NIG-Levy process in asset price modeling: case of Estonian companies

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Abstract. As an asset is traded at fair value, its varying price trace an interesting trajectory reflecting in a general way the asset's value and underlying economic activities. These trajectory exhibit jumps, clustering and a host of other properties not usually captured by Gaussian based models. Levy processes offer the possibility of distinguishing jumps, diffusion, drift and the laxity to answer questions on frequency, continuity, etc. An important feature of normal inverse Gaussian-Levy (NIG-Levy) model is its path richness: it can model so many small jumps in a way that eliminates the need for a Gaussian component; hence, limitations arising from Gaussian based models are almost eliminated. Secondly, the characteristics listed above are reflected in the Levy triplet and are easily introduced in the modeling picture through estimated Levy parameters. Thirdly, knowledge of NIG-Levy parameters enables us to use NIG-Levy models as underlying asset price models for pricing financial derivatives.
We use the R open software to calculate Levy parameters for 12 Estonian companies and choose good NIG-Levy asset price models by the method proposed by Käärik and Umbleja (2011). We observe that not all financial data of Estonian companies trading on the Tallinn Stock Exchange between 01 Jan 2008 – 01 Jan 2012 can be effectively modeled by NIG-Levy process, despite having Levy parameters. Those positively modeled are recommended as underlying assets for pricing financial derivatives.
Keywords: NIG, goodness of fits test, fitting price process.

JEL Classification: C68, C29
AMS Classification: 60G51, 65C20

1 Introduction

As an asset is traded at fair value, its varying price trace an interesting trajectory reflecting in a general way the asset's value and underlying economic activities [4]. These trajectory exhibit jumps, clustering and a host of other properties not usually captured by Gaussian based models. Levy processes offer the possibility of distinguishing jumps diffusion, drift and the laxity to answer questions on frequency, continuity, etc. An important feature of normal inverse Gaussian-Levy (NIG-Levy) model is its path richness: it can model so many small jumps in a way that eliminates the need for a Gaussian component; hence, limitations arising from Gaussian based models are almost eliminated [2, 6, 7].

Brownian motion or Gaussian based models are known to have serious limitations like light tails, inability to effectively capture jumps, model stochastic volatility, clustering, and a host of others extensively discussed in [2, 3]. Particularly important is their inability to make a clear distinction between rare large jumps and small but frequent jumps; one of the motivations calling out for the use of more efficient functions to model asset prices. One may even guess that in a small economy like Estonia, asset prices may not very much reflect global trends like market data collected from big economies with uncountable number of factors affecting asset prices. In this light, there is a possibility to think local elections may be reflected more in the prices of assets than Iran's current turbulent relation with the west affecting the prices of oil on the world market. As this event based approach is the case, we need more responsive functions to reflect such trends than ordinary Gaussian based models.

We calculate NIG-Levy process parameters by maximum likelihood method and apply Käärik and Umbleja [5] proposed strategy for selecting good models. We observe that the asset price trajectories of two companies (Baltika and Ekpress Grupp) trading on the Tallinn stock exchange between 01 Jan 2008 – 01 Jan 2012 can be modeled by NIG-Levy process. Thus their stochastic properties are effectively described by NIG-Levy model and these price models can be used by investors in constructing optimal portfolios.

The following section answers the question why we need normal inverse Gaussian Levy process for modelling asset prices. It briefly touches on general characteristics of Levy processes before expanding with the NIG-

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Levy process. Analysis of data and selection of model is the subject of section three. Some useful graphs are appended in the appendix.

2 Why normal inverse Gaussian Levy process for modeling asset prices?

The normal inverse Gaussain Levy process is a member of the general class of Levy processes. By definition, a stochastic process X_t is a Levy process if has stationary and independent increments, and has stochastically independent paths. Mathematically speaking, a Levy process is a continuous time stochastic process $X = \{X_t: t \geq 0\}$ defined on the probability space (Ω, F, P) with the following basic properties:

1) $P(X_0 = 0) = 1$ i.e the process starts at zero;
2) $\forall\, s, t \geq 0, X_{s+t} - X_t$ is distributed as X_s i.e. stationary increments;
3) $\forall\, s, t \geq 0, X_{s+t} - X_t$ is independent of X_u, $s \leq t \leq u$ i.e. independent increments;
4) $t \to X_t$ is a.s. right continuous with left limits.

The above properties clearly describe closing price properties, and their study in the general framework of Levy processes reveals useful properties. Levy processes present us with models that eliminate most of the weaknesses of Gaussian models. The make clear distinctions between large and small jumps and are able to capture small jumps in a way that we do not need a Brownian motion component. Further, they present possibility of answering questions related to frequency, size, continuity as well as distinguishing between drift, Brownian motion and Poisson based components making each of these only a limiting case [6,7]. Associated with Levy processes are infinitely divisible probability distributions, with the possibility of skewed shapes and slow decaying tails that perfectly fit log-return data [1, 2, 6]. Some of these properties can be seen by applying the famous Levy-Ito decomposition theorem and arriving at the Levy-Khinchine representation of a general Levy process as can be seen clearly in [1, 6] and for the NIG-Levy case presented in equation (3) below.

NIG-Levy process definition and basic characteristics

NIG-Levy process with parameters $\alpha, \beta, \mu, \delta$ and condition $-\alpha < \beta < \alpha$ denoted $NIG(\alpha, \beta, \delta, \mu)$ can be defined as follows:
Consider a bivariate Brownian motion (u_t, v_t) starting at point $(u, 0)$ and having constant drift vector (β, γ) with $\gamma > 0$ and let z denote the time at which v_t hits the line $v = \delta > 0$ for the first time (u_t, v_t are assumed independent). Then letting $\alpha = \sqrt{\beta^2 + \gamma^2}$, the law of u_z is $NIG(\alpha, \beta, \delta, \mu)$ [1]. This distribution has probability density of the form

$$f_{NIG}(x: \alpha, \beta, \delta, \mu) = \frac{\alpha\delta}{\pi} e^{\left(\delta(\alpha^2-\beta^2)^{1/2} - \beta(x-\mu)\right)} \frac{K_1\left(\alpha(\delta^2 + (x-\mu)^2)^{1/2}\right)}{(\delta^2 - (x-\mu)^2)^{1/2}} \tag{1}$$

where $K_1(x) = \frac{x}{4}\int_0^\infty e^{\left(t + \frac{\gamma^2}{4t}\right)} t^{-2}\, dt$ is a modified Bessel function of the third kind. $\delta > 0$ is scale, $\beta \geq 0$ is symmetry, μ is location and $\alpha > 0$ is tail heaviness. It has a moment generating function of the form

$$M(x: \alpha, \beta, \delta, \mu) = exp\left[\delta\left\{\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + x)^2}\right\} + x\mu\right], \tag{2}$$

mean $= \mu + \frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}$, variance $= \alpha^2\delta(\alpha^2 - \beta^2)^{-3/2}$, skewness $= 3\beta\alpha^{-1}\delta^{-1}(\alpha^2 - \beta^2)^{-1/4}$ and kurtosis $= 3\left(1 + \frac{\alpha^2 + 4\beta^2}{\delta\alpha^2\sqrt{\alpha^2-\beta^2}}\right)$ [6]. A useful property is the fact that if $x_1, x_2, x_3, \ldots, x_m$ are independent normal inverse Gaussian random numbers with common parameters α, β but having individual location and scale parameters u_i and $d_i (i = 1, \ldots, m)$, then $x_+ = x_1 + \cdots + x_m$ is again distributed according to a normal inverse Gaussian law, with parameters $(\alpha, \beta, \delta_+, u_+)$ [1]. Thus closing prices can be drawn from such a distribution in two ways. First, each closing price can be drawn from a different NIG-Levy distribution with individual location and scale parameters but all the corresponding NIG-Levy distributions have common α, β parameters. Second, all the closing prices can be drawn from the same NIG-Levy distribution with unique $(\alpha, \beta, \delta, \mu)$ parameters. We employ the second method in this work.

The characteristic function of $NIG(\alpha, \beta, \delta, \mu)$ can be represented as $E[e^{iuX_t}] = e^{t\psi(u)}$ giving us the Levy-Khinchine representation as

$$\psi(u) = \int_{|x| \geq 1} (1 - e^{iux}) f(x: \alpha, \beta, \delta)\, dx + \int_{|x| < 1} (1 - e^{iux} - iux) f(x: \alpha, \beta, \delta)\, dx + iu\rho \tag{3}$$

where $f(x{:}\,\alpha,\beta,\delta) = \frac{\delta\alpha}{\pi|x|}\exp(\beta x)K_1(\alpha|x|)$ and $\rho = \frac{2\delta\alpha}{\pi|x|}\int_0^1 \sinh(\beta x)\, K_1(\alpha x)$ and $\mu = 0$ here. The first term of $\psi(u)$ (equation 3) capture big jumps while the second term captures small jumps [1].

Considering that closing prices of assets can be drawn from a Levy process with defined characteristics, the above mentioned stylized facts not generally captured by Brownian models are incorporated in Levy models through the characteristic triplet. This in itself means that Levy models are more effective than pure Gaussian models. Secondly, determination of Levy parameters means we can represent asset price process with Levy model. It must be injected here that this says nothing about the effectiveness of the model, only that it is better than pure Gaussian models. Thus, we still need goodness of fits test and other strategies to select best model.

3 Selecting best fit asset price model

We used the method proposed by Käärik and Umbleja [5] for selecting best asset price models. Below, the method is outlined.

Käärik and Umbleja proposed method for selecting best models

1. choose a suitable class of distributions (using general or prior information about the specific data) ;
2. estimate the parameters (by finding maximum likelihoods);
3. estimate goodness of fit;
   a) visual estimation,
   b) classical goodness-of-fit tests (Kolmogorov-Smirnov, chi-squared with equiprobable classes),
   c) probability or quantile-quantile plots.

This method was proposed as a general method in the fitting of insurance claim data to distributions. The distributions they used had tails, with some having really heavy-tails and even belonging to the class of subexponential distributions. But the proposed method is general. It turns out that NIG-Levy process has semi heavy tails, thus a good candidate for using the model. The subject of proving whether NIG-Levy process is subexponential or not is still being investigated, even though there is considerable literature showing that certain aspects of the NIG-Levy process can be restricted to fulfill subexponentiality. As well, analysis of data for different companies revealed the presence of tails. These and other observable properties are discussed below.

Company	**Alpha (α)**	**Beta (β)**	**Delta (δ)**	**Mu (μ)**
Arco Vara	468.90	468.86	0.03	0.02
Baltika	7.06	6.62	0.22	0.52
Ekpress Grupp	2.68	2.15	0.49	0.85
Harju Elekter	3.20	-2.07	0.72	2.95
Tallinna Kaubamaja	145.95	-145.06	0.55	9.75
Tallinna Vesi	107.80	106.86	0.85	2.92
Trigon	2953.24	2952	0.01	0.21
Nordecon	4.78	4.39	0.38	0.86
Viisnurk	208.64	-203.95	0.32	2.70
Olympic Entertainment Grupp	1107.89	1106.56	0.07	-0.03
Silvano Fashion Grupp	35.31	-29.589	8.11	14.43
Tallink	1588.86	356.46	49.54	-10.82

Table 1 Estimated NIG-Levy parameters for different companies trading on Tallinn stock exchange between 01/01/2008 and 01/01/2012

Analysis and selecting best models

To implement step one; we analyzed the kurtoses and skews of different companies. The skews of four companies (Harju Elekter, Tallinna Kaubamaja, Trigon, and Viisnurk) were negative indicating left tails while the remaining were positive indicating right tails. Thus data told us to use distribution that can capture tails. This is very good as the NIG-Levy process can capture tails. Theoretically, this feature comes to light when asymptotic relations for Bessel functions are applied. Next, kurtoses were looked at. None of the companies had a zero kurtosis thus data was non-normal. Of the positive kurtoses, Baltika and Ekpress Grupp had highest peaks. Hence, data suggested using distribution that has peaks and can capture tails. Peaks are essentially captured by NIG-Levy process as these represent jumps of the process. Both large and small jumps are captured.

Parameters for NIG-Levy process were estimated using maximum likelihoods as suggested and results for different companies are displayed on Table 1.

Company	**χ2 statistic**	**χ2 p-value**	**K-S D-value**	**K-S p-value**	**Skew**	**Kurtosis**
Arco Vara	2251.60	P < 0.00001	0.23	p < 0.00001	0.38	-1.53
Baltika	1771.12	P < 0.00001	0.06	p = 0.05723	1.67	2.33
Ekpress Grupp	1194.24	P < 0.00001	0.07	P = 0.01198	1.70	2.53
Harju Elekter	1345.87	P < 0.00001	0.09	p = 0.00027	-0.82	-0.05
Tallinna Kaubamaja	919.71	P < 0.00001	0.94	P < 0.00001	-1.18	-1.21
Tallinna Vesi	1225.08	P < 0.00001	1	P < 0.00001	1.11	1.05
Trigon	927.70	P < 0.00001	0.62	P < 0.00001	-0.03	-1.48
Nordecon	2287.88	P < 0.00001	0.89	P < 0.00001	0.004	-0.92
Viisnurk	741.17	P < 0.00001	0.92	P < 0.00001	-0.52	-0.72
Olympic Entertainment Grupp	1661.61	P < 0.00001	1	P < 0.00001	0.42	-0.68
Silvano Fashion Grupp	1430.68	P < 0.00001	0.68	P < 0.00001	1.58	1.63
Tallink	937.89	P < 0.00001	0.86	P < 0.00001	1.37	0.77

Table 2 Chi-square, Kolmogorov –Smirnov test values and sample skew and kurtosis for different companies trading on Tallinn stock exchange between 01/01/2008 and 01/01/2012

We observed that chi-square test rejected the null hypothesis (theoretical distribution suits described data) for all the companies while Kolmogorov-Smirnov (KS) test had positive feedback for three companies (Baltika, Ekpress Grupp, and Harju Elekter). Visual estimation of the fitted density and log density plots (see appendix for plots) led to selecting four companies (Nordecon, Trigon, Baltika and Ekpress Grupp) as best candidates. Further visual estimation by studying Q-Q plots eliminated Trigon and Nordecon. We were thus left with Baltika and Ekpress Grupp as the only suitable candidates for NIG-Levy asset price model.

4 Conclusion

We have reviewed the essential properties of Levy processes that make them more suitable for asset price modelling than general Gaussian processes and applied the NIG-Levy process to modeling asset prices for assets traded on the Tallinn stock exchange between 01 January 2008 and 01 January 2012. Selecting best NIG-Levy asset price model using the strategy proposed by Käärik and Umbleja [2011], we concluded that prices of Baltika and Ekpress Grupp can be modeled with NIG-Levy process; thus can be used as underlying assets in pricing financial deriveatives.

Acknowledgements

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Appendix

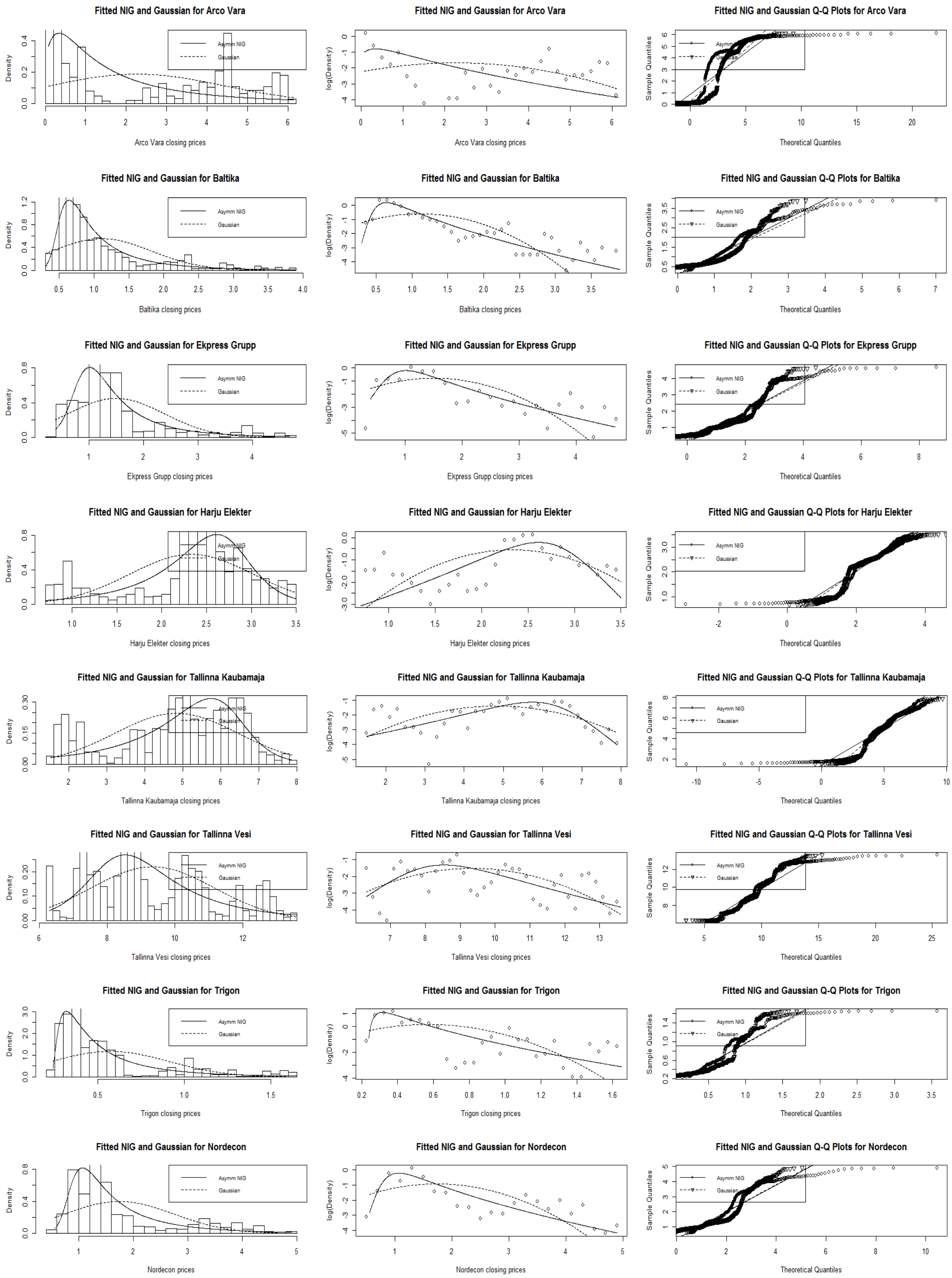

Figure 1 Plots of fitted NIG-Levy and Gaussian densities, log densities and Q-Q plots to the closing prices of 8 companies trading on the Tallinn stock exchange between 01/01/2008 and 01/01/2012

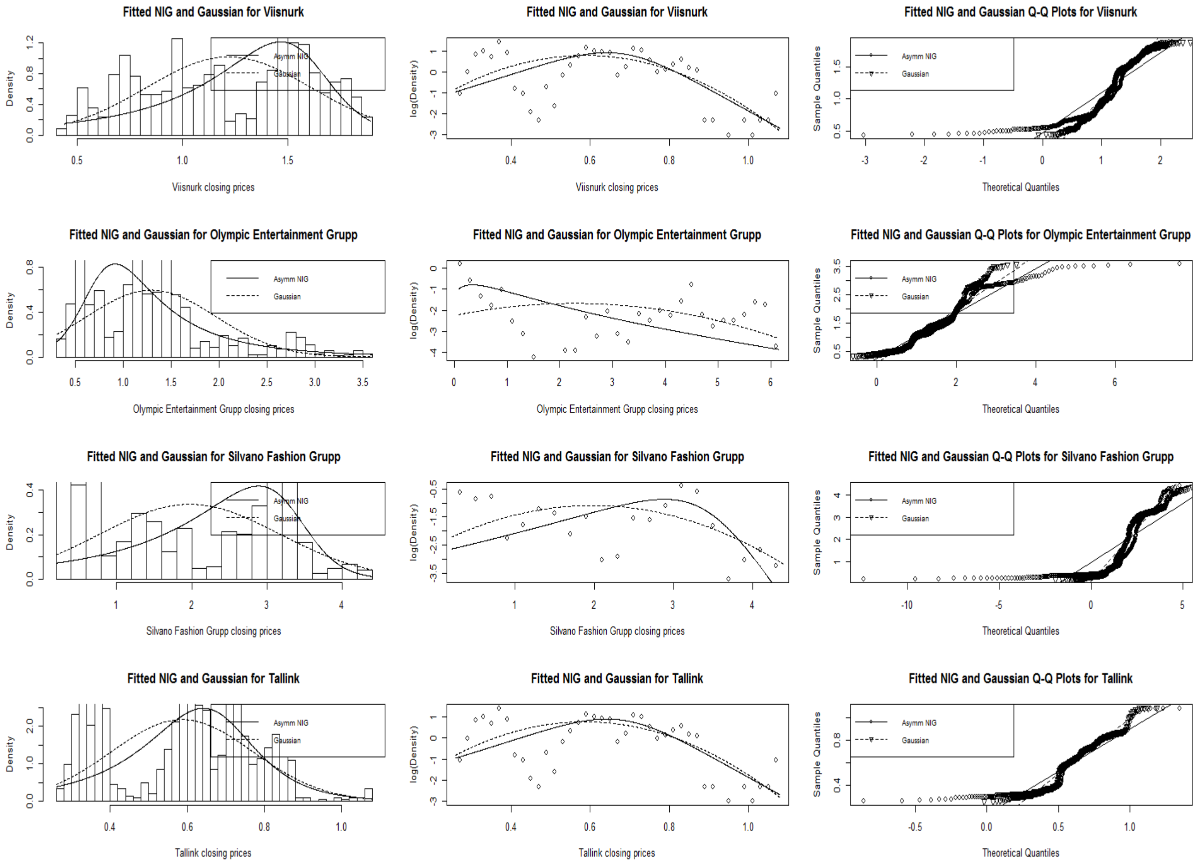

Figure 2 Plots of fitted NIG-Levy and Gaussian densities, Log densities and Q-Q plots to the closing prices of 4 companies trading on the Tallinn stock exchange between 01/01/2008 and 01/01/2012

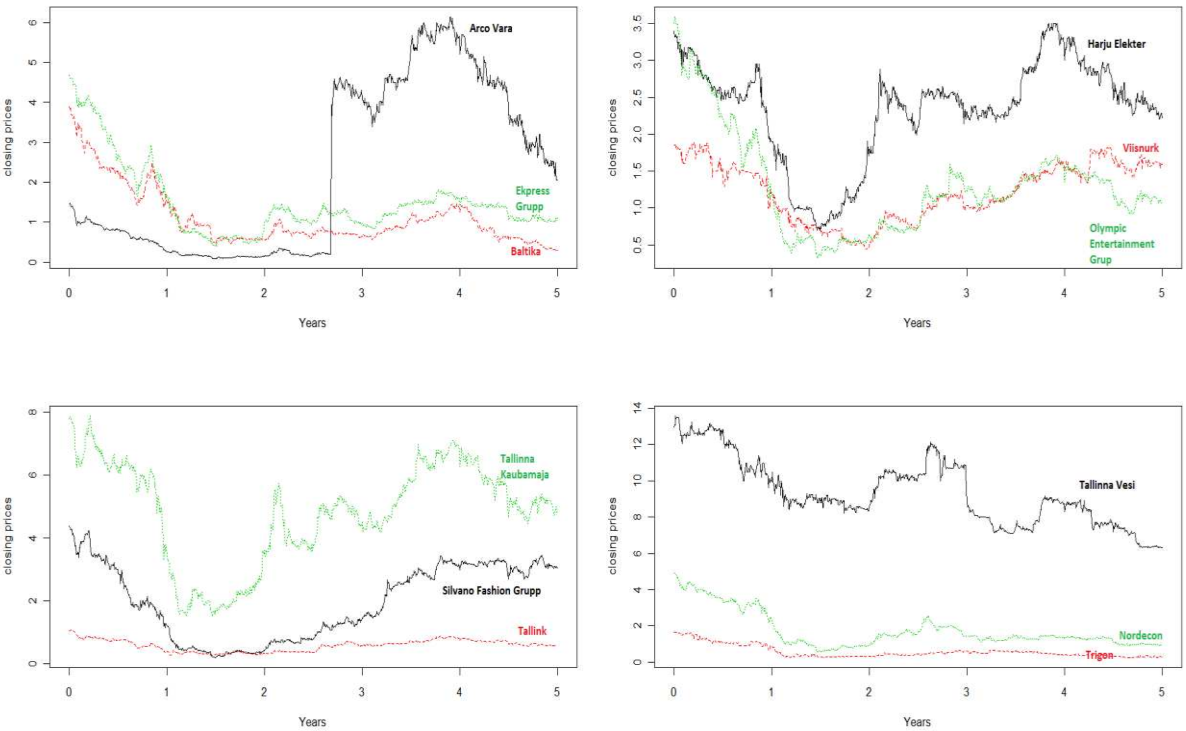

Figure 3 Plots of closing prices (thousands of euros) of 12 companies trading on the Tallinn stock exchange between 01/01/2008 and 01/01/2012