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# Modeling and forecasting foreign exchange daily closing prices with normal inverse Gaussian

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**Abstract.** We fit the normal inverse Gaussian(NIG) distribution to foreign exchange closing prices using the open software package R and select best models by Käärrik and Umbleja (2011) proposed strategy. We observe that daily closing prices (12/04/2008 - 07/08/2012) of CHF/JPY, AUD/JPY, GBP/JPY, NZD/USD, QAR/CHF, QAR/EUR, SAR/CHF, SAR/EUR, TND/CHF AND TND/EUR are excellent fits while EGP/EUR and EUR/GBP are good fits with a Kolmogorov-Smirnov test p-value of 0.062 and 0.08 respectively. It was impossible to estimate normal inverse Gaussian parameters (by maximum likelihood; computational problem) for JPY/CHF but CHF/JPY was an excellent fit. Thus, while the stochastic properties of an exchange rate can be completely modeled with a probability distribution in one direction, it may be impossible the other way around. We also demonstrate that foreign exchange closing prices can be forecasted with the normal inverse Gaussian (NIG) Lévy process, both in cases where the daily closing prices can and cannot be modeled by NIG distribution.

**Keywords:** NIG, modeling, forecasting, foreign exchange, goodness of fits tests

**PACS:** S02.50.Ey, 02.70.Rr, 02.50.Cw

## INTRODUCTION

The foreign exchange (FX) market is the largest and only major round the clock financial market with an average daily turn-over in excess of four trillion dollars [10]. Exchange rates affect governments, importers/exporters, salaries and spending decisions of travelers/international workers, etc. talk less financial derivatives written on them. It can even be said they affect everyone using money due to globalization. Since their stochastic properties are determined by a host of factors ranging from market participants to governmental decisions, modeling and predicting them accurately is of vital importance. Researchers continue to grapple with the problem of developing accurate FX models; their problem compounded by the collapse of the Bretton Woods agreement setting in motion floating exchange rates [3].

We use maximum likelihoods method to estimate normal inverse Gaussian parameters and apply Käärrik and Umbleja (2011) model selection technique [4] to choose excellent and good models. Results show that daily closing prices (12/04/2008 - 07/08/2012) of CHF/JPY, AUD/JPY, GBP/JPY, NZD/USD, QAR/CHF, QAR/EUR, SAR/CHF, SAR/EUR, TND/CHF and TND/EUR are excellent fits while EGP/EUR and EUR/GBP are good fits with a Kolmogorov-Smirnov test p-value of 0.062 and 0.08 respectively [7]. This means there are theoretical probability distributions capable of capturing the underlying stochastic properties of FX closing prices. The NIG distribution and NIG-Lévy process have been studied extensively in [1][5][2]. In this work, we present just the unique qualities of

NIG-distribution that mimic consecutive differences in closing prices, making this theoretical probability distribution stand out in this context. We do this through the generalized inverse Gaussian distribution; a member of the class of generalized hyperbolic distributions introduced in 1977 by Ole E. Barndorff-Nielsen [1]. Being able to forecast FX prices with NIG-Lévy process simply mean FX traders need not worry much about turmoil or governmental decisions but concentrate on prices given by model to gain advantage in trading. As well, it has been demonstrated in [6] that the underlying stochastic properties of some assets trading on the Tallinn stock exchange can be captured by such a distribution, and also in [7] that closing prices of FX can be modeled by such a distribution. Forecasting these closing prices by NIG-Lévy process is a natural extension of the results in [7], which we elaborate on in this work.

Section two introduces a general Lévy process of which the NIG-Lévy process is a subclass. This is to demonstrate that it has properties similar to FX closing prices. Then we outline NIG-distributions unique properties; after presenting it as a special case of generalized hyperbolic distribution (GHYP). Model selection strategy and analysis is the subject of section 3.

## GENERAL CHARACTERISTICS OF NIG

### General Lévy process

A Lévy process is a continuous time stochastic process  $X = \{X_t : t > 0\}$  defined on the probability space

$(\Omega, F, P)$  with the following basic characteristics:

1.  $P(X_0) = 1$  i.e. the process starts at zero;
2.  $\forall s, t \geq 0, X_{s+t} - X_t$  is distributed as  $X_s$  i.e. stationary increments;
3.  $\forall s, t \geq 0, X_{s+t} - X_t$  is independent of  $X_u, s \leq u$ , i.e. independent increments;
4.  $t \rightarrow X_t$  is a.s. right continuous with left limits.

It should be noted that through the application of the Lévy-Khinchine theorem and Lévy-Itô formula, a general Lévy process is seen to contain components that can capture both small and big jumps, and drift. The above are all characteristics of FX closing prices and especially for the NIG-Lévy process,  $\forall s, t \geq 0, X_{s+t} - X_t$  is NIG distributed [1][5][2].

### NIG-distribution; presentation through GHYP

A random variable  $Z$  has a GHYP distribution with parameters  $(\lambda, \alpha, \beta, \delta, \mu)$  if the conditional distribution is equal to

$$Z|Y = y \sim N(\mu + \beta y, y) \quad (1)$$

where

$$Y \sim \frac{1}{2} \left\{ \frac{\sqrt{\alpha^2 - \beta^2}}{\delta} \right\}^{\lambda/2} K_{\lambda}^{-1}(\sqrt{\delta \sqrt{\alpha^2 - \beta^2}} y)^{\lambda-1} * e^{\{-\frac{1}{2}((\alpha^2 - \beta^2)y + \delta^2 y^{-1})\}}, y > 0, \quad (2)$$

$$K_{\lambda}(y) = \frac{1}{2} \int_0^{\infty} u^{\lambda-1} e^{-\frac{1}{2}y(u+u^{-1})} du, y > 0 \quad (3)$$

and  $N(\mu + \beta y, y)$  is the normal distribution with mean  $\mu + \beta y$  and variance  $y$ . Thus  $Z$  has a probability density function[1]

$$f(z; \lambda, \alpha, \beta, \delta, \mu) = a_{\lambda}(\alpha, \beta, \delta) \{ \sqrt{(\delta^2 + (z - \mu)^2)} \}^{\lambda-1/2} *$$

$$K_{\lambda-1/2}(\alpha \sqrt{\delta^2 + (z - \mu)^2}) e^{\beta(z - \mu)} \quad (4)$$

where  $a_{\lambda}(\alpha, \beta, \delta)$  is a normalizing constant of the form

$$a_{\lambda}(\alpha, \beta, \delta) = \frac{(\sqrt{\alpha^2 - \beta^2})^{\lambda}}{\sqrt{2\pi} \delta^{\lambda} \alpha^{\lambda-1/2} K_{\lambda}(\delta \sqrt{\alpha^2 - \beta^2})}. \quad (5)$$

To get NIG distribution, we simply let  $\lambda = -\frac{1}{2}$  above with the restrictions  $\delta > 0, 0 \leq |\beta| \leq \alpha$  and  $\mu \in R$ . The parameters  $\alpha, \beta, \delta, \mu$  play different roles.  $\alpha$  determines how flat the density function is. It takes on positive values.  $\beta$  determines the skewness of the distribution. If  $\beta = 0$ , we get a symmetric distribution.  $\delta$  corresponds to the scale of the distribution while  $\mu$  is responsible for the shift of the probability density function [5].

### NIG-Lévy process

The NIG-Lévy process with parameters  $\alpha, \beta, \delta, \mu$  represented  $NIG(\alpha, \beta, \delta, \mu)$  can be defined as

$$NIG(\alpha, \beta, \delta, \mu) = \beta \delta^2 f_{IG}(t^{(a,b)}; a, b) + \delta W_{f_{IG}(t^{(a,b)}; a, b)} \quad (6)$$

where  $a = 1, b = \delta \sqrt{\alpha^2 - \beta^2}$  with  $\alpha > 0$  tail heaviness,  $\beta \geq 0$  symmetry,  $\delta > 0$  scale and  $\mu > 0$  location parameter.  $\mu = 0$  above. We note that

$$f_{IG}(x; a, b) = \frac{ax^{-\frac{3}{2}}}{\sqrt{2\pi}} e^{ab} e^{\{-\frac{1}{2}(\frac{a^2}{x} + xb^2)\}}. \quad (7)$$

Thus  $NIG(\alpha, \beta, \delta, \mu = 0)$  can be easily simulated noting that  $W_s$  is distributed as  $N(0, s)$  and  $X_t^{NIG}$  has an  $NIG(\alpha, \beta, t\delta, t\mu = 0)$  law.

The probability density function of  $NIG(\alpha, \beta, \delta, \mu)$  looks complicated but it has a simple moment generating function of the form

$$M_Z(t) = \exp\{t\mu + \delta(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + t)^2})\} \quad (8)$$

from which we get the mean, variance, skewness and kurtosis as  $\mu + \delta \frac{\beta}{\sqrt{\alpha^2 - \beta^2}}, \delta \frac{\alpha^2}{\{\sqrt{\alpha^2 - \beta^2}\}^3}, \frac{3\beta}{\alpha \sqrt{\delta \sqrt{\alpha^2 - \beta^2}}}$

and  $3 \frac{(1+4(\frac{\beta^2}{\alpha^2}))}{\delta \sqrt{\alpha^2 - \beta^2}}$  respectively[5].

### More NIG useful properties for closing price modeling

1. If  $Z \sim NIG(\alpha, \beta, \delta, \mu)$ , then  $Y = kZ \sim (\alpha/k, \beta/k, \delta/k, \mu/k)$ .
2. If  $Z_1 \sim NIG(\alpha, \beta, \delta_1, \mu_1)$  and  $Z_2 \sim NIG(\alpha, \beta, \delta_2, \mu_2)$  are independent, then the sum  $Y = Z_1 + Z_2 \sim NIG(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2)$ .
3. If  $Z_i \sim NIG(\alpha, \beta, \delta, \mu)$ , ( $i = 1, 2, \dots, n$ ) are independent, then the sample mean  $\bar{Z} = \frac{1}{n} \sum_{i=1}^n Z_i \sim NIG(n\alpha, n\beta, \delta, \mu)$ .
4. If  $Z \sim NIG(\alpha, \beta, \delta, \mu)$ , then the variable  $Y = (Z - \mu)/\delta \sim NIG(\alpha\delta, \beta\delta, 0, 1)$ , the standard NIG distribution.

Property 2 above is unique for NIG distribution.

### MODEL SELECTION AND ANALYSIS

#### Käärrik and Umbleja (2011) proposed model selection strategy

1. choose a suitable class of distributions (using general or prior information about the specific data) ;

2. estimate the parameters (by finding maximum likelihoods);
3. estimate goodness of fit;
  - visual estimation
  - classical goodness-of-fit tests (Kolmogorov-smirnov, chi-squared with equiprobable classes),
  - probability or quantile-quantile plots [4].

## Analysis of data

Data is from the UK, and these are the quoted daily closing prices of currency trades or foreign exchange (FX) closing prices. This data is known to be interfered with by governmental policies on a regular basis. Since it covers parts of the recent financial bubble, fiscal stimulus decisions by governments are reflected in the prices. The period is from 12 April 2008 to 07 August 2012. Being able to forecast FX prices with NIG-Lévy process will simply mean FX traders need not worry much about turmoil or governmental decisions but concentrate on prices forecasted by model to gain advantage in trading. In other words, the price at time  $t + 1$  given FX closing prices up to time  $t$  is generated by letting the NIG-Lévy process propagate up to time  $t + 1$ ; a simple forecasting model without dependencies on actions of other players in the market.

Looking at skews and kurtoses of the FX closing prices (Table 2) suggest using a distribution which is skewed and can capture tails; something the NIG is excellent at since its skewness and kurtosis depend on parameters which can be varied easily. As well, figures 1-3 show the fits of NIG-FX models. Results of the Chi-square test can totally be neglected here as these depend on how the classes are chosen; something our software does automatically. Kolmogorov-Smirnov test was really positive for AUD/JPY, CHF/JPY, GBP/JPY, NZD/USD, QAR/CHF, QAR/EUR, SAR/CHF, SAR/EUR, TND/CHF and TND/EUR with excellent p-values. As well EGP/EUR and EUR/GBP had good p-values OF 0.062 and 0.08 respectively and can be considered good models.

## Conclusion

Closing prices of some foreign exchanges can be modeled with normal inverse Gaussian distribution. This suggest there may be other theoretical probability distributions capable of capturing the underlying stochastic properties of bad or impossible to estimate models considered in this work. Thus, it has been shown that daily closing prices (12/04/2008 - 07/08/2012)

CHF/JPY, GBP/JPY, QAR/EUR, SAR/EUR, TND/CHF, EGP/EUR, EUR/GBP and TND/EUR can be modeled with normal inverse Gaussian distribution and their future prices forecasted with NIG-Lévy process. As well, NZD/USD, QAR/CHF, and SAR/CHF closing prices could not be captured by NIG distribution, but future prices can be forecasted with NIG-Lévy process; almost the same conclusion with world indexes studied in [8].

## Simple forecasting code in R

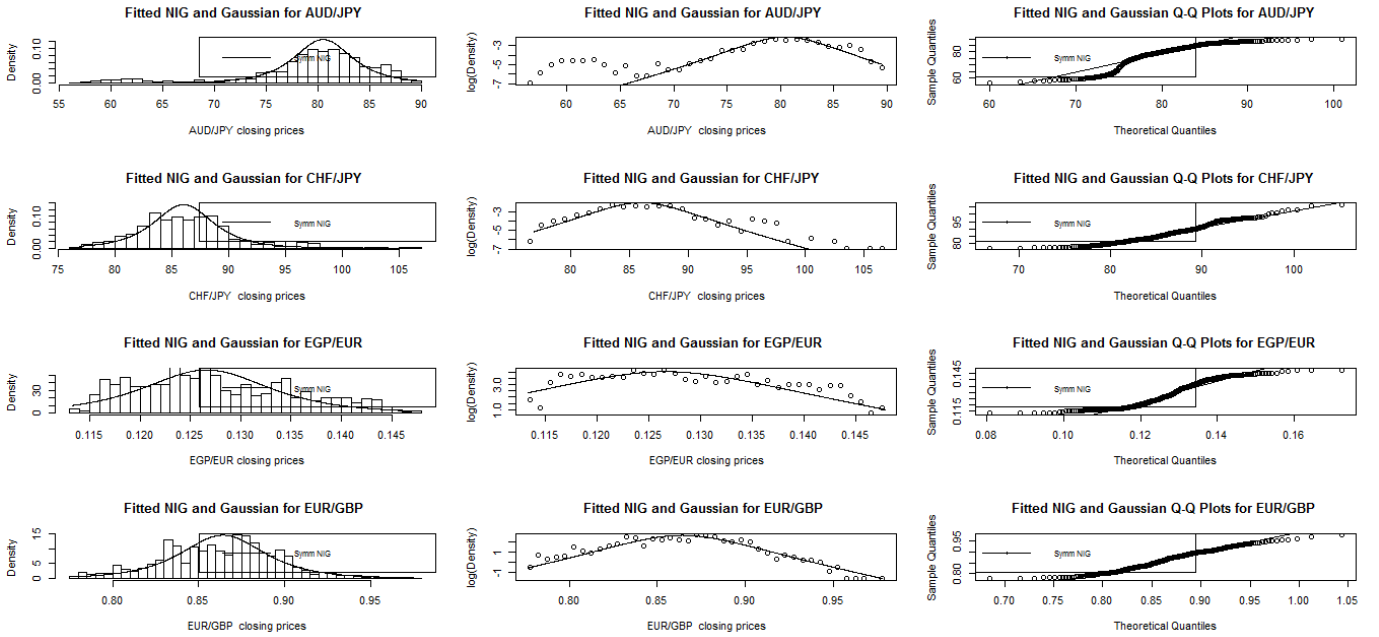
1. Estimated parameters  $\alpha, \beta, \delta, \mu$  from historical return data
2. Denote last day's closing price by  $s_0$ . We average  $m$  generate prices
  - library(fBasics)
  - forecaster=function(alpha,beta,delta,mu,s0,m){
  - matah<- rep(0,m)
  - for (i in 1:m){
  - matah[i]<-rnig(1,alpha,beta,delta,mu)+s0}
  - return(mean(matah))}
  - forecaster(1.2, -0.2, 0.9, 0.17, 84.266, 10000)
  - [1] 84.2976
  - forecaster(1.2, -0.2, 0.9, 0.17, 84.266, 100000)
  - [1] 84.27799
  - forecaster(1.2, -0.2, 0.9, 0.17, 84.266, 1000000)
  - [1] 84.2842

## ACKNOWLEDGMENTS

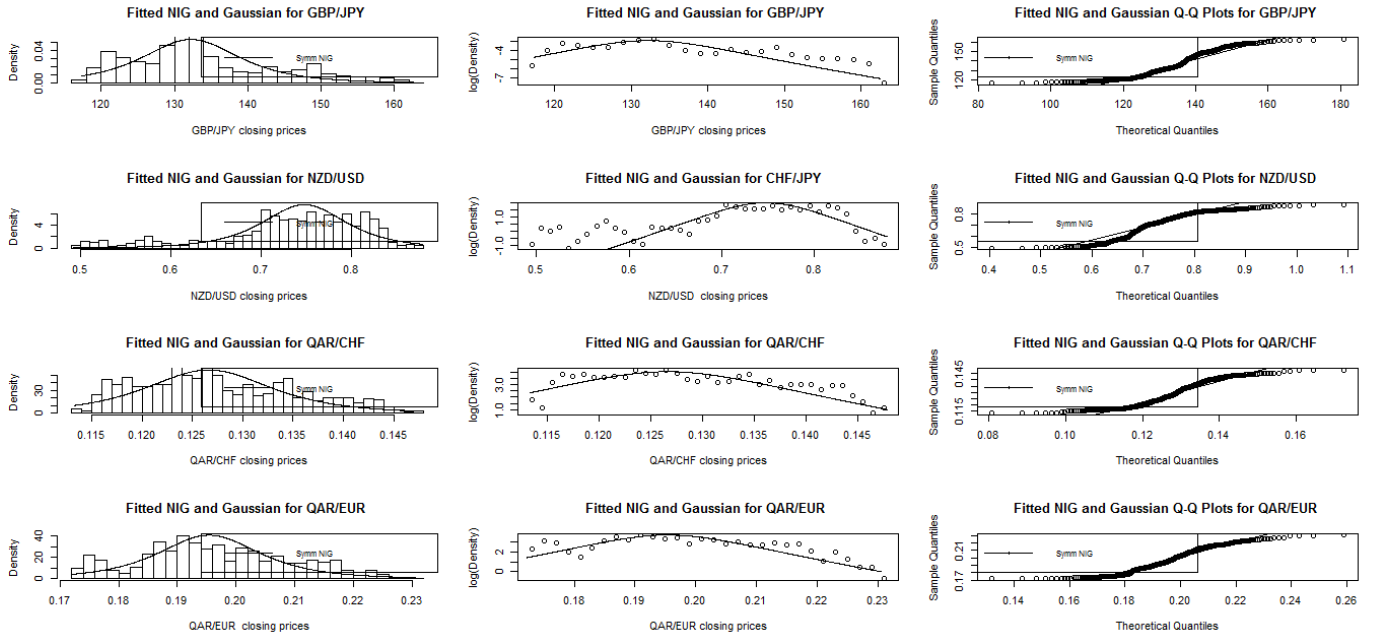
Research was financed by Estonian Science Foundation grant 8802 and Estonian Doctoral School in Mathematics and Statistics. Conference visit financed by European Social Fund and Foundation Archimedes Dora 8 grant.

**TABLE 1.** Estimated NIG parameters, Skews, Kurtoses, and Kolmogorov-Smirnov(KS) test results for NIG-Lévy process models: profits/losses

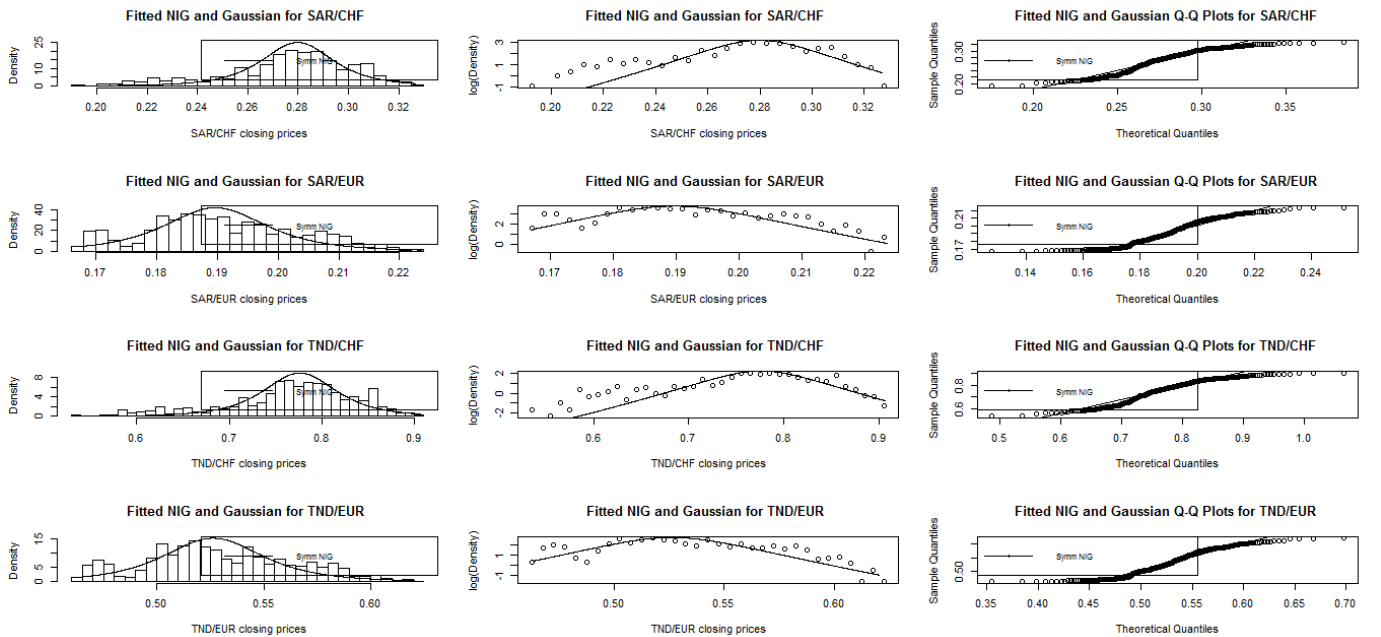
FX	alpha( $\alpha$ )	beta( $\beta$ )	delta( $\delta$ )	mu( $\mu$ )	Skew	Kurtosis	KS p-value	KS D-value
AUD/JPY	1.2	-0.2	0.9	0.17	-0.55	2.27	0.69	0.032
CAD/JPY	1.54	-0.23	1.01	0.16	-0.4	1.81	0.572	0.035
CHF/EUR	77.86	0.002	0.002	0.0002	-3.27	47.52	0.79	0.029
CHF/GBP	139.44	6.1	0.004	9.69e-05	-1.41	20.37	0.18	0.049
CHF/JPY	1.44	-0.19	0.84	0.12	-1.18	11.67	0.43	0.039
EGP/CHF	816.43	21.01	0.002	-1.1e-04	0.33	5.43	0.4	0.04
EGP/EUR	1349.23	-27.43	0.002	2.86e-05	-0.18	1.21	0.16	0.05
EUR/GBP	209.8	-1.51	0.006	-4.97e-05	-0.134	2.94	0.54	0.036
EUR/JPY	1.26	-0.24	1.38	0.24	-0.35	1.51	0.95	0.023
GBP/JPY	0.68	-0.023	1.12	0.024	-0.25	2.53	0.15	0.05
JOD/GBP	161.27	15.38	0.009	-0.0006	0.48	2.3	0.57	0.035
JOD/JPY	0.85	-0.026	1.02	-0.02	0.15	3.88	0.98	0.021
JPY/EUR	93.79	16.77	0.007	-0.0009	-0.05	6.15	0.29	0.004
JPY/GBP	92.61	7.13	0.006	-4.9e-05	0.27	6.25	0.83	0.028
NZD/USD	227.48	-30.68	0.01	0.002	-0.31	1.75	0.34	0.042
QAR/EUR	847.89	14.99	0.002	-2.56e-05	-0.16	1.33	0.26	0.045
SAR/EUR	846.27	33.75	0.002	-6.9e-05	-0.12	1.36	0.83	0.028
SAR/GBP	791.6	61.45	0.003	-6.31e-05	0.54	2.92	0.57	0.035
TND/CHF	194.77	6.01	0.008	-0.0004	0.48	7.35	0.61	0.034
TND/EUR	317.45	6.42	0.005	-8.2e-05	-0.17	1.33	0.34	0.042
TND/GBP	275.73	19.04	0.004	-0.0001	0.53	3.11	0.313	0.043
TND/JPY	1.64	-0.04	0.51	-0.012	0.02	4.03	0.29	0.04
USD/EUR	253.5	4.7	0.008	-0.0001	-0.16	1.32	0.83	0.028
USD/GBP	204.18	13.59	0.005	-0.0002	0.53	3.12	0.61	0.034
XAU/USD	0.051	-0.007	11.96	2.48	-0.67	4.61	0.91	0.025



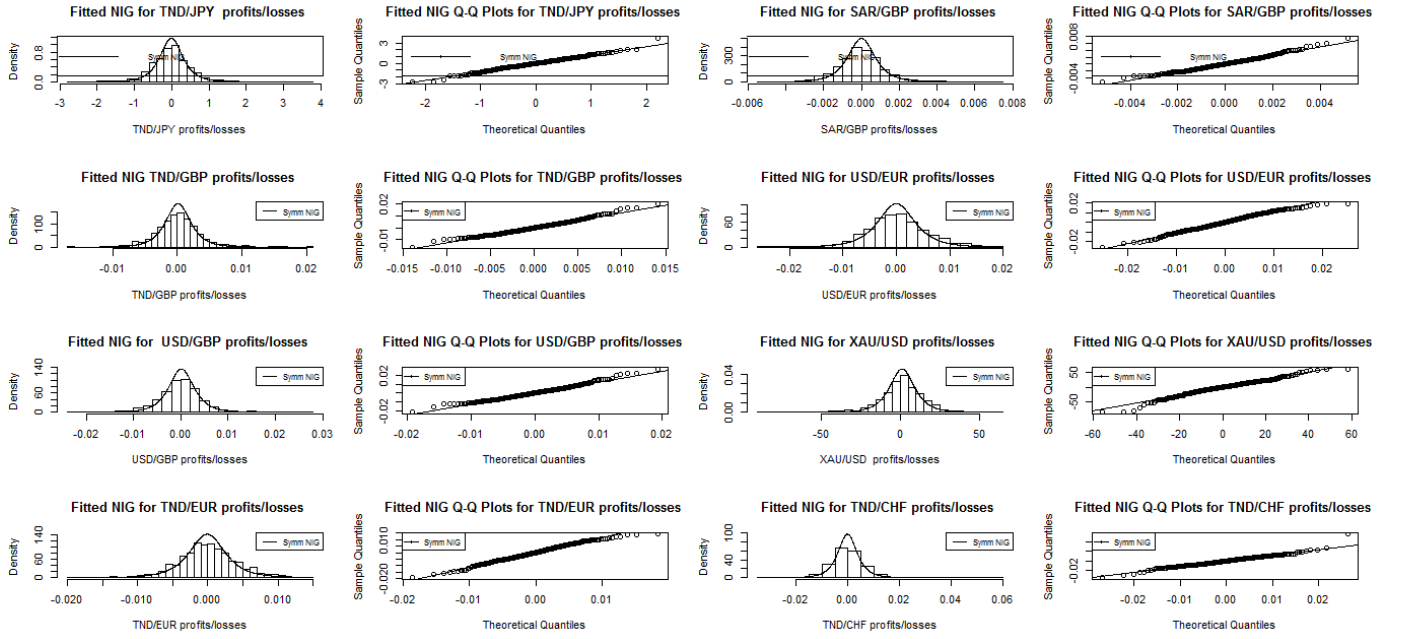
**FIGURE 1.** Fitted NIG density, log density and Q-Q plots for AUD/JPY, CHF/JPY, EGP/EUR and EUR/GBP NIG-FX models: daily closing prices



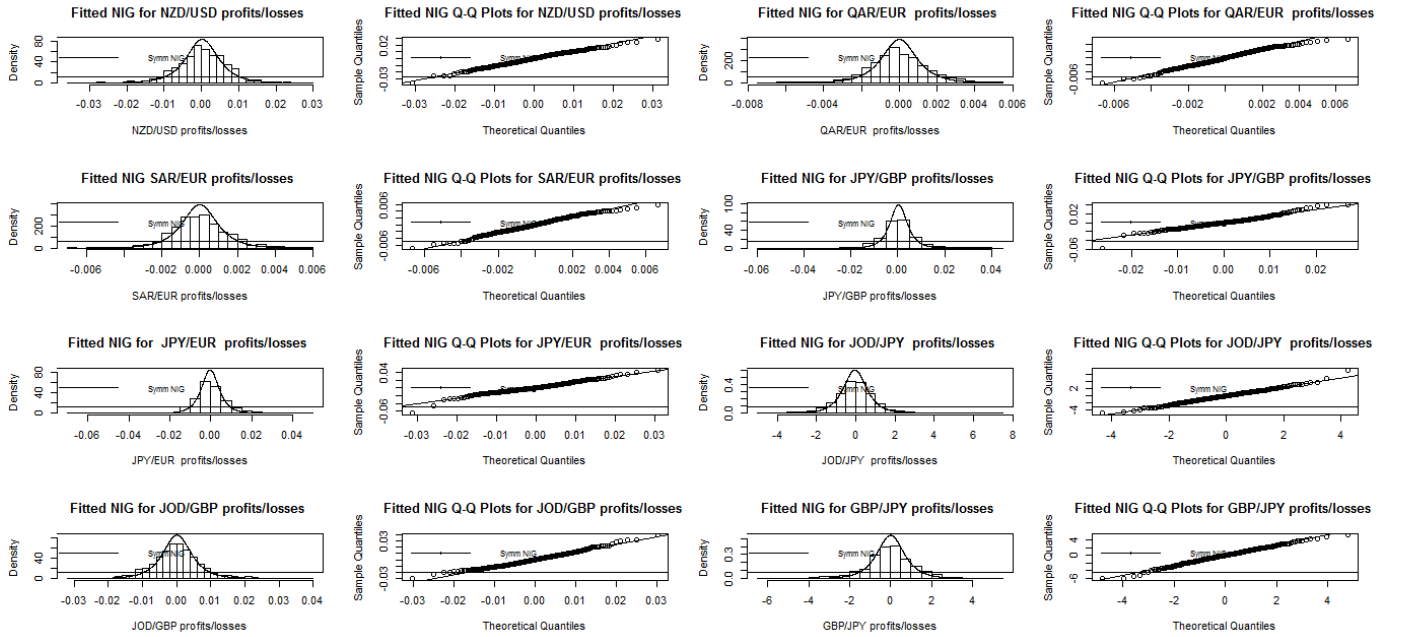
**FIGURE 2.** Fitted NIG density, log density and Q-Q plots for GBP/JPY, NZD/USD, QAR/CHF and QAR/EUR NIG-FX models: daily closing prices



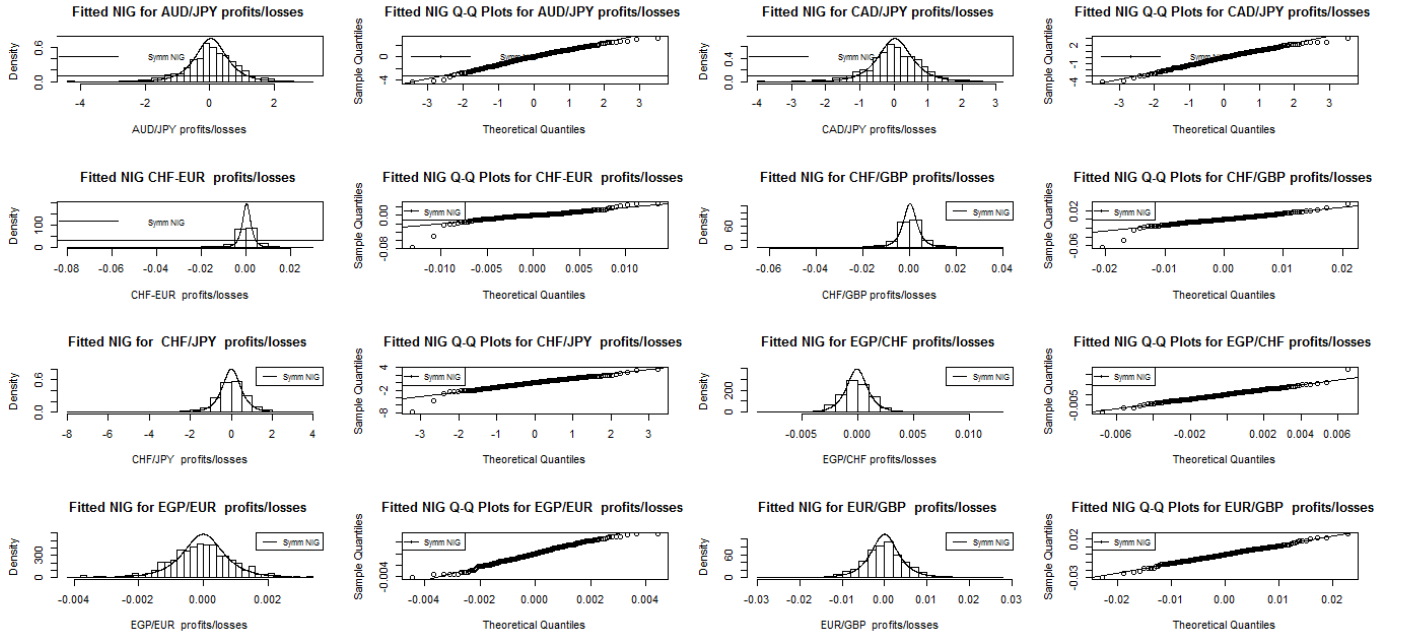
**FIGURE 3.** Fitted NIG density, log density and Q-Q plots for SAR/CHF, SAR/EUR, TND/CHF and TND/EUR NIG-FX models: daily closing prices



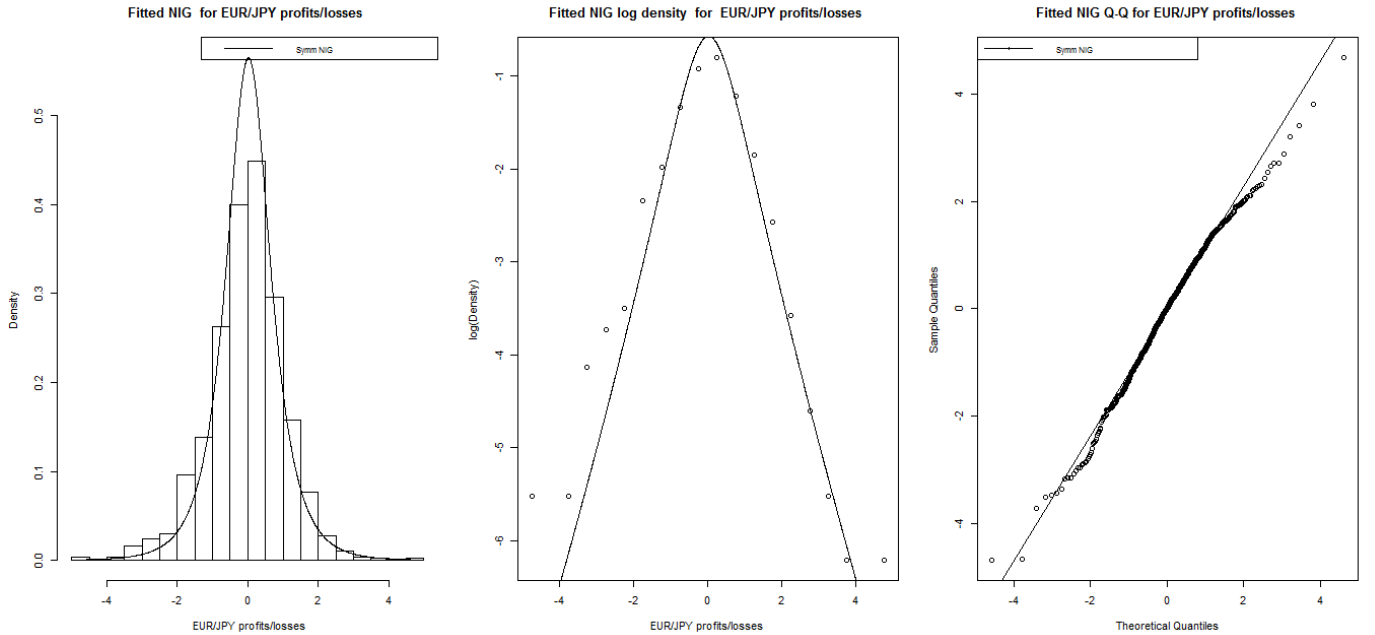
**FIGURE 4.** Fitted NIG density and Q-Q plots for eight NIG-Lévy process FX models: Profits/losses



**FIGURE 5.** Fitted NIG density and Q-Q plots for eight NIG-Lévy process FX models: profits/losses



**FIGURE 6.** Fitted NIG density and Q-Q plots for eight NIG-Lévy process FX models: Profits/losses



**FIGURE 7.** Fitted NIG density, log density and Q-Q plots for EUR/JPY NIG-Lévy process FX model: Profits/losses

**TABLE 2.** Estimated NIG Parameters, Skews, Kurtoses, and Kolmogorov-Smirnov(KS) test results for NIG-distributed FX models: daily closing prices

FX	Alpha ( $\alpha$ )	Beta( $\beta$ )	Delta ( $\delta$ )	Mu ( $\mu$ )	Skew	Kurtosis	KS p-value	KS D-value
AUD/JPY	0.33	-0.23	5.07	84.3	-1.53	2.49	0.4	0.04
CHF/JPY	0.54	0.26	7.31	82.41	0.76	1.16	0.22	0.047
EGP/EUR	18215.6	18011.2	0.004	0.1	0.38	-0.76	0.062	0.059
EUR/GBP	2194.25	-412.12	2.41	1.32	-0.01	-0.07	0.08	0.057
GBP/JPY	8.43	8.31	4.32	108.3	0.65	-0.3	0.12	0.054
NZD/USD	354.97	-342.22	0.048	0.91	-0.98	0.44	0.24	0.046
QAR/CHF	2152.2	-2092.4	0.02	0.37	-0.77	0.29	0.12	0.053
QAR/EUR	1364.5	1022.2	0.07	0.12	0.19	-0.6	0.37	0.041
SAR/CHF	2656.5	-2594.4	0.02	0.36	-0.77	0.28	0.16	0.05
SAR/EUR	2702.86	2331.53	0.054	0.099	0.19	-0.6	0.16	0.05
TND/CHF	1088.3	-1065.6	0.047	0.99	-0.76	0.28	0.341	0.042
TND/EUR	1014.79	878.64	0.153	0.27	0.18	-0.6	0.341	0.042

**TABLE 3.** Impossible to estimate (by maximum likelihoods) and bad NIG-distributed FX models: daily closing prices

Impossible to Estimate	Bad Models				
AUD/USD	USD/EUR	CHF/EUR	CHF/GBP	TND/GBP	TND/GBP
CHF/USD	EGP/USD	EUR/JPY	TND/JPY	JOD/GBP	USD/GBP
EUR/JOD	JOD/JPY	JPY/EUR	JPY/GBP	SAR/USD	USD/JOD
JPY/CHF	EGP/CHF	QAR/GBP	QAR/USD	USD/JPY	XAU/USD
JPY/USD	CAD/JPY	TND/USD			

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