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Halkos, George and Papageorgiou, George

Department of Economics, University of Thessaly

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George E. Halkos and George J. Papageorgiou
University of Thessaly, Department of Economics,
Korai 43, 38333, Volos, Greece

Abstract

This paper is concerned with the classic topic of intertemporal resource economics: the optimal harvesting of renewable natural resources over time by one and several resource owners with conflicting interests. The traditional management model, dating back to Plourde (1970), is extended towards a two–state model in which harvesting equipment is treated as a stock variable. As a consequence of this extension, equilibrium dynamics with bifurcations and limit cycles occur. We also discuss conflicts as a game with two types of players involved: the traditional fishermen armed with the basic equipment and the heavy equipment users. Both players have a common depletion function, considered as harvesting, which is dependent together on personal effort and on intensity of equipment's usage.

Keywords: Renewable resources; exploitation of natural resources; optimal control; differential games.

JEL classifications: C61; C62; Q32.

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1. Introduction

Intertemporal economic problems can be formulated either as optimal management models or as dynamic games. A basic difference between the two types of formulation is that, in the former case there is only one strategically acting agent, i.e. the regulator, while in the latter there are more than one strategically interacting agents, choosing their actions that determine the current and future levels of utility. Consider, for example, a single stock of an exhaustible or reproductive resource that is simultaneously exploited by several agents that do not cooperate.

Each agent chooses an extraction strategy to maximize the discounted stream of future utility. Then, the actions taken determine not only their utility levels but also the level of the stock. There are several implications of the above formulation. First, the actions taken by agents determine the size of a single capital stock that fully describes the current state of the economic system. Second, if there is no mechanism that forces players to coordinate their actions, they will act strategically and play a non–cooperative game. Third, the equilibrium outcome will critically depend on the strategy spaces available to the agents.

There is a wide choice of possible actions (strategies) taken by the players. They may choose a simple time profile of actions and pre-commit themselves to these fixed actions over the entire planning horizon. Players then use open–loop strategies. Alternatively players might choose feedback or closed–loop or Markov strategies conditioning their actions on the current state of the system and reacting immediately every time the state variable changes, hence they are not required to pre-commit. If fisheries use open–loop strategies they specify a time path of fishing effort in the beginning of the game and commit themselves to stick to these preannounced actions over the entire planning horizon.

Alternatively, if they use feedback strategies they choose decision rules that determine current actions as a function of current stock of the resource. Feedback decision rules capture the strategic interactions present in a dynamic game. If a rival fishery makes a catch today that necessarily results in a lower level of the fish stock, the opponents react with actions that take this change in the stock into account. In that sense closed–loop strategies capture all the features of strategic interactions.

In these lines, the main contribution of the paper relies on the results obtained firstly, in the optimal management of the two state variable model in which the harvesting is treated as a function of accumulated equipment and secondly, in the Nash equilibrium of the game for which the players compete having a common harvesting (depletion) function. In equilibrium terms we find first, the conditions under which equilibrium with limit cycles occur in the management problem and second, we find the relation between the player's discount factors in order to ensure equilibrium with limit cycles, as well.

The structure of the paper is the following. Section 2 reviews the existing related literature and comments about the importance of the cyclical strategies in harvesting management. Section 3 discusses the two dimensional optimal control, as the one state model is well known. The next section concerns the conflicts as dynamic game with two players and with a common harvesting function. The last section concludes the paper.

2. Literature review

In environmental economics' vast literature, one given important meaning is connected with the exploitation of natural resources. According to this literature strand a regeneration function is involved, which is necessary to model the interactions between the nature and the human activities. In an important paper, Strobele (1988) considers the whole environment as renewable natural resource and the damage done to nature is described by a downward shift in the regeneration function due to the industrial waste emission. In the same, but more restrictive, way Hannesson (1983) compares the optimality of the monopolistic and social planning extractions, finding that the monopolistic standing optimal stock of the resource (say the nature) may either be larger or smaller than under the social planning.

Strobele and Wacker (1995) extend the one specie exploitation to multiple species in a predator–prey model. They derive a modified golden rule of harvesting, applying optimal control theory. Their conclusions about the modified golden rule in the steady state, is related with the "additional productivity effects". Farmer (2000), reconsidering Mourmouras' type overlapping generations' model with renewable natural resources, shows that there exists a non trivial stationary state which exhibits, by definition, intergenerational natural – capital equality.

Finally, natural resources harvesting differs from production. Renewable resources economic literature, based on the foundations of Gordon (1954), Scott (1955) and Smith (1969), suggests particular properties of the open access natural resources which requires tools of analysis beyond those supplied by elementary economic theory. Such an appropriate tool is the optimal control theory and the use of differential equations in dynamic systems (either in a continuous or a discrete framework), which are of common use in most models that explain the optimal management of natural resources extraction. These systems depend on more than one parameter that measures different economic and biological characteristics of the exploited resource. So the structural stability is a key point to study in order to explore whether the qualitative dynamical properties of the system persist when its structure is perturbed. In this context, the study of the structural stability is the first step to follow the analysis of the system.

On the other hand, it is reasonable to consider the stock of any renewable resource as a capital stock and treat the exploitation of that resource in much the same way as one would treat accumulation of a capital stock. This has been done to some extent by Clark (1990) and Clark and Munro (1975), whose papers contain a discussion of this point of view. However, the analysis is much simpler than it appears in the literature especially since the interaction between markets and the natural biology dynamics has not been made clear. Furthermore renewable resources are commonly analyzed in the context of models where the growth of the renewable resource examined is affected by two factors: the size of the resource itself and the harvesting rate. This specification does not take into account that human activities other than harvesting may have an impact on the growth of the natural resource (Levhari and Withagen, 1992).

Some externalities may arise in maximum sustained yield programs of replenishable natural resource exploitation followed by two fundamental problems. The first is that the existence of a social discount factor (or interest rate) may cause the maximum sustained yield program to be non-optimal (Plourde, 1970). The second problem relates to many externalities which may be present in harvesting resources. The most significant of these externalities is the stock externality in production. That is, there is a potential misallocation of inputs in the production of natural resource product due to the fact that one input, the natural resource, contributes to production but may not receive payment, as nobody owns the resource.

An analysis of the biomass harvesting (like fisheries) must take into account the biological nature of fundamental capital, the renewable resource, and must recognize the common property feature of land or sea, so it must allow that the fundamental capital is the subject of exploitation. The problem of fishing industry has been tackled by economists giving attention to the common property characteristics associated with both the open access and the lack of proper property rights to the fishery industry (Gordon, 1954; Björndal, 1987). A number of existing studies on fishery economics have paid attention to the form of properties: full rights or no rights at all (Smith, 1969; Plourde, 1971). Both cases lead to unique Nash non-cooperative outcomes with the social planner's outcome in the case of full rights and the open access in the case of no rights. The latter is the result of the tragedy of commons (for discussion see Clark and Munro, 1975).

The fishery model with adjustment costs, arisen from changes in control variable, has been solved by Liski *et al.* (2001), thus providing a link between stable limit cycle policies and increasing returns in harvesting.

The management model, presented here, is close to a Wirl's (1995) paper which analyses the stability of optimal renewable resource extraction programs. In the complementary Wirl's paper the second state variable is the capital, while the harvesting function thought as a function of effort, capital and resource stock. Choosing the cost parameter as a bifurcation variable he shows that the cyclical exploitation of renewable resources may be optimal. The crucial condition that drives this result is the possibility of growth of the biomass, which implies that the stock falls below the level that maximizes the sustainable yield.

2.1 Cyclical strategies in harvesting management

In the fishery economics management vast literature two possible optimal strategies are considered under stationary conditions (e.g. Plourde, 1971; Clark, 1990). The first ones are the continuous time strategies, whereby the renewable resource is exploited at all times. Following this type of strategy, the resource stock is considered dependent on both economic and social conditions as regards the resource, the discount rate and finally the initial resource stock size. The implications of that strategy depend on the stationary size of the resource, for which the harvesting rate is decided (Lewis and Schmalensee, 1979). As it becomes clear -and as it is well known- the above strategy does not take into account (or neglects) the role of capital inflows taking place in the harvesting industry.

As already mentioned, one can consider as capital inflows the available fleets and the human capital employed, but a more interesting aspect is the ability to modify all the above capital factors involved in the harvesting. Another important reason to deviate from the original continuous time strategy is the argument raised by some authors (e.g. Clark, 1990; Dawind and Kopel, 1997), which states that harvesting strategies that stabilize the stock of the renewable resource to a usual steady state level may be replaced by policies involving the abandonment or cyclical utilization of the resource (Liski *et al.*, 2001).

This second strategy, already discussed in harvesting management literature, involves extinction or abandonment policies, implying that, after a finite time, harvesting is abandoned forever (Lewis and Schmalensee, 1979). A first valuable insight for this type of optimal strategy is the fact that cyclical fishing policies are observed in practice. Moreover, Björndal (1987) uses data covering years 1952–1972 to show a relatively regular cyclical pattern for harvesting. In harvesting management, one possible optimal cyclical harvest policy, well known as chattering strategies or pulse fishing (Liski *et al*., 2001), is incorporated with the fleets' withdrawal and reentry as well as by hiring and firing workers, thus implying cost fluctuations.

Chattering strategies, in fishery management, are also subject to adjustment costs since the harvest rate and the costs incurred (startup and reentry costs) are independent of each other. The size of adjustment costs plays a crucial role in the optimal harvesting policies. Especially in the case of relatively modest adjustment costs, it has been shown that all conceivable policies will exhibit a limit cycle policy over time, which not only exists but it is also stable. One important result obtained from the above chattering policy is the fact that the cyclical utilization of the resource is related to smooth fluctuations and not to complete shutdowns of the fishery management. The economic implication of the cyclical harvesting policy existence is summarized as the profitable advantage of increasing returns by temporarily harvesting excessive quantities and stabilizing the stock of fish over time by cutting the harvest rate back after each period of excessive harvesting (Liski *et al.*, 2001).

It is also worth noting that the above given implication is valid only for low adjustment costs; otherwise, for high adjustment costs, it is preferable to follow the saddle point stability with a constant harvest rate. The importance of cyclical policies in harvesting, also known as pulse fishing policies, is confirmed by Wirl's model (Wirl, 1995), whereby the resulting cyclical strategy is related to the positive externality of the stock. Moreover, in the same paper, Wirl obtains saddle point stability for low adjustment costs contrary to the results obtained by Liski *et al* (2001).

While the importance of pulse fishing policies is well understood in the management context, the possibility of limit cycle policies in the conflicting approach has not been previously addressed, at least to our knowledge. Therefore, in order to achieve realism, we suggest a simple game model between two types of players: the crowd of the negligible capital investment fishermen, using a single boat for their fishing effort, and the heavily equipped players, using a fleet of vessels, hence facing adjustment costs. As it becomes clear, the crowd of the first type, thought as one player, has all the prerequisites (i.e. negligible adjustment costs and increasing returns) to follow the profitable cyclical patterns as identified in the management case, but it is not certain that their costs remain negligible due to the presence of the heavily equipped rivals in the same harvesting arena. Supposing that they adhere to the cyclical fishing policy, as the proven profitable solution for them, they take the corresponding substantial risk.

On the other side, the heavily equipped players, using fleets and workingmen and therefore facing adjustment costs, decide to follow the same profitable cyclical pattern, lowering their adjustment costs as much as possible. Since the low adjustment cost is the basic prerequisite to follow a cyclical pattern it is reasonable to internalize the above cost inside the intensity of their fleet usage. In the suggested conflict between the two types of players, the basic supposition is that the players adhere to the cyclical patterns, as they are considered the only profitable policies that stabilize the resource stock. In this way, the suggested model contributes to the existing harvesting management literature in the conflicting sense, clarifying the conditions under which the desired cyclical policies are obtained.

3. A management of commercial harvesting

In the traditional case model, also studied for instance by Clark and which goes back to the very simple Gordon-Schaefer model, $x(t)$ is the resource stock at time t, $\phi(t)$ the resource's harvesting function and $g(x(t))$ the regeneration function of the natural resource. With these functions in the model one obtain the system dynamics, as

$$\dot{x}(t) = g(x(t)) - \phi(t) \qquad (1)$$

It is assumed that the regeneration function $g:[0,\infty) \rightarrow \mathbb{R}$ is continuous, twice continuously differentiable on $(0,\infty)$ and strictly concave. In addition, it is assumed that $g(0)=0$, $\lim_{x\to 0} g'(x)=\infty$, and that there exists a unique resource stock $\bar{x}>0$ such that $g(\bar{x})=0$. This implies that $g'(x)>0$ for all $x\in(0,\bar{x})$ and $g'(x)<0$ for all $x>\bar{x}$.

The goal of the decision maker is to maximize the discounted utility derived over the infinite planning interval $[0,\infty)$. That is, the objective functional is given as:

$$\int_0^\infty e^{-\rho t} U(\phi(t))dt \qquad (2)$$

where $U:[0,\infty)\rightarrow \mathbb{R}$ is the utility function. Concerning equilibrium, in this reference one state model, it has been shown that the optimal management admits a unique equilibrium path which converges to the saddle point (see for example Dockner *et al*, 2000).

One the other hand, commercial extraction of natural resources in an intensive rate requires sometimes improvements on the harvesting equipment in order to be efficient. But better equipment is subject to adjustment costs, for instance electronic machines, vessels, boats and workmen hiring are some of these adjustment costs. The supposition of quadratic adjustment costs simplifies the arithmetic calculations but is not essential. With these additional assumptions one can treat the harvesting effort not as an instantaneous control but rather as a stock variable. Integrating over past adjustments the new control variable $E(t)$ enters into the model, describing the evolution of the harvesting effort.

3.1. Adjustment costs

Considering harvesting as a stock variable, some modifications are necessary to made in the objective functional, that is the introduction of the adjustment costs $C(E(t))$, for the new stock. In this subsection, as the analysis it is well known e.g. Liski *et al.* (2001), we briefly discuss a concave natural resources regeneration function $g(x)$. The concavity of the function $g(x)$ states that the law of diminishing returns applies here too. Moreover the utility enjoyed by the representative agent is a function depending on the harvest $\phi(t)$ and on the existing resource stock, as well. With these modifications the optimal management problem becomes

$$\max_{E(t)} \int_0^\infty e^{-\rho t} \left[U(\phi(t), x(t)) - C(E(t)) \right] dt \qquad (3)$$

subject to

$$\dot{x}(t) = g(x(t)) - \phi(t), \quad x(0) = x_0 \qquad (4)$$

$$\dot{\phi}(t) = E(t), \qquad \phi(0) = \phi_0 \qquad (5)$$

Model $(3)-(5)$ is an optimal control with two state and one control variable and with a quadratic cost function. The necessary conditions required by the maximum principle, provide the following four dimensional system of equations:

$$\dot{x}(t)=g(x(t))-\phi(t), \quad (6) \qquad \dot{\phi}(t)=E(t) \quad (7)$$
$$\dot{\lambda}_1=-\frac{\partial H}{\partial x}+\rho\lambda_1, \quad (8) \quad \dot{\lambda}_2=-\frac{\partial H}{\partial \phi}+\rho\lambda_2 \quad (9)$$

together with the optimality $$\frac{\partial H}{\partial E}=-C'+\lambda_1=0 \qquad (10).$$

The function $$H=U(\phi(t),x(t))-C(E(t))+\lambda_1\left[g(x(t))-\phi(t)\right]+\lambda_2 E(t)$$

is the Hamiltonian current value of the problem $(3)-(5)$ and λ_1, λ_2 are the costate variables.

According to Hartman (1963), the behavior of the trajectories of system $(6)-(9)$ around certain equilibrium points can be deduced from the qualitative study of the linear system $\dot{y}=Jy$, where J is the Jacobian matrix given by the partial derivatives of the functions of the right hand side of system $(6)-(9)$ with respect to each variable. The possibility of limit cycles appearance in models with two state variables was established by Dockner and Feichtinger (1991).

Now, one can use an explicit quadratic formula for the adjustment cost function that helps the qualitative analysis of the system $(6)-(9)$. Using the quadratic cost function $C(E)=1/2\gamma E^2$ with $\gamma>0$, then expression (10) becomes $E=\lambda_1/\gamma$ and finally the conditions that determine the optimal plan of a central decision maker, after the appropriate substitutions, are (time is neglected to avoid notational overburdening):

$$\begin{aligned}
\dot{x}&=g(x)-\phi, && x(0)=x_0 && (11)\\
\dot{\phi}&=\lambda_1/\gamma, && \phi(0)=\phi_0 && (12)\\
\dot{\lambda}_1&=(\rho-g')\lambda_1-U_x && && (13)\\
\dot{\lambda}_2&=\rho\lambda_2-U_\phi+\lambda_1 && && (14)
\end{aligned}$$

The study of the dynamic properties of system $(11)-(14)$ includes stability of the system which is restricted to saddle point stability, i.e. to a two dimensional manifold in the four dimensional space of state and costates. According to Dockner's explicit formula (Dockner, 1985) the four eigenvalues $r_i,\ i=1,...,4$ of the linearized dynamics of the canonical equations are given by:

$$r_{1,2,3,4}=\rho/2\pm\sqrt{\rho^2/4-\Psi/2\pm(1/2)\sqrt{\Psi^2-4\det J}} \qquad (16)$$

and the magnitude Ψ is the sum of determinants of submatrices of the Jacobian J expressed as:

$$\Psi=\begin{Vmatrix}\dfrac{\partial\dot{x}}{\partial x} & \dfrac{\partial\dot{x}}{\partial\lambda_1}\\ \dfrac{\partial\dot{\lambda}_1}{\partial x} & \dfrac{\partial\dot{\lambda}_1}{\partial\lambda_1}\end{Vmatrix}+\begin{Vmatrix}\dfrac{\partial\dot{\phi}}{\partial\phi} & \dfrac{\partial\dot{\phi}}{\partial\lambda_2}\\ \dfrac{\partial\dot{\lambda}_2}{\partial\phi} & \dfrac{\partial\dot{\lambda}_2}{\partial\lambda_2}\end{Vmatrix}+2\begin{Vmatrix}\dfrac{\partial\dot{x}}{\partial\phi} & \dfrac{\partial\dot{x}}{\partial\lambda_2}\\ \dfrac{\partial\dot{\lambda}_1}{\partial\phi} & \dfrac{\partial\dot{\lambda}_1}{\partial\lambda_2}\end{Vmatrix} \qquad (17)$$

From Dockner's formula (16), it is well known that sufficient conditions for the saddle point are first the positive determinant of the Jacobian matrix and secondly the negativity of the coefficient Ψ given by (17). A positive determinant of the Jacobian is crucial for stability, because a negative determinant restricts the stability to a one dimensional manifold of initial conditions (with one negative eigenvalue, the other three are positive or have positive real parts) and the generic solution is unstable. Figure 1 classifies the eigenvalues depending on the determinant of J ($\det J$) and Ψ .

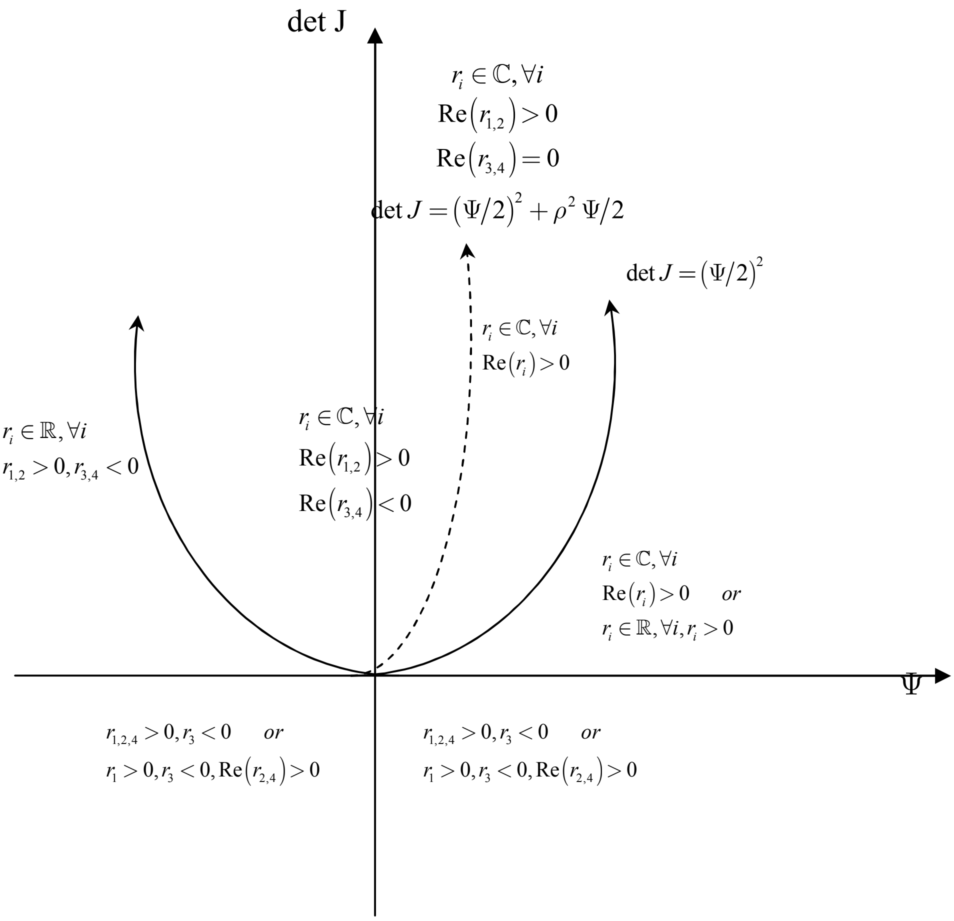

Figure 1: Classification of the eigenvalues depending on $\det J$ and Ψ.

As one can see in Figure 1, also published by Dockner and Feichtinger (1991), there not exists at least one case for which all eigenvalues are negative numbers, the latter means that complete stability is impossible. Dockner and Feichtinger (1991) show that a necessary and sufficient condition for the eigenvalues to be pure imaginary numbers is $\det J > \left(\frac{1}{2}\Psi\right)^2$ and $\det J = \left(\frac{1}{2}\Psi\right)^2 + \frac{1}{2}\rho^2\Psi$. Moreover, the necessary and sufficient conditions correspond to the eigenvalues $r_{3,4}$ that cross the imaginary axis when they go from one side of the dashed curve to the other. Considering the discount rate ρ as a parameter, the values of ρ for which the conditions are met, are possible Hopf bifurcations[1] (Kuznetsov, 1997) and a limit

[1] Hopf bifurcations occur when there are two pure imaginary eigenvalues of the Jacobian matrix. Hopf bifurcations, so called bifurcations of co–dimension one, are related to the existence of a simple real

cycle will emerge if the complex eigenvalues $r_{3,4}$ cross the imaginary axis with non-zero velocity at $\rho = \rho_0$, i.e. $\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\mathrm{Re}\left(\rho_{3,4}\right)\right)\Big|_{\rho=\rho_0} \neq 0$

Following formula (16) one can compute the Jacobian J of equations $(11)-(14)$ at the equilibrium:

$$J = \begin{bmatrix} g' & -1 & 0 & 0 \\ 0 & 0 & 0 & 1/\gamma \\ -U_{xx} - g''U_{\phi} & -U_{x\phi} & \rho - g' & 0 \\ -U_{x\phi} & -U_{\phi\phi} & 1 & \rho \end{bmatrix} \qquad (18)$$

and the determinant of J is:

$$\det\ J = \frac{(\rho - 2g')U_{x\phi} + g'(\rho - g')U_{\phi\phi} - U_{xx} - g''U_{\phi}}{\gamma} \qquad (19)$$

Considering stability, the one dimensional control problem without adjustment costs, studied by Berck (1981), served as a good benchmark for comparisons with the two state variables model. The Hamiltonian current value of the one dimensional problem is: $H_1 = U(\phi, x) + \lambda(g(x) - \phi)$ and the optimality conditions are given as:

$$H_{\phi} = U_{\phi} - \lambda = 0 \qquad (20)$$

$$\dot{\lambda} = (\rho - g')\lambda - U_x \qquad (21)$$

$$\dot{x} = g(x) - \phi \qquad (22)$$

Setting the optimal control $\phi = \chi(x, \lambda)$, the derivatives with respect to x, λ are $\phi_x = \chi_x = -U_{x\phi}/U_{\phi\phi}$ and $\phi_{\lambda} = \chi_{\lambda} = 1/U_{\phi\phi}$. The Jacobian matrix $\hat{J}$ of the one dimensional model without adjustment costs, after these calculations, becomes

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eigenvalue of Jacobian matrix equal to zero. The dynamic change produced by values of the parameter higher than the bifurcation value has the result of closed trajectories (limit cycles). The equilibrium point for which there exist any of these two types of eigenvalues is known as non hyperbolic equilibrium point.

$$\hat{J}=\begin{bmatrix}\dfrac{\partial\dot{x}}{\partial x} & \dfrac{\partial\dot{x}}{\partial\lambda}\\ \dfrac{\partial\dot{\lambda}}{\partial x} & \dfrac{\partial\dot{\lambda}}{\partial\lambda}\end{bmatrix}=\begin{bmatrix}g'+U_{x\phi}/U_{\phi\phi} & -1/U_{\phi\phi}\\ -U_{xx}-g''U_{\phi}+U_{x\phi}^{2}/U_{\phi\phi} & (\rho-g')-U_{x\phi}/U_{\phi\phi}\end{bmatrix}$$

Calculating the determinant of the Jacobian $\hat{J}$ one can see that it finally becomes

$$\det\ \hat{J}=\frac{(\rho-2g')U_{x\phi}+g'(\rho-g')U_{\phi\phi}-g''U_{xx}}{U_{\phi\phi}} \qquad (23)$$

Comparing (19) and (23) it can be seen that the relation between these determinants

is
$$\det\ J=\frac{\det\ \hat{J}\ U_{\phi\phi}}{\gamma} \qquad (24)$$

Some simple conclusions from the above discussion are drawn below:

- First, relation (24), between Jacobian determinants, implies that any instability arisen from the one dimensional problem cannot stabilize the two dimensional problem even with the introduction of adjustment costs and second stock variable into the model.
- Second, looking at (17) we realize that the three terms summed up are essential. The first and second terms are the determinants of the one dimensional problem without adjustment costs, while the third term measures the interaction between the first two terms.

Application of (17) yields
$$\Psi=g'(\rho-g')+U_{\phi\phi}/\gamma \qquad (25)$$

Hence, in the case of growth $g'>0$, the suppositions of the logistic growth ($g''=-2$), $U_{\phi\phi}=U_{x\phi}=0$ and $g'>\rho>0$ are sufficient to ensure saddle point stability, $(\det\ J>0,\ \Psi<0)$, but the local monotonicity is not implied.

3.2. The incentive for fleet modifications

As a continuation of the previous discussion about commercial harvesting, the basic two dimensional management problem consisting of equations $(3)-(5)$, can also be modified in the case the available equipment is subject to expansions or reductions. As already mentioned, harvesting equipment can be considered the available fleet, electronic machines, boats, nets, workmen hiring and so forth. Equipment's modifications are also highly dependent on the existing renewable resource stock and it can be seen as a stock as well, which affects directly the harvesting function ϕ.

Therefore one can treat the harvesting function $\phi(t)$ as a function of the accumulated equipment, $\phi(E)$. The accumulated equipment E, does not, however, remain at a fixed level, but is also subject to depreciation, which entails at a simple depreciation rate and moreover it is reasonable to argue that the renewable resource extractor enjoys utility from the decision to modify equipment. The modifications that are possible to the original model are first, in the objective functional which enters in an additively separable utility form and, second in the two equations of motion. Setting harvesting equipment as a state variable, the decision to expand (or to reduce) would be now the new control which enters into the system.

After all the simplified assumptions, the original optimal control problem $(7)-(9)$ now becomes

$$\max_{u} \int_{0}^{\infty} e^{-\rho t}\left[U_1(x)+U_2(u)\right]dt \qquad (26)$$

subject to $\quad \dot{x}(t)=g\left(x(t)\right)-\phi(E), \quad x(0)=x_0 \qquad (27)$

and $\quad \dot{E}(t)=u-\delta E, \quad E(0)=E_0 \qquad (28)$

$U_1(x)$, $U_2(u)$ represent utility in separable form, consisting of the utility derived from the existing renewable resource stock and from agent's decision u to modify his equipment. The harvesting function $\phi(E)$ is denoted as a function of the available equipment, while δ is the equipment's depreciation rate. The control u influences directly equipment's changes, but also has an indirect effect on the renewable resource stock via the harvesting $\phi(E)$.

Moreover the representative agent faces an intertemporal trade–off between the benefits associated with the stock $U_1(x)$ and the benefits resulting from fleet's expansion or reduction $U_2(u)$. It is worth noting that the second part of utility $U_2(u)$ is the net value, which captures all the costs associated with the expansion or reduction. Finally, the decision to modify equipment, u, is maybe positive in the case of expansion or negative in the case of reduction, which also means that the depreciation parameter δ can be set to zero at the steady state equilibrium implying $u_\infty = 0$, i.e. no equipment's modification made in equilibrium.

We proceed by analyzing the continuous–time optimization problem for which the extractor of the renewable resource seeks to maximize the discounted stream of benefits expressed by (26). The Hamiltonian is:

$$H = U_1(x) + U_2(u) + \lambda_1 \dot{x} + \lambda_2 \dot{E},$$

where λ_1, λ_2 are the adjoint variables of the states x, E respectively. The Hamiltonian is concave in both states and control because the utility functions are both concave as well as the equations of motion for the states $\dot{x}$, $\dot{E}$. The concavity implies that the sufficient optimality conditions are met if additionally the limiting transversality conditions are satisfied

$$\lim_{t\to\infty} e^{-\rho t}\lambda_1 x = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\lambda_2 E = 0$$

The Hamiltonian maximizing condition for control values lying in the interior is given by

$$H_u = U_2'\left(u^*\right) + \lambda_2 = 0 \qquad (29)$$

and moreover the Hamiltonian's strict concavity implies $H_{uu} = U_1''(u) < 0$

Applying the inverse function theorem the inverse function $h(\lambda_2) = \left(U_2'\right)^{-1}(\lambda_2)$ already exists, satisfying the optimality condition $H_u\left(x, E, h(\lambda_2), \lambda_1, \lambda_2\right) = 0$

The following two equations determine the evolution of the costates λ_1, λ_2

$$\dot{\lambda}_1 = \left(\rho - g'(x)\right)\lambda_1 - U_1'(x) \qquad (30)$$

$$\dot{\lambda}_2 = (\rho + \delta)\lambda_2 + \lambda_1\phi'(E) \qquad (31)$$

Equations (30), (31) together with the two equations of motion (27), (28) constitutes the following canonical system of necessary conditions

$$\dot{x}(t) = g\left(x(t)\right) - \phi(E) \qquad (32) \qquad\qquad \dot{E}(t) = h(\lambda_2) - \delta E \qquad (33)$$

$$\dot{\lambda}_1 = \left(\rho - g'(x)\right)\lambda_1 - U_1'(x) \qquad (34) \qquad\qquad \dot{\lambda}_2 = (\rho+\delta)\lambda_2 + \lambda_1\phi'(E) \qquad (35)$$

and the corresponding Jacobian becomes

$$J = \begin{pmatrix} g'(x) & -\phi'(E) & 0 & 0 \\ 0 & -\delta & 0 & -\dfrac{1}{U_2''(u)} \\ -\dfrac{g''(x)U_1'(x)}{\rho - g'(x)} - U_1''(x) & 0 & \rho - g'(x) & 0 \\ 0 & \dfrac{\phi''(E)U_1'(x)}{\rho - g'(x)} & \phi'(E) & \rho + \delta \end{pmatrix}$$

Again we may apply formula (16) to compute the four eigenvalues of the above Jacobian, which are crucial to characterize the local dynamics of the linear ODE that approximates the canonical equations (32) - (35). But now formula's (16) coefficient Ψ reduces to

$$\Psi = g'(x)\left(\rho - g'(x)\right) - \delta(\rho+\delta) + \frac{\phi''(E)U_1'(x)}{U_2''(u)\left(\rho - g'(x)\right)} \qquad (36)$$

and the determinant of the Jacobian evaluated at the equilibrium is given by

$$\det J = -g'\delta\left(\rho - g'\right)(\rho+\delta) + \frac{g''U_1'\phi'^2}{U_2''\left(\rho - g'\right)} + \frac{\phi'^2 U_1''}{U_2''} + \frac{g'\phi''U_1'}{U_2''} \quad (37)$$

We consider the strictly concave optimal control problem (26) - (28) and assume that an optimal, interior solution u^* and a stationary equilibrium exist. The stability properties of this optimally controlled system depends on the sign of the growth's function rate of change g' (evaluated at the steady state) and on the other model characteristics in the following way.

<u>Case 1</u>: $g' \leq 0$ and the long–run equilibrium is a saddle point. The result follows directly from (37), since $g' \leq 0$ implies $\det J > 0$ and $\Psi < 0$. Therefore, two eigenvalues must have negative real parts.

<u>Case 2</u>: $0 < g'(x) < \rho$, the long–run equilibrium is characterized by all different cases, i.e. saddle point stability, locally unstable spirals and instability such that convergence to the equilibrium is restricted to a one dimensional set of initial conditions. According to Poincare–Andronov–Hopf (PAH) theorem, the transition from a domain of stable to locally unstable may give rise to limit cycles.

Under the supposition of growth, $g' > 0$ (Case 2), and a diffusion process with one and only one point $\tilde{x}$ such that $g'(\tilde{x}) = 0$, it is well known that the time path of the renewable resource level consists of a convex segment $(\text{if } x < \tilde{x})$ and a concave segment (if $x > \tilde{x}$). In other words, the domain of the low level $(x < \tilde{x})$

exhibits increasing returns and the domain of high level is characterized by diminishing returns. It is plausible that diminishing returns lead to stable equilibrium, whereas increasing returns favour complexities, i.e. limit cycles. The reason is that a low level of resource may increase to a certain threshold so it may be rational for the agent to expand his equipment to gain future benefits.

Specifications

We assume benefits stemming from the existing renewable resource stock to be proportional to its current level. Moreover the growth of benefits associated with the current accumulated level of equipment's expansion is, however, not unrestricted but rather reaches a maximum level. After all we specify the functional forms as follows:

$$U_1(x) = a_1 x, \qquad a_1 > 0 \tag{38}$$

$$U_2(u) = \beta_1 u - \tfrac{1}{2}\beta_2 u^2, \qquad \beta_1 > 0,\ \beta_2 \geq 0 \tag{39}$$

$$g(x) = x(1-x) \tag{40}$$

$$\phi(E) = \gamma E, \qquad \gamma > 0 \tag{41}$$

The last two equations represent the fact that a maximum level of the resource exists towards which x grows in the absence of harvesting, while the decline of the resource's level is proportional to the accumulated level of equipment E. But, in the long–run, the decision for modifications has a relative small meaning due to the high depreciation that has been made on to the past accumulated equipment. That is, at the steady state, the decision, u^*, tends to zero and this result is attained only setting the depreciation rate very close to zero, $\delta \approx 0$. With the last supposition and under specifications (38) - (41) the determinant of the Jacobian (37) and coefficient Ψ

(36) reduces to

$$\det J = \frac{g''(x)U_1'(x)\phi'^2(x)}{U_2''(u^*)(\rho - g'(x))} = \frac{2\rho\beta_1\gamma}{\beta_2} \qquad (42)$$

$$\Psi = g'(x)(\rho - g'(x)) = \frac{a_1\gamma(\rho^2\beta_1 - a_1\gamma)}{\beta_1^2\rho^2} \qquad (43)$$

Having the set of necessary requisites for a pair of purely imaginary eigenvalues existence, i.e. $\det\ (J) - \left(\frac{\Psi}{2}\right)^2 - \frac{\rho^2\Psi}{2} = 0$, $\Psi > 0$ and $\det\ (J) > 0$, we continue choosing a_1 as the bifurcation point for the certain parameter values $\beta_1 = \beta_2 = 1$, $\rho = 0.01$, $\gamma = 0.071$. It can be shown numerically (Grass *et al.*, 2008), for the above values of parameters, the conditions for complex eigenvalues with positive real parts are met for $a_1 \in (6.69\ ,\ 7.595)$, and moreover stable limit cycles exist, at least in the right–hand vicinity of $a_1 = 6.69$.

Figure 2 shows the phase portrait in the modification – stock plane that corresponds to the above values of a_1 In figure 2 the four phases **I – IV** characterize the cycle as optimal strategy in the management problem. That is:

Phase I: $\dot{x} > 0$ and $\dot{u} > 0$ Phase II: $\dot{x} > 0$ and $\dot{u} < 0$

Phase III: $\dot{x} < 0$ and $\dot{u} < 0$ Phase IV: $\dot{x} < 0$ and $\dot{u} > 0$

Starting with a minimum level of renewable resource stock, Phase I is characterized by reduction in equipment $u < 0$ but at a diminishing rate $\dot{u} > 0$. This process implies that, in the same Phase I, decision u becomes positive at some time instant and continues to grow for sufficient level of the resource stock. In Phase II equipment expands yet when resource stock is still rising to its peak. In Phase III since the renewable resource stock peaks its maximum value the agent exploits the large stock, but equipment's high expansion now affects the resource stock which declines, so a decision to reduce equipment is taken. Finally, in Phase IV, decision u becomes negative, meaning equipment's reduction, and the resource stock stops the downward fall.

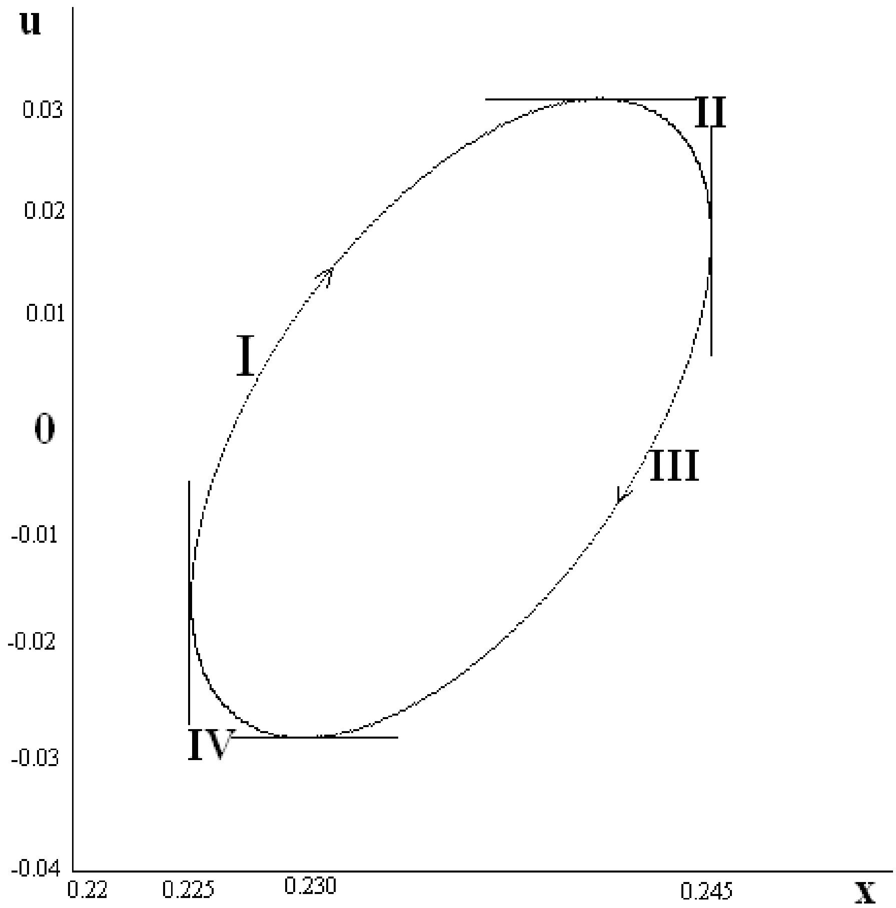

Figure 2. Phase portrait of the example of a cyclical strategy in a decision–stock plane.

4. Conflicts with a common harvesting function

Let us, as before, denote by $x(t)$ the instantaneous renewable resource which is in common access at time t. Without any harvesting taking place the stock of resources grows according to the function $g(x)$, obviously depending on the resource itself, satisfying the conditions $g(0)=0$, $g(x)>0$ for all $x \in (0,K)$, $g'(x)<0$ for all $x \in (K, \infty)$, $g''(x) \leq 0$. In the game that follows we assume that two types of players are involved. First is the renewable resource extractors (players) acting with the traditional mode in the sense of Clark (1990), with the latter implying that they are armed with the basic equipment, usually harvests only personally, but there is a crowd of this type of players. Second are the commercial heavy equipment users with a lot of vessels usually acting as factories. Carrying out harvesting is costly for the second type of players, e.g. damages in the available equipment, payroll for workingmen, also reducing its financial capital.

Considering now the depletion of the renewable resource stock (the harvesting function), one can think that however, does not only depend on the intensive usage $\nu(t)$ of the heavy equipped player, but is also influenced by the other players' overall effort $u(t)$ which act traditionally. We set as instrument variables the intensity of equipment and the personal harvesting effort respectively i.e. for the heavy equipped player (player type 2) the intensity of the harvesting equipment's usage $\nu(t)$, and the for traditional fishermen (players of kind 1) its personal effort $u_i(t)$, both assumed non-negatives $\nu(t) \geq 0, \quad u_i(t) \geq 0$.

We denote the overall harvesting function by $\phi(u, \nu)$, also depending on both overall effort $u(t) = \sum_i u_i(t)$ and on intensity. Combining the growth $g(x)$ with the harvesting function $\phi(u, \nu)$ the state dynamics can be written as

$$\dot{x} = g(x) - \phi(u, \nu), \qquad x(0) = x_0 > 0 \tag{44}$$

Along a trajectory the non negativity constraint is imposed, that is

$$x(t) \geq 0 \quad \forall t \geq 0 \tag{45}$$

A higher intensity of harvesting equipment usage (for player 2) and also the effort of the crowd of traditionally acting fishermen (player 1) certainly leads to stronger depletion of the renewable resource, so it is enough reasonable to assume that the partial derivatives of the harvesting function to be positive with respect to the parameters, i.e. $\phi_u > 0$, $\phi_\nu > 0$. Moreover the law of diminishing returns is applied only for the type 1 player's effort undertaken, that is $\phi_{uu} < 0$ and for simplicity we assume $\phi_{\nu\nu} = 0$. Additionally, we assume that the Inada conditions, which guarantee that the optimal strategies are nonnegative, holds true, i.e.

$$\begin{aligned} &\lim_{u\to 0}\phi_u(u,\nu)=\infty, \qquad \lim_{u\to\infty}\phi_u(u,\nu)=0 \\ &\lim_{\nu\to 0}\phi_\nu(u,\nu)=0, \qquad \lim_{\nu\to\infty}\phi_\nu(u,\nu)=\infty \end{aligned} \tag{46}$$

The utility functions the two players want to maximize are defined as follows: *Player 1*, the representative traditional fisherman, derives instantaneous utility, on one hand from its own harvesting product, but its personal effort $u(t)$ gives rise to increasing and convex costs $a(u)$, and on the other hand from the high stock of renewable resource also denoted by the increasing function $\varphi(x)$. After all the present value of player's 1 utility is described by the following functional

$$J_1 = \int_0^\infty e^{-\rho_1 t}\left[\phi(u,\nu)+\varphi(x)-a(u)\right]dt \tag{47}$$

Player 2, the heavy equipped, enjoys utility $\upsilon(x)$ from the renewable resource stock $x(t)$, but also from their equipment's intensity of use ν, which is described by the function $\beta(\nu)$. For the utilities $\upsilon(x)$ and $\beta(\nu)$ we assume that they are monotonically increasing functions with decreasing marginal returns, that is $\upsilon'(x)>0$, $\beta'(\nu)>0$ and $\upsilon''(x)<0$, $\beta''(\nu)<0$. We also assume that the individually acting players' overall effort u has no impact on player's 2 utility. So, player's 2 utility function is defined, in additively separable form, as:

$$J_2 = \int_0^\infty e^{-\rho_2 t}\left[\upsilon(x)+\beta(\nu)\right]dt \tag{48}$$

4.1. Periodic Solutions

Let us now explore whether periodic solutions are possible, starting with steady state and stability analysis of necessary conditions. As it is clear the problem can be treated as a differential game with two controls and one state. Corresponding Hamiltonians, optimality conditions and adjoint variables for the problem under consideration are respectively:

$$H_1 = \phi(u,\nu)+\varphi(x)-a(u)+\lambda_1\left(g(x)-\phi(u,\nu)\right)$$

$$H_2 = \upsilon(x)+\beta(\nu)+\lambda_2\left(g(x)-\phi(u,\nu)\right)$$

$$\frac{\partial H_1}{\partial u} = (1-\lambda_1)\phi_u(u,\nu)-a'(u)=0 \tag{49}$$

$$\frac{\partial H_2}{\partial \nu} = \beta'(\nu)-\lambda_2\phi_\nu(u,\nu)=0 \tag{50}$$

$$\dot{\lambda}_1 = \rho_1\lambda_1 - \frac{\partial H_1}{\partial x} = \lambda_1\left[\rho_1 - g'(x)\right]-\varphi'(x) \tag{51}$$

$$\dot{\lambda}_2 = \rho_2\lambda_2 - \frac{\partial H_2}{\partial x} = \lambda_2\left[\rho_2 - g'(x)\right]-\upsilon'(x) \tag{52}$$

where subscripts denote player 1 and player 2 respectively for Hamiltonias H_i and adjoints λ_i $i=1,2$. Steady state solutions for the state, adjoints and controls are solutions of the system of equations:

$$g(x)=\phi(u,\nu),\ \ \lambda_1\left[\rho_1-g'(x)\right]-\varphi'(x)=0,\ \ \lambda_2\left[\rho_2-g'(x)\right]-\upsilon'(x)=0$$

$$(1-\lambda)\phi_u(u,\nu)-a'(u)=0\,,\quad \beta'(\nu)-\mu\phi_\nu(u,\nu)=0\,.$$

The Jacobian matrix of the system of optimality conditions is the following

$$J=\begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \lambda_1} & \frac{\partial \dot{x}}{\partial \lambda_2} \\ \frac{\partial \dot{\lambda}_1}{\partial x} & \frac{\partial \dot{\lambda}_1}{\partial \lambda_1} & \frac{\partial \dot{\lambda}_1}{\partial \lambda_2} \\ \frac{\partial \dot{\lambda}_2}{\partial x} & \frac{\partial \dot{\lambda}_2}{\partial \lambda_1} & \frac{\partial \dot{\lambda}_2}{\partial \lambda_2} \end{pmatrix} = \begin{pmatrix} g'(x) & -\frac{\partial \phi(u,\nu)}{\partial \lambda_1} & -\frac{\partial \phi(u,\nu)}{\partial \lambda_2} \\ -\lambda_1 g''(x)-\varphi''(x) & \rho_1 - g'(x) & 0 \\ -\lambda_2 g''(x)-\upsilon''(x) & 0 & \rho_2 - g'(x) \end{pmatrix}$$

which also gives: $\operatorname{tr}(J)=\rho_1+\rho_2-g'(x)$ and

$$\det(J)=g'(x)\big(\rho_1-g'(x)\big)\big(\rho_2-g'(x)\big)-\frac{\partial \phi(u,\nu)}{\partial \lambda_1}\big(\lambda_1 g''(x)+\varphi''(x)\big)\big(\rho_2-g'(x)\big)-$$
$$-\frac{\partial \phi(u,\nu)}{\partial \lambda_2}\big(\lambda_2 g''(x)+\upsilon''(x)\big)\big(\rho_1-g'(x)\big)$$

According to Wirl (1997) (Proposition 4) the existence of a pair of purely imaginary eigenvalues requires that the following conditions are satisfied:

$$\operatorname{tr}(J)>0,\quad \det(J)>0,\quad w>0,\quad \det(J)=w\operatorname{tr}(J)$$

where coefficient w is the result of the sum of the following determinants

$$w=\begin{vmatrix} g'(x) & -\frac{\partial \phi(u,\nu)}{\partial \lambda_1} \\ -\lambda_1 g''(x)-\varphi''(x) & \rho_1-g'(x) \end{vmatrix} + \begin{vmatrix} \rho_1-g'(x) & 0 \\ 0 & \rho_2-g'(x) \end{vmatrix} +$$

$$+\begin{vmatrix} g'(x) & -\frac{\partial \phi(u,\nu)}{\partial \lambda_2} \\ -\lambda_2 g''(x)-\upsilon''(x) & \rho_2-g'(x) \end{vmatrix} =$$

$$=\rho_1\rho_2-\left[g'(x)\right]^2-\frac{\partial \phi(u,\nu)}{\partial \lambda_1}\left[\lambda_1 g''(x)+\varphi''(x)\right]-\frac{\partial \phi(u,\nu)}{\partial \lambda_2}\left[\lambda_2 g''(x)+\upsilon''(x)\right]$$

From now on the crucial condition for cyclical strategies (precisely for Hopf bifurcations to occur) is that $w>0,\quad w=\dfrac{\det(J)}{\operatorname{tr}(J)}$

which after simple algebraic calculations reduces to

$$\begin{aligned}&\rho_1\rho_2\left[\rho_1+\rho_2-2g'(x)\right]=\\&=\frac{\partial\phi(u,\nu)}{\partial\lambda_1}\left[\lambda_1 g''(x)+\varphi''(x)\right]\rho_1+\frac{\partial\phi(u,\nu)}{\partial\lambda_2}\left[\lambda_2 g''(x)+\upsilon''(x)\right]\rho_2\end{aligned}\tag{53}$$

4.2. Specifications for the game

We specify the functions of the game as follows: a diffusion process for the renewable resource growth function, that is $g(x)=rx(1-x)$, a Cobb–Douglas type function for the harvesting $\phi(u,\nu)=u^{\gamma}\nu$ and the utility function stemming from equipment's intensive use of player 2 in the form $\beta(\nu)=A-\nu^{(\xi-1)}/(1-\xi)$. Note that the utility function $\beta(\nu)$ with $A>0$ and $\xi\in(0,1)$ exhibits constant relative risk aversion in the sense of Arrow–Pratt measure of risk aversion. All the other functions are left in a linear form, i.e. both utilities stemming from the existing renewable resource stock are for player 1 $\varphi(x)=\varphi x$ and for player 2 $\upsilon(x)=\upsilon x$, while the player's 1 effort cost in the linear fashion $a(u)=au$, as well. Note that all the involved coefficients, i.e. the intrinsic growth rate r and the slopes φ, υ and a are positive real numbers, but $\gamma\in(0,1)$ and $A>0$ and $\xi\in(0,1)$, as already mentioned. With the above specifications the following result holds true.

Proposition 4.1

A necessary condition for cyclical strategies in the game between traditionally acting and heavy equipped players, as described above, is the heavy equipped players are more impatient than the simple traditionally acting.

Proof: See in the Appendix

The intuition behind proposition 4.1 is straightforward. We start with a rather low and increasing intensity of equipment usage on behalf of the heavy equipped players. The traditionally acting players operate at a low effort, as well, because the increasing effort incurs costs, but they are worrying about the renewable resource level, consequently for their jobs, by reason of the player 2 presence. Now suppose that the heavy equipped react as a farsighted, he would increase the equipment's intensity only moderately and the dynamical system would approach a stable steady state. But, due to their impatience they behave myopically and react by strongly increasing the intensity of their machines. At this time the crowd of the traditionally acting players, has only two choices: to loose their jobs or to increase their overall effort. Suppose that they stay in the harvesting increasing their overall effort, but the latter means that the combination of high intensity on behalf of the heavy equipped and the higher effort on behalf of the crowd leads to a strong reduction of the renewable resource stock.

But the low level of the resource stock is unprofitable for the heavy equipped to work at a high intensity, therefore they have to decrease intensity and the cycle is closed. A new cycle starts again, possibly in another place because of the stock's reduction, but with the same results also described. In our opinion the crucial point of this intuitive explanation is that player's 1 strategic variable u lags behind player's 2 strategic variable ν and both are lagged behind the state variable, the renewable resource's stock x.

4.3 The linear example

In this subsection we calculate the Nash equilibrium of the harvesting differential game. The concept of open loop Nash equilibrium is based on the fact that every player's strategy is the best reply to the opponent's exogenously given strategy. Obviously, equilibrium holds if both strategies are simultaneously best replies.

Following Dockner *et al.* (2000), we formulate the current value Hamiltonians for both players, as follows

$$H_1 = \phi(u,\nu) + \varphi(x) - a(u) + \lambda_1\left(g(x) - \phi(u,\nu)\right)$$

$$H_2 = \upsilon(x) + \beta(\nu) + \lambda_2\left(g(x) - \phi(u,\nu)\right)$$

The first order conditions, for the maximization problem, are the following system of differential equations for both players. First, the maximized Hamiltonians are

$$\frac{\partial H_1}{\partial u} = (1-\lambda_1)\phi_u(u,\nu) - a'(u) = 0 \tag{54}$$

$$\frac{\partial H_2}{\partial \nu} = \beta'(\nu) - \lambda_2\phi_\nu(u,\nu) = 0 \tag{55}$$

and second, the costate variables are defined by the equations

$$\dot{\lambda}_1 = \rho_1\lambda_1 - \frac{\partial H_1}{\partial x} = \lambda_1\left[\rho_1 - g'(x)\right] + \varphi'(x) \tag{56}$$

$$\dot{\lambda}_2 = \rho_2\lambda_2 - \frac{\partial H_2}{\partial x} = \lambda_2\left[\rho_2 - g'(x)\right] + \upsilon'(x) \tag{57}$$

The Hamiltonian of player 1, H_1, is concave in the control u as far as $\lambda_1 < 1$ and is guaranteed by the assumptions on the signs of the derivatives, i.e. $\phi_{uu} < 0,\ \phi_{\nu\nu} = 0$ and from the decreasing marginal returns on the player's 2 utilities, i.e. $\upsilon''(x) < 0,\ \beta''(\nu) < 0$. Moreover, optimality condition (54) implies that the adjoint variable λ_1 is positive only if player's 1 marginal utility ϕ_u exceeds marginal costs, since $\lambda_1 = \left(\phi_u\left(u,\nu\right) - a'\left(u\right)\right)/\phi_u\left(u,\nu\right)$.

We also assume linearity of the model. A linear population growth function, despite the critique as a fairly unrealistic model, is a good approximation for the exponential growth of human population since 1900 (Murray, 2002). To be more precise we specify the following functions of the game in linear form:

i. the renewable resource's growth function in the form $g\left(x\right) = \omega \cdot x$, where ω is the growth rate,

ii. the utility function, $\varphi\left(x\right)$, which stems from the high stock of the renewable resource, in the form $\varphi\left(x\right) = \varphi \cdot x$

iii. the function that measures player's 1 effort cost in the form $u\left(t\right) = a \cdot u$

and all the constants involved are positive numbers, that is $\omega, \varphi,\ a > 0$. From the second player's side, the functions that are maximized are specified linear, i.e. the utilities arisen from the resource stock and high intensity realizations are written as $\upsilon\left(x\right) = \upsilon \cdot x\left(t\right)$ and $\beta\left(\nu\right) = \beta \cdot \nu\left(t\right)$ respectively.

After the above simplified specifications the canonical system of equations (54) – (55) can be rewritten as:

$$\frac{\partial H_1}{\partial u} = \left(1 - \lambda_1\right)\phi_u\left(u,\nu\right) - a = 0 \tag{58}$$

$$\frac{\partial H_2}{\partial \nu} = \beta - \lambda_2 \phi_\nu\left(u,\nu\right) = 0 \tag{59}$$

$$\dot{\lambda}_1 = \rho_1 \lambda_1 - \frac{\partial H_1}{\partial x} = \lambda_1\left[\rho_1 - \omega\right] - \varphi \tag{60}$$

$$\dot{\lambda}_2 = \rho_2 \lambda_2 - \frac{\partial H_2}{\partial x} = \lambda_2\left[\rho_2 - \omega\right] - \upsilon \tag{61}$$

and the limiting transversality conditions has to hold

$$\lim_{t\to\infty} e^{-\rho_1 t} x(t)\lambda_1(t)=0, \quad \lim_{t\to\infty} e^{-\rho_2 t} x(t)\lambda_2(t)=0 \tag{62}$$

The analytical expressions of the adjoint variables (λ_1, λ_2), solving equations (62)-(61), are respectively:

$$\lambda_1(t)=\frac{\varphi}{\rho_1-\omega}+e^{(\rho_1-\omega)t}C_1 \tag{63}$$

$$\lambda_2(t)=\frac{\upsilon}{\rho_2-\omega}+e^{(\rho_2-\omega)t}C_2 \tag{64}$$

In order for the transversality conditions to be satisfied it is convenient to choose the constant steady state values, and therefore the adjoint variables collapses to the following constants

$$\lambda_1=\frac{\varphi}{\rho_1-\omega}, \quad \lambda_2=\frac{\upsilon}{\rho_2-\omega} \tag{65}$$

To ensure certain signs for the adjoints (65) we impose another condition on the discount rates, which claim that discount rates are greater than the resource's growth, i.e. we impose the condition $\rho_i>\omega, \quad i=1,2$ thus, the constant adjoint variables have both positive signs.

The above condition seems to be restrictive but can be justified as otherwise optimal solutions do not exist. Indeed, choosing $\rho_2<\omega$, player's 2 discount rate to be lower than the resource's growth rate, their objective functional becomes unbounded in the case they choose to carry out no harvesting. Similarly, choosing player's 1 discount rate to be lower than the growth rate the associated adjoint variable λ_1 becomes a positive quantity in the long run. As a shadow price is implausible to be positive for optimal solutions, the above reasoning is sufficient for the assumption $\rho_i>\omega, \quad i=1,2$.

Once the concavity of the Hamiltonians, with respect to the strategies and for both players, is satisfied the first order conditions guarantee its maximization. Now, we choose the harvesting function's $\phi(u,\nu)$ specification, i.e. the specification of the function that reduces the renewable resource. This function is depending on both effort and intensity. We choose a similar to Cobb – Douglas production function specification, which characterized by constant elasticities, in the following form:

$$\phi(u,\nu) = u^{\sigma}\nu^{\zeta} \qquad 0 < \sigma < 1 < \zeta$$

Let us next present the calculations of the explicit formulas at the Nash equilibrium.

4.4. Optimal Nash Strategies

Applying first order conditions for the chosen specification function

$$\phi_u(u,\nu) = \frac{a}{1-\lambda_1} \quad \Leftrightarrow \quad \sigma u^{\sigma-1}\nu^{\zeta} = \frac{a}{1-\lambda_1} \tag{66}$$

$$\phi_\nu(u,\nu) = \frac{\beta}{\lambda_2} \quad \Leftrightarrow \quad \zeta u^{\sigma}\nu^{\zeta-1} = \frac{\beta}{\lambda_2} \tag{67}$$

The combination of (66) and (67), using the Cobb–Douglas type of specification, reveals an existing interrelationship between the strategies, that is

$$\phi(u^*,\nu^*) = (u^*)^{\sigma}(\nu^*)^{\zeta} \quad \Leftrightarrow \quad \frac{au^*}{\sigma(1-\lambda_1)} = \frac{\beta\nu^*}{\zeta\lambda_2} \quad \Leftrightarrow \quad \nu^* = u^*\frac{a\zeta\lambda_2}{\sigma(1-\lambda_1)\beta} \tag{68}$$

Expression (68) now predicts the interrelationship between the player's Nash strategies, for which the result of comparison between them is dependent on the constant parameters and on the constant adjoint variables, as well.

Substituting back (68) into (67) we are able to find the analytical expressions of the strategies, after the following algebraic calculations. Expression (67) now becomes:

$$\left(u^*\right)^{\sigma+\zeta-1} = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{1-\zeta} \left(\frac{\zeta\lambda_2}{\beta}\right)^{1-\zeta} \left(\frac{\lambda_2\zeta}{\beta}\right)^{-1} = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{1-\zeta} \left(\frac{\lambda_2\zeta}{\beta}\right)^{-\zeta}$$

and from the latter the analytical expressions for the equilibrium strategies is derived in a more comparable form now, as:

$$u^* = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{\frac{1-\zeta}{\sigma+\zeta-1}} \left(\frac{\zeta\lambda_2}{\beta}\right)^{\frac{-\zeta}{\sigma+\zeta-1}} \quad (69)$$

$$\nu^* = \left[\frac{a}{\sigma\left(1-\lambda_1\right)}\right]^{\frac{\sigma}{\sigma+\zeta-1}} \left(\frac{\zeta\lambda_2}{\beta}\right)^{\frac{\sigma-1}{\sigma+\zeta-1}} \quad (70)$$

Further substitutions in the equation of the resource's accumulation, $\dot{x} = \omega x - u^\sigma \nu^\zeta$, yield the following steady state value of the stock

$$x_{SS} = \frac{1}{\omega}\left[\frac{a}{\left(1-\lambda_1\right)\sigma}\right]^{\frac{\sigma}{\sigma+\zeta-1}} \left(\frac{\zeta\lambda_2}{\beta}\right)^{\frac{-\zeta}{\sigma+\zeta-1}} \quad (71)$$

We summarize the above discussion in a proposition.

Proposition 4.2:

Assuming the harvesting function to exhibit constant elasticity and all the other functions to be linear, then the harvesting game yields constant optimal Nash strategies. The analytical expressions of the strategies are given by (69) and (70) for the traditional fishermen and the heavy equipped respectively. The steady state value of the resources' stock is given by the expression (71).

Proposition 4.2 seems to be with a little economic meaning caused by the linearity of the paradigm. But the constancy of the resulting strategies can be seen in connection with the concept of time consistency, a central property in economic theory. By the large, time consistency is a minimal requirement for a strategy's credibility, but in general open loop strategies they have not the time consistency property by default, since these strategies are time, and not state, dependent functions. Nevertheless, a constant strategy may be a time consistent one, since the crucial characteristic for time consistency, i.e. the independency of any initial state x_0, is met for the above constant strategies.

5. Conclusions

Overfishing is caused due to the imposition of externalities on national and international levels. The Tragedy of the Commons occurring in the fishing industry is related to the inefficient allocation of resources in harvesting and the resulting reduction of the available limited resource stocks due to the common access competition. This common access competition requires regulation of fishing efforts and fishing capacity to attain sustainability. This regulation may be limited due to the absence of appropriate international authorities as well as due to the occurring Prisoners' dilemma situations as governments tend to support their fisheries.

In Environmental Economics the exploitation of renewable resources is a well overlooked field since the original model dated back to Schäfer (1994). As known, the analysis concentrates on the two basic factors that affect the fishing industry, namely the size of the resource itself and the rate of human harvesting. The above specification does not take into account any other human activities which affect biomass, for example coastlines pollution.

Concerning long–run equilibrium, as it is well known, the simplest case of the saddle–point type stability requires only one characteristic of the renewable resource's growth function that is the negative growth. But even the supposition of negative growth is sufficient for the saddle-point stability, the local monotonicity is not implied i.e. transient cycles may occur.

On the other hand, harvesting management is not restricted in the traditional way of the renewable resource extraction in the sense of one man show. Commercial harvesting often requires investment and disinvestment in equipment, and the undertaken decision to expand or to reduce equipment obeys onto the state variable which is the existing renewable resource stock. Therefore, concerning harvesting, as a stock variable, equilibrium dynamics become more complex, and much richer, also including saddle–point stability. The dynamics of such equilibrium reveal cyclical policies as optimal strategies.

The emphasis given in our paper is not restricted on the stability properties of the optimal management program, but we also focus on the stability properties of the induced nonzero sum game between two types of players which share a common depletion function thought as a harvesting. Precisely, the game set up between a crowd of weakly armed and a strongly armed player with a common depletion function yields an economic result, for which the discount rate plays the crucial role for periodic solutions. That is, the condition for periodic solutions is that the strong equipped player to be more impatient than the weak. Finally, for the supplement linear example of the same game we compute the optimal Nash strategies for both players, which are constant expressions.

Appendix

Proof of proposition 4.1.

With the specifications, given in subsection 4.2, one can compute

$g'(x) = r(1-2x)$, $g''(x) = -2r$, $\phi_u(u,\nu) = \gamma u^{\gamma-1}$, $\phi_\nu(u,\nu) = u^\gamma$, $a'(u) = a$, $\beta'(\nu) = \nu^{\xi-2}$, $\varphi'(x) = \varphi$, $\upsilon'(x) = \upsilon$

$$\frac{\partial H_1}{\partial u} = 0 \;\Leftrightarrow\; (1-\lambda_1)\phi_u(u,\nu) = a'(u) \;\Leftrightarrow\; (1-\lambda_1)\gamma u^{\gamma-1}\nu = a \qquad (A.1)$$

$$\frac{\partial H_2}{\partial \nu} = 0 \;\Leftrightarrow\; \beta'(\nu) = \lambda_2\phi_\nu(u,\nu) \;\Leftrightarrow\; \lambda_2 u^\gamma = \nu^{\xi-2} \qquad (A.2)$$

Combining $(A.1)$ and $(A.2)$ the optimal strategies take the following forms

$$u^* = \lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{(\xi-2)/[1+(1-\xi)(1-\gamma)]} \qquad (A.3),$$

$$\nu^* = \lambda_2^{(\gamma-1)/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma/[1+(1-\gamma)(1-\xi)]} \qquad (A.4)$$

and the optimal harvesting becomes

$$\phi(u^*,\nu^*) = \lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma(\xi-1)/[1+(1-\gamma)(1-\xi)]} \qquad (A.5)$$

with the following partial derivatives

$$\frac{\partial \phi}{\partial \lambda_1} = \frac{\lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma(\xi-1)/[1+(1-\gamma)(1-\xi)]}}{(1-\lambda_1)}\frac{\gamma(\xi-1)}{1+(1-\xi)(1-\gamma)} = \qquad (A.6)$$
$$= \frac{\phi(u^*,\nu^*)}{(1-\lambda_1)}\frac{\gamma(\xi-1)}{1+(1-\xi)(1-\gamma)}$$

$$\frac{\partial \phi}{\partial \lambda_2} = \frac{\lambda_2^{-1/[1+(1-\gamma)(1-\xi)]}\left[\frac{a}{\gamma(1-\lambda_1)}\right]^{\gamma(\xi-1)/[1+(1-\gamma)(1-\xi)]}}{\lambda_2}\frac{-1}{1+(1-\xi)(1-\gamma)} = \qquad (A.7)$$
$$= \frac{\phi(u^*,\nu^*)}{\lambda_2}\frac{-1}{1+(1-\xi)(1-\gamma)}$$

Both derivatives (A.6), (A.7) are negatives due to the assumptions on the parameters $\gamma,\ \xi \in (0,1)$ and on the signs of derivates, that is $\phi_u > 0,\ \phi_\nu > 0,\ \upsilon'(x) > 0,\ \varphi'(x) > 0$, which ensures the positive sign of the adjoints $\lambda_1,\ \lambda_2$.

Condition $w = \dfrac{\det(J)}{\operatorname{tr}(J)}$ now becomes

$\rho_1\rho_2\left[\rho_1+\rho_2-2g'(x)\right] = \lambda_1\rho_1 g''(x)\dfrac{\partial\phi}{\partial\lambda_1} + \lambda_2\rho_2 g''(x)\dfrac{\partial\phi}{\partial\lambda_2}$, which after substituting the values from (A.6), (A.7) and making the rest of algebraic manipulations, finally yields (at the steady states)

$$\frac{\phi(u_\infty,\nu_\infty)g''(x)}{1+(1-\xi)(1-\gamma)}\left[\rho_1\gamma(1-\xi)\frac{\varphi}{\varphi+g'(x)-\rho_1}-\rho_2\right]-\rho_1\rho_2\left[\rho_1+\rho_2-2g'(x)\right]=0 \qquad (\text{A.8})$$

Where we have set $\dfrac{\lambda_1}{1-\lambda_1} = \dfrac{\varphi}{\rho_1 - g'(x) - \varphi}$ stemming from the adjoint equation $\dot{\lambda}_1 = \lambda_1\left(\rho_1 - g'(x)\right) - \varphi'(x)$, which at the steady states reduces into $\lambda_1 = \varphi'(x)/\left(\rho_1 - g'(x)\right)$.

Condition $w > 0$ after substitution the values from (A.6), (A.7) becomes

$$w = \rho_1\rho_2 - \left[g'(x)\right]^2 + \frac{\phi(u,\nu)g''(x)}{1+(1-\xi)(1-\gamma)}\left[\gamma(1-\xi)\frac{-\varphi}{g'(x)+\varphi-\rho_1}+1\right] > 0 \qquad (\text{A.9})$$

The division (A.8) by ρ_1 yields

$$\frac{\phi(u_\infty,\nu_\infty)g''(x)}{1+(1-\xi)(1-\gamma)}\left[\gamma(1-\xi)\frac{\varphi}{\varphi+g'(x)-\rho_1}-\frac{\rho_2}{\rho_1}\right]-\rho_2\left[\rho_1+\rho_2-2g'(x)\right]=0 \ (\text{A.10})$$

The sum $(\text{A.9})+(\text{A.10})$ must be positive, thus after simplifications and taking into account that $\phi(u_\infty,\nu_\infty) = g(x)$, we have:

$g(x)g''(x)\dfrac{\rho_1-\rho_2}{\rho_1\left[1+(1-\xi)(1-\gamma)\right]} > \left[\rho_2 - g'(x)\right]^2$ and the result $\rho_2 > \rho_1$ follows from the strict concavity of the logistic growth $g'' < 0$.

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