benefit Area ($\mathrm{BA}_{\mathrm{EE}}$) is evaluated as

$$\mathrm{BA}_{EE} = \int_0^{z_0} (\varphi(z) - g(z))dz = \frac{\beta_0}{\beta_1}\left(e^{\beta_1 z_0} - 1 \right) - \frac{\theta_0}{\theta_1}\left(e^{\theta_1 z_0} - 1 \right) \tag{23}$$

Case 7

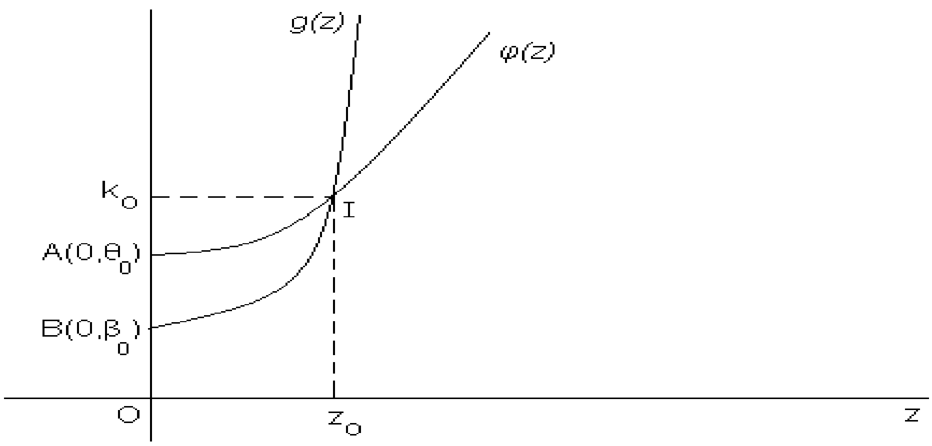


Figure 8: $\theta_0 > \beta_0$, MAC and MD functions exponential

Finally, the case that $\varphi(z)$ is exponential and $g(z)$ is of second order is the dual of case 6a. In this way we have investigated all possible cases modeling MAC and MD. A compact view of all the cases is presented in Table 2. The results obtained above have been adopted in an empirical application discussed in the next section.

Table 2: The compact presentation of the results

Case	MD=φ(z)	MAC=g(z)	$z_0>0$	BA	Restrictions
1	$\alpha+\beta z$	$\beta_0+\beta_1 z$	$-\dfrac{\beta_0-\alpha}{\beta_1-\beta}$	(3)	$\alpha>\beta_0$ $\beta_1>\beta$
2	$\alpha+\beta z$	$\beta_0+\beta_1 z+\beta_2 z^2$	$-\dfrac{\beta_1-\beta}{2\beta_2}$	(7), (8)	$\beta_2>0$ $\beta>\beta_1$ $\alpha>\beta_0$
3	$\alpha+\beta z$	$\beta_0 e^{\beta_1 z}$	Numerically (10)	(11)	$\beta_1>0$
4	$\alpha z^2+\beta z+\gamma$	$\beta_0+\beta_1 z$			$0<\beta_0<\gamma$
4a	$\alpha>0$ D=0		$Z_0=\dfrac{\beta_1-\beta}{2\alpha}$	(15)	$\beta_1>\beta$ $\beta<0$ $0<\beta_0<\gamma$
4b	$\alpha>0$ D>0				$\beta<0<\alpha\gamma<(\beta/2)^2$ $\beta_1>\beta_0$
4c	$\alpha<0$ D>0				$\beta_1<\beta$ $\alpha\gamma<0$
5	$\alpha z^2+\beta z+\gamma$	$\beta_0+\beta_1 z+\beta_2 z^2$			$\alpha\neq 0$ $\beta_2\neq 0$
5a	$\alpha>0$ D=0	$\beta_2>0$ $d\leq 0$	$Z_0=\dfrac{\beta-\beta_1}{2(\alpha-\beta_2)}$	(20)	$\beta_1>\beta$ $\alpha>\beta_2$
5b	$\alpha>0$ D>0	$\beta_2>0$ $d\leq 0$			
6	$\alpha z^2+\beta z+\gamma$	$\beta_0 e^{\beta_1 z}$		(20)	$\beta_0>0$ $\alpha>0$
6a	$\alpha>0$, D>0		Numerically	(22)	$\gamma>\beta_0$
7	$\theta_0 e^{\theta_1 z}$	$\beta_0 e^{\beta_1 z}$	$z_0=\dfrac{1}{\beta_1-\theta_1}\ln\dfrac{\theta_0}{\beta_0}$	(23)	$\beta_1\neq\theta_1$

4. An empirical application

In our empirical application we shall use estimates for the available data for different European countries. For this purpose we discuss how the two curves, the abatement cost g(z) and the damage cost φ(z), can be approximated.

Starting with the abatement cost function these measures the cost of reducing tonnes of emissions of a pollutant, like sulphur (S), and differs from country to country depending on the local costs of implementing best practice abatement techniques as well as on the existing power generation technology. For abating sulphur emissions various control methods exist with different cost and applicability levels (Halkos 1993, 1995, 1996, 1997):

(a) gas oil desulphurisation,

(b) heavy fuel oil desulphurisation,

(c) hard coal washing,

(d) in furnace direct limestone injection,

(e) flue gas desulphurisation and

(f) fluidised bed combustion.

To calculate total emissions from each source (TE_p) the annual emissions for a given pollutant in each sector for each European country are calculated. Total emissions are then determined as:

$$TE_p = \Sigma\ [PR_{ijt} \text{ x } (1\text{-}\alpha_t)\ E_{pij} \text{ x } AR_{ijtf}] \qquad (24)$$

where **i** stands for country, **j** for sector, **t** for technology, **f** for fuel and **p** for pollutant. Similarly, **PR** stands for production levels; $\boldsymbol{\alpha_t}$ for the abatement efficiency of method t and **AR** for the application rate (Halkos, 2010).

In the same way, given the generic engineering capital and operating control cost functions for each efficient abatement technology, total and marginal costs of different levels of pollutant's reduction at each individual source and in the national (country) level can be constructed. According to Halkos (1993, 1995, 2010), the cost of an emission abatement option is given by the total annualized cost (**TAC**) of this abatement option, including capital and operating cost components. Specifically:

$$TAC = \{(TCC)[r/(1-(1+r)^{-n})]\} + VOMC + FOMC \qquad (25)$$

Where **TCC** is the total capital cost; **VOMC** and **FOMC** stand for the variable and fixed operating and maintenance cost respectively; **r/[1-(1+r)$^{-n}$]** is the capital recovery factor at real discount rate **r**, which converts a capital cost to an equivalent stream of equal annual future payments, considering the time value of money (represented by r). Finally, **n** stands for the economic life of the asset (in years).

For every European country a least cost curve is derived by finding the technology on each pollution source with the lowest marginal cost per tone of pollutant removed in the country and the amount of pollutant removed by that method on that pollution source. In this way the first step on the country's abatement curve is constructed. Iteratively the next highest marginal cost is found and is added to the country curve with the amount of pollutant (say sulphur) removed on the X-axis. In the national cost curve each step corresponds to a control measure that leads to an emission reduction of an extra unit at the least cost. Figure 9 presents the steps in deriving control cost curves; while Figure 10 shows the Total and Marginal Abatement Cost curves for Austria in the year 2000.

For analytical purposes, it is important to approximate the cost curves of each country adopting a functional form. Extending the mathematical models described above to stochastic models (as the error term from the Normal distribution with mean zero and variance σ^2) we have found that least squares equations of the form

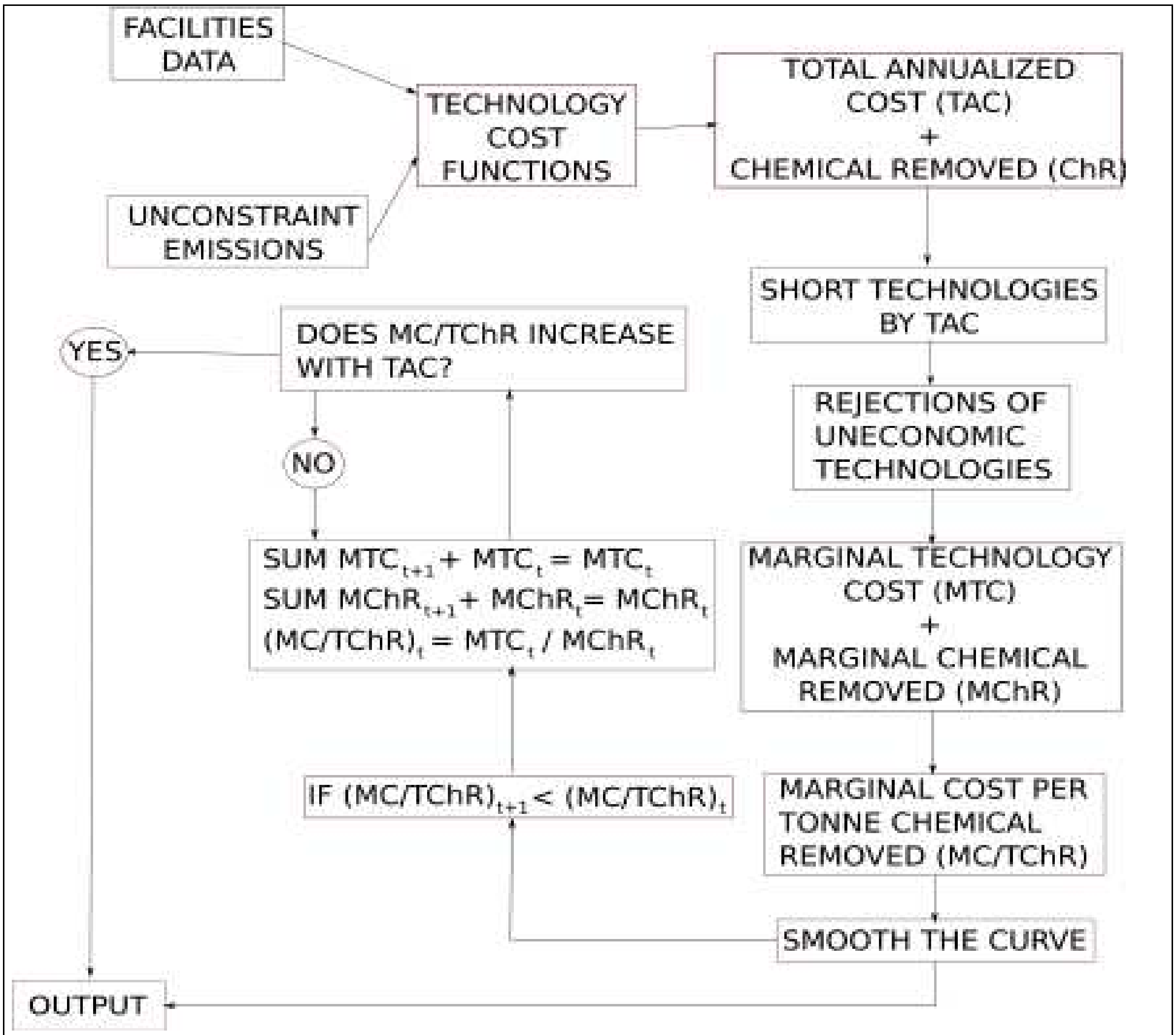
$$g(z) = AC_i = \beta_{0i} + \beta_{1i} SR_i + \varepsilon_i \qquad (26.1)$$

or

$$g(z) = AC_i = \beta_{0i} + \beta_{1i} SR + \beta_{2i} SR_i^2 + \varepsilon_i \qquad (26.2)$$

lead to satisfactory approximations for all the countries analyzed in this paper[4]. In these equations **SR$_i$** represents sulphur removed in country i, **AC$_i$** abatement cost in country i and **ε_i** the disturbance term with the usual hypotheses (Halkos, 2006).

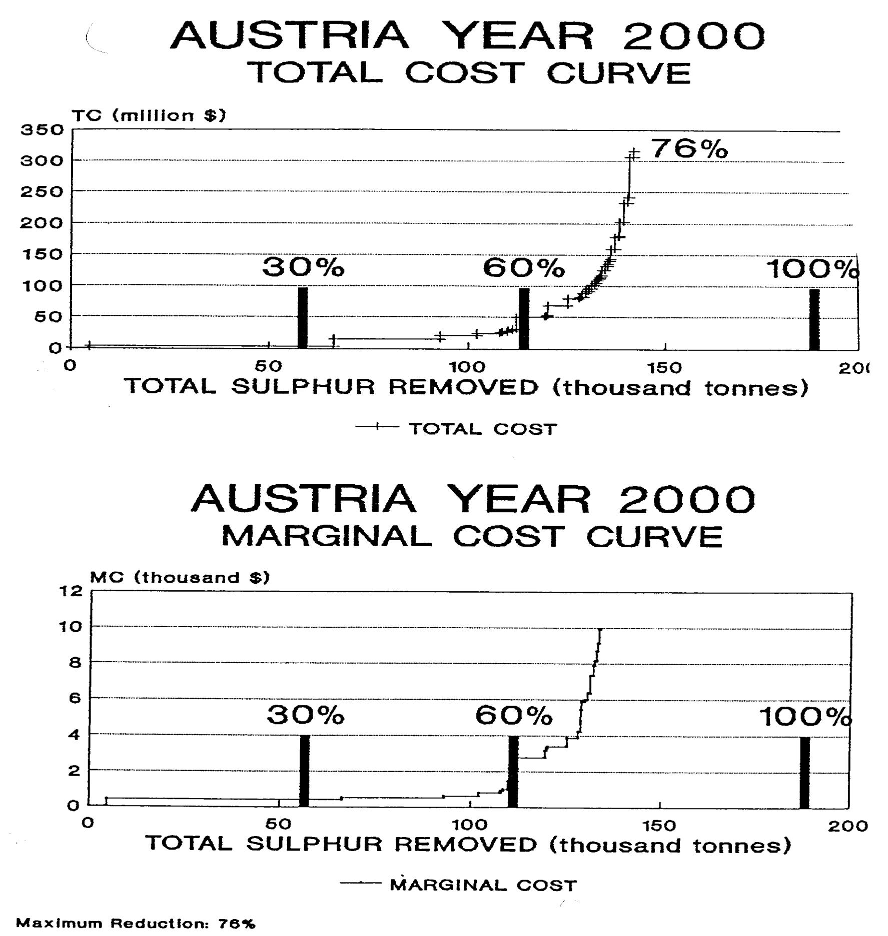
Figure 9: Steps in constructing Abatement Cost Curves



Source: Halkos (2010)

[4] Equations were fitted across the range 5-55% of maximum feasible abatement. The estimated coefficients of both specifications were statistically significant in all cases with only exception in the estimate of β_1 in the quadratic specification of Spain.

Figure 10: Total and Marginal Abatement Cost curves for Austria in the year 2000



Source: Modified form Halkos (1992)

Next, the calculation of the damage function φ(z) is necessary. The problem of estimating damage cost functions is a lot more difficult compared to the estimation of abatement costs, as the effects of pollution cannot be identified with any accuracy and sometimes it takes a long time to realize the consequences. In our case and in order to extract the damage estimates we use the case of acidification which is related to transboundary pollution, requiring that the model takes account of the distribution of the externality among the various countries (victims).

Each country receives a certain number of pollutant's units whose deposition is due to the other countries' emissions as well as to its own emissions. The deposition of sulphur in country i is given by

$$D_i = B_i + d_{ii}(1-\alpha_i)E_i + \Sigma_{i\neq j} d_{ij}(1-\alpha_j)E_j = B_i + \Sigma_j d_{ij}(1-\alpha_j)E_j \qquad (27)$$

where $\mathbf{E_j}$ is the total annual sulphur emission in country j; $\mathbf{D_i}$ is the total Annual sulphur Deposition in country i; $\boldsymbol{\alpha_i}$ is the abatement efficiency coefficient in country i and $\mathbf{d_{ij}}$ is the transfer coefficient from country j to i, indicating what proportions of emissions from any source country is ultimately deposited in any receiving country; $\mathbf{B_i}$ is the level of the so-called background deposition attributable to natural sources (such as volcanoes, forest fires, biological decay, etc) in receptor-country i, or to pollution remaining too long in the atmosphere to be tracked by the model, i.e. is probably attributable not only to natural sources but also to emissions whose origin cannot be determined. This assignment is summarized in the European Monitoring and Evaluation Programme transfer coefficient matrix (EMEP).

Due to difficulties, we do not directly estimate the damage function, but instead we infer its parameters assuming that countries currently equate national marginal damage cost with national marginal abatement cost[5]. The restrictions on the derivatives of the damage cost function are important. The total cost resulting from a specific level of pollutant (like sulphur) for country i is,

$$TC_i = \text{abatement cost} + \text{damage cost} = AC_i + DC_{i.}$$

[5] Rabl *et al*. (2013) approximate the damage function by a linear function of the pollution emissions and they claim that linearity is found to be appropriate approximation in the case of PM, SO_2 and NO_X emissions, while for CO_2 linearity is probably acceptable for emissions reductions in the "foreseeable" future period.

As mentioned, abatement costs are estimated by linear and quadratic functions of sulphur removed (as in 26.1 and 26.2) and we also assume that damage costs are linear and quadratic in deposits. Damage depends on deposits, which depend on the $[d_{ij}]$ matrix as explained before. In the more complex case of both quadratic abatement and damage cost functions the total cost function may be expressed as:

$$TC_i = [\beta_{0i} + \beta_{1i}\, SR_i + \beta_{2i}\, SR_i^2] + [\gamma_{0i} + \gamma_{1i}\, D_i + \gamma_{2i}\, D_i^2] \qquad i=1,2,\ldots,n \qquad (28)$$

It follows that total cost is minimized when

$$\gamma_{1i} = [(\beta_{1i}+2\beta_{2i}SR_i-2\gamma_{2i}d_{ii}^2SR_i)/d_{ii}] \quad \text{and} \quad \gamma_{2i}=[(\beta_{1i}+2\beta_{2i}SR_i-\gamma_{1i}d_{ii})/2d_{ii}^2\, SR_i]$$

where $\mathbf{d_{ii}}$ the transfer coefficient[6].

This is the only information available to "calibrate" the damage function, on the assumption that national authorities act independently (as Nash partners in a non-cooperative game with the rest of the world), taking as given deposits originating in the rest of the world. If we assume linear abatement and quadratic damage cost functions then $\gamma_{1i}= (\beta_{1i}-2\gamma_{2i}d_{ii}^2\, SR_i)/d_{ii}$ and $\gamma_{2i}=[(\beta_{1i}-\gamma_{1i}d_{ii})/2d_{ii}^2\, SR_i]$

Finally in the case of linearity for both abatement and damage cost functions $\gamma_{1i}=\beta_{1i}/d_{ii}$.

Table 3 presents the estimated damage and abatement cost coefficients in the more complex case of assuming both cost functions are quadratic[7]. Similarly, Table 4 presents the corresponding "calibrated" Benefit Area (**BA**^{c}) indexes evaluated from the available parameter estimates in the case of linear-quadratic (**LQ**) and quadratic-quadratic (**QQ**) damage and abatement cost functions respectively for 20 European countries.[8] In the first of the two case studies the associated efficiency index[9] is

[6] If like Mäler (1989, 1990) and Newbery (1990) we set $\gamma_{2i}=0$ then $\gamma_{1i} =(\beta_{1i}+2\beta_{2i}SR_i)/d_{ii}$.
[7] Estimates of c_0 were derived by assuming countries act in a Nash behavior.
[8] The empirical results presented are indicative and very sensitive to the assumptions of calibration.

presented[10]. This is a measure of what percentage of the adopted policy covers that policy which provides the maximum benefit area. Similarly, the evaluation of the optimal damage reduction, z_0 as has been denoted in this paper, provides evidence, that the larger it is the better the adopted environmental policy.

As it can be seen, countries with high optimal damage reductions are the UK and France (in both cases LQ and QQ) and Former Czechoslovakia, Spain and Turkey (in the LQ case). On the other hand, countries with low damage reductions are Greece, Hungary, Italy, Romania (in the case of LQ cost functions), Finland, Sweden, Turkey (in the case of QQ) and Norway and Switzerland in both cases. Large industrial upwind counties (like Denmark, France and the UK) seem to have a very large benefit area. Looking at the EMEP transfer coefficients matrix it can be seen that the countries with large benefit areas are those with large numbers on the diagonal. This shows the importance of the domestic sources of pollution. The large off-diagonal transfer coefficients indicate in general the major effects of one country on another, and especially the externalities imposed by the Eastern European countries on the others.

Similarly downwind or near to the sea countries seem to have small benefit areas. Additionally, the damage caused by acidification depends on where the depositions occur. In the case of occurrence over the sea it is less likely to have much harmful effect, as the sea is naturally alkaline. In the same way, if it occurs over sparsely populated areas with acid tolerant soils then the damage is low (Newbery, 1990).

---

[9] Following Halkos and Kitsos (2005) the **efficiency (Eff)** of the benefit area, in comparison with the maximum evaluated from the sample of countries under investigation, can be estimated using as measure of efficiency the expression:

$$\text{Eff} = \left[\frac{BA}{\max BA}\right]*100$$

[10] Germany dominates the picture in Europe as it has a very high initial abatement level (≈42%) and its calibrated damage function ensures high abatement levels (Hutton and Halkos, 1995). For this reason the efficiency index was constructed on the second highest benefit area (former Czechoslovakia).

Table 3: Coefficient estimates in the case of quadratic MD and MAC functions

Countries	c_0	c_1	c_2	b_0	b_1	b_2
Albania	0,7071	0,01888	0,0001397	-3,3818	0,015	0,0048
Austria	8,57143	0,055012	0,0001145	3,274	-0,221	0,004
Belgium	2,2424	0,03869	0,0001688	0,497	-0,124	0,003
Former Czechoslovakia	37,794	0,100323	0,000059	11,241	0,2358	0,00018
Denmark	10	0,1923	0,0060811	-2,49	0,099	0,0053
Finland	4,021	0,0781	0,00014587	2,343	-0,098	0,0046
France	33,158	0,277352	0,000197	42,374	-0,053	0,0018
Greece	3,7373	0,034133	0,0000491	-1,614	0,342	0,0006
Hungary	5,101	0,031488	0,0000417	2,506	0,216	0,0004
Italy	21,01	0,030036	0,0000191	12,5	0,36	0,0003
Luxembourg	0,421	0,3161	0,0272381	-0,7272	0,01	0,09234
Netherlands	8,353	0,19513	0,00351442	-6,18	0,41	0,0009
Norway	1,421	0,07852	0,00017008	0,94	-0,244	0,0164
Poland	6,212	0,023153	0,000071	-8,023	0,324	0,00009
Romania	9,091	0,011364	0,00006237	5,502	0,19	0,0001
Spain	11,7	0,007288	0,00497419	10,21	-0,021	0,00014
Sweden	2,4	0,06423	0,0000932	4,074	-0,252	0,004
Switzerland	2,4	0,56027	0,002803	5,7543	-1,6289	0,11203
Turkey	14,9	0,01781	0,00001223	8,0622	0,011	0,00036
UK	19,1	0,06879	0,0000467	15,54	0,0264	0,0003

Table 4: Calculated "calibrated" Benefit Areas (BA^c)

	Linear - Quadratic						**Quadratic-Quadratic**	
Countries	D	z_0	$g(z_0)$	$G(z_0)$	BA	Eff	z_0	BA
Albania	0,07852	29,594	-3,38	-52,05	81,24	3,5872	0,416282	1,460084
Austria	0,160942	84,649	3,3	294,1	628,6	27,756	35,51795	34,38386
Belgium	0,047413	63,406	0,5	37,2	182,8	8,0715	28,73163	58,58427
Former Czechoslovakia	0,037791	160,988	11,24	5119,8	2264,6	100		
Denmark	0,273493	58,138	-2,5	369,72	536,7	23,698	-507,79	66016,29
Finland	0,061886	46,182	2,4	154,72	114,3	5,0453	19,76705	1,578593
France	0,042777	149,22	42,4	7726,3	309,2	13,65	103,0412	950,4364
Greece	0,107625	16,83	-1,62	22,23	45,5	2,0095		
Hungary	0,038197	13,66	2,51	54,72	17,9	0,7901		
Italy	0,119088	25,22	12,5	431,19	108,1	4,7726		
Luxembourg	0,517796	5,56	-0,73	1,4	5,8	0,2572	2,351007	2,359444
Netherlands	0,098488	54,98	-6,18	329,7	424,4	18,741	-73,6549	85,02478
Norway	0,135573	21,056	0,94	16,75	30,6	1,3508	9,93597	2,085391
Poland	0,095634	46,67	-8,03	-18,57	333,7	14,734		
Romania	0,033347	19,87	5,5	147,1	35,8	1,5803		
Spain	0,001635	245,43	10,2	2563,2	527,8	23,305	-2,92603	11,43726
Sweden	0,073218	73,35	4,1	147,1	201,7	8,9075	40,47174	55,68209
Switzerland	3,289337	17,87	5,56	55,8	76,5	3,378	10,02119	24,05788
Turkey	0,009866	147,82	8,1	1698,5	698,65	30,851	9,819189	58,17368
UK	0,006069	200,5	15,6	4452,1	759,9	33,551	83,67548	1813,895

5. Conclusions and Policy Implications

The analysis of the effectiveness of environmental programs and regulations requires the comparison of damage and control costs associated with the reduction of different pollutants. It is worth mentioning that although there is an obvious uncertainty in damage costs we cannot ignore that uncertainty is also present in the abatement cost functions due to abatement efficiencies that may differ between countries and between adopted scenarios.

The typical approach to define the optimal pollution level has been to equate the marginal damage of an extra unit of pollution with the corresponding marginal abatement cost. An efficient level of emissions maximizes the net benefit that is the difference between abatement and damage costs. Therefore the identification of this efficient level shows the level of benefits maximization, which is the output level resulting if external costs (damages) are fully internalized.

In this paper the corresponding optimal cost and benefit points were evaluated analytically. It is shown that this is feasible in the linear and quadratic cases while in the exponential case only approximated values can be obtained. The explicit evaluation of the benefit area was also discussed and analytical forms for this particular area were calculated for different policies. In this way the optimal level was also evaluated.

We show that the optimal pollution level can be evaluated only under certain conditions as were derived is section 3. Specifically, it is required that in all of the cases $\alpha>\beta_o$ if we assume that MAC is an increasing function and MD is a decreasing function. That is, the constant term in the damage cost function (we may think of the background deposition) is bigger than the abatement cost at level z=0 (we may think of fixed costs of operating an abatement method at level z=0). In cases of both linear or both quadratic functions we have $\beta > \beta_1$. That is the slope of the benefit function must be greater than the marginal abatement cost at level z=0. For the quadratic case it is required that $\beta_2 > 0$ while for the exponential case β_0, $\beta_1 > 0$. Both the quadratic and the exponential cases obey to the same form of evaluating the benefit area.

From our empirical findings is clear that the evaluation of the "calibrated" Benefit Area, as it was developed, provides an index to compare the different policies adopted from different countries on the basis of how large calibrated Benefit Area provides eventually. In this way a comparison of different policies can be performed. Certainly the policy with the maximum Benefit Area is the best, and the one with the minimum is the worst. Clearly the index BA^c provides a new measure for comparing the adopted policies.

An important finding (in the case of transboundary pollution) is that domestic pollution sources are important while big industrial upwind counties seem to have a very large benefit area. On the other hand, downwind near to the sea countries or over sparsely populated areas with acid tolerant soils seem to have small benefit areas. As mentioned the empirical results derived are only indicative and very sensitive to the assumptions of calibration.

Policy makers may have multiple objectives with efficiency and sustainability to be of high priority. Environmental policies should consider that economic development is not uniform across regions and may differ significantly (Halkos and Tzeremes, 2010). At the same time reforming economic policies to cope with EU enlargement may face problems and this may in turn affect their economic efficiencies (Halkos and Tzeremes, 2009).

REFERENCES

Barbier E. (1998). *The Economics of Environment and Development: Selected Essays*, Edward Elgar, London UK.

Bureau of Labor Statistics (2007). Fatal Occupational Injuries, Employment, and Rates of Fatal Occupational Injuries, 2006. (report, BLS [www.bls.gov], US Department of Labor, 2007), www.bls.gov/iif/oshwc/cfoi/CFOI_Rates_2006.pdf

Bjornstad D. and Kahn J. (1996). *The contingent valuation of environmental resource: Methodological issues and research needs*. Edward Elgar, London UK.

Bosello F., Eboli F. and Pierfederici R. (2012). Assessing the economic impacts of climate change. *Review of Environment Energy and Economics*, 1–9.

Brandt J., Silver J.D., Christensen J.H., Andersen M.S., Bønløkke J.H., Sigsgaard T., Geels C., Gross A., Hansen A.B., Hansen K.M., Hedegaard G.B., Kaas E., and Frohn L.M. (2013). Contribution from the ten major emission sectors in Europe and Denmark to the health-cost externalities of air pollution using the EVA model system – an integrated modelling approach. *Atmospheric Chemistry and Physics Discussions*, **13**, 5871–5922.

Cohen M.A. (2010a). Taxonomy of Oil Spill Costs—What Are the Likely Costs of the *Deepwater Horizon* Spill? (report, Resources for the Future [www.rff.org], www.rff.org/rff/documents/RFF-BCK-Cohen-DHCosts_update.pdf.

Cohen M.A. (2010b). *Deterring Oil Spills: Who Should Pay and How Much?* (publication of Resources for the Future [www.rff.org], www.rff.org/Publications/Pages/PublicationDetails.aspx?PublicationID=21161.

Department of Energy and Climate Change (2009). *Carbon Valuation in UK Policy Appraisal: a Revised Approach*. Climate Change Economics, London.

EMEP (various years). Airborne transboundary transport of sulphur and nitrogen over Europe: model description and calculations, EMEP/MSC-W Reports.

Fankhauser S. (1994). The economic costs of global warming damage: a survey. *Global Environmental Change*, **4(4)**, 301–309.

Fankhauser S. (1995). *Valuing Climate Change: The Economics of the Greenhouse*, 1st ed. EarthScan, London.

Farmer A., Kahn J.R., McDonald J.A. and O'Neill R. (2001). Rethinking the optimal level of environmental quality: justifications for strict environmental policy. *Ecological Economics*, **36**, 461-473.

Fisher A.C. and Hanemann W.M. (1990). Information and the dynamics of Environmental protection: the concept of the critical period. *The Scandinavian Journal of Economics*, **92,** 399-414.

Freeman A. III (1993). *The measurement of environmental and resource values: Theory and methods*. Resources for the Future, Washington DC.

Georgiou S., Pearce D., Whittington D. and Moran D. (1997). *Economic values and the environment in the developing world.* Edward Elgar, London UK.

Gollier C., Bruno J., and Treich N. (2000). Scientific Progress and Irreversibility: An Economic Interpretation of the Precautionary Principle. *Journal of Public Economics,* **75,** 229–253.

Greenstone M., Kopitsy E., and Wolverton A. (2013). Developing a Social Cost of Carbon for US Regulatory Analysis: A Methodology and Interpretation. *Review of Environmental Economics and Policy*, **7(1)**, 23–46.

Halkos G. (1992). *Economic perspectives of the acid rain problem in Europe.* DPhil Thesis, Department of Economics and Related Studies, University of York.

Halkos G. (1993). An evaluation of the direct costs of abatement under the main desulphurisation technologies, MPRA Paper 32588, University Library of Munich, Germany.

Halkos G. (1995). An evaluation of the direct cost of abatement under the main desulfurization technologies. *Energy Sources*, **17(4)**, 391-412.

Halkos G. (1996). Incomplete information in the acid rain game, *Empirica Journal of Applied Economics and Economic Policy,* **23(2)**, 129-148.

Halkos G. (1997). Modeling optimal nitrogen oxides abatement in Europe, MPRA Paper 33132, University Library of Munich, Germany.

Halkos G. (2000). Determining optimal air quality standards: Quantities or prices?, MPRA Paper 42849, University Library of Munich, Germany.

Halkos G. and Kitsos C.P. (2005). Optimal Pollution Level: A theoretical identification. *Applied Economics,* **37**, 1475-1483.

Halkos G.E. (2006). *Econometrics: Theory and practice.* Giourdas Publications.

Halkos G. (2010). Construction of abatement cost curves: The case of F-gases, MPRA Paper 26532, University Library of Munich, Germany.

Halkos G.E. and Tzeremes N.G. (2009). Economic efficiency and growth in the EU enlargement. *Journal of Policy Modeling*, **31(6),** 847-862.

Halkos G.E. and Tzeremes N.G. (2010). Measuring regional economic efficiency: the case of Greek prefectures. The Annals of Regional Science, **45(3)**, 603-632.

Halkos G.E. and Jones N. (2012). Modeling the effect of social factors on improving biodiversity protection, Ecological Economics, **78(C)**, 90-99.

Halkos G.E. and Matsiori S. (2012). Determinants of willingness to pay for coastal zone quality improvement, *The Journal of Socio-Economics*, **41(4)**, 391-399.

Heal G.M. and Kriström B. (2002). Uncertainty and climate change. *Environmental and Resource Economics,* **22**, 3-39.

Hutton J.P. and Halkos G. (1995). Optimal acid rain abatement policy in Europe: an analysis for the year 2000, *Energy Economics*, **17(4)**, 259-275.

Interagency Working Group on Social Cost of Carbon (2010). *Social Cost of Carbon for Regulatory Impact Analysis under Executive Order 12866*. Final Rule Technical Support Document: Energy Efficiency Program for Commercial and Industrial Equipment: Small Electric Motors. Appendix 15A. US Department of Energy, Washington, DC.

Jacoby H.D. and Ellerman A.D. **(**2004). The safety valve and climate policy. *Energy Policy,* **32(4)**, 481-491.

Kneese A. V. (1972). Rationalizing decisions in the quality management of water supply in urban-industrial areas, in *Readings in Urban Economics* (Ed.) M. Edel and J. Rothenberg, New York, The MacMillan Company.

Kolstad C.D. (1996). Fundamental irreversibilities in stock externalities. *Journal of Public Economics*, **60**, 221–233.

Ortega J. M. and Rheinboldt W.C. (1970). *Iterative solution of non-linear equations in several variables*, Academic Press Inc.

Pindyck R. S. (2000). Irreversibilities and the timing of environmental policy. *Resource and Energy Economics,* **22**, 233-59.

Pindyck R.S. (2002). Optimal timing problems in environmental economics. *Journal of Economic Dynamics and Control,* **26**, 1677-97.

Pindyck R.S. (2007). Uncertainty in Environmental Economics. *Review in Environmental Economics and Policy,* **1(1)**, 45-65.

Pizer W.A. (2002). Combining price and quantity controls to mitigate global climate change. *Journal of Public Economics,* **85**, 409-434.

Pizer W.A. (2003). *Climate change catastrophes. 2003. Resources for the Future Working Paper 03–31, May 2003.*

Pizer W. A. (2006). The evolution of a global climate change agreement. *American Economic Review*, **96(2),** 26-30.

Litman T. (2013). Full Cost Analysis of Petroleum Consumption *Transport* Beyond Oil, Chapter 3 pp 51-69. In: John L. Renne and Billy Fields *Transport Beyond Oil Policy Choices for a Multimodal Future*10.5822/978-1-59726-242-2_4© Island Press 2013

Maddison, D.J. (2003). The amenity value of the climate: the household production function approach. *Resource and Energy Economics*, **25(2)**, 155–175.

Maddison, D., Rehdanz, K. (2011). The impact of climate on life satisfaction. *Ecological Economics*, **70(12)**, 2437–2445.

Mäler, K.G. (1989) The acid rain game, in *Valuation, methods and policy making in environmental economics,* (Ed.) H. Folmer and E. Ireland, Chapter 12.

Mäler, K. G. (1990). International environmental problems. *Oxford Review of economic policy,* **6(1)**, 80-108.

Mendelsohn, R.O., Morrison, W.N., Schlesinger, M.E., Andronova, N.G. (2000a). Country-specific market impacts of climate change. *Climatic Change*, **45(3–4)**, 553–569.

Mendelsohn, R.O., Schlesinger, M.E.,Williams, L.J. (2000b).Comparing impacts across climate models. *Integrated Assessment*, **1(1)**, 37–48.

Miyoshi C., Mason K.J. (2013). The damage cost of carbon dioxide emissions produced by passengers on airport surface access: the case of Manchester Airport. *Journal of Transport Geography*, **28**, 137–143.

Newbery, D. (1990). Acid rain. *Economic Policy,* **11**, 288-346.

Nordhaus W.D. (1991). To slow or not to slow: the economics of the greenhouse effect. *Economic Journal*, **101(444)**, 920–937.

Nordhaus W.D. (1994a). Expert opinion on climate change. *American Scientist,* **82(1)**, 45–51.

Nordhaus W.D. (1994b). *Managing the Global Commons: The Economics of Climate Change*. The MIT Press, Cambridge.

Nordhaus W.D. (2006). Geography and macro economics: new data and new findings. *Proceedings of the National Academy of Science*, **103(10)**, 3510–3517.

Nordhaus W.D. (2008). *A Question of Balance: Weighing the Options on Global Warming Policies*. Yale University Press, New Haven.

Nordhaus W.D. (2011). The economics of tail events with an application to climate change. *Review of Environmental Economics and Policy*, **5(2)**, 240–257.

Perman R., Ma Y., Common M., Maddison D. and McGilvray J. (2011). *Natural Resources and Environmental Economics*. 4th Edition, Addison Wesley.

Rabl A., Spadaro J.V. and van der Zwaan B. (2005) . Uncertainty of Air Pollution Cost Estimates: To What Extent Does It Matter? Environmental Science and Technology, **39(2),** 399-408.

Rehdanz K. and Maddison D.J. (2005). Climate and happiness. *Ecological Economics,* **52(1)**, 111–125.

Roberts M. J. and Spence A. M. (1976). Effluent charges and licenses under uncertainty. *Journal of Public Economics,* **5,** 193-208.

Roughgarden T. and Schneider S. H. (1999). Climate change policy: quantifying uncertainties for damages and optimal carbon taxes. *Energy Policy,* **27*:*** 415-29.

Tol R.S.J. (1995). The damage costs of climate change toward more comprehensive calculations. *Environmental and Resource Economics*, **5(4)**, 353–374.

Tol R.S.J. (2002a). Estimates of the damage costs of climate change: part I: benchmark estimates. *Environmental and Resource Economics,* **21(1)**, 47–73.

Tol R.S.J. (2002b). Estimates of the damage costs of climate change: part II: dynamic estimates. *Environmental and Resource Economics*, **21(2)**, 135–160.

Tol R.S.J. (2013). Targets for global climate policy: An overview. *Journal of Economic Dynamics & Control,* **37**, 911–928.

Ulph A. and Ulph D. (1997). Global warming, irreversibility and learning. *The Economic Journal,* **107**, 636-50.

Weitzman M. L. (1974). Prices vs. Quantities. *Review of Economic Studies,* **41**, 477-91.

Weitzman M. L. (1978). Optimal rewards for economic regulation. *American Economic Review,* **68**, 683-91.