Outperforming the naïve Random Walk forecast of foreign exchange daily closing prices using Variance Gamma and normal inverse Gaussian Levy processes

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26 June 2013

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Abstract. This work demonstrates that forecast of foreign exchange (FX) daily closing prices using the normal inverse Gaussian (NIG) and Variance Gamma (VG) Levy processes outperform the naïve Random Walk model. We use the open software R to estimate NIG and VG distribution parameters and perform several classical goodness–of -fits test to select best models. Seven currency pairs can be forecasted by both Levy processes: TND/GBP, EGP/EUR, EUR/GBP, EUR/JPY, JOD/JPY, USD/GBP, and XAU/USD, while USD/JPY and QAR/JPY can be forecasted with the VG process only. RMSE values show that NIG and VG forecast are comparable, and both outperform the naïve Random Walk out of sample. Appended R-codes are original.

Keywords: Levy process, NIG, VG, forecasting, goodness of fits, foreign exchange.

JEL Classification: C16
AMS Classification: 60G51

1 Introduction

There are many models purporting to describe the evolution of exchange rates as a function of different macro-economic fundamentals like prices, money, interest rates, productivity differentials, government debts, terms of trade, net foreign assets, etc. These models are typically limited by the fact that all macro-economic fundamentals cannot be included; thus such models are usually a simplification of the economy [1]. Fundamentals are capable of explaining exchange rates, but hardly are good in forecasting. Also, factors relating to the behavior of market participants are usually neglected; something captured, modeled and replicated in the framework of standard asset pricing model in [8]. It must be pointed out that the famous Meese-Rogoff [3,10] paper advocating the Random Walk model as the best model for exchange rate forecasting was never really about ex-ante forecasting in its true sense i.e. using time t information to forecast exchange rates at time $t+1$. In fact, Meese-Rogoff's regressions were more about concurrent explanations, with the only "forecasting" element being their reliance on ex-ante data to estimate equation parameters [3,10].

Micro based models in contrast, focus on the processes through which information affect prices. First, transaction flows contain information relevant to fundamentals and second, market markers adjust prices based on information in such a way that these are reflected in spot prices [1]. Our approach is inspired by Charles Engel and Kenneth West (2004) [3] who suggests an approach based on asset prices. They advocate that if prices are I(1), the exchange rate will follow a process arbitrarily close to a Random Walk (RW). This is because I(1) processes can be split into RW and stationary processes. The class of Levy processes we use here can be approximated by RW's. These Levy processes have dynamics capable of capturing FX dynamics arising from exceptional circumstances.

Modeling and forecasting daily prices with NIG-Levy process has been demonstrated for index prices in [14] and for FX in [15]. Considering that FX prices often exhibit unique qualities not typical of other assets prices (see [9],[11] for detailed exposition), we extend results of [15] by demonstrating that seven currency pairs can be forecasted by both NIG and VG processes: TND/GBP, EGP/EUR, EUR/GBP, EUR/JPY, JOD/JPY, USD/GBP, and XAU/USD, while USD/JPY and QAR/JPY can be forecasted with the VG process only. We also report that forecast of daily closing prices of USD/GBP for a sixty day period using NIG and VG processes outperform the naïve Random Walk approach looking at RMSE values.

The section below reviews the construction of NIG and VG processes starting with the definition of a general Levy process, time changes and approximation of Levy processes by RW's. Next, we propose an implement a forecasting mechanism. Appended R codes are original.

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2 NIG and VG processes

Definition 1 (Levy process). *A real valued stochastic process* $X_t = X(t,\omega), t \geq 0, \omega \in \Omega$ *defined on a probability space* (Ω, F, P) *is called a one dimensional Levy process if it satisfies the conditions:* $X_0 = 0$ *a.s., X has independent and stationary increments with stochastically continuous paths, and X has paths continuous from the right with left-sided limits. In fact, for any time t > s, the distribution of the increments* $(X_t - X_s)$ *depends only on the length of the interval (t-s) and* $(X_t - X_s)$ *is independent of* $(X_u, u \leq s)$. Standard and linear Brownian motion, Poisson and compound Poisson processes are examples of Levy processes [6,13].

Definition 2 (Infinite divisibility). *A real value random variable X is infinitely divisible if for each* $n \in N$, *there exist a sequence of i.i.d. random variables* $X_{1,n}, X_{2,n}, \ldots, X_{n,n}$ *such that X is distributed as* $X_{1,n} + X_{2,n} + \ldots + X_{n,n}$. Complete characterization of an infinitely divisible distribution in terms of its characteristic exponent is given by the Levy-Khintchine theorem defined below.

Theorem 1 (Levy-Khintchine theorem). *A random variable* X_t *is infinitely divisible if and only if there exist a triplet (b, c, v) such that the characteristic exponent of* X_t *is of form* $Ee^{iuX_t} = e^{t\psi(u)}$ *where*

$$\psi(u) = ibu - \frac{(uc)^2}{2} + \int_{-\infty}^{+\infty} \left(e^{iux} - 1 - iux1_{\{|x|\leq 1\}}\right) v(dx) \tag{1}$$

$b, c \in \mathcal{R}$ and b is a drift term, c a diffusion term and v a positive measure on $\mathcal{R}/\{0\}$ such that $\int_{\mathcal{R}/\{0\}} (1 \wedge |x|^2) v(dx) < \infty$. $\psi(u)$describes the path properties of a Levy process [13]. A well known result of Levy processes is that any Levy process can be represented as an independent sum of Brownian motion ($\mu t + \sigma B_t$) and a compound Poisson-like process [12]. Without dwelling on the technicalities and restriction imposed on $\psi(u)$, we turn our focus to random walk approximation before stating another important theorem necessary for understanding time changes.

Random Walk approximation: Let us consider the Levy process X_t and apply the Levy-Khintchine formula above. Suppose $c = 0$ in $\psi(u)$ and we have an infinite Levy measure v. Fixing the time domain $[0, T), n \geq 1$ and letting $h = \frac{T}{n}$, we generate increments $\Delta_i^h X = X(jh) - X\big((j-1)h\big)$ as independently identical random variables [8] with distribution $P_h(.) = P(X(h) \in .)$, $j = 1, \ldots, n-1$. Let

$$X^h(t) = \begin{cases} 0, & if\ 0 \leq t < h, \\ \Delta_1^h X + \cdots + \Delta_j^h X, & if\ jh \leq t < (j+1)h, \end{cases}$$

then the process $\{X^h(t)\}_{0 \leq t < T}$ is a RW approximation to $\{X(t)\}_{0 \leq t < T}$. This approximation illuminates the connection with I(1) processes.

Theorem 2. *Let* X_t *for* $t \geq 0$ *be a Levy process with characteristic exponent* ψ *and let* τ_s, $s \geq 0$ *be an independent subordinator*[2] *with characteristic exponent* Φ. *Then the process* $Y_s = X_{\tau_s}$ *is again a Levy process with characteristic exponent* $\Phi o \psi$. This theorem is the basis of definition and use of time changes below[7].

Definition 3 (Time Change). *Consider the Levy process* $X(t), t \geq 0$ *and let T(t) be a subordinator. Then the process described by* $X(t) = Z(T(t))$ *is called a subordinated process, with the process Z called a directing process.* This directing process usually provides the link between business time and real time i.e. at real time $t \geq 0$, $T(t)$ of business time units passed and the value of our asset is positioned at $Z\big(T(t)\big)$. The directing process we use is Brownian motion. This is because Levy processes are semi-martingales and any semi-martingale can be written as a time changed Brownian motion. There are also many directing processes that can be used. A detailed analysis is given in [7].

Deviating slightly, the no-arbitrage assumption implies the existence of a probability measure under which the discounted stock prices are martingales. This means stock prices are martingales under real probability measure; implying $Ln(S(t)) = Z(T(t))$. This clearly represents how asset prices respond to information arrival [6]. Some days, very little good or bad news is released causing trading to typically slow and causing prices to barely fluctuate. Other days, trading is brisk and price evolution accelerates as traders adjust their expectations to arrival of new information [2,6]. Thus, the price process is seen as instantaneously and continuously adjusting to exogenous demand and supply shocks [7]. This accounts for some of the reasons why uncertainty in the economy is often represented by a filtered probability space $(\Omega, F, F_t, \mathrm{P})$ where F_t is the filtration of information available at time *t* and *P* is the real probability measure [2, 6]. There is also a net flow of information from long to short time scales and the behavior of long-term traders influences those of short-term traders. This is the basis for [8]. Let

[2] A subordinator is a strictly non-decreasing Levy process.

us proceed with defining the inverse Gaussian distribution and Gamma process; basis to define and understand the NIG and VG processes.

Definition 4 (Inverse Gaussian distribution). *A random variable X has inverse Gaussian (IG) distribution with parameters* υ *and* λ *if its density is of form*

$f(x;\upsilon,\lambda)=\sqrt{\frac{\lambda}{2\pi x^3}}\,e^{-\frac{\lambda(x-\upsilon)^2}{2x\upsilon^2}}$ *and* $\lambda > 0$ *is shape parameter,* $\upsilon > 0$ *is its mean while* $x > 0$.

Definition 5 (Gamma process). *The density of a gamma (G) process*[3] *with mean rate t and variance* υt *is given by*

$f(g)=\frac{g^{\left(\frac{t}{\upsilon}-1\right)}e^{\left(-\frac{g}{\upsilon}\right)}}{\Gamma\left(\frac{t}{\upsilon}\right)\upsilon^{\frac{t}{\upsilon}}}$ *where* $\Gamma(x)$ *denotes the classical gamma function.*

Armed with definitions 1-5 and theorems 1 and 2, we are now ready to fully define NIG and VG processes.

NIG process: Consider a Brownian motion Z with drift θ and volatility σ. If we consider a random time change which follows $IG(t;\upsilon,1)$, NIG process can be defined as

$$X_{NIG}(t;\sigma,\upsilon,\theta)=\theta IG(t;\upsilon,1)+\sigma Z\big(IG(t;\upsilon,1)\big). \quad (2)$$

$Ee^{iuX_{NIG}(t)}=e^{\left(-t\sigma\sqrt{\left(\frac{\upsilon}{\sigma}\right)^2+\frac{\theta^2}{\sigma^4}-\left(\frac{\theta}{\sigma^2}+iu\right)^2}-\frac{\upsilon}{\sigma}\right)}$. Using the substitutions $\beta=\frac{\theta}{\upsilon^2},\sigma=\delta,\alpha^2=\left(\frac{\upsilon}{\sigma}\right)^2+\frac{\theta^2}{\sigma^4}$, the NIG process can be rewritten as

$$X_{NIG}(t;\alpha,\beta,\delta,\upsilon=0)=\beta\delta^2 IG\big(t;\delta\sqrt{\alpha^2-\beta^2},1\big)+\delta Z\left(IG\big(t;\delta\sqrt{\alpha^2-\beta^2},1\big)\right), \quad (3)$$

more suitable for computations. With (3), we get the mean = $\frac{\beta\delta}{\sqrt{\alpha^2-\beta^2}}$, variance = $\alpha^2\delta(\alpha^2-\beta^2)^{\frac{-3}{2}}$,

skew = $\frac{3\beta(\alpha\delta)^{-1}}{4\sqrt{\alpha^2-\beta^2}}$ and kurtosis = $3\left(1+\frac{\alpha^2+4\beta^2}{\delta\alpha^2\sqrt{\alpha^2-\beta^2}}\right)$.

VG process: Alternatively, if we consider a random time change which is a Gamma process with mean rate unity and variance rate , we get VG process as

$$X_{VG}(t;\sigma,\upsilon,\theta)=\theta G(t;\upsilon,1)+\sigma Z\big(G(t;\upsilon,1)\big). \quad (4)$$

$Ee^{iuX_{VG}(t)}=\left(\frac{1}{1-i\theta\upsilon u+\upsilon\frac{(\upsilon\sigma)^2}{2}}\right)^{\frac{1}{\upsilon}}$ with $\lambda=\frac{(\sigma u)^2}{2}-i\theta u$, variance = $\theta\upsilon^2+\sigma^2$, skew = $2(\upsilon\theta)^2\theta+3\theta\upsilon(\sigma)^2$, and

kurtosis = $3variance^2+12\sigma+3\upsilon\sigma^4$.

It is easy to see that with proper parameters (σ,υ,θ), the processes 3 and 4 propagate with time. This means if we know (σ,υ,θ) at time *t*, we can let the process propagate to time *t+1*, providing us with values out of sample. This model intrinsically assumes there will be no changes to the values of estimated parameters before time *t+1* or beyond. Using the same estimated parameters for long periods i.e. beyond a day in our case will not reflect the arrival of new information, neither ongoing changes in the market and will put traders at risk. We consider daily closing prices only and forecast of next day closing prices on a rolling basis. Data used for parameter estimation are for daily closing prices between 29/11/2007 and 22/07/2011; covering parts of the recent financial crises. In sample forecast is of little appeal to us. We use data from 29/11/2007 to 28/04/2011 (900 data points) as our training sample and the remaining (60 data points) as test sample. Below, we outline our procedure for out of sample forecast.

3 Model selection, analysis and forecasting

We follow a similar procedure as in [14,15] and it is outlined in simple terms as follows:
1) Select distribution based on known characteristics of data. 2) Estimate parameters of distribution. 3) Carry out classical goodness of fits test i.e. Kolmogorov-Smirnov and Anderson Darling. 4)Select good models i.e. those passing classical goodness of fits test. 5) Develop forecasting codes. 6) check RMSE values.

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[3] A gamma process is a random process with independent gamma distributed increments.

Table A.2 outlines the summary statistics of our return data. With these, it is clear that our distribution must be heavy tailed, and just like the NIG, the VG has semi heavy tails. Parameters are estimated using nigFit,vgFit functions of the ghyp and VarianceGamma packages in R-version 2.15.3 respectively. These can be varied so as to get better fit. Tables A.1 and A.3 display estimated parameters, Kolmogorov-Smirnov and Anderson-Darling test results for VG and NIG processes respectively. Looking at the P-values, there is clear significance at the 5% level; quite intriguing as these two distributions are quite different. Forecast is of 60 values are done on a rolling basis. RMSE[4] values are then calculated. Using our basic code, results for NIG and VG forecast respectively are

```
> RiMSE(60,100)
[1] 0.003184383
> RiMSE(60,1000)
[1] 0.003246613
> RiMSE(60,10000)
[1] 0.003235002
```

```
> RiMSE(60,100)
[1] 0.00330961
> RiMSE(60,1000)
[1] 0.003242958
> RiMSE(60,10000)
[1] 0.003230029
```

Random walk forecast is estimated using the "forecast" package of the newly released R-version 3.0.1 in RStudio interface.Simple code for RW forecast is

```
> USD.GBP1 <- read.delim("C:/Users/Rehoboth/Desktop/Exchange rates/USD-GBP1.txt")
>   View(USD.GBP1)
> library(forecast)
> plose1<-USD.GBP1$CLOSE
> dlose1<-ts(plose1)
> fit1<-rwf(dlose1[1:900],h=60)
> accuracy(fit1,dlose1[901:960])
                       ME       RMSE         MAE      MPE      MAPE     MASE
Training set 0.0001263626 0.00497182 0.003595106 2.032913 57.925703 1.000000
Test set     0.0177733333 0.01891583 0.017773333 2.873276  2.873276 4.943758
                   ACF1
Training set 0.05147218
Test set             NA
```

It is clear from the RMSE values above that both NIG and VG forecast outperform the naive RW. It is also crystal clear that forecast from NIG and VG Levy processes are comparable up to four decimal places. This results are a clear indication on the inefficiency of RW model as compared to more robust estimates.

FX	USD/ JPY	EGP/ EUR	EUR/ GBP	EUR/ JPY	JOD/ JPY	QAR/ GBP	TND/ GBP	USD/ GBP	XAU/ USD
vgC	0.0003	-3.11e-5	1.83e-5	0.002799	-4.149e-4	-0.0011	-0.001	-0.001	0.0052
sigma	0.0069	7.941e-3	5.98e-3	0.008827	7.846e-3	0.0074	0.0074	0.0074	0.0109
theta	0.0001	5.152e-5	-1.13e-4	-0.00298	8.918e-5	0.0014	0.0015	0.0016	-0.004
nu	1.1803	.2997	.1586	0.2743	.2500	0.3015	0.3311	0.30	0.1256
KS D	0.035	0.04	0.039	0.028	0.042	0.033	0.037	0.048	0.048
KS P	0.572	0.3998	0.4318	0.8275	0.3404	0.6469	0.4997	0.199	0.1991
AD P	0.4448	0.3166	0.3891	0.4804	0.30697	0.5066	0.424	0.244	0.191

Table A.1 Estimated VG parameters, Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test values

FX	Mean	Variance	Skew	Kurtosis
EGP/EUR	-4.29e-5	3.865e-6	-0.431	11.05
EUR/GBP	-0.0002	8.20e-5	-0.262	1.1422
EUR/JPY	-0.0028	7.75e-5	1.204	15.10
JOD/JPY	0.00048	0.00012	0.1752	5.491
QAR/GBP	-1.62e-6	4.034e-7	0.219	7.141
TND/GBP	0.0003	5.875e-5	0.5061	2.604
USD/GBP	0.000297	5.86e-5	0.5031	2.6171
XAU/USD	0.000789	0.000127	-0.31995	1.9890
USD/JPY	-0.00013	4.189e-5	0.274	2.715

Table A.2 Summary statistics of FX returns

Code fragment for forecasting:

This code is used after first estimating relevant parameters by maximum likelihoods. *s0* is the last closing price while m is the number of prices we average. We set m=10,000 for all the computations above.

[4] RiMSE below is just RMSE. We use this in our program code to calculate RMSE values.

VG forecast: library(VarianceGamma); library(fBasics)
forecaster1=function(s0,m){ matah<- rep(0,m); for (i in 1:m){ matah[i]<-(1+rvg(1, vgC = 0, sigma = 1, theta = 0, nu = 1,param = c(vgC,sigma,theta,nu)))*s0}; return(mean(matah))}
NIG forecast [14,15]: forecaster=function(alpha,beta,delta,mu,s0,m){ matah<- rep(0,m); for (i in 1:m){ matah[i]<-(1+rnig(1,alpha,beta,delta,mu))*s0}; return(mean(matah))}

FX	TND/ GBP	EGP/ EUR	EUR/ GBP	EUR/ JPY	JOD/ JPY	USD/ GBP	XAU/ USD
alpha	151.045	205.1585	201.7434	173.3203	124.4768	*151.2416*	85.0122
beta	16.631	-0.9756	0.2773	-30.845	-0.805378	16.2792	-8.948
delta	0.00865	0.0130	0.007583	0.0136	0.00831	0.00865	0.01088
mu	-0.00066	7.88e-05	-0.0001	0.00227	-0.000271	-0.00064	0.00195
KS D	0.024	0.0311	0.0361	0.0361	0.019	0.0371	0.0431
KS P	0.9351	0.7214	0.5348	0.5348	0.9936	0.4991	0.3124
ADP	0.62032	0.36019	0.53921	0.48450	0.62478	0.40622	0.36348

Table A.3 Estimated NIG parameters, Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) test values

4 Conclusion

We have demonstrated that seven currency pairs: TND/GBP, EGP/EUR, EUR/GBP, EUR/JPY, JOD/JPY, USD/GBP, and XAU/USD, can be forecasted with both the NIG and VG processes, while USD/JPY and QAR/JPY can be forecasted with the VG process only. We have also demonstrated that NIG and VG forecast beat the naïve Random Walk approach. Further research will concentrate on exploring and forecasting with other Levy processes and comparing these with traditional forecasts results.

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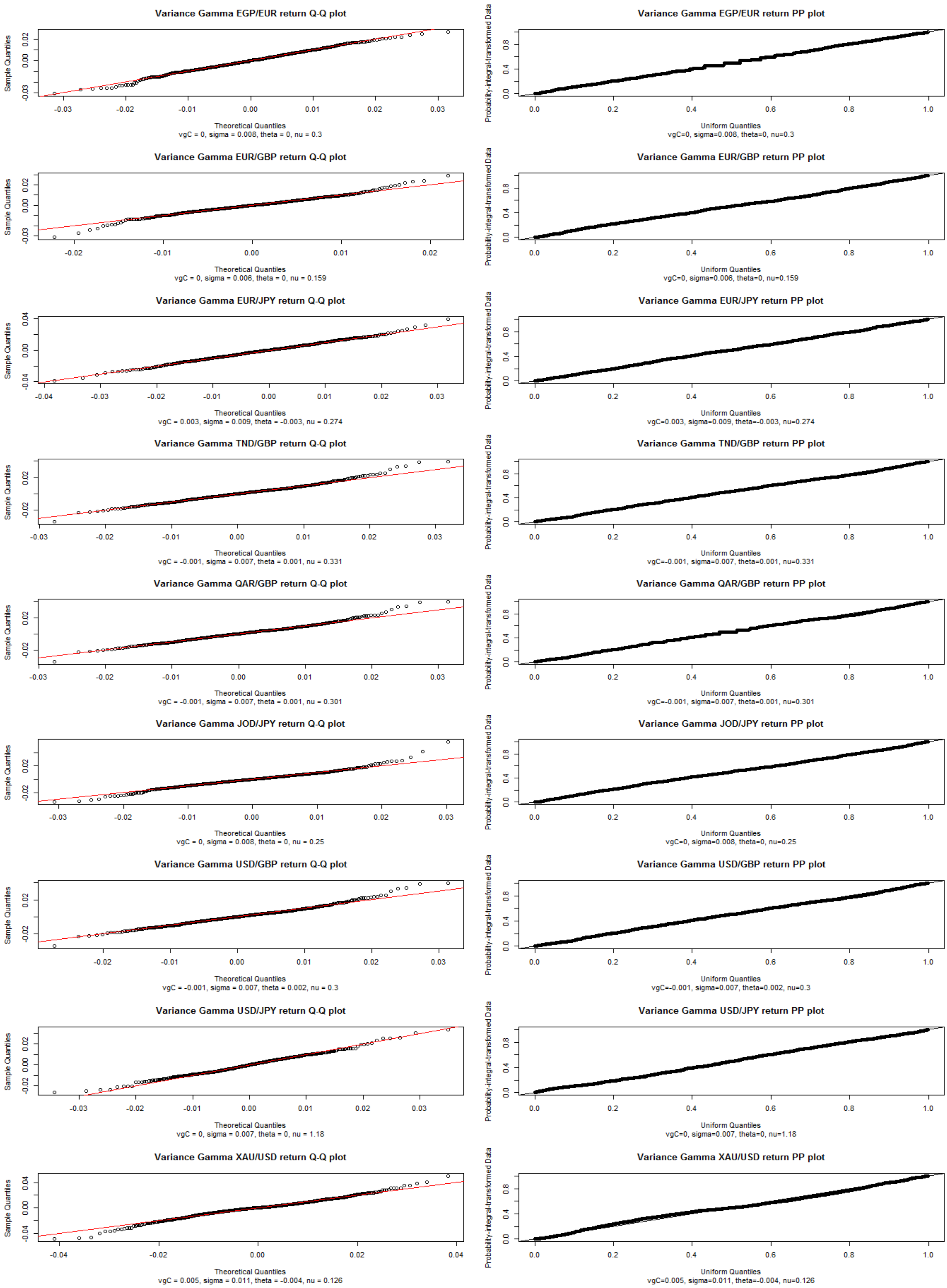

Figure A Variance Gamma QQ and PP plots for nine FX currency pairs