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Abstract

We show that the standard analysis of vertical relationships transposes directly to investment dynamics. Thus, when a firm undertaking a project requires an outside supplier (*e.g.*, an equipment manufacturer) to provide it with a discrete input to serve a growing but uncertain demand, and if the supplier has market power, investment occurs too late from an industry standpoint. The distortion in firm decisions is characterized by a Lerner-type index. Despite the underlying investment option, greater volatility can result in a lower value for both firms. We examine several contractual alternatives to induce efficient timing, a novel vertical restraint being for the upstream to sell a call option on the input. We also extend the model to allow for downstream duopoly. When downstream firms are engaged in a preemption race, the upstream firm sells the input to the first investor at a discount such that the race to preempt exactly offsets the vertical distortion, and this leader invests at the optimal time. These results are illustrated with a case study drawn from the pharmaceutical industry.

JEL classification: C73; D43; D92; L13

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1 Introduction

In dynamic models of irreversible investment under uncertainty, such as market entry or R&D, the investment cost (which constitutes the strike price of a so-called investment option) is often tacitly taken to reflect economic fundamentals closely. This assumption seems reasonable in industries such as real estate development, or when the investment is performed largely in-house, as may occur with R&D. However, there are many other cases in which a firm contemplating investment depends on an outside firm with market power to provide it with a discrete input (e.g., a key equipment) it needs to start producing and selling. Thus, a local hospital must decide when to buy diagnostic imaging equipment from an outside firm, an oil company that decides to drill offshore must acquire a platform from a specialized supplier, or an aeronautics firm will coordinate aircraft development with an engine manufacturer. In addition, strategic issues can arise if several firms seek to invest in an industry, and call upon the same supplier. To illustrate, at the end of the paper we outline the case of a market for a new vaccine, where demand is related to the diffusion of an emerging pathogen, and firms must invest in a factory constructed to exact specifications before starting operations.

This paper uses advances in irreversible investment and in duopoly investment games to build a model of vertical relationships in which the cost of a firm's investment is endogenous. Thus, our aim is to contribute in a growing research area that straddles industrial organization and corporate finance. We believe our key originality lies in the integration of two research streams that had seemed heretofore distinct: modern treatments of irreversible investment choices, as in Dixit and Pindyck [8], and the classic representation of vertical relationships as described, e.g., by Tirole [28]. Also, we extend this framework to include similar strategic specifications downstream to those of models by Smit and Trigeorgis [26], Mason and Weeds [20], and Boyer, Lasserre and Moreaux [2],[1] but with an upstream equipment supplier that prices with market power. The most closely related work we have identified is in corporate finance and studies the impact of agency on option exercise, most notably Grenadier and Wang [12] (corporate governance), and Lambrecht[16], Lambrecht and Myers [17] (takeovers).[2]

Specifically, we show that the standard analysis of vertical relationships translates directly to investment timing, with the level of investment trigger replacing price as the decision variable of

[1] For recent surveys of game theoretic real options models, see Boyer, Gravel, and Lasserre [3], and Huisman, Kort, Pawlina, and Thijssen [13].

[2] See also Lambrecht, Pawlina, and Teixeira [18] and Patel and Zavodov [22] for alternative approaches to real options in vertical structures, and Yoshida [31] for a discussion of the impact of strategic complementarity on investment timing.

the downstream firm. When an upstream supplier exercises market power, a vertical effect akin to double marginalization causes the downstream firm to unduly delay its investment relative to the optimal exercise threshold for the industry. This distortion increases with both market growth and volatility and decreases with the interest rate. The industry earns lower value under separation than under integration. In contrast with the standard real option framework, greater volatility decreases upstream and downstream firm value near the exercise threshold, because of the simultaneous presence of two effects: the option value of delay is balanced by a greater mark-up choice by the upstream firm.

The study of vertical relationships typically examines contractual restraints, by which an upstream firm can improve on a fixed input price. We verify that an upstream firm that can contract on the state of final demand achieves the integrated outcome, but also find that, provided demand volatility is low, a simple time-dependent pricing rule suffices to approximate the industry optimum. Alternatively, if spacing out payments is feasible, an option or downpayment restores efficiency. This latter explanation of use of restraints appears to rationalize existing practices in some industries, notably Airbus' approach to marketing aircraft.

Without such contractual alternatives, the upstream firm benefits from the presence of a second downstream firm, although this possibly occurs at the expense of aggregate industry value. We find that the race between downstream firms to preempt one another exactly balances the incentive to delay caused by the upstream firm's mark-up, so the leader invests at the optimal integrated threshold (as in the reference case with a single integrated firm), whereas the follower invests at the separation threshold (for duopoly profits), a type of "no distortion at the top" result. The leader receives a discounted price, and this discount increases with volatility and decreases with competition in the downstream product market. The comparison of industry value under different structures reveals that the three-firm industry structure may be more desirable than both bilateral monopoly (even if adding a second downstream firm decreases downstream industry profits) and preemption between vertically integrated firms (even if double marginalization induces firms to delay entry).

The remainder of the paper is organized as follows. In Section 2 we describe the model, with one upstream supplier and one downstream firm, and investigate the basic vertical distortion. This is done by comparing the equilibrium outcomes in the integrated case, which we use as a benchmark, with the outcomes of the separated case. In Section 3, we discuss contractual alternatives that aim to restore the industry optimum and relate them to an industry case. In Section 4, we introduce a second downstream firm and study equilibrium pricing and investment decisions, then compare with the outcomes under alternative industry structures. In Section 5, we illustrate the analysis by examining the case of an emerging market for a new vaccine. Section 6 concludes. All the proofs and derivations are in the appendix.

2 The Basic Vertical Distortion

Investment in a discrete input is necessary to operate on a final market. It can be produced and used by the same firm (integration), or produced by an upstream supplier and used by a downstream firm (separation). The cost of producing the input is positive and denoted by I. The flow profit resulting from investment is $Y_t \pi_M$ where π_M is the instantaneous monopoly profit per unit of Y_t, and $Y_t > 0$ is a scale parameter assumed to follow a geometric Brownian motion with drift, $dY_t = \alpha Y_t dt + \sigma Y_t dZ_t$. The non-negative parameters α and σ represent the market's expected growth rate (or "drift") and volatility, respectively, and Z_t is a standard Wiener process.[3] A lowercase $y = Y_t$ is used to denote the current level of the state variable, and it is assumed throughout the paper that the initial market size is positive and sufficiently small so firms prefer to delay rather than to invest immediately.[4] We let y_i denote a decision variable which is a threshold that, when attained by Y_t for the first time and from below at a stochastic future date, triggers the investment in the discrete input. The discount rate $r > \alpha$ is common to all firms.[5]

2.1 Integrated case

Suppose that a single firm produces the discrete input, is able to observe the current market size, and thus may decide at which future threshold to invest so as to earn the subsequent flow profit. Given the investment cost I and the current market size y, the value of a firm that decides to invest when the market reaches size $y_i \geq y$ is:

$$V\left(y, y_i, I\right) = \left(\frac{\pi_M}{r-\alpha} y_i - I\right)\left(\frac{y}{y_i}\right)^{\beta}, \tag{1}$$

where $\beta\left(\alpha, \sigma, r\right) \equiv \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$ is a function of parameters, referred to as β for conciseness, that occurs throughout the paper. The expressions of $V\left(y, y_i, I\right)$ in (1), and of β, are standard

[3]The geometric Brownian motion is derived from $Y_t = Y_0 \exp\left[\left(\alpha - \frac{1}{2}\sigma^2\right)t + \sigma Z_t\right]$ by using Itô's lemma.

[4]Specifically, we suppose that $Y_0 < \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M} I$, where β is a function of parameters defined in Section 2.1.

[5]A firm may delay investment indefinitely if $r \leq \alpha$.

in real option models (see Dixit and Pindyck [8], Chapter 5, or Chevalier-Roignant and Trigeorgis, Chapters 11-12).[6] We will use the property that β is decreasing in α and in σ, and increasing in r, throughout the paper.

The integrated firm's decision problem is $\max_{y_i \geq y} V(y, y_i, I)$. Since the objective is quasiconcave, differentiating (1) gives the value-maximizing investment trigger, $y^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M} I$, which serves as a benchmark throughout the analysis. The current value of the firm that invests at the optimal threshold y^* is:

$$V(y, y^*, I) = \frac{I}{\beta - 1}\left(\frac{y}{y^*}\right)^{\beta}. \tag{2}$$

2.2 Separated case

Suppose that the input production and investment decisions are made by distinct firms. In this case a vertical externality arises. The following assumptions are made in order to describe this externality simply and distinctly. First, the upstream firm, as an input producer on the intermediate market, does not observe the state of the system (the downstream market size y) at any date, including $t = 0$. However it knows the structural parameters of the demand process. Its only choice consists of the input price $p_S \geq I$ (thereby determining the terms of the downstream firm's investment option). The input price is taken to be constant, although the upstream may generally prefer to have its price increase over time in order to hasten downstream investment (see Section 3.2). Second, the downstream firm is assumed to be a price-taker in the intermediate market.[7] Given p_S, it observes the current size of the final market, and decides at which threshold y_i to invest. To establish the equilibrium in (y_i, p_S) we begin with the downstream firm's optimization problem.

The value of a downstream firm that decides to invest when the market reaches size y_i, given the investment cost p_S and the current market size y, is:

$$V(y, y_i, p_S) = \left(\frac{\pi_M}{r - \alpha} y_i - p_S\right)\left(\frac{y}{y_i}\right)^{\beta}, \tag{3}$$

all $y \leq y_i$. The separated firm's decision problem is $\max_{y_i \geq y} V(y, y_i, p_S)$, and the associated value-maximizing investment trigger is $y_S(p_S) = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M} p_S$, which is increasing in p_S, with $y_S(I) = y^*$.

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[6] The term $\left(\frac{y}{y_i}\right)^{\beta}$ in (1) reads as the expected discounted value, measured when $Y_t = y$, of receiving one monetary unit when Y_t reaches y_i for the first time. In the certainty case $\sigma = 0$, we have $\beta = \frac{r}{\alpha}$ and $\left(\frac{y}{y_i}\right)^{\beta} = e^{-r(t_i - t)}$, which is the standard continuous time discounting term.

[7] As in Tirole [28] it is "for simplicity" that we "assume that the manufacturer chooses the contract" (p. 173).

That is, when it is charged the true input cost, the downstream firm invests at the same trigger as the integrated firm.

At the current market size y, the upstream firm's value is:

$$W(y, p_S) = (p_S - I)\left(\frac{y}{y_S(p_S)}\right)^{\beta}, \tag{4}$$

all $y \leq y_S$. Given $y_S(p_S)$, the upstream firm's decision problem is $\max_{p_S} W(y, p_S)$, leading to the optimal price which is to set $p_S^* = \frac{\beta}{\beta-1}I$. In what follows, let $y_S^* \equiv y_S(p_S^*)$. We find:

Proposition 1 *In the separated case, the optimum investment trigger and input price are:*

$$y_S^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_M} I \text{ and } p_S^* = \frac{\beta}{\beta-1} I. \tag{5}$$

Substituting back (5) into (3-4), we obtain the firm values under separation:

$$V(y, y_S^*, p_S^*) = \frac{\beta I}{(\beta-1)^2}\left(\frac{y}{y_S^*}\right)^{\beta} \text{ and } W(y, p_S^*) = \frac{I}{\beta-1}\left(\frac{y}{y_S^*}\right)^{\beta}. \tag{6}$$

From (6) we obtain that $\frac{V(y, y_S^*, p_S^*)}{W(y, p_S^*)} = \frac{\beta}{\beta-1}$, implying that the downstream value is higher than the upstream's. Using (2) we find that:

$$\frac{V(y, y_S^*, p_S^*) + W(y, p_S^*)}{V(y, y^*, I)} = (2\beta - 1)\frac{(\beta-1)^{\beta-1}}{\beta^{\beta}}, \tag{7}$$

with $f(\beta) \equiv (2\beta - 1)\frac{(\beta-1)^{\beta-1}}{\beta^{\beta}} \in \left(\frac{2}{e}, 1\right)$.[8] Hence the industry value is lower under separation than under integration, as is to be expected.

The decision problem is depicted in Figure 1 for specific parameter values ($\beta = 2$, $I = \frac{\pi_M}{r-\alpha} = 1$, and $y = 1$). The downstream isovalue curves are concave in (y_i, p_S) space, and the dashed line is the locus of optimal responses to given upstream prices, $y_S(p_S)$. For example, if the input is priced at cost, the downstream firm's value-maximizing investment trigger is $y^* = 2$. The dashed line effectively constitutes the constraint for the upstream firm's optimization problem. The upstream isovalue curves are convex in (y_i, p_S) space (the ordering of the curves follows from the monotonicity of the value functions V and W in p_S). Because p_S^* maximizes $W(y, p_S)$, the point (y_S^*, p_S^*) lies at a tangency of

[8] The expression $f(\beta)$ occurs several times in the paper and is characterized in Appendix A.2.

an upstream isovalue with the locus $y_S(p_S)$. The separated outcome is reached at $(y_S^*, p_S^*) = (4, 2)$ (point A), yielding $V(1, y_S^*, p_S^*) = \frac{1}{8}$ and $W(1, p_S^*) = \frac{1}{16}$. The industry value under separation is lower than the integrated value, $V(1, y^*, I) = \frac{1}{4}$. We return in Section 3 to the gains that firms achieve by moving to the contract curve.

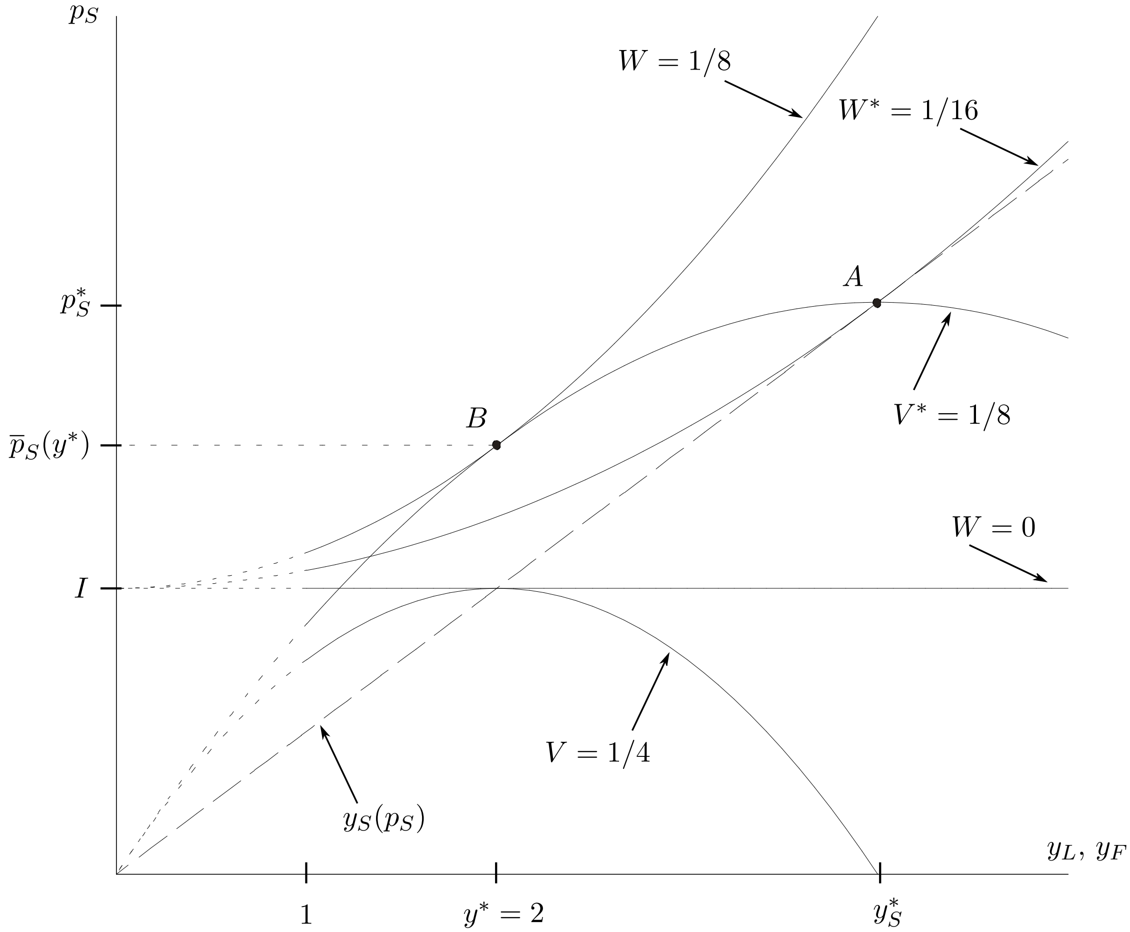

Figure 1: Upstream and downstream isovalues ($\beta = 2$, $y = I = \frac{\pi_M}{r-\alpha} = 1$). Point A describes the separated equilibrium, in which the upstream firm charges $p_S^* = 2$, and the downstream firm enters at $y_S^* = 4$. Points B describes a joint-value maximizing contract (see Section 3) in which the upstream firm chooses the investment level $y^* = 2$ and charges the input price $\overline{p}_S(y^*) = 1.5$ in order to maximize its own value under the constraint that the downstream firm earns no less than $V^* = \frac{1}{8}$.

2.3 The vertical distortion

Dixit, Pindyck, and Sødal [9] observe that there is a formal analogy between the real option model presented in Section 2.1 and monopoly pricing with isoelastic demand. The isomorphism is obtained by taking $Q = aP^{-b}$ as demand, and a constant marginal cost of production c, with $P \equiv y_i$, $a \equiv \frac{\pi_M}{r-\alpha} y^\beta$, $b \equiv \beta$, $c \equiv \frac{r-\alpha}{\pi_M} I$. Thus, the optimal investment rule of the integrated firm ($y^* = \frac{\beta}{\beta-1} \frac{r-\alpha}{\pi_M} I$) has the same form as a monopoly price. This analogy extends to the separated case that we have introduced in Section 2.2, and is useful in order to understand the equilibrium. In comparison with the baseline model of vertical externality[9], the investment trigger substitutes for the final price as the downstream decision variable so that the model is formally similar to the model of double marginalization.

The vertical externality may be gauged as follows. First note that both the input price and the investment trigger are greater in the separated case than under integration ($\frac{p_S^*}{I} = \frac{y_S^*}{y^*} = \frac{\beta}{\beta-1}$).[10] Then, for the upstream and downstream firm decisions, we can define the following magnitudes:

$$L_p \equiv \frac{p_S^* - I}{p_S^*} = \frac{1}{\beta}, \qquad L_y \equiv \frac{y_S^* - y^*}{y_S^*} = \frac{1}{\beta}. \tag{8}$$

The expressions (8) have a similar form and interpretation as the Lerner index, which is generally taken to measure market power. As noted above, β plays the same role as the elasticity of demand, and fully characterizes L_p and L_y. As with double marginalization (where a more elastic demand results in more competitive pricing), here a higher β results in more efficient input pricing and more timely downstream entry. The degree of distortion therefore increases with a higher growth rate or volatility, and decreases with a higher interest rate.

Note that the industry value under separation relative to integration, $f(\beta)$ (see (7) above), decreases with β. Thus, although a higher α or σ or a lower r result in greater distortion in decisions, they are also associated with less distortion in payoffs compared with integration. This is surprising in appearance only, and similar contrasting effects exist in the successive monopolies model with constant elasticity demand (with a highly inelastic demand, large distortions in decisions need not result in large distortions in payoffs). To summarize:

[9] See Tirole [28] for a description of the externality identified by Spengler [27].

[10] In a model of hostile takeovers with sequential decisions, Lambrecht [16] derives a very similar result to the one presented here. Notably, the expression for the additional mark-up on the target firm's value has an analogous form (merger timing is optimal, whereas hostile takeovers occur inefficiently late).

Proposition 2 *The industry value is lower under separation than under integration. The distortion in firm decisions, as measured by L_p and L_y, is increasing in market growth rate and volatility and decreasing in the interest rate, whereas the distortion in separated and integrated payoffs is decreasing in market growth rate and volatility and increasing in the interest rate.*

2.4 Sensitivity analysis

For the sensitivity analysis we successively consider changes in the interest rate, the growth rate and volatility. Most parameters have effects that are typical for a real options framework, but there are two notable exceptions. These exceptions arise because of the simultaneous presence of option effects and the vertical distortion. Thus, an increase in the interest rate has an ambiguous effect on the investment trigger, and an increase in volatility can reduce (rather than increase) firm value.

□ *Interest rate.* With respect to the decision variables, we find first $\frac{dp_S^*}{dr} < 0$. The effect on the investment trigger is ambiguous. To see why, note that $\frac{dy_S^*}{dr} = \frac{\partial y_S^*}{\partial r} + \frac{\partial y_S^*}{\partial p_S}\frac{dp_S^*}{dr}$, so that a change in the interest rate has a direct and an indirect effect. All else equal, a higher interest rate leads the downstream firm to delay its investment ($\frac{\partial y_S^*}{\partial r} > 0$), but also results in a lower input price ($\frac{dp_S^*}{dr} < 0$), which in turn lowers the investment threshold. The latter effect can dominate, in particular when the interest rate is sufficiently high.

As for firm values, letting $V^* \equiv V(y, y_S^*, p_S^*)$ and $W^* \equiv W(y, p_S^*)$, we find the following elasticity expressions:

$$\varepsilon_{V^*/r} = r\left(-\frac{\beta}{r-\alpha} + \left(\ln\frac{y}{y_S^*} + \frac{1}{\beta}\right)\frac{d\beta}{dr}\right) < \varepsilon_{W^*/r} = r\left(-\frac{\beta}{r-\alpha} + \left(\ln\frac{y}{y_S^*} + \frac{1}{\beta-1}\right)\frac{d\beta}{dr}\right) < 0, \quad (9)$$

for all $y \leq y_S^*$. Consider the first of these. A change in the interest rate affects the value of the downstream option, both directly and indirectly through the input price, so that $\frac{dV^*}{dr} = \frac{\partial V^*}{\partial r} + \frac{\partial V}{\partial p_S}\frac{dp_S^*}{dr}$ (by the envelope theorem, $\frac{\partial V}{\partial y_S}$ drops out of this expression). In this case the negative direct (or option) effect dominates the positive indirect (or vertical) one. The reasoning for the upstream firm is formally similar.[11]

□ *Growth.* The effect on the decision variables are $\frac{dp_S^*}{d\alpha} > 0$ and $\frac{dy_S^*}{d\alpha} > 0$. For firm values, we find:

$$\varepsilon_{V^*/\alpha} = \alpha\left(\frac{\beta}{r-\alpha} + \left(\ln\frac{y}{y_S^*} + \frac{1}{\beta}\right)\frac{d\beta}{d\alpha}\right) > \varepsilon_{W^*/\alpha} = \alpha\left(\frac{\beta}{r-\alpha} + \left(\ln\frac{y}{y_S^*} + \frac{1}{\beta-1}\right)\frac{d\beta}{d\alpha}\right) > 0, \quad (10)$$

[11] Strictly speaking, only the downstream firm receives an option value. The upstream firm's value results from dynamic optimization, but is not an option value in the regular sense.

for all $y \leq y_S^*$. Here $\frac{dV^*}{d\alpha} = \frac{\partial V^*}{\partial \alpha} + \frac{\partial V}{\partial p_S}\frac{dp_S^*}{d\alpha}$. A change in the growth rate results in a positive direct (or option) effect, and in a smaller negative indirect (or vertical) effect. Again, the effect on the upstream firm's value is similar.

□ *Volatility.* The effect of volatility on the decision variables is unambiguous, with $\frac{dp_S^*}{d\sigma} > 0$ and $\frac{dy_S^*}{d\sigma} > 0$. However, the effect of volatility on firm values is ambiguous. We find:

$$\varepsilon_{V^*/\sigma} = \sigma\left(\ln\frac{y}{y_S^*} + \frac{1}{\beta}\right)\frac{d\beta}{d\sigma}, \qquad \varepsilon_{W^*/\sigma} = \sigma\left(\ln\frac{y}{y_S^*} + \frac{1}{\beta - 1}\right)\frac{d\beta}{d\sigma}, \tag{11}$$

with $\varepsilon_{V^*/\sigma}$ positive (zero) if and only if $y < (=) y_S^* \exp\left(-\frac{1}{\beta}\right) \equiv \check{y}^V$, and $\varepsilon_{W^*/\sigma}$ positive (zero) if and only if $y < (=) y_S^* \exp\left(-\frac{1}{\beta-1}\right) \equiv \check{y}^W$ (note that $\check{y}^W < \check{y}^V < y_S^*$ for all $\beta > 1$). Thus, and in contrast with typical real option models, firm value *decreases* with volatility near the investment threshold.

To see this better, consider the value of the upstream's investment option. Similarly to above, the effect of a change in volatility consists of a direct (or option) effect and an indirect (or vertical effect):

$$\frac{dV^*}{d\sigma} = \underbrace{\frac{\partial V^*}{\partial \sigma}}_{\text{option effect } (>0)} + \underbrace{\frac{\partial V}{\partial p_S}\frac{dp_S^*}{d\sigma}}_{\text{vertical effect } (<0)}. \tag{12}$$

When volatility changes, the vertical effect in elasticity terms (equal to $(1/\sigma\beta)(d\beta/d\sigma)$) is constant over time, whereas the magnitude of the option effect (equal to $(\ln(y/y_S^*)/\sigma)(d\beta/d\sigma)$) depends on the current market size y. The option effect dominates at low market sizes, but as demand evolves and the upstream firm nears its investment threshold, option value is relatively less important. Rather, the firm's payoff is then more sensitive to the vertical pricing distortion (which is increasing in volatility). Figure 2 illustrates the behavior of $V(y, y_S^*, p_S^*)$ over $[0, y_S^*]$ for several levels of β.

Moreover, a similar ambiguity exists for the effect of volatility on upstream value, although the market size thresholds at which $\varepsilon_{W^*/\sigma}$ turns negative is lower. A corollary is that there exists a range of market sizes, $(\check{y}^V, \check{y}^W)$, over which the two firms have divergent preferences with respect to volatility (greater volatility lowers upstream value and raises downstream value in this range). We thus have:

$$\begin{cases} 0 \leq \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} & \text{if } y \leq \check{y}^W; \\ \varepsilon_{W^*/\sigma} < 0 < \varepsilon_{V^*/\sigma} & \text{if } \check{y}^W < y < \check{y}^V; \\ \varepsilon_{W^*/\sigma} < \varepsilon_{V^*/\sigma} \leq 0 & \text{if } \check{y}^V \leq y. \end{cases} \tag{13}$$

The following proposition summarizes the sensitivity results.

Proposition 3 *In the separated case, the impact of the interest rate and the demand process parameters on firm decisions and values are as follows:*

(i) the investment threshold (y_S^*) increases with market growth and volatility, whereas the effect of the interest rate is ambiguous. The input price (p_S^*) increases with market growth and volatility, and decreases with the interest rate.

(ii) the upstream and downstream values at optimum (W^* and V^*) are increasing in market growth and decreasing in the interest rate. Greater volatility raises firm values when the market size is low enough, and lowers firm values when market size approaches the investment threshold.

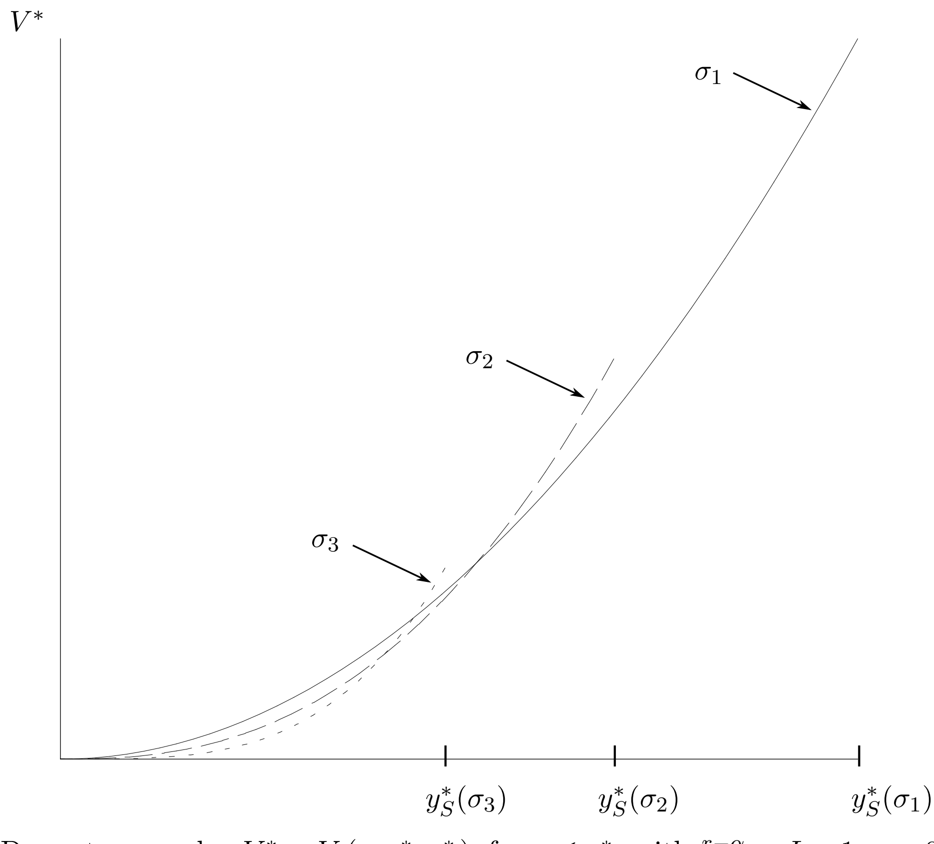

Figure 2: Downstream value $V^* = V\left(y, y_S^*, p_S^*\right)$, for $y \leq y_S^*$, with $\frac{r-\alpha}{\pi_M} = I = 1$, $r = 0.2$, $\alpha = 0.05$ and $\sigma_1 > \sigma_2 > \sigma_3$ such that $\beta(\sigma_1) = 2$ (solid), $\beta(\sigma_2) = 2.5$ (dash), $\beta(\sigma_3) = 3.5$ (dots). For large initial market sizes, greater uncertainty (i.e., a lower β) reduces firm value.

Together, Propositions 1, 2, and 3 describe a vertical externality arising with a downstream demand evolving stochastically over time. The suboptimality feature of the separated case results from a dynamic analog to a well-known static economic effect (double marginalization), and can occur whenever an input supplier with market power distorts the cost of the input, i.e. the strike price of the downstream investment option, independently of the specifications of the model. To readers versed in industrial organization, this intuitive result should come as no surprise, although we are not aware of other work that draws this parallel. Moreover, the specification that is presented here allows us to offer new insights on some aspects of supplier relationships, such as the relative weight of option effects and vertical distortion.

3 Contractual Alternatives

It is well-established in industrial organization that various contractual alternatives or vertical restraints such as resale price maintenance, quantity discounts, and two-part tariffs, can allow a separated structure to realize the integrated profit. Similar contractual mechanisms apply in the dynamic setting, albeit with differences in implementation and interpretation. For simplicity, throughout the section we fix the current time to be $t = 0$, assume that it is the upstream firm that sets the contract which the downstream firm can accept or reject, and that such a contract is enforceable. Moreover, the upstream firm cannot credibly commit *not* to sell the input at p_S^* at a future date when the trigger y_S^* is reached. Thus, unless otherwise specified (see Section 3.2), the outside option of the downstream firm is the value that it would realize in the separated case described in Section 2.2, which we denote by $V_0(y) \equiv V(y, y_S^*, p_S^*)$. Also, the contractual alternatives we examine require that the upstream firm has some information about downstream demand. There are two polar cases, depending on whether it observes the value of downstream demand at all times (Y_t, all t), or only at one point in time (Y_0). Three contractual alternatives appear especially noteworthy.

3.1 State-dependent pricing

Suppose that the upstream firm continuously and verifiably observes the state of downstream demand. It can then specify both the investment trigger and the input price, in a contract analogous to resale price maintenance in the static vertical framework. By dictating the trigger that maximizes industry value ($y_i = y^*$), the upstream firm can appropriate all the benefits above the downstream firm's reservation value. All that remains is to identify the input price $p_S(y_i)$. Formally, for any $y \le y^*$, and by slightly abusing notation to include the downstream investment trigger y_i as an argument of the function W,[12] the upstream firm's problem is:

$$\max_{y_i, p_S} W(y, y_i, p_S) \qquad \text{s.t.} \quad V(y_i, y_i, p_S) \ge V_0(y_i). \tag{14}$$

The downstream participation constraint in (14) determines a maximum input price $\overline{p}_S(y_i)$ such that, at the time the contract investment trigger y_i is reached, the downstream firm prefers to invest immediately rather than waiting until y_S^* is reached and investing then at the price p_S^* described in Section 2.2 (with $\overline{p}_S(y_i)$ defined by $V(y_i, y_i, \overline{p}_S(y_i)) = V(y_i, y_S^*, p_S^*)$). Total value maximization by the upstream firm yields an optimal contract $(y_i^*, p_S^*(y_i^*)) = (y^*, \overline{p}_S(y^*))$ with $\overline{p}_S(y^*) = \frac{\beta}{\beta-1}\left(1 - \frac{(\beta-1)^{\beta-1}}{\beta^\beta}\right) I$ (with an infinite input price at all other times than when the trigger y^* is reached for the first time).[13]

State-dependent pricing can be readily illustrated, as in Figure 1 above. With the parameter values of the figure ($\beta = 2$, $y = I = \frac{\pi_M}{r-\alpha} = 1$), the input supplier chooses $y_i = y^* = 2$. The participation constraint in (14) reduces to $p_S \le \frac{3}{2}$, so the upstream firm charges $p_S^* = \overline{p}_S(2) = \frac{3}{2}$. Point B describes the optimal contract.

3.2 Dynamic pricing rule[14]

Even if it is unable to practice state dependent pricing, an upstream firm can still improve on a fixed price contract if it has some information regarding the state of the stochastic process. Suppose that although it cannot observe and contract on the state y for $t > 0$, the upstream firm observes and can base its contract on Y_0 by proposing a price schedule $P_S(t)$ that makes the input price grow over time.[15] Put informally, such pricing is effective because it counteracts the downstream firm's tendency to delay investment. We show that when volatility is low enough, the upstream firm approaches the integrated value with a simple constant growth rate pricing rule.

[12] That is, we define $W(y, y_i, p_S) = (p_S - I)\left(\frac{y}{y_i}\right)^\beta$, all $y \le y_i$.

[13] As we assume for simplicity that bargaining power is distributed so the upstream captures all additional surplus, we can omit the upstream participation constraint $W(y, y_i, p_S) \ge W(y, y_S^*, p_S^*)$.

[14] We are grateful to a referee for suggesting this contractual alternative to us.

[15] The characterization of an optimal price schedule is a complex enough dynamic agency problem to be beyond our scope here. For a general treatment of this question, the reader may refer to Kruse and Stack [15], who show that the first-best outcome for the industry can in fact be implemented with a time-varying transfer.

Assume that the upstream firm uses a pricing rule of the form $P_S(t) = P_0 e^{\gamma t}$, where P_0 and γ are non-negative and chosen at $t = 0$. In order for the downstream option to be well-defined, we suppose that the rate of growth is capped ($\gamma < r$). Where there is no danger of confusion, we denote the current price at time t by p_S. Setting such a contract requires determining the initial level P_0, and therefore that the upstream observes the market state at the time the pricing policy is chosen. Moreover, the upstream firm is assumed to have a form of myopic behavior in the sense that it sets the pricing rule at $t = 0$ and does not subsequently revise it (e.g., it does not learn from the downstream's behavior over time). Finally, in contrast with the other contractual alternatives studied in this section, the downstream firm's reservation value is set to zero.[16]

Under these assumptions, the solution of the dynamic pricing problem runs as follows (see Section A.4 of the appendix for the full derivation). To begin with, the downstream's investment decision (faced with the input price rule p_S) has a known form[17] that is very similar to the one described in Section 2.2. Given the input price growth rate (γ) set by the upstream firm, the optimal exercise policy depends on the ratio of market size to input price $\widehat{y} \equiv \frac{y}{p_S}$, and consists of investing when this ratio reaches a trigger $\widehat{y}_S^* = \frac{\widehat{\beta}}{\widehat{\beta}-1}\frac{r-\alpha}{\pi_M}$ where $\widehat{\beta} \equiv \beta(\alpha-\gamma, \sigma, r-\gamma)$.[18] It therefore either invests immediately if $\widehat{y} \geq \widehat{y}_S^*$, or waits until the trigger $\widehat{y}_S^*$ is reached. A higher growth rate in the input price reduces the deterministic advantage to waiting and hastens investment ($\frac{d\widehat{y}_S^*}{d\gamma} < 0$). As a result, the upstream's payoff at the decision time ($t = 0$) can be shown to be:

$$\widehat{W}(P_0, \gamma) = \begin{cases} P_0^{-(\widehat{\beta}-1)} \left(\frac{Y_0}{\widehat{y}_S^*}\right)^{\widehat{\beta}} - I P_0^{-\widehat{\beta}'} \left(\frac{Y_0}{\widehat{y}_S^*}\right)^{\widehat{\beta}'} & \text{if } Y_0/P_0 < \widehat{y}_S^*, \\ P_0 - I & \text{if } Y_0/P_0 \geq \widehat{y}_S^*, \end{cases} \tag{15}$$

where $\widehat{\beta}' \equiv \beta(\alpha-\gamma, \sigma, r)$. Note that in (15), $\widehat{\beta}$ (hence $\widehat{y}_S^*$) as well as $\widehat{\beta}'$ are functions of the decision variables P_0 and γ. The upstream firm's optimization problem is thus:

$$\max_{P_0,\gamma} \widehat{W}(P_0, \gamma) \qquad \text{s.t.} \quad V(\widehat{y}, \widehat{y}_S^*, p_S) \geq 0, \tag{16}$$

[16] The stationary price studied in Section 2.2 is a special case of the pricing rule ($\gamma = 0$). A downstream firm that does not purchase the input at $P_S(t)$ is assumed not to purchase at all, whereas a downstream firm that rejects the other contractual alternatives of this section (state-dependent price, option on the input) presumably can purchase the input at p_S^*.

[17] See McDonald and Siegel [21] and Dixit and Pindyck [8], Chapter 6.

[18] It is ratio y/p_S that is relevant for the optimal investment decision here because the value of the downstream's investment option is homogeneous of degree one in market size and input price (y, p_S).

where the constraint binds in the deterministic case but not generally.

As $\widehat{W}\left(P_0, \gamma\right)$ is continuous in σ and in its arguments, when volatility is low (as $\sigma \downarrow 0$), the optimal constant-growth pricing rule p_S^* converges to the solution of the deterministic case ($\sigma = 0$), as well as to the state-dependent pricing rule. Moreover, in the deterministic case, the optimal pricing scheme $(P_0^*, \gamma^*) = \left(\frac{\pi_M}{r-\alpha} Y_0, \alpha\right)$ achieves the investment decision and outcome of the integrated firm. Thus, when the scale parameter process is not too volatile, an input price that increases at a rate of about α results in investment timing near the optimal threshold.

3.3 Option (or downpayment) on the input

A third contractual alternative is a policy that is reminiscent of a two-part tariff. As in Section 3.2, the upstream firm is assumed to observe the initial state of the demand process, Y_0, which need not be verifiable here. However, it is not constrained to a spot transaction, and may collect a payment both at the time of contracting and when the input is delivered. Then, the integrated value is realized by means of an up-front option offered to the downstream firm on the specific input at a suitable exercise price, p_S. We know from Section 2.2 that the input buyer maximizes its private value by exercising the option when Y_t reaches $y_S\left(p_S\right)$. The objective of the upstream supplier involves inducing the choice of the efficient investment trigger by the input buyer, and appropriating the value in excess of the downstream reservation level, which we assume to be the value from the separated case, i.e. $V_0(y)$. This is done through an initial transfer payment, t_S, which we interpret as the option premium, and which also corresponds to a non-refundable deposit on the specific input.

Formally, the upstream problem is then:

$$\max_{p_S, t_S} W\left(y, p_S\right) + t_S \qquad \text{s.t.} \quad V\left(y, y_S\left(p_S\right), p_S\right) - t_S \geq V_0(y), \tag{17}$$

where the downstream participation constraint in (17) determines an upper bound on the transfer payment. With the joint-value maximizing input price $p_S^* = I$, the downstream firm chooses to invest at $y_S\left(I\right) = y^*$, and value maximization by the upstream yields a transfer payment $\bar{t}_S^*(y)$ such that the downstream firm's participation constraint in (17) is exactly satisfied.[19] The optimal option contract

[19] As we assume the bargaining power is distributed so that the upstream firm captures all the surplus, the participation constraint of the upstream firm ($W\left(y, p_S\right) + t_S \geq W\left(y, p_S^*\right)$) can be omitted. Otherwise, $\bar{t}_S^*$ constitutes an upper bound on admissible option premiums.

is $\left(p_S^*, \bar{t}_S^*(Y_0)\right) = (I, V(Y_0, y^*, I) - V_0(Y_0))$, which results in the integrated outcome.[20] As with the dynamic pricing problem discussed above, note that some information about the demand state at the contracting date (Y_0) is necessary in order to write the contract.

The aircraft industry provides an illustration of the results here and in Section 2. In Antikarov and Copeland [1], the case of Airbus Industrie is carefully narrated by one of its marketing directors, John Stonier. To begin with, manufacturers explicitly market option contracts to their customers, who otherwise have a natural deferral option. In addition, aircraft manufacturers are economically sophisticated and know that their customers differ with respect to the volatility of their revenue streams (cf. p. 39, "The options were more valuable to some airlines than to others, and we could segment the market in this way"). The marketing approach adopted by Airbus can thus be rationalized by appealing to the dynamic vertical externality we have identified. The sale of options to customers whose revenue stream exhibits greater volatility (those for whom the deferral option is more valuable) is consistent with Proposition 2. It is exactly when volatility is high that the distortion associated with dynamic double marginalization is greatest, therefore yielding more incentive to provide contractual alternatives.

3.4 Example and synthesis

To illustrate, suppose that parameters are set as in Figure 1, (i.e., $\beta = 2$ and $I = \frac{\pi_M}{r-\alpha} = 1$), so the efficient investment trigger for the industry is $y^* = 2$. Moreover, the current (initial) market size is assumed to be $Y_0 = 1$. Recall that the integrated value is $V(1, y^*, I) = 1/4$. The different contractual alternatives described in the section are then:

☐ *State-dependent pricing.* The efficient contract is straightforward to identify: $(y^*, \bar{p}_S(y^*)) = (2, 1.5)$. Note that this contractual alternative requires that the input supplier have more information than in the two other contractual alternatives we study, as it must monitor the state of demand continuously until the trigger y^* is reached.

☐ *Dynamic pricing rule.* Keeping $\beta = 2$ as in the rest of the examples, we fix $\alpha = 0.04995$, $\sigma = 0.01$, and $r = 0.1$. The optimal pricing scheme is then $(P_0^*, \gamma^*) \simeq (0.99, 0.045)$ (note that the initial price

[20] The relative option premium (as a share of industry profit) is a function of β only: $\bar{t}_S^*(y)/V(y, y^*, I) = 1 - \left(\frac{\beta-1}{\beta}\right)^{\beta-1}$. Note that $\bar{t}_S^*(y)/V(y, y^*, I)$ is decreasing in σ.

is lower than the investment cost I in this case). This results in firm values $\widehat{W}(\widehat{y}, p_S) \simeq 0.18$ and $\widehat{V}(\widehat{y}, \widehat{y}^*, p_S) \simeq 0.04$ whose sum is between the industry values under integration and separation (3/16).

☐ *Option on input.* The efficient contract is: $\left(p_S^*, \bar{t}_S^*(Y_0)\right) = (1, 0.125)$ where the first term is either interpreted as a strike price or a payment upon completion, and the second as an option price or as a downpayment. Of the alternatives that we examine, this contractual alternative has a lower informational requirement (Y_0) and achieves the efficient outcome.

The following proposition summarizes our results concerning contractual alternatives.

Proposition 4 *Suppose that the upstream firm chooses the contract and has some information regarding market size. Then, if the upstream firm can observe and contract on the state Y_t, it induces efficient investment timing and the integrated value. If the upstream firm can only observe the initial demand state Y_0, then:*

(i) it improves upon constant pricing by using a constant growth rate rule $P(t) = P_0 e^{\gamma t}$ and, as volatility approaches zero, it approaches efficient investment timing and the integrated value;

(ii) by selling an option (downpayment) on the input with strike price I it induces efficient investment timing and the integrated value.

As has been noted, these alternatives are similar to known vertical restraints in industrial economics. When such alternatives are not available to the upstream firm, the presence of downstream competition may act as a substitute. The earlier investment implied by the resulting race to preempt in the downstream industry actually fully counteracts the double marginalization distortion for the first firm that invests.

4 Downstream Duopoly with an Upstream Supplier

In this section we build on the analysis of Fudenberg and Tirole [10] (preemption), and (*inter alia*) Smets [25], Grenadier [11], Weeds [29], Mason and Weeds [20], Boyer, Lasserre, and Moreaux [2], Huisman, Kort, and Thijssen [14] (preemption under uncertainty)[21], by integrating the upstream decision and a downstream preemption race.

[21] A comprehensive discussion of these contributions can be found in Chevalier-Roignant and Trigeorgis [7].

We have established above that, in the separated case, and in the absence of sufficient contractual instruments, investment occurs too late from an industry standpoint. However it is well known that in preemption races the first firm invests too early, leading to rent dissipation. In this section, we study the interaction of these two effects. In particular, we show that the race to preempt downstream can counteract the double marginalization distortion, and thereby functions as a substitute for the vertical restraints examined in Section 3.

The structural assumptions are those of Section 2.2, except that on the intermediate market the upstream firm faces not one but two potential downstream buyers, that decide non-cooperatively when to enter the final market. Thus, as in Section 2 we once again suppose that the upstream firm does not observe Y_t at any date and does not practice the contractual alternatives described in Section 3. Moreover, the upstream is assumed not to learn from the decisions of downstream firms.[22] However, it may charge different prices at different dates (intertemporal price discrimination is allowed).

With these assumptions the flow profits depend on the number of active firms. Now Y_t describes an industry-wide shock, and flow profits are $Y_t\pi_M$ (monopoly profit) if a single firm has entered the downstream market, and $Y_t\pi_D$ (duopoly profit) for each firm if both have invested. We assume $0 < \pi_D \leq \pi_M$, reflecting either the usual case ($\pi_D < \pi_M/2$) or strong spillovers/complementarities ($\pi_D \geq \pi_M/2$). A key parameter in this section is the ratio π_M/π_D, which we take as a measure of competition in the product market, and which is also viewed as an indicator of first-mover advantage in the downstream investment game (Pawlina and Kort [23]).

4.1 Equilibrium of the pricing and entry game

The main result of the section is the characterization of the Nash equilibrium of the pricing and entry game played by the upstream and downstream firms. In duopoly entry games, two types of equilibrium may arise, namely one in which firms invest sequentially (the "preemption" equilibrium, where firms are equally likely to invest first), and equilibria in which firms invest simultaneously. However in our model, only equilibria of the first type arise (see proof of Proposition 5 below), so we do not discuss the second type any further here.

[22] That is, it does not infer Y_t from downstream entry decisions, either because it has myopic behavior, or because it observes only the degree of competition in the downstream market (as measured by the ratio π_M/π_D) and not the downstream flow profit levels.

The upstream firm's strategy consists of two prices p_L and p_F. This is because it is assumed to condition the spot price on its information in the intermediate product market (the number of inputs demanded). In what follows p_L denotes the spot price charged to the first firm to invest (the "leader"), and p_F denotes the spot price for the second firm (the "follower"). Thus, the choice of input price is closed loop (without commitment as to future prices), and the upstream firm charges a constant price over time until the first entry occurs, and another constant price over time thereafter.

Downstream firms are assumed to be price takers in the intermediate market. Their strategic space therefore effectively consists of two entry thresholds y_P and y_F. Here y_P denotes the preemption trigger, which is the market size at which the first of the two firms enters, and y_F denotes the follower's investment trigger at which the second firm enters. The dynamics of industry structure downstream therefore consist of three successive phases, namely no firm, monopoly starting when market size reaches y_P for the first time, and duopoly after it hits y_F. The reader may note that the strategies we describe are shorthand for the "simple" strategies defined in Fudenberg and Tirole [10], and more recently by Huisman, Kort, and Thijssen [14], although as such they are sufficient for the equilibrium characterization that we provide.[23]

Our assumptions are such that everything happens as if firms played the following four-stage game:

- Stage 1: the upstream firm chooses the first input price (p_L);
- Stage 2: downstream firms observe p_L and engage in a preemption race that determines the first entry threshold (y_P);
- Stage 3: the upstream firm chooses the second input price (p_F);
- Stage 4: the remaining downstream firm observes p_F and chooses the second entry threshold (y_F).

In order to determine the equilibrium of the pricing and entry game, it suffices to determine the input prices (p_L, p_F) and the two investment triggers (y_P, y_F). In preemption equilibrium, the identity

[23] The full specification of the investment game is rather technical, but this is necessary in order to properly deal with "ties" in which both downstream firms seek to invest simultaneously. Thus, the true strategy space of downstream firms consists of a pair of real-valued functions describing both investment threshold and investment "intensity" (see Huisman, Kort, and Thijssen [14]). However, this does not have a direct bearing on the form of the equilibrium investment thresholds derived in our model.

of the leader and follower are indeterminate, in that either firm effectively invests first, with equal probability.

Proceeding by backward induction, once the leader has invested, the subgame between the upstream firm and the follower is identical to that in Section 2.2. When the current market size is y, the value of a follower that invests at a threshold $y_F \geq y$ and pays a price p_F is:

$$F(y, y_F, p_F) = \left(\frac{\pi_D}{r-\alpha} y_F - p_F\right) \left(\frac{y}{y_F}\right)^{\beta}. \tag{18}$$

Then, the optimal second spot price for the upstream firm is $p_F^* = \frac{\beta}{\beta-1} I$, and the optimal follower investment threshold is $y_F^* = \left(\frac{\beta}{\beta-1}\right)^2 \frac{r-\alpha}{\pi_D} I$. Compared with the case where the specific input is produced internally ($p_F = I$), the follower invests at a level of y that is $\frac{\beta}{\beta-1}$ times higher.

Remark 1 $F(y, y_F^*, p_F^*)$ *does not depend on* (p_L, y_P).

Indeed, what the firm takes into account when it chooses an investment trigger, as a follower, is the profit flow it may expect in the future. This flow is *not* impacted by the investment cost of the leader, nor by its exact investment date.

To determine the preemption threshold y_P, given p_L, it is necessary to refer to the value of a firm that invests immediately at the current market size y, given that its rival invests optimally as a follower. Let $L(y, p_L)$ denote this value, which has a different form from the $V(\cdot)$ expressions in the rest of the paper:

$$L(y, p_L) = \frac{\pi_M}{r-\alpha} y - p_L - \frac{\pi_M - \pi_D}{r-\alpha} y_F^* \left(\frac{y}{y_F^*}\right)^{\beta}, \tag{19}$$

all $y \leq y_F^*$. Although this function is commonly used in preemption models, it is also useful to consider a somewhat more general expression of (19), which is $\tilde{L}(y, y_L, y_F, p_L) = \left(\frac{\pi_M}{r-\alpha} y_L - p_L\right) \left(\frac{y}{y_L}\right)^{\beta} - \frac{\pi_M - \pi_D}{r-\alpha} y_F \left(\frac{y}{y_F}\right)^{\beta}$, all $y \leq y_L \leq y_F$. The function $\tilde{L}(y, y_L, y_F, p_L)$ measures the value, at the current market size y, of a firm that is free to invest at the future trigger y_L as a leader with a follower that invests at y_F.[24] We have $L(y, p_L) = \tilde{L}(y, y_L, y_F^*, p_L)$ when the constraint $y_L \equiv y$ is imposed.

Remark 2 $\arg\max_{y_L} \tilde{L}(y, y_L, y_F^*, I) = \{y^*\}$.

In other words, when it incurs the "true" cost of investment $p_L = I$, a firm that is free to choose y_L invests at the same date as in the integrated case (with a single firm). This is another illustration of the "myopic" behavior as coined by Leahy [19].

[24] See Reinganum [24].

The analysis of the investment game based on the functions (18) and (19) closely follows that of existing models. The threshold y_P, which is defined by $L(y_P\left(p_L\right), p_L) = F\left(y_P(p_L), y_F^*, p_F^*\right)$, is a function of p_L. We define $y_P^* \equiv y_P(p_L^*)$, where p_L^* denotes the upstream supplier's value-maximizing price. We find:

Proposition 5 *In the separated case with two downstream firms, there is a unique equilibrium characterized by:*

$$\textit{(i) downstream triggers} \quad : \quad y_P^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M}I, \qquad y_F^* = \left(\frac{\beta}{\beta-1}\right)^2\frac{r-\alpha}{\pi_D}I, \tag{20}$$

$$\textit{(ii) upstream prices} \quad : \quad p_L^* = \left(1-\Delta\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)p_F^*, \qquad p_F^* = \frac{\beta}{\beta-1}I, \tag{21}$$

with $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{1-\beta} - \left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{-\beta} \in \left(0, \frac{(\beta-1)^{\beta-1}}{\beta^\beta}\right)$.

There are two salient features in this proposition. First, the upstream firm induces the first downstream firm to enter at the efficient threshold ($y_P^* = y^*$). The intuition behind this is as follows. In a preemption equilibrium, rent equalization implies that, for any investment cost chosen upstream, including $p_L = I$, the leader's value is pegged on the follower payoff $F\left(y, y_F^*, p_F^*\right)$. The latter value does not depend on p_L (Remark 1). By raising the price p_L above I, the upstream firm increases the cost of leading the sequence of investments, and thereby raises the preemption equilibrium trigger $y_P\left(p_L\right)$. It also appropriates any additional monetary gain on top of the value $\tilde{L}\left(y, y_P\left(p_L\right), y_F^*, p_L\right) = F\left(y, y_F^*, p_F^*\right)$ retained by the downstream leader. Therefore, the supplier's best strategy is to maximize the joint value of the two vertically related units, by setting y_P equal to the investment trigger y^*. This is the same investment trigger as the one chosen by the leader when it incurs the "true" cost I (Remark 2). A noteworthy feature of the specification with vertical separation and two downstream firms, compared with similar real option games, is that the solution in the preemption scenario is analytic. The closed-form expression of y_P^* facilitates the comparative statics, which are consistent with the interpretation of the model given in Section 2.3. As for the follower's investment threshold y_F^*, recall that it is chosen analogously to the separated case (like y_S^* in Section 2.2).

Second, the way that the upstream firm induces entry at the efficient threshold is by discounting the input for the first downstream firm, thus exacerbating the preemption race. The level of the discount $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)$ results from the interplay of two effects, option effects (β) on the one hand, and downstream competition effects (π_M/π_D) on the other. Intuitively, when downstream competition is severe (high π_M/π_D), preemption leads firms to invest earlier, and it is less necessary to discount the first input. Similarly, when option effects are substantial (low β) so that firms tend to enter relatively late, it is necessary to discount the first input more in order to induce efficient entry. These remarks are captured in the next proposition.

Proposition 6 *For all admissible parameter values, the discount is decreasing in β and $\frac{\pi_M}{\pi_D}$:*

$$\frac{d\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\beta} < 0, \quad \frac{d\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\frac{\pi_M}{\pi_D}} < 0. \tag{22}$$

More can be said about the equilibrium values of decision variables in Proposition 5 regarding comparative statics. For example, the price elasticities with respect to the parameters of the stochastic process have tractable forms that are easily ranked ($\varepsilon_{p_F^*/\beta} < \varepsilon_{p_L^*/\beta} < 0$ and $\varepsilon_{p_F^*/\frac{\pi_M}{\pi_D}} = 0 < \varepsilon_{p_L^*/\frac{\pi_M}{\pi_D}}$, on this see the appendix, Section A.6.1).

As we have seen, a key prediction of the model is that the upstream supplier induces an efficient investment threshold for the first firm that is identical to the integrated case, analogously to a "no discrimination at the top" result. These points are illustrated in Figure 3 for specific parameter values.

The two solid curves refer to the separated case. The quasi concave one represents $\tilde{L}\left(y, y_L, y_F^*, p_L^*\right)$, that is the value of the leader as a function of y_L, and measured at a given y (specifically, $y = 1$) provided that the follower invests at the optimal threshold y_F^*, and for an upstream value-maximizing price p_L^* (with $\beta = 2$, $I = r - \alpha = \pi_M = 1$, $\pi_D = \frac{1}{2}$). The other solid curve is a graph of the follower value $F\left(y, y_F, p_F^*\right)$, which has the same expression as in (18). Note that, when $y = y_F^*$, the leader value is higher since $p_L^* = \frac{13}{8} < 2 = p_F^*$. The preemption threshold y_P^* is determined by the condition that firms are indifferent at that point between investing as a leader or waiting to invest as a follower. In this figure, the dashed curve represents the upstream firm's optimization problem. It describes the reference (or "true") leader value, based on the actual investment cost I (i.e., $p_L = I$) for all possible $y_L \leq y_F^*$, and for $y_F = y_F^*$ (i.e., $p_F = p_F^* > I$). This is the graph of $\tilde{L}\left(y, y_L, y_F^*, I\right)$, which reaches a maximum when $y_L = y^*$.

Consider now the upstream value in the preemption equilibrium of Proposition 5, that is

$$\tilde{W}\left(y, p_L^*, p_F^*\right) = \left(p_L^* - I\right)\left(\frac{y}{y_P\left(p_L^*\right)}\right)^{\beta} + \left(p_F^* - I\right)\left(\frac{y}{y_F^*}\right)^{\beta}. \tag{23}$$

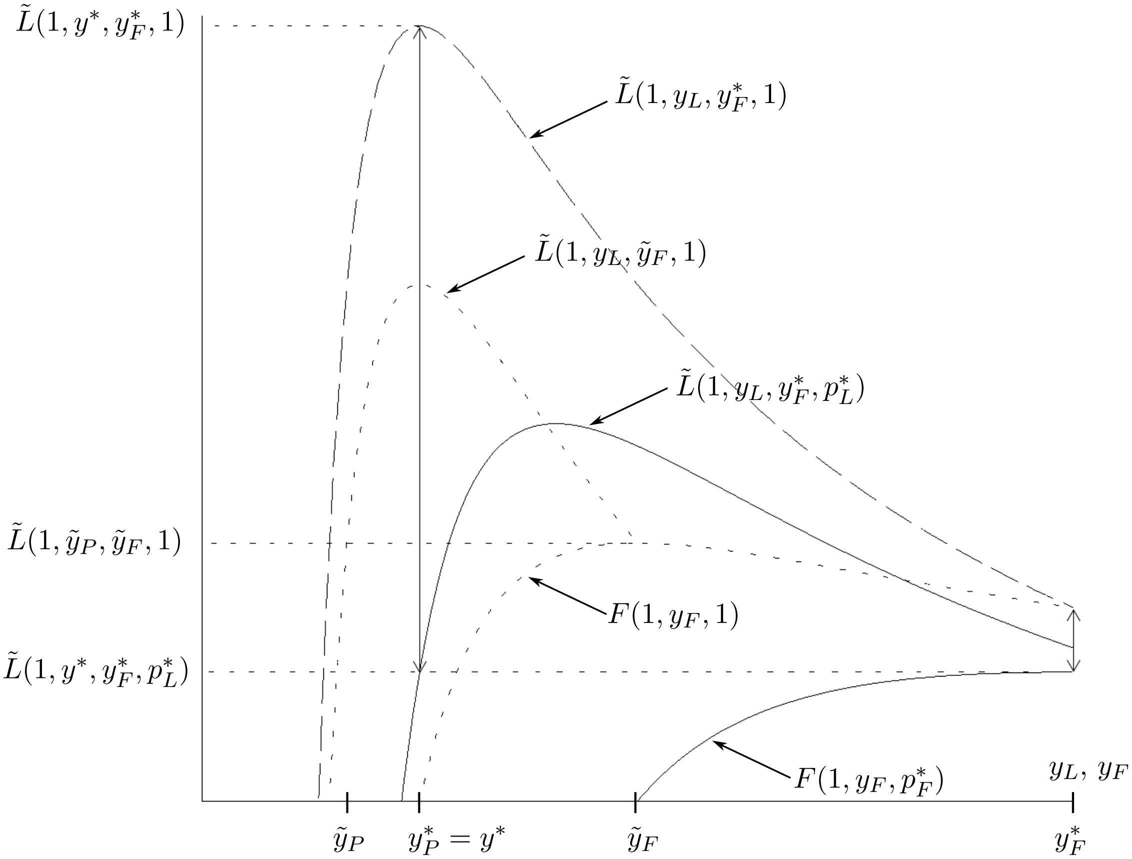

Figure 3: Leader and follower values at current market size $y = 1$ as a function of investment triggers y_L, y_F (with $\beta = 2$, $I = r - \alpha = \pi_M = 1$, $\pi_D = \frac{1}{2}$). The preemption trigger $y_P^* = y^*$ in the separated case maximizes the reference leader value (that is, $\tilde{L}(1, y_L, y_F^*, 1)$), and is greater than the trigger under preemption when both downstream firms face the true investment cost. By charging $p_L^* > 1$, the upstream firm appropriates the value differential $\tilde{L}\,(1, y^*, y_F^*, 1) - \tilde{L}\,(1, y^*, y_F^*, p_L^*)$. By charging $p_F^* > 1$, it also earns the difference $F\,(1, y_F^*, 1) - F\,(1, y_F^*, p_F^*)$.

This value can be visualized in Figure 3 by reinterpreting each term on the right hand side of the equality sign in (23) as follows. On the one hand, the supplier chooses p_L, shifting the leader value function, to maximize the difference between the reservation value that must be given to the leader and the reference leader value (with the true cost I) at the preemption trigger $y_P(p_L)$. By charging exactly $p_L^* > I$, so that the leader invests at $y_P^* = y_P\,(p_L^*) = y^*$, the supplier appropriates the value differential

$$\tilde{L}\left(y, y^*, y_F^*, I\right) - \tilde{L}\left(y, y^*, y_F^*, p_L^*\right) = \left(p_L^* - I\right)\left(\frac{y}{y_P\left(p_L^*\right)}\right)^\beta. \tag{24}$$

In addition, the upstream supplier earns the difference between the value that the follower would earn as an integrated firm, and the value it earns as a separate entity, with $y_F = y_F^*$ in both cases. Formally, by charging $p_F^* > I$, the supplier appropriates

$$F\left(y, y_F^*, I\right) - F\left(y, y_F^*, p_F^*\right) = \left(p_F^* - I\right)\left(\frac{y}{y_F^*}\right)^\beta. \tag{25}$$

The magnitudes (24) and (25) are represented by the vertical arrows in the figure.

Finally, Figure 3 also shows the investment triggers that arise in the standard investment model, or equivalently with two integrated downstream firms (i.e., $p_L = p_F = I$), which are denoted $\tilde{y}_P$ and $\tilde{y}_F$ respectively. We discuss this comparison further in Section 4.2 below.

Extension: exclusive dealing

Interesting outcomes can arise if some of the assumptions of the pricing and entry game are relaxed. For simplicity, assume that downstream complementarities are not too strong ($\pi_M/\pi_D > 2$), so the entry of a second firm reduces flow profits for the industry. Then, instead of supposing that the upstream firm cannot commit to future prices, suppose that it can credibly threaten to charge $p_F = \infty$, or (equivalently) that it can add an enforceable exclusivity clause when it sells the input to the first firm. Then, by arguments similar to above, the upstream can induce efficient timing for the first investment (y^*) and capture the integrated value by charging $p_L^{\text{excl}} = V\left(y, y^*, I\right)$. To see this, consider Figure 3 again. With exclusive dealing, the relevant follower payoff $F\left(y, y_F^*, \infty\right)$ tends to the horizontal axis asymptotically ($y_F^* = \infty$). The supplier then appropriates the value differential, which is now simply $\tilde{L}\left(y, y^*, \infty, I\right)$, and is greater than $\tilde{W}\left(y, p_L^*, p_F^*\right)$ if $\pi_M/\pi_D > 2$. The combination of (potential) downstream preemption and exclusive dealing constitutes an effective vertical restraint in this case.

4.2 Comparison of industry structures

We compare our model with three relevant benchmarks: the industry optimum, the standard preemption framework, as well as the bilateral monopoly model of Section 2.2.[25]

[25] As is common in this literature our objective here is descriptive rather than normative, so we focus on industry value and leave aside the characterization of the consumer surplus.

□ *Industry optimum.* The first benchmark we consider is the industry optimum, in which the entry thresholds are chosen so as to maximize the joint value $\tilde{L}(y, y_1, y_2, I) + F(y, y_2, I)$ over $(y_1, y_2) \in [y, \infty) \times [\max\{y, y_1\}, \infty)$. The industry value at optimum is trivially greater than the one realized in the scenario of Proposition 5. There are two main subcases to consider, depending on the degree of competition π_M/π_D. To begin with, note that in all cases, the timing of the first investment is the same in our preemption scenario and in the industry optimum ($y_P^* = y_1^* = y^*$).[26] Next, when the degree of competition is large ($\pi_M/\pi_D \geq 2$), the second entry exerts a negative externality on industry payoffs, the industry optimum involves a single downstream investment ($y_2^* = \infty$), and necessarily the second investment occurs too early under preemption. When the degree of competition is small ($\pi_M/\pi_D < 2$), the second investment exerts a positive externality (because of downstream complementarities). In that case, the second investment under preemption may occur either too early or too late ($y_F^* \lesseqgtr y_2^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{2\pi_D-\pi_M}I$), depending on the primitives of the model ($\beta(\pi_M/\pi_D - 1) \lesseqgtr 1$). Two independent effects are involved: with double marginalization, a larger option effect (low β) leads the second firm to delay too much, whereas more complementarities (low π_M/π_D) generate a greater positive externality of the follower's entry on industry value.

□ *Standard preemption.* We know from existing models with similar specifications that, in a preemption equilibrium without any upstream mark-up, the leader invests first at $\tilde{y}_P$, which is strictly less than the efficient threshold y^*. Then the follower invests at $\tilde{y}_F = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_D}I$. Note that in Figure 3, the two dotted curves represent the leader and follower payoff functions $\tilde{L}(y, y_L, \tilde{y}_F, I)$ and $F(y, y_F, I)$ respectively. It is straightforward to check that entry occurs generally earlier in standard preemption ($\tilde{y}_P < y_P^* < \tilde{y}_F < y_F^*$). The comparison of equilibrium values over the two industry structures hinges on several effects and is ambiguous. With respect to industry value, the first firm's entry is less efficient in the standard preemption model, but the second firm's entry can be more efficient. Moreover, industry value is greater with a separate input supplier when option effect is low (high β) or downstream competition is tough (high π_M/π_D), but standard preemption can dominate as well.

□ *Bilateral monopoly.* A final comparison is with the bilateral monopoly case that we studied in section 2.2. With respect to investment thresholds, the only relevant comparison is for the first investment ($y_P^* = y^* < y_S^*$), so the first firm's entry is more efficient with downstream preemption. With respect to industry value, the presence of a second firm affects the structure and potential surplus of the industry, either positively or negatively (as $\pi_M/\pi_D \lesseqgtr 2$), hindering direct comparison. Again, the comparison of industry structures with respect to value is ambiguous. For a small option effect (high β) and tough

[26] Note that $\tilde{L}(y, y_1, y_2, I)$ is separable in y_1 and y_2 (see also Remark 2 above).

competition (high π_M/π_D), preemption with an input supplier dominates bilateral monopoly, but the opposite can also hold.

To summarize:

Proposition 7 *Entry thresholds and industry value are such that:*

(i) for the first entry threshold, $\tilde{y}_P < y_P^* = y_1^* = y^* < y_S^*$*, and for the second entry threshold (when defined),* $\tilde{y}_F < y_F^*$*,* $\tilde{y}_F < y_2^*$ *and* $y_F^* \lesseqgtr y_2^*$ *when* $\beta\left(\pi_M/\pi_D - 1\right) \lesseqgtr 1$*.*

(ii) the industry structure {preemption with upstream} may either dominate or be dominated by the two alternative structures {bilateral monopoly} and {standard preemption}. For large enough $\left(\beta, \frac{\pi_M}{\pi_D}\right)$*, {preemption with upstream} is an optimal structure for the industry.*

The objective of this section has been to describe a complex industrial structure, in which an upstream sells an input to two successive entrants. Entry and pricing decisions in this structure reflect the interplay of two conflicting mechanisms: preemption (which leads downstream firms to enter "too early") and separation (which leads them to enter "too late"). Taken in isolation, each of these mechanisms is detrimental to industry value, either via rent dissipation, or via double marginalization. But in the three-firm structure, the two can balance out and yield greater industry value. Notably, by Proposition (7), when downstream competition is tough (large π_M/π_D), preemption is severe and leads to entry much too early by the first firm. Moreover, it is exactly when downstream competition is tough that the second firm's entry reduces industry flow profits (the industry optimum is for a single downstream firm to be active in this case). Yet, balanced with the upstream's pricing decision, preemption among downstream firms then has the effect of a vertical restraint, and is sufficiently effective as such for greater industry value to result.

5 An Example: Investments in the Vaccine Industry

In developing the model presented here, our thinking was guided by an example of investment in production facilities for a new vaccine against dengue fever that illustrates many of the theoretical aspects. Dengue is a disease caused by any of four closely related virus serotypes transmitted by mosquitoes. It strikes people with low levels of immunity, with symptoms that include intense joint and muscle pain, headache, nausea, and fever. The most severe form of the disease is the dengue hemorrhagic fever. Although it occurs mostly in Asian and Latin American countries, where it is a leading cause of hospitalization, the disease spreads to new parts of the globe each year, including countries such as Australia and the United States (Puerto Rico, the U.S. Virgin Islands, the Texas-Mexico border, Pacific islands, and most recently Florida).[27]

To address this problem, one or more vaccines are due to arrive over the next years. With clinical studies reaching their final phase, Sanofi Pasteur, the vaccines division of Sanofi SA, was the first to launch the construction of a new plant north of Lyon (France) in 2009, with an annual capacity of 100 million doses per year. The total investment amounts to $477 million (see Carroll [4]). The main facility, which concentrates the firm's production technology, has been specifically designed for the processing of the novel vaccine. In line with the irreversible investment framework, most components of the expenditure, including fees and wages, capital amortization, and the costs for the safety and quality qualification procedure, are unrecoverable. The equipment is sourced on an intermediate market from specialized input providers, and represents on the order of 35-40% of the plant construction cost. The customized lyophilisators, which use liquid nitrogen refrigeration for freeze drying operations, constitute a central piece of equipment in the mass production process. The suppliers of pharmaceutical freeze drying technology are highly concentrated, suggesting the possibility of market power. In Europe, lyophilisators are supplied by firms such as Usifroid, a subsidiary of Telstar SA which recently claimed a French market share of 80% in freeze drying equipment solutions for the pharmaceutical industry. In 2009, GlaxoSmithKline Plc (GSK), another leading vaccine producer, announced that it would develop and manufacture another dengue vaccine with a Brazilian partner.[28] GSK is thus likely to also invest in additional production capacities in the foreseeable future, at some point in time that will depend on demand forecasts.

The potential demand for the vaccines is clearly growing, though future levels are uncertain. The number of reported cases, as measured annually, is a simple indicator of the magnitude of future demand.[29] According to the World Health Organization (WHO) "[a]n estimated 2.5 billion people live in over 100 endemic countries and areas where dengue viruses can be transmitted. Up to 50 million infections occur annually with 500,000 cases of dengue hemorrhagic fever and 22,000 deaths mainly among children". The number of countries reporting cases is another demand indicator. The

[27] See World Health Organization [30] and Center for Disease Control [5].

[28] Sources on Telstar's market shares in Europe and GSK's project to manufacture a vaccine for dengue fever are http://www.telstar-lifesciences.com/en/ and http://www.gsk.com/media/pressreleases/2009, respectively.

[29] The issue of demand forecasts for new vaccines is discussed thoroughly in Center for Global Development [6].

WHO indicates that "[p]rior to 1970, only 9 countries had experienced cases of dengue hemorrhagic fever; since then the number has increased more than 4-fold and continues to rise".[30]

This real-world situation, where firms' choice of investment timing depends on the cost of a key equipment, which is necessary in order to serve a growing though uncertain demand, and is delivered by an upstream supplier with market power, is thus emblematic of the many market cases captured by our model specifications.

To flesh out the idea that double marginalization leads to delayed entry, note that the demand for the vaccine can be proxied by the number of reporting countries, insofar as governments constitute firms' effective clients in this industry. As a rough approximation, for the forty year period up to 2005, this demand is fit by a geometric Brownian motion with drift and volatility parameters $\alpha = 0.05$ and $\sigma = 0.21$.[31] The upstream mark-up and downstream delay are measured by Lerner-based market power indices that we have defined (L_p and L_y in (8)).[32] Letting the cost of capital range from 10 to 20 percent consistently with a moderate to high premium on historical stock market returns, these measures range from 0.3 to 0.6. A back of the envelope calibration exercise thus suggests that the dynamic double marginalization phenomenon identified in Section 2.2 could result in a downstream firm like Sanofi SA setting an investment threshold up to two or three times greater than the industry optimum.

On the other hand, another salient feature of this case is rival firm GlaxoSmithKline Plc's impending entry in the market. To the extent that, as seems likely, Sanofi SA could anticipate its rival's arrival into the market, the main result of Section 4 relating to the downstream duopoly case (Proposition 5) suggests that despite any upstream mark-up on the investment cost, Sanofi's entry should occur at the efficient point from the standpoint of the industry as a whole. Exactly where the truth lies between the two extremes predicted by the bilateral monopoly and downstream preemption scenarios we study, as well as the comparison with the social optimum, is a question that we leave open.

6 Conclusion

In this paper, we have studied investment timing when firms depend on an outside supplier to provide a discrete input (e.g., a key equipment), developing a dynamic version of a heretofore static model. The

[30]Source: http://www.who.int/csr/disease/dengue/impact/en/index.html.

[31]Data available at http://www.who.int/csr/disease/dengue/denguenet/en/.

[32]These measures range from 0 (no distortion) to 1 (full distortion).

upstream firm's mark-up depends on the stochastic process followed by downstream flow profits. A vertical distortion arises because the upstream firm's pricing induces the downstream firm to delay the exercise of its investment option. The distortion, measured by a Lerner index, increases with market growth and volatility and decreases in the interest rate. Downstream firm value is more sensitive to the market growth rate and the interest rate than upstream firm value, and in contrast with the standard real option framework, greater volatility decreases firm values near the exercise threshold. Aside from forcing efficient investment timing directly when it can contract on the state of final demand, an input supplier that knows the initial demand state can approach the integrated value with a simple dynamic pricing rule when volatility is low, or, if such contracts are feasible, induce optimal investment timing by means of an option on the input, as aircraft manufacturers do. Otherwise, the presence of a second downstream firm results in a preemption race and acts as a substitute for vertical restraints. The input is then sold to the downstream leader at a discount that reflects the interaction of the option effect with both vertical effects (double marginalization) and horizontal effects (preemption). The first firm invests optimally as a result, and industry value is higher than in comparable industry structures when option effects are low or when downstream competition is high.

The model developed in this paper may be viewed alternatively as an extension of the existing real options literature, or as the extension of the classic industrial organization analysis of vertical relationships to a stochastic dynamic setting. Relatively to other work, we hope to have shown how not only preemption races, but also other insights of industrial economics fit naturally with option theory, that is to say that an integrated vision of firm investment exists at the intersection of economics and finance, which likely involves options and games.

The model we study rests on several assumptions that may be relaxed. One direct extension is to allow for upstream competition. If suppliers compete in prices and there is a single downstream firm, then the integrated optimum is restored. On the other hand, in an industry with two upstream and two downstream firms, upstream competition presumably results in a standard preemption race downstream, and the leader invests too early. Also, a qualitative prediction of the model is that the first input is sold at a discount under downstream duopoly, but learning effects which decrease the upstream firm's production cost for the second input supplied, could reduce the apparent discount that is offered to the leader. Many of the key results hold if demand is taken to follow a process other than geometric Brownian motion, but we have not studied the consequences of weakening our assumptions on the cost of capital, which could differ between firms, for instance. Our analysis has adhered to the classical assumption that the contract terms are decided by the upstream firm, and one could also envisage that it is the downstream firms that have market power in the input market, and therefore write the contract. Finally, our focus has been on describing firm choices and firm profits, and a broader analysis would integrate a consumer welfare measure as well.

A Appendix

A.1 Sensitivity of β to r, α and σ

The derivatives of β with respect to the growth and volatility parameters α and σ arise throughout the paper, and have the following expressions:

$$\frac{d\beta}{dr} = \frac{1}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2} > 0, \quad \frac{d\beta}{d\alpha} = \frac{-\beta}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2} < 0, \quad \frac{d\beta}{d\sigma} = \frac{-2\left(r - \alpha\beta\right)}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^3} \leq 0. \quad (26)$$

With respect to the sign of the latter expression, note that $r \geq \alpha\beta$, with equality only if $\sigma = 0$. Moreover, we have $\frac{d\beta}{dr} = -\frac{1}{\beta}\frac{d\beta}{d\alpha}$. ■

A.2 Behavior of $f(\beta)$

We use $f(\beta) \equiv (2\beta - 1)\frac{(\beta-1)^{\beta-1}}{\beta^\beta}$ several times in the paper. As f is continuous on $[1, \infty)$ and $\lim_{1^+}(\beta - 1)^{\beta-1} = 1$, $f(1) = 1$. To show that $\lim_\infty f(\beta) = \frac{2}{\mathrm{e}}$, note that $f(\beta) = \left(1 + \frac{\beta}{\beta-1}\right)\left(1 - \frac{1}{\beta}\right)^\beta$ and recall that $\lim_\infty \left(1 - \frac{1}{\beta}\right)^\beta = \mathrm{e}^{-1}$. Finally, $f'(\beta) = \left(2 - (2\beta - 1)\ln\frac{\beta}{\beta-1}\right)\frac{(\beta-1)^{\beta-1}}{\beta^\beta}$. Define $x \equiv \frac{\beta}{\beta-1} \in (1, \infty)$ so after substituting and rearranging $f'(\beta)$ has the sign of $\Gamma(x) \equiv 2(x - 1) - (x + 1)\ln x$. Then $\lim_1 \Gamma(x) = 0$ and $\Gamma'(x) = 1 - \ln x - \frac{1}{x}$. Since $\ln x > \frac{x}{x-1}$, $\Gamma'(x) < -\frac{x^2+x-1}{x(x-1)} < 0$, so $\Gamma(x)$ is negative on $[1, \infty)$ and therefore $f'(\beta) < 0$. ■

A.3 Proof of Proposition 3 *(Sensitivity analysis in bilateral monopoly)*

A.3.1 Decision variables (y_S^*, p_S^*)

Investment threshold:

$$\begin{aligned}
\frac{dy_S^*}{dr} &= y_S^* \left(\frac{1}{r-\alpha} - \frac{2}{\beta\,(\beta-1)} \frac{d\beta}{dr} \right) \lesseqgtr 0, && (27)\\
\frac{dy_S^*}{d\alpha} &= -y_S^* \left(\frac{1}{r-\alpha} + \frac{2}{\beta\,(\beta-1)} \frac{d\beta}{d\alpha} \right) && (28)\\
&= \frac{y_S^*}{r-\alpha} \frac{\frac{1}{2} + \frac{\alpha}{\sigma^2}}{\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)} > 0, && (29)\\
\frac{dy_S^*}{d\sigma} &= -\frac{2y_S^*}{\beta\,(\beta-1)} \frac{d\beta}{d\sigma} > 0. && (30)
\end{aligned}$$

To establish that $\frac{dy_S^*}{dr}$ has an ambiguous sign (negative for low r and positive otherwise), note that it has the sign of $\Theta\,(r) \equiv \beta\,(\beta-1)\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)\sigma^2 - 2\,(r-\alpha)$. We find that Θ is a convex function of r, as $\Theta''(r) = \frac{2\frac{\alpha^2}{\sigma^4} + 3\frac{2r-\alpha}{\sigma^2} + 1}{\left(\beta - \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)\right)^3 \sigma^2} > 0$ (since $\alpha < r$). Also, at $r = \alpha$, $\Theta(\alpha) = 0$, $\Theta'(\alpha) = -1$, and $\lim_{r\to\infty} \Theta\,(r) = \infty$. We conclude that $\Theta\,(r) = 0$ has a unique root in (α, ∞).

Input price:

For r, α, and σ we have $\frac{dp_S^*}{d\cdot} = -\frac{I}{(\beta-1)^2} \frac{d\beta}{d\cdot}$, where $\frac{d\beta}{d\cdot}$ is given by (26) above.

A.3.2 Firm values (V^*, W^*)

We successively focus on α and r only since the effect of a change in σ on V^* and W^* follows directly from the expressions given in the text.

Growth:

It is useful to begin by signing $\frac{dW^*}{d\alpha}$. After simplification, computations yield, for all $y \leq y_S^*$:

$$\begin{aligned}
\frac{dW\,(y, p_S^*)}{d\alpha} &= \left(\frac{\beta}{r-\alpha} + \left(\frac{1}{\beta-1} + \ln \frac{y}{y_S^*} \right) \frac{d\beta}{d\alpha} \right) W\,(y, p_S^*) && (31)\\
&\geq \left(\frac{\beta}{r-\alpha} + \frac{1}{\beta-1} \frac{d\beta}{d\alpha} \right) W\,(y, p_S^*)\,. && (32)
\end{aligned}$$

Thus, $\frac{dW\left(y,p_S^*\right)}{d\alpha} > 0$ if $\Lambda\left(\alpha,r,\sigma\right) \equiv \left(\beta-1\right)\left(\beta-\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)\right)\sigma^2-\left(r-\alpha\right) > 0$. Taking $r=\alpha$, $\Lambda\left(\alpha,\alpha,\sigma\right)=0$, and $\frac{d\Lambda(\alpha,r,\sigma)}{dr}=\left(\beta-1\right)\left(\beta-\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)\right)^{-1} > 0$. Therefore, $\frac{dW\left(y,p_S^*\right)}{d\alpha} > 0$ for all admissible parameter values.

For the sign of $\frac{dV^*}{d\alpha}$, similarly after simplification, computations yield, for all $y \leq y_S^*$:

$$\frac{dV\left(y,y_S^*,p_S^*\right)}{d\alpha} = \left(\frac{\beta}{r-\alpha}+\left(\frac{1}{\beta}+\ln\frac{y}{y_S^*}\right)\frac{d\beta}{d\alpha}\right)V\left(y,y_S^*,p_S^*\right) \tag{33}$$

$$\geq \left(\frac{\beta}{r-\alpha}+\frac{1}{\beta}\frac{d\beta}{d\alpha}\right)V\left(y,y_S^*,p_S^*\right). \tag{34}$$

As we have established $\frac{dW\left(y,p_S^*\right)}{d\alpha} > 0$ and since $\frac{1}{\beta}\frac{d\beta}{d\alpha} > \frac{1}{\beta-1}\frac{d\beta}{d\alpha}$, we have $\frac{dV\left(y,y_S^*,p_S^*\right)}{d\alpha} > 0$ for all admissible parameter values.

Interest rate:

We begin with $\frac{dV}{dr}$. For all $y \leq y_S^*$:

$$\frac{dV\left(y,y_S^*,p_S^*\right)}{dr} = \left(-\frac{\beta}{r-\alpha}+\left(\frac{1}{\beta}+\ln\frac{y}{y_S^*}\right)\frac{d\beta}{dr}\right)V\left(y,y_S^*,p_S^*\right), \tag{35a}$$

$$= -\left(\frac{\beta}{r-\alpha}+\frac{1}{\beta}\left(\frac{1}{\beta}+\ln\frac{y}{y_S^*}\right)\frac{d\beta}{d\alpha}\right)V\left(y,y_S^*,p_S^*\right), \tag{35b}$$

$$= -\left(\frac{\beta}{r-\alpha}V\left(y,y_S^*,p_S^*\right)+\frac{1}{\beta}\frac{dV\left(y,y_S^*,p_S^*\right)}{d\alpha}\right) < 0, \tag{35c}$$

where the second equality follows from $\frac{d\beta}{dr}=-\frac{1}{\beta}\frac{d\beta}{d\alpha}$, and the final inequality from the result $\frac{dV\left(y,y_S^*,p_S^*\right)}{d\alpha} > 0$ established above.

Next, we evaluate $\frac{dW}{dr}$. For all $y \leq y_S^*$, using $\frac{d\beta}{dr}=-\frac{1}{\beta}\frac{d\beta}{d\alpha}$ again and rearranging yields:

$$\frac{dW\left(y,p_S^*\right)}{dr} = -\left(\frac{\beta-1}{r-\alpha}W\left(y,p_S^*\right)+\frac{1}{\beta}\frac{dW\left(y,p_S^*\right)}{d\alpha}\right) < 0. \tag{36}$$

It remains to rank the elasticities. A simple reorganization of terms together with $\frac{1}{\beta} < \frac{1}{\beta-1}$ directly leads to $\varepsilon_{W^*/\alpha} < \varepsilon_{V^*/\alpha}$ and $\varepsilon_{W^*/r} > \varepsilon_{V^*/r}$. ■

A.4 Derivation of $\widehat{W}\left(\widehat{y}, p_S\right)$ *(Dynamic pricing problem)*

Deterministic case:

For $\sigma = 0$, with the pricing rule $P_S(t) = P_0 e^{\gamma t}$, the downstream payoff has the simpler form $\widehat{V}\left(y, y_i, p_S\right) = \frac{\pi_M}{r-\alpha} y_i \left(\frac{y}{y_i}\right)^{\frac{r}{\alpha}} - P_0 \left(\frac{y}{y_i}\right)^{\frac{r-\gamma}{\alpha}}$, and the optimal downstream investment policy is:

$$y_i^*(p_S) = \begin{cases} y & \text{if } P_0 < \frac{\pi_M}{r-\alpha} y, \gamma \geq \alpha, \\ [y, \infty) & \text{if } P_0 = \frac{\pi_M}{r-\alpha} y, \gamma = \alpha, \\ \left(\frac{(r-\gamma) P_0}{\pi_M}\right)^{\frac{\alpha}{\alpha-\gamma}} y^{-\frac{\gamma}{\alpha-\gamma}} & \text{if } \gamma < \alpha, \\ \infty & \text{if } P_0 > \frac{\pi_M}{r-\alpha} y, \gamma = \alpha \text{ or } P_0 \geq \frac{\pi_M}{r-\alpha} y, \gamma > \alpha. \end{cases} \tag{37}$$

The upstream payoff is $\widehat{W}\left(y, p_S\right) = P_0 \left(\frac{y}{y_i^*(p_S)}\right)^{\frac{r-\gamma}{\alpha}} - I \left(\frac{y}{y_i^*(p_S)}\right)^{\frac{r}{\alpha}}$. At the maximum described in the text, $(P_0^*, \gamma^*) = \left(\frac{\pi_M}{r-\alpha} Y_0, \alpha\right)$, the downstream is indifferent between all investment triggers in $[y, \infty)$, and the upstream appropriates all the surplus. It can then be checked directly that no other exponential price rule p'_S simultaneously satisfies the two conditions $y_i^*(p'_S) = y^*$ (efficient investment trigger) and $\widehat{W}\left(y, p'_S\right) = V\left(y, y^*, I\right)$ (upstream capture of the integrated value).

Optimal downstream policy, stochastic case:

If the input price follows a rule $P_S\left(t\right) = P_0 e^{\gamma t}$, the downstream firm faces a real option with demand and cost uncertainty (see Dixit and Pindyck [8], Chapter 6). The investment trigger is then an optimum ratio of market size to input cost $Y_t / P_S(t)$ that has the form $\widehat{y}_S^* = \frac{\widehat{\beta}}{\widehat{\beta}-1} \frac{r-\alpha}{\pi_M}$, where $\widehat{\beta} > 1$ is the unique positive root to the fundamental quadratic associated with the differential equation satisfied by the option value $\widehat{V}$, i.e. $\widehat{\beta} = \frac{1}{2} - \frac{\alpha-\gamma}{\sigma^2} + \sqrt{\left(\frac{\alpha-\gamma}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r-\gamma)}{\sigma^2}}$. The trigger is well-defined so long as $\alpha, \gamma < r$. For $\widehat{y} < \widehat{y}_S^*$, the optimal policy is a (stochastic) stopping time $\widetilde{\tau} = \inf\left\{\tau > t, \widehat{y} \geq \widehat{y}_S^*\right\}$ (and immediate investment otherwise). The downstream option value is therefore:

$$\widehat{V}\left(\widehat{y}, \widehat{y}_S^*, p_S\right) = \mathbb{E}_t \left(\frac{\pi_M}{r-\alpha} Y_{\widetilde{\tau}} - P_S\left(\widetilde{\tau}\right)\right) e^{-r\widetilde{\tau}}, \tag{38a}$$

$$= \frac{p_S}{\widehat{\beta}-1} \left(\frac{\widehat{y}}{\widehat{y}_S^*}\right)^{\widehat{\beta}}, \tag{38b}$$

all $\widehat{y} < \widehat{y}_S^*$. Finally, note that $\frac{d\widehat{y}_S^*}{d\gamma} = -\frac{1}{\left(\widehat{\beta}-1\right)^2} \frac{r-\alpha}{\pi_M} \frac{d\widehat{\beta}}{d\gamma} < 0$ since $\frac{d\widehat{\beta}}{d\gamma} = \frac{\widehat{\beta}-1}{\left(\widehat{\beta} - \left(\frac{1}{2} - \frac{\alpha-\gamma}{\sigma^2}\right)\right)\sigma^2} > 0$.

Upstream payoff, stochastic case:

Given the downstream firm's trigger, and supposing that $\widehat{y} < \widehat{y}_S^*$ so the downstream delays investment (otherwise $\widehat{W}(P_0, \gamma) = P_0 - I$ directly), the upstream value at $t = 0$ is defined by:

$$\widehat{W}(P_0, \gamma) = \mathbb{E}_0 \left(P_S(\widetilde{\tau}) - I\right) e^{-r\widetilde{\tau}}, \tag{39}$$

where $\widetilde{\tau}$ is the downstream's stochastic stopping time. To obtain the expression (15) in the text, note that the first summand in (39), $\mathbb{E}_0 P_S(\widetilde{\tau}) e^{-r\widetilde{\tau}}$, follows directly from the downstream stopping rule (38a) and for the second term, the expected discount rate is $\mathbb{E}_0 e^{-r\widetilde{\tau}} = \left(\frac{\widehat{y}}{\widehat{y}_S^*}\right)^{\widehat{\beta}'}$ with $\widehat{\beta}' = \frac{1}{2} - \frac{\alpha-\gamma}{\sigma^2} + \sqrt{\left(\frac{\alpha-\gamma}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$ since $\widehat{y} = (Y_0/P_0)\exp\left[\left(\alpha - \gamma - \frac{\sigma^2}{2}\right)t + \sigma Z_t\right]$.

Upstream optimization, stochastic case:

To summarize, the upstream's problem is $\max_{(P_0,\gamma)\in\mathbb{R}\times[0,r)} \widehat{W}(\widehat{y}, p_S)$, with $\widehat{W}$ continuous in σ and in its arguments. The deterministic model ($\sigma = 0$) is a special case of the general problem and has a unique solution that achieves the integrated firm value for the upstream firm. Therefore, as $\sigma \downarrow 0$ the upstream firm can approach the integrated value with an exponential pricing rule. ■

A.5 Proof of Proposition 5 *(Equilibrium with upstream and downstream duopoly)*

The optimal follower investment threshold y_F^* and second spot price p_F^* having been discussed in the text, only the first spot price p_L^* and the preemption threshold y_P^* remain to be established.

Preemption threshold (Stage 2):

As p_L and p_F^* are exogenous for price-taking downstream firms, only parameters of the value functions are altered as compared with standard real option game models (specifically, the investment cost, which is asymmetric for the first and second firm to invest), and existing arguments (Fudenberg and Tirole [10], Huisman et al. [14]) apply to establish the existence of preemption equilibrium. In particular, so long as the equation $L(y, p_L) = F(y, y_F^*, p_F^*)$ has a root $y_P \in [y, y_F^*)$, downstream firms seek to invest immediately when the market size reaches the threshold y_P, with either firm equally likely to effectively then invest as a leader.

As $F(y, y_F^*, p_F^*)$ is non-negative, increasing, and convex while $L(y, p_L)$ is increasing and concave in y, and moreover since $L(y, p_L)$ decreases with p_L, the threshold y_P is well-defined for a range of input prices $p_L \in \left[\underline{p}, \overline{p}\right]$. The lower bound does not constrain the preemption outcome and upstream optimization so long as the initial market size is sufficiently small, so we focus on the upper bound $\overline{p}$. Setting $\frac{dL(y,p_L)}{dy} = \frac{dF\left(y,y_F^*,p_F^*\right)}{dy}$, we find that tangency occurs at $\overline{y} = \left(\frac{\pi_M}{\beta\pi_M-(\beta-1)\pi_D}\right)^{\frac{1}{\beta-1}} y_F^*$. Note that $\overline{y} < y_F^*$ so long as $\pi_D < \pi_M$. Then, $\overline{p}$ is defined implicitly by the condition $L\left(\overline{y}, \overline{p}\right) = F\left(\overline{y}, y_F^*, p_F^*\right)$.

Thus, for $p_L \in \left[\underline{p}, \overline{p}\right]$, a preemption equilibrium exists at the threshold $y_P\left(p_L\right)$ that verifies $L\left(y_P\left(p_L\right), p_L\right) = F\left(y_P\left(p_L\right), y_F^*, p_F^*\right)$. Specifically, $y_P\left(p_L\right)$ is defined implicitly by:

$$\frac{\pi_M}{r-\alpha} y_P - p_L - \frac{\beta}{\beta-1}\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D} - 1\right) I \left(\frac{y_P}{y_F^*}\right)^{\beta} = 0. \tag{40}$$

Upstream optimization (Stage 1):

The value of the upstream firm when the current market size is y is:

$$\tilde{W}\left(y, p_L, p_F^*\right) = \left(p_L - I\right)\left(\frac{y}{y_P(p_L)}\right)^{\beta} + \left(p_F^* - I\right)\left(\frac{y}{y_F^*}\right)^{\beta}. \tag{41}$$

From (40) we obtain an expression of $p_L - I$, which is plugged into (41). This leads to:

$$\tilde{W}\left(y, p_L, p_F^*\right) = \left(\frac{\pi_M}{r-\alpha} y_P - I\right)\left(\frac{y}{y_P}\right)^{\beta} - \frac{\beta}{\beta-1}\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D} - 1\right) I \left(\frac{y}{y_F^*}\right)^{\beta} + \left(p_F^* - I\right)\left(\frac{y}{y_F^*}\right)^{\beta}. \tag{42}$$

Note that the second and third summands in (42) are independent of y_P, and the first summand is identical to the integrated payoff (1) of Section 2.1. The upstream firm's decision problem is thus that of the integrated firm, and the first-order condition is satisfied at $y_P^* = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_M} I$.

Substituting y^* into (40) gives the optimal first spot price $p_L^* = \left(1 - \Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)\right)\frac{\beta}{\beta-1} I$, with $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{1-\beta} - \left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{-\beta}$. Since $\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D} > 1$, $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right) > 0$. Also, $\Delta\left(\beta, 1\right) = \frac{(\beta-1)^{\beta-1}}{\beta^{\beta}} < \frac{1}{\beta}$. We have $\frac{d\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\frac{\pi_M}{\pi_D}} = \beta\left(\frac{\pi_D}{\pi_M} - 1\right)\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{-\beta} < 0$ because $\pi_M > \pi_D$, so $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right) < \frac{1}{\beta}$. In addition, it can be verified that $y_P^* < \overline{y}$ if and only if $\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right) < \frac{1}{\beta}$, so the equilibrium preemption trigger is in the admissible range.

Absence of simultaneous investment equilibrium in Stage 2:

We have described an equilibrium in which firms invest sequentially, but it is also necessary to rule out simultaneous investment equilibria, as these can arise in real option games. Note first that in the sequential equilibrium, the upstream firm earns greater value from selling the input to the leader than to the follower. After rearrangement, $(p_L^* - I)\left(\frac{y}{y_P(p_L^*)}\right)^\beta > (p_F^* - I)\left(\frac{y}{y_F^*}\right)^\beta$ if and only if:

$$\Xi\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^\beta - \beta\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right) + \beta - 1 > 0. \tag{43}$$

Here, $\Xi(\beta, 1) = (2\beta - 1)\left(\frac{1}{f(\beta)} - 1\right) > 0$ (see Section A.2), and $\frac{\partial \Xi}{\partial \frac{\pi_M}{\pi_D}} = \frac{\beta^2}{\beta-1}\left(\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{\beta-1} - 1\right) > 0$, so $\Xi\left(\beta, \frac{\pi_M}{\pi_D}\right)$ is positive for all admissible parameter values.

For a given price p_{sim} charged by the upstream firm in the event of simultaneous investment downstream (we allow $p_{\text{sim}} \neq p_L, p_F$), the optimal simultaneous entry threshold is $y_{\text{sim}}(p_{\text{sim}}) = \frac{\beta}{\beta-1}\frac{r-\alpha}{\pi_D}p_{\text{sim}}$. The upstream firm's payoff in that case is $W_{\text{sim}}(y, p_{\text{sim}}) = 2(p_{\text{sim}} - I)\left(\frac{y}{y_{\text{sim}}(p_{\text{sim}})}\right)^\beta$, and the optimal input price is $p_{\text{sim}}^* = \frac{\beta}{\beta-1}I\ (= p_F^*)$. Then, the downstream firms both enter at the same date as a follower under preemption ($y_{\text{sim}}(p_{\text{sim}}^*) = y_F^*$). The upstream value at the candidate optimal simultaneous investment equilibrium is therefore:

$$W_{\text{sim}}(y, p_{\text{sim}}^*) = 2(p_F^* - I)\left(\frac{y}{y_F^*}\right)^\beta < (p_L^* - I)\left(\frac{y}{y_P(p_L^*)}\right)^\beta + (p_F^* - I)\left(\frac{y}{y_F^*}\right)^\beta = \tilde{W}(y, p_L^*, p_F^*), \tag{44}$$

where the inequality follows from the fact that the upstream makes greater profits off the leader's input than the follower's. ■

A.6 Proof of Proposition 6 *(Comparative statics of the discount)*

We have established that $\frac{d\Delta}{d\frac{\pi_M}{\pi_D}} = \beta\left(\frac{\pi_D}{\pi_M} - 1\right)\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{-\beta} < 0$ for all $\pi_M > \pi_D$ in the proof of Proposition 5. The other derivative is $\frac{d\Delta}{d\beta} = \frac{1}{\beta-1}\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)^{-\beta}\left(\frac{\pi_M}{\pi_D} - 1 - \ln\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)\left(\beta\frac{\pi_M}{\pi_D} - (\beta-1)\right)\right)$. The sign of $\frac{d\Delta}{d\beta}$ is that of $\Upsilon\left(\beta, \frac{\pi_M}{\pi_D}\right) \equiv \frac{\pi_M}{\pi_D} - 1 - \ln\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right)\left(\beta\frac{\pi_M}{\pi_D} - (\beta-1)\right)$. We have $\Upsilon(\beta, 1) = -\ln\left(\frac{\beta}{\beta-1}\right)$, and $\frac{d\Upsilon}{d\frac{\pi_M}{\pi_D}} = (\beta-1)\left(\frac{\pi_D}{\pi_M} - 1\right) - \beta\ln\left(\frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}\right) < 0$, so $\Upsilon\left(\beta, \frac{\pi_M}{\pi_D}\right) < 0$. Thus, $\frac{d\Delta}{d\beta} < 0$. ■

A.6.1 Additional comparative statics of input prices

To establish the ranking of the price elasticities described in the text ($\varepsilon_{p_F^*/\beta} = \varepsilon_{p_S^*/\beta} < \varepsilon_{p_L^*/\beta} < 0$), note first that $\varepsilon_{p_F^*/\beta} = -\frac{1}{\beta-1} < 0$. Moreover, by Proposition 6, $\frac{d\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)}{d\beta} < 0$ so $\varepsilon_{\Delta\left(\beta, \frac{\pi_M}{\pi_D}\right)/\beta} < 0$. It follows that $\varepsilon_{p_L^*/\beta} = \varepsilon_{\left(1-\Delta\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)/\beta} + \varepsilon_{p_F^*/\beta} > \varepsilon_{p_F^*/\beta}$. Finally, we establish that $\varepsilon_{p_L^*/\beta} < 0$ by showing that $\frac{dp_L^*}{d\beta} < 0$. Let $x \equiv \frac{\beta}{\beta-1}\frac{\pi_M}{\pi_D}$ for compactness and first evaluate this expression:

$$\frac{dp_L^*}{d\beta} = -\frac{I}{(\beta-1)^2}\left(1 + x^{-\beta}\left(-\beta\left(\beta-1\right)\left(x-1\right)\ln x + \left(\beta-2\right)x - \left(\beta-1\right)\right)\right). \tag{45}$$

Consider $\frac{dp_L^*}{d\beta}$ as a function of $\frac{\pi_M}{\pi_D}$. After simplification,

$$\frac{\partial^2 p_L^*}{\partial\beta\partial\frac{\pi_M}{\pi_D}} = -\frac{\beta x^{-\beta}\left(\pi_M - \pi_D\right)\left(\beta\ln x - 2\right)I}{\left(\beta-1\right)\pi_M}, \tag{46}$$

so $\frac{dp_L^*}{d\beta}$ is quasiconcave in $\frac{\pi_M}{\pi_D}$ and attains a global maximum at $\frac{\beta-1}{\beta}\mathrm{e}^{\frac{2}{\beta}}$. Then, again after simplification,

$$\left.\frac{\partial p_L^*}{\partial\beta}\right|_{\frac{\pi_M}{\pi_D}=\frac{\beta-1}{\beta}\mathrm{e}^{\frac{2}{\beta}}} = -\frac{\left(1+\left(\beta-1\right)\mathrm{e}^{-2} - \beta\mathrm{e}^{-2\frac{\beta-1}{\beta}}\right)I}{\left(\beta-1\right)^2}. \tag{47}$$

The sign of the numerator in (47) depends on the sign of the expression $\Phi(\beta) \equiv 1 + (\beta-1)\mathrm{e}^{-2} - \beta\mathrm{e}^{-2\frac{\beta-1}{\beta}}$. We have $\Phi(1) = 0$ and $\Phi'(\beta) = \mathrm{e}^{-2} - \frac{\beta-2}{\beta}\mathrm{e}^{-2\frac{\beta-1}{\beta}} > 0$. Therefore, $\left.\frac{\partial p_L^*}{\partial\beta}\right|_{\frac{\pi_M}{\pi_D}=\frac{\beta-1}{\beta}\mathrm{e}^{\frac{2}{\beta}}} < 0$, from which it follows that $\frac{dp_L^*}{d\beta} < 0$. For completeness, note that trivially $\varepsilon_{p_F^*/\frac{\pi_M}{\pi_D}} = 0 < \varepsilon_{\left(1-\Delta\left(\beta,\frac{\pi_M}{\pi_D}\right)\right)/\frac{\pi_M}{\pi_D}} = \varepsilon_{p_L^*/\frac{\pi_M}{\pi_D}}$. ■

A.7 Proof of Proposition 7 *(Industry structure comparisons)*

Industry optimum:

First, we formally state the value maximization problem for the industry that is described informally in the text, that is $\max_{(y_1,y_2)\in[y,\infty)\times[\max\{y,y_1\},\infty)} \tilde{L}(y,y_1,y_2,I) + F(y,y_2,I)$ where:

$$\tilde{L}(y,y_1,y_2,I) + F(y,y_2,I) = \left(\frac{\pi_M}{r-\alpha}y_1 - I\right)\left(\frac{y}{y_1}\right)^\beta + \left(\frac{2\pi_D - \pi_M}{r-\alpha}y_2 - I\right)\left(\frac{y}{y_2}\right)^\beta. \tag{48}$$

This objective is separable and quasi-concave in its arguments over its domain of definition. If $\pi_M/\pi_D < 2$, the optimum solution is interior, whereas if $\frac{\pi_M}{\pi_D} \geq 2$, $\frac{\partial\left(\tilde{L}+F\right)}{\partial y_2} > 0$ for all $y_2 > y$, so $y_2^* = \infty$. When the solution is interior, $\frac{y_F^*}{y_2} = \frac{\beta}{\beta-1}\left(2 - \frac{\pi_M}{\pi_D}\right)$ from which the corresponding expression in the text follows.

Threshold rankings:

The rankings for the first firm triggers ($\tilde{y}_P < y_P^* = y_1^* = y^* < y_S^*$) follow directly from Propositions 1 and 5, as well as the properties of standard preemption races. The follower trigger rankings similarly result from the comparison of the values that are given in the text.

Industry structure rankings:

The industry value under preemption with an upstream input supplier is:

$$\begin{aligned}\tilde{L}\left(y, y_P^*, y_F^*, p_L^*\right) + F\left(y, y_F^*, p_F^*\right) + \tilde{W}\left(y, p_L^*, p_F^*\right) &= \tilde{L}\left(y, y_P^*, y_F^*, I\right) + F\left(y, y_F^*, I\right) \\ &= \frac{I}{\beta - 1}\left(\frac{y}{y_P^*}\right)^{\beta} + \left(\left(\frac{\beta}{\beta-1}\right)^2\left(2 - \frac{\pi_M}{\pi_D}\right) - 1\right) I \left(\frac{y}{y_F^*}\right)^{\beta}. \end{aligned} \tag{49}$$

It is this expression that we successively compare with industry value under bilateral monopoly and standard preemption, as follows.

(i) After simplification, $\tilde{L}\left(y, y_P^*, y_F^*, I\right) + F\left(y, y_F^*, I\right) > V\left(y, y_S^*, p_S^*\right) + W\left(y, p_S^*\right)$ if and only if:

$$(2\beta - 1)\left(\frac{1}{f(\beta)} - 1\right)\left(\frac{\pi_M}{\pi_D}\right)^{\beta} - \beta^2\frac{\pi_M}{\pi_D} + \beta^2 + 2\beta - 1 > 0. \tag{50}$$

Let $\Psi\left(\beta, \frac{\pi_M}{\pi_D}\right)$ denote the expression on the left-hand side of the inequality (50), and recall that $\lim_{\infty} f\left(\beta\right) = \frac{2}{\mathrm{e}}$. Since the first term dominates, $\lim_{\beta \to \infty} \Psi\left(\beta, \frac{\pi_M}{\pi_D}\right) = \lim_{\frac{\pi_M}{\pi_D} \to \infty} \Psi\left(\beta, \frac{\pi_M}{\pi_D}\right) = \infty$. To see that $\Psi\left(\beta, \frac{\pi_M}{\pi_D}\right)$ can take negative values, observe that $\Psi\left(\beta, \frac{\pi_M}{\pi_D}\right)$ is continuous and $\Psi\left(1, \frac{\pi_M}{\pi_D}\right) = 2 - \frac{\pi_M}{\pi_D}$.

(ii) After simplification, $\tilde{L}\left(y, y_P^*, y_F^*, I\right) + F\left(y, y_F^*, I\right) > 2\tilde{V}\left(y, I, I\right) \left(= \tilde{L}\left(y, \tilde{y}_P, \tilde{y}_F, I\right) + F\left(y, \tilde{y}_F, I\right)\right)$ if and only if:

$$\frac{\beta^{\beta}}{(\beta-1)^{\beta-1}}\left(\left(\frac{\pi_M}{\pi_D}\right)^{\beta} - 2\right) - \beta^2\frac{\pi_M}{\pi_D} + \beta^2 + 2\beta - 1 > 0. \tag{51}$$

Let $\Omega\left(\beta, \frac{\pi_M}{\pi_D}\right)$ denote the expression on the left-hand side of the inequality (51). The first term dominates so $\lim_{\beta \to \infty} \Omega\left(\beta, \frac{\pi_M}{\pi_D}\right) = \lim_{\frac{\pi_M}{\pi_D} \to \infty} \Omega\left(\beta, \frac{\pi_M}{\pi_D}\right) = \infty$. To see that the inequality can be violated, take $\Omega(\beta, 1) = (2\beta - 1)\left(1 - \frac{1}{f(\beta)}\right)$, which is negative since $f\left(\beta\right) \in \left(\frac{2}{\mathrm{e}}, 1\right)$. ■

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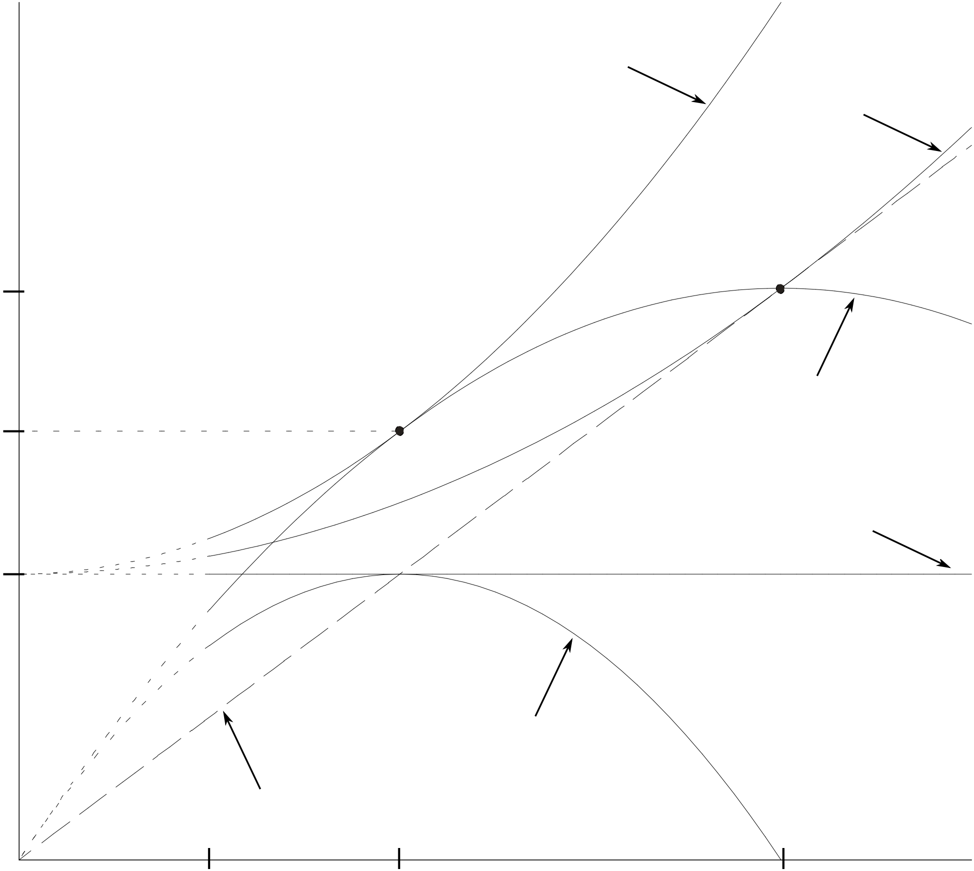
Figure(s)



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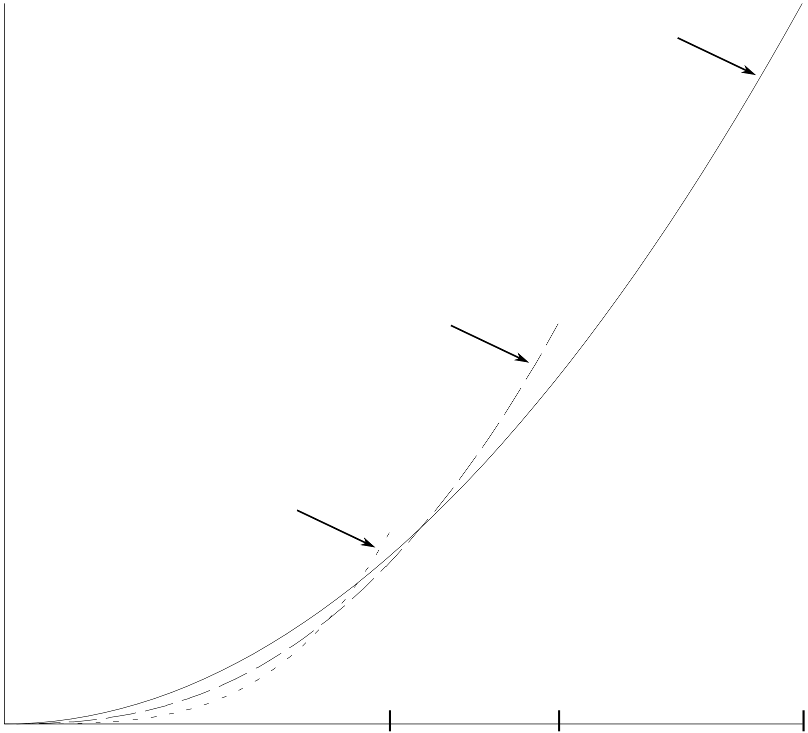

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